Programming Language Concepts, cs2104 Lecture 07 (2003-09-26)

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Reading Suggestions

- Chapter 3
  - Sections 3.1 – 3.4 [careful]
    but skip or browse [3.4.4,3.4.7,3.4.8]
  - Sections 3.6, 3.7 [careful]
- And of course the handouts!

Organizational

- Midterm exam: next lecture (8)

Making Computations Iterative

Accumulators

This Lecture

- Making computations iterative

  [last lecture]

- Higher-order programming
- Abstract data types

Using Accumulators

- Accumulator stores intermediate result
- Finding an accumulator amounts to finding a state invariant
  - recursion maintains state invariant
  - state invariant must hold initially
  - result must be obtainable from state invariant

Summary So Far

- Use accumulators
  - yields iterative computation
  - find state invariant
- Exploit structural properties
  - for example: $x^{2n} = (x \times x)^n$
- Exploit both kinds of knowledge
  - on how programs execute **(abstract machine)**
  - on application/problem domain

More on Accumulators

- Accumulators on numbers, fine…
- How about lists?
- List type is a form of recursive data type
- How about recursive data types?

Type Notation

- Type information simplifies program development
- Many functions are designed based on the type of the input arguments
- Let us have some type notation
- The list type is subtype of the record type
- Other useful types are trees, etc

Type of List

- Based on the type hierarchy
  - ⟨Value⟩, ⟨Record⟩,…
  - ⟨Record⟩ ⊂ ⟨Value⟩
    - The Record type is a subtype of the Value type
  - List is either `nil` or `X|Xr` where `Xr` is a list where X is an arbitrary value
  - ⟨List⟩ ::= `nil` | ⟨Value⟩'|'⟨List⟩

On Types

- A list whose elements are of given type

  ⟨List T⟩ ::= `nil` | ⟨T⟩ '|' ⟨List T⟩

  - T is a type variable
  - ⟨List T⟩ is a type function
  - ⟨List ⟨Int⟩⟩ : a list whose elements are integers
  - ⟨List ⟨Value⟩⟩ is equal to ⟨List⟩

On Types (trees)

- Binary trees

```
⟨BTree T⟩ ::= leaf
            | tree(key:⟨Literal⟩ value:T
                   left: ⟨BTree T⟩
                   right: ⟨BTree T⟩ )
```

  - Binary tree representing a dictionary mapping keys to values
  - Binary tree is either a leaf (atom leaf), or
  - An internal node with label tree, with left and right subtrees, a key and a value
  - The key is of literal type and the value of type T

On Types: procedures and functions

- The type of a procedure is

  ⟨`proc {$` T_1 … T_n`}`⟩

  - T_1 … T_n are the types of the arguments

On Types: procedures and functions

- The type of a function is

  ⟨`fun {$` T_1 … T_n`}:`T⟩

  - T_1 … T_n are the types of the arguments, and T is the type of the result
  - ⟨`fun {$` ⟨List⟩ ⟨List⟩ `}:` ⟨List⟩ ⟩ is a function that takes two lists and returns a list

On Types: procedures and functions

- The type of a function is

  ⟨`fun {$` T_1 … T_n`}:`T⟩

  - T_1 … T_n are the types of the arguments, and T is the type of the result
  - ⟨`fun {Append` ⟨List⟩ ⟨List⟩ `}:` ⟨List⟩ ⟩ is the type of Append

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Back to Recursive and Iterative Computation

- We will use type definition to construct programs
- Reversing a list
- Type of `Reverse` is ⟨`fun {$` ⟨List⟩`}:` ⟨List⟩⟩
- How to reverse the elements of a list

  `{Reverse [a b c d]}`

  returns

  `[d c b a]`

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Reversing a List

- How to reverse the elements of a list
- Reverse of nil is nil
- Reverse of X|Xs is Z, where
  reverse of Xs is Ys, and
  append Ys and [X] to get Z

Question

- What is correct

```
{Append {Reverse Xr} X}
```

or

```
{Append {Reverse Xr} [X]}
```

Naïve Reverse Function

```
fun {NRev Xs}
   case Xs
   of nil  then nil
   [] X|Xr then
      {Append {NRev Xr} [X]}
   end
end
```


Question

- What is the problem with naïve reverse?
- Possible answers
  - not tail recursive
  - and also `Append` is costly

Cost of Naïve Reverse

- Suppose a recursive call `{NRev Xs}`
  - where `{Length Xs}`=n
  - assume cost of `{NRev Xs}` is c(n)

    number of function calls
  - then c(0) = 0

    c(n) = c(`{Append {NRev Xr}` …}) + c(n-1)

    = n + c(n-1)

    = n + (n-1) + c(n-2)
  - this yields: c(n) = n/2(n+1)
- For list of length n, `NRev` uses approx. n^2 calls!

