Thermal Computations for Electronic Equipment

An Intensive Short Course

Thermal Computations, Inc.
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Newberg, Oregon 97132-6909

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The following pages include numerous references to TCEE, referencing either equations, e.g. E 2.21, figures, e.g. TCEE Fig. 3-7, or perhaps tables. This material is reproduced with permission from the major reference for this work: Gordon N. Ellison, Thermal Computations for Electronic Equipment, Robert E. Krieger Publishing Company, Malabar, Florida, 1989.

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Thermal Analysis Fundamentals for Electronics

Some Technology Trends

Computer Technology Generations

(Hannemann, R., 1990. Table 1.1, page 3. Used with permission)

1945-1955	First generation: Vacuum tube electronics
1955-1965	Second generation: Transistors (10^0 circuits/chip)
1965-1975	Third generation: Integrated circuits (SSI) (10^1 circuits/chip)
1975-1985	Fourth generation: Large scale integration (LSI) (10^3 circuits/chip)
1985-	Fifth generation: LSI......VLSI......ULSI ($10^4 – 10^6$ circuits/chip)

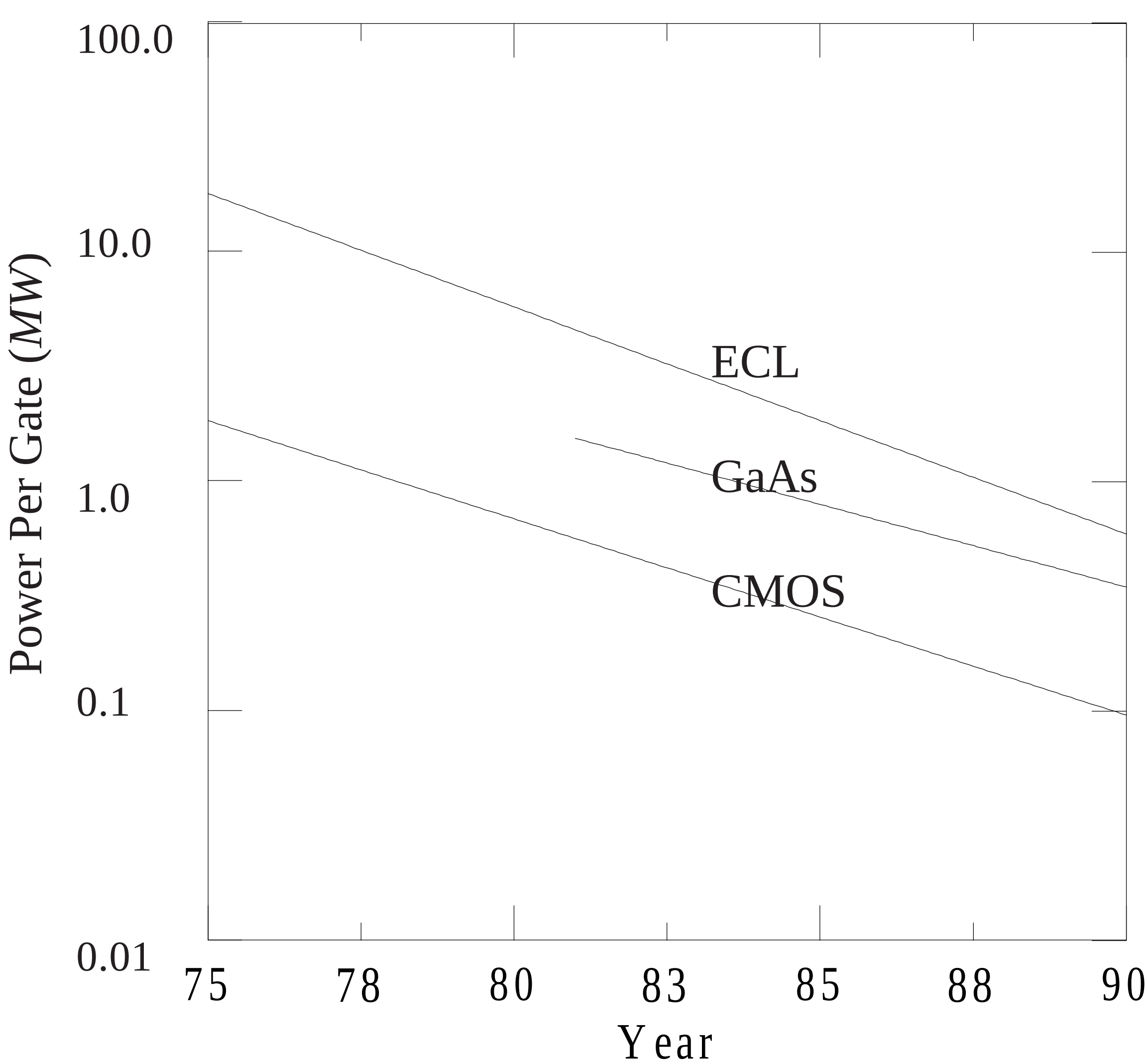
Hannemann, R., ASME Series on Advances in Thermal Modeling of Electronic Components and Systems, Editors: Avram-Bar Cohen and Allan D. Kraus, Vol. II, Table 1.1, page 3, ASME Press, New York, N.Y., 1990. Copyright © The American Society of Mechanical Engineers, 345 East 47th Street, N.Y., N.Y. 10017. Used With Permission.

IC Technology Trends:
(Hannemann, R., 1990, Table 1.2, page 4. Used with permission)

	1980	1988	1996
Semiconductor technology	TTL	CMOS	CMOS
Relative density	1	200	1250
Chip power (W)	2	10	40
Chip power density (W/cm^2)	8	17	25
Chip area (cm^2)	0.25	1.0	1.6
Pins/chip (max)	64	200	400
System clock (MHZ)	5	25	125

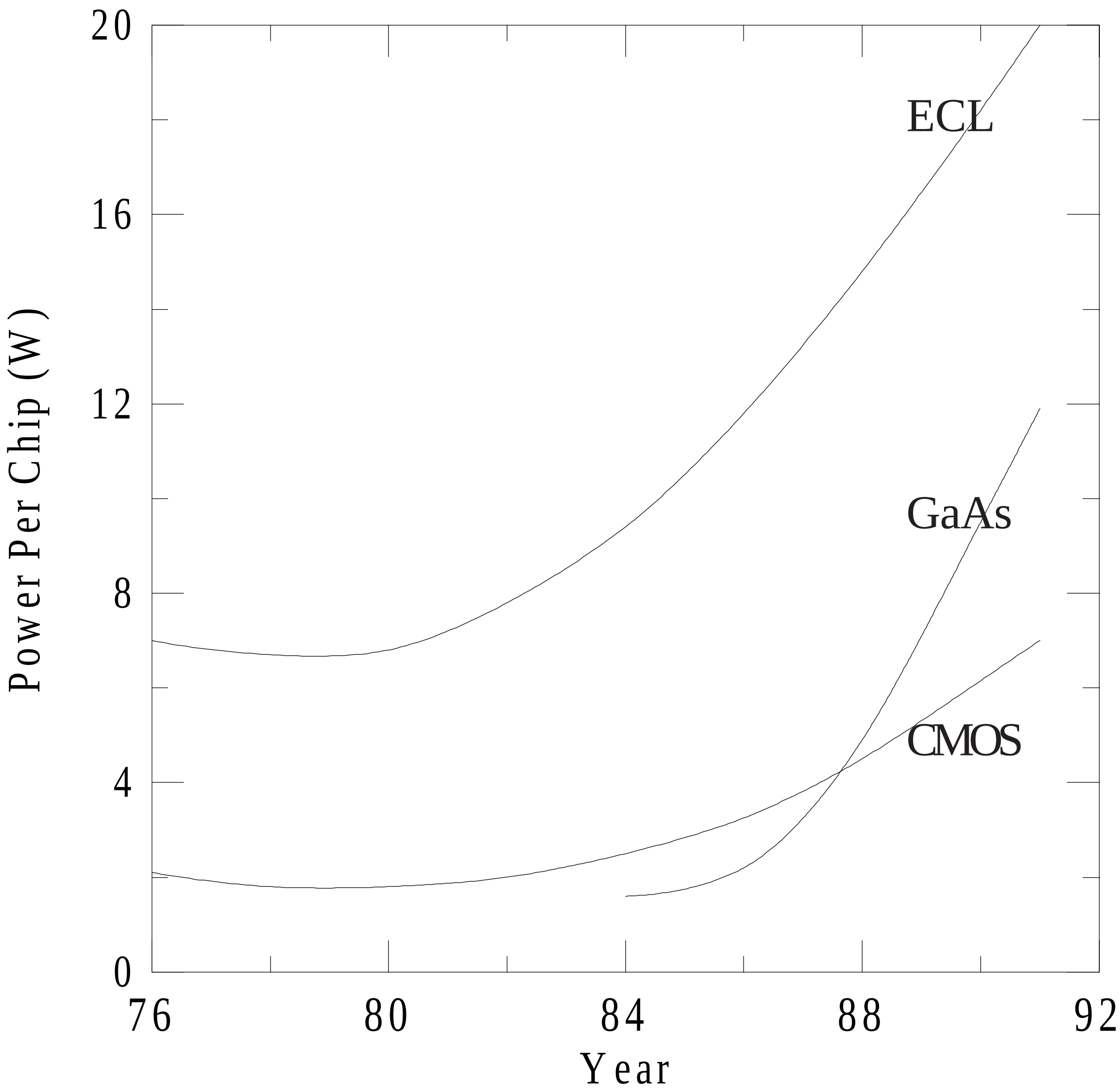
Power Dissipation Trends

(Adapted from Hannemann, 1990 Figure 1.4, page 10. Used with permission.)

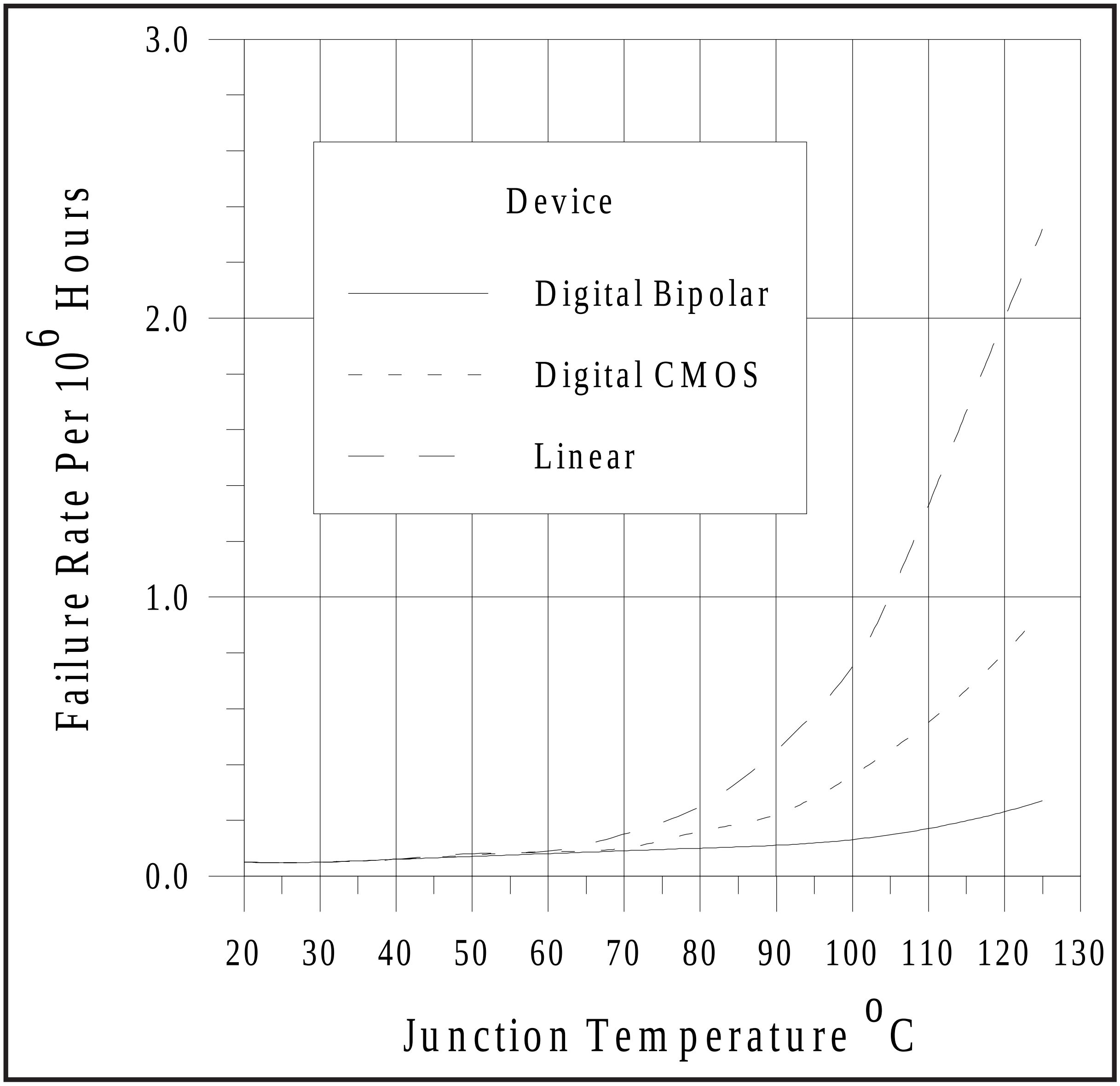


Power Dissipation Trends

(Adapted from Hannemann, 1990, Figure 1.4, page 10. Used with permission.)



Why Thermal Analysis?

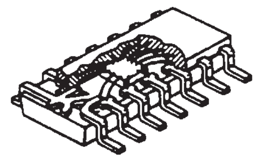
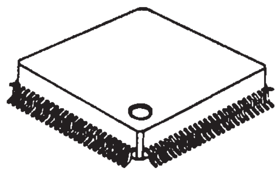
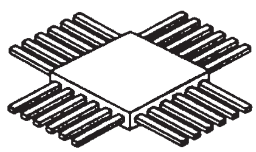
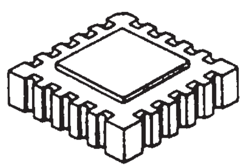
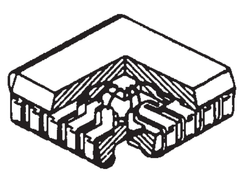
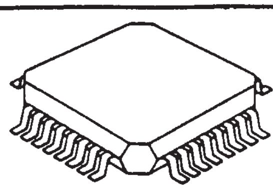


Component failure rates vs. temperature for digital and analog components. Data from MIL-HDBK-217.

A Short Survey of Electronic Components and Systems

Characteristics of Single Chip Packages:

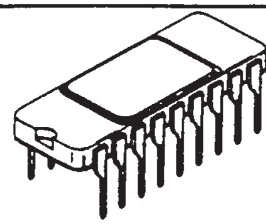
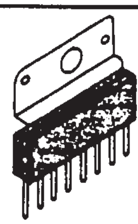
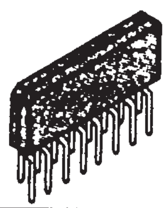
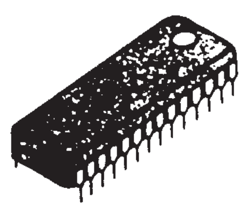
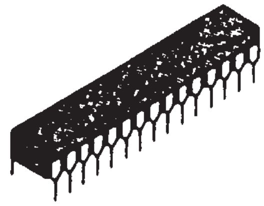
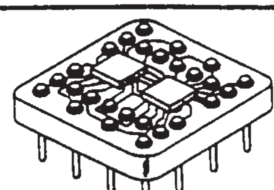
(Bar-Cohen, Avram, 1993, Table 1.1, page 22. Use with permission.)

I. SURFACE MOUNTED	SHAPE	TYPICAL FEATURES		
		MATERIAL	LEAD PITCH	# OF I/O PINS
SOP SMALL OUTLINE PACKAGE		PLASTIC	•1.27mm (50MIL) •2 direction lead	8-40
QFP QUAD FLAT PACKAGE		PLASTIC	• 1.0mm •0.8mm •0.65mm •4 direction lead	88-200
FPG FLAT PACKAGE OF GLASS		CERAMIC	•1.27mm (50MIL) •.762mm (30MIL) •2 direction lead •4 direction lead	20-80
LCC LEADLESS CHIP CARRIER		CERAMIC	•1.27mm (50MIL) •1.016mm(40MIL) •.762mm (30MIL)	20-40
PLCC PLASTIC LEADED CHIP CARRIER		PLASTIC	•1.27mm (50MIL) • J-shaped bend • 4 direction lead	18-124
VSQF VERY SMALL QUAD FLAT PACKAGE		PLASTIC	•0.5mm	32-100

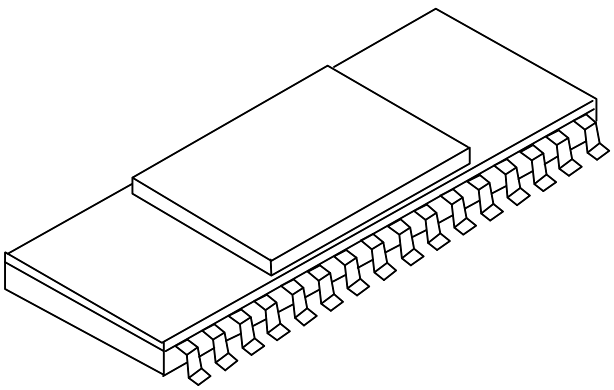
Bar-Cohen, Avram, ASME Series on Advances in Thermal Modeling of Electronic Components and Systems, Editors: Avram-Bar Cohen and Allan D. Kraus, Vol. III, Table 1.1, page 22, ASME Press, New York/IEEE Press, New York, 1993. Copyright © The American Society of Mechanical Engineers, 345 East 47th Street, N.Y., N.Y. 10017. Used With Permission.

Characteristics of Single Chip Packages - Continued:

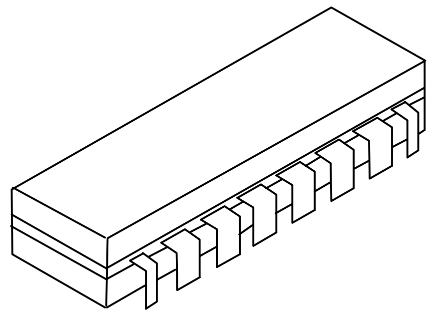
(Bar-Cohen, Avram, 1993, Table 1.1, page 22. Use with permission.)

II. THROUGH HOLE MOUNTED	SHAPE	TYPICAL FEATURES		
		MATERIAL	LEAD PITCH	# OF I/O PINS
DIP DUAL INLINE PACKAGE		CERAMIC PLASTIC	• 2.54mm(100MIL)	8-64
SIP SINGLE INLINE PACKAGE		PLASTIC	• 2.54mm(100MIL) • 1 direction lead	3-25
ZIP ZIGZAG INLINE PACKAGE		PLASTIC	• 2.54mm(100MIL) • 1 direction lead	16-24
S-DIP SHRINK DIP		PLASTIC	• 1.778mm(70MIL)	20-64
SK-DIP SKINNY DIP		CERAMIC PLASTIC	• 2.54mm • Half-size pitch in the width direction	24-32
PGA PIN GRID ARRAY		CERAMIC PLASTIC	• 2.54mm(100MIL)	

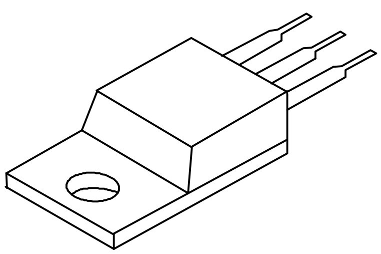
IC Packages:



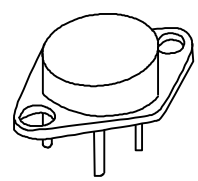
(a) Ceramic Gull-Wing
Dual-In-Line



(b) 16-Lead Plastic DIP

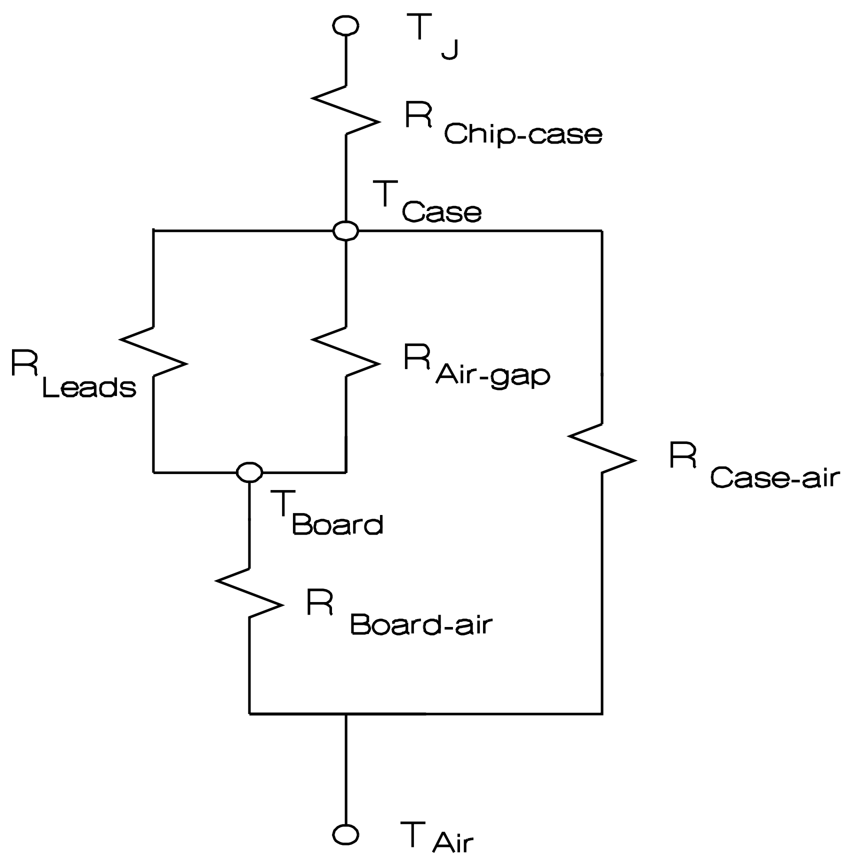


(c) 3-Lead Plastic TO-220



(d) 2-Lead TO-3

Heat-flow paths for dual-in-line packages:



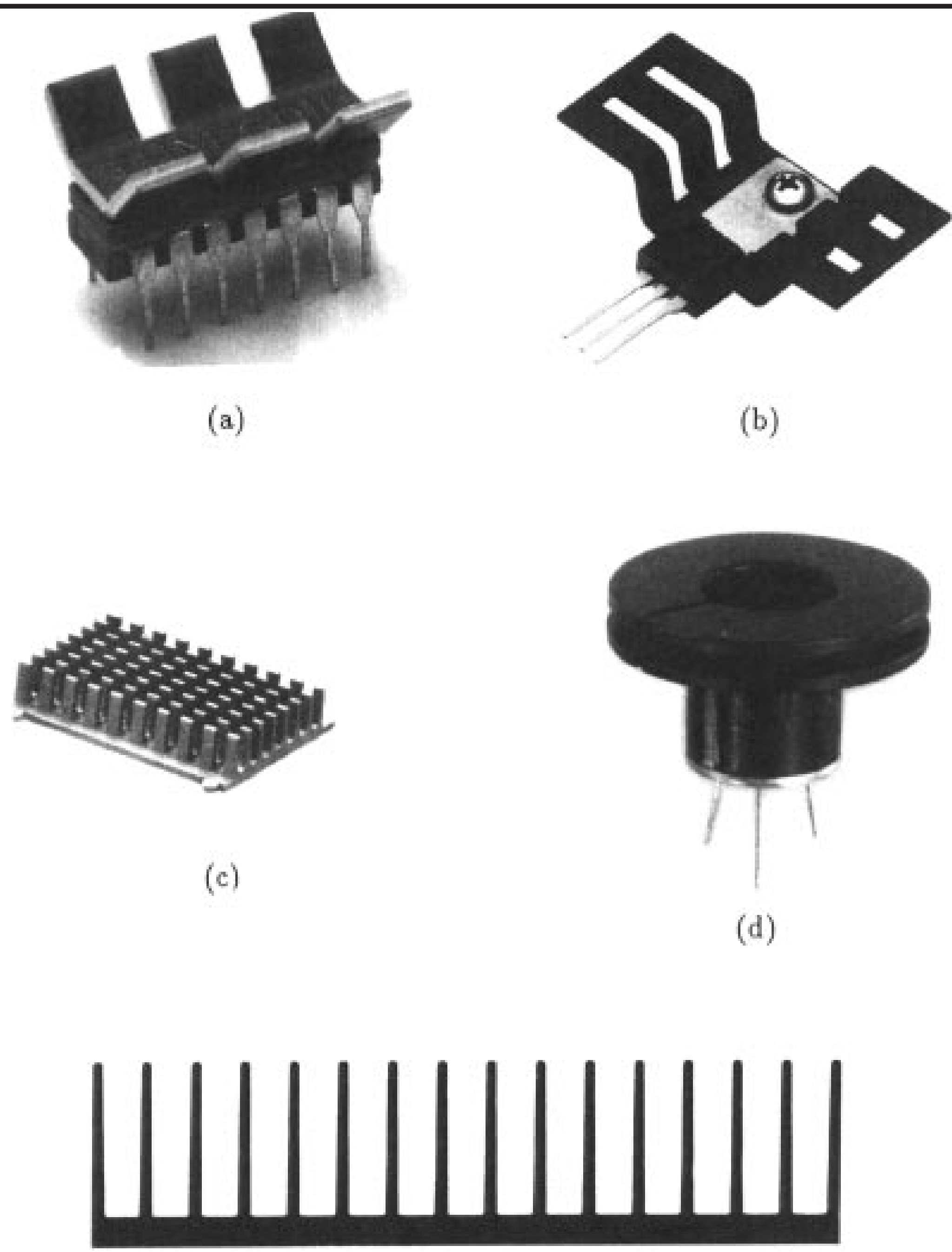
T_J = Chip junction temperature

T_{Case} = Package case temperature

T_{Board} = Board temperature

T_{Air} = Local ambient air temperature

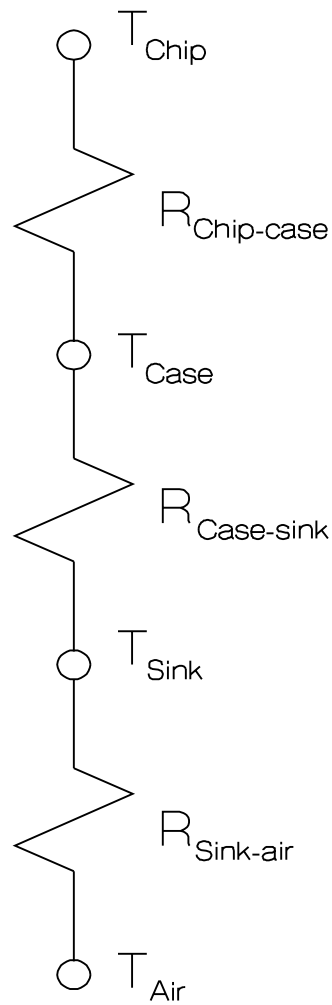
Ellison, Gordon N., ASME Series on Advances in Thermal Modeling of Electronic Components and Systems, Editors: Avram-Bar Cohen and Allan D. Kraus, Vol. III, Figure 3.2, page 157, ASME Press, New York/IEEE Press, New York, 1993. Copyright © The American Society of Mechanical Engineers, 345 East 47th Street, N.Y., N.Y. 10017. Used With Permission.


(a) (b)

(c) (d)

(e)

Figure 3.4 Component heat sinks (a) bond-on DIP heat sink for 14- or 16-pin packages, (b) power transistor heat sink for TO-202 packages, (c) pin grid array heat sink, (d) press-on sink for TO-5, 8 packages, (e) extrusion, end view [Courtesy of Thermalloy, Inc.].
Ellison, Gordon N., 1993, Figure 3.4, page 159. Used with permission.



Heat-flow paths for a typical power transistor chip package mounted on a heat a sink.

(Ellison, G.N., 1993, Figure 3.3, page 158. Used With Permission.)

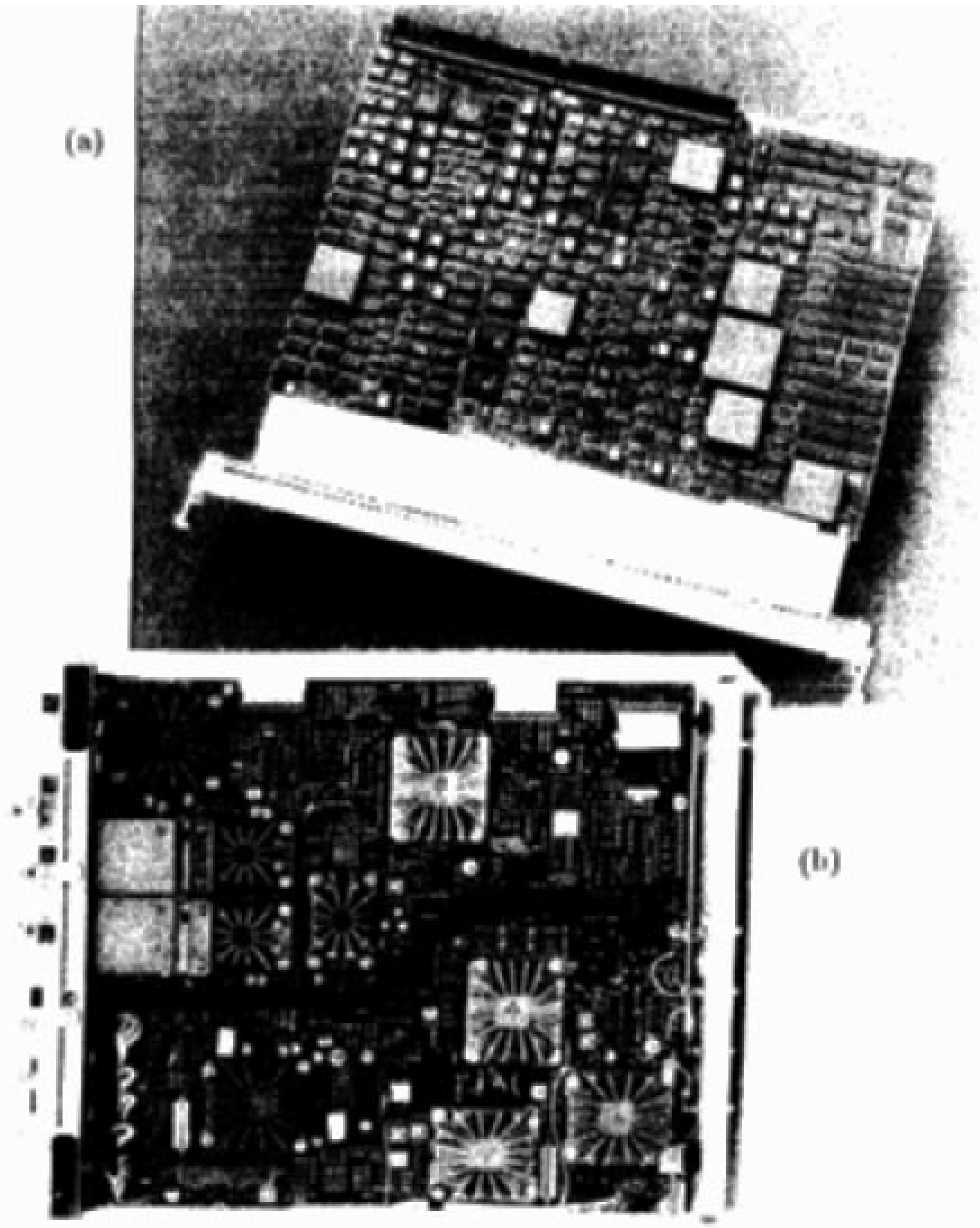
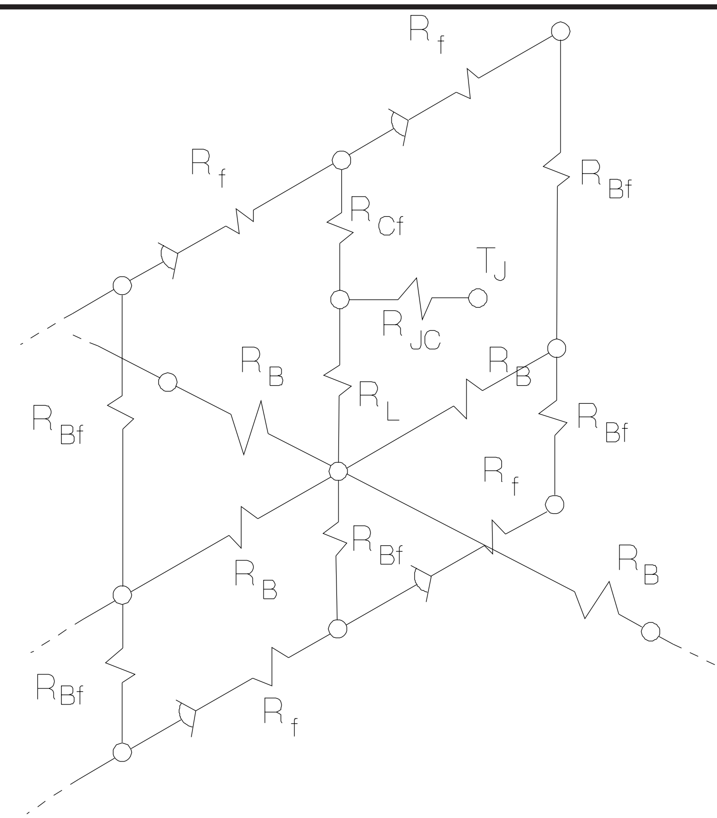


Figure 3.5 Circuit boards: (a) computer graphics workstation board, (b) portable oscilloscope main board [Courtesy of Tektronix, Inc.].

Ellison, Gordon N., 1993, Figure 3.5, page 160. Used with permission.

Thermal network schematic for printed circuit board heat transfer problem:



R_B = Board conduction resistance

R_L = Component case-to-board resistance

R_{Cf} = Component case-to-fluid resistance

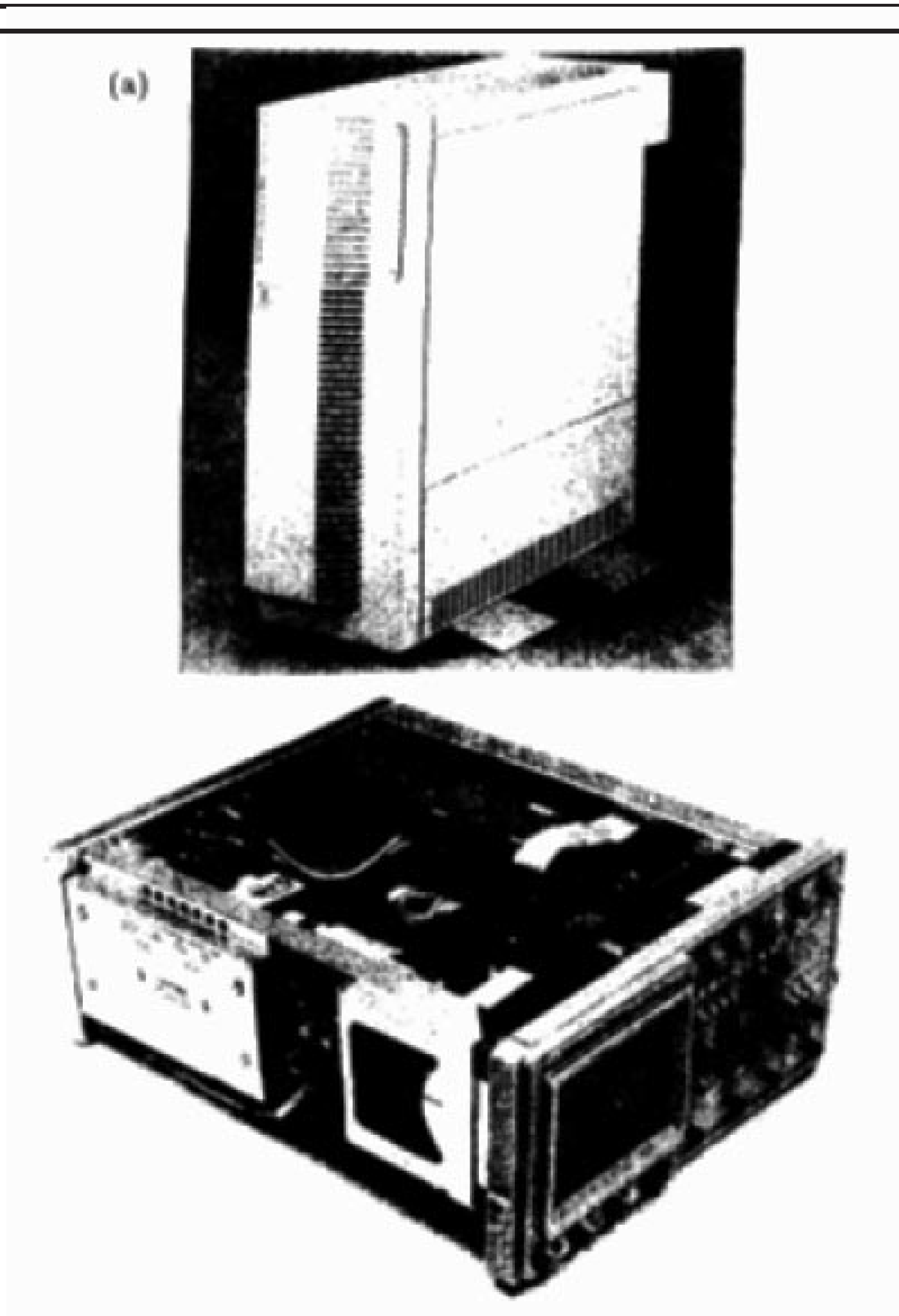
R_{Bf} = Board surface-to-fluid resistance

R_f = One-way fluid resistance

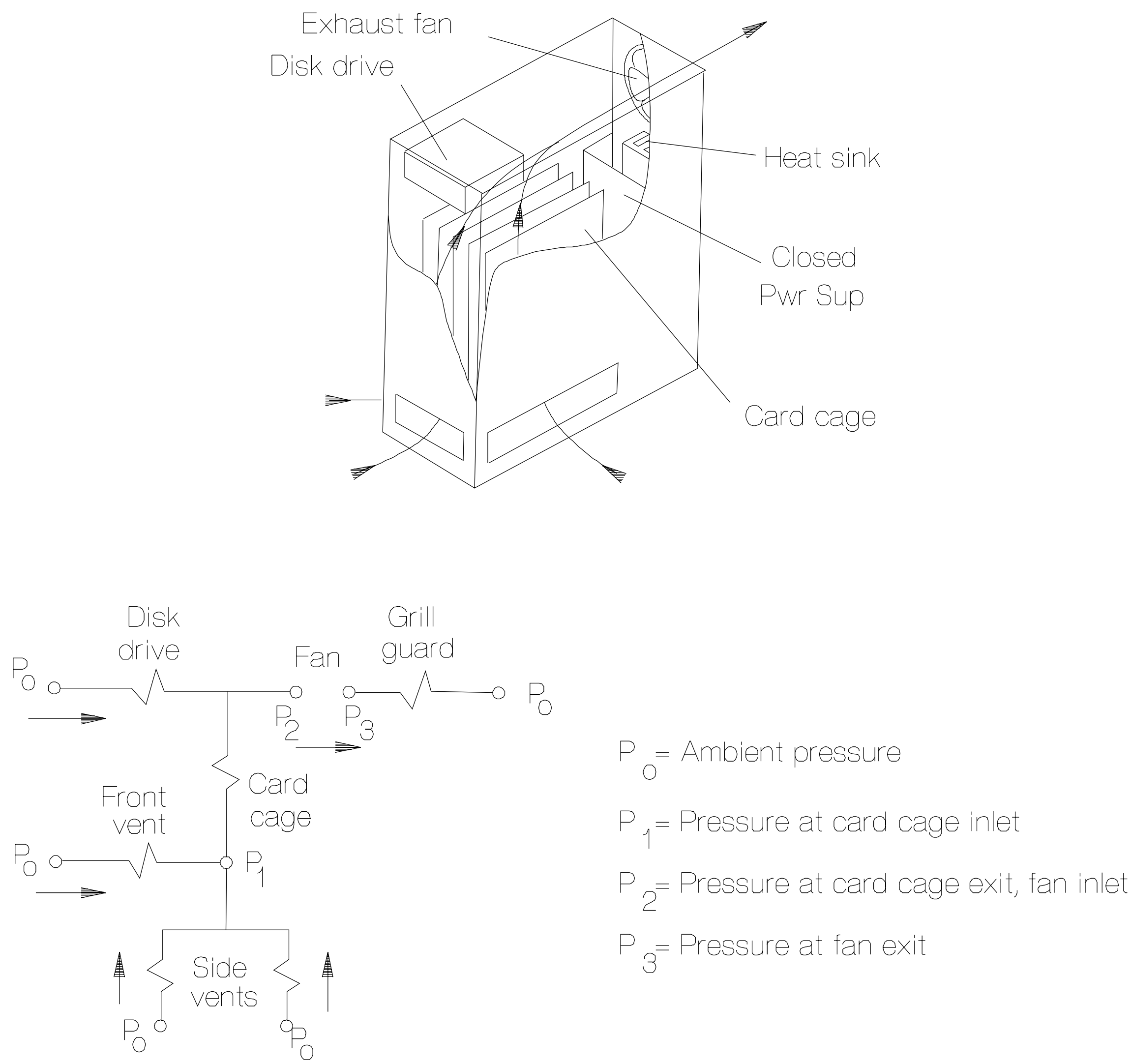
R_{JC} = Chip junction-to-case resistance

Denotes direction for which fluid node "sees" upstream fluid node

(Ellison, G.N., 1993, Figure 3.6, page 161. Used With Permission.)

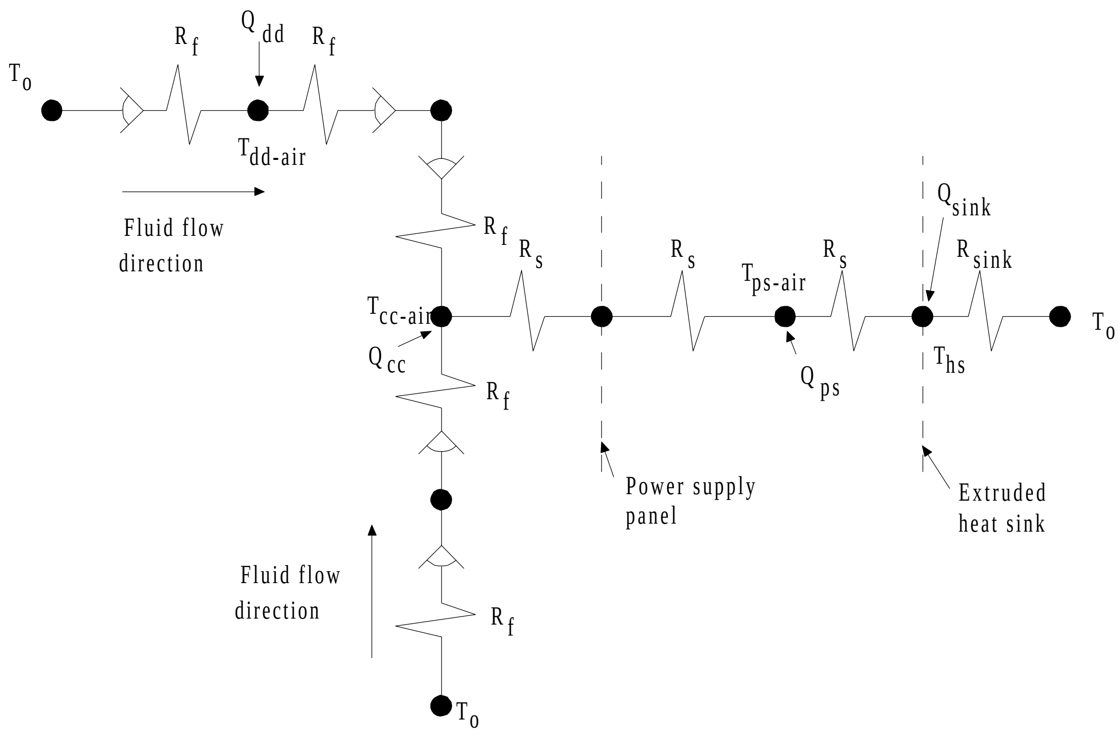


Packaged electronic systems: (a) computer workstation, (b) portable oscilloscope.
Ellison, Gordon N., 1993, Figure 3.7, page 163. Used with permission. Permission.



Pressure/air flow analog for generic workstation.

(Ellison, G.N., 1993, Figures 3.8, 3.9, page 164. Used With Permission.)



