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OPTIMAL CONTROL OF OVERHEAD CRANES

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Abstract. A nonlinear dynamic model of an overhead crane which represents simultaneous travel, traverse, and hoisting/lowering motions is considered. Nonlinear feedback forms of control are studied, and numerical results are obtained in such a way that specified boundary conditions and the functional constraints for the states and controls are satisfied while minimizing the sway and final time. The results show that the crane can be transferred to a desired position in the shortest possible time while minimizing the sway of the load not only during transfer but also at final time using the suggested control scheme. Several numerical results of the controls, objectives, and of the states are presented which indicate that the proposed method works well.

Key Words. Feedback; Nonlinear systems; Optimal control; Optimization; Simulations.

NOMENCLATURE

$a_i(t)$	reference control inputs.
c_{ij}	feedback Parameters.
F_1, F_2, F_l	normalized driving forces in x, y, and l direction.
$F_x, F_y F_l$	x, y, and l direction forces.
g	gravitation acceleration.
q_i, r_i	weighting factors.
I_1, I_2, I_3	equivalent moments of inertia for the bridge motor, trolley motor, and hoisting motor, respectively.
l	length of cable.
M	mass of the load.
m_1, m_2	masses of the bridge and the trolley, respectively.
r_1, r_2, r_3	radii of the bridge motor pinion, the trolley motor pinion, and the hoisting drum, respectively.
T_1, T_2, T_3	driving torques generated by the bridge, trolley, and hoisting drive motors, respectively.
t	time, variable
x, y, z	coordinate system.
$x_l, x_2, \ldots x_{10}$	state-space variables.
x_G, y_G, z_G	x, y, and z coordinates of the load, respectively.
θ, ψ	swing angles.

1. INTRODUCTION

Overhead cranes are widely used to transport objects in many factories, ports, and work places. The fundamental motions of an overhead crane can be described as: object hoisting or lowering, trolley travel, and bridge traverse. In order to increase the productivity of the system, it is necessary that all these motions of the crane should take place at high speeds. This could however, lead to undesirable sway of the suspended object if proper control action is not taken. This sway is particularly serious, because it could cause damage to the system and the surroundings. Also, if the sway of the suspended object continues up to the end of the transfer then it becomes quite difficult to place the object at the desired final location. Therefore, it is a fundamental requirement to seek a satisfactory control method to minimize the sway during the load transfer and at the final time. Also, minimizing the time of such transfer will bring about a large cost saving.

There are different control methods which can be used in achieving the above goals. Open-loop methods such as continuous sway control and discontinuous sway control are used in some applications (Al-Zinger and Brozovic, 1983; Carbon, 1976). Open-loop control schemes are found to be unsuitable in a real working environment, because they cannot make any compensation for external disturbances such as wind. Therefore, closed-loop control schemes are being employed in most of the work places. Different types of closed-loop control methods can be used, such as optimal control, variable structure control and linear/nonlinear feedback form control.

Sakawa and Shindo (1982) studied the simultaneous motion of hoisting of the load and traversing of the trolley. Their aim was to minimize the sway of the container during as well as at the end of the transfer, without paying much consideration to the transfer time in the performance index. Karihaloo (1982) suggested an optimal control law producing high accelerations during the trajectory, which results in oscillations except at the final position, but produces a minimum course time. Moustafa and Ebeid (1988) developed a nonlinear dynamic model that takes into account the simultaneous travelling of the bridge and the traversing of the trolley. Ebeid *et al.* (1992) presented a non-linear electro-mechanical model describing the dynamic behavior of overhead cranes which includes the dynamics of the two driving motors. Moustafa and Emara -Shabaik (1992) have recently developed a feedback control strategy to minimize the sway of the suspended object for the overhead crane model presented in Ebeid *et al.* (1992) by using pole-assignment and perturbation techniques. An acceleration model has been used by Khan (1993) to formulate an optimized linear feedback scheme to control the motion of the crane so that the suspended load arrives at its final destination in the shortest possible time with the minimum load sway. However, this developed feedback control was linear, and the study considers only small changes in the cable length.