Doing Better for Reverse

- Use an accumulator, of course
  - technique: already reversed part of list
- How does `Reverse` compute?
- Some abbreviations
  - `{IR Xs}` for `{IterRev Xs}`
  - `Xs ++ Ys` for `{Append Xs Ys}`

Computing NRev

```
{NRev [a b]}                =
{NRev [b]} ++ [a]           =
({NRev nil} ++ [b]) ++ [a] =
(nil ++ [b]) ++ [a]     =
[b] ++ [a]              =
[b a]
```


IterRev (IR) State Transformation

```
  Xs      Rs
[a b c] nil        ⇒
[b c]   a|nil      ⇒
[c]     b|a|nil    ⇒
nil     c|b|a|nil = [c b a]
```

IterRev (IR) State Transformation

```
  Xs      Rs
[a b c] nil         ⇒
[b c]   a|nil       ⇒
[c]     b|a|nil     ⇒
nil     c|b|a|nil = [c b a]
```

- The general pattern:

```
{IR X|Xr Rs} ⇒ {IR Xr X|Rs}
```


Using an Accumulator for IterRev (IR)

```
{IR [a b]} ++ nil               =
{IR [b]}   ++ ([a] ++ nil)      =
{IR [b]}   ++ ([a])             =
{IR nil}   ++ ([b] ++ [a])      =
{IR nil}   ++ ([b a])           =
nil        ++  ([b a])          =
[b a]
```

Reverse Intermediate Step

```
fun {IterRev Xs Ys}
   case Xs
   of nil  then Ys
   [] X|Xr then
      {IterRev Xr X|Ys}
   end
end
```

- Is tail recursive now

State Invariant for Reverse

- Now it is easy to see that

  {Reverse [$x_1 \ldots x_n$]}

  =

  {Reverse [$x_{i+1} \ldots x_n$]} ++ [$x_i \ldots x_1$]

  as we go along

Reverse Proper

```
local
   fun {IterRev Xs Ys}
      case Xs
      of nil  then Ys
      [] X|Xr then {IterRev Xr X|Ys}
      end
   end
in
   fun {Rev Xs} {IterRev Xs nil} end
end
```


Summary

- Accumulators for lists work as they do for integers
- Think of the problem as state transformation
- Essential: finding state invariant
  - find it by some hand computation

Constructing Programs by Following the Type

- Observe, programs so far that takes lists, has a form that corresponds to the list type

- ⟨List T⟩ ::= `nil` | ⟨T⟩ '|' ⟨List T⟩

- ```
  case Xs
  of nil then ⟨expr⟩ % base case
  [] X|Xr then ⟨expr⟩ % recursive call
  end
  ```


Constructing Programs by Following the Type

- This helps us when the type gets complicated

- Nested lists are lists whose elements can be lists
- Example Xs: `[[1 2]  4 nil [[5] 10]]`

- Find the number of elements in a nested list
- `{Length Xs} = 5`
  ```

Constructing Programs by Following the Type

- Nested lists

- ⟨NList T⟩ ::= `nil`
  | ⟨NList T⟩'|'⟨NList T⟩
  | T '|' ⟨NList T⟩ (T is not nil nor a cons)

•

```
case Xs
of nil then ⟨expr⟩ % base case
[] X|Xr andthen {IsList X} then
   ⟨expr⟩ % recursive calls for X and Xr
[] X|Xr then
    ⟨expr⟩ % recursive call for Xr
end
```


Constructing Programs by Following the Type

- Nested lists

•

```
fun {IsList L}
   L == nil orelse
   {Label L}=='|' andthen {Width L}==2
end
```

•