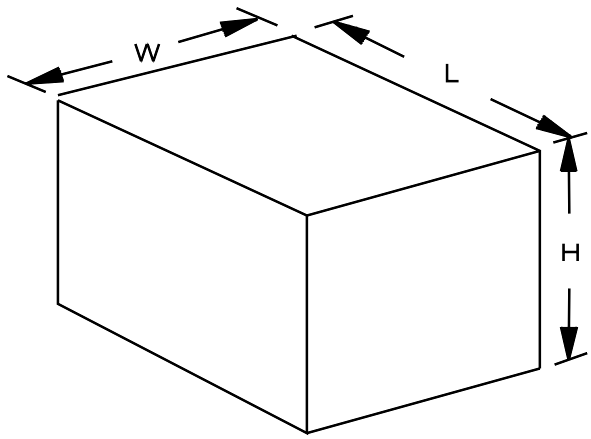
$T_{ps\text{-}air}$ = Power supply air temp.	R_f = Thermal fluid resistance
T_{hs} = Heat sink temp.	R_s = Surface resistance (conv./rad.)
$T_{cc\text{-}air}$ = Card cage air temp.	R_{sink} = Heat sink resistance (conv./rad.)
$T_{dd\text{-}air}$ = disk drive air temp.	Q_{dd} = Disk drive dissipation
T_o = Ambient air temp	Q_{sink} = Heat sink dissipation
Q_{cc} = Card cage dissipation	Q_{ps} = Power supply dissipation

Denotes downstream node "sees" only upstream node

Thermal analog for generic workstation.

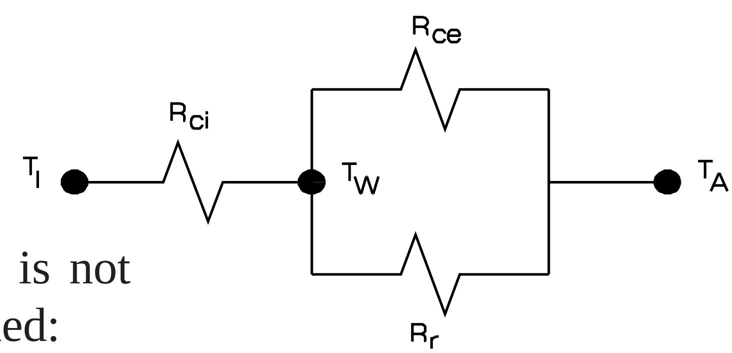
(Ellison, G.N., 1993, Figure 3.10, page 165. Used With Permission.)

Basic enclosure cooling consideration for sealed enclosure:



(a) Enclosure geometry

This model is not recommended:



(b) Simple thermal circuit

Q = total heat dissipation

T_A = Ambient air temp.

T_W = Average wall temp.

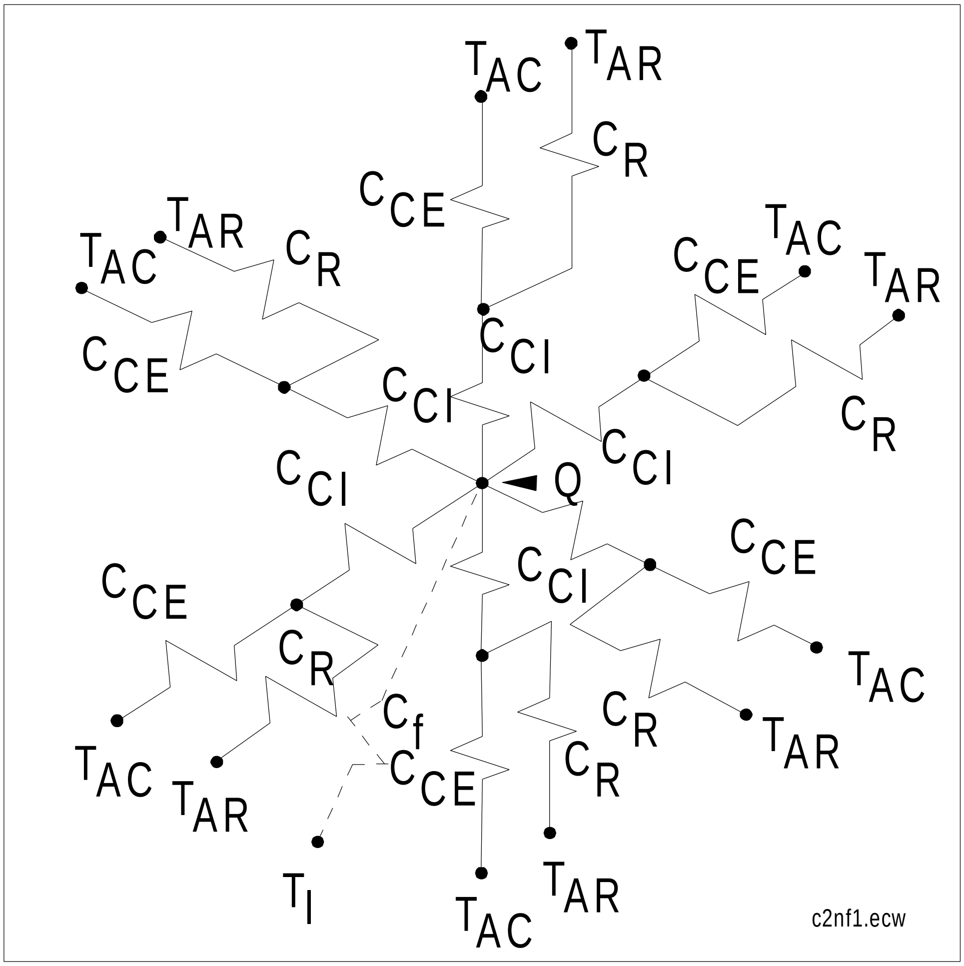
T_I = Internal air temp.

R_{CI} = Convection resistance, internal

R_{CE} = Convection resistance, external

R_r = Radiation resistance

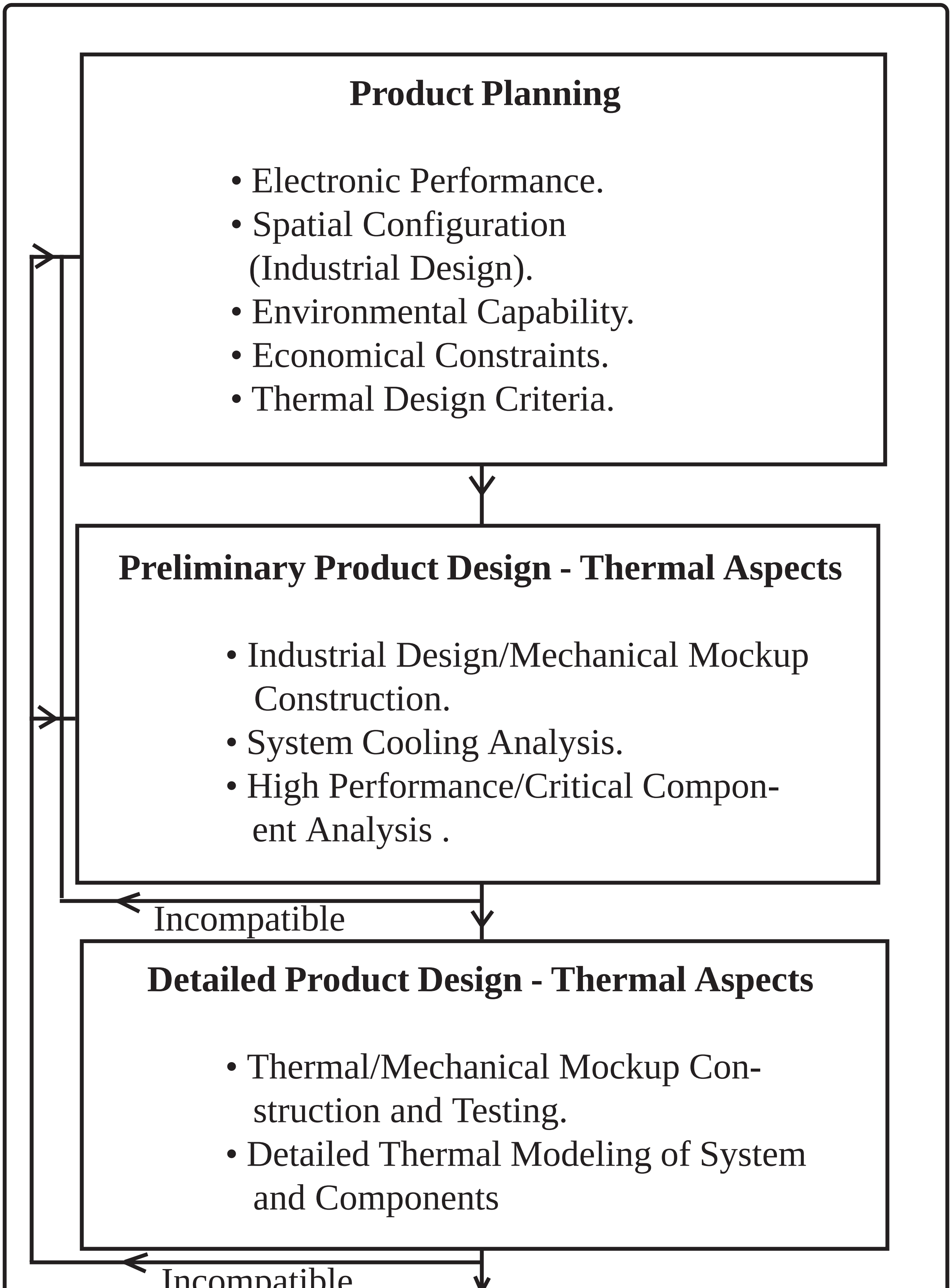
A recommended model for an enclosure with wall thickness effects neglected.



C_{CI}: Internal convection
C_{CE}: External convection
C_R: Radiation
C_f: Vent fluid

Q: Internal dissipation
T_{AC}: Ambient for convection
T_{AR}: Ambient for radiation

A Thermal Design Methodology

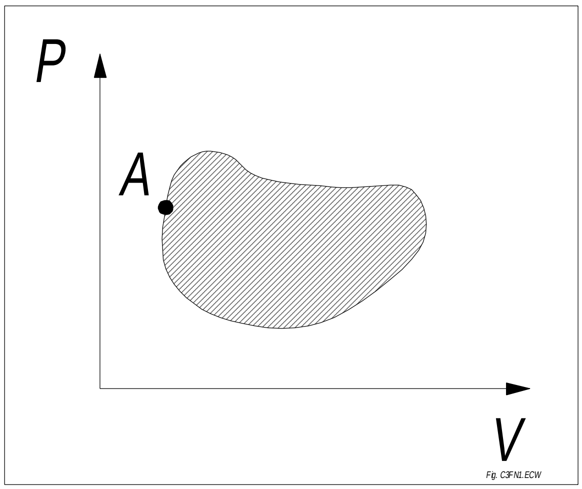

Product Planning
• Electronic Performance.
• Spatial Configuration (Industrial Design).
• Environmental Capability.
• Economical Constraints.
• Thermal Design Criteria.
Preliminary Product Design - Thermal Aspects
• Industrial Design/Mechanical Mockup Construction.
• System Cooling Analysis.
• High Performance/Critical Component Analysis .
Incompatible
Detailed Product Design - Thermal Aspects
• Thermal/Mechanical Mockup Construction and Testing.
• Detailed Thermal Modeling of System and Components
Incompatible

Air Temperature Rise in a Duct For Forced and Buoyancy Driven Air Flow

Thermodynamics of a Heated Fluid*

The first law of thermodynamics -

In a reversible (the process produces exactly the same result taken in either direction), cyclic process such as that shown in the following figure,



a quantity of work

$$W = \oint PdV$$

appears or disappears each cycle around the path. Experiment has also shown that an equivalent amount of heat given by

$$q = \oint \bar{d}q$$

appears or disappears also (the barred symbol $\bar{d}$ means that the result depends on the path taken).

* The author is indebted to Carolyn Roos for her suggestions concerning the development of the enthalpy change of a heated, moving fluid.

The conservation of energy in a cyclic closed system is then

$$\oint \left(\bar{d}q - \bar{d}W \right) = 0$$

The preceding equation cannot be proven analytically, but it has never been shown to be false if all known forms of work are included.

In an open process heat minus work is not conserved, i.e.

$$Heat\,into\,system - \;Work\,done\,by\,system \neq \;0$$

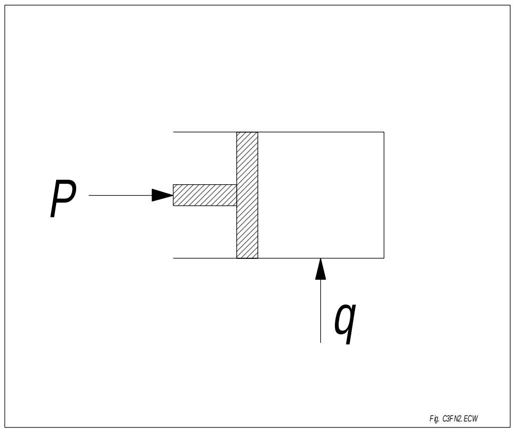
but this is no problem because the substance has a change of state. The equivalent mathematical statement is

First Law of Thermodynamics
for reversible processes

$$dE = \bar{d}q - \bar{d}W$$

where E is defined as the *total system energy*.

Now consider the illustrated quasi-static process where the only pressures are hydrostatic. If the piston is very slowly moved an infinitesimal distance *L*, the internal pressure will very nearly be equal to the external hydrostatic pressure. A static system implies the internal force equals the external force and *P* therefore is nearly constant.



The total energy *E* is the sum of the internal (thermal) engery *U*, the kinetic energy *KE*, and potential energy *PE*, etc. Then

$$E = U + KE + PE + \ldots$$

We shall consider thermodynamic systems where the only <u>changes</u> are in the internal (thermal) energy.

$$dE = dU$$

Then

$$\bar{d}q = dE + \bar{d}W = dU + \bar{d}W$$

$$dU = \bar{d}q - \bar{d}W = \bar{d}q - PdV$$

Heat capacity as a Fluid Property

$$\bar{d}q = dU + PdV$$

If we assume that an equation of state (analytical or graphical) exists relating the thermodynamic variables *T, P,* and *V*, any of the three variables is quantifiable in terms of the other two (one independent and two independent variables). We may then write

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

$$\begin{aligned}
\bar{d}q &= dU + PdV \\
&= \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + PdV \\
&= \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T dV + P\right] dV
\end{aligned}$$

Heat capacity at constant volume

$$c_V \equiv \left(\frac{\bar{d}q}{dT}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$

It is more common in practice to use

$\bar{U} \equiv$ internal energy per unit mass

$C_P \equiv$ Heat capacity per unit mass $= \left(\frac{\partial \bar{U}}{\partial T}\right)_V$

Heat capacity at constant pressure

Defining enthalpy

$$H = U + PV$$
$$U = H - PV$$

$$\begin{aligned} dU &= dH - PdV - VdP \\ &= \left(\frac{\partial H}{\partial T}\right)_P dT + \left(\frac{\partial H}{\partial P}\right)_T dP - PdV - VdP \end{aligned}$$

Using

$$\bar{d}q = dU + PdV$$

$$\begin{aligned} \bar{d}q &= \left(\frac{\partial H}{\partial T}\right)_P dT + \left(\frac{\partial H}{\partial P}\right)_T dP - PdV - VdP + PdV \\ &= \left(\frac{\partial H}{\partial T}\right)_P dT + \left[\left(\frac{\partial H}{\partial P}\right)_T - V\right] dP - PdV + PdV \\ &= \left(\frac{\partial H}{\partial T}\right)_P dT + \left[\left(\frac{\partial H}{\partial P}\right)_T - V\right] dP \end{aligned}$$

We now define the heat capacity at constant pressure as

$$c_P \equiv \left(\frac{\bar{d}q}{dT}\right)_P = \left(\frac{\partial H}{\partial T}\right)_P$$

Using the usual heat capacity per unit mass (specific heat) C_P and enthalpy in units of energy per unit mass

$$C_P = \left(\frac{\partial \overline{H}}{\partial T}\right)_P$$

Note: In the case of C_P we can relax hydrostatic internal and external pressure as now, not exactly equal, and go back to our previous

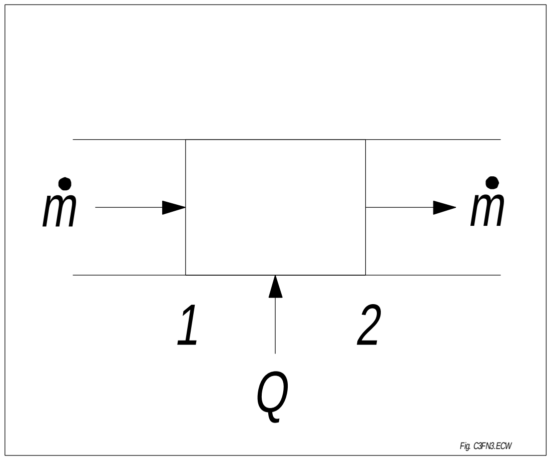
$$\begin{aligned} \bar{d}q &= dU + \bar{d}W = dU + \bar{d}(PV) \\ &= dU + PdV + VdP \end{aligned}$$

but now *VdP=0* and we still have

.

$$\bar{d}q = dU + PdV$$

Consider a steady-flow, or *quasi-static*, reversible process on the following open, single stream system:



In this system we apply conservation of energy with

Transport of internal energy into control volume + net heat rate added + Work to push volume $\overline{V}_1$ into control volume = Transport of internal energy out of control volume + Work to push volume $\overline{V}_2$ out of control volume

$\dot{m} \equiv$ mass flow rate

$\overline{U} \equiv$ internal energy per unit mass

$\overline{V} \equiv$ volume per unit mass ($\overline{V}_1 = \overline{V}_2$)

$Q \equiv$ net heat rate input

$$\dot{m}\overline{U}_1 + Q + \dot{m}P_1\overline{V}_1 = \dot{m}\overline{U}_2 + \dot{m}P_2\overline{V}_2$$

$$\dot{m}\left(\overline{U}_1 + P_1\overline{V}_1\right) + Q = \dot{m}\left(\overline{U}_2 + P_2\overline{V}_2\right)$$

But the enthalpy <u>per unit mass</u> is

$$\overline{H} = \overline{U} + P\overline{V}$$

so that we have

$$\dot{m}\overline{H}_1 + Q = \dot{m}\overline{H}_2$$

$$Q = \dot{m}\left(\overline{H}_2 - \overline{H}_1\right) = \dot{m}\Delta\overline{H}$$

$$\Delta\overline{H} = Q/\dot{m}$$

If a gas obeys the ideal gas law, as air certainly does to a sufficient approximation, the internal energy and enthalpy are functions of temperature only (see Holman, J.P., Thermodynamics, McGraw Hill Publishing Co., 1974, pages 75, 200 or Fermi, E., Thermodynamics, Dover Publications, Inc. 1937, page 23).

Then $\left(\frac{\partial\overline{H}}{\partial T}\right)_P = \frac{d\overline{H}}{dT}$ and since $C_P = \frac{d\overline{H}}{dT}$

$$\Delta\overline{H} = \overline{H}_2 - \overline{H}_1 = \int_{T_1}^{T_2} C_P dT$$

and if C_p is constant with temperature

$$\Delta\overline{H} = C_P(T_2 - T_1)$$

setting $\Delta\overline{H} = Q/\dot{m}$ equal to the preceding equation for $\Delta\overline{H} = C_P(T_2 - T_1)$,

$$\boxed{Q = \dot{m}C_P(T_2 - T_1)}$$

Derivation of Air Temperature Rise

$$Q = \dot{m} C_p \Delta T = \rho G C_p \Delta T$$

$$\Delta T = Q \big/ \rho G C_p$$

where

$$\dot{m} \equiv \text{ mass flow rate}$$

$$C_p \equiv \text{ specific heat}$$

$$Q \equiv \text{ heat convected into air flow}$$

$$\rho \equiv \text{ air density}$$

$$G \equiv \text{ volumetric air flow rate}$$

Using the ideal gas law

$$PV = \left(\frac{m}{M} \right) RT'$$

$$m \equiv \text{mass}, \; M \equiv \text{molecular weight}$$

$$R \equiv \text{gas constant}, \; T' \equiv \text{absolute temperature}$$

$$\rho = \left(\frac{m}{V} \right) = \frac{PM}{RT'}, \qquad \rho_o = \frac{PM}{RT_0'}$$

$$\rho = \rho_o \left(\frac{T_o + 273.15}{T + 273.15} \right), \qquad \rho_o, \; \rho \quad \text{density at } T_o \left[{}^oC \right], T \left[{}^oC \right]$$

At sea level conditions and $T_o = 0\,^oC$

$$\rho_o = 0.021\ gm/in.^3\,,\ \ C_p = 1.01\ joules/gm \cdot K$$

Then

$$\rho = \rho_o\, 5.736/(T + 273.15)$$

$$G\left[\frac{in.^3}{s}\right] = G\left[\frac{ft^3}{\text{min.}}\right]\left(\frac{\text{min.}}{60\,s}\right)\left(\frac{12\,in.}{ft.}\right)^3$$

$$\Delta T = Q\left(\frac{1}{\rho}\right)\left(\frac{1}{G}\right)\left(\frac{1}{C_p}\right)$$

$$\Delta T\left[^oC\right] = Q[W]\left(\frac{T\left[^oC\right] + 273.15}{5.736}\right)\left[\frac{60}{(12)^3 G}\right]\frac{1}{1.01}$$

$$\Delta T = \frac{5.99x10^{-3}(T + 273.15)Q}{G}$$

$\Delta T,\ T$ in c

Q in W

G in $ft^3/\text{min.}$

which is TCEE, E2.28 (almost).

Table 2-1. Physical properties of air at atmospheric pressure with units converted from [2]. Table A-3 (p. 636) from *Principles of Heat Transfer*, 3rd Edition by Frank Kreith. Copyright © 1958, 1965, 1973 by Harper & Row, Publishers, Inc. Reprinted by permission of the publisher.

T (°C)	T (°F)	ρ (gm/in.^{3})	C_p (joule/gm · °C)	μ (10^{-4} gm/in. · sec)	ν in.^{2}/sec	k (10^{-4} watt/in. · °C)	Pr	β (10^{-3}/°C)	$g\rho^2/\mu^2$ (10^6/in.^{3})	$g\beta\rho^2/\mu^2$ (10^3/in.^{3} · °C)
−18	0	0.023	1.00	4.195	0.0187	5.846	0.73	3.916	1.12	4.38
0	32	0.021	1.01	4.403	0.0209	6.153	0.72	3.661	0.899	3.29
38	100	0.019	1.01	4.856	0.0259	6.769	0.72	3.216	0.569	1.83
93	200	0.016	1.01	5.442	0.0344	7.648	0.72	2.729	0.324	0.885
149	300	0.014	1.02	6.085	0.0441	8.483	0.71	2.370	0.195	0.462
204	400	0.012	1.03	6.614	0.0544	9.318	0.689	2.094	0.128	0.269
260	500	0.0108	1.03	7.143	0.0655	10.15	0.683	1.876	0.243	0.166

Identification of *T* in TCEE, E2.28 by Using Differential Forms of ΔT, ΔQ:

TCEE E2.28 identifies *T* as the average bulk temperature over the length of the duct. This section addresses an attempt to circumvent using *T* as the average bulk temperature.

We begin by recognizing that the defining air temperature rise equation can be written with differentials:

$$dT = \frac{C}{G}(T + 273.15)dQ$$

where

$$C = 5.99x10^{-3}$$

The differential in temperature is the temperature rise across a small length segment of the duct from which the differential heat dissipation is transferred into the duct. With this definition, the temperature *T* is exactly the air temperature in a short section of the duct. Rearranging the preceding equation slightly:

$$\frac{dT}{(T + 273.15)} = C\frac{dQ}{G}$$

Making a variable transformation,

$$u = T + 273.15$$

$$du = dT$$

$$\int \frac{du}{u} = \frac{C}{G} \int dQ + B$$ where B is a constant of integration

$$\ln(u) = \frac{C}{G} Q + B$$

$$\ln(T + 273.15) = \frac{C}{G} Q + \ln(A)$$ where A is also a constant

$$\ln\left(\frac{T + 273.15}{A}\right) = \frac{C}{G} Q$$

$$\frac{T + 2731.5}{A} = e^{\frac{CQ}{G}}$$

$$T = Ae^{\frac{CQ}{G}} - 273.15$$

At $Q = 0$, $T = T_I$, where T_I is the air inlet temperature. Then

$$A = T_I + 273.15$$

$$T = (T_I + 273.15)e^{\frac{CQ}{G}} - 273.15$$

$$(T + 273.15) = (T_I + 273.15)e^{\frac{CQ}{G}}$$

Consider the expansion

$$e^{\frac{CQ}{G}} = 1 + \left(\frac{CQ}{G}\right) + \frac{1}{2!}\left(\frac{CQ}{G}\right)^2 + \ldots$$

Then

$$(T + 273.15) = (T_I + 273.15)\left[1 + \frac{CQ}{G} + \ldots.\right]$$

$$\cong (T_I + 273.15)\left(1 + \frac{CQ}{G}\right)$$

$$\cong T_I\left(1 + \frac{CQ}{G}\right) + 273.15\left(1 + \frac{CQ}{G}\right)$$

After a little algebra

$$\Delta T = T - T_I = \frac{CQ}{G}(T_I + 273.15)$$

$$\Delta T \cong \frac{5.99x10^{-3}}{G} Q(T_I + 273.15)$$

which is slightly different than TCEE, E2.28 in the identification of the temperature T as the inlet temperature T_I.

$$\text{If } T_I = 20\ {}^oC,$$

$$\Delta T \cong \frac{1.76Q}{G}$$

Identification of *T* in TCEE, E2.28 as Average Bulk Temperature $\overline{T}_B$:

$$\Delta T = \frac{5.99x10^{-3}}{G} Q(T + 273.15) = \frac{C}{G} Q\left(\overline{T}_B + 273.15\right)$$

where $C=5.99x10^{-3}$.

$$\overline{T}_B = \frac{T_I + T_E}{2} = \frac{T_I + T_I + \Delta T}{2} = \frac{2T_I + \Delta T}{2}$$

for inlet and exit temperatures, T_I and T_E.

Then, solving for ΔT

$$\Delta T = \frac{CQ}{G}\left(\frac{\Delta T + 2T_I}{2} + 273.15\right)$$

$$\left(\frac{G}{CQ}\right)\Delta T = T_I + 273.15 + \frac{\Delta T}{2}$$

$$\Delta T\left(\frac{G}{CQ} - \frac{1}{2}\right) = T_I + 273.15$$

$$\Delta T\left(\frac{2G}{CQ} - 1\right) = 2(T_I + 273.15)$$

$$\Delta T = \frac{2(T_I + 273.15)}{\left(\frac{2G}{CQ} - 1\right)}$$

$$\Delta T = \frac{2(T_I + 273.15)}{\left(\frac{2G}{CQ} - 1\right)} = 2(T_I + 273.15)\left(\frac{2G}{CQ} - 1\right)^{-1}$$

$$= 2(T_I + 273.15)\left(\frac{CQ}{2G}\right)\left(1 - \frac{CQ}{2G}\right)^{-1}$$

$$= \frac{2(T_I + 273.15)Q}{333.9G}\left(1 - \frac{Q}{333.9G}\right)^{-1}$$

Expanding the $()^{-1}$ as a binomial expansion for sufficiently small Q and G,

$$\Delta T = \frac{2(T_I + 273.15)Q}{333.9G}\left(1 + \frac{Q}{333.9G} + \cdots\right)$$

$$\cong \frac{2(T_I + 273.15)Q}{333.9G}$$

$$\cong \frac{5.99x10^{-3}}{G} Q(T_I + 273.15)$$

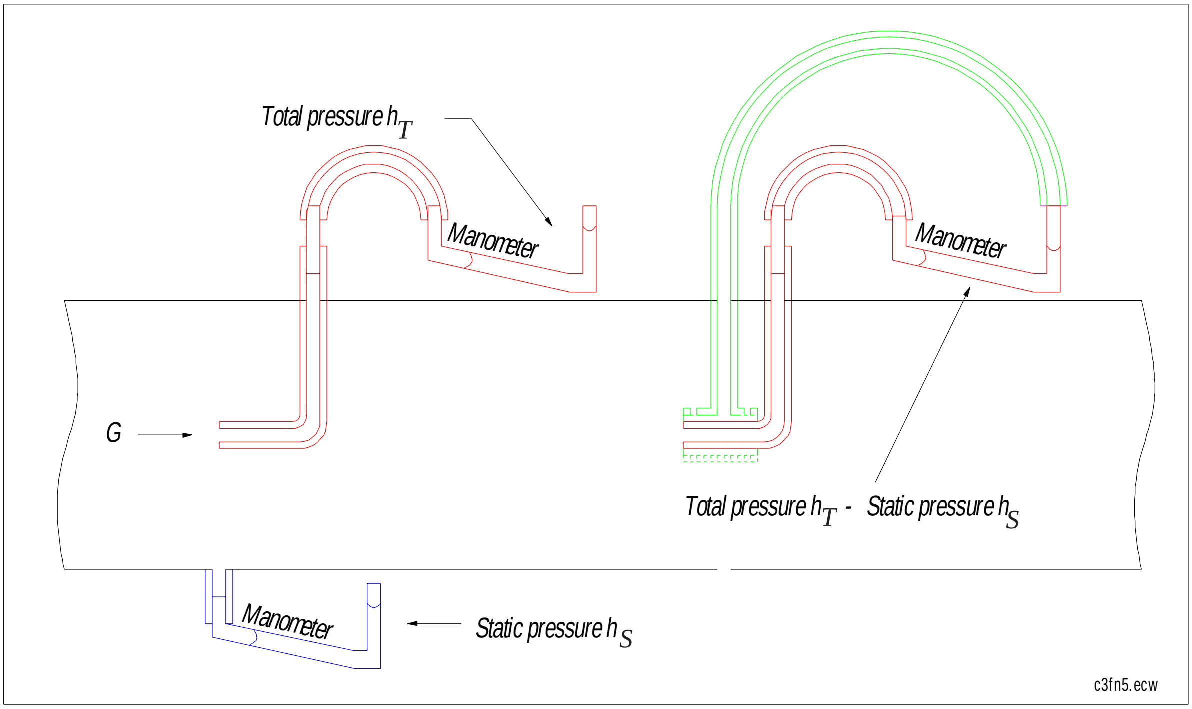
which is identical to the previous method. Neither method has an advantage.

In either case, if $T_I = 20.0\ {}^oC$, then

$$\Delta T \cong 1.76\frac{Q}{G}, \quad \frac{Q}{G} \leq 30$$

Forced Air Flow - Mostly Systems

Pressure Head Basics



In ambient: $h_S = 0,\ h_V = 0$

In duct:

$h_S =$ static pressure

$h_V =$ velocity pressure

$h_T =$ total pressure $= h_V + h_S$

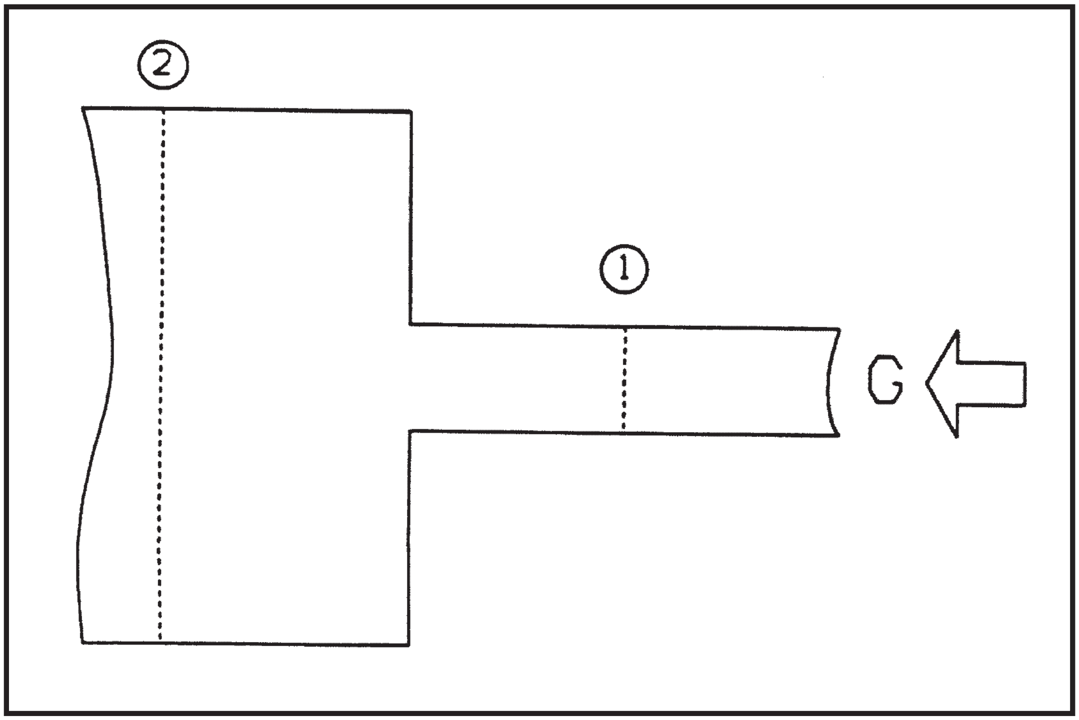
Note:

$$h_V = 1.29x10^{-3}\left(G^2/A^2\right)\ [in.\ H_2O]$$

$$G = \text{airflow}\left[ft.^3/\text{min.}, i.e.\ CFM\right]$$

$$A = \text{duct cross-sectional area}\left[in.^2\right]$$

Bernoulli's Equation with Losses



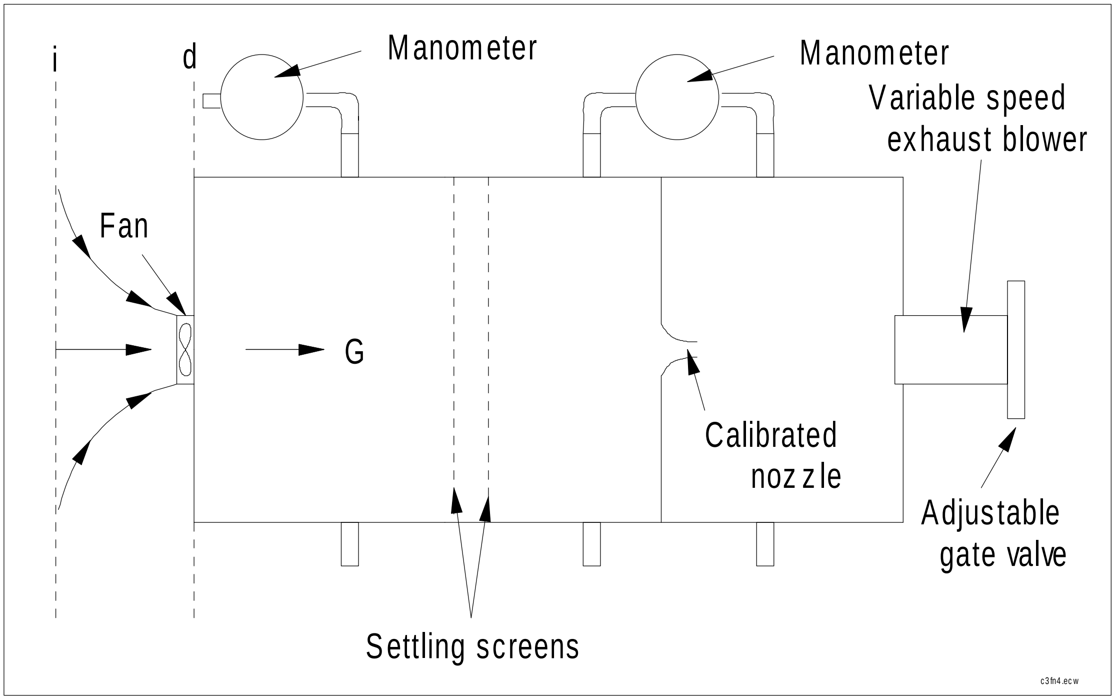
$$h_T = \text{ total pressure}$$
$$= h_V + h_S$$

$$h_{T1} = h_{T2} + h_L$$
$$h_{V1} + h_{S1} = h_{V2} + h_{S2} + h_L$$

$$h_L = \text{ total pressure loss}$$

Fan Testing

A fan test system -



$h_{fs} \equiv$ fan static pressure head

$h_{sd} \equiv$ static pressure head at the fan discharge plane

$h_{si} \equiv$ static pressure head at fan inlet

$h_{vi} \equiv$ velocity pressure head at fan inlet

Fan static pressure,

$$h_{fs} \equiv \Delta h_T - \Delta h_V = (h_{Td} - h_{Ti}) - (h_{Vd} - h_{Vi})$$

$$= (h_{Vd} + h_{sd} - h_{Vi} - h_{si}) - (h_{Vd} - h_{Vi})$$

$$= h_{sd} - h_{si} \text{ and since for this test setup, } h_{si} = 0$$

$$h_{fs} = h_{sd}$$

Estimate of Possible Error by Measuring "Downstream" Static Pressure Instead of at Fan Discharge

Defining

h_{T2} = total pressure head in test system at indicated pressure taps

h_L = total pressure loss between fan discharge plane and indicated pressure taps

$$\begin{aligned} h_{Td} &= h_{T2} + h_L \\ h_{sd} + h_{vd} &= h_{s2} + h_{v2} + h_L \end{aligned}$$

$$\begin{aligned} \Delta h_s &= h_{sd} - h_{s2} \\ &= h_{v2} - h_{vd} + h_L \end{aligned}$$

The total pressure loss by expansion from the discharge plane to the static pressure taps is given by a standard expression

$$h_L = h_{vd}\left(1 - \frac{A_d}{A_2}\right)^2 \quad \text{for a velocity head } h_{vd} = \frac{1}{2}\rho V_d^2$$

at the fan discharge.

Returning to the error Δh_s in static pressure measurement,

$$\Delta h_s = h_{v2} - h_{vd} + h_L = h_{v2} - h_{vd} + h_{vd}\left(1 - \frac{A_d}{A_2}\right)^2$$

$$= h_{v2} + h_{vd}\left[\left(1 - \frac{A_d}{A_2}\right)^2 - 1\right]$$

Case A: Chamber diameter = Fan diameter, i.e. $A_2 = A_d$

$$\Delta h_s = h_{v2} + h_{vd}\left[\left(1 - \frac{A_d}{A_2}\right)^2 - 1\right] = h_{vd} + h_{vd}\left[\left(1 - \frac{A_d}{A_d}\right)^2 - 1\right] = 0$$

just as one would expect

Case B: The chamber diameter is very large compared to the fan diameter, i.e. $A_2 >> A_d$

$$\Delta h_s = h_{v2} + h_{vd}\left[\left(1 - \frac{A_d}{A_2}\right)^2 - 1\right] = h_{v2} + 0$$

$$= h_{v2} = \frac{1}{2}\rho V_2^2 = \frac{1.29x10^{-3}}{A_2^2} G^2$$

for Δh in units of *in.* H_2O, *G* in units of ft^3/min.

Suppose the large diameter chamber had a diameter of about three feet or $r_2 \cong 20in.$.

Then

$$\Delta h_s = \frac{1.29x10^{-3}}{A_2^2}G^2 = \frac{1.29x10^{-3}}{\left[\pi(20)^2\right]^2}G^2 \approx 1x10^{-9}G^2$$

G (ft^3/min.)	Δh_s (in. H_2O)
1	10^{-9}
10	10^{-7}
100	10^{-5}
1000	*0.001*

so the conclusion is that the error in Δh_s is trivial.

Fan and System Matching - Summary of Basic Systems

Fan curves: provided by fan vendor or measured by designer.

System curve: $H_L = \sum System\ losses$.

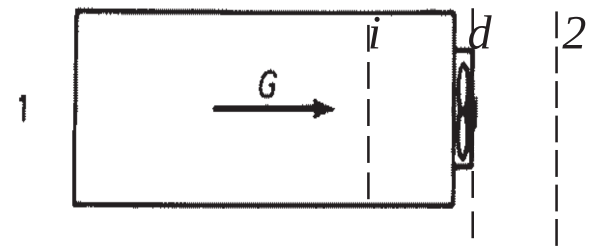
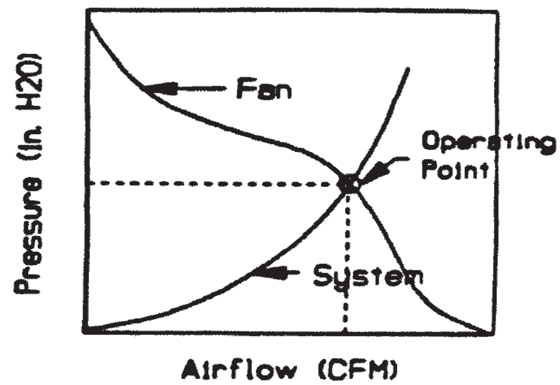


Fig. 6-7. Exhaust fan system.

$$\boxed{h_{fs} = H_L}$$



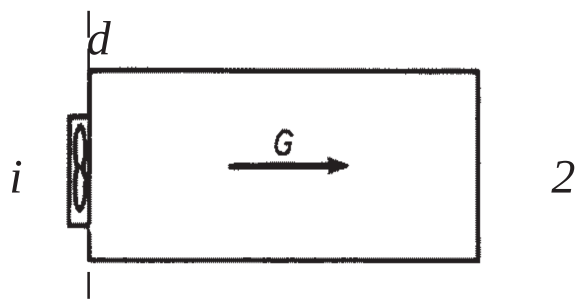
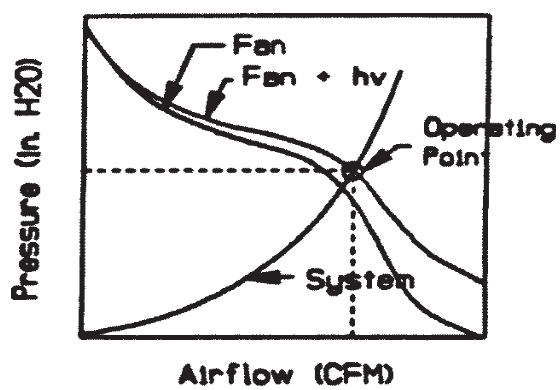


Fig. 6-6. Blower fan system.

$$\boxed{h_{fs} + h_{vd} = H_L}$$



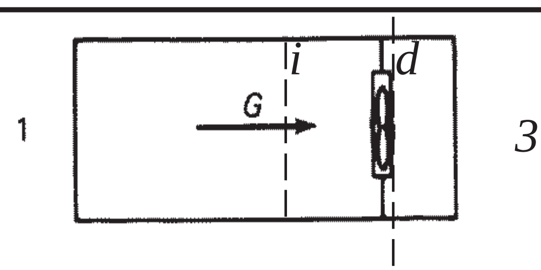
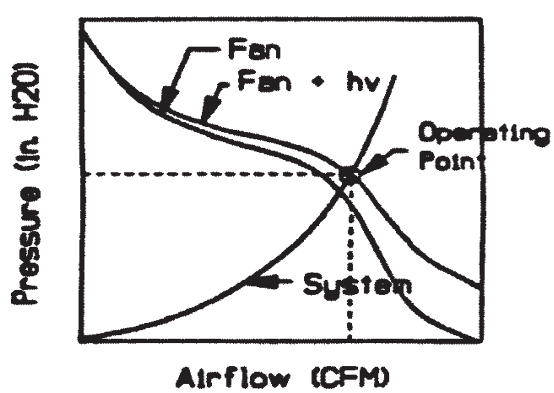


Fig. 6-8. Intermediate fan system.

$$\boxed{h_{fs} + h_{vd} = H_L}$$



Details of Operating Point Conditions - Some General Conditions

Fan total pressure h_T

Referring to any of the basic system illustrations :

$$h_T = h_{Td} - h_{Ti}$$

Fan static pressure h_{fs}

$$\begin{aligned} h_T &= h_{Td} - h_{Ti} \\ &= (h_{Vd} + h_{sd}) - (h_{Vi} + h_{si}) \\ &= h_{Vd} - h_{Vi} + (h_{sd} - h_{si}) \\ h_{sd} - h_{si} &= h_T - (h_{Vd} - h_{Vi}) \end{aligned}$$

But the fan static pressure is

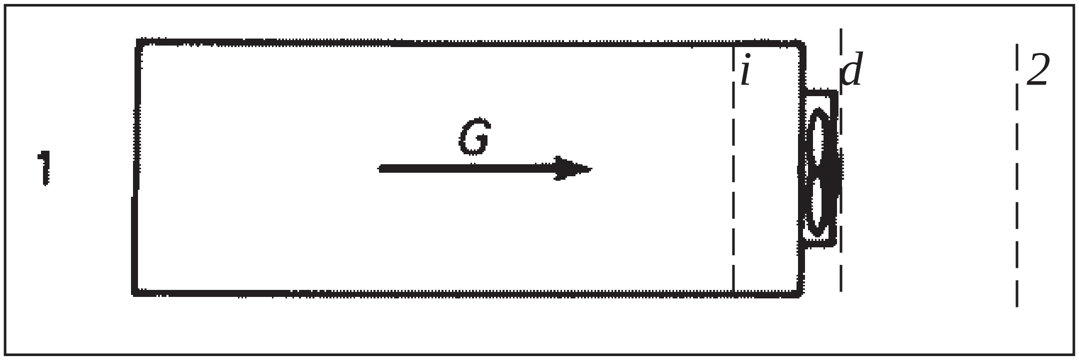
$$h_{fs} = h_{sd} - h_{si}$$

so that

$$h_{fs} = h_T - (h_{Vd} - h_{Vi})$$

$$h_{fs} = (h_{Td} - h_{Ti}) - (h_{Vd} - h_{Vi})$$

Details of Operating Point Conditions - A Negatively Pressurized System



From *1* to *i*

Bernoulli's equation with losses H_L

$$h_{T1} = h_{Ti} + H_L$$

$$h_{V1} + h_{S1} = h_{Ti} + H_L$$

$$h_{V1} \equiv 0, \quad h_{S1} \equiv 0$$

$$h_{Ti} = H_L \qquad (1)$$

From *d* to *2*

Bernoulli's equation with losses $H_{Ld\text{-}2}$

$$h_{Td} = h_{T2} + H_{Ld-2}$$

$$= h_{V2} + h_{S2} + H_{Ld-2}$$

$$h_{V2} \equiv 0, \quad h_{S2} \equiv 0$$

$$h_{Td} = H_{Ld-2} \qquad (2)$$

Substracting (1) from (2)

$$h_{Td} - h_{Ti} = H_{Ld-2} + H_L \qquad (3)$$

Substituting (3) into

$$h_{fs} = (h_{Td} - h_{Ti}) - (h_{Vd} - h_{Vi})$$
$$= H_{Ld-2} + H_L - (h_{Vd} - h_{Vi})$$

It will be shown later (when individual loss elements are intoduced) that for the infinite expansion from the fan discharge plane

$$H_{Ld-2} = h_{Vd}$$

Then

$$h_{fs} - h_{Vi} = H_L$$

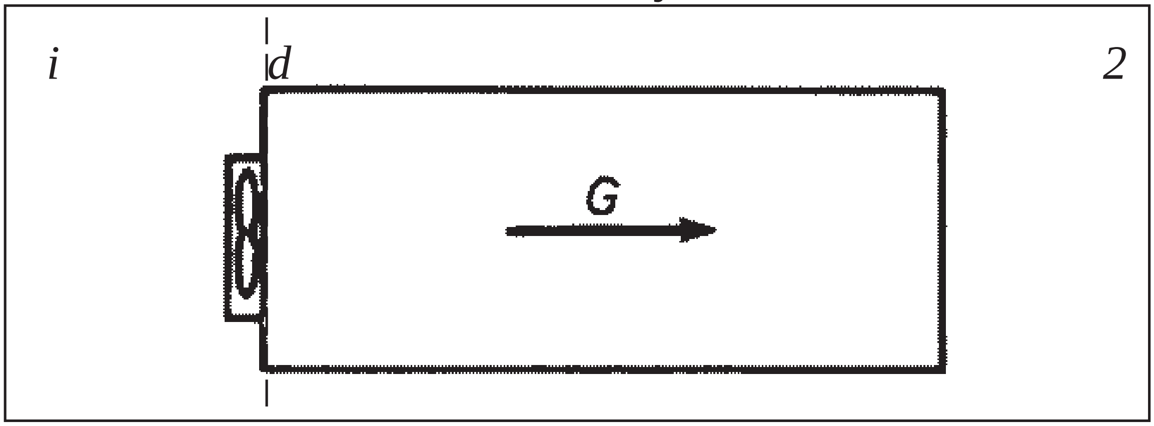
Neglecting h_{Vi} for most applications ($h_{Vi} \propto V_i^2$ where V_i is the smallest velocity in the system)

$$\boxed{h_{fs} = H_L} \qquad \textit{Exhaust System}$$

Note: For enclosures >> fan (>> means on a flow area basis) h_{Vi} is small and h_{fs}=H_L is a valid approximation.