In the present study, the most general case of crane motions, i.e. simultaneous travelling of the bridge, traversing of the trolley, and load hoisting/lowering, are considered. A detailed description of an optimized linear/nonlinear feedback form control including the performance index with minimizing final time is given. Selection criteria, control constraints, and state- variable constraints which minimize sway and final time are presented. A numerical optimization technique is proposed for solving the given control problem. A control scheme for the nonlinear acceleration-control model of the crane is developed to minimize the object sway during the transfer process. In addition, the object is required to be transferred in the minimum time, and to arrive at its final destination with minimum sway.

2. DYNAMIC MODEL OF THE OVERHEAD CRANE

Consider the model of the overhead crane as shown in Fig. 1, which was also used in Khan (1993). For simplicity, assume the following:

1. The elasticity of the crane elements, dissipation effects like rolling resistance and losses in the drive mechanism, as well as effects of wind forces, are negligible.
2. The load is assumed to be concentrated at a point and hanging at the end of a massless cable with negligible length changes due to load swing.

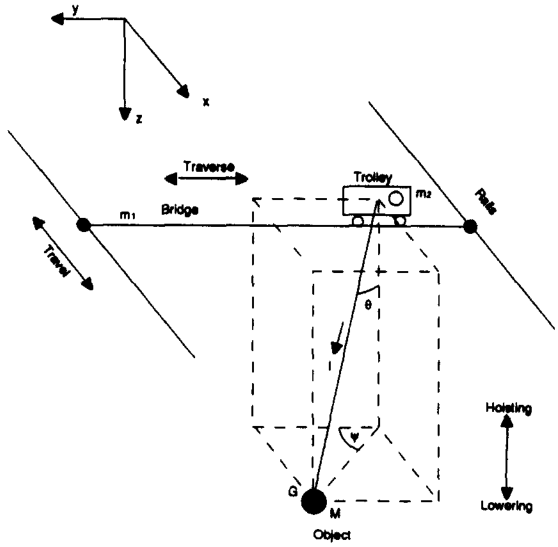

Fig. 1. Simplified model of the overhead crane.

The system will thus have five degrees of freedom, namely: travelling motion of the bridge (x), traversing motion of the trolley (y), sway in the plane of motion (θ), oscillation of the plane determined by the cable and the vertical axis through the suspension point about an equilibrium position (ψ), and the length of the cable (l).

The position vector of the point of suspension, 0, with respect to the fixed-axis coordinate system is

$$r_o = x\vec{i} + y\vec{j} + z\vec{k}. \tag{1}$$

The absolute position vector of the load is

$$r_G = X_G\vec{i} + Y_G\vec{j} + Z_G\vec{k}, \tag{2}$$

where

$$\begin{aligned} x_G &= x + l\sin\theta\sin\psi \\ y_G &= y + l\sin\theta cos\psi \\ z_G &= z + l\cos\theta. \end{aligned} \tag{3}$$

The kinetic and potential energies of the system are, respectively, given by

$$\begin{aligned} T = \;& \tfrac{1}{2}M(\dot{x}_G^2 + \dot{y}_G^2 + \dot{z}_G^2) + \tfrac{1}{2}m_1\dot{x}^2 + \\ & \tfrac{1}{2}m_2(\dot{x}^2 + \dot{y}^2) + \tfrac{1}{2}I_1(\dot{x}/r_1)^2 + \\ & \tfrac{1}{2}I_2(\dot{y}/r_2)^2 + \tfrac{1}{2}I_3(\dot{l}/r_3)^2 \end{aligned} \tag{4}$$

$$V = -Mgl\cos\theta, \tag{5}$$

where I_1 is the equivalent moment of inertia of the bridge motor, I_2 is the equivalent moment of inertia of the trolley motor, I_3 is the equivalent moment of inertia of the hoisting motor, M is the mass of the load, m_1 is the mass of the bridge, m_2 is the mass of the trolley, r_1 is the radius of the bridge motor pinion, r_2 is the radius of the trolley motor pinion, and r_3 is the radius of the hoisting drum. I_1, I_2, and I_3 include the effects of the rotating parts attached to their respective motors.