```
case Xs
of nil then ⟨expr⟩ % base case
[] X|Xr andthen {IsList X} then
   ⟨expr⟩ % recursive calls for X and Xr
[] X|Xr then
    ⟨expr⟩ % recursive call for Xr
end
```

Constructing Programs by Following the Type

- Nested lists: ⟨fun {Length ⟨NList T⟩}:⟨Int⟩⟩

- ```
  fun {Length Xs}
      case Xs
      of nil then 0 % base case
      [] X|Xr andthen {IsList X} then
           {Length X} + {Length Xr}
      [] X|Xr then
           1+{Length Xr}
      end
  end
  ```


Higher-Order Programming
  ```

Higher-Order Programming

- Higher-order programming = the set of programming techniques that are possible with procedure values (lexically-scoped closures)
- higher-order programming is the foundation of secure data abstraction component-based programming and object-oriented programming

Higher-Order Programming

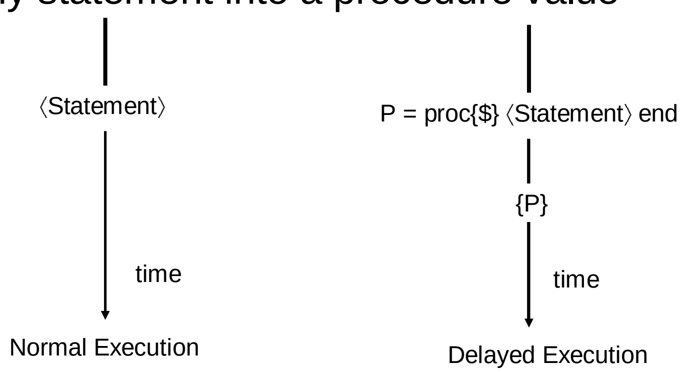
- Basic operations
  - Procedural abstraction: the ability to convert any statement into a procedure value
  - Genericity: the ability to pass procedure values as arguments to a procedure call
  - Instantiation: the ability to return procedure values as results from a procedure call
  - Embedding: the ability to put procedure values in data structures

Higher-order programming

- Control abstractions
  - The ability to define control constructs
  - Integer and list loops, accumulator loops, folding a list (left and right)

Procedural Abstraction

- Procedural abstraction is the ability to convert any statement into a procedure value



Procedural Abstraction

- A procedure value is usually called a closure, or more precisely, a lexically-scoped closure
  - A procedure value is a pair: it combines the procedure code with the contextual environment
- Basic scheme:
  - Consider any statement <s>
  - Convert it into a procedure value:
    `P = proc {$} <s> end`
  - Executing `{P}` has exactly the same effect as executing `<s>`

The same holds for expressions

- A procedure value is usually called a closure, or more precisely, a lexically-scoped closure
  - A procedure value is a pair: it combines the procedure code with the contextual environment
- Basic scheme:
  - Consider any expression `<E>`
  - Convert it into a function value:
    `F = fun {$} <E> end`
  - Executing `X = {F}` has exactly the same effect as executing `X = E`

Example

Suppose we want to define the operator `andthen` (&& in Java) as a function:

```
fun {AndThen B1 B2}
   if B1 then B2 else false end
end

If {AndThen X>0 Y>0} then … else …
```


Example

Does not work because both `X>0` and `Y>0` are evaluated

```
fun {AndThen B1 B2}
   if B1 then B2 else false end
end

If {AndThen X>0 Y>0} then … else …
```

Example: use procedural abstractions

```
fun {AndThen B1 B2}
   if {B1} then {B2} else false end
end

If {AndThen
    fun{$} X>0 end  fun{$}Y>0 end}
then … else … end
```


Genericity

- To make a function generic is to let any specific entity (i.e., any operation or value) in the function body become an argument of the function.
- The entity is abstracted out of the function body.

Genericity

- Replace specific entities (zero 0 and addition +) by function arguments

```
fun {SumList Ls}
   case Ls
   of nil then 0
   [] X|Lr then X+{SumList Lr}
   end
end
```


Genericity

```
fun {SumList L}
   case L
   of nil then 0
   [] X|L2 then X+{SumList L2}
   end
end
```

⇩

```
fun {FoldR L F U}
   case L
   of nil then U
   [] X|L2 then {F X  {FoldR L2 F U}}
   end
end
```

Genericity SumList

```
fun {SumList Ls}
    {FoldR L
     fun {$ X Y} X+Y end 0}
end
```