For enclosures ≅ fan, then h_{fs}-h_{Vi}=H_L should perhaps be considered, but for most problems h_{Vi} is still small enough and h_{fs}=H_L is valid.

Details of Operating Point Conditions - A Positively Pressurized System



From *d* to *2*

Bernoulli's equation with losses H_L

$$\begin{aligned} h_{Td} &= h_{T2} + H_L \\ &= h_{V2} + h_{S2} + H_L \end{aligned}$$

$$h_{V2} \equiv 0, \quad h_{S2} \equiv 0$$

$$h_{Td} = H_L \tag{1}$$

But it was previously shown that

$$h_{fs} = (h_{Td} - h_{Ti}) - (h_{Vd} - h_{Vi})$$

so that $$h_{Td} = h_{fs} + h_{Ti} + (h_{Vd} - h_{Vi}) \tag{2}$$

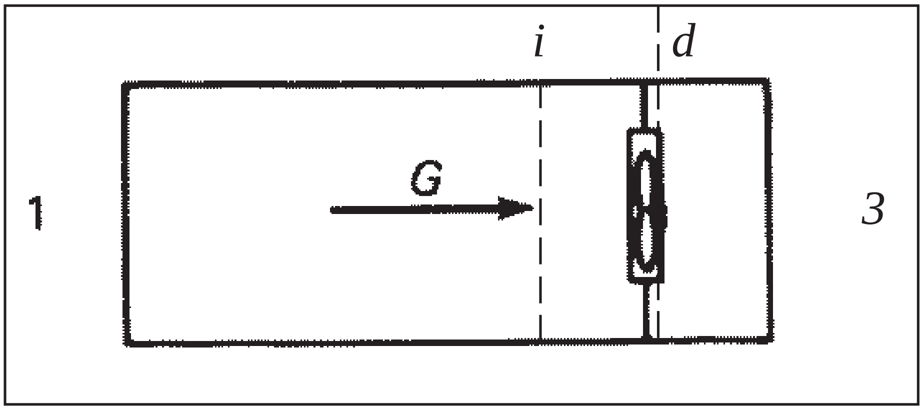
Combining (1) and (2)

$$\begin{aligned} h_{fs} + h_{Vd} &= H_L - h_{Ti} + h_{Vi} \\ &= H_L - (h_{Vi} + h_{Si}) + h_{Vi} \\ &= H_L - h_{Si} \end{aligned}$$

and since $h_{Si}=0$

$$\boxed{h_{fs} + h_{Vd} = H_L}$$ *Blower System*

Details of Operating Point Conditions - A Positively/ Negatively Pressurized System (Intermediate Fan)



From *1* to *i*

Bernoulli's equation with losses $H_{L1\text{-}i}$

$$
\begin{aligned}
h_{T1} &= h_{Ti} + H_{L1-i} \\
h_{V1} + h_{S1} &= h_{Ti} + H_{L1-i} \\
h_{V1} \equiv 0, &\quad h_{S1} \equiv 0 \\
h_{Ti} &= -H_{L1-i} \qquad (1)
\end{aligned}
$$

From *d* to *3*

Bernoulli's equation with losses $H_{Ld\text{-}3}$

$$
\begin{aligned}
h_{Td} &= h_{T3} + H_{Ld-3} \\
&= h_{V3} + h_{S3} + H_{Ld-3} \\
h_{V3} \equiv 0, &\quad h_{S3} \equiv 0 \\
h_{Td} &= H_{Ld-3} \qquad (2)
\end{aligned}
$$

Subtracting (1) from (2)

$$h_{Td} - h_{Ti} = H_{Ld-3} - (-H_{L1-i})$$
$$= H_L \qquad (3)$$

i.e. H_L is the sum of all losses <u>up to</u>, but not including fan inlet, plus all losses from fan discharge plane to system exit.

But

$$h_{fs} = h_T - (h_{Vd} - h_{Vi})$$
$$= h_{Td} - h_{Ti} - (h_{Vd} - h_{Vi})$$
$$h_{Td} - h_{Ti} = h_{fs} + (h_{Vd} - h_{Vi}) \qquad (4)$$

Combining (3) and (4)

$$h_{fs} + (h_{Vd} - h_{Vi}) = H_L$$

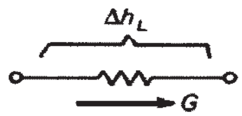
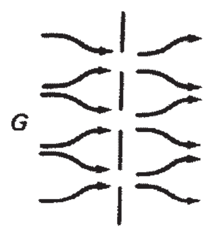
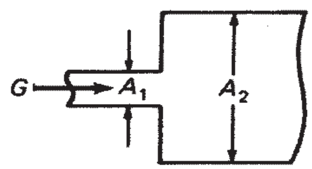
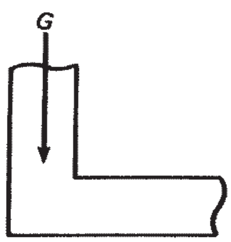
Neglecting h_{Vi} for most applications ($h_{Vi} \propto V_i^2$, where V_i is the smallest velocity in the system)

$$\boxed{h_{fs} + h_{Vd} = H_L} \qquad \textit{Intermediate Fan}$$

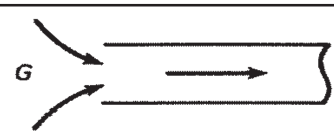
Note: For enclosures >> fan, $h_{Vd} >> h_{Vi}$ and $h_{fs}+h_{Vd}=H_L$ is a valid approximation.

For enclosures $\cong$ fan, $h_{Vd} \cong h_{Vi}$ and $h_{fs}=H_L$ is the most appropriate.

Common Elements

(a)		Basic element	R
(b)		Perforated or slotted plate	$2.4 \times 10^{-3}/A^2$
		Recommended is $R = 2.0x10^{-3} / A_f^2$	
(c)		Sudden expansion	$1.290 \times 10^{-3}\left[\frac{1}{A_1}\left(1-\frac{A_1}{A_2}\right)\right]^2$
(d)		Sharp cornered turn	$1.81 \times 10^{-3}/A^2$

Not recommended

(e)		Contraction	$1.2 \times 10^{-3}/A^2$

Recommended

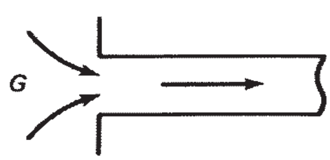
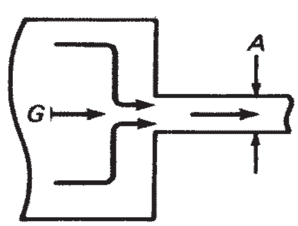
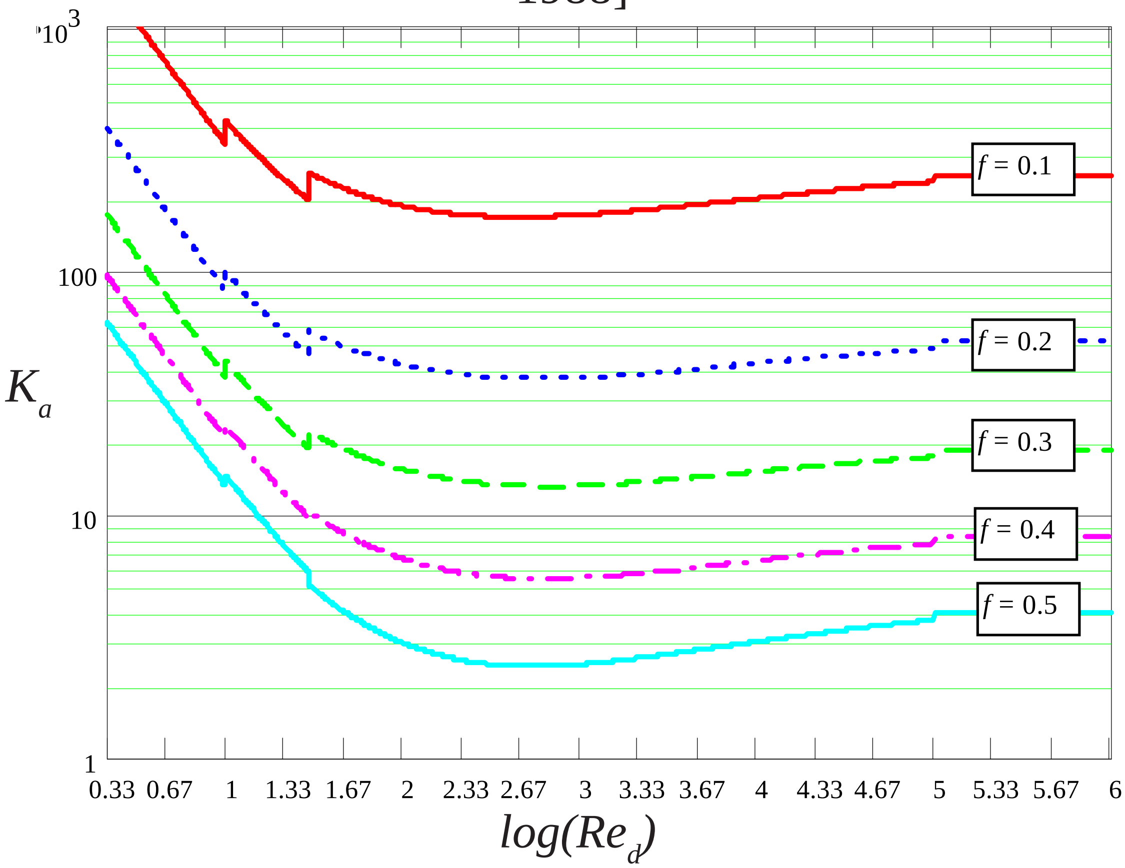
(f)		Contraction	$0.63 \times 10^{-3}/A^2$
(g)		Contraction	$0.63 \times 10^{-3}/A^2$

Fig. 6-9. Airflow resistance formulae.

Additional Detail on Perforated Plate Resistance

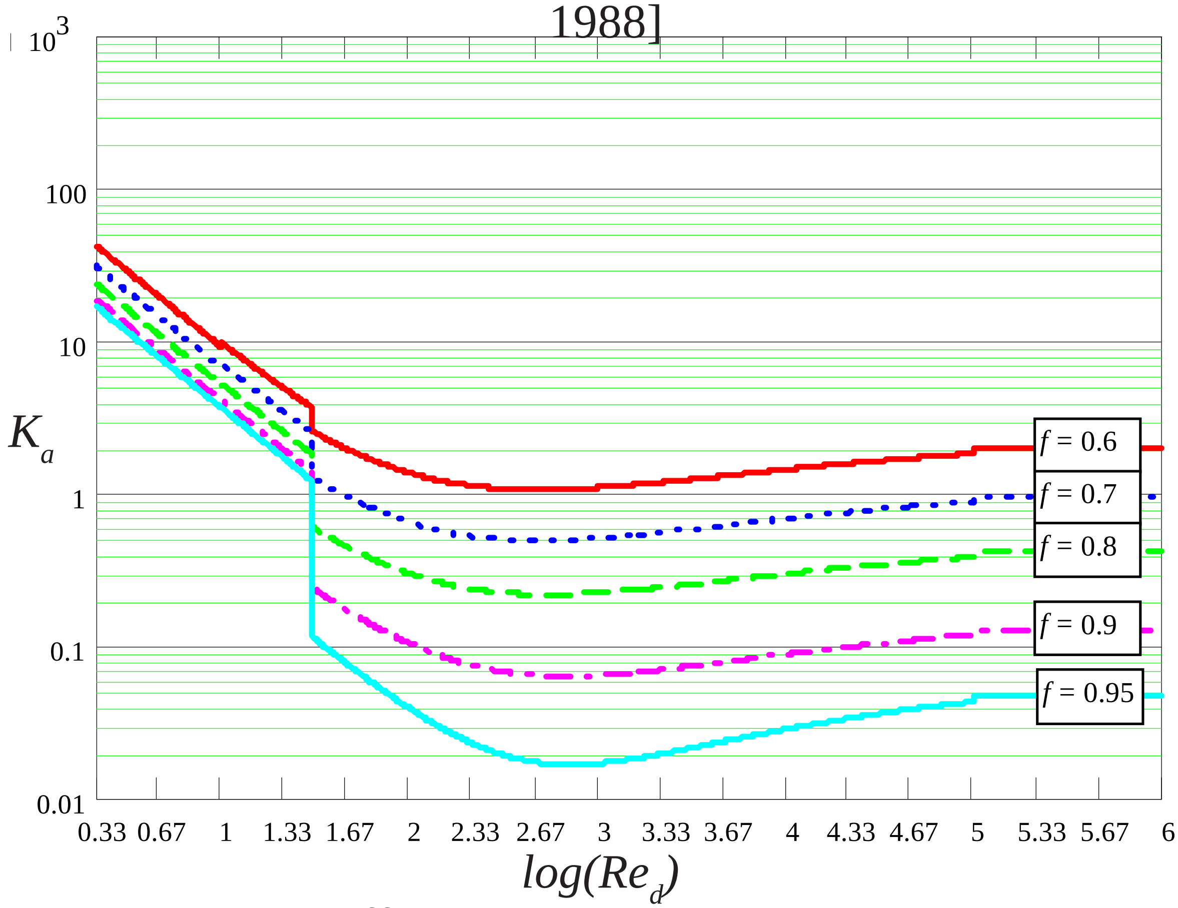
[plotted results from Adam, Johannes, 1998 based on Idelchik, 1988]



Pressure drop coefficient K_a (based on approach velocity) of perforated plates function of *log(Re$_d$)* (Reynold's number $Re_d = V_d d/\upsilon$, d=hole diameter, V_d= velocity of air in holes) and free area ratio f, for $0.1 \le f \le 0.5$. Note the logarithmic scaling on the vertical axis. f = free area ratio (f = total hole area/total plate area) and $K_d = K_a f^2$. Also $\mathrm{Re}_d < 10$, $K_d = 30/\mathrm{Re}_d$. An airflow resistance R is calculated by multiplying K_d times the resistance of one velocity head, i.e. $R_{Perf} = K_d\left[1.29x10^{-3}/A_f^2\right]$. The Reynold's number is calculated using the "device velocity" or $V = G/A_f$ and *d=hole diameter.*

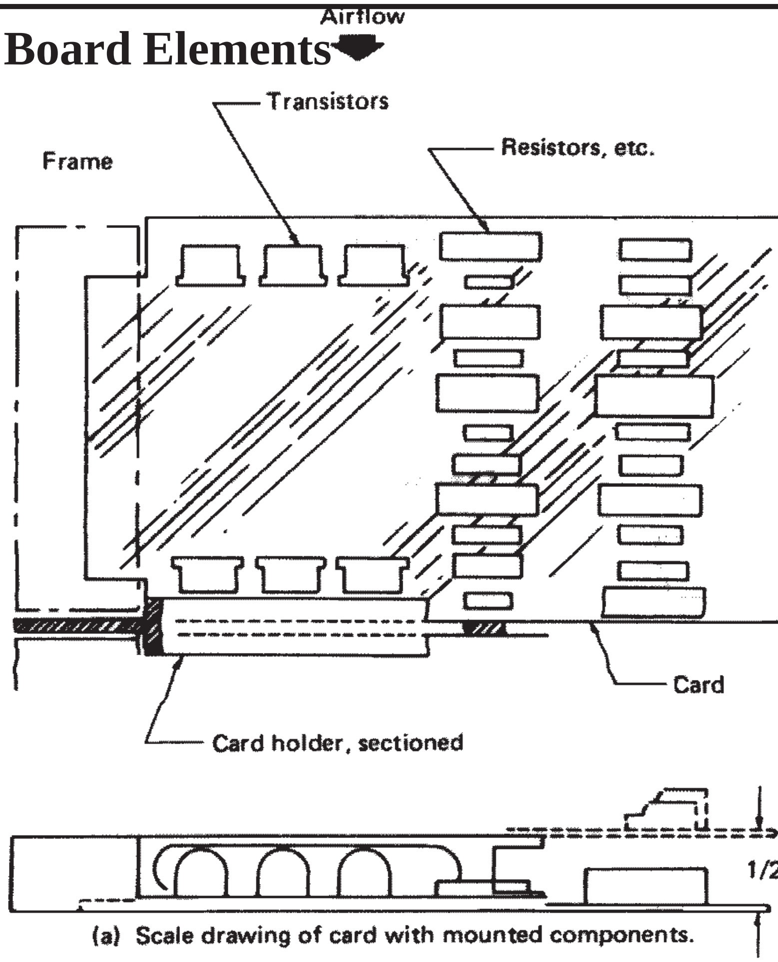
Additional Detail on Perforated Plate Resistance - Continued

[plotted results from Adam, Johannes, 1998 based on Idelchik, 1988]

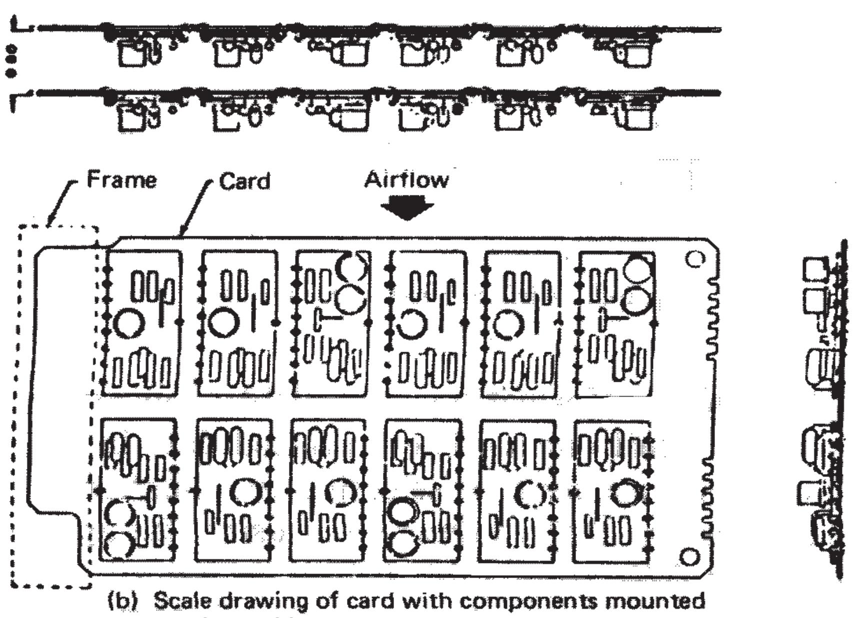


Pressure drop coefficient K_a (based on approach velocity)of perforated plates function of $log(Re_d)$ (Reynold's number $Re_d = V_d d/\upsilon$, d=hole diameter, V_d= velocity of air in holes) and free area ratio f, for $0.6 \le f \le 0.95$. Note the logarithmic scaling on the vertical axis. f = free area ratio (f = total hole area/total plate area) and $K_d = K_a f^2$. Also $\mathrm{Re}_d < 10$, $K_d = 30/\mathrm{Re}_d$. An airflow resistance R is calculated by multiplying K_d times the resistance of one velocity head, i.e. $R_{Perf} = K_d\left[1.29x10^{-3}/A_f^2\right]$. The Reynold's number is calculated using the "device velocity" or $V = G/A_f$ and *d=hole diameter.*.

True Circuit Board Elements

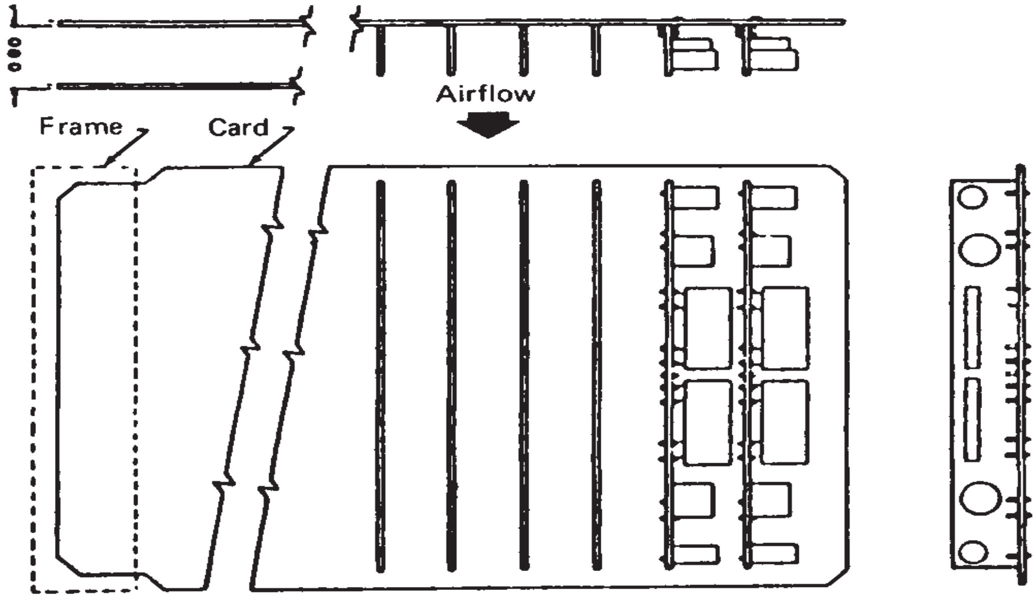


(a) Scale drawing of card with mounted components.



(b) Scale drawing of card with components mounted on elevated base.

Fig. 6-10. Circuit card geometry referring to Table 6-1. Reprinted from [28]. Author: Donald Hay, McLean Engineering Division of Zero Corporation, Princeton Junction, N.J. 08550.



(c) Scale drawing of card with components mounted on perpendicular daughter boards.

Fig. 6-10. (*Continued*)

Table 6-1. Circuit card airflow resistance formulae referred to Fig. 6-10. Reprinted from [28]. Author: Donald Hay, McLean Engineering Division of Zero Corporation, Princeton Junction, N.J. 08550.

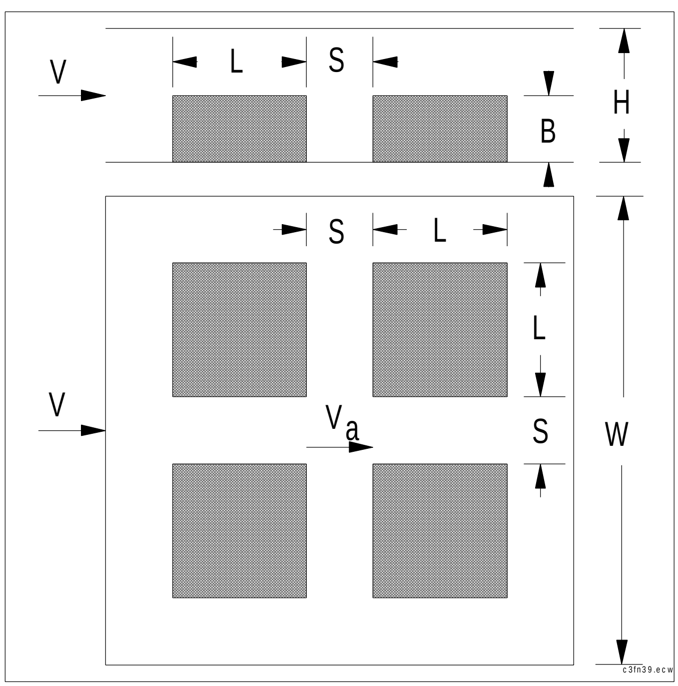
Card Geometry	Reference Figure 6-10	Free Passage	Card Spacing (in.)	R_L Formulae
Childless	a	62%	½	$1.35nL10^{-3}(1/A)^{(2.00-0.03n)}$
Childless	a	81%	1	$3.08nL10^{-4}(1/A)^{(2.00-0.01n)}$
Childless	a	70%*	½	$1.93nL10^{-3}(1/A)^{(2.00-0.03n)}$
// daughter	b	74%	0.80	$1.95nL10^{-3}(1/A)^2$
// daughter	b	87%	1.60	$1.43nL10^{-3}(1/A)^2$
⊥ daughter	c	58%	0.80	$5.18nL10^{-4}(1/A)^2$
⊥ daughter	c	79%	1.60	$3.24nL10^{-4}(1/A)^2$

*This formulae includes the pressure drop caused by the card holder while this is omitted in those above.
n = no. of card rows through which air flows.
L = card dimension (in.) parallel to flow.
A = total cross-sectional area (in.^{2}) at entrance including card edges.

Modeled Circuit Board Elements

Teertstra, et al. 1997 presented an analytical model that predicts pressure loss for fully developed flow for air in a parallel plate channel with an array of uniformly sized and spaced cuboid blocks on one wall. They used a composite solution, based on the laminar and turbulent smooth wall channel limiting cases. The results are offered in the form of a friction factor that is applicable to a full range of Reynold's numbers. They quote an accuracy to within 15 % of other authors experimental data.

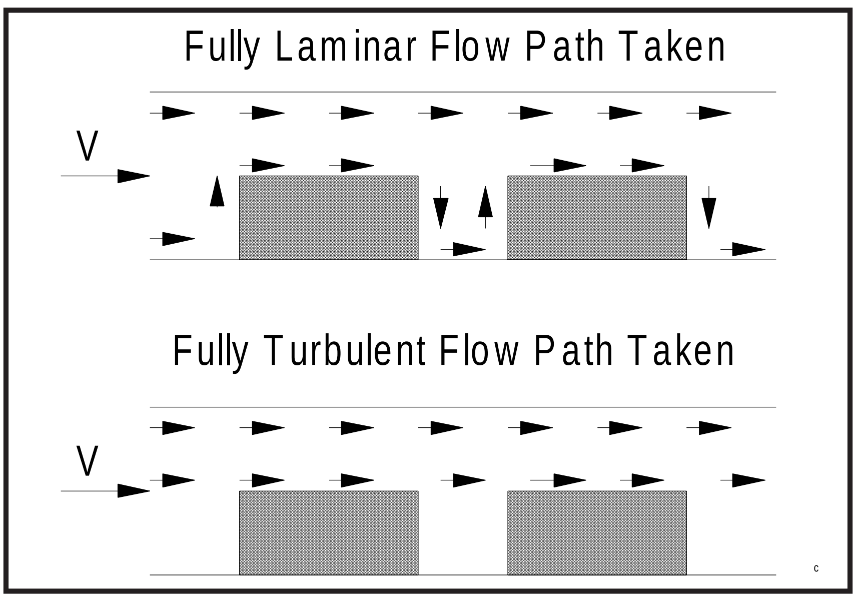
Although some confusion is possible, a notation nearly identical to that of Teerstra is used to permit easier study of the original article, which is highly recommended, particularly because this writer believes a typographical error exists in Teerstra's definition of the friction factor f_{D_h} for the array. The following geometry is used:



Some noteworthy comments:

1. V = approach velocity
2. V_a= "array" velocity, where in this case, array is based on airflow both above and between the cuboids.
3. An array of uniformly sized and spaced cuboid blocks is assumed. The user must determine the block size and spacing that best characterizes the actual values.
4. The model assumes fully developed flow and does not include pressure loss due to entrance effects or developing flow in the first few rows.
5. The model is assumes an infinitely wide channel.

The mathematical model constructed by Teerstra uses only simple algebra. In addition to constructing a composite solution from laminar and turbulent asymptotes, the friction path length is calculated from a total "in and out" path for fully laminar flow, a "top-surface" only path for fully turbulent flow, and a *B/H* biasing for intermediate flow (see following illustration).



A dimensionless channel friction factor f_{2H} is used:

$$f_{2H} = \frac{-\left(\frac{dp}{dx}\right)2H}{\frac{1}{2}\rho V^2}$$

where x is the straight-through flow path.

The study concludes with

$$f_{2H} = \left[\left(\frac{96\mathrm{A}}{\mathrm{Re}_{2H}}\right)^3 + \left(\frac{0.347\mathrm{B}}{\mathrm{Re}_{2H}^{1/4}}\right)^3\right]^{1/3}, \ \mathrm{Re}_{2H} = \frac{V2H}{\upsilon} \qquad (a)$$

which is a fit to the laminar (first term in parentheses if A=1) and turbulent (second term in parentheses if B=1) asymptotes.

$$\mathrm{A} = \frac{\gamma^2}{\varsigma^3\chi}, \ \mathrm{B} = \frac{\gamma^{5/4}}{\varsigma^3\xi}$$

$$\gamma = 1 + \frac{B}{H}\frac{H}{L}\frac{L}{L+S}, \ \varsigma = \left[1 - \frac{B}{H}\frac{L}{L+S}\right] \qquad (b), (c)$$

$$\chi = \left[\frac{B}{H} + \left(1 - \frac{B}{H}\right)\left(1 + \frac{2B}{H}\frac{H}{L}\frac{L}{L+S}\right)\right] \qquad (d)$$

$$\xi = \frac{B}{H} + \left(1 - \frac{B}{H}\right)\frac{L}{L+S} \qquad (e)$$

The pressure gradient is

$$\frac{dp}{dx} = -\frac{f_{2H}\left(\frac{1}{2}\rho V^2\right)}{2H}$$

Assuming that the pressure loss is uniform for the entire card length, L_{Card}, the pressure gradient is

$$\frac{\Delta p}{L_{Card}} = -\frac{f_{2H}\left(\frac{1}{2}\rho V^2\right)}{2H}$$

Keeping in mind that entrance pressure loss effects are not included,

$$\left|\frac{\Delta p}{\frac{1}{2}\rho V^2}\right| = \frac{L_{Card}}{2H} f_{2H}$$

Since $(1/2)\rho V^2$ is one velocity head (h_V), based on approach velocity,

$$\Delta h_{Card} = \frac{L_{Card}}{2H} f_{2H} h_V$$

The card pressure head loss in units of *in.* H_2O, based on an inlet area, i.e. card width *x* card-to-card spacing, $A=WH$, is

$$\Delta h_{Card} = \frac{1.29x10^{-3}}{A^2} \frac{L_{Card}\, f_{2H}}{2H} G^2 \text{ using definitions (a) - (e)}$$

and

$$R_{Card} = \frac{1.29x10^{-3}}{A^2} \frac{L_{Card}\, f_{2H}}{2H}$$

Note: Do not confuse the above area A with Teerstra's $\mathrm{A} = \gamma^2 / (\varsigma^3 \chi)$.

Combining Elements

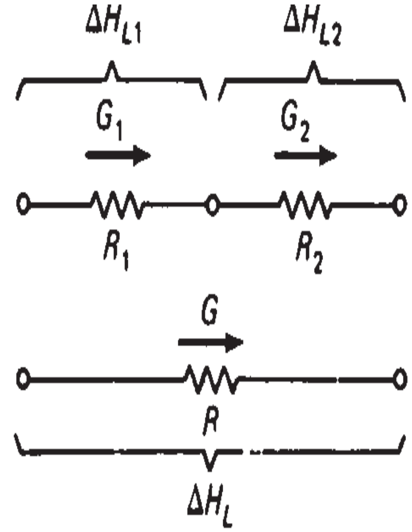


Fig. 6-11. Series addition of airflow resistances.

Hence:

$$\boxed{R = R_1 + R_2}$$

E6.10
Series airflow elements

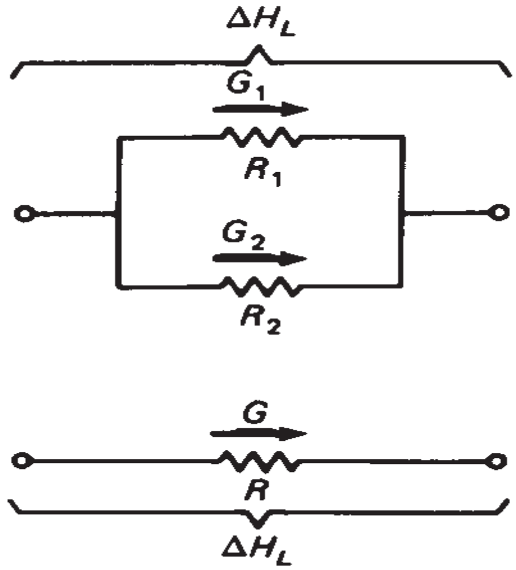


Fig. 6-12. Parallel addition of airflow elements.

$$\frac{1}{\sqrt{R}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}}$$

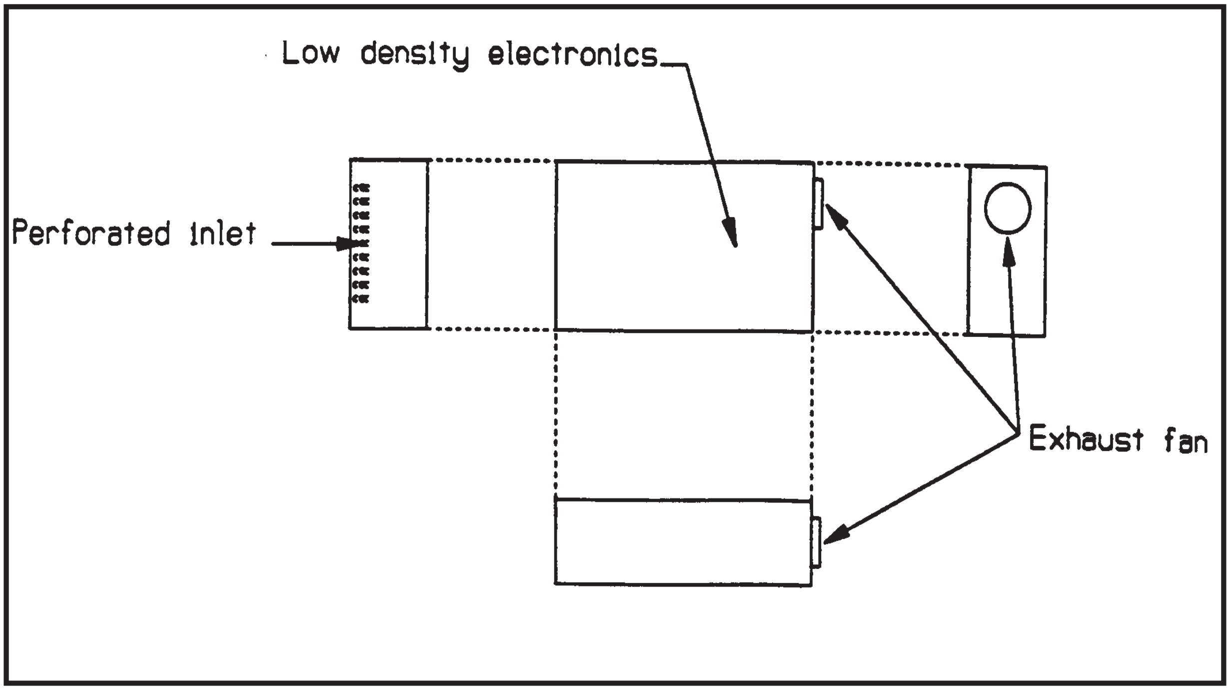
E6.11
Parallel airflow elements

$$\frac{G_2}{G} = \begin{cases} \dfrac{1}{1+\sqrt{\dfrac{R_2}{R_1}}} \\[2ex] \sqrt{\dfrac{R}{R_2}} \end{cases}$$

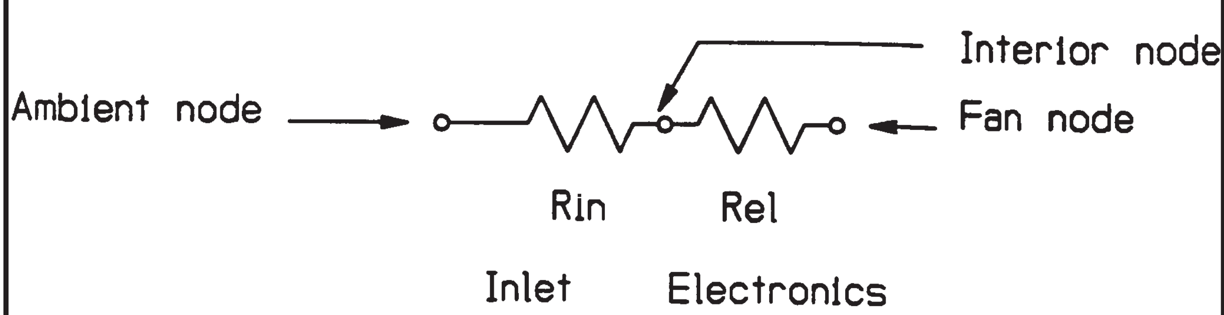
E6.12
Branch airflow in parallel circuit

Example

Simple Negatively Pressurized Enclosure



Airflow Model



Pressure Loss Calculations

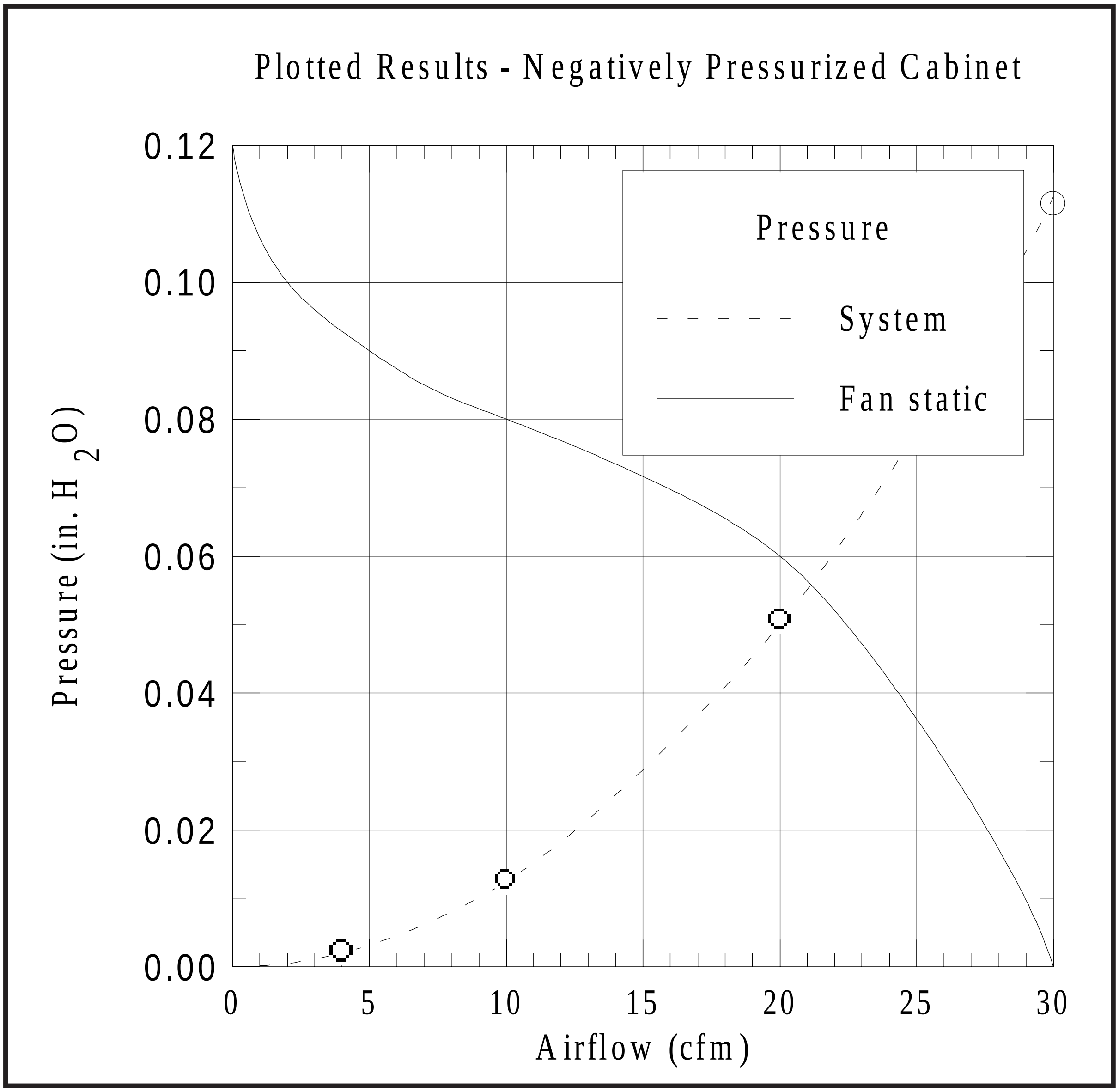
$$R_{in} = \frac{2.0x10^{-3}}{A^2} = \frac{2.0x10^{-3}}{(10\ in. x1.0\ in. x0.4)^2}$$

$$= 1.25x10^{-4}\ in. H_2O / (cfm)^2$$

$$R_{el} = \text{small, i.e. } 0$$

$$H_L = (R_{in} + R_{el})G^2 = 1.25x10^{-4} G^2$$

$G\ [cfm]$	$H_L\ [in.\ H_2O]$
1	$1.25x10^{-4}$
2	$5.0x10^{-4}$
4	$2.0x10^{-3}$
10	$1.25x10^{-2}$
20	0.05
30	0.1125



○ Indicates calculated system pressure

System Operating Point is at $G = 21\,cfm$.