The Lagrangian function is given as

$$L = T - V.$$

Taking x, y, θ, ψ and l as the generalized coordinates, the equations of motion can respectively be derived by using Lagrangian function as follows:

$$\begin{aligned} \ddot{x} + b_1\ddot{l}\sin\theta\sin\psi + b_1 l\ddot{\psi}\sin\theta\cos\psi &+ \\ b_1 l\ddot{\theta}\cos\theta\sin\psi - b_1 l(\dot{\theta}^2 + \dot{\psi}^2)\sin\theta\sin\psi &+ \\ 2b_1 l\dot{\theta}\dot{\psi}\cos\theta\cos\psi + 2b_1\dot{l}\dot{\theta}\cos\theta\sin\psi &+ \\ 2b_1\dot{l}\dot{\psi}\sin\theta\cos\psi &= F_1 \end{aligned} \tag{6}$$

$$\begin{aligned} \ddot{y} + b_2\ddot{l}\sin\theta\cos\psi - b_2 l\ddot{\psi}\sin\theta\sin\psi &+ \\ b_2 l\ddot{\theta}\cos\theta\cos\psi - b_2 l(\dot{\theta}^2 + \dot{\psi}^2)\sin\theta\cos\psi &- \\ 2b_2 l\dot{\theta}\dot{\psi}\cos\theta\sin\psi + 2b_2\dot{l}\dot{\theta}\cos\theta\cos\psi &- \\ 2b_2\dot{l}\dot{\psi}\sin\theta\sin\psi &= F2 \end{aligned} \tag{7}$$

$$\begin{aligned} l\ddot{\theta} + 2\dot{l}\dot{\theta} - \frac{1}{2}l\dot{\psi}^2\sin(2\theta) + g\sin\theta &+ \\ \ddot{x}\sin\psi\cos\theta + \ddot{y}\cos\psi\cos\theta &= 0 \end{aligned} \tag{8}$$

$$\begin{aligned} l\sin\theta\ddot{\psi} + 2\dot{l}\sin\theta\dot{\psi} + 2l\dot{\psi}\dot{\theta}\cos\theta &+ \\ \ddot{x}\cos\psi - \ddot{y}\sin\psi &= 0 \end{aligned} \tag{9}$$

$$\begin{aligned} \ddot{l} - b_3 l\dot{\theta}^2 - b_3 l\dot{\psi}^2\sin^2\theta - b_3 g\cos\theta &+ \\ b_3\ddot{x}\sin\theta\sin\psi + b_3\ddot{y}\sin\theta\cos\psi &= F_3 \end{aligned} \tag{10}$$

where

$$\begin{array}{ll} F_x = T_1/r_1 & F_1 = F_x/M_1 \\ F_y = T_2/r_2 & F_2 = F_y/M_2 \\ F_l = T_3/r_3 & F_3 = F_l/M_3 \\ M_1 = M + m_1 + m_2 + I_1/r_1^2 & b_1 = M/M_1 \\ M_2 = M + m_2 + I_2/r_2^2 & b_2 = M/M_2 \\ M_3 = M + I_3/r_3^2 & b_3 = M/M_3. \end{array}$$

In the above T_1, T_2, and T_3 are the driving torques generated by the bridge drive motor, the trolley drive motor, and the hoisting motor.