Genericity ProductList

```
fun {ProductList Ls}
    {FoldR L
     fun {$ X Y} X*Y end 1}
end
```

Genericity Some

```
fun {Some Ls}
    {FoldR L
     fun {$ X Y} X orelse Y end
     false}
end
```

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List Mapping

- Mapping
  - each element recursively
  - *calling function for each element*
  - Snoctruct list that takes output
- Separate function calling by passing function as argument

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Other generic functions: Map

```
fun {Map Xs F}
   case Xs
   of nil then nil
   [] X|Xr then
      {F X}|{Map Xr F}
   end
end
{Browse {Map [1 2 3]
         fun {$ X} X*X end}}
```


Other generic functions: Filter

```
fun {Filter Xs P}
   case Xs
   of nil then nil
   [] X|Xr andthen {P X} then
      X|{Filter Xr P}
   [] X|Xr then {Filter Xr P}
   end
End
{Browse {Filter [1 2 3] IsOdd}}
```

Instantiation

- Instantiation: the ability to return procedure values as results from a procedure call
- A factory of specialized functions

```
fun {Add X}
   fun {$ Y} X+Y end
end

Inc = {Add 1}
{Browse {Inc 5}} % shows 6
```


Instantiation: An application (protected values)

- Given a value (e.g. [1 2 3]) we would like to seal it for any other client except for a person that has a key
- A procedure that has the key can access the list
- No other procedures can access the value

Instantiation: An application (protected values)

```
[1 2 3] Wrap     →    [1 2 3]

[1 2 3]  UnWrap  →    [1 2 3]
```


Instantiation: An application (protected values)

New basic data type is names
A name is unforgable atom:
- Cannot be displayed
- The only possible operation is comparison

X = {NewName}
Y = {NewName}
- X and Y are different

Wrappers

```
proc {NewWrapper Wrap Unwrap}
      Key={NewName}
in
      fun {Wrap X} … end
      fun {Unwrap W} {W Key} end
end
```


Wrappers

```
proc {NewWrapper ?Wrap ?Unwrap}
      Key={NewName}
in
      fun {Wrap X} … end
      fun {Unwrap W} {W Key} end
end
```

- ? Indicate that Wrap and UnWrap is output arguments (just a comment)

Wrappers

- ```
  declare W UW
  {NewWrapper W UW}
  RL = {W [1 2 3]}
  {Browse RL}  % cannot see
  {Browse {UW RL}} % shows [1 2 3]
  ```
- `NewWrapper` is a factory that creates two related procedures

Instantiation: An application (protected values)

`{Wrap [1 2 3]}` → $\boxed{\texttt{[1 2 3]}}$

`{UnWrap` $\boxed{\texttt{[1 2 3]}}$ `}` → `[1 2 3]`
  ```

Wrappers

- ```
  proc {NewWrapper ?Wrap ?Unwrap}
        Key={NewName}
  in
     fun {Wrap X}
        fun{$ K} if K==Key then X end
     end
     fun {Unwrap W} {W Key} end
  end
  ```


Instantiation: An application (protected values)

`[1 2 3]` `Wrap` → $\boxed{\texttt{[1 2 3]}}$

$\boxed{\texttt{[1 2 3]}}$ `UnWrap` → `[1 2 3]`
  ```

Embedding

- Embedding is when procedure values are put in data structures
- Embedding has many uses:
  - Modules: a module is a record that groups together a set of related operations
  - Software components: a software component is a generic function that takes a set of modules as its arguments and returns a new module. It can be seen as specifying a module in terms of the modules it needs.

Control Abstractions

```
proc {For I J P}
   if I > J then skip
   else {P I} {For I+1 J P}
   end
end

{For 1 10 Browse}

for I in 1..10 do {Browse I} end
```

Control Abstractions

```
proc {ForAll Xs P}
   case Xs
   of nil then skip
   [] X|Xr then
      {P X} {ForAll Xr P}
   end
End

{ForAll [a b c d]
  proc{$ I}
     {System.showInfo "the item is: " # I}
  end}
```


Control Abstractions

```
proc {ForAll Xs P}
   case Xs
   of nil then skip
   [] X|Xr then
      {P X} {ForAll Xr P}
   end
end

for I in [a b c d] do
   {System.showInfo "the item is: " # I}
end
```