Remembering that we calculated the perforated plate resistances (the only resistances considered) using

$$R = \frac{2.0x10^{-3}}{A_f^2}$$

we shall see what the result would be using Adam, Fried and Idelchick correlation:

$$V_d = \frac{G}{A_f} = \frac{21cfm}{\frac{10\,in.\,x1.0\,in.\,0.4}{144}} = 756\ ft/\text{min.}$$

$$\text{Re}_d = \frac{V_d d}{5\upsilon} = \frac{(756\ ft/\text{min.})(0.188\,in.)}{5(0.023)} = 1236$$

The loss coefficient plots indicate $K_d = K_a f^2 = (5.5)(0.4)^2 = 0.88$ for perforated plate resistances based on $R = K_d\left(1.29x10^{-3}/A_f^2\right)$. This implies

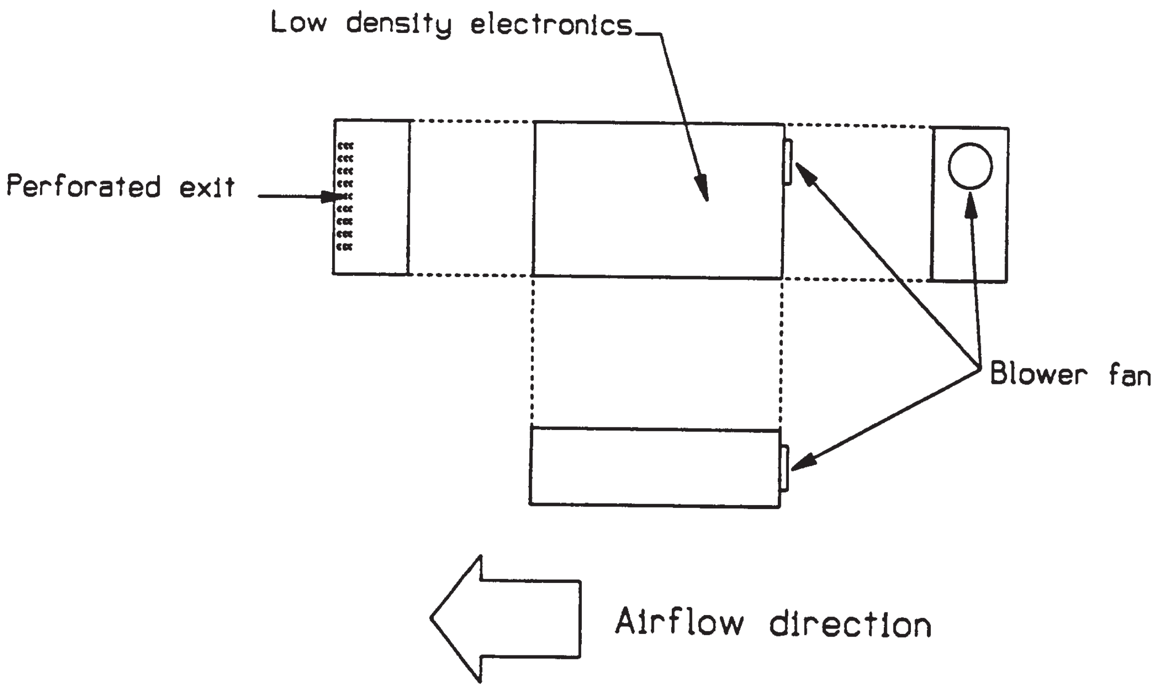
$$R = K_d\left(1.29x10^{-3}/A_f^2\right) = (0.88)\left(1.29x10^{-3}/A_f^2\right) = 1.14x10^{-3}/A_f^2$$

or about one-half of the calculated system pressure loss, which upon examination of the pressure plots, imples a total system airflow of $24\,cfm$. For this problem, we shall stay with the more conservative result of *21 cfm*. If the internal dissipation is 100 *W*, the overall air temperature rise is

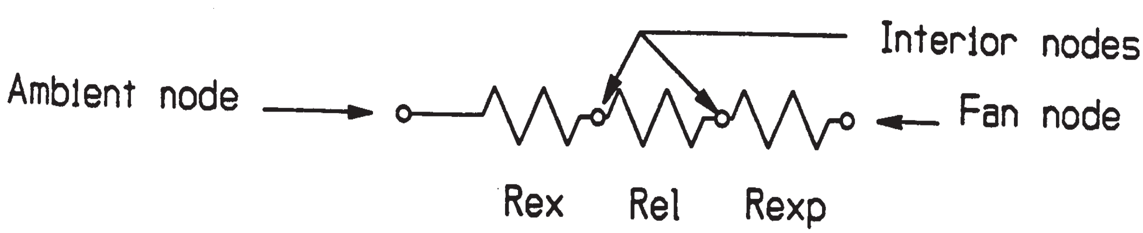
$$\Delta T = 1.76\frac{Q}{G} = 1.76\frac{(100\ W)}{(21\,cfm)} = 8\ {}^oC$$

well mixed air temperature rise in the cabinet.

Application Example:
Simple Positively Pressurized Enclosure



Airflow Model



Rex = exit resistance
Rel = electronics resistance
Rexp = fan expansion resistance

Pressure Loss Calculations

$$R_{\exp} = 1.29x10^{-3}\left[\frac{1}{A_1}\left(1-\frac{A_1}{A_2}\right)\right]^2$$

$$A_1 = \text{fan discharge area}$$

$$= \pi\frac{d^2}{4} = \pi\frac{(3.5\ in.)^2}{4}$$

$$= 9.62\ in.^2$$

$$A_2 = \text{downstream area}$$

$$= 10\ in.\ x 5\ in.$$

$$= 50\ in.^2$$

$$R_{\exp} = 1.29x10^{-3}\left[\frac{1}{9.62}\left(1-\frac{9.62}{50}\right)\right]^2$$

$$= 9.09x10^{-6}$$

$$R_{el} = 0$$

$$R_{ex} = 1.25x10^{-4} \quad \text{as before}$$

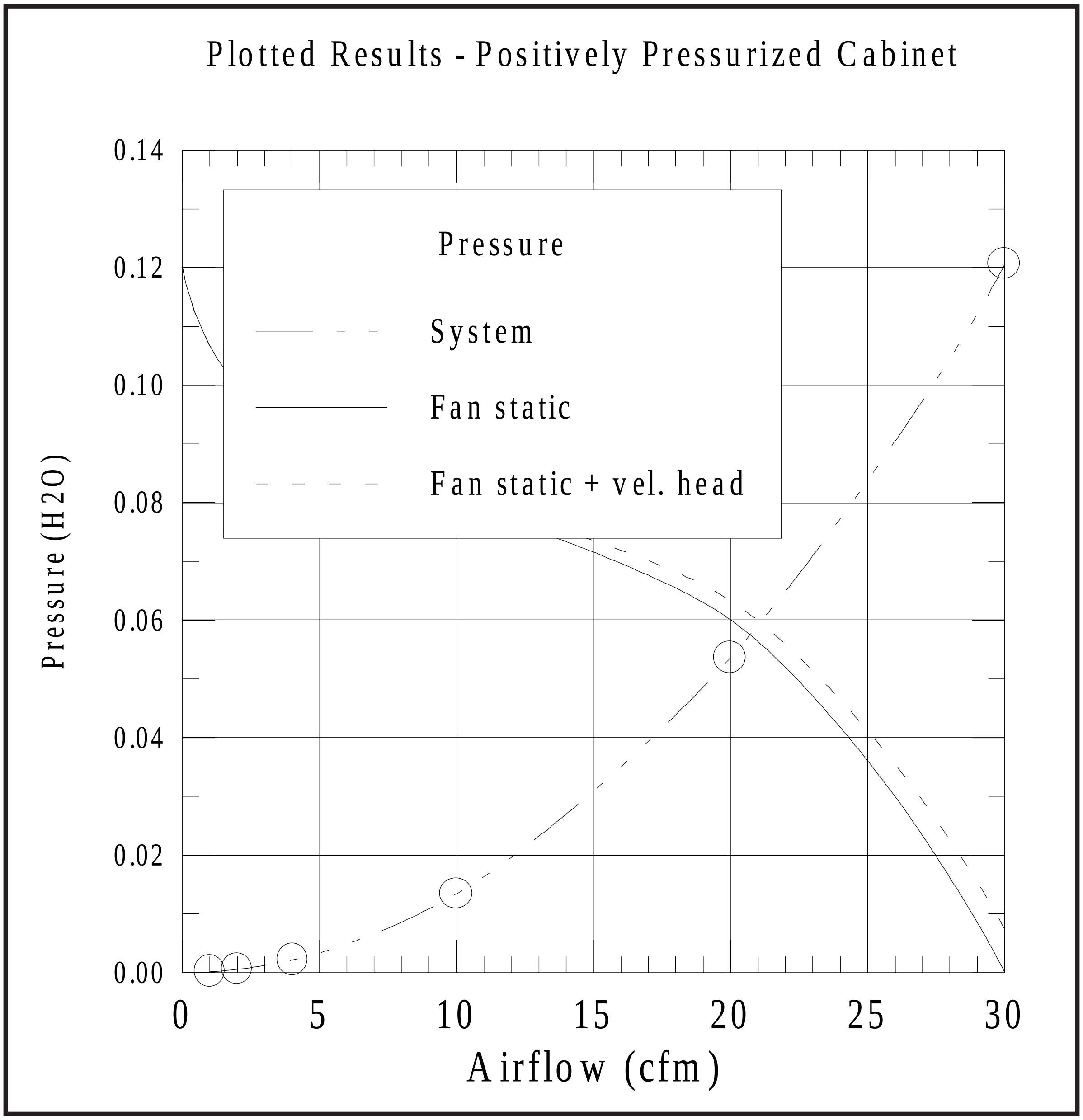
$$H_L = \left(R_{\exp} + R_{el} + R_{ex}\right)G^2$$
$$= \left(9.09x10^{-6} + 0 + 1.25x10^{-4}\right)G^2$$
$$= 1.34x10^{-4}G^2$$

Remember velocity pressure

$$h_v = 1.29x10^{-3}\frac{G^2}{A^2} = 1.29x10^{-3}\frac{G^2}{\left[\pi\frac{(4.0\ in.)^2}{4}\right]^2}$$

$$= 8.2x10^{-6}G^2$$

$G\ [cfm]$	$H_L\ (in.\ H_2O)$	$h_v\ [in.\ H_2O]$
1	$1.34x10^{-4}$	$8.20x10^{-6}$
2	$5.36x10^{-4}$	$3.28x10^{-5}$
4	$2.14x10^{-3}$	$1.31x10^{-4}$
10	$1.34x10^{-2}$	$8.20x10^{-4}$
20	$5.36x10^{-2}$	$3.28x10^{-3}$
30	$1.21x10^{-1}$	$7.38x10^{-3}$



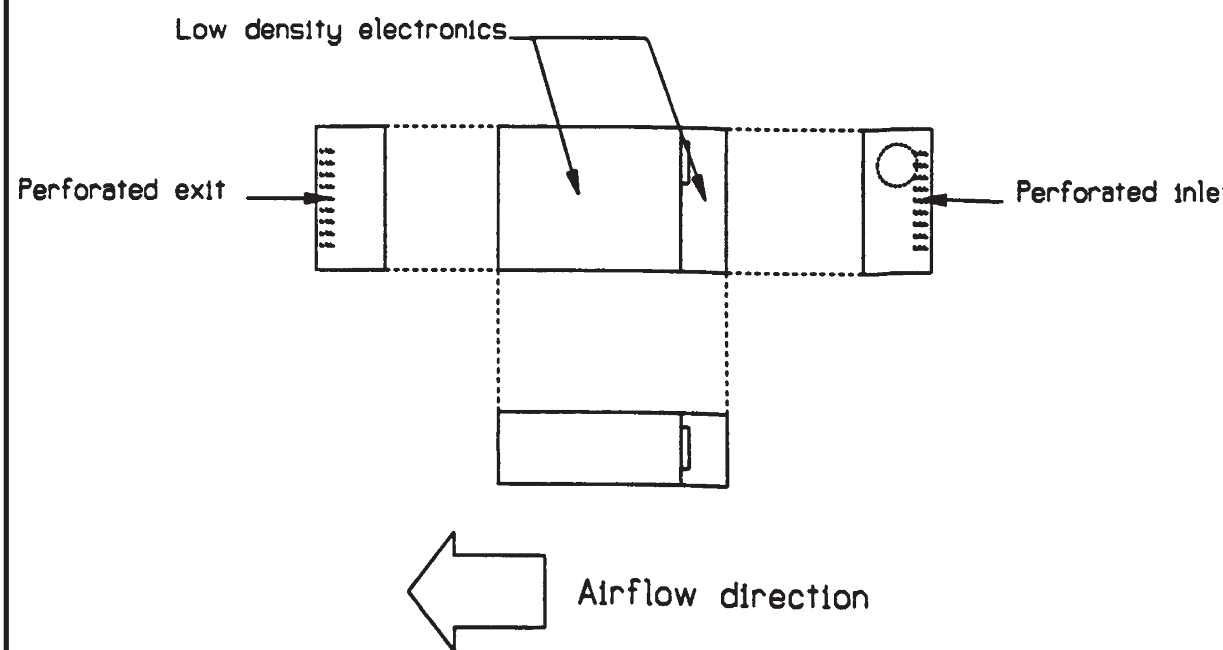
○ Indicates calculated system pressure

System Operating Point Still Appears to be about

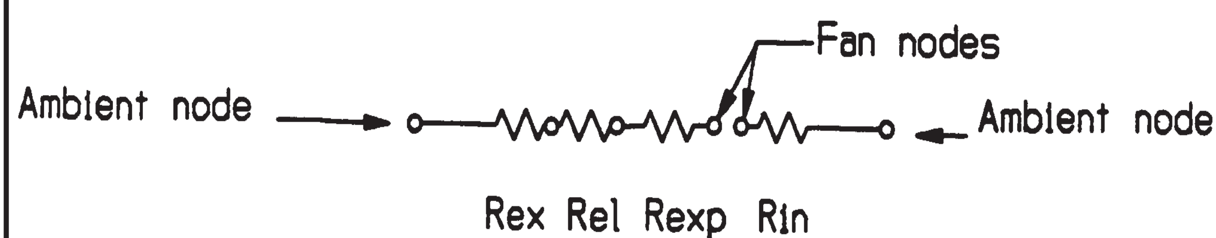
$$G = 21\,cfm$$

when $R = 2.0x10^{-3} / A_f^2$ is used for the perforated plate resistance. There is no need to re-calculate the results using the perforated plate loss factor of Adam, Fried and Idlechick because it would be exactly the same as was obtained for the negatively pressurized enclosure, i.e. *G = 24 cfm*.

Application Example:
Simple Positively/Negatively (Intermediate Fan) Pressurized Enclosure



Airflow Model



Rex - exit resistance
Rel- electronics resistance
Rexp - fan expansion resistance
Rin - inlet resistance

Pressure Loss Calculations

$$R_{in} = 1.25x10^{-4} \text{ as before}$$

$$R_{\exp} = 9.09x10^{-6} \text{ as before}$$

$$R_{el} = 0 \text{ as before}$$

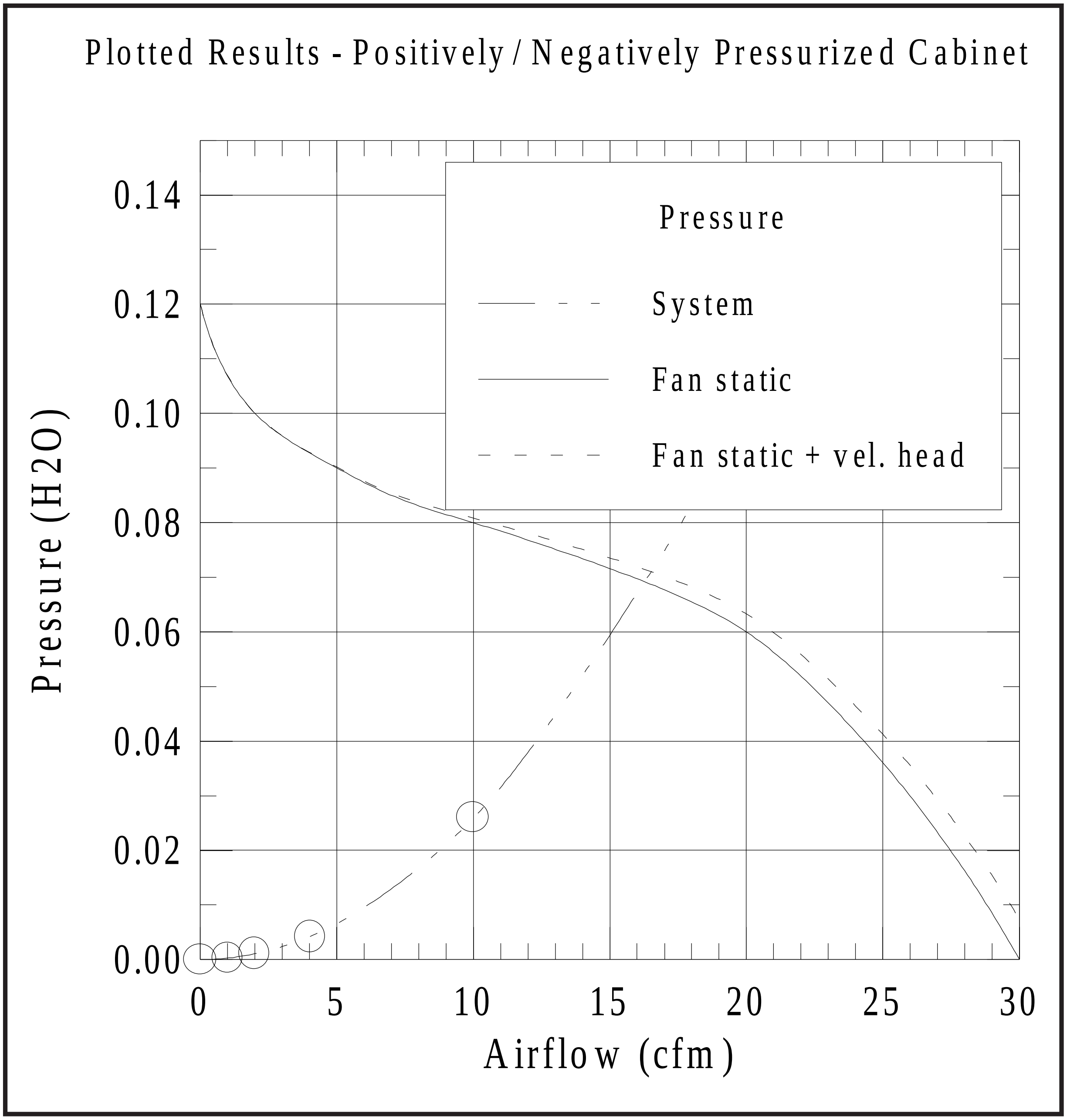
$$R_{ex} = 1.25x10^{-4} \text{ as before}$$

$$H_L = \left(R_{in} + R_{\exp} + R_{el} + R_{ex}\right)G^2$$

$$= \left(1.25x10^{-4} + 9.09x10^{-6} + 0 + 1.25x10^{-4}\right)G^2$$

$$= 2.59x10^{-4}G^2$$

G [cfm]	H_L [$in.\ H_2O$]	h_v [$in.\ H_2O$] as before
1	$2.59x10^{-4}$	$8.2x10^{-6}$
2	$1.04x10^{-3}$	$3.28x10^{-5}$
4	$4.15x10^{-3}$	$1.31x10^{-4}$
10	$2.59x10^{-2}$	$8.20x10^{-4}$
20	0.1036	$3.28x10^{-3}$
30	0.2330	$7.38x10^{-3}$



○ Indicates calculated system pressure

System Operating Point is at $G = 16\ \ cfm$ and the well mixed air temperature rise at the enclosure exit is

Remembering that we calculated the perforated plate resistances (the only resistances considered) using

$$R = \frac{2.0x10^{-3}}{A_f^2}$$

we shall see what the result would be using Adam, Fried and Idelchick correlation:

$$V_d = \frac{G}{A_f} = \frac{16\,cfm}{\frac{10\,in.\,x\,1.0\,in.\,0.4}{144}} = 576\ ft/\text{min.}$$

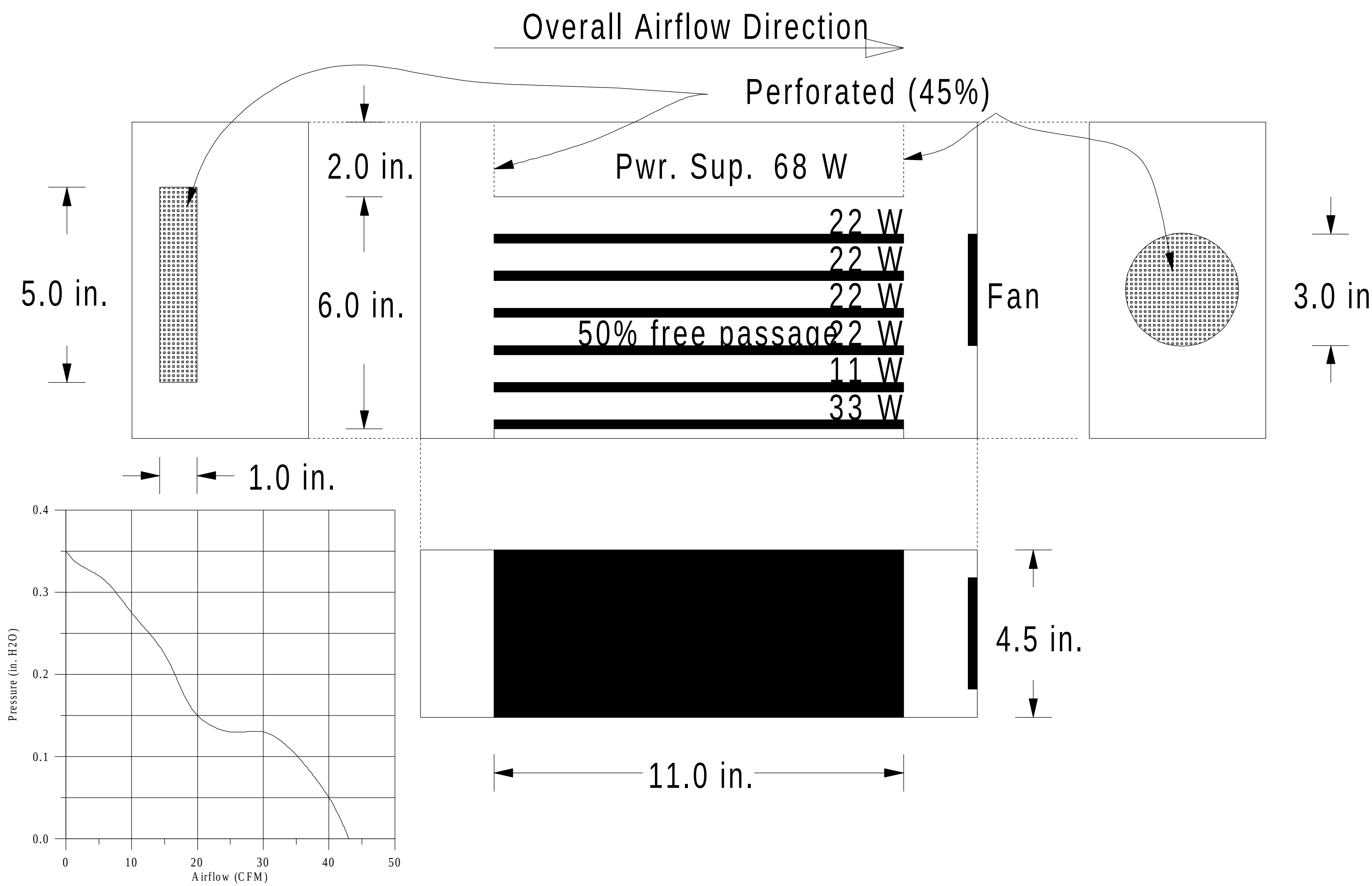
$$\text{Re}_d = \frac{V_d d}{5\upsilon} = \frac{(576\ ft/\text{min.})(0.188\,in.)}{5(0.023)} = 942$$

The loss coefficient plots indicate $K_d = K_a f^2 = (5.5)(0.4)^2 = 0.88$ for perforated plate resistances based on $R = 1.29x10^{-3} / A_f^2$. This suggests implies one-half of the calculated system pressure loss, which upon examination of the pressure plots, imples a total system airflow of $20\,cfm$. For this problem, we shall stay with the more conservative result of *16 cfm*. If the internal dissipation is 100 *W*, the overall well mixed air temperature rise is

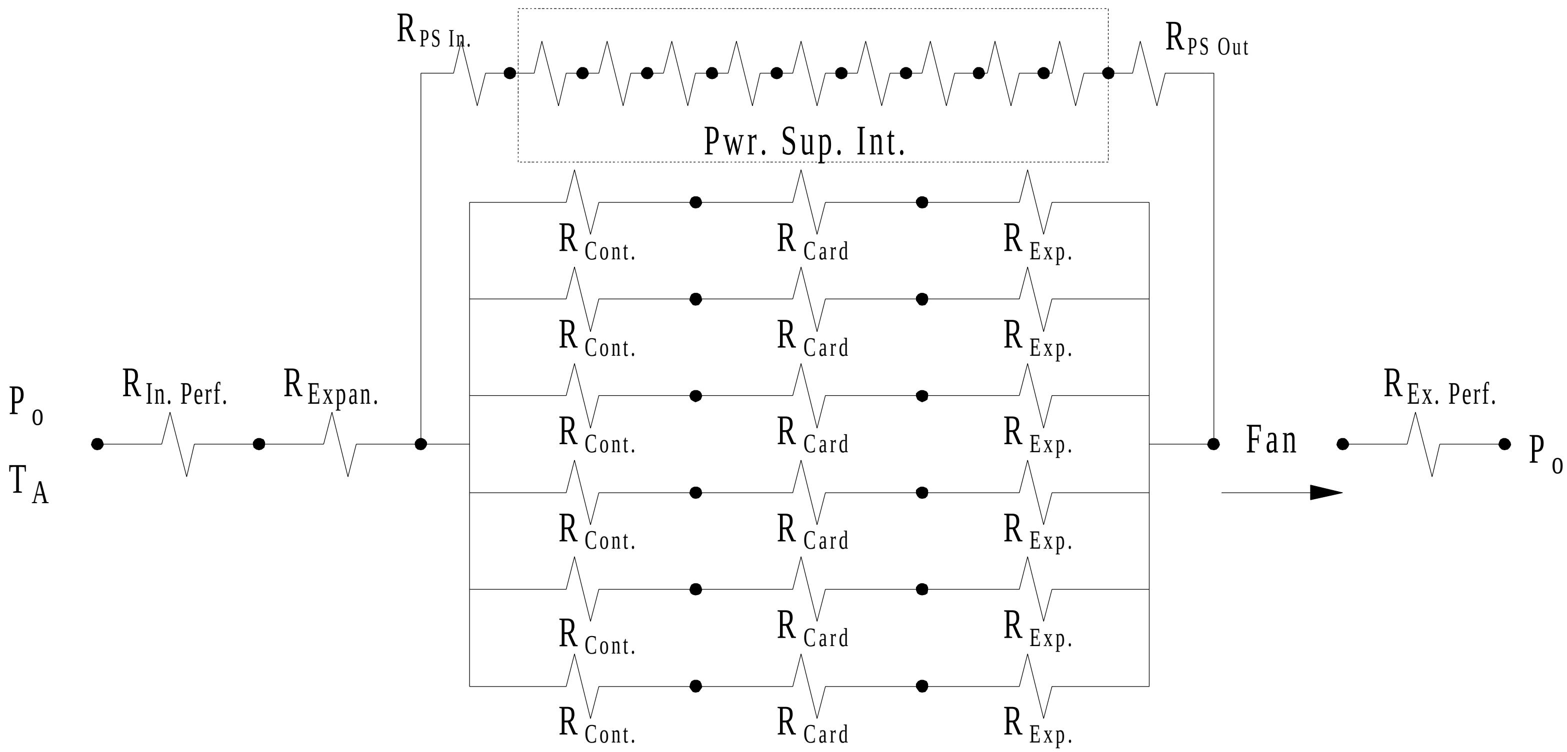
$$\Delta T = 1.76\frac{Q}{G} = 1.76\frac{(100\ W)}{(16\ cfm)} = 11\ {}^oC$$

Example

A More Complex Enclosure



Airflow Circuit



Near Inlet

$$R_{IntPerf} = \frac{2.0x10^{-3}}{A_f^2} = \frac{2.0x10^{-3}}{(5.0\,x\,1.0\,x\,0.45)^2} = 3.95x10^{-4}$$

$$R_{Expan} = 1.29x10^{-3}\left[\frac{1}{A_1}\left(1-\frac{A_1}{A_2}\right)\right]^2$$

$$= 1.29x10^{-3}\left[\frac{1}{5.0\,x\,1.0}\left(1-\frac{5.0\,x\,1.0}{4.5\,x\,8.0}\right)\right]^2 = 3.83x10^{-5}$$

Circuit Boards Taken *One At a Time* (All 6 Boards Could Be Modeled As One Resistor)

$$R_{Cont} = \frac{0.63x10^{-3}}{A_f^2} = \frac{0.63x10^{-3}}{(4.5x1.0x0.5)^2} = 1.24x10^{-4}$$

The best circuit board formula seems to be

$$R_{Card} = \frac{5.18nLx10^{-4}}{A^2} = \frac{5.18(1)(11.0)x10^{-4}}{(4.5x1.0)^2} = 2.81x10^{-4}$$

$$R_{Exp} = 1.29x10^{-3}\left[\frac{1}{A_1}\left(1-\frac{A_1}{A_2}\right)\right]^2$$

$$= 1.29x10^{-3}\left[\frac{1}{4.5x1.0x0.5}\left(1-\frac{4.5x1.0x0.5}{4.5x1.0}\right)\right]^2$$

$$= 6.37x10^{-5}$$

Card Cage: R_{CC}

$$R_{Brd} = R_{Cont} + R_{Card} + R_{Exp} = 1.24x10^{-4} + 2.8x10^{-4}$$
$$+6.37x10^{-5}$$
$$= 4.68x10^{-4}$$

$$\frac{1}{\sqrt{R_{CC}}} = \frac{6}{\sqrt{R_{Brd}}}, \; R_{CC} = \frac{R_{Brd}}{36} = 1.3x10^{-5}$$

Power Supply

$$R_{PSIn} = \frac{2.0x10^{-3}}{A_f^2} = \frac{2.0x10^{-3}}{(2.0x4.5x0.45)^2} = 1.22x10^{-4}$$

$$R_{PSOut} = R_{PSIn} = 1.22x10^{-4}$$

Represent internals by 9, contraction - expansions, 50% open:

$$R_{PSInt} = 9\left\{\frac{0.63x10^{-3}}{A_f^2} + 1.29x10^{-3}\left[\frac{1}{A_f}\left(1 - \frac{A_f}{A_2}\right)\right]^2\right\}$$
$$= 9\left\{\frac{0.63x10^{-3}}{(2.0x4.5x0.5)^2} + 1.29x10^{-3}\left[\frac{1}{2.0x4.5x0.5}(1 - 0.5)\right]^2\right\}$$
$$= 9\left\{3.11x10^{-5} + 1.59x10^{-5}\right\} = 4.70x10^{-4}$$

Total power supply resistance: R_{PS}

$$\begin{aligned} R_{PS} &= R_{PSIn} + R_{PSInt} + R_{PSOut} \\ &= 1.22x10^{-4} + 4.70x10^{-4} + 1.22x10^{-4} \\ &= 7.14x10^{-4} \end{aligned}$$

Near Fan

$$R_{Ex\ Perf} = \frac{2.0x10^{-3}}{A_f^2} = \frac{2.0x10^{-3}}{\left[\pi\left(\frac{3.0}{2}\right)^2 x\,0.45\right]^2} = 1.98x10^{-4}$$

Card Cage + Power Supply: $R_{Box\ Int}$

$$\frac{1}{\sqrt{R_{Box\ Int}}} = \frac{1}{\sqrt{R_{CC}}} + \frac{1}{\sqrt{R_{PS}}} = \frac{1}{\sqrt{1.3x10^{-5}}} + \frac{1}{\sqrt{7.14x10^{-4}}}$$

$$R_{Box\ Int} = 1.01x10^{-5}$$

Total System Resistance: R_{Sys}

$$R_{Sys} = R_{Int\,Perf} + R_{Expan} + R_{Box\,Int} + R_{Ex\,Perf}$$
$$= 3.95x10^{-4} + 3.83x10^{-5} + 1.01x10^{-6} + 1.98x10^{-4}$$
$$= 6.41x10^{-4}$$

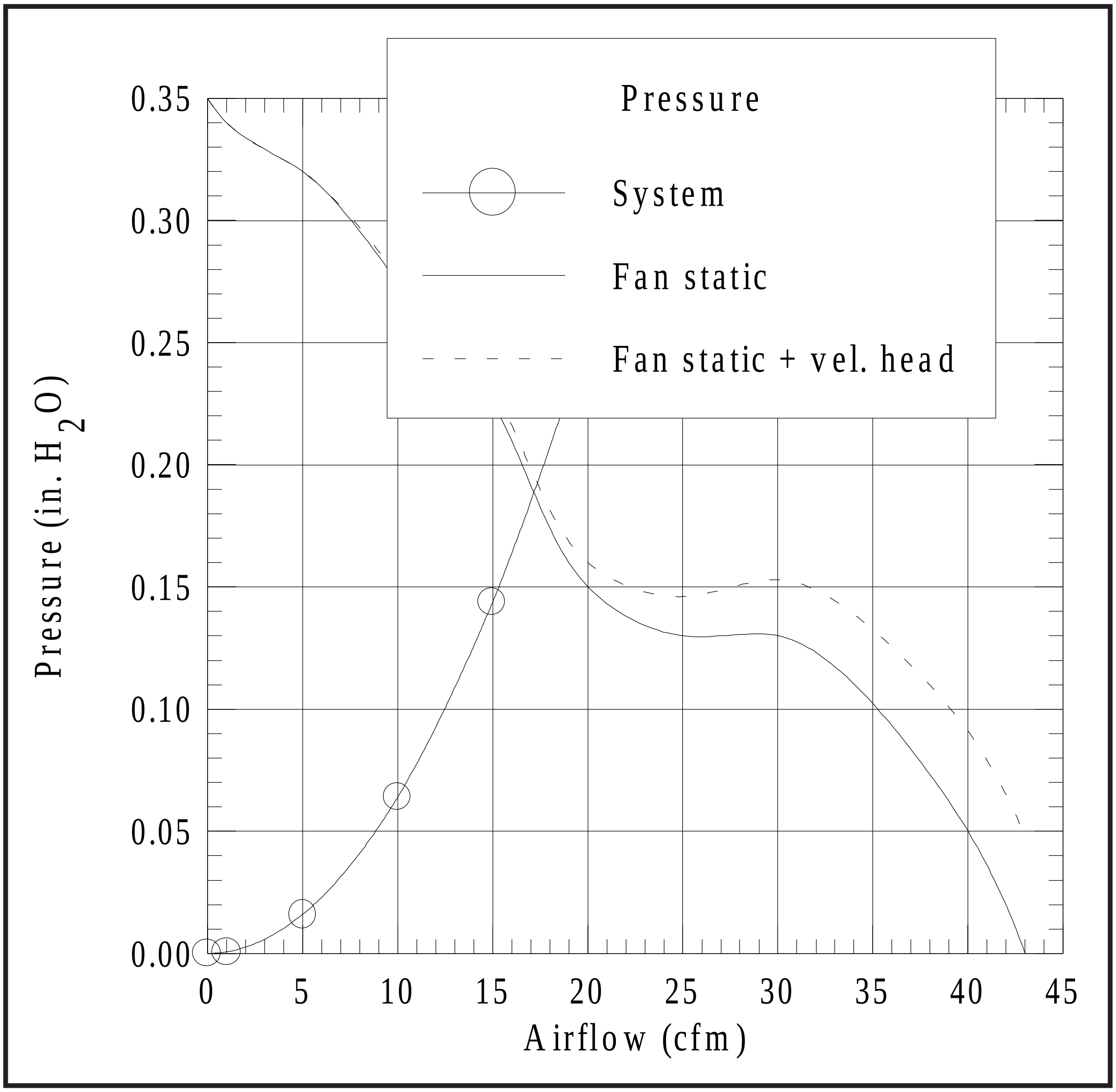
Total System Pressure Loss Calculated -

$$P_{sys} = R_{Sys}G^2 = 6.41x10^{-4}G^2$$

$$h_{Vd} = \frac{1.29x10^{-3}}{A_d^2}G^2 = \frac{1.29x10^{-3}}{\left[\pi\left(\frac{3.0}{2}\right)^2\right]^2}G^2 = 2.58x10^{-5}G^2$$

G (ft^3/min.)	P_{Sys} (in. H_2O)	h_{Vd} (in. H_2O)
1	6.41×10^{-4}	2.58×10^{-5}
5	1.60×10^{-2}	6.45×10^{-4}
10	6.41×10^{-2}	2.58×10^{-3}
15	0.144	5.81×10^{-3}
20	0.257	0.010
25	0.41	0.016
30	0.58	0.023
40	1.22	0.041
43	1.19	0.048

Plotted Airflow Results -



System operating point is at an airflow of G=17 ft^3/min.

Card Cage Results

$$G_{CC} = G\sqrt{\frac{R_{BoxInt}}{R_{CC}}} = 17\sqrt{\frac{1.01x10^{-5}}{1.3x10^{-5}}} = 14.9\ ft^3/\text{min.}$$

$$G_{Card} = G_{CC}/6 = 14.9/6 = 2.5\ ft^3/\text{min.}$$

Power Supply Results

$$G_{PS} = G - G_{CC} = 17 - 14.9 = 2.1\ ft^3/\text{min.}$$

Continuing with more complex example, re-evaluate card airflow using Teerstra's pressure loss model.

This method usually requires a few iterations because of the dependence that the friction factor has on the Reynold's number. However, the current problem has already been solved using a different circuit board airflow, so we shall use the existing result and then iterate as necessary.

The existing solution indicated an airflow in each card channel of *2.5* ft^3/min. Remembering that a Reynold's number using velocity dimensions of *ft/min.*, length dimensions *in.*, and kinematic velocity dimensions of $in.^2/sec$,

$$\mathrm{Re}_{2H} = \frac{2HV}{5\upsilon}$$

$$= \frac{2(1.0\,in.)\left[\left(2.5\,ft^3/\mathrm{min}\right)/\left(1.0\,in.\,x\,4.5\,in./\left(144\,in.^2/ft^2\right)\right)\right]}{5\left(0.023\,in.^2/\mathrm{sec}\right)}$$

$$= 696$$

Using *B=0.5 in.*, *H=1.0 in.*, *L=0.5 in.*, *S=0.5 in.* (the component dimensions *B, L, S* are kind of hypothetical as they were not indicated in the original problem statement).

$$\gamma = 1 + \left(\frac{B}{H}\right)\left(\frac{H}{L}\right)\left(\frac{L}{L+S}\right) = 1 + \left(\frac{0.5}{1.0}\right)\left(\frac{1.0}{0.5}\right)\left(\frac{0.5}{0.5+0.5}\right) = 1.5$$

$$\varsigma = 1 - \left(\frac{B}{H}\right)\left(\frac{L}{L+S}\right) = 1 - \left(\frac{0.5}{1.0}\right)\left(\frac{0.5}{0.5+0.5}\right) = 0.75$$

$$\chi = \left(\frac{B}{H}\right) + \left(1 - \frac{B}{H}\right)\left[1 + \left(\frac{2B}{H}\right)\left(\frac{H}{L}\right)\left(\frac{L}{L+S}\right)\right]$$

$$= \left(\frac{0.5}{1.0}\right) + \left(1 - \frac{0.5}{1.0}\right)\left[1 + \left(\frac{2x0.5}{1.0}\right)\left(\frac{1.0}{0.5}\right)\left(\frac{0.5}{0.5+0.5}\right)\right] = 1.5$$

$$\xi = \left(\frac{B}{H}\right) + \left(1 - \frac{B}{H}\right)\left(\frac{L}{L+S}\right) = \left(\frac{0.5}{1.0}\right) + \left(1 - \frac{0.5}{1.0}\right)\left(\frac{0.5}{0.5+0.5}\right)$$

$$= 0.75$$

$$\mathrm{A} = \frac{\gamma^2}{\varsigma^3\chi} = \frac{(1.5)^2}{(0.75)^3(1.5)} = 3.56, \quad \mathrm{B} = \frac{\gamma^{5/4}}{\varsigma^3\xi} = \frac{(1.5)^{5/4}}{(0.75)^3(0.75)} = 5.25$$

$$f_{2H} = \left[\left(\frac{96\mathrm{A}}{\mathrm{Re}_{2H}}\right)^3 + \left(\frac{0.347\mathrm{B}}{\mathrm{Re}_{2H}^{1/4}}\right)^3\right]^{1/3}$$

$$= \left[\left(\frac{96x3.56}{696}\right)^3 + \left(\frac{0.347x5.25}{696^{1/4}}\right)^3\right]^{1/3}$$

$$= 0.546$$

$$R_{Card} = \left(\frac{1.29x10^{-3}}{A^2}\right)\left(\frac{L_{Card}}{2H}\right) f_{2H}$$

$$= \left[\frac{1.29x10^{-3}}{(4.5x1.0)^2}\right]\left[\frac{(11.5)}{2(1.0)}\right](0.546)$$

$$= 1.9x10^{-4}\ in.\,H_2O \Big/ \left(ft^3/\text{min.}\right)^2$$

The single card resistance using the McLean card resistance was determined to be *2.81x10*$^{-4}$. We should perform at least one more iteration to correct the results.

<u>Iteration using new card resistance from Teerstra's model</u>

$$R_{Brd} = R_{Cont} + R_{Card} + R_{Exp}$$

$$= 1.24x10^{-4} + 1.9x10^{-4} + 6.37x10^{-5}$$

$$= 3.78x10^{-4}$$

$$\frac{1}{\sqrt{R_{CC}}} = \frac{6}{\sqrt{R_{Brd}}} = \frac{6}{\sqrt{3.78x10^{-4}}}$$

$$R_{CC} = 1.05x10^{-5}$$

$$\frac{1}{\sqrt{R_{BoxInt}}} = \frac{1}{\sqrt{R_{CC}}} + \frac{1}{\sqrt{R_{PS}}}$$

$$= \frac{1}{\sqrt{1.05x10^{-5}}} + \frac{1}{\sqrt{6.54x10^{-4}}}$$

$$R_{BoxInt} = 8.27x10^{-6}$$

$$R_{Sys} = R_{IntPerf} + R_{Expan} + R_{BoxInt} + R_{ExtPerf}$$

$$= 3.95x10^{-4} + 3.83x10^{-5} + 8.27x10^{-6} + 1.98x10^{-4}$$

$$= 6.40x10^{-4}$$

This re-calculation of the system resistance is almost exactly identical to the first value calculated using the McLean card model.

We shall therefore not re-calculate the total airflow, but only calculate the new internal airflow distribution and resultant temperatures.

$$G_{CC} = G\sqrt{\frac{R_{BoxInt}}{R_{CC}}} = 17\sqrt{\frac{8.27x10^{-6}}{1.05x10^{-5}}} = 15.1\ ft^3/\text{min.}$$

$$G_{Card} = G_{CC}/6 = 15.1/6 = 2.5\ ft^3/\text{min.}$$

An additional iteration of the card airflow is not necessary.

We shall instead proceed directly to the final calculation of the air temperature rises.