As far as the industrial applications are concerned, acceleration control models (Marttinen *et al.*, 1990; Marttinen and Virkkunen, 1987; Khan, 1993) are preferred in places where accuracy is of prime importance. Also, from the operation point of view, they are easier to control. The mathematical formulation of the acceleration model, which is adopted in this paper, is given below in state-space form. Assuming the available control variables to be the accelerations of the bridge, the trolley, and the winch, i.e.,

$$u_1 = \ddot{x}, \qquad u_2 = \ddot{y} \qquad u_3 = \ddot{l} \tag{11}$$

and introducing the state variables $x_1 \ldots x_{10}$ as

$$\begin{array}{lll} x_1 = x & x_2 = \dot{x} & x_3 = y \\ x_4 = \dot{y} & x_5 = \theta & x_6 = \dot{\theta} \\ x_7 = \psi & x_8 = \dot{\psi} & x_9 = l \\ x_{10} = \dot{l} & & \end{array} \tag{12}$$

then by definition and from equations (8) and (9), the following state space representation can be obtained

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u_1 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= u_2 \\ \dot{x}_5 &= x_6 \\ \dot{x}_6 &= -2\frac{x_{10}}{x_9}x_6 + \frac{1}{2}x_8^2\sin(2x_5) \\ &\quad -\frac{g}{x_9}\sin x_5 - \frac{u_1}{x_9}\cos x_5\sin x_7 \\ &\quad -\frac{u_2}{x_9}\cos x_5\cos x_7 \\ \dot{x}_7 &= x_8 \\ \dot{x}_8 &= -2\frac{x_{10}}{x_9}x_8 - 2x_8x_6\cot x_5 \\ &\quad -\frac{u_1\cos x_7}{x_9\sin x_5} + \frac{u_2\sin x_7}{x_9\sin x_5} \\ \dot{x}_9 &= x_{10} \\ x_{10} &= u_3. \end{aligned} \tag{13}$$

The initial conditions are chosen as

$$\begin{array}{ll} x_1(0) = x_2(0) = 0 & x_7(0) = \psi_i \\ x_3(0) = x_4(0) = 0 & x_8(0) = \dot{\psi}_i \\ x_5(0) = \theta_i & x_9(0) = l_s \\ x_6(0) = \dot{\theta}_i & x_{10}(0) = 0 \end{array} \quad (14)$$

where l_s is the length of the cable at the start of the operation, θ_i, ψ_i, are their initial swing angles, and $\dot{\theta}_i, \dot{\psi}_i$, are their initial rates of change. The required final states are

$$\begin{array}{ll} x_1(t_f) = X & x_6(t_f) = 0 \\ x_2(t_f) = 0 & x_7(t_f) = 0 \\ x_3(t_f) = Y & x_8(t_f) = 0 \\ x_4(t_f) = 0 & x_9(t_f) = l_f \\ x_5(t_f) = Y & x_{10}(t_f) = 0 \end{array} \quad (15)$$

where X, Y are the desired final positions of the bridge and the trolley respectively, and l_f is the desired length of the cable at the target position.

3. CONTROL SCHEME

A control scheme for transporting the object from some initial position to a desired final position with minimum object sway, in minimum time,will be developed. The control problem under consideration thus consists of finding the controls u_1, u_2, and u_3, which transfer the dynamic system from the initial state (14) to the final state (15) while a given objective function is minimized and a number of constraints on both control and state variables are satisfied. In order to solve this problem, a numerical technique, based on an optimization method, is used to provide the optimized feedback laws for the sway control.

3.1. *Performance index*

In the present study, the purpose of the control scheme is to transport an object from an initial position to a target one under the following conditions:

1. minimizing the load sway at the final time.
2. minimizing the load sway in the transfer process; and
3. making the transfer time as short as possible.

Therefore, the following performance index which takes into account the above three problems is proposed

$$J = \tfrac{1}{2}\sum_{i=5}^{8} q_i x_i^2(t_f) + \int_0^{t_f}\{\lambda + \sum_{i=5}^{8} r_i x_i^2(t)\}dt \quad (16)$$

where $q_i, r_i; i = 5, 6, 7, 8$ are positive constants assigning different weights to the sway angles and their derivatives. λ is a positive number to be chosen to weight the relative importance of the elapsed time. For very large λ, the optimal solution will resemble a time-optimal problem. In this study, different values for the weighting factors are tried out to study their effect on the optimization process.