List Mapping

- Mapping
  - each element recursively
  - *calling function for each element*
  - Snoctruct list that takes output
- Separate function calling by passing function as argument

Other Examples

- `{Filter Xs F}`
  returns all elements of `Xs` for which `F` returns true
- `{Some Xs F}`
  tests whether `Xs` has an element for which `F` returns true
- `{All Xs F}`
  tests whether `F` returns true for all elements of `Xs`

Folding Lists

- Consider computing the sum of list elements
  - …or the product
  - …or all elements appended to a list
  - …or the maximum
  - …
- What do they have in common?
- Snocider example: `SumList`

`SumList`: Naïve

```
fun {SumList Xs}
   case Xs
   of nil  then 0
   [] X|Xr then {SumList Xr}+X
   end
end
```

- First step: make tail-recursive with accumulator

SumList: Tail-Recursive

```
fun {SumList Xs N}
   case Xs
   of nil  then N
   [] X|Xr then {SumList Xr N+X}
   end
end
{SumList Xs 0}
```

- Question:
  - what is about computing the sum?
  - what is generic?

SumList: Tail-Recursive

```
fun {SumList Xs N}
   case Xs
   of nil  then N
   [] X|Xr then {SumList Xr N+X}
   end
end
{SumList Xs 0}
```

- Question:
  - what is about computing the sum?
  - what is generic?

How Does SumList Compute?

```
{SumList [2 5 7] 0}          =
{SumList [5 7]   0+2}        =
{SumList [7]    (0+2)+5}     =
{SumList nil   ((0+2)+5)+7} = …
```


SumList Slightly Rewritten…

```
{SumList [2 5 7] 0}          =
{SumList [5 7]   {F 0 2}}        =
{SumList [7]    {F {F 0 2} 5}}     =
{SumList nil   {F {F {F 0 2} 5} 7}=
 …
```

with

```
  fun {F X Y} X+Y end
```

Left-Folding

- Two values define “folding”
  - initial value `0` for `SumList`
  - binary function `+` for `SumList`
- Left-folding `{FoldL [`$x_1 \ldots x_n$`] F S}`

  `{F … {F {F S` x_1`}` x_2`} …` x_n`}`

  or

  `(…((S` $\otimes_F$ x_1`)` $\otimes_F$ x_2`) …` $\otimes_F$ x_n`)`

Left-Folding

- Two values define “folding”
  - initial value `0` for `SumList`
  - binary function `+` for `SumList`
- Left-folding `{FoldL [`$x_1 \ldots x_n$`] F S}`

  left is here!

  `{F … {F {F S` x_1`}` x_2`} …` x_n`}`

  or

  `(…((S` $\otimes_F$ x_1`)` $\otimes_F$ x_2`) …` $\otimes_F$ x_n`)`

FoldL

```
fun {FoldL Xs F S}
   case Xs
   of nil  then S
   [] X|Xr then {FoldL Xr F {F S X}}
   end
end
```


SumList with FoldL

```
local
   fun {Plus X Y} X+Y end
in
   fun {SumList Xs}
      {FoldL Xs Plus 0}
   end
end
```

Properties of `FoldL`

- Tail recursive
- First element of list if folded first…
  - what does that mean?

What Does This Do?

```
local
   fun {Snoc Xr X}
      X|Xr
   end
in
   fun {Foo Xs}
      {FoldL Xs Snoc nil}
   end
end
```

How Does Foo Compute?

```
{Foo [a b c]} =
{FoldL [a b c] Snoc nil} =
{FoldL [b c] Snoc {Snoc nil a}} =
{FoldL [b c] Snoc [a]} =
{FoldL [c] Snoc {Snoc [a] b}} =
{FoldL [c] Snoc [b a]} =
{FoldL nil Snoc {Snoc [b a] c}} =
{FoldL nil Snoc [c b a]} = [c b a]
```


Right-Folding

- Two values define "folding"
  - initial value
  - binary function
- Right-folding `{FoldR` [$x_1 \ldots x_n$] `F S}`

  `{F` x_1 `{F` x_2 … `{F` x_n `S}` …`}}`

  or

  $x_1 \otimes_F (x_2 \otimes_F (\ldots (x_n \otimes_F S) \ldots))$

Right-Folding

- Two values define “folding”
  - initial value
  - binary function

- Right-folding {FoldR [x

  {F x_1 {F x_2 … {F x_n

  or

  $x_1 \otimes_F (x_2 \otimes_F (\ldots (x_n \otimes_F S) \ldots))$

right is here!