22 Watt Cards - Air Temperature Rise Above Inlet"

$$\Delta T = \frac{1.76 Q_{Card}}{G_{Card}} = \frac{1.76(22)}{2.5} = 16\ ^{o}C$$

11 Watt Card -

$$\Delta T = \frac{1.76 Q_{Card}}{G_{Card}} = \frac{1.76(11)}{2.5} = 8\ ^{o}C$$

33 Watt Card -

$$\Delta T = \frac{1.76 Q_{Card}}{G_{Card}} = \frac{1.76(33)}{2.5} = 23\ ^{o}C$$

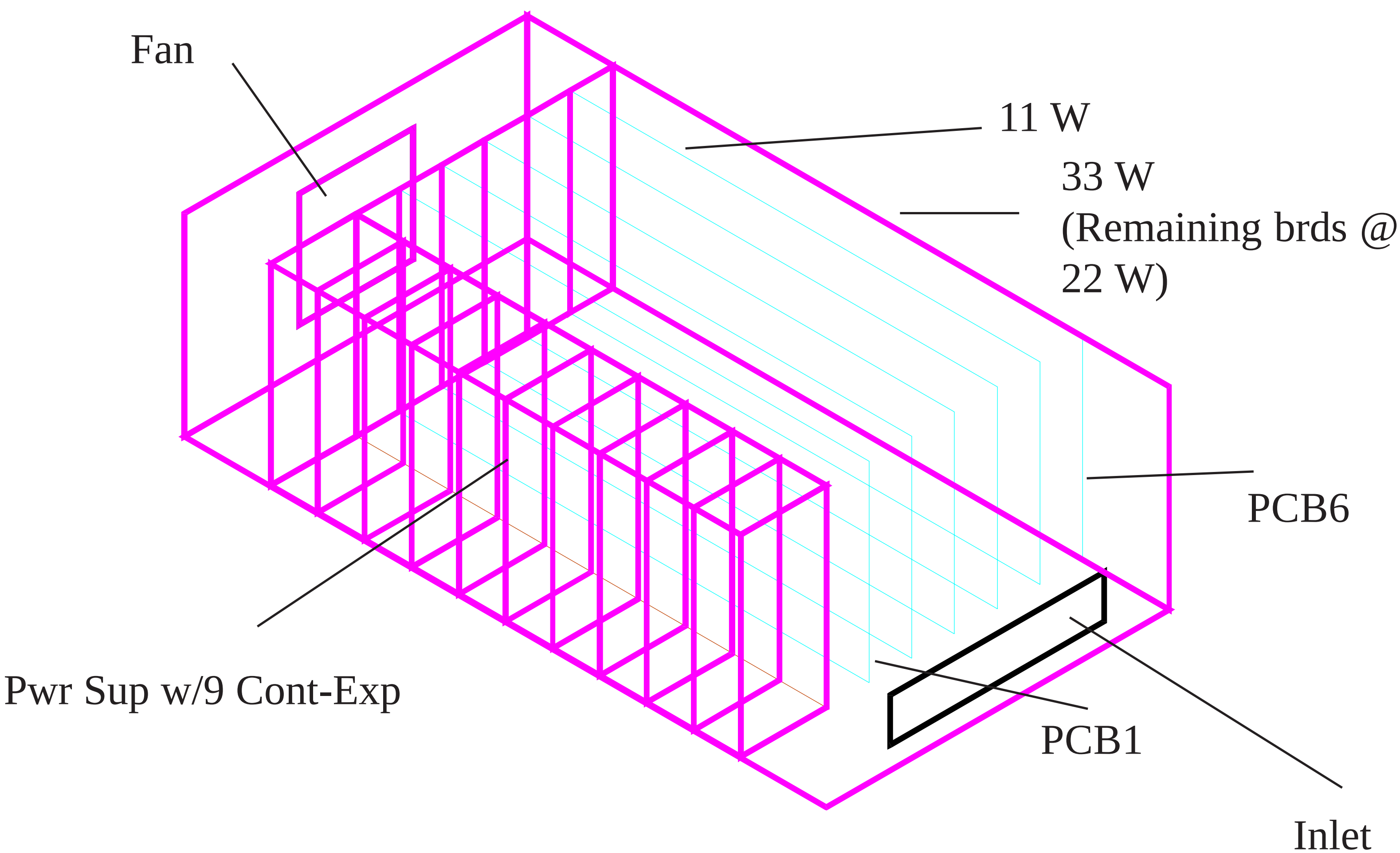
Power Supply -

$$G_{PS} = G - G_{CC} = 17 - 15 = 2.0\ ft^3/\text{min.}$$

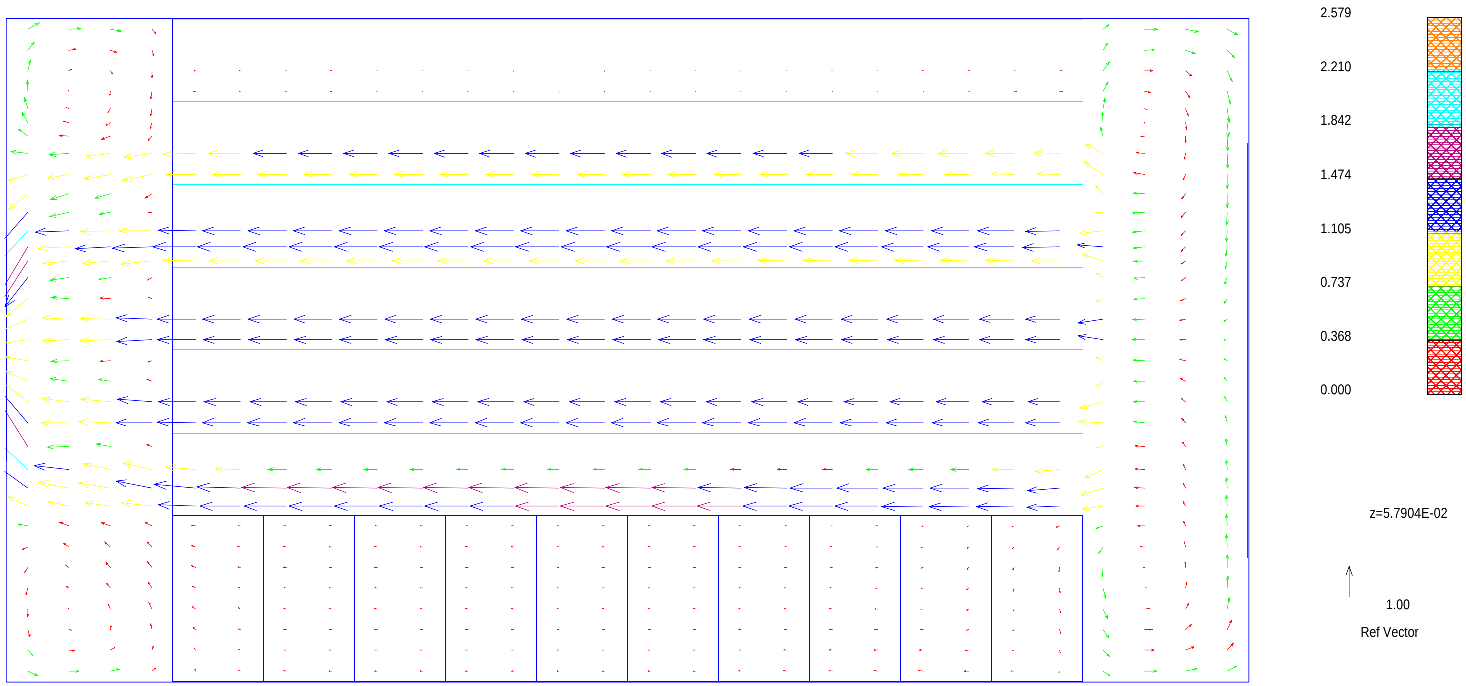
$$\Delta T = \frac{1.76 Q_{PS}}{G_{PS}} = \frac{1.76(68)}{2.0} = 60\ ^{o}C$$

Comments: This design has some thermal problems. Before a thermal mockup or more detailed modeling (such as CFD) is attempted, such design changes need to be considered. This design could profit by a fan change, which needs to be considered along with the system curve. The power supply internal resistance could be examined more carefully, but an airflow resistance *measurement* would be preferred. The power supply inlet and exit could be considered as candidates for improvement. In the card cage, perhaps some of the 11 watt card air could be channeled into the 33 watt card channel.

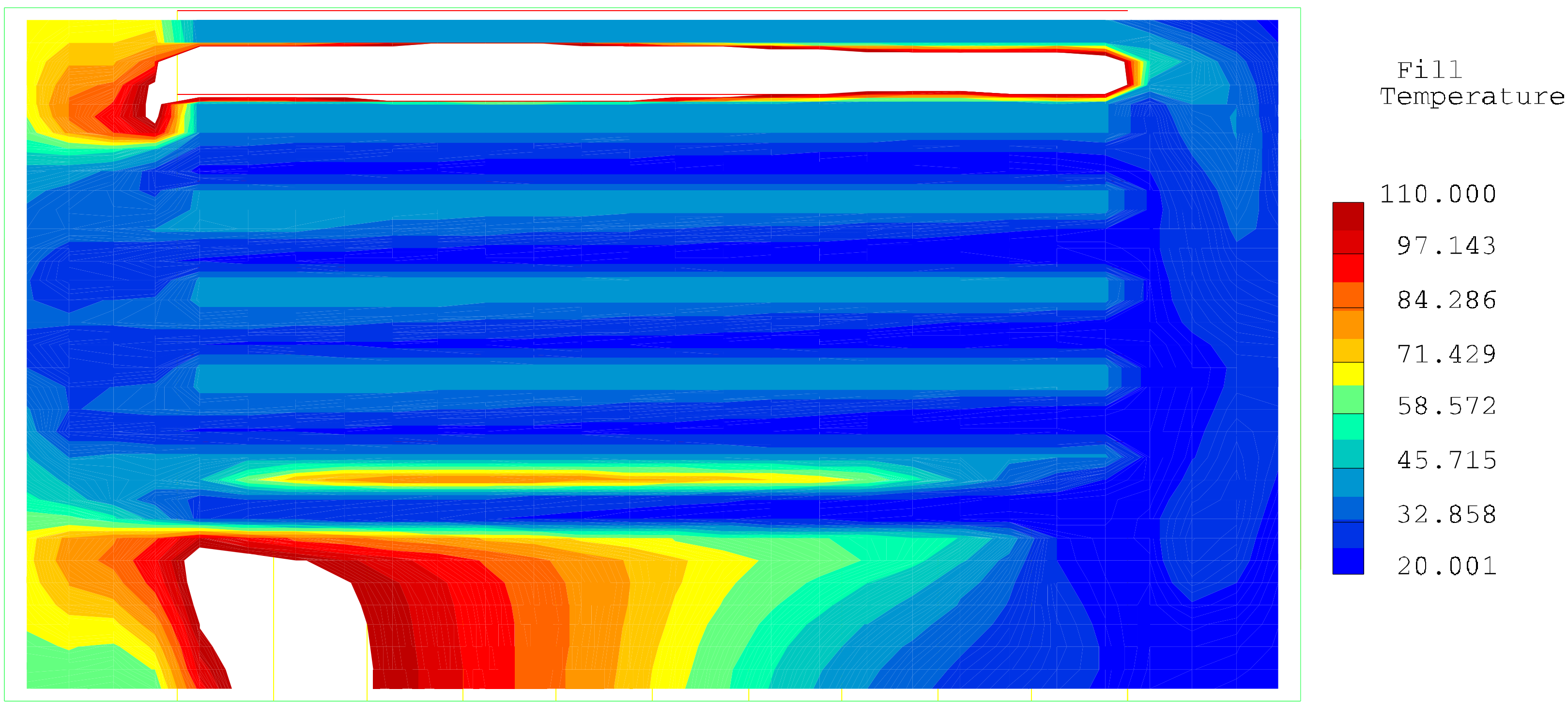
FLOTHERM® Model of Fan Cooled Enclosure



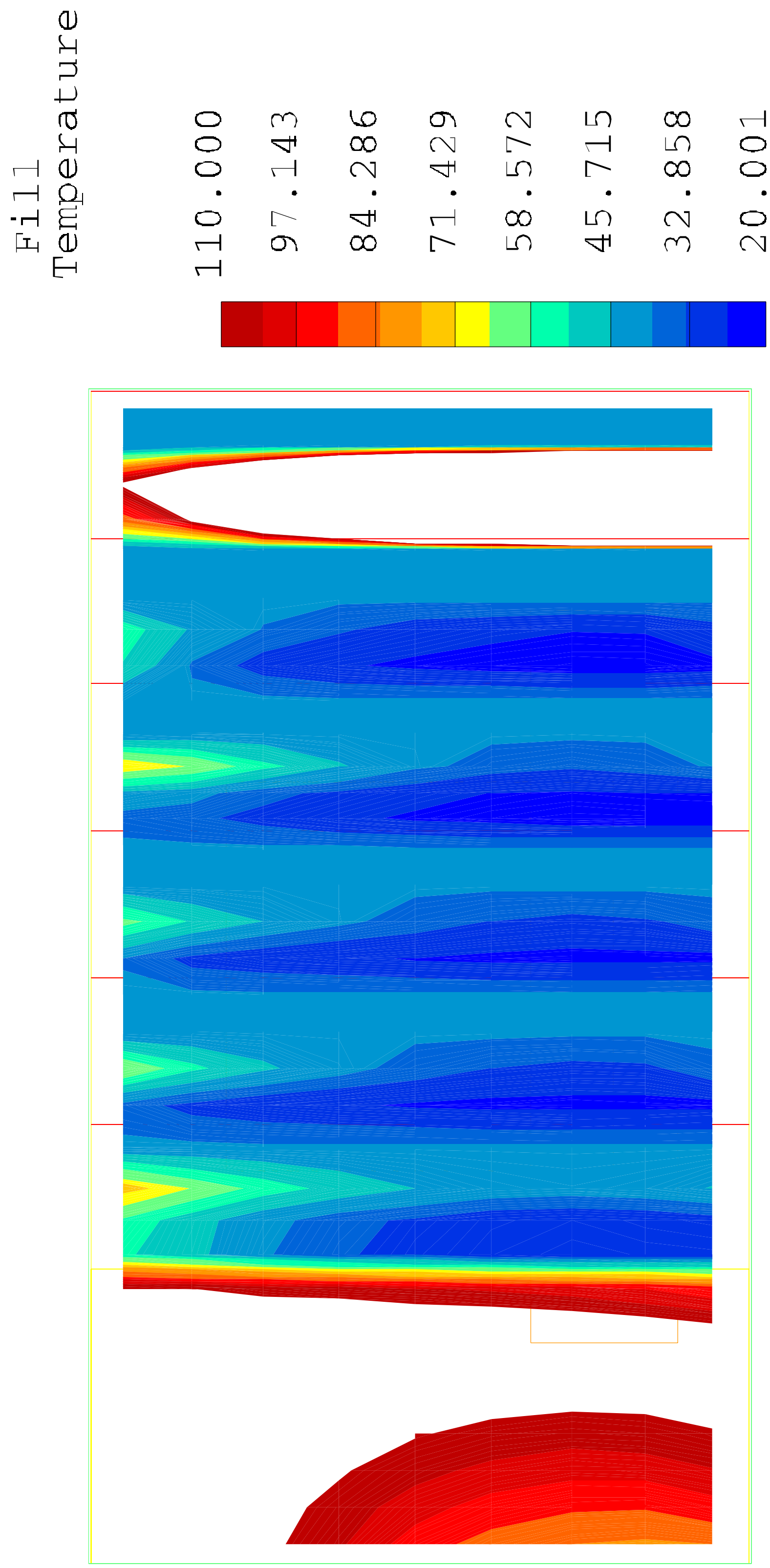
FLOTHERM® Velocity Results Mid-Way Between Top and Bottom



FLOTHERM® Results Mid-Way Between Top and Bottom



FLOTHERM® Results at Card Cage Exit Plane



Comparison Between Network and FLOTHERM® Results

Location	**Network Airflow (*ft³/min.*)**	**FLOTHERM Airflow (*ft³/min.*)**	**Network Air Temp. Rise (*°C*)**	**FLOTHERM Air Temp. Rise (*°C*)**
Pwr. Sup. Exit	2.0	1.0	60	91*
PCB1 Exit	2.5	3.7	16	16
PCB2 Exit	2.5	3.5	16	13
PCB3 Exit	2.5	3.5	16	13
PCB4 Exit	2.5	3.5	16	13
PCB5 Exit	2.5	2.7	8	9
PCB6 Exit	2.5	0.05	23	404*
Fan Inlet	17	18	21	20

(*) Positive and negative flow indicated in results. This air ΔT was selected from the maximum of inflow and outflow at the region exit (mean flow region in FLOTHERM®).

Forced Air Flow - Mostly Ducts and Extrusions

The Airflow Problem

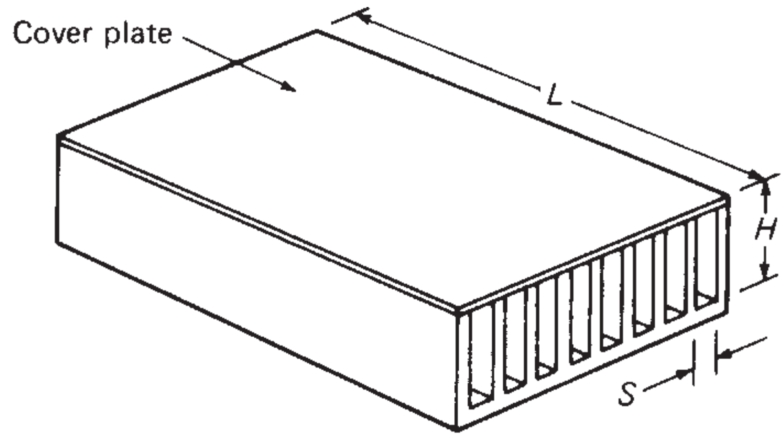


Fig. 6-16. Geometry for forced convection cooled extruded heat sink.

$$R = \frac{1.29x10^{-3}}{N_p^2 A_c^2}\left[K_c + K_e + 4\bar{f}\,\frac{L}{D_H}\right],\ in.H_2O\Big/\left(ft^3/\text{min.}\right)^2$$

for $H_L = RG^2,\ in.H_2O$

N_p =Number of parallel channels

A_c = Cross-sectional area of each channel, $in.^2$

K_c = Contraction loss coefficient

K_e = Expansion loss coefficient

$\bar{f}$ = Average friction coefficient for length L

$$D_H = \text{ hydraulic diameter } = \frac{2SH}{(S+H)}$$

$$\mathrm{Re}_D \equiv \text{Reynold's number } = \frac{VD_H}{\upsilon}$$

or in mixed English units

$$\mathrm{Re}_D == \frac{VD_H}{5\upsilon}$$

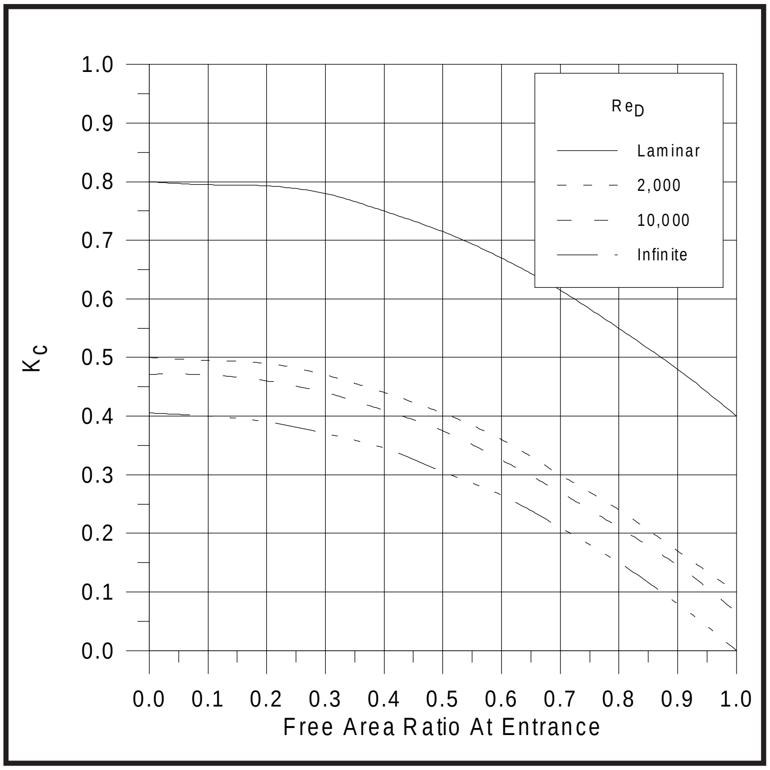
$V[ft/\mathrm{min.}] =$ air velocity in duct

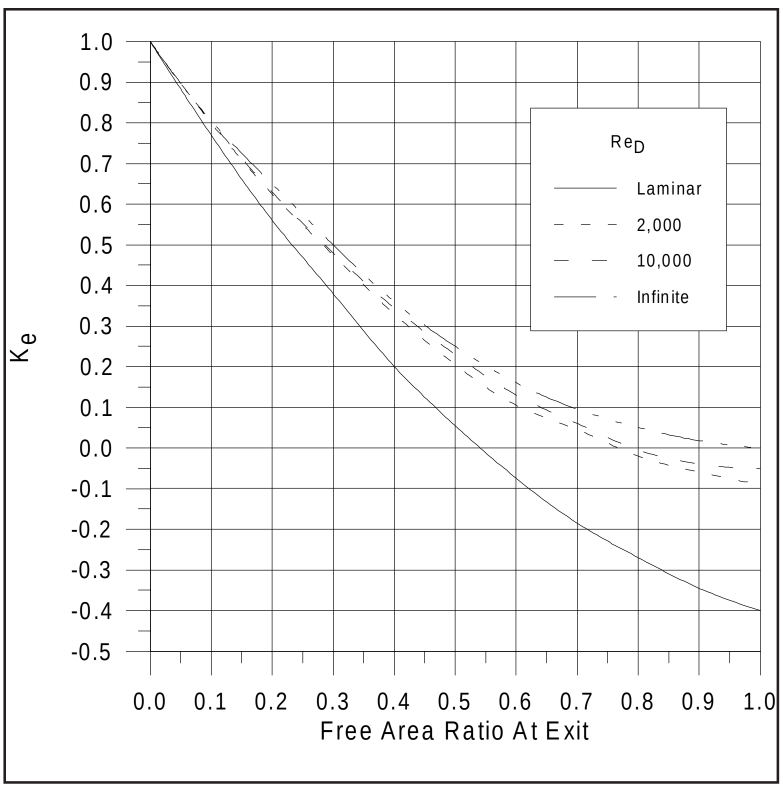
$\upsilon[in./\mathrm{sec}] \equiv$ Kinematic viscosity

Laminar Flow - $\mathrm{Re}_D \leq 2000$

Transition Flow - $2000 < \mathrm{Re}_D < 10000$

Turbulent Flow - $\mathrm{Re}_D \geq 10000$





Adapted from Kays & London, 1964.

Friction coefficient:

Table 6-3. Mean friction coefficients for laminar flow between parallel plates. From *Handbook of Heat Transfer* by W. M. Rohsenow and J. P. Hartnett, [3]. Copyright © 1973 by McGraw-Hill, Inc. Used with the permission of McGraw-Hill Book Company.

$L/(DRe_D)$	$\bar{f}Re_D$
0.0_4431	168.4
0.0_3209	88.89
0.0_3354	73.14
0.0_3686	57.60
0.00159	42.63
0.00260	35.73
0.00338	32.60
0.00448	29.78
0.00529	28.76
0.00567	27.83
0.00644	26.97
0.00733	26.21
0.00845	25.56
0.00910	25.27
0.00983	25.01
0.01067	24.77
0.01114	24.67
0.01165	24.57
0.01221	24.47
0.01283	24.40
0.01352	24.32
0.01427	24.25
0.01518	24.20
0.01611	24.14
0.01695	24.10
0.02059	24.06
∞	24.00

Turbulent flow - $\mathrm{Re}_D \geq 10{,}000$

$$R = \frac{1.29x10^{-3}}{N_p^2 A_c^2}\left[K_c + K_e + 4\bar{f}_{app}\frac{L}{D_H}\right],\ in.H_2O\big/\left(ft^3/\mathrm{min.}\right)^2$$

See previous figures for contraction and expansion loss constants, K_c and K_E.

$\bar{f}_{app} \equiv$ apparent friction coefficient

Friction coefficient:

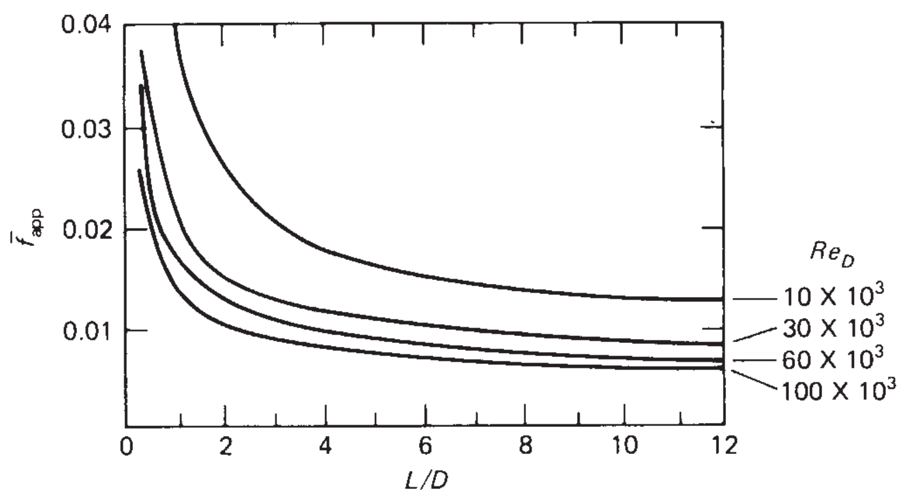
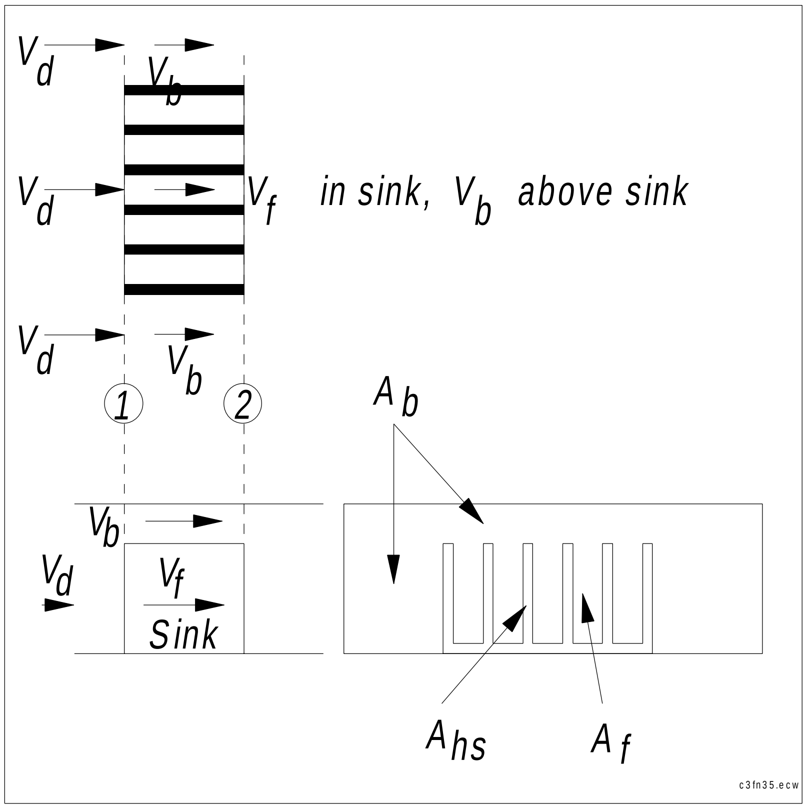


Fig. 6-18. Friction coefficient for turbulent flow in the hydrodynamic entry length of a circular tube. From *Handbook of Heat Transfer*, Editors, W. M. Rohsenow and J. P. Hartnett, Fig. 4, p. 7-7, copyright © 1973 by McGraw-Hill, Inc. Used with permission of McGraw-Hill Book Co.

Consideration of Flow By-Pass Effects (Simons and Schmidt, 1997)



Applying Bernoulli's Equation with losses at control surfaces *1* and *2* out of sink,

$$\frac{\rho V_d^2}{2} + p_1 = \frac{\rho V_b^2}{2} + p_2 + \Delta p_b$$

and *1* and *2* in sink,

$$\frac{\rho V_d^2}{2} + p_1 = \frac{\rho V_f^2}{2} + p_2 + \Delta p_f$$

These Bernoulli's Equations require correct and consistent units, i.e. $\rho\left[slugs/ft^3\right], p\left[lb_f / ft^2\right], \Delta p\left[lb_f / ft^2\right], V\left[ft/s\right].$

Subtracting the second equation from the first

$$\frac{\rho V_b^2}{2} - \frac{\rho V_f^2}{2} + \Delta p_b - \Delta p_f = 0$$

$$\Delta p_b << \Delta p_f$$

$$\frac{\rho V_b^2}{2} - \frac{\rho V_f^2}{2} - \Delta p_f = 0$$

Using conservation of mass flux for constant fluid density, i.e. conservation of air flow

$$G_d = G_b + G_f$$

$$V_d A_d = V_b A_b + V_f A_f$$

$$V_b = \frac{V_d A_d - V_f A_f}{A_b}$$

$$V_b^2 = \frac{V_d^2 A_d^2 + V_f^2 A_f^2 - 2 V_d V_f A_d A_f}{A_b^2}$$

Substituting V_b^2 into the substracting Bernoulli's Equations performing a little algebra

$$aV_d^2 + bV_d + c = 0$$

$$a = \left(\frac{A_d}{A_b}\right)^2,\ b = -2\left(\frac{A_d}{A_b}\right)\left(\frac{A_f}{A_b}\right)V_f,\ c = -\left\{V_f^2\left[1 - \left(\frac{A_f}{A_b}\right)^2\right] + \frac{2\Delta p_f}{\rho}\right\}$$

which is, of course, a quadratic equation solved using

$$V_d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

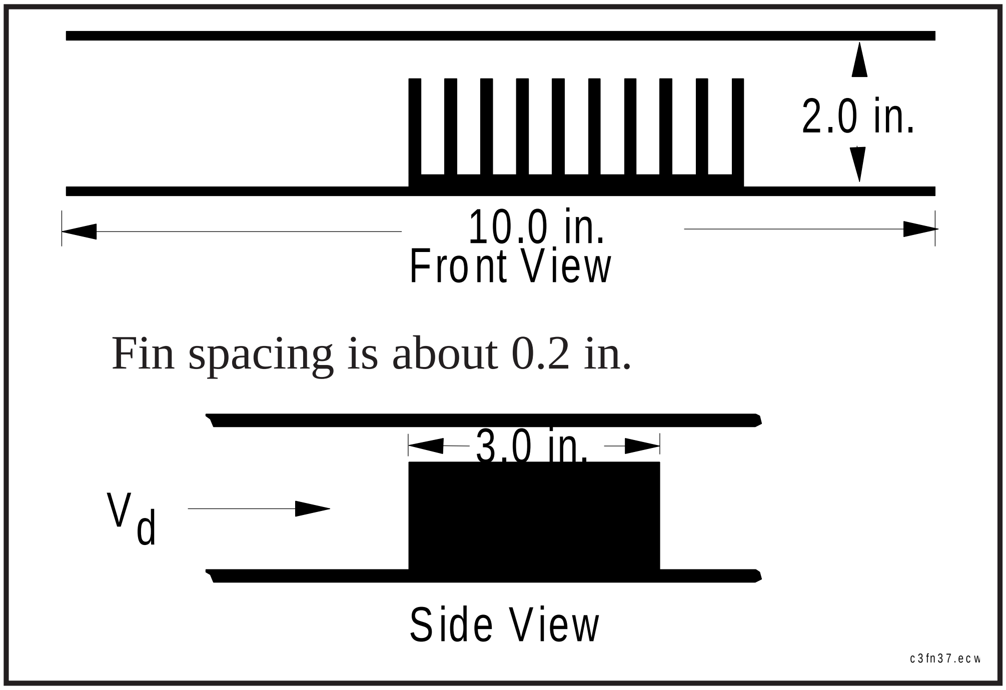
where the only realistic solution is obtained using the + sign.

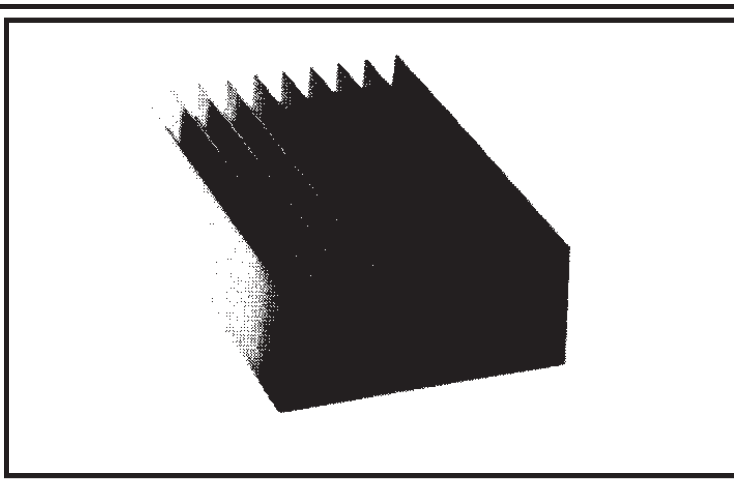
The following procedure may be used:

1. Determine the heat sink thermal resistance necessary for a good design.
2. Select a heat sink and determine the necessary air velocity.
3. Calculate or measure the heat sink pressure loss Δp_f for the air velocity.
4. Solve the quadratic equation to get the required duct channel air velocity, V_d and ultimately the required duct flow, $G_d = V_d A_d$, necessary to cool the heat sink.

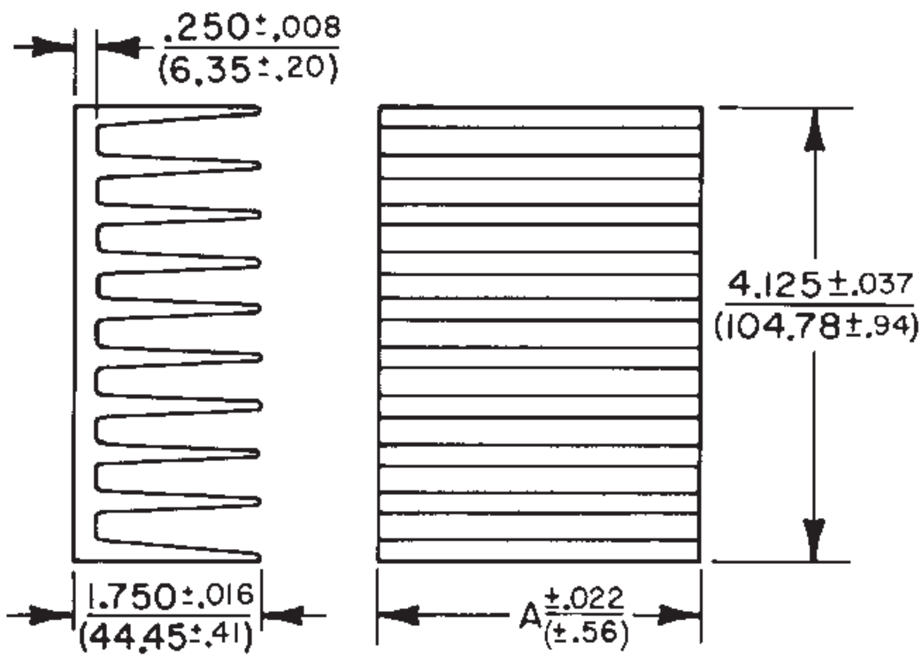
Application Example

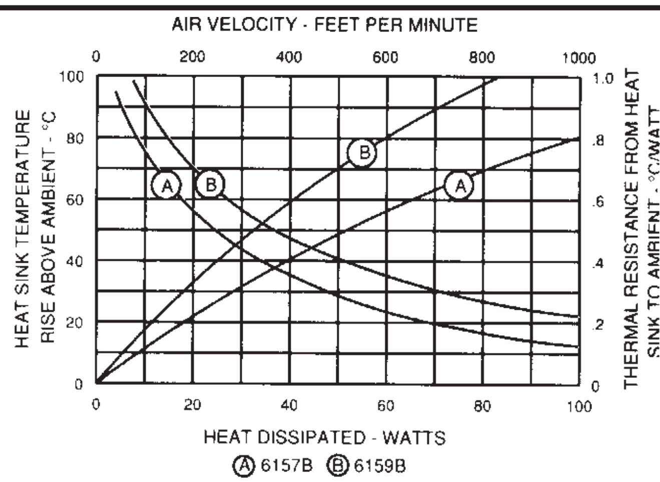
An extrusion is used to cool a microprocessor dissipating *25 W*. The heat sink is in a circuit board channel with the indicated dimensions. The goal is to determine how much airflow in the board channel is required to keep the heat sink at a *20 °C* rise above the channel inlet temperature. Neglect the dissipation on the remainder of the PCB.





Heat sink properties from vendor data sheet:





Model	Dim. A	Weight (lbs./grams)
6157B	6.00	1.68 (762.04)
6159B	3.00	0.84 (381.02)

Determination of required heat sink airflow:

Required heat sink resistance

$$R_{\sin k} = \frac{T_{\sin k} - T_{inlet}}{Q} = \frac{20}{25} = 0.8\,{}^{o}C/W$$

Required heat sink air velocity is determined from vendor graph to be $V_f = 140\ ft./\text{min.}$.

Calculation of heat sink pressure drop:

Hydraulic diameter of a fin channel is

$$D_H = \frac{2SH}{S+H} = \frac{2(0.2)(1.5)}{0.2+1.5} = 0.353\,in.$$

The Reynold's number for the fin channels is then

$$\text{Re} = \frac{V_f D_H}{5\upsilon} = \frac{(140\ ft/\text{min.})(0.353\,in.)}{5(0.023)} = 430$$

which is certainly indicative of laminar flow.

We shall use TCEE, Table 6-3 to get the laminar friction factor. To use the table we must calculate

$$L/(D_H\,\mathrm{Re}) = 3.0\,in./(0.353\,in.\cdot 430) = 0.02.$$

From TCEE, Table 6-3 we find $f = 24/\mathrm{Re} = 0.056$.

The contraction coefficient is found from the preceding graphs:

$$\sigma = A_f \big/ \left(A_f + A_{hs}\right) = 0.2\,in.\,x\,1.5\,in.\,x\,9 \big/ \left(A_f + A_{hs}\right)$$
$$= 2.7\,in.^2 / (4.125\,in.\,x\,1.75\,in.) = 0.37$$

At Re_D=430, K_C=0.77, K_e=0.24.

The heat sink airflow resistance is then

$$R_{af} = \frac{1.29x10^{-3}}{N_p^2 A_c^2}\left[K_c + K_e + 4\,\frac{fL}{D_H}\right]$$
$$= \frac{1.29x10^{-3}}{(9)^2 (0.2\,in.\cdot 1.5\,in.)^2}\left[0.77 + 0.24 + 4\,\frac{(0.056)(3.0\,in.)}{(0.353\,in.)}\right]$$
$$= 5.16x10^{-4}\,in.\,H_2O \big/ \left(ft^3/\mathrm{min.}\right)^2$$

The heat sink airflow resistance is used to calculate the heat sink presssure loss

$$G_f = \frac{N_p A_c}{144} V_f = \frac{N_p SH}{144} V_f$$

$$= \frac{(9)(0.2\,in.)(1.5\,in.)}{144\,in.^2 / ft^2}(140\ ft./\text{min.}) = 2.63\ ft.^3/\text{min.}$$

$$\Delta p = R_{af} G_f^2 = 5.16x10^{-4}(2.63)^2 = 3.57x10^{-3}\ in.\,H_2O$$

Calculation of duct airflow:

The quantities needed to solve for the duct airflow are now all available, we must be careful of units.

Areas in $in.^2$ are safe to use as long as ratios of areas are used.

Pressure loss must be converted from $in.\ H_2O$ to lb_f/ft^2.

$$\Delta p = 3.57x10^{-3}\,in.\,H_2O\left(0.0361\frac{lb_f}{in.^2} \Big/ in.\,H_2O\right)\left(144\,in.^2 / ft^2\right)$$

$$= 0.019\,lb_f / ft^2$$

or

$$\Delta p = \left(3.57x10^{-3} in.\ H_2O\right)(5.198\ / in.\,H_2\,O) = 0.019\frac{lb_f}{ft.^2}$$

The velocity V_f must be converted from *ft/min.* to *ft/s.*

$$V_f = \frac{140\ ft/\text{min.}}{60\, s/\text{min.}} = 2.33\ ft/s$$

We also need the density of air

$$\begin{aligned}\rho &= 0.075\, lb_m / ft^3 \\ &= \left(0.075\, lb_m / ft^3\right) \Big/ \left(\frac{32.2\, slugs}{lb_m}\right) = 2.33x10^{-3}\, slugs / ft^3\end{aligned}$$

We are finally ready to calculate the duct velocity:

$$A_d = 10\, in.\, x\, 2\, in. = 20\, in.^2\,,\ \ A_f = 0.2\, in.\, x 1.5\, in.\, x\, 9 = 2.7\, in.^2$$

$$A_{hs} = 4.125\, in.\, x\, 1.75\, in. - A_f = 4.52\, in.^2$$

$$A_b = A_d - A_f - A_{hs} = 12.8\, in.^2$$

$$a = \left(\frac{A_d}{A_b}\right)^2 = \left(\frac{20\, in.^2}{12.8\, in.^2}\right)^2 = 2.44$$

$$b = -\left[2\left(\frac{A_d}{A_b}\right)\left(\frac{A_f}{A_b}\right)V_f\right] = -\left[2\left(\frac{20\, in.^2}{12.8\, in.^2}\right)\left(\frac{2.7\, in.^2}{12.8\, in.^2}\right)2.33\right] = -1.54$$

$$c = -\left\{ V_f^2 \left[1 - \left(\frac{A_f}{A_b} \right)^2 \right] + \frac{2\Delta p}{\rho} \right\}$$

$$= -\left\{ (2.33)^2 \left[1 - \left(\frac{2.7}{12.8} \right)^2 \right] + \frac{2\left(0.023\, lb_f / ft^2\right)}{2.33x10^{-3}\, slugs / ft^3} \right\}$$

$$= -24.9$$

$$V_d = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1.54) + \sqrt{(-1.54)^2 - 4(2.44)(-24.9)}}{2(2.44)}$$

$$= 3.53\ ft/s$$

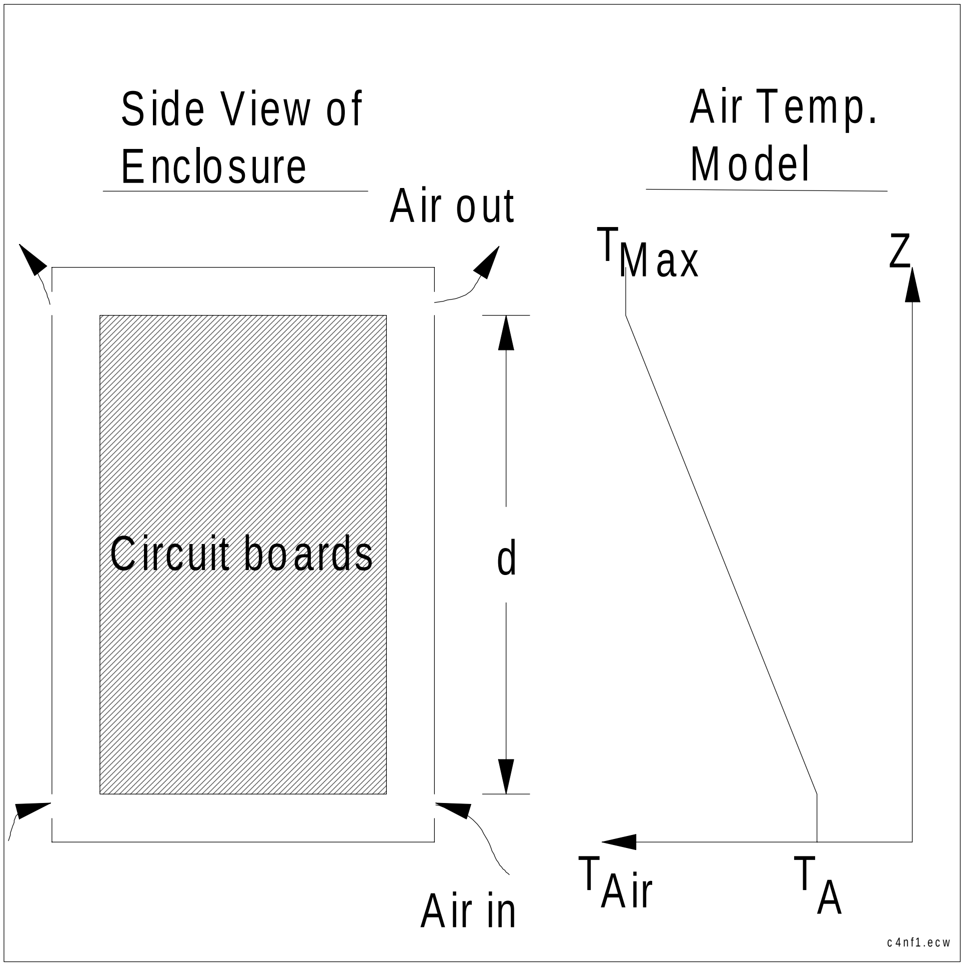
$$V_d = 3.53\ ft/s = (3.53\ ft/s)(60\, s/\text{min.}) = 212\ ft/\text{min.}$$

$$G_d = V_d A_d = (212\ ft/\text{min.})\left(\frac{10\, in.\, x\, 2\, in.}{144\, in.^2 / ft^2} \right) = 29.4\ ft^3/\text{min.}$$

In summary then, we about *29 ft³/min.* in the circuit board channel to get *2.6 ft³/min.* through the heat sink.

Buoyancy Driven Air Draft

The Internal Model



$$H_B = H_B(\Delta T_{Air} = T_{Max} - T_A)$$

See TCEE for details.

Fluid temperature rise $(T_{Max} - T_A)$:

$$Q_f = \dot{m}C_p(T_{Max} - T_A) = \rho G C_p \Delta T$$

$$\Delta T = Q_f \Big/ \left(\rho G C_p\right)$$

Assuming ideal gas behavior,

$$\rho = \rho_o\left(\frac{T_o + 273}{T + 273}\right),\ T_o = \text{ some reference temperature}$$

Using $T = T_A + \Delta T$ it can be shown that (see TCEE)

$$\Delta T = \frac{2(T_A + 273.15)}{\left(\frac{CG}{Q} - 1\right)}, \qquad \text{where now } C = 333.9$$

Derivation of a Non-Iterative Style Airdraft (not in TCEE):

Returning to the exact formula for air temperature rise,

The required pressure equations are:

System loss: $H_L = RG^2$ (TCEE page 169)

It can also be shown that (see TCEE) the buoyancy pressure is -

$$H_B = 0.0012d\left[\frac{(\Delta T/2)}{(\Delta T/2)+T_I+273.15}\right]$$ (TCEE page 168)

Equate the system pressure head loss to the buoyancy pressure,

$$H_L = H_B$$

$$H_L = H_B$$

$$RG^2 = 0.0012d\left[\frac{(\Delta T/2)}{(\Delta T/2) + T_I + 273.15}\right]$$

$$= 0.0012d\left[\frac{1}{1 + \left(\dfrac{T_I + 273.15}{(\Delta T/2)}\right)}\right]$$

But

$$\left(\frac{\Delta T}{2}\right) = \frac{(T_I + 273.15)}{\left(\dfrac{CG}{Q} - 1\right)}$$

then

$$RG^2 = \frac{0.0012d}{\left\{1 + \dfrac{(T_I + 273.15)}{\left[(T_I + 273.15)\Big/\left(\dfrac{CG}{Q} - 1\right)\right]}\right\}}$$

$$= \frac{0.0012d}{\left[1 + \left(\dfrac{CG}{Q} - 1\right)\right]}$$

$$= \frac{0.0012dQ}{CG}$$

Solving for *G*,

$$G = 1.53x10^{-2}\left(\frac{Qd}{R}\right)^{1/3}$$

which is preferred over the iterative method in TCEE, pp. 169-170.

Application Example: An Equipment Rack Cooled Only by Airdraft

Consider a 6 ft. high, 20 inches deep, and 20 inches wide rack and panel system filled with electronics. The electronics consists of five separate card cages, each ten inches in height. Each card cage has boards on one inch centers with 81% free area for airflow. Each card cage dissipates 100 W. The inlet and exit to the rack and panel system are identical and are spread over the entire 20 *in. x* 20 *in.* top and bottom panels. Each panel has a 35% perforation pattern with 0.188 inch diameter holes.

If convection and radiation losses from the system are neglected, can the cabinet be adequately cooled without using a blower, i.e. is the overall air temperature rise < 20 ^{o}?