3.2. *Control and state variables, constraints*

Due to the limited effects of the driving motors, the control variables as well as the bridge, trolley, and hoisting velocities are bounded. These constraints, in general, depend on the characteristics of the electric motors. In this study, these constraints have been assumed to be of simple bounded magnitudes.

3.2.1. *Control-Functional constraints.* Since the torque of the electrical driving motors is limited, it is natural to assume that, for $0 \le t \le t_f$,

$$|u_i(t)| \le 0.30 m/s^2, \qquad i = 1, 2$$
$$|u_3(t)| \le 0.50 m/s^2$$

3.2.2. *State variable- Functional constraints* . The velocities of the bridge, the trolley and the winch are bounded, due to practical limitations. It is assumed that these variables, respectively, satisfy the following constraints, for $0 \le t \le t_f$,

$$|x_2(t)| \le 2.0 m/s$$
$$|x_4(t)| \le 1.0 m/s$$
$$|x_{10}(t)| \le 0.5 m/s$$

The above control and state variable-constraints have been selected from (Weaver, 1979; Kulwiec, 1983).

3.2.3. *Terminal constraints on state variables.* For all $0 \le t \le t_f$, the terminal constraints are given by equation (15) with $X = 20m, Y = 10m$ and $l_f = 9.5m$.

4. SIMULATION AND OPTIMIZATION PROCEDURE

In order to find the control law, which minimizes the given performance index J and satisfies the given constraints, extensive numerical computations were done by using a simulation and optimization package Al-Garni and Nizami (1992). The package uses the Gear method in the simulator Gear (1971) and Feasible Sequential Quadratic

Programming (FSQP) in the optimization, which is based on the routines developed in Zhou and Tits (1992). The package is self-contained and has both the simulator and the optimization routines linked together through an interface code which can provide the control law and the other parameters.

To convert the infinite-dimensional optimization problem to a finite-dimensional optimization problem explicit equations are introduced for the controls. These equations are parametric in nature, where the parameters are determined by the optimization process. The controls u_1, u_2, and u_3 are formulated as a combination of some state variables and their rates. The state variables x_5, x_6, x_7, x_8 (the sway angles θ and ψ and their rates) will be selected in the present study for feedback because they can be easily measured, and also their values are required at every moment by the control scheme so that the corresponding control actions can be taken.

The controls are assumed to be generated as follows:

$$\begin{aligned} u_1 &= c_{11}a_1(t) + c_{12}x_5(t) + c_{13}x_6(t) \\ &\quad + c_{14}x_7(t) + c_{15}x_7^2(t) \\ u_2 &= c_{21}a_2(t) + c_{22}x_5(t) + c_{23}x_6(t) \\ &\quad + c_{24}x_7(t) + c_{25}x_7^2(t) \\ u_3 &= c_{31}a_3(t). \end{aligned} \tag{17}$$

Here, $a_1(t), a_2(t)$ and $a_3(t)$ are reference control inputs for the travelling, transverse and hoisting motions, respectively. They are usually selected to suit a desired nominal transport plan and are manually given in ordinary cases. The coefficients c_{ij} are feedback parameters to be calculated by the optimization program such that the constraints are satisfied.

Assuming that each of the desired hoisting and lowering times each equals $5sec$, the desired reference control strategy is defined as follows

$$a_1(t), a_2(t) = \begin{cases} 0 & 0 < t \le 5 \\ +1 & 5 < t \le t_f/2 \\ -1 & t_f/2 < t \le (t_f - 5) \\ 0 & (t_f - 5) < t \le t_f \end{cases} \tag{18}$$

$$a_3(t) = \begin{cases} -1 & 0 < t \le 2.5 \\ +1 & 2.5 < t \le 5 \\ 0 & 5 < t \le (t_f - 5) \\ +1 & (t_f - 5) < t \le (t_f - 2.5) \\ -1 & (t_f - 2.5) < t \le t_f \end{cases} \tag{19}$$

It should be noted that the reference controls are selected in the above form because a large class of dynamic systems are known to exhibit a bang-bang type of optimal trajectory when minimum transport time is sought.