FoldR

```
fun {FoldR Xs F S}
   case Xs
   of nil  then S
   [] X|Xr then {F X {FoldR Xr F S}}
   end
end
```

Properties of `FoldR`

- Not tail-recursive
- Elements folded in order

`FoldL` or `FoldR`?

- `FoldL` and `FoldR` compute same value, if function F commutes:

```
{F X Y}=={F Y X}
```

- If function commutes: `FoldL`
  - `FoldL` tail-recursive
- Otherwise: `FoldL` or `FoldR`
  - depending on required order of result

Example: Appending Lists

- Given: list of lists
  `[[a b] [1 2] [e] [g]]`
- Task: compute all elements in one list in order

- Solution:

```
fun {AppAll Xs}
   {FoldR Xs Append nil}
end
```

- Question: What would happen with `FoldL`?

Tuples and Records…

- Techniques for lists explored here of course applicable…
  - …see tutorial

Summary

- Many operations can be partitioned into
  - pattern implementing
    - recursion
    - application of operations
  - operations to be applied
- Typical patterns
  - `Map` mapping elements
  - `FoldL/FoldR` folding elements
  - `Filter` filtering elements
  - `Sort` sorting elements
  - …

Abstract Data Types

Preview

Data Types

- Data type
  - set of values
  - operations on these values
- Primitive data types
  - records
  - numbers
  - …
- Abstract data types
  - completely defined by its operations (interface)
  - implementation can be changed without changing use

Example: Lab Assignment 2

- Abstract data type for Huffman-trees
- Different implementations
- Same interface

Motivation

- Sufficient to understand interface only
- Software components can be developed independently
  - as long as only interface is used
- Developers need not to know implementation details

Outlook

- How to define abstract data types
- How to organize abstract datatypes
- How to use abstract datatypes

Example: Dictionaries

- Designing the interface

  `{MakeDict}`

  returns new dictionary

  `{DictMember D F}`

  tests whether feature `F` is member of dictionary `D`

  `{DictAccess D F}`

  return value of feature `F` in `D`

  `{DictAdjoin D F X}`

  return dictionary with value `X` at feature `F` adjoined

- Interface depends on purpose, could be richer (for example, `DictCondSelect`)

Using the `Dict` ADT

- Now we can write programs using the ADT without even having an implementation for it
- Implementation can be provided later
- Eases software development in large teams

Implementing the `Dict` ADT

- Now we can decide on a possible implementation for the `Dict` ADT
  - based on pairlists
  - based on records

- Regardless on what we decide, programs using the ADT will work!
  - the interface is a ***contract*** between use and implementation

`Dict`: Pairlists

```
fun {MakeDict}
   nil
end
fun {DictMember D F}
   case D
   of nil then false
   [] G#X|Dr then
      if G==F then true
      else {DictMember Dr F}
      end
   end
end
…
```

Dict: Records

```
fun {MakeDict}
   mt
end
fun {DictMember D F}
   {HasFeature D F}
end
fun {DictAccess D F}
   D.F
end
fun {DictAdjoin D F X}
   {AdjoinAt D F X}
end
```


Example Frequency Counting

```
local
   fun {Inc D X}
      if {DictMember D X} then
         {DictAdjoin D X {DictAccess D X}+1}
      else {DictAdjoin D X 1}
      end
   end
in
   fun {Cnt Xs}
      % returns dictionary
      {FoldL Xs Inc {MakeDict}}
   end
end
```

Example Frequency Counting

```
local
   fun {Inc D X}
      if {DictMember
         {DictAdjoin
      else {DictAdjoi
      end
   end
in
   fun {Cnt Xs}
      % returns dictionary
      {FoldL Xs Inc {MakeDict}}
   end
end
```

homework:
understand and try
this example!

Evolution of ADTs

- Important aspect of developing ADTs
  - start with simple (possibly inefficient) implementation
  - refine to better (more efficient) implementation
  - refine to carefully chosen implementation
    - hash table
    - search tree
- All of evolution is local to ADT
  - no change of programs using ADT is needed!

Next Lecture

- Midterm exam
- More on abstract data types
- Software components, modules, functors

See You Next Week!

Have a Nice Weekend