We shall begin the airflow resistance calculation using $R_{inlet}=R_{Exit}$ of

$$R_{inlet} = \frac{2.0x10^{-3}}{A_f^2} = \frac{2.0x10^{-3}}{(20\,in.\,x\,20\,in.\,x\,0.35)^2} = 1.02x10^{-7}$$

$$R_{Cards} = 5R_{Contractions} + 5R_{Expansions} + R_{CC}$$

$$= 5\left[\frac{0.63x10^{-3}}{(20\,in.\,x\,20\,in.\,0.81)^2}\right]$$

$$+5\left\{1.29x10^{-3}\left[\frac{1}{20\,in.\,x\,20\,in.}(1-0.81)\right]^2\right\}$$

$$+\frac{3.08(5)(10.0\,in.)10^{-4}}{(20\,in.\,x\,20\,in.)^2}$$

$$= 3.08x10^{-8} + 1.46x10^{-9} + 9.6x10^{-8} = 1.28x10^{-7}$$

$$R_{af} = 2R_{Inlet} + R_{Cards}$$

$$= 2\left(1.02x10^{-7}\right) + 1.28x10^{-7} = 3.32x10^{-7}$$

We shall use a dissipation height d equal to the total height of the five card cages, i.e., $d = 50\ in.$

$$G = 1.53x10^{-2}\left(\frac{Qd}{R_{af}}\right)^{1/3} = 1.53x10^{-2}\left(\frac{500\,W\,x\,50\,in.}{3.32x10^{-7}}\right)^{1/3}$$

$$= 65\ ft^3/\text{min.}$$

Using the Adam, Fried & Idelchick perforated plate plots for the inlet and exit,

$$\mathrm{Re}_d = \frac{V_D D}{5\upsilon} = \frac{\left(G/A_f\right)D}{5\upsilon} = \frac{\left[65 \Big/ \left(\frac{20x20x0.35}{144}\right)\right](0.188)}{5(0.023)}$$

$$= 109 \qquad \Rightarrow \quad K_d = K_a f^2 = (10)(0.35)^2 = 1.2$$

Then

$$R_{Inlet} = R_{Exit} = K_d h_V = (1.2)\left(\frac{1.29x10^{-3}}{A_f^2}\right)$$

$$= (1.2)\left[\frac{1.29x10^{-3}}{(20x20x0.35)^2}\right] = 8.0x10^{-8}$$

$$R_{af} = 2\left(8.0x10^{-8}\right) + 1.28x10^{-7} = 2.9x10^{-7}$$

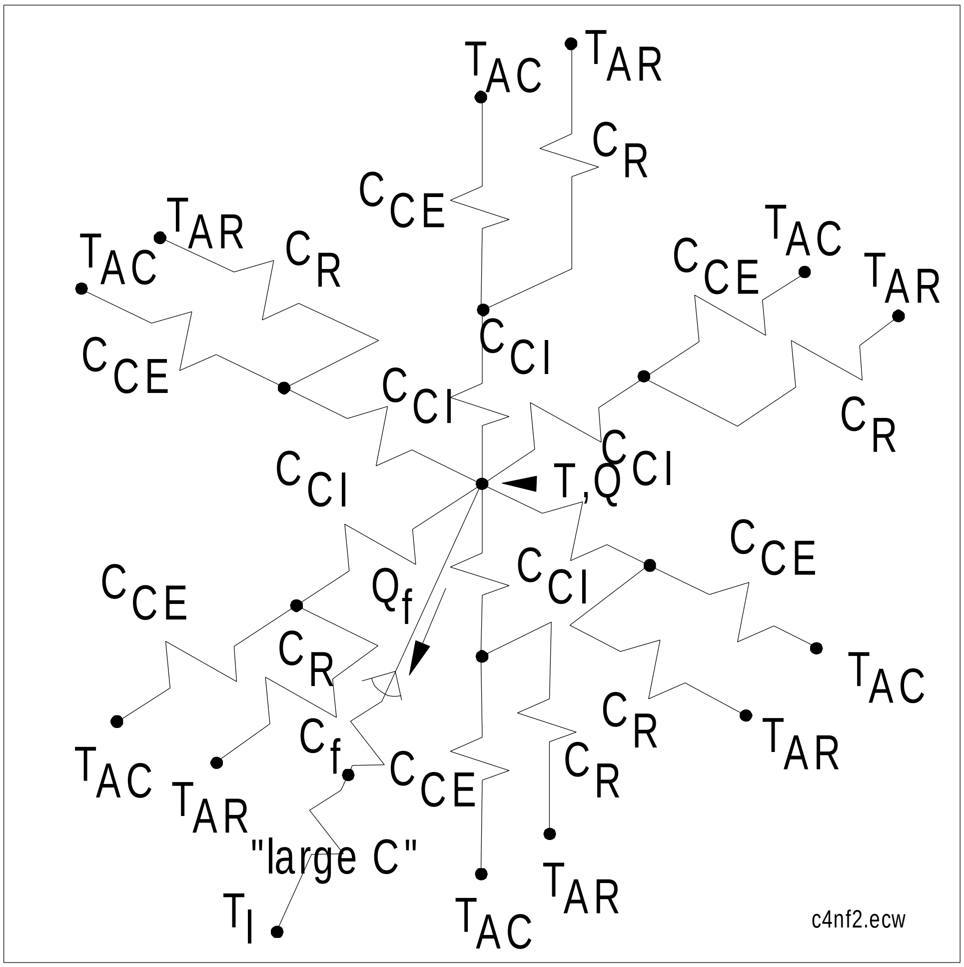
$$G = 1.53x10^{-2}\left(\frac{500x50}{2.9x10^{-7}}\right)^{1/3} = 68\ ft^3/\text{min.}$$

which is sufficiently close to the original 65 $ft^3/$min. to not require further iteration.

$$\Delta T = 1.76\frac{Q}{G} = 1.76\left(\frac{500}{68}\right) = 13\ ^oC$$

It appears that the enclosure is probably adequately cooled.

The Recommended System Model for a Vented Enclosure



Note the "one way" thermal fluid conductance element C_f.

$$Q_f = \text{Heat carried away by the air draft}$$

$$= \dot{m}C_p(T - T_A) = \rho G C_p(T - T_A)$$

$$\dot{m} \equiv \text{mass flow rate}$$

$$\rho \equiv \text{fluid (air) density}$$

$$G \equiv \text{volumetric fluid flow rate}$$

$$C_p \equiv \text{specific of fluid (air)}$$

Then $Q_f = C_f\left(T - T_A\right), \quad C_f = \rho G C_p$

Using $\Delta T = 1.76 Q/G$, it is trivial to show

$$C_f = G/1.76$$

The method of solving the vented enclosure problem with TNETFA or similar programs is:

1. Solve circuit for a sealed enclosure, i.e. G very small,.e.g.0.001 ft^3/min.

2. Calculate the airflow circuit resistance R_a for the system.

3. Guess Q_f, *e.g.* $Q_f=(1/4)Q$.

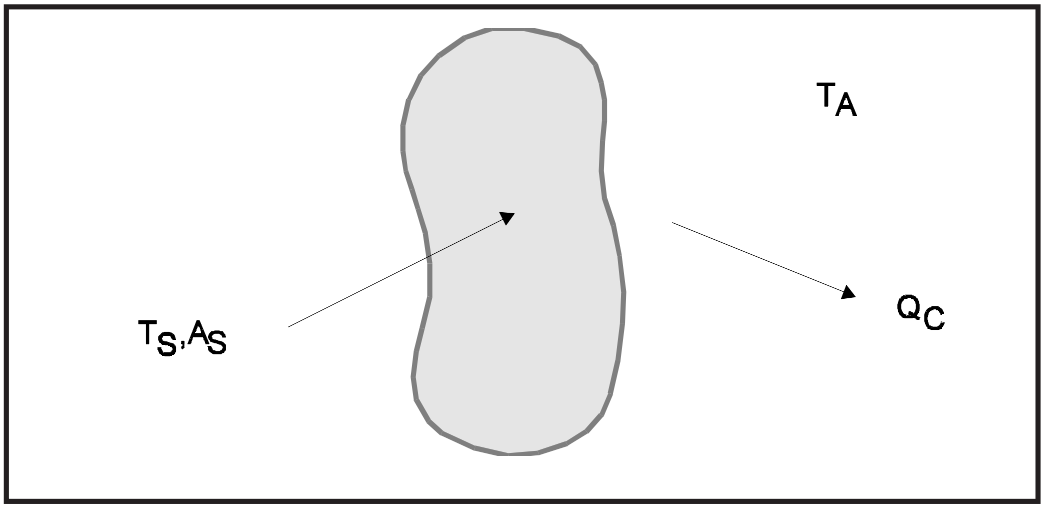
4. Calculate $G=1.53x10^{-2}(Q_f d/R_a)^{1/3}$, revise C_f in model file.

5. Solve circuit to get new Q_f.

6. Repeat steps 4, 5 until solution has converged.

Forced Convection Heat Transfer

Convection from a Surface



Newtonian Cooling -

$$Q_c = h_c A_s (T_s - T_A)$$

$h_c \equiv$ convective heat transfer coefficient

Convection Conductance -

$$C_c = h_c A_s$$

Convection Resistance -

$$R_c = \frac{1}{C_c} = \frac{1}{h_c A_s}$$

Units -

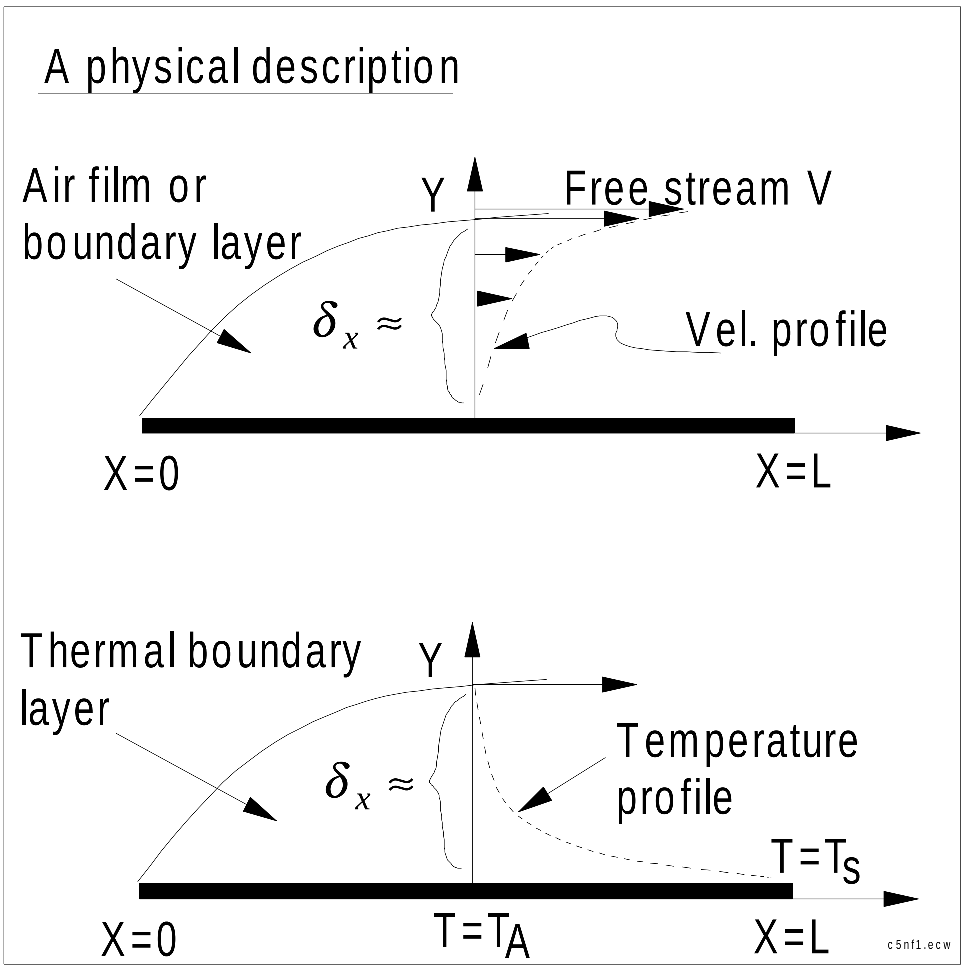
If $Q_c[W], A_s\left[m^2\right], T\left[{}^oC\right]$

$$C_c\left[W/{}^oC\right], h_c\left[W/in.^2 \cdot {}^oC\right]$$

or any consistent set of units.

Convective Heat Transfer Coefficient

$h_c = C_c / A_s$, i.e. a surface conductance per unit area



The thickness of the air aerodynamic and thermal boundary layers develop at about the same rate and very approximately, $h_x \approx k_{air\ film} / \delta_x$.

Nusselt Number

Perhaps a little surprisingly, we find experimental and theoretical expressions for the h_x using a geometric length (for the hardware) using not δ_x, but for example, x, and a multiplying factor Nu_x.

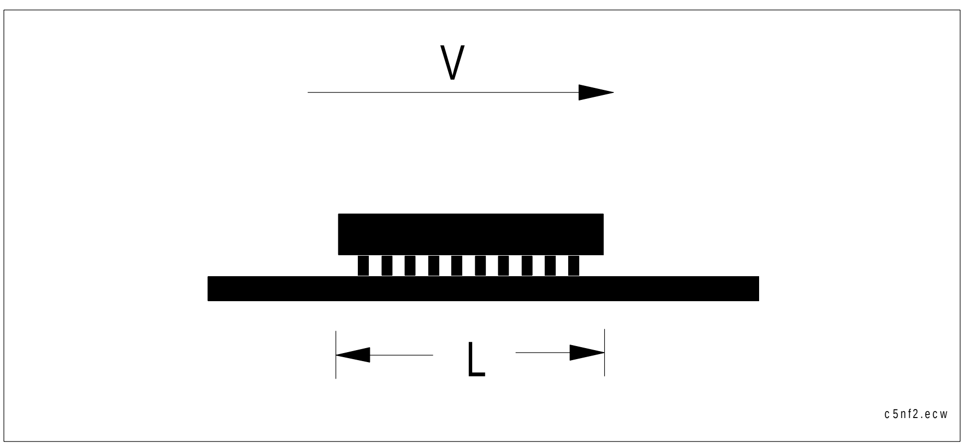
$$h_x = \frac{k}{x} Nu_x$$

$Nu_x \equiv$ Nusselt number, dimensionless

$k =$ thermal conductivity of air

$x =$ a length dimension

For a forced air cooled electronic component



$$\bar{h}_L = \frac{1}{L}\int_0^L h_x dx$$

Preliminaries

Mean air temperature rise -

$$\Delta T\left[{}^{o}C\right] = 1.76\,\frac{Q[W]}{G\left[ft^3\ /\ \min\right]}$$

$G \equiv$ channel flow

$Q \equiv$ heat transferred into flow

Flow velocity -

$$G = (VA)/144$$

$V \equiv$ average velocity, $ft./\min$

$A \equiv$ cross - sectional area of flow, $in.^2$

$$V = 144\,G/A$$

Reynold's number -

Sometimes used to characterize possible flow regime.

$$\mathrm{Re}_p = VP(\rho/\mu)(1/5)$$

$V[ft./\mathrm{min.}]$

$\rho \equiv$ fluid density

$\mu \equiv$ fluid viscosity

$\upsilon = \mu/\rho \equiv$ kinematic viscosity

$= 0.024\ in.^2/\mathrm{sec.},\ 30\ {}^oC$

$P \equiv$ characteristic length, $in.$

Flat plate laminar-turbulent transition probably at

$$\mathrm{Re}_L \approx 5x10^5$$

Duct flow is considered as probably laminar to

$$\mathrm{Re}_{D_H} < 2000$$

and turbulent (fully)

$$\mathrm{Re}_{D_H} > 10{,}000$$

Flat Plate

Nusselt Number -

A classic correlation for flat plate laminar flow is:

$$Nu_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \qquad \text{TCEE 2.20}$$

The local heat transfer coefficient is

$$h_x = \left(\frac{k}{x}\right) Nu_x$$

The average heat transfer coefficient is

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x dx = \frac{1}{L}\int_0^L \left(\frac{k}{x}\right) Nu_x dx = \left(\frac{k}{L}\right)\int_0^L \frac{1}{x}(0.332)\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}\,dx$$

$$= \left(\frac{k}{L}\right)\int_0^L \frac{1}{x}(0.332)\left(\frac{Vx}{\upsilon}\right)^{1/2} \mathrm{Pr}^{1/3}\,dx$$

$$= (0.332)\left(\frac{k}{L}\right)\left(\frac{V}{\upsilon}\right)^{1/2} \mathrm{Pr}^{1/3}\int_0^L \frac{1}{x} x^{1/2} dx$$

$$= (0.332)\left(\frac{k}{L}\right)\left(\frac{V}{\upsilon}\right)^{1/2} \mathrm{Pr}^{1/3}\int_0^L \frac{1}{\sqrt{x}} dx$$

$$= (0.332)\left(\frac{k}{L}\right)\left(\frac{V}{\upsilon}\right)^{1/2} \mathrm{Pr}^{1/3}\left(2\sqrt{L}\right) = 2\left(\frac{k}{L}\right)(0.332)\left(\frac{VL}{\upsilon}\right)^{1/2} \mathrm{Pr}^{1/3}$$

$$= 2\left(\frac{k}{L}\right)(0.332)\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\bar{h}_L = 2h_L$$

Using the properties k, υ for air at $T = 50\ ^oC$,
where $V\left[ft./\text{min.}\right]$, $L[in.]$, $h\left[W/in.^2 \cdot {}^oC\right]$.

$$\boxed{\bar{h}_L = 0.001092\sqrt{\frac{V}{L}}}$$

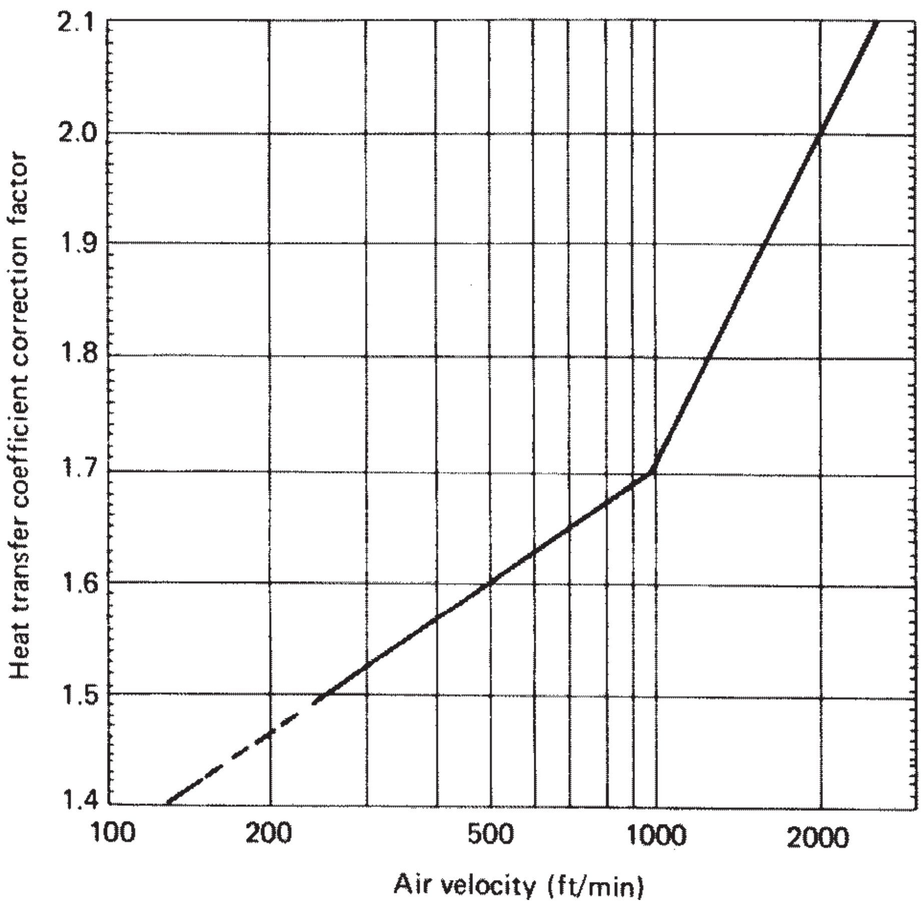
Classical h_L
TCEE E2.24

Note:

$$\overline{Nu_L} \equiv \left(\frac{L}{k}\right)\bar{h}_L = \left(\frac{L}{k}\right)2h_L = 2\left(\frac{L}{k}h_L\right) = 2\,Nu_L$$

Experimental correction -

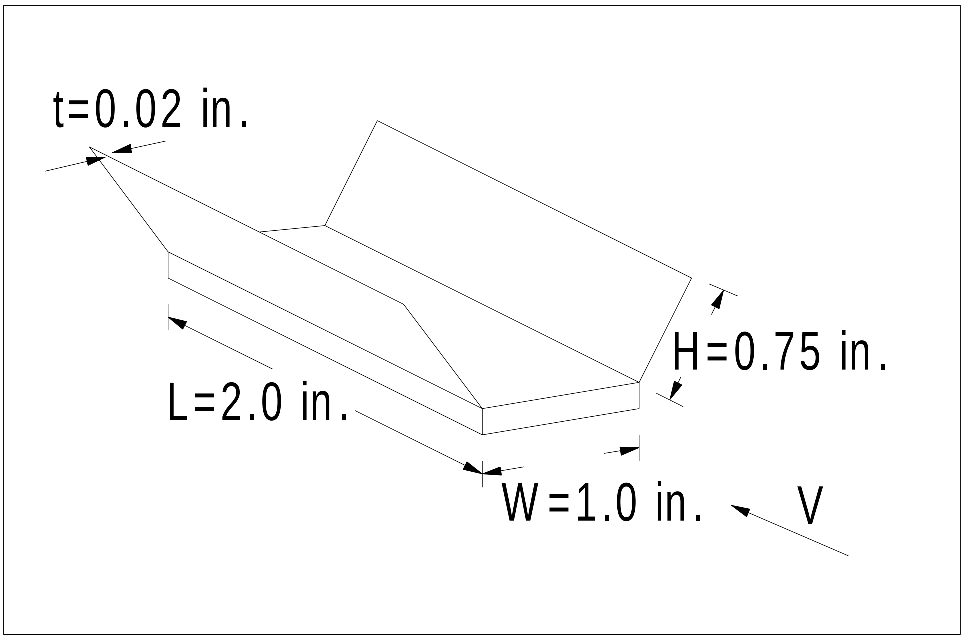
$$\overline{h}_L = \overline{h}_{Classical} \cdot f$$



TCEE Fig. 2-12. correction factor applicable to heat transfer coefficient for laminar flow over a *2.0 in. x 2.0 in. x 0.025 in.* (thick) ceramic substrate.

Example

Winged Aluminum Heat Sink



V = 100 ft./min., Q = 5.0 W dissipated by IC in package.

Objective is to calculate temperature at base of heat sink.

$$A_{Sink} = 4HL + WL = 4(0.75)(2.0) + (1.0)(2.0) = 8\,in.^2$$

$$\begin{aligned}\bar{h}_L &= f\bar{h}_{Classical} = f(0.001092)\left(\sqrt{V/L}\right) \\ &= 1.36(0.001092)\left(\sqrt{100/2.0}\right) = 0.011\,W\big/in.^2\cdot{}^oC\end{aligned}$$

$$R_{Sink} = \frac{1}{\bar{h}_L A_{Sink}} = \frac{1}{(0.011)(8.0)} = 11.4\,{}^oC\big/W$$

The worst case model neglects conduction to board (a poor assumption) and radiation.

$$\Delta T_{Sink\,Base-Ambient} = R_{Sink}Q = (11.4)(5) = 57\,{}^oC$$

Without the heat sink,

$$R_{Case-Ambient} = \frac{1}{\bar{h}A_{Case}} = \frac{1}{(0.011)(1.0)(2.0)} = 45^oC\big/W$$

and worst case,

$$\Delta T_{Case-Ambient} = R_{Case-Ambient}Q = (45)(5) = 225\,{}^oC$$

Check on the fin efficiency for the heat sink fins -

$$C_k = \frac{kLt}{H} = \frac{\left(4.0\,W/in.^2\cdot{}^oC\right)(2.0\,in.)(0.02\,in.)}{0.75\,in.} = 0.21\,W/{}^oC$$

$$C_S = \bar{h}A_{fin} = 2\bar{h}LH$$

$$= 2\left(0.011W/in.^2\cdot{}^oC\right)(2.0\,in.)(0.75in.) = 0.033\,W/{}^oC$$

$$C_S/C_k = 0.033/0.21 = 0.16$$

This indicates a fin efficiency of about $\eta = 0.9$. The corrected heat sink thermal resistance and temperature rise are therefore

$$R_{Sink} = \frac{1}{\eta\bar{h}_L A_{Sink}} = \frac{1}{(0.9)(0.011)(8.0)} = 12.6\,{}^oC/W$$

$$\Delta T_{Sink\,Base-Ambient} = R_{Sink}Q = (12.6)(5) = 63\,{}^oC$$

"Flatpack" Components on PCB

(C.C. Tai and V.T. Lucas (IBM))

$$Nu_x \equiv \text{Nusselt Number} = \frac{hx}{k}$$

$x \equiv$ distance from inlet

$k \equiv$ air thermal conductivity

$h \equiv$ convective heat transfer coefficient

$$Nu_x = C\left\{\left[1+\left(\frac{x}{D_H}\right)^{-0.886}\right]\mathrm{Re}_{D_H}\right\}^n$$

$D_H \equiv$ hydraulic diameter $= 4\,A_c / Perimeter$

$\mathrm{Re}_{D_H} < 2000;\ C = 0.072,\ n = 0.70$

$2000 < \mathrm{Re}_{D_H} < 10{,}000;\ C = 0.0056,\ n = 1.02$

Paper is not very clear on where velocity in Re_{D_H} and A_c, *Perimeter* in D_H are determined, but paper seems to indicate that location is over array, i.e. within component channel and not the approach region.

Not much choice except to assume well mixed air to calculate temperature rise.

Using

$$k_{air} = 6.8x10^{-4} \text{ watt / in.}\cdot {}^{o}C, \upsilon_{air} = 0.0259 \text{ in.}^2/\sec \text{ at } 40\ {}^{o}C$$

and

$$\text{Re}_{D_H} = \frac{VD_H}{5\upsilon_{air}} \text{ for } V[ft/\text{min.}], D_H[in.], \upsilon_{air}\left[in.^2/\sec\right]$$

$$h_{Laminar} = \frac{4.9x10^{-5}}{D_H}\left\{7.7VD_H\left[1+\left(\frac{x}{D_H}\right)^{-0.836}\right]\right\}^{0.70},$$

$$h_{Turbulent} = \frac{3.8x10^{-6}}{D_H}\left\{7.7VD_H\left[1+\left(\frac{x}{D_H}\right)^{-0.836}\right]\right\}^{1.02},$$

x and D_H in $in.$, V in $ft/\text{min.}$, h in $W/in.^2\cdot {}^{o}C$

"DIP" Components on PCBS

(Wills, Martin, May 1983)

$h \equiv$ convective heat transfer coefficient, $W/m^2 \cdot K$

$L \equiv$ component length in airstream direction, m

$P \equiv$ component pitch in airstream direction, m

$N \equiv$ row number

$V \equiv$ air velocity in free stream between component tops opposite board, m/s

$\rho_{air} \equiv$ air density, kg/m^3

Without card guides,

$$h = 5.3\frac{L}{p} + 7.6\frac{V^{0.8}}{(Np)^{0.2}}\left[\left(\frac{p}{L}\right) - 1\right]^{0.13}$$

With card guides,

$$h = 5.3\frac{L}{p} + 6.2\frac{V^{0.8}}{(Np)^{0.36}}\left[\left(\frac{p}{L}\right) - 1\right]^{0.13}$$

Again, not much choice except to assume well mixed air to calculate temperature rise.

In other units,

$h \equiv$ convective heat transfer coefficient, $W/in.^2 \cdot {}^oC$

$L \equiv$ component length in airstream direction, $in.$

$p \equiv$ component pitch in airstream direction, $in.$

$N \equiv$ row number

$V \equiv$ air velocity in free stream between component tops opposite board, $ft./\text{min.}$

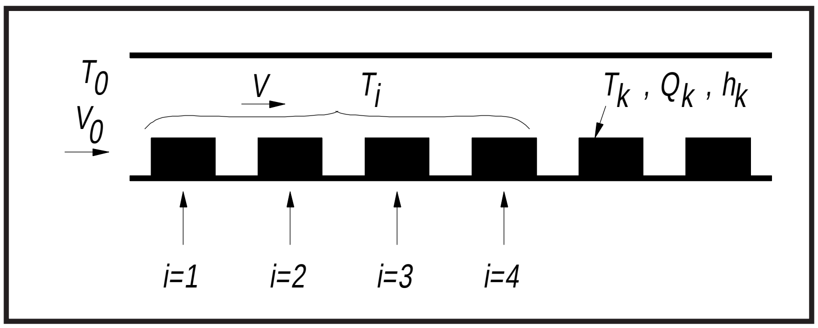
Without card guides,

$$h = 3.42x10^{-3}\frac{L}{p} + 1.60x10^{-4}\frac{V^{0.8}}{(Np)^{0.2}}\left[\left(\frac{p}{L}\right) - 1\right]^{0.13}$$

With card guides,

$$h = 3.42x10^{-3}\frac{L}{p} + 2.35x10^{-4}\frac{V^{0.8}}{(Np)^{0.36}}\left[\left(\frac{p}{L}\right) - 1\right]^{0.13}$$

Adiabatic Heat Transfer Coefficients and Adiabatic Reference Temperature



The preceding correlations for heat transfer coefficients did not use an optimum methodology for determining heat transfer coefficients and local reference temperatures. The method used in the next two correlations is more accepted within the electronics cooling industry.

The preferred experimental method consists of using a test board constructed in such a manner that the heat dissipated by each component is totally convected to the local air, i.e. the board is made of thermal insulation material.

The heat transfer coefficient at each component site is measured by dissipating heat only within the component of interest. The heat transfer coefficient is then calculated from measurements using:

$$h_{ad} = \frac{Q}{A(T_S - T_{ad})}$$

where

Q = heat dissipation by single component of interest

T_S = surface temperature of single component of interest

A = component surface area

T_{ad} = adiabatic reference temperature = inlet temperature during h_{ad} measurement

A thermal "wake function" is defined as the fractional temperature rise (above inlet temperature) of any component surface due to upstream heating by some other element. The thermal wake function for the *kth* element is:

$$\theta_{k-i} = \left. \frac{T_k - T_0}{T_i - T_0} \right|_{\substack{q_i \neq 0 \\ q_k = 0}}$$

where

T_k = adiabatic (surface) temperature of the *kth* element, unheated

T_i = temperature of the *ith* upstream block

T_0 = channel inlet temperature

The package elements are counted from the first row, beginning with one.

The surface temperature of the k^{th} element in an array is computed from:

$$T_k - T_0 = \left(\frac{Q_k}{h_k A_k} \right) + \sum_{i=1}^{k-1} \theta_{k-i} (T_i - T_0) \qquad i < k$$

and

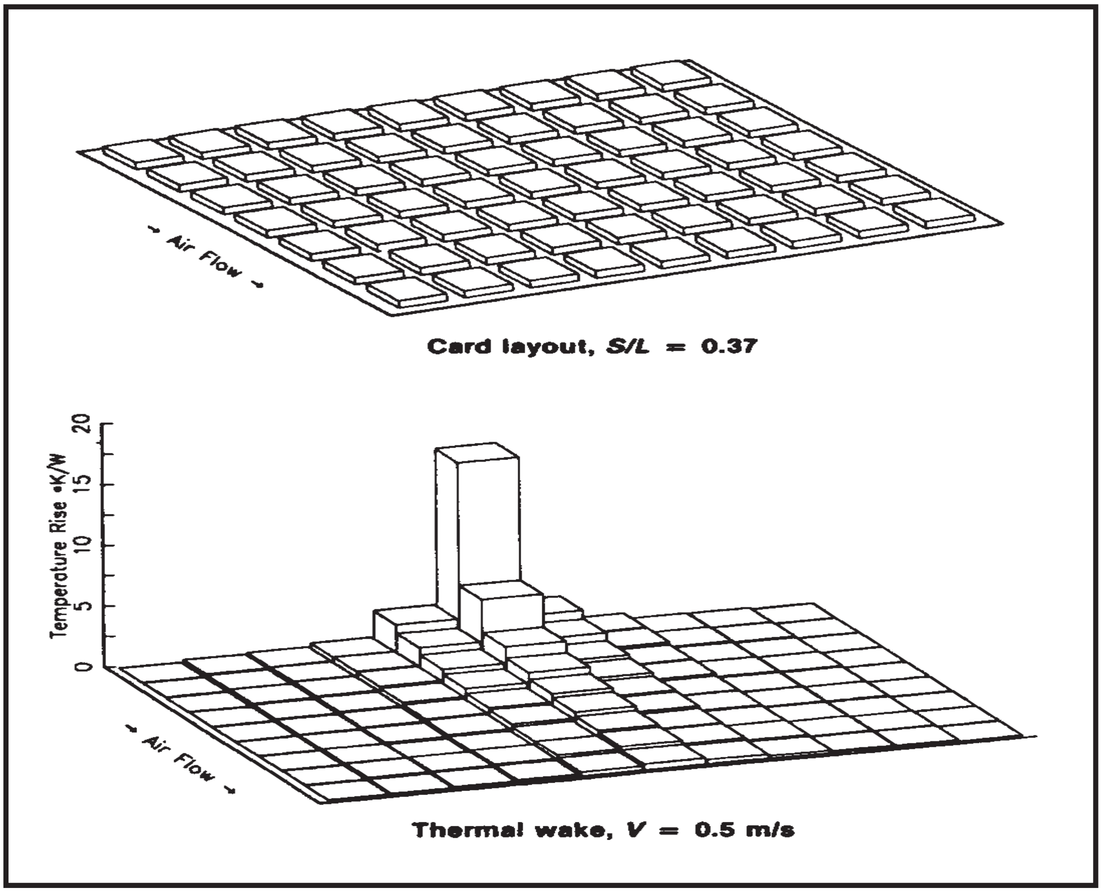
$$h_k = \left. \frac{Q_k}{A_k (T_k - T_0)} \right|_{\substack{only \\ Q_k \neq 0}}$$

A Correlation That Should Be Applicable to a Greater Variation of Component Styles (*B=0.23 in., L=1.46 in.)*

Copeland,David, "Effects of Channel Height and Planar Spacing on Air Cooling of Electronic Components", Journal of Electronic Packaging, Trans. ASME, Vol. 114, Dec. 1992, pp. 420-424. Used With Permission.

This paper includes not only the effects of module self-heating, but also the thermal wake effect (downstream air heating) as pioneered by R. Moffat and studied by his numerous students.

Illustration of thermal wake effect from Copeland's paper (*S*=component edge-to-edge spacing, *L*=component length):



Copeland's heat transfer coefficient

$V \equiv$ approach velocity, m / s

$H \equiv$ channel height, component surface to channel wall, mm

$B \equiv$ component height, mm

$h \equiv$ heat transfer coefficient, $W/m^2 \cdot K$

$Row \equiv$ row number of heated component

$$h = \left[25.9 + 4.1V_{app}\right]\left[1 + 0.15e^{(1-Row)}\right](H/B)^{0.02}; \qquad S/L = 0.03$$

$$h = \left[20.0 + 7.0V_{app}\right]\left[1 + 0.14e^{(1-Row)}\right](H/B)^{0.02}; \qquad S/L = 0.37$$

$$h = \left[18.5 + 7.1V_{app}\right]\left[1 + 0.02e^{(1-Row)}\right](H/B)^{0.11}; \qquad S/L = 1.05$$

Copeland's thermal wake function

Thermal wake at first row after heated component where *Row*= row number of component contributing this part of wake.

$\theta \equiv$ temperature rise at component in wake per watt heat dissipation at heated component, ${}^{o}C/W$

$$\theta_1 = \left[1 - 0.09e^{(1-Row)}\right](H/B)^{-0.10} \Big/ \left[0.14 + 0.03V_{app}\right];\ S/L = 0.03$$

$$\theta_1 = \left[1 - 0.13e^{(1-Row)}\right](H/B)^{-0.16} \Big/ \left[0.12 + 0.06V_{app}\right];\ S/L = 0.37$$

$$\theta_1 = \left[1 - 0.03e^{(1-Row)}\right](H/B)^{-0.61} \Big/ \left[0.09 + 0.05V_{app}\right];\ S/L = 1.05$$

Thermal wake further downstream of heated component where *N*= number of rows past the heated component

$$\theta_N / \theta_1 = 1/N$$

Copeland's heat transfer coefficient in other units

$V \equiv$ approach velocity, ft / min

$H \equiv$ channel height, component surface to channel wall, in,

$B \equiv$ component height, $in.$

$h \equiv$ heat transfer coefficient, $W/in.^2 \cdot {}^oC$

$Row \equiv$ row number of heated component

$$h = \left[1.67x10^{-2} + 1.34x10^{-5} V_{app}\right]\left[1 + 0.15e^{(1-Row)}\right]\left(\frac{H}{B}\right)^{0.02}; \frac{S}{L} = 0.03$$

$$h = \left[1.29x10^{-2} + 2.29x10^{-5} V_{app}\right]\left[1 + 0.14e^{(1-Row)}\right]\left(\frac{H}{B}\right)^{0.02}; \frac{S}{L} = 0.37$$

$$h = \left[1.19x10^{-2} + 2.33x10^{-5} V_{app}\right]\left[1 + 0.02e^{(1-Row)}\right]\left(\frac{H}{B}\right)^{0.11}; \frac{S}{L} = 1.05$$

Copeland's thermal wake function in other units

Thermal wake at first row after heated component where *Row*= row number of component contributing this part of wake.

$$\theta \equiv \text{temperature rise at component in wake per watt heat dissipation at heated component, } {}^{o}C/W$$

$$\theta_1 = \left[1 - 0.09e^{(1-Row)}\right]\left(\frac{H}{B}\right)^{-0.10} \Big/ \left[0.14 + 1.52x10^{-4}V_{app}\right]; \frac{S}{L} = 0.03$$

$$\theta_1 = \left[1 - 0.13e^{(1-Row)}\right]\left(\frac{H}{B}\right)^{-0.16} \Big/ \left[0.12 + 3.05x10^{-4}V_{app}\right]; \frac{S}{L} = 0.37$$

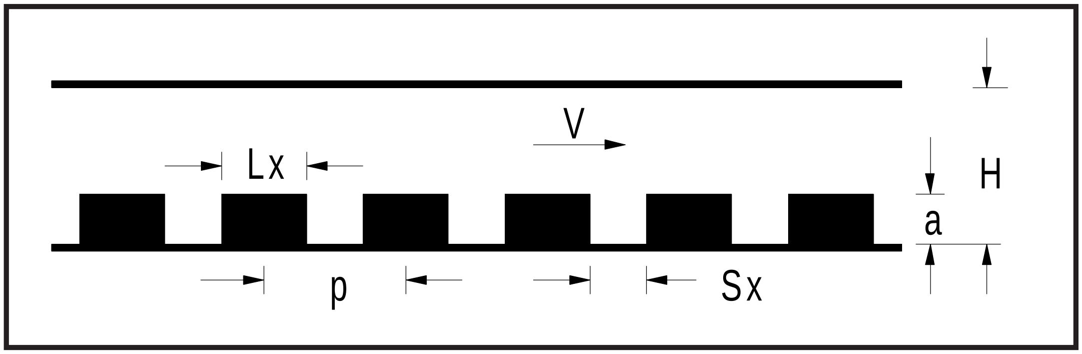
$$\theta_1 = \left[1 - 0.03e^{(1-Row)}\right]\left(\frac{H}{B}\right)^{-0.61} \Big/ \left[0.09 + 2.54x10^{-4}V_{app}\right]; \frac{S}{L} = 1.05$$

Thermal wake further downstream of heated component where *N*= number of rows past the heated component

$$\theta_N / \theta_1 = 1/N$$

M. Faghri's Et Al. Heat Transfer Coefficient and Thermal Wake Function

(Faghri, M., 1996)



The range of parameters for which the study was conducted are:

$$S_x / L_x = 0.128: \qquad (H - a)/L_x = 0.128 \rightarrow 0.765$$

$$S_x / L_x = 0.33: \qquad (H - a)/L_x = 0.25 \rightarrow 1.0$$

$$Nu = \left[1 + 0.0786\left(\frac{x}{D_h}\right)^{-1.099}\right]\left[\frac{\left(\dfrac{H-a}{L_x}\right)^{-0.670}}{2.912\,\mathrm{Re}^{-0.607}\left(\dfrac{S_x}{L_x}\right)^{-0.295}}\right]$$

where $\mathrm{Re} = \dfrac{V(H-a)}{\nu}, \qquad D_h = \dfrac{2WH}{(W+H)}$

In - line effects:

$$\theta_1 = \frac{\left(\frac{S_x}{L_x}\right)^{-0.540}}{2.681 \mathrm{Re}^{0.168}}$$

$$\frac{\theta_N}{\theta_1} = 0.151 + 0.849 N^{-1.314}$$

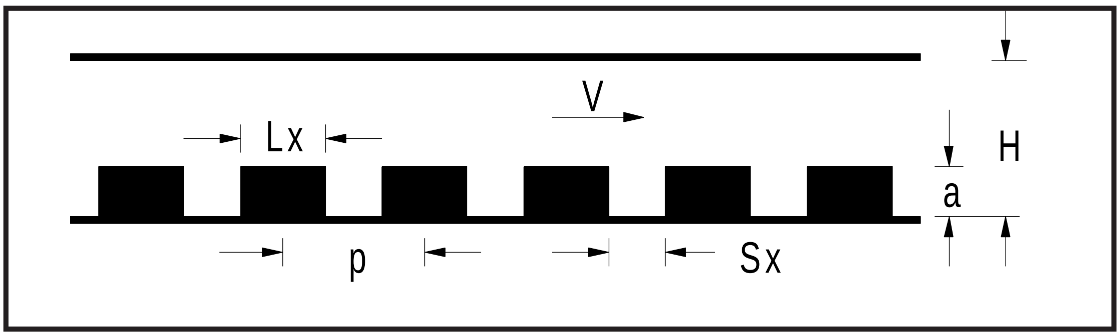
Flank effects:

$$\frac{\theta_{fN}}{\theta_1} = 0.0575 N^{1.128}$$

Wirtz's Summary of Heat Transfer Coefficients and Wake Function

(Wirtz, R., 1996)

Wirtz defines a low profile package as one for which the package top represents at least 50% of the total package area.



Remembering that

$$T_k - T_0 = \left(\frac{Q_k}{h_k A_k} \right) + \sum_{i=1}^{k-1} \theta_{k-i} (T_i - T_0) \qquad i < k$$

and

$$h_k = \left. \frac{Q_k}{A_k (T_k - T_0)} \right|_{\substack{only \\ Q_k \neq 0}}$$

Wirtz recommends:

$$Nu_{L_x} = 0.6\,\mathrm{Re}_{L_x}^{0.5}\,\mathrm{Pr}^{0.33}, \quad \mathrm{Re}_{L_x} \le 5000$$

$$Nu_{L_x} = 0.082\,\mathrm{Re}_{L_x}^{0.72}, \quad \mathrm{Re}_{L_x} > 5000$$

and for very low profile packages,

$$Nu_{L_x} = 0.07\,\mathrm{Re}_{L_x}^{0.718}, \quad \mathrm{Re}_{L_x} > 5000$$

where *Pr* is the coolant Prandtl number and the Reynold's number is $\mathrm{Re}_{L_x} = \rho V L_x / \mu = V L_x / \upsilon$ for V, L_x, L_y and υ are the by-pass velocity, component length in flow direction, component length in transverse direction, and kinematic viscosity, respectively.

The range of geometric data from which the Nusselt Numbers are constructed is as follows:

Author	***L_x (mm)***	***L_x (in.)***	***L_x/a***	***Sigma***	***H/a***
Wirtz & Dykshoorn (1)	25.4	1.0	4.00	0.25	1.5-4.6
Sparrow et al. (2)	26.7	1.05	2.67	0.64	2.7
Anderson & Moffat (3)	37.5	1.48	3.95	0.59	1.5-4.6
Wirtz et al. (4)	56.0	2.21	8.75	0.49	1.5-10
Wirtz & Mathur (5)	69.8	2.75	6.0	0.45	2.0
Wirtz & Colban (6)	69.8	2.75	6.0	0.45-0.69	2.0

$$\sigma \equiv \text{Packing density} = \frac{L_x L_y}{(L_x + S_x)(L_y + S_y)}$$

Wirtz recommends Row 1, $h_k = 1.25(k/L_x)Nu_{L_x}$

Row 2, $h_k = 1.10(k/L_x)Nu_{L_x}$

Rows 3..., $h_k = (k/L_x)Nu_{L_x}$

For a thermal wake function, Wirtz notes that Kang (Kang, S.S., *The Thermal Wake Function for Rectangular Electronic Modules*, J. Electron. Packag., Vol. 116, pp. 55-59) presents formulae for both laminar and turbulent flow, but that the wake function for laminar flow is not yet substantiated by extensive experimental data. Both are given here:

Laminar flow -

$$\theta_{1L} = 0.42\left(\frac{p}{L_x}\right)^{-0.5}$$

Turbulent flow -

$$\theta_{1T} = 7.19C\,\mathrm{Pr}^{-0.5}\left(\frac{L_x}{p}\right)^{0.5}\left(\frac{L_x}{H}\right)^{0.44}\left(1-\frac{a}{H}\right)^{0.5}\mathrm{Re}_{0,L}^{n-0.94}$$

where $\mathrm{Re}_{0,L}$ is the package Reynold's number based on the inlet average velocity, V_0.