5. RESULTS AND DISCUSSION

The performance of the overhead crane model with the optimum controller being applied was studied by computer simulations. All computations were carried out using a main-frame computer system type AMDAHL-5850. The feedback parameters of the controller of equations (17)-(19), together with the final time t_f, are calculated by the optimization program such that the problem constraints are satisfied. Three cases have been simulated by using the obtained optimal controllers for different performance indices as given in Table 1. In case 1, all the weighting factors are chosen to be equal, i.e., no relative importance is assigned to any of the variables. In cases 2, and 3, more weight is put on minimizing the load oscillation during transport and on minimizing the final time. The controller parameters for the three cases are given in Tables 2 and 3; also the value of $c_{31} = 0.5$ for all the three cases.

Table 1 Weighting factors and CPU times.

	Case 1	Case 2	Case 3
λ	1	100	100
q_5	1	1	1
q_6	1	2	2
q_7	1	1	1
q_8	1	2	2
r_5	1	10	50
r_6	1	20	50
r_7	1	10	50
r_8	1	20	50
CPU Time	62.8	34.2	28.3

Table 2 Controller feedback parameters.

	Case 1	Case 2	Case 3
c_{11}	0.985E-02	0.112E-01	0.994E-02
c_{12}	-0.112E+01	0.139E+01	-0.930E+00
c_{13}	0.149E+01	0.149E+01	0.127E+01
c_{14}	-0.230E-03	-0.100E-01	-0.194E-03
c_{15}	0.122E+01	0.112E+01	-0.339E+00

The simulation results are shown in Figs 2-9,where C1 = case 1, C2 = case 2 and C3 = case 3. The optimal controls, as obtained by the optimization procedure, are shown in Figs 2-4. It should be noted that the trolley acceleration control does not return to zero at the final destination, as seen

Table 3 Controller feedback parameters.

	Case 1	Case 2	Case 3
c_{21}	-0.499E-02	-0.489E-02	-0.499E-02
c_{22}	-0.432E+00	-0.639E+00	-0.463E+00
c_{23}	0.939E-01	0.209E+00	0.233E+00
c_{24}	-0.667E-03	-0.547E-02	-0.682E-02
c_{25}	0.149E+01	0.142E+01	0.819E+00

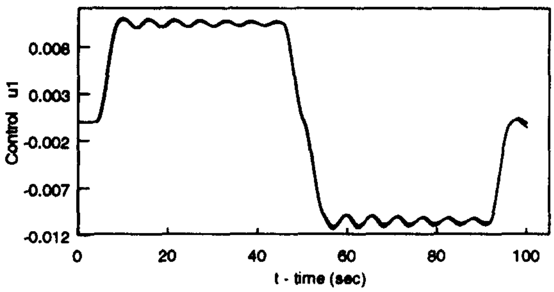

Fig. 2. Bridge control acceleration (u_1).

in Fig. 3. Such an undesirable error is introduced because the swing angle at the end of the travel is not zero, forcing the controller to produce some corrective action. This can be easily avoided by feeding back the trolley velocity to the optimum controller. Such a problem was not dealt with, however, in this present research. Nevertheless, the swing angle at the end of the travel as obtained in this work is small, as can be seen in Fig. 7.

Figures 5, 6 and 9 show the crane trajectory, while Fig. 7 shows the load oscillations during and at the end of transport. It is clear that the optimal controller works well since it produces small swing angles (maximum value of swing is approximately 0.045°). It can also be noticed that case 3 gives the best performance as regarding load sway because a heavier weighting is assigned to the rate of the swing angles. Figure 8 shows the behavior of the load sway, ψ. It can be noticed that the maximum value of ψ is at the end of the travel and it is less than 3° for all cases. However, when θ is small, the magnitude of ψ is not of much importance, as seen in these results.