The constant C and exponent n, are the coefficient and exponent, respectively, for the power-law for the Nusselt number for the packages $Nu_{L_x} = C\,\mathrm{Re}_{0,L_x}^{n}$

The thermal wake function for the column packages for both laminar and turbulent flow is given by

$$\theta_{k-i} = \theta_1 \left(\frac{1}{k-i} \right)^m,$$

$$m \cong 0.5 + 0.335 e^{\left(-\frac{Pe}{20}\right)} + 0.105 e^{\left(-\frac{Pe}{100}\right)} + 0.06 e^{\left(-\frac{Pe}{2500}\right)}$$

$$Pe = \mathrm{Re}\,\mathrm{Pr}$$

where Pe is the Peclet number (product of Reynold's and Prandtl numbers).

The Peclet number is based on the inlet velocity and a length scale $\frac{L_y^2}{p}$.

For laminar flow

$$Pe_L = \frac{V_0 \left(\frac{L_y^2}{p} \right)}{\upsilon} \mathrm{Pr}$$

L_x, L_y, p are the component package length in the flow direction, the width in the transverse direction, and the pitch in the flow direction.

For turbulent flow, Kang offers a Peclet number using a derived thermal diffusivity α_t for turbulent flow based on the molecular diffusivity $\alpha = k / (C_p \rho)$. Using consistent units,

$$Pe_t = \frac{VL_y^2}{0.006p}\left(\frac{C_p\rho}{k}\right)\left(\frac{1}{\mathrm{Re}_H^{0.88}}\right)$$

For units of *V[ft/min.], L_y[in.], p[in.], C_p[J/gmK], k[W/inK], ρ [gm/in^3], and Re_H* based on the bypass velocity *V*, and the card-to-card spacing *H:*

$$Pe_t = \frac{VL_y^2}{0.006p}\left(\frac{C_p\rho}{k}\right)\left(\frac{1}{\mathrm{Re}_H^{0.88}}\right)\left(\frac{1}{5}\right)$$

Application Example Using Wirtz's Summary - 7 Rows of DIPS on 8.05 in. Wide x5.3 in. Long PCB

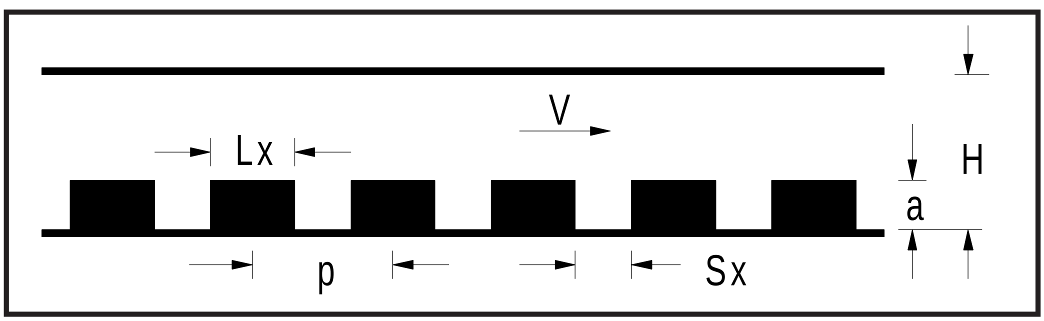
Component height, bottom to top = 0.15 in.
a = component height including air gap = 0.163 in.
H = board surface to board surface spacing=1.0 in.
W = board width = 8.05 in.
p = pitch = 0.7
L_x = component length = 0.24 in.
L_y=component width=0.82 in.
S_x = component spacing = 0.46 in.
V = by-pass velocity (above components) = 305 ft./min.
Q_{Chip}=0.33 W
T_0 = inlet temperature ^oC

$$L_x / a = 0.24 / 0.163 = 1.47$$

$$H / a = 1.0 / 0.163 = 6.14$$

$$\sigma = \frac{(0.24)(0.82)}{(0.24 + 0.46)(0.82 + 0.0)} = 0.34$$

In this example we shall assume an insulating board which implies that all of each package heat dissipation is convected directly from the package surface area.



$$\upsilon = 0.025 in^2 / s, \ \mathrm{Pr} = 0.72, \ k = 6.5x10^{-4}\, W / in.\cdot{}^{o}C$$

$$S_x = 0.46\, in., \ L_x = 0.24\, in., \ L_y = 0.82\, in., \ H = 1.0\, in.$$

$$a = 0.123 + 0.04 \ in.$$

$$V = 305 \ ft / \min, \ T_0 = 0$$

$$Q_1 = Q_2 ...... = Q_7 = 0.33W$$

$$A = L_x L_y + 2(0.82x0.123 + 0.24x0.123) = 0.458\, in.^2$$

The Reynold's number based on component length is

$$\mathrm{Re}_{L_x} = \frac{VL_x}{5\nu} = \frac{(305)(0.24)}{5(0.025)} = 586$$

which certainly indicates laminar flow.

The laminar flow Peclet Number (RePr) is and *m* are

$$Pe_L = \frac{\left(\frac{H-a}{H}\right)V\left(\frac{L_y^2}{p}\right)}{5\nu}\mathrm{Pr} = \frac{\left(\frac{1.0-0.163}{1.0}\right)(305)\frac{(0.82)}{0.7}}{5(0.025)} = 1412$$

$$m_L = 0.5 + 0.335e^{-\frac{Pe_L}{20}} + 0.105e^{-\frac{Pe_L}{100}} + 0.06e^{-\frac{Pe_L}{2500}}$$

$$= 0.5 + 7.14x10^{-32} + 7.71x10^{-8} + 3.41x10^{-2} = 0.534$$

$$\theta_{1L} = 0.42\left(\frac{p}{L_x}\right)^{-0.5} = 0.42\left(\frac{0.7}{0.24}\right)^{-0.5} = 0.246$$

Using $$\theta_{k-i} = \theta_1\left(\frac{1}{k-i}\right)^m = 0.246\frac{1}{(k-i)^{0.534}}$$

$$\theta_2 = 0.1699$$
$$\theta_3 = 0.1368$$
$$\theta_4 = 0.1173$$
$$\theta_5 = 0.1042$$

Each heat transfer coeficient is calculated using the "fully developed" value,

$$h_{Laminar} = \frac{k}{L_x} C_{Laminar} \mathrm{Re}_{L_x}^{n_{Laminar}} \mathrm{Pr}^{0.33}$$
$$= \left(\frac{6.5x10^{-4} W/in.\cdot {}^oC}{0.24 in.}\right)(0.6)(586)^{0.5}(0.7)^{0.33}$$
$$= 0.35 W/in.^2 \cdot {}^oC$$

First Row -

$$h_{L1} = 1.25 h_{Laminar} = 0.044 W / in^2 \cdot {}^oC$$

$$R_1 = 1 / h_{L1}A = 49.0\,{}^oC/W$$
$$T_1 - T_0 = \frac{Q_1}{h_{L1}A} = 16.2$$

Second Row -

$$h_{L2} = 1.10 h_{La\min ar} = 0.039 W / in^2 \cdot {}^oC$$

$$\Delta T_{2Air} = \theta_{1L} \left(T_1 - T_0\right) = 0.246\left(16.2\right) = 3.99\,{}^oC$$

$$T_2 - T_0 = \frac{Q_2}{h_{L2}A} + \Delta T_{2Air} == \frac{0.33}{(0.039)(0.458)} + 3.99 =$$
$$= 18.47 + 3.99 = 22.5\,{}^oC$$

Third Row -

$$h_{L3} = h_{La\min ar} = 0.035 W / in^2 \cdot {}^oC$$

$$\Delta T_{3Air} = \theta_{3-1}\left(T_1 - T_0\right) + \theta_{3-2}\left(T_2 - T_0\right) = \theta_2\left(T_1 - T_0\right)$$
$$+ \theta_{1L}\left(T_2 - T_0\right)$$
$$= 0.1699\left(16.2\right) + 0.246\left(22.5\right) = 2.75 + 5.54 = 8.29\,{}^oC$$

$$T_3 - T_0 = \frac{Q_3}{h_{L3}A} + \Delta T_{3Air} = \frac{0.33}{(0.035)(0.458)} + 8.29 = 20.6 + 8.29$$
$$= 28.9\,{}^oC$$

Fourth Row -

$$h_{L4} = h_{Laminar} = 0.035 W / in^2 \cdot {}^oC$$
$$R_4 = 1/h_4 A = 61.2\, {}^oC/W$$

$$\Delta T_{4Air} = \theta_{4-1}(T_1 - T_0) + \theta_{4-2}(T_2 - T_0) + \theta_{1L}(T_3 - T_0)$$
$$= 13.1^oC$$

$$T_4 - T_0 = \frac{Q_4}{h_{L4} A} + \Delta T_{4Air} = 33.6\, {}^oC$$

Fifth Row -

$$h_{L5} = h_{Laminar} = 0.035 W / in^2 \cdot {}^oC$$
$$R_5 = 1/h4A = 61.2\, {}^oC/W$$

$$\Delta T_{5Air} = \theta_{5-1}(T_1 - T_0) + \theta_{5-2}(T_2 - T_0) + \theta_{5-3}(T_3 - T_0)$$
$$+ \theta_{1L}(T_4 - T_0) = 18.2\, {}^oC$$

$$T_5 - T_0 = \frac{Q_5}{h_{L5} A} + \Delta T_{5Air} = 38.6\, {}^oC$$

Sixth Row -

$$h_{L6} = h_{Laminar} = 0.035 W / in^2 \cdot {}^oC$$
$$R_6 = 1 / h_{Laminar} A = 61.2 \, {}^oC/W$$

$$\Delta T_{6Air} = \theta_{6-1}(T_1 - T_0) + \theta_{6-2}(T_2 - T_0) + \theta_{6-3}(T_3 - T_0) + \\ + \theta_{6-4}(T_4 - T_0) + \theta_{1L}(T_5 - T_0) \\ = 23.5 \, {}^oC$$

$$T_6 - T_0 = \frac{Q_6}{h_{L6} A} + \Delta T_{6Air} = 43.9 \, {}^oC$$

Seventh Row -

$$h_{L7} = h_{Laminar} = 0.035 W / in^2 \cdot {}^oC$$
$$R_7 = 1 / h_{Laminar} A = 61.2 \, {}^oC/W$$

$$\Delta T_{7Air} = \theta_{7-1}(T_1 - T_0) + \theta_{7-2}(T_2 - T_0) + \theta_{7-3}(T_3 - T_0) + \\ + \theta_{7-4}(T_4 - T_0) + \theta_{7-5}(T_5 - T_0) + \theta_{1L}(T_6 - T_0) \\ = 29.2 \, {}^oC$$

$$T_7 - T_0 = \frac{Q_7}{h_{L7} A} + \Delta T_{7Air} = 49.6 \, {}^oC$$

Application Example - 7 Rows of DIPS: Duct Flow Using A "Revised" Correlation (Details of Correlation Discussed in Following "Duct Section" of Notes) -

$$D_H = \frac{4(W \cdot H)}{2(W+H)} = \frac{4(1.0\,in. - 0.06\,in. - 0.15\,in.)(8.05\,in.)}{2\left[(1.0\,in. - 0.06\,in. - 0.15\,in.) + 8.05\,in.\right]} = 1.44\,in.$$

$$\mathrm{Re}_{D_H} = \frac{VD_H}{5\mu} = \frac{(305\,ft./\min.)(1.44\,in.)}{5 \cdot 0.026} = 3.4x10^3$$

which suggests turbulent flow (actually transitional), i.e. for fully developed flow,

$$Nu_{D_H} = 0.023\,\mathrm{Re}_{D_H}^{0.8}$$

and

$$\frac{\bar{h}_L}{h_{D_H}} = 1 + 1.68\left(\frac{D_H}{L}\right)^{0.58},\ 2 \le \frac{L}{D_H} = \frac{5.1\,in.}{1.66\,in.} = 3.1 \le 20$$

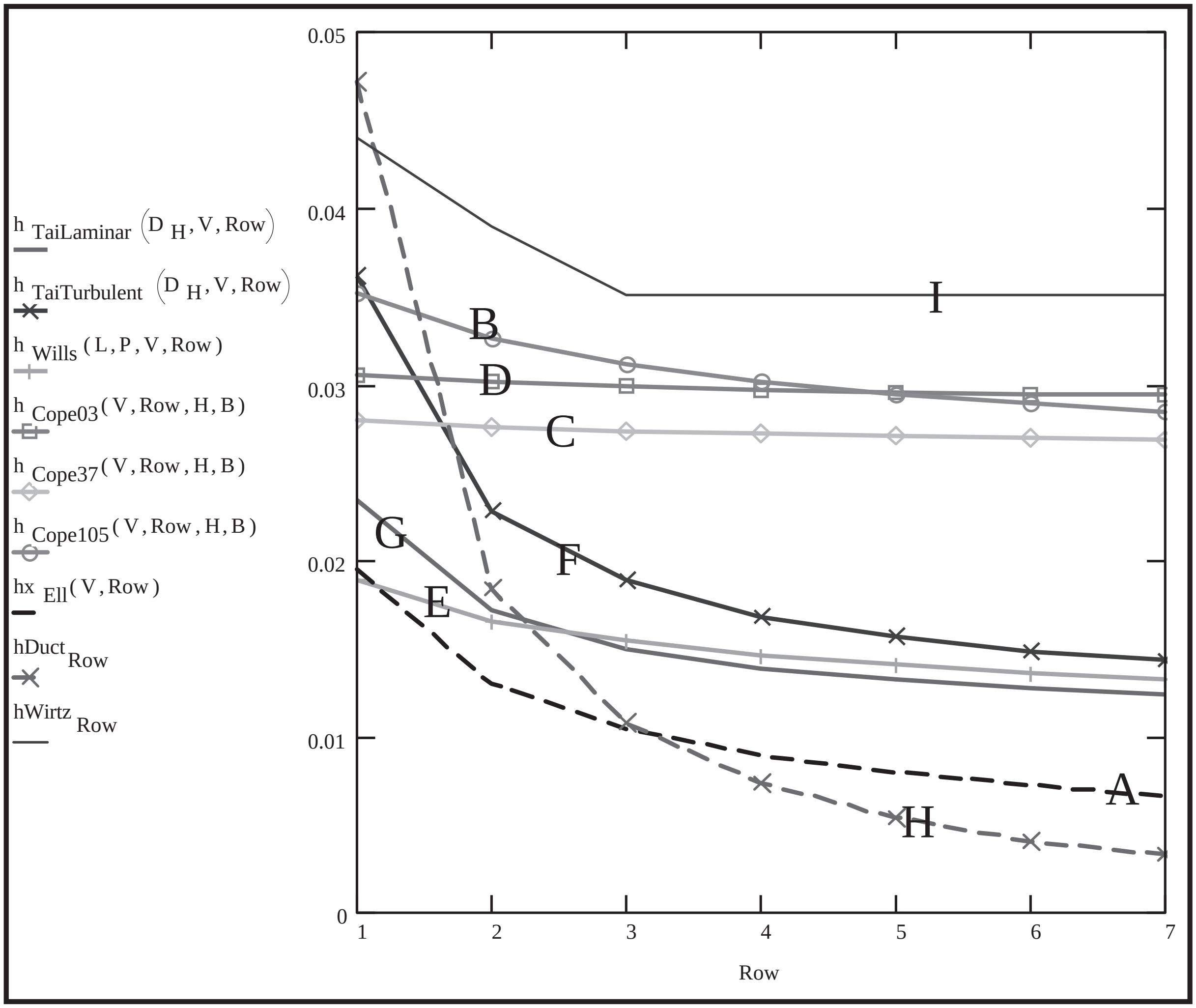
for a mean *h*.

The local *h* is calculated using the TCEE (revised) Fig. 2-19 , Nu_{D_H}, and

$$Nu_{D_H} = h_\infty \frac{D_H}{k_{air}},\ Nu_x = h_x \frac{x}{k_{air}}$$

$$\frac{h_x}{h_\infty} = \left(\frac{D_H}{x}\right)\left(\frac{Nu_x}{Nu_{D_H}}\right)$$

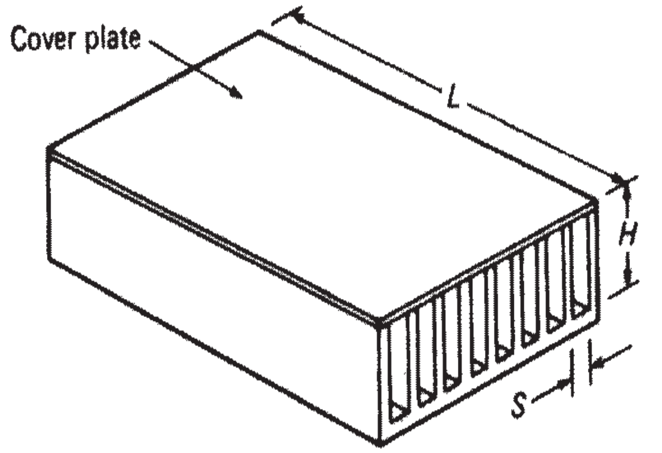
Plot of heat transfer coefficients



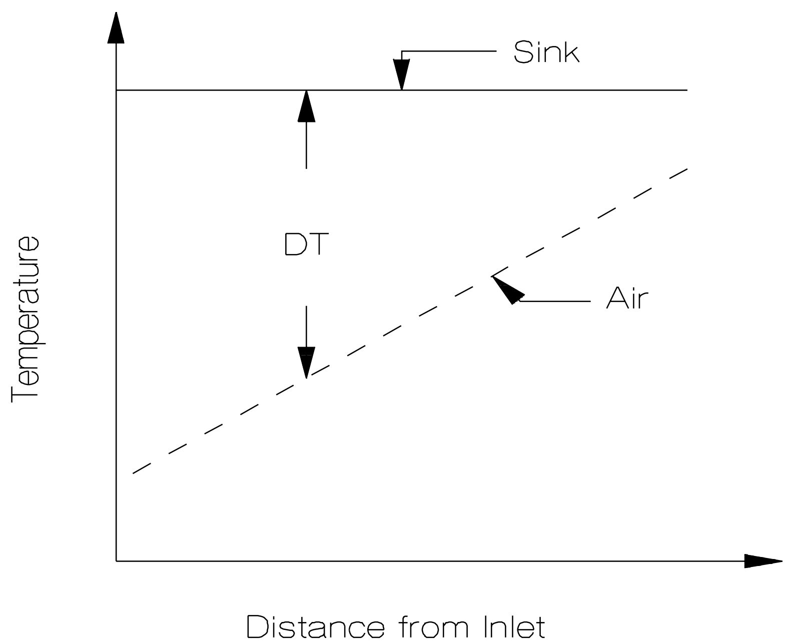
A: Ellison flat plate
B: Copeland, S/L=1.05
C: Copeland, S/L=0.37
D: Copeland, S/L=0.03
E: Wills
F: Tai & Lucas, turbulent
G: Tai & Lucas, laminar
H: "Revised" duct, turbulent
I: Wirtz

Ducts and Finned Heat Sinks - A Model

The Problem



Model



Circuit Representations of Model

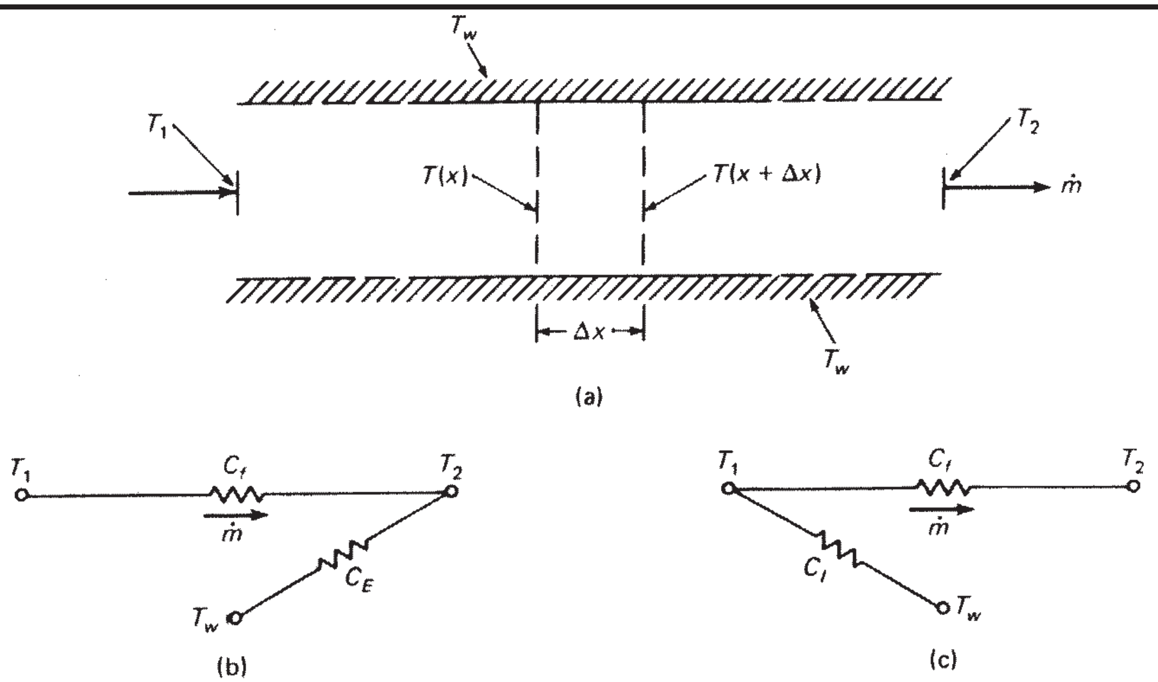


Fig. 2-16. Representation of duct flow heat transfer models. (a) Continuous model of duct section. (b) Two-element model referenced to exit temperature. (c) Two-element model referenced to inlet temperature.

Information on model solutions may be found in text.

Referring to Fig. 2-16 (a), the heat ΔQ_{wf} transferred from a short element Δx of wall is:

$$\Delta Q_{wf} \cong hP\Delta x\left[T_w - T(x)\right]$$

for a duct perimeter P.

The heat ΔQ_a absorbed by the air element with a mass flow $\dot{m}$ rate is:

$$\Delta Q_a \cong \dot{m}C_p\left[T(x+\Delta x) - T(x)\right]$$

In the limit $\Delta x \rightarrow 0$:

$$dQ_{wf} = hP(T_w - T)dx$$
$$dQ_a = \dot{m}C_p dT$$

Conservation of energy requires that:

$$dQ_{wf} = dQ_a$$
$$hP(T_w - T)dx = \dot{m}C_p dT$$
$$\int_{T_1}^{T_2} \frac{dT}{T - T_w} = -\int_0^L \frac{hP}{\dot{m}C_p} dx$$
$$\ln\left(\frac{T_2 - T_w}{T_1 - T_w}\right) = -\frac{hA_s}{\dot{m}C_p}$$
$$T_w - T_2 = (T_w - T_1)e^{-\beta}, \quad \beta = \frac{hA_s}{\dot{m}C_p}$$

Adding T_1 to both sides of

$$T_w - T_2 = (T_w - T_1)e^{-\beta}$$

$$T_1 + (T_w - T_2) = T_1 + (T_w - T_1)e^{-\beta}$$

$$T_1 - T_2 = (T_1 - T_w) + (T_w - T_1)e^{-\beta} = (T_w - T_1)\left(e^{-\beta} - 1\right)$$

$$T_2 - T_1 = (T_w - T_1)\left(1 - e^{-\beta}\right)$$

and since

$$T_2 - T_1 = \frac{Q}{\dot{m}C_p}$$

$$\frac{Q}{\dot{m}C_p} = (T_w - T_1)\left(1 - e^{-\beta}\right)$$

$$\frac{Q}{(T_w - T_1)} = \left(1 - e^{-\beta}\right)$$

Using the definition of C_I

$$C_I \equiv \frac{Q}{(T_w - T_1)}$$

$$C_I = \left(\frac{hA_s}{\beta}\right)\left(1 - e^{-\beta}\right)$$

$$\frac{C_I}{hA_s} = \frac{1 - e^{-\beta}}{\beta}$$

TCEE E2.32

Returning to

$$T_w - T_2 = (T_w - T_1)e^{-\beta}$$

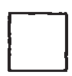
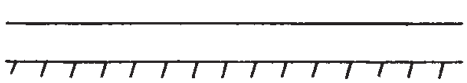
$$C_E \equiv \frac{Q}{T_w - T_2} = \frac{Q}{(T_w - T_1)}e^{\beta}$$

$$= C_I e^{\beta} = hA_s \frac{\left(1 - e^{-\beta}\right)}{\beta} e^{\beta}$$

$$\frac{C_E}{hA_s} = \frac{\left(e^{\beta} - 1\right)}{\beta} \qquad \text{TCEE E2.31}$$

Laminar Duct Flow - Fully Developed

Table 2-9. Nusselt numbers for fully developed velocity and temperature profiles in tubes of various cross sections with laminar flow. The constant-heat-rate solutions are based on constant axial heat rate, but with constant temperature around the tube periphery. Nusselt numbers are averages with respect to tube periphery. $Nu_{(H)}$ for constant heat rate, $Nu_{(T)}$ for constant temperature. From *Convective Heat and Mass Transfer*, 2nd Edition, by W. M. Kays and M. E. Crawford. Copyright © 1980, 1966 by McGraw-Hill, Inc. Used with the permission of McGraw-Hill Book Company.

Cross Section Shape	b/a	$Nu_{(H)}$	$Nu_{(T)}$
		4.364	3.66
	1.0	3.63	2.98
	1.4	3.78	
	2.0	4.11	3.39
	3.0	4.77	
	4.0	5.35	4.44
	8.0	6.60	5.95
	∞	8.235	7.54
		5.385	4.86
		3.00	2.35

Mean Nusselt Number With Respect to Duct Length - Plot for Constant Wall Temperature:

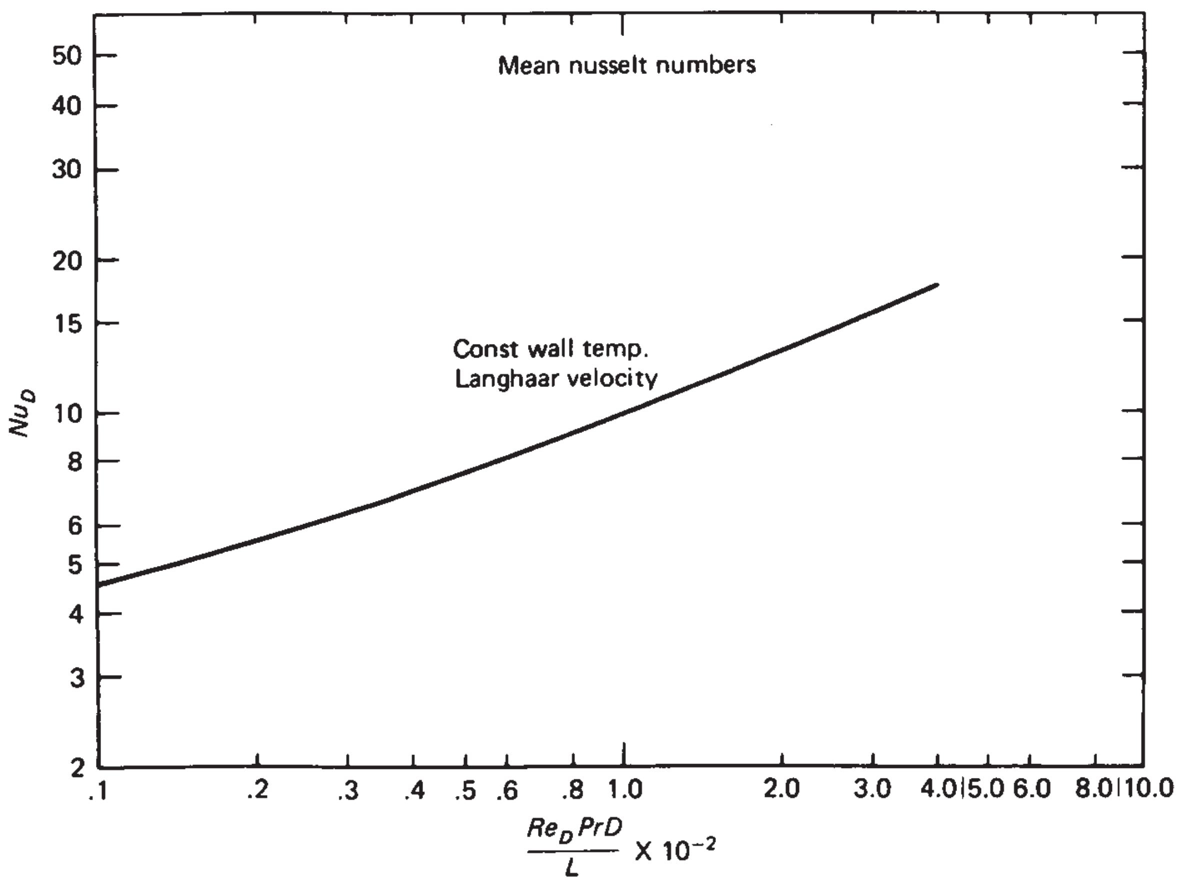


Fig. 2-18. Mean Nusselt numbers with respect to tube length. From [13], reprinted by permission of The American Society of Mechanical Engineers.

Mean *Nu* With Respect to Duct Length, Constant Wall Temperature, Langhaar Velocity Profile: TCEE 2.34.

$$\overline{N}u_{D_H} = 3.66 + \frac{0.104\left(\dfrac{\mathrm{Re}_{D_H}\,\mathrm{Pr}}{L/D_H}\right)}{1+0.016\left(\dfrac{\mathrm{Re}_{D_H}\,\mathrm{Pr}}{L/D_H}\right)^{0.8}}$$

Application? Extruded heat sink.

Local *Nu*, Constant Heat Input, Langhaar Velocity Profile:

$$\overline{N}u_{x} = 4.36 + \frac{0.036\left(\dfrac{\mathrm{Re}_{D_H}\,\mathrm{Pr}}{x/D_H}\right)}{1+0.0011\left(\dfrac{\mathrm{Re}_{D_H}\,\mathrm{Pr}}{x/D_H}\right)}$$

Application? Circuit board cooling,

Recommended Procedure for Applying Circular Cross-Section Duct Nu to Rectangular Cross-Section Ducts:

1. If $\mathrm{Re}_{D_H} < 2100$, calculate Nu from appropriate formula for laminar flow with entry length effects.

2. Using TCEE Table 2-9, determine fully developed Nu for correct rectangular duct aspect ratio.

3. Get ratio . $r_{Nu} = \dfrac{Nu - \text{rectangular}}{Nu - \text{circular}}$

4. Multiply Nu from step 1 by step 3 result, i.e.

$$Nu(\text{rectangular duct}) = Nu_{D_H}(\text{circular duct}) \cdot r_{Nu}$$

Turbulent Duct Flow

Fully Developed:

$$Nu_{D_H} = 0.023\,\mathrm{Re}_{D_H}^{0.8} \qquad \text{TCEE E2.37}$$

Local Entry Length Effects:

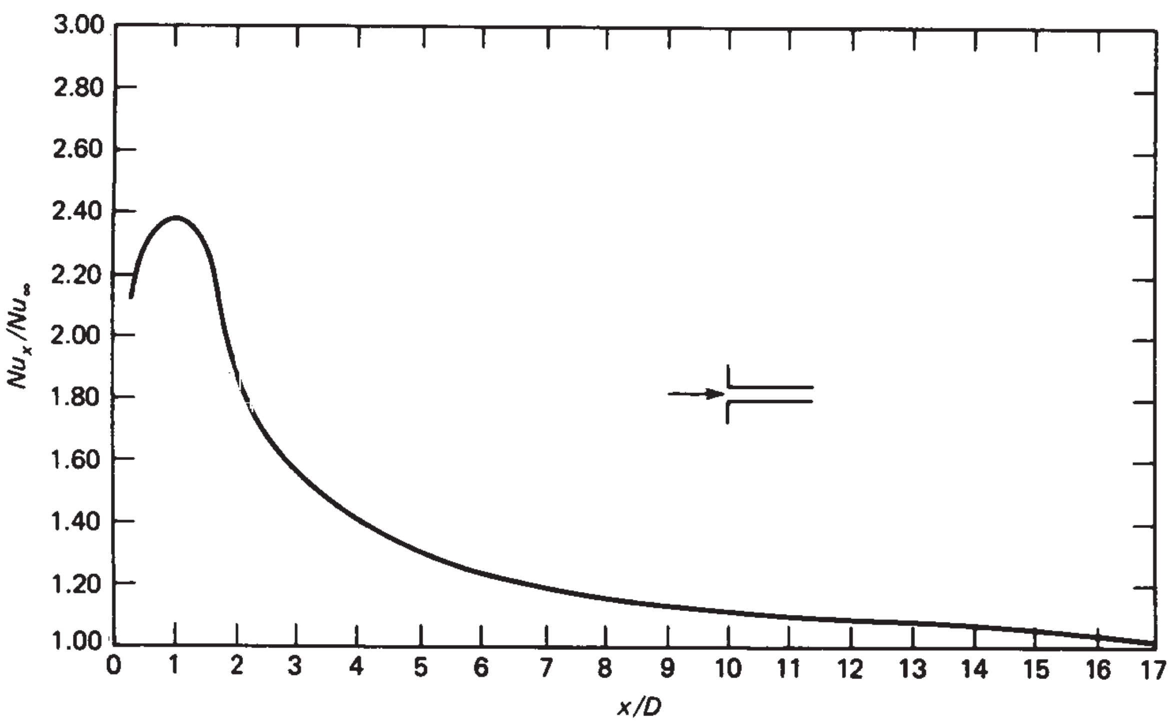


Fig. 2-19. Local Nusselt number for turbulent flow in the entry length of a circular tube with a right angle edge entrance, [12]. From *Convective Heat and Mass Transfer*, 2nd Edition, by W. M. Kays and M. E. Crawford, Fig. 13-10, copyright © 1980 by McGraw-Hill, Inc. Used with the permission of McGraw-Hill Book Company.

Integration and Curve Fit of Entry Length Effects:

$$\frac{\bar{h}_L}{h_{D_H}} = 1 + 1.68\left(\frac{D_H}{L}\right)^{0.58}; \; 2 \le \frac{L}{D_H} \le 20 \qquad \text{TCEE E2.38}$$

$$\frac{\bar{h}_L}{h_{D_H}} = 1 + 6\left(\frac{D_H}{L}\right); \quad 20 \le \frac{L}{D_H} \qquad \text{TCEE E2.39}$$

where h_{D_H} is the fully developed value (TCEE E2.37)

Recommended Procedure for Applying Circular Cross-Section Duct *Nu* to Rectangular Cross-Section Ducts:

1. Use turbulent flow *Nu* for $2100 < \mathrm{Re}_{D_H}$.

2. Decide if local *h* or average *h* is required.

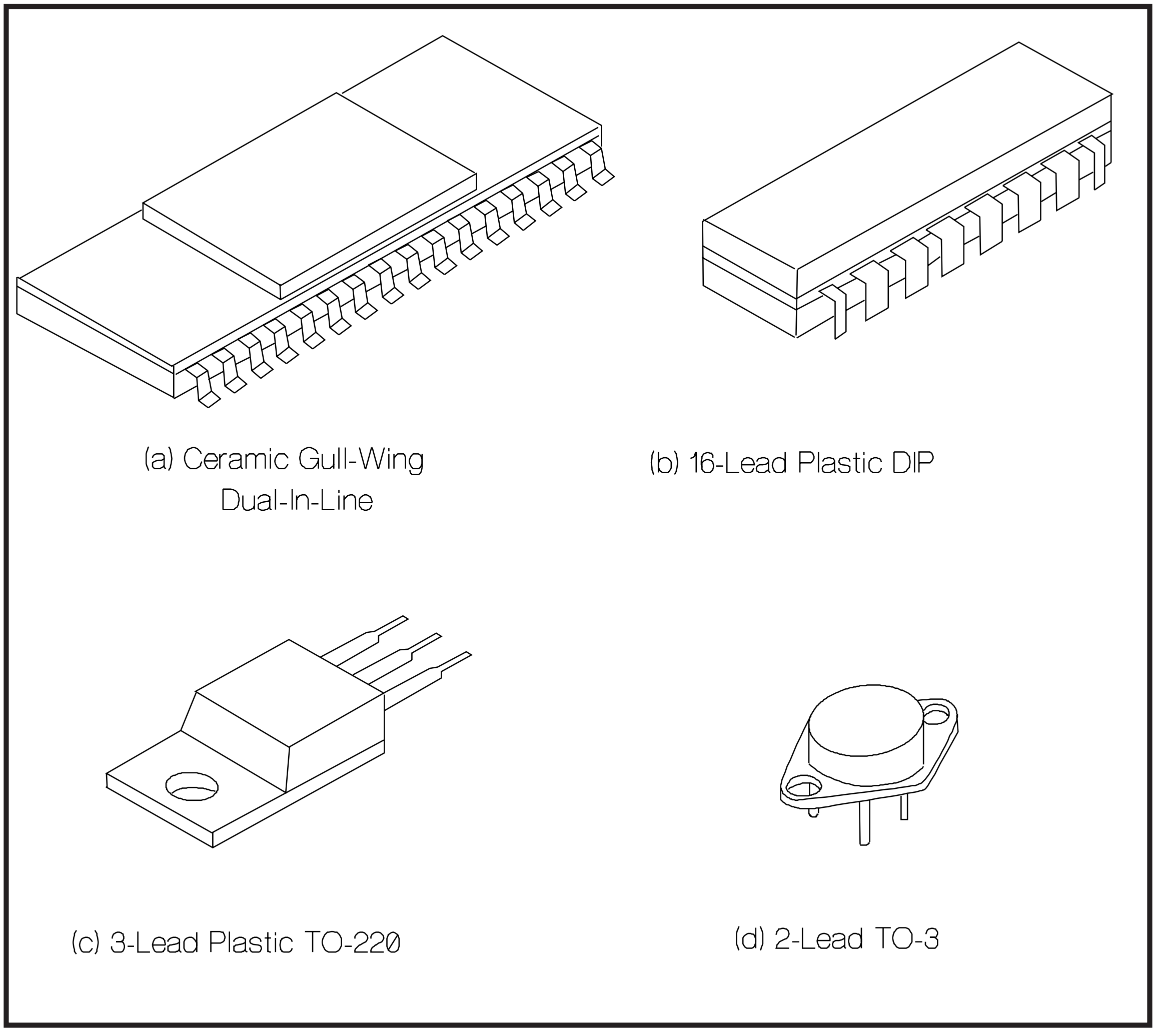
3. Calculate h_{DH} for fully developed flow (TCEE E2.37).

4. Calculate h_x using step 3 result and TCEE Fig. 2-19 or calculate $\bar{h}_L$ using step 3 and TCEE E2.38/E2.39.

Note: circular to rectangular duct correction not required.

Vendor Component Data

Some component styles:



(a) Ceramic Gull-Wing Dual-In-Line

(b) 16-Lead Plastic DIP

(c) 3-Lead Plastic TO-220

(d) 2-Lead TO-3

Data for 16 lead "through-hole" DIPS:

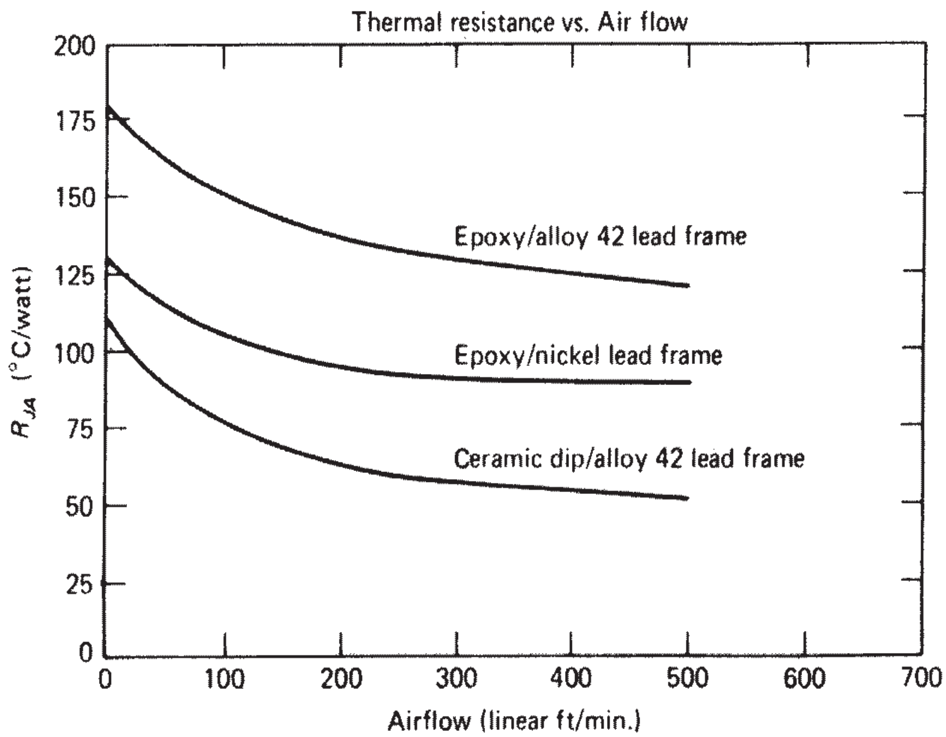
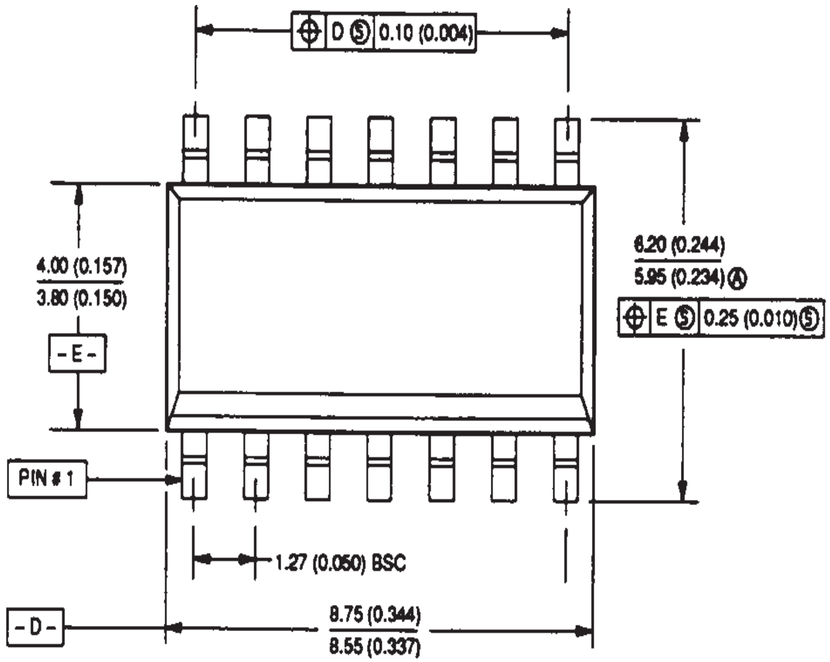


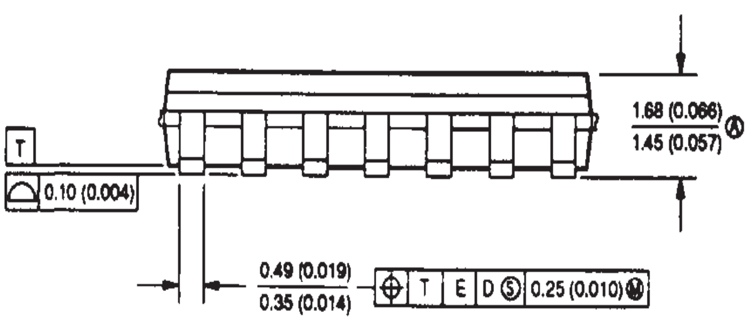
Fig. 2-22. Thermal resistance, junction to ambient, of some 16 lead dual-in-line packages, [15]. Reprinted from *Electronics*, October 31, 1974. Copyright © McGraw-Hill Inc. 1982. All rights reserved.

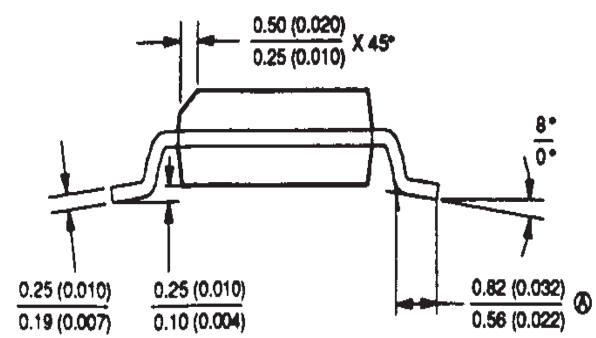
14-Pin plastic SO (Small Outline) dual-in-line package: Package body is 0.344 in. (8.75 mm) x 0.157 in. (4.0 mm). Data from "Signetics Surface Mount Process and Application Notes", 1991. The drawings and data are not guaranteed to be the most recent. Permission granted by Philips Semiconductors.



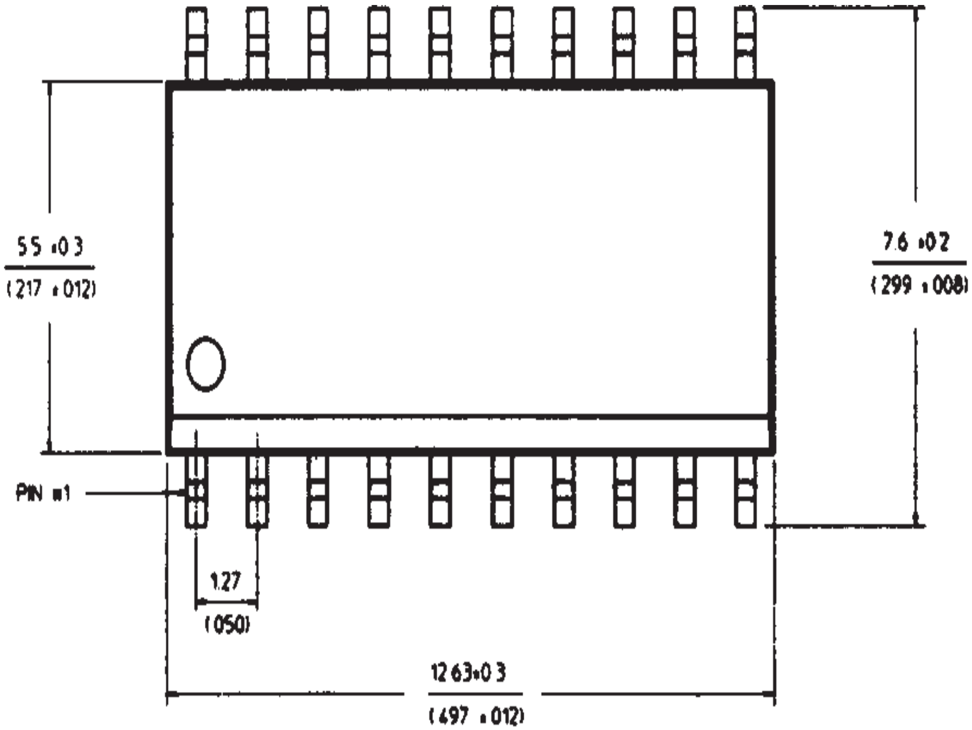
NOTES

1. Package dimensions conform to JEDEC Specification MS-012-AB for standard Small Outline (SO) package, 14 leads, 3.75mm (0.150") body width (Issue A, June 1985).
2. Controlling dimensions are mm. Inch dimensions in parentheses.
3. Dimensioning and tolerancing per ANSI Y14.5M-1982.
4. "T", "D", and "E" are reference datums on the molded body and do not include mold flash or protrusions. Mold flash or protrusions shall not exceed 0.15mm (0.006") on any side.
5. Pin numbers start with Pin #1 and continue counterclockwise to Pin #14 when viewed from the top.
6. Signetics ordering code for a product packages in a plastic Small Outline (SO) package is the suffix D after the product number.





20-Pin plastic SOL (Small Outline Large) dual-in-line package: Package body is 0.497 in. (12.63 mm) x 0.217 in. (5.5 mm).

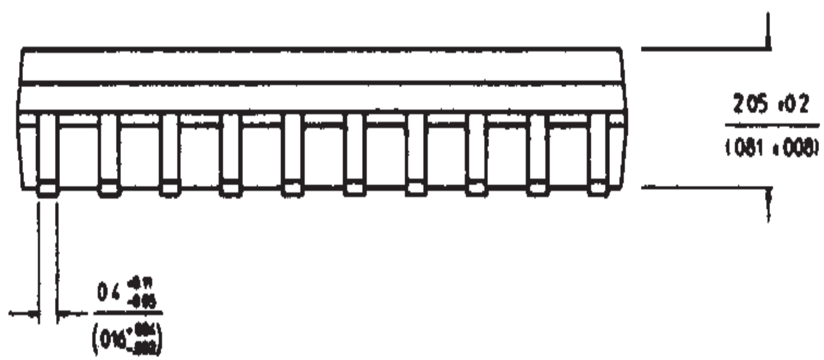


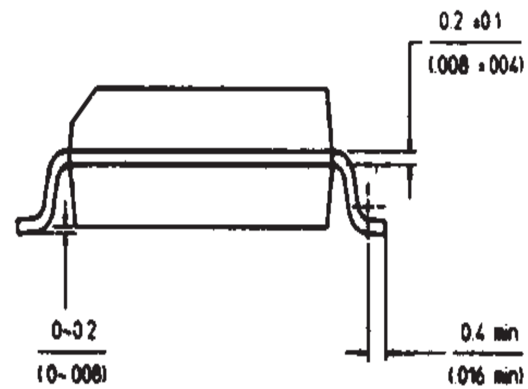
NOTES

1. CONTROLLING DIMENSIONS ARE IN mm
DIMENSIONS IN PARENTHESES ARE IN INCHES.

2. PIN NUMBERS START WITH PIN #1 AND CONTINUE COUNTERCLOCKWISE TO PIN #20 WHEN VIEWED FROM TOP.

Data from "Signetics Surface Mount Process and Application Notes", 1991. The drawings and data are not guaranteed to be the most recent. Permission granted by Philips Semiconductors.

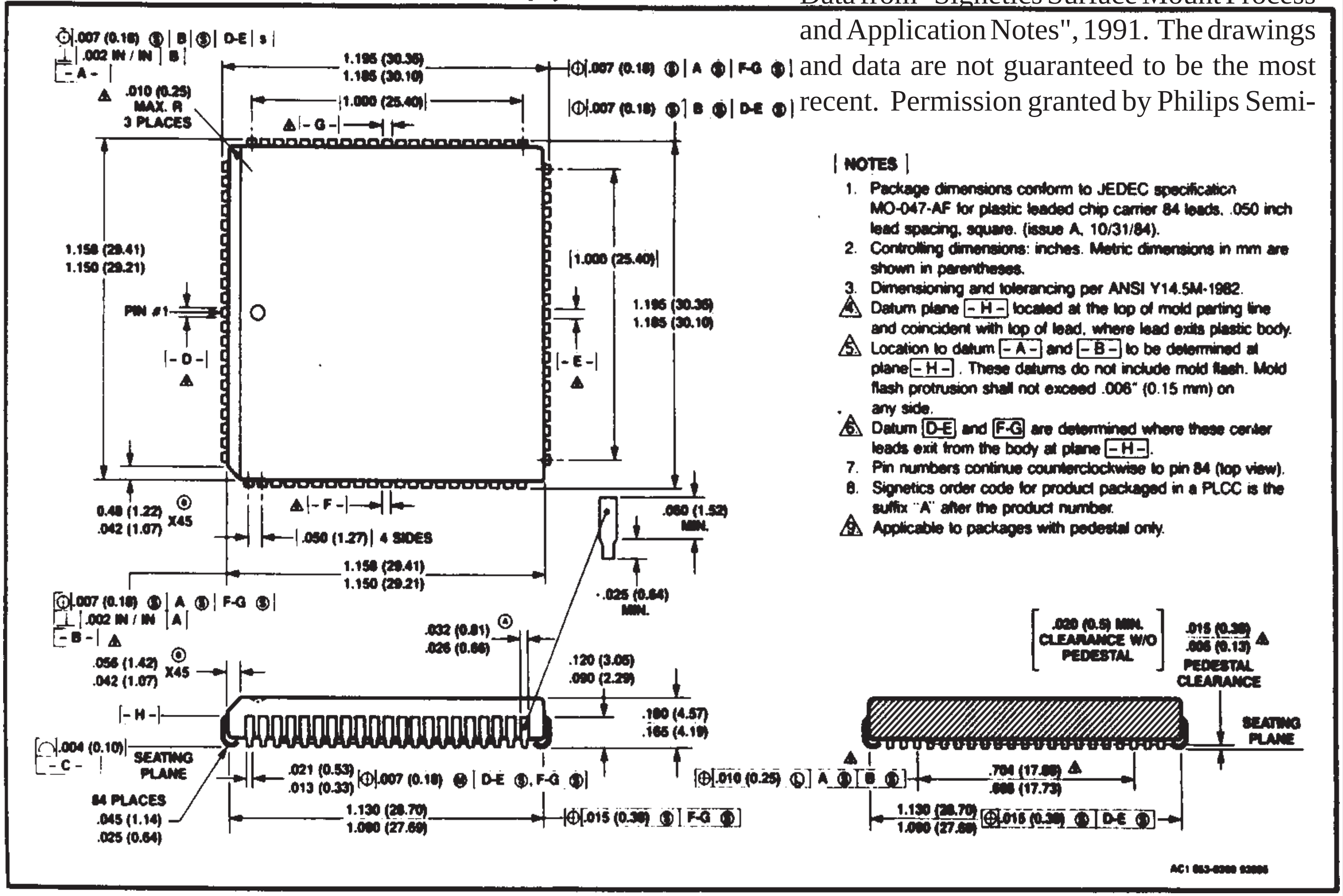




84-Pin Plastic Leaded Chip Carrier (PLCC) package: Package body is 1.195 in. (29.4 mm) x 1.158 in. (29.4 mm).

84-PIN PLASTIC LEADED CHIP CARRIER (A) PACKAGE

Data from "Signetics Surface Mount Process and Application Notes", 1991. The drawings and data are not guaranteed to be the most recent. Permission granted by Philips Semi-

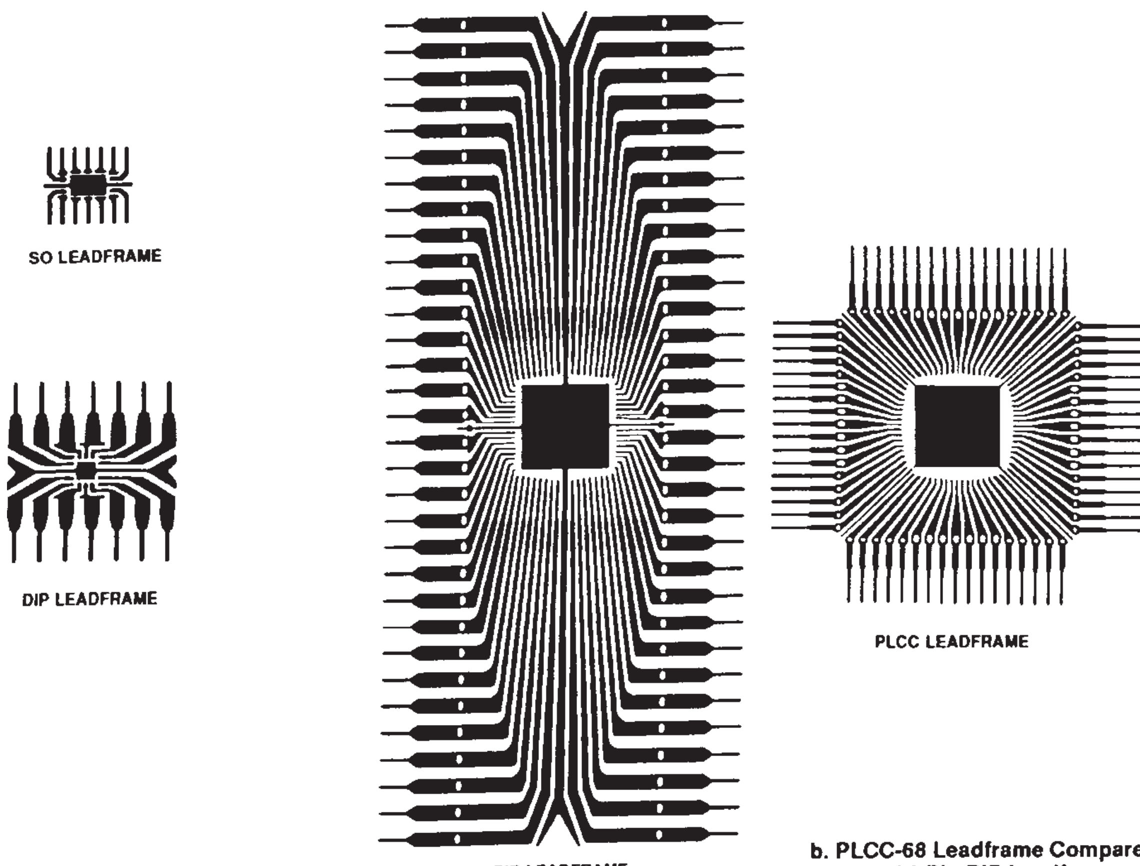


| NOTES |

1. Package dimensions conform to JEDEC specification MO-047-AF for plastic leaded chip carrier 84 leads, .050 inch lead spacing, square. (issue A, 10/31/84).
2. Controlling dimensions: inches. Metric dimensions in mm are shown in parentheses.
3. Dimensioning and tolerancing per ANSI Y14.5M-1982.
4. Datum plane -H- located at the top of mold parting line and coincident with top of lead, where lead exits plastic body.
5. Location to datum -A- and -B- to be determined at plane -H- . These datums do not include mold flash. Mold flash protrusion shall not exceed .006" (0.15 mm) on any side.
6. Datum D-E and F-G are determined where these center leads exit from the body at plane -H-.
7. Pin numbers continue counterclockwise to pin 84 (top view).
8. Signetics order code for product packaged in a PLCC is the suffix "A" after the product number.
9. Applicable to packages with pedestal only.

AC1 853-0399 93005

Some lead frame geometries:

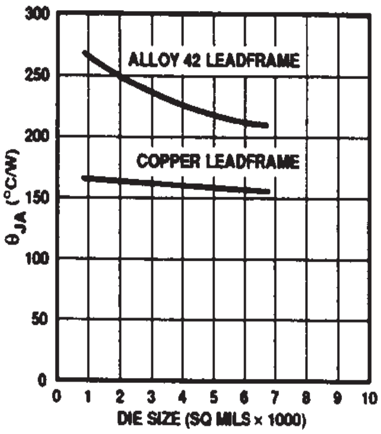


b. PLCC-68 Leadframe Compared to a 64-Pin DIP Leadframe

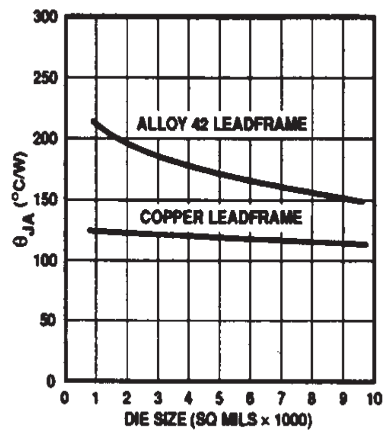
Data from "Signetics Surface Mount Process and Application Notes", 1991. The drawings and data are not guaranteed to be the most recent. Permission granted by Philips Semiconductors.

Thermal resistance, junction-to-ambient vs. die size for natural convection:

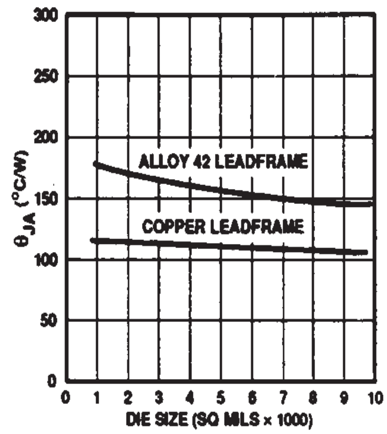
Typical θ_{JA} Data SO-8[1]



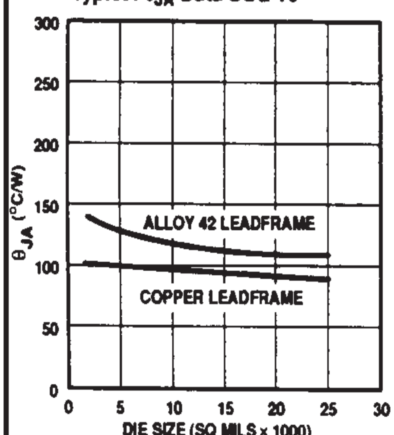
Typical θ_{JA} Data SO-14[1]



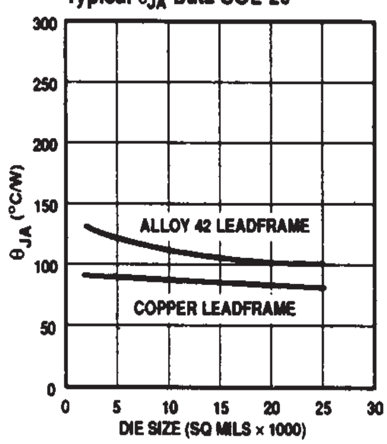
Typical θ_{JA} Data SO-16[1]



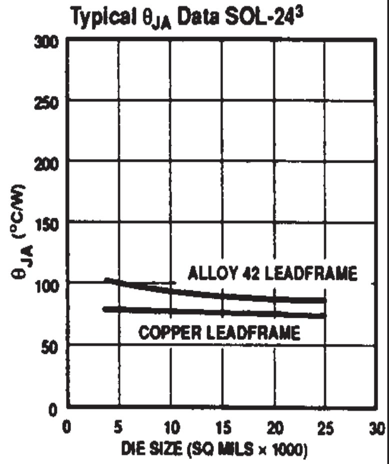
Typical θ_{JA} Data SOL-16[2]



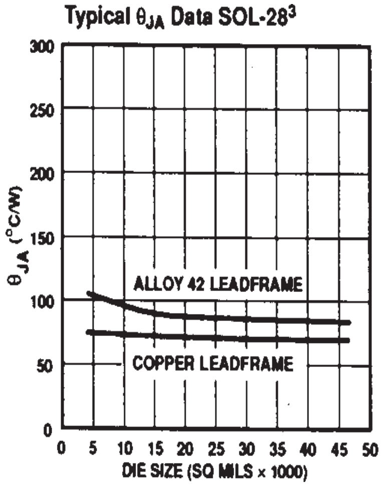
Typical θ_{JA} Data SOL-20[3]



Typical θ_{JA} Data SOL-24[3]



Typical θ_{JA} Data SOL-28[3]



NOTES:

1. TEST CONDITIONS:

Test ambient	Still air
Power dissipation:	0.5W
Test fixture:	Philips PCB (1.12" × 0.75" × 0.059")
Accuracy:	±15%

2. TEST CONDITIONS:

Test ambient	Still air
Power dissipation:	0.5W
Test fixture:	Philips PCB (1.58" × 0.75" × 0.059")
Accuracy:	±15%

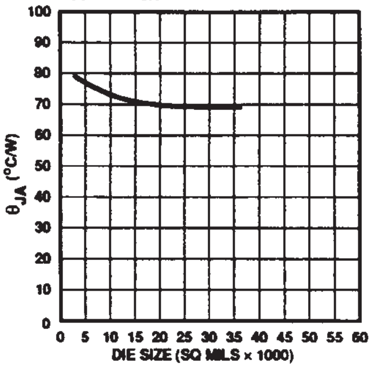
3. TEST CONDITIONS:

Test ambient	Still air
Power dissipation:	0.7W
Test fixture:	Philips PCB (1.58" × 0.75" × 0.059")
Accuracy:	±15%

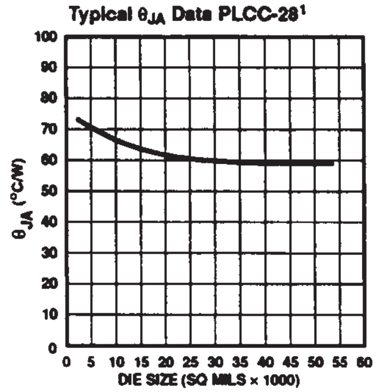
Data from "Signetics Surface Mount Process and Application Notes", 1991. The drawings and data are not guaranteed to be the most recent. Permission granted by Philips Semiconductors.

Thermal resistance, junction-to-ambient vs. die size for natural convection:

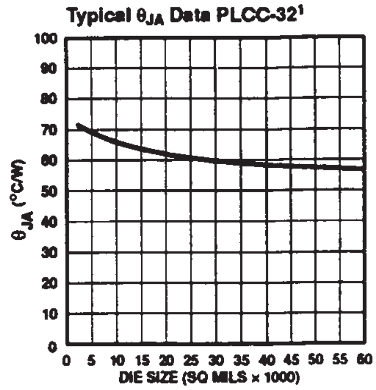
Typical θ_{JA} Data PLCC-20[1]



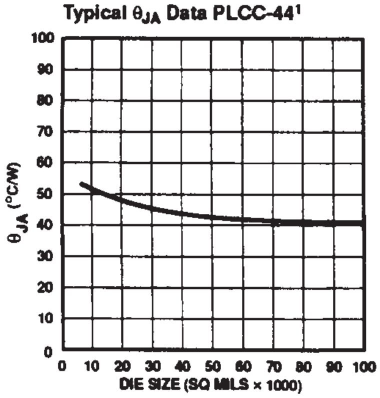
Typical θ_{JA} Data PLCC-28[1]



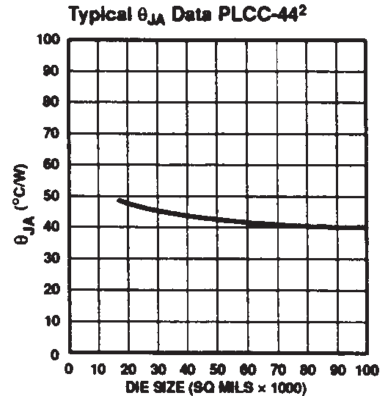
Typical θ_{JA} Data PLCC-32[1]



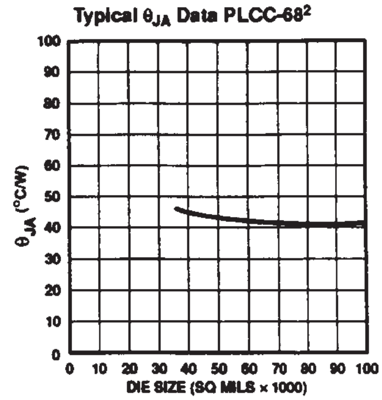
Typical θ_{JA} Data PLCC-44[1]



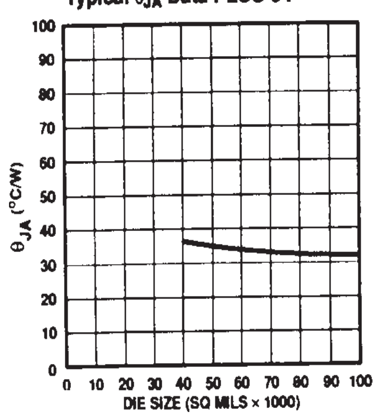
Typical θ_{JA} Data PLCC-44[2]



Typical θ_{JA} Data PLCC-68[2]



Typical θ_{JA} Data PLCC-84[3]



NOTES:

1. TEST CONDITIONS:

Test ambient	Still air
Power dissipation:	0.75W
Test fixture:	Signetics PCB (2.24" × 2.24" × 0.062")
Accuracy:	±15%

2. TEST CONDITIONS:

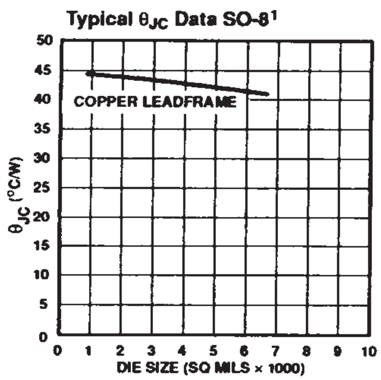
Test ambient	Still air
Power dissipation:	1.0W
Test fixture:	Signetics PCB (2.24" × 2.24" × 0.062")
Accuracy:	±15%

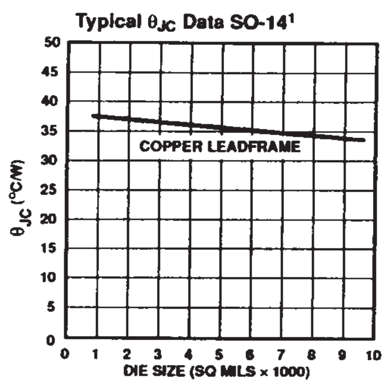
3. TEST CONDITIONS:

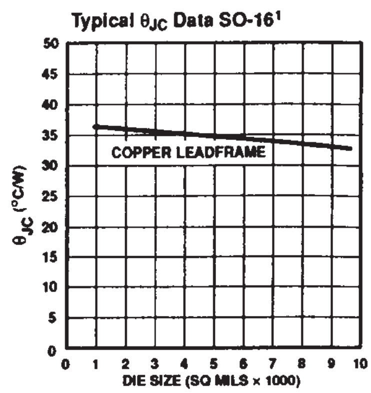
Test ambient	Still air
Power dissipation:	1.5W
Test fixture:	Signetics PCB (2.24" × 2.24" × 0.062")
Accuracy:	±15%

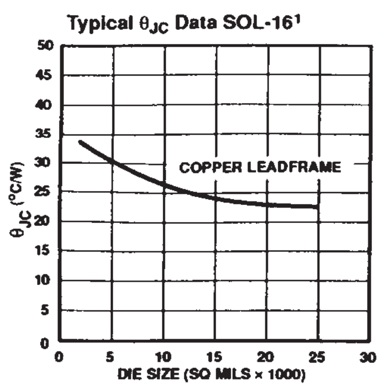
Data from "Signetics Surface Mount Process and Application Notes", 1991. The drawings and data are not guaranteed to be the most recent. Permission granted by Philips Semiconductors.

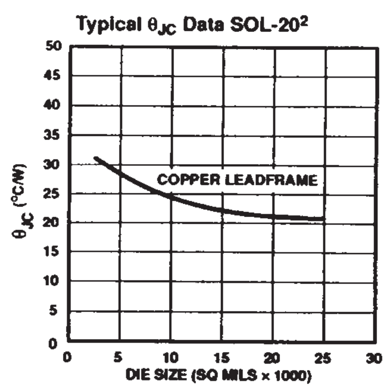
Thermal resistance, junction-to-case vs. die size for natural convection:

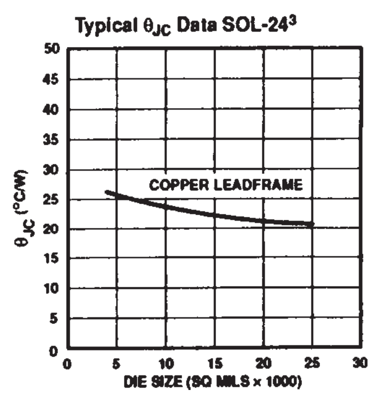


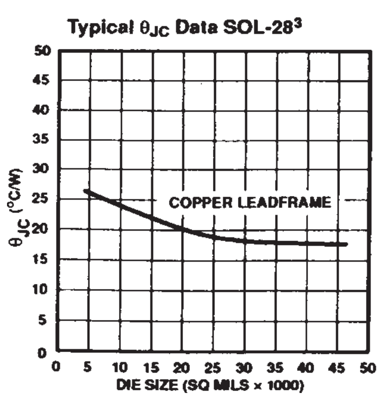












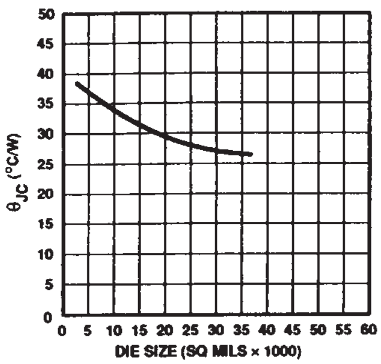
NOTES:

1. TEST CONDITIONS:
   Power dissipation: 0.5W
   Test fixture: "Infinite" heat sink
   Accuracy: ±15%
2. TEST CONDITIONS:
   Power dissipation: 0.7W
   Test fixture: "Infinite" heat sink
   Accuracy: ±15%
3. TEST CONDITIONS:
   Power dissipation: 1.0W
   Test fixture: "Infinite" heat sink
   Accuracy: ±15%

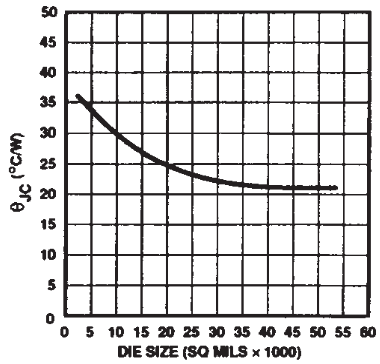
Data from "Signetics Surface Mount Process and Application Notes", 1991. The drawings and data are not guaranteed to be the most recent. Permission granted by Philips Semiconductors.

Thermal resistance, junction-to-case vs. die size
for natural convection:

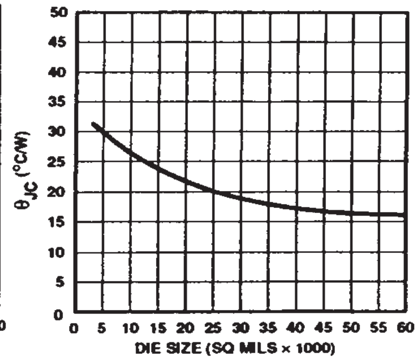
Typical θ_{JC} Data PLCC-20[1]



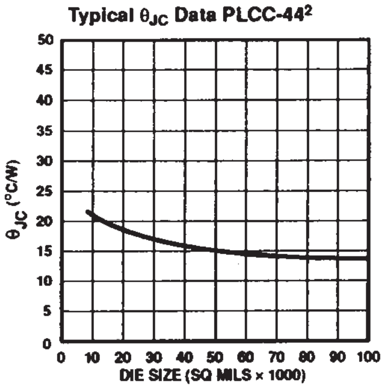
Typical θ_{JC} Data PLCC-28[2]



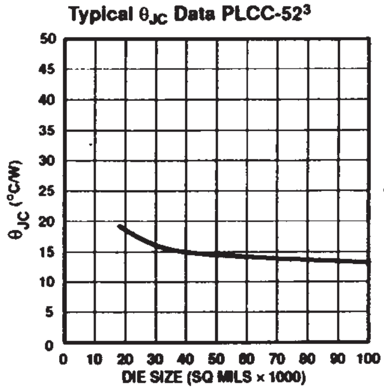
Typical θ_{JC} Data PLCC-32[2]



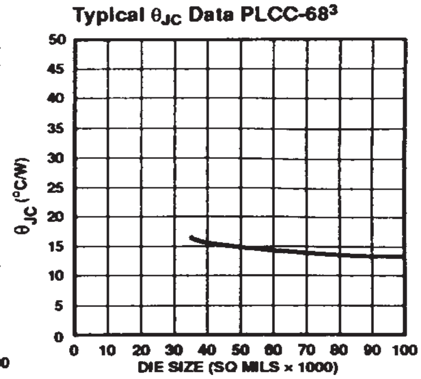
Typical θ_{JC} Data PLCC-44[2]



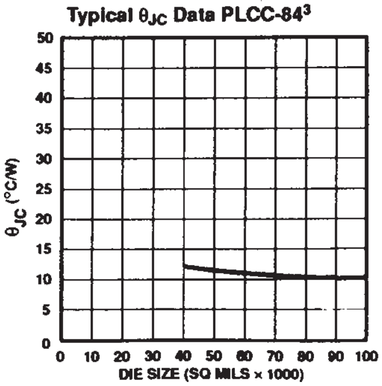
Typical θ_{JC} Data PLCC-52[3]



Typical θ_{JC} Data PLCC-68[3]



Typical θ_{JC} Data PLCC-84[3]



NOTES:

1. TEST CONDITIONS:
   - Power dissipation: 0.75W
   - Test fixture: "Infinite" heat sink
   - Accuracy: ±15%
2. TEST CONDITIONS:
   - Power dissipation: 1.0W
   - Test fixture: "Infinite" heat sink
   - Accuracy: ±15%
3. TEST CONDITIONS:
   - Power dissipation: 2.0W
   - Test fixture: "Infinite" heat sink
   - Accuracy: ±15%

Data from "Signetics Surface Mount Process and Application Notes", 1991. The drawings and data are not guaranteed to be the most recent. Permission granted by Philips Semiconductors.

Thermal resistance, junction-to-ambient vs. die size for forced convection:

Data from "Signetics Surface Mount Process and Application Notes", 1991. The drawings and data are not guaranteed to be the most recent. Permission granted by Philips Semiconductors.

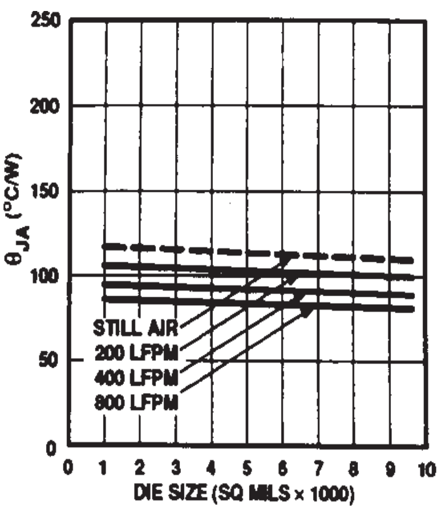


Figure 15. Results of Air Flow on θ_{JA} on SO-14 with Copper Leadframe

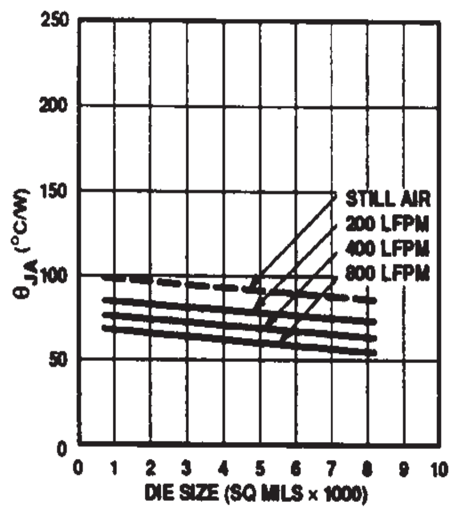


Figure 16. Results of Air Flow on θ_{JA} on SOL-16 with Copper Leadframe

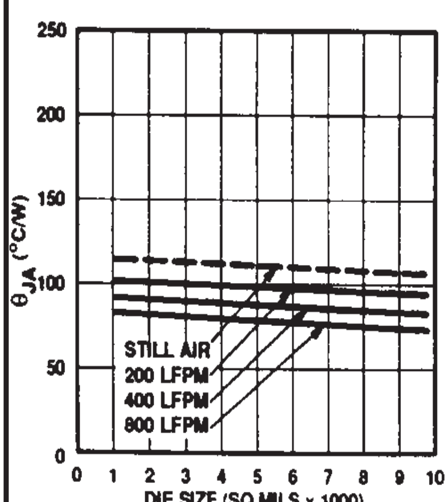


Figure 17. Results of Air Flow on θ_{JA} on SO-16 with Copper Leadframe

20?

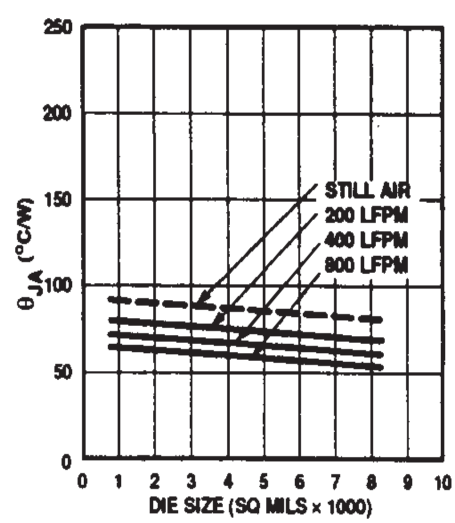


Figure 18. Results of Air Flow on θ_{JA} on SOL-20 with Copper Leadframe

24?

Natural Convection

Natural Convection - Flat Plates

Flat Plates

$$Nu_P \equiv \text{Nusselt Number} = \frac{P}{k} h_c$$

$$Nu_P = C(Gr_P \Pr)^n, \; P \equiv \text{Length Scale}$$

$$Gr_P \equiv \text{Grashof Number} = \frac{g\rho^2}{\mu^2} \beta(T_S - T_A)P^3$$

$$\Pr \equiv \text{Prandtl Number} = \frac{\mu C_p}{k}$$

$$Ra \equiv \text{Rayleigh Number} = Gr \Pr$$

Vertical Plate: $P = H$

Laminar flow: $10^4 < Ra_P < 10^9$, $C = 0.59, n = 0.25$

Turbulent flow: $10^9 < Ra_P < 10^{12}$, $C = 0.13, n = 0.33$

Horizontal Plate $\uparrow$: $P = WL / [2(W + L)]$

Laminar flow: $2.2x10^4 \le Ra_P \le 8x10^6$, $C = 0.54, n = 0.25,$

Turbulent flow: $8x10^6 \le Ra_P \le 1.6x10^9$, $C = 0.15, n = 0.33$

Horizontal Plate $\downarrow$: $P = WL / [2(W + L)]$

Laminar flow: $3.0x10^5 \le Ra_P \le 3x10^{10}$, $C = 0.27, n = 0.25$

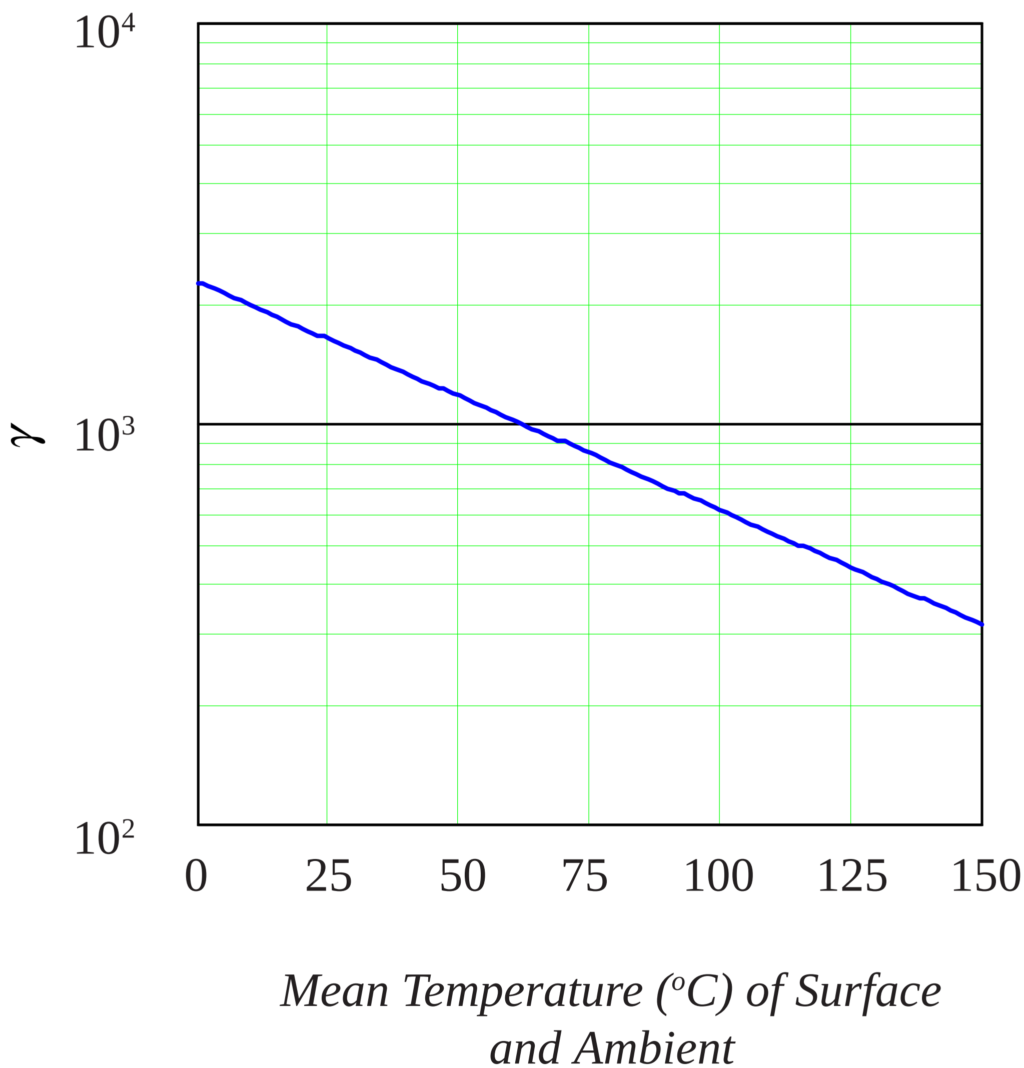
Turbulent flow: Nothing offered

Flat Plates Simplified

Simplified Calculation of Rayleigh Number Ra_P

$$Ra = \gamma \cdot \Delta T \cdot P^3, \ \gamma = \left(\frac{g\beta\rho^2}{\mu^2} \right) \Pr$$

where ΔT is in ^oC, P is in inches



Simplifying the flat plate formulae for practical application:

$$h = \frac{k}{P} Nu_P$$

Vertical plate, laminar flow - $10^4 < Ra_H < 10^9$

$$h = \frac{k}{H}(0.59)(Gr\,\mathrm{Pr})^{1/4} = 0.59\,\frac{k}{H}\left[\left(\frac{g\beta\rho^2 \Delta T H^3}{\mu^2}\right)\mathrm{Pr}\right]^{1/4}$$

$$= 0.59\,\frac{k}{H}\left(\frac{g\beta\rho^2}{\mu^2}\right)^{1/4} \mathrm{Pr}^{1/4}\,\Delta T^{1/4}\left(H^3\right)^{1/4}$$

$$= 0.59k\left(\frac{g\beta\rho^2}{\mu^2}\right)^{1/4} \mathrm{Pr}^{1/4}\,\Delta T^{1/4}\left(\frac{H^3}{H^4}\right)^{1/4}$$

$$= \alpha\left(\frac{\Delta T}{H}\right)^{0.25},\ \alpha = 0.59k\left(\frac{g\beta\rho^2}{\mu^2}\right)^{1/4} \mathrm{Pr}^{1/4}$$

T(C)	**Alpha**	**Ra (DT=1,H=1)**	**Ra (DT=5,H=2)**	**Ra (DT=100,H=10)**
0	0.0025	2260	$9\text{x}10^4$	$2\text{x}10^8$
25	0.0025	1629	$7\text{x}10^4$	$2\text{x}10^8$
50	0.0024	1173	$5\text{x}10^4$	$1\text{x}10^8$
75	0.0023	846	$3\text{x}10^4$	$9\text{x}10^7$
100	0.0023	609	$2\text{x}10^4$	$6\text{x}10^7$
125	0.0022	439	$2\text{x}10^4$	$4\text{x}10^7$
150	0.0021	316	$1\text{x}10^4$	$3\text{x}10^7$

Vertical plate, turbulent flow - $10^9 < Ra < 10^{12}$

$$h = \frac{k}{H}(0.13)(Gr\,\mathrm{Pr})^{1/3} = 0.13\,\frac{k}{H}\left[\left(\frac{g\beta\rho^2\Delta T H^3}{\mu^2}\right)\mathrm{Pr}\right]^{1/3}$$

$$= 0.13\,\frac{k}{H}\left(\frac{g\beta\rho^2}{\mu^2}\right)^{1/3}\mathrm{Pr}^{1/3}\,\Delta T^{1/3}\left(H^3\right)^{1/3}$$

$$= 0.13k\left(\frac{g\beta\rho^2}{\mu^2}\right)^{1/3}\mathrm{Pr}^{1/3}\,\Delta T^{1/3}\left(\frac{H^3}{H^3}\right)^{1/3}$$

$$= \alpha\Delta T^{0.33},\ \alpha = 0.13k\left(\frac{g\beta\rho^2}{\mu^2}\right)^{1/3}\mathrm{Pr}^{1/3}$$

T(C)	**Alpha**
0	0.001000
25	0.000977
50	0.000931
75	0.000884
100	0.000837
125	0.000790
150	0.00744

Horizontal plate facing up, laminar flow -

$$2.2x10^4 < Ra_P < 8x10^6,\ \ P = WL/[2(W+L)]$$

$$h = \frac{k}{P}(0.54)(Gr\,\mathrm{Pr})^{1/4} = 0.54\,\frac{k}{P}\left[\left(\frac{g\beta\rho^2\Delta TP^3}{\mu^2}\right)\mathrm{Pr}\right]^{1/4}$$

$$= 0.54\,\frac{k}{P}\left(\frac{g\beta\rho^2}{\mu^2}\right)^{1/4}\ \mathrm{Pr}^{1/4}\ \Delta T^{1/4}\left(P^3\right)^{1/4}$$

$$= 0.54k\left(\frac{g\beta\rho^2}{\mu^2}\right)^{1/4}\ \mathrm{Pr}^{1/4}\ \Delta T^{1/4}\left(\frac{P^3}{P^4}\right)^{1/4}$$

$$= \alpha\left(\frac{\Delta T}{P}\right)^{0.25},\ \alpha = 0.54k\left(\frac{g\beta\rho^2}{\mu^2}\right)^{1/4}\ \mathrm{Pr}^{1/4}$$

T(C)	**Alpha**	**Ra (DT=1,H=1)**	**Ra (DT=5,H=2)**	**Ra (DT=100,H=2.5)**
0	0.0023	2260	$9x10^4$	$4x10^6$
25	0.0022	1629	$7x10^4$	$3x10^6$
50	0.0022	1173	$5x10^4$	$2x10^6$
75	0.0021	846	$3x10^4$	$1x10^6$
100	0.0021	609	$2x10^4$	$1x10^6$
125	0.0020	439	$2x10^4$	$7x10^5$
150	0.0019	316	$1x10^4$	$5x10^5$

Horizontal plate facing down, laminar flow -

$$3x10^{5} < Ra_{P} < 3x10^{10},\ P = WL/[2(W+L)]$$

$$h = \frac{k}{P}(0.27)(Gr\,\mathrm{Pr})^{1/4} = 0.27\frac{k}{P}\left[\left(\frac{g\beta\rho^{2}\Delta T