It should be noted that the minimization of load

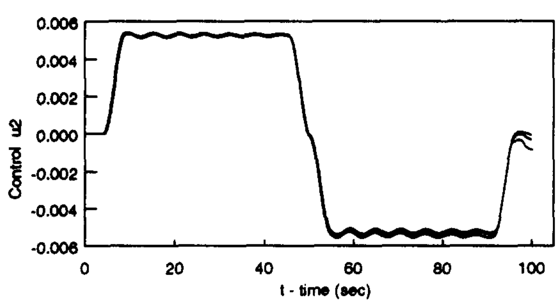

Fig. 3. Trolley control acceleration (u_2).

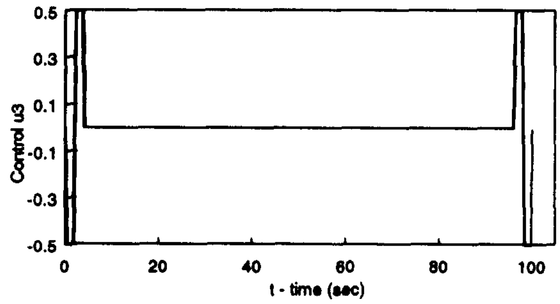

Fig. 4. Hoist control acceleration (u_3).

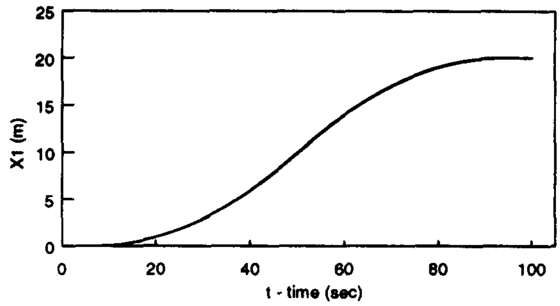

Fig. 5. Bridge displacement ($x_1 = x$).

oscillations during transport is of prime practical importance. Therefore, it is desirable, in real life to choose λ of the performance index not to be too large, to assign less weight to the transport time minimization. In this case, the optimum control problem resembles a free final time one. Such a situation naturally leads to a greater transfer time and consequently small accelerations. This explains the small maximum values of u_1 and u_2 obtained by the optimization program as compared with their upper limits of their corresponding constraints.

Table 1 also shows the CPU time needed to obtain the optimal controllers for the different cases considered. It is seen that the time required for the optimization procedure decreases by increasing the weighting factors λ. However, for very large λ, the optimum controller produces high accelerations, which lead to the drawback of large load oscillations.

The CPU time required by the optimization pro-

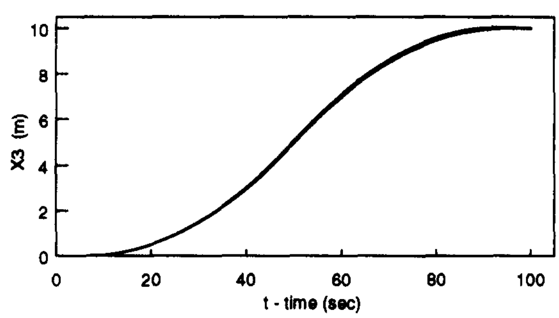

Fig. 6. Trolley displacement ($x_3 = y$).

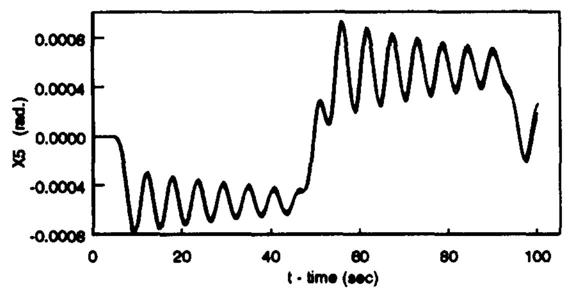

Fig. 7. Swing angle ($x_5 = \theta$).

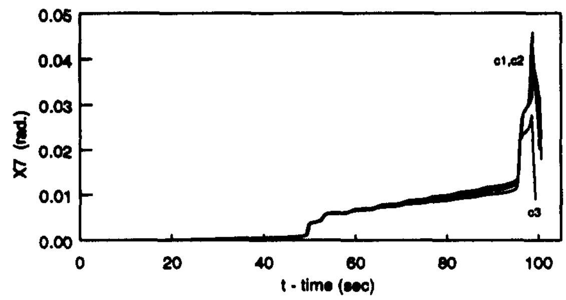

Fig. 8. Oscillation angle ($x_7 = \psi$).

gram to obtain the optimal controller for a given setting varies from 62.8 minutes for case 1 to 28.3 minutes for case 3. It should be noted that the optimization procedure is done off-line and the simulation of the actual travel of the crane to its final destination, using the obtained optimal controller, takes less than 10 seconds for all cases.

It is known that a stiffness problem could be encountered during simulation because of the large differences that exist between the time derivatives of some state variables, as well as the discontinuities present in the acceleration trajectory. To avoid this problem and to improve the numerical accuracy, a cosine taper function is recommended to be used as a weighting function for the reference controls (Newland, 1991). An example of such function is the Bingham window, which can be written as

$$d(t) = \frac{1}{2}(1 \pm \cos \pi(t - t_s)/\Delta t),$$

where t_s is the switching time, and Δt is the time duration in which the switch occurs. The positive

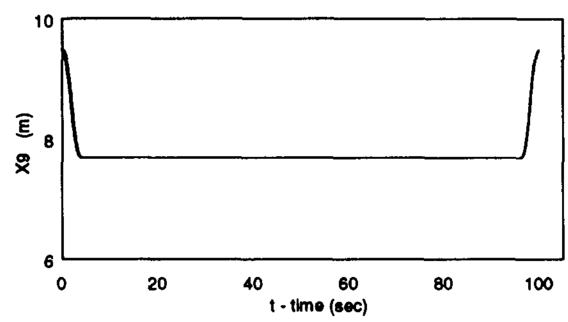

Fig. 9. Cable length ($x_9 = l$).

sign is used when the switch is made to a lower value (negative switch) and the negative sign is used when the switch is made to a higher value (positive switch). It is obvious that the weighting function $d(t)$, for the case of a positive switch, is zero at $t = t_s$ and rises smoothly, along a cosine curve, to a maximum at $t = t_s + \Delta t$. Similar behavior is obtained for a negative switch. It should also be noted that the introduction of $d(t)$ makes the control more physically viable, since making a switch instantaneously is not practical.

The power rating of the driving motors depends in the first place on the desired nominal trajectory. Obviously, a small transport time forces the optimum controller to produce very high accelerations. This requires motors with considerably high rating. Therefore, a compromise between minimizing the transport time and minimizing the load oscillations is advised, depending on the practical considerations of the actual crane.

6. CONCLUSIONS

The most general case of crane motions, i.e., simultaneous travelling of the bridge, traversing of the trolley, and load hoisting is considered. A nonlinear model, derived by using the Lagrange's equation of motion, is presented. The nonlinear acceleration control model is used to develop an optimized control law for controlling the motion of the crane. The object can be transported to the desired destination in a state of rest in the minimum possible time, while minimizing the load sway during and at the end of the transport.

A detailed description of an optimized feedback control method including the performance index selection criteria, the control, constraints, and the state-variable constraints is provided. The control law uses only those states which are of prime importance and can be measured easily. A numerical technique is used for solving the given problem. The simulation results show the effectiveness of the optimal control scheme as applied to the nonlinear acceleration-control model. They also show the practicability of controlling the crane motion to damp the load sway during the transfer and at the final destination.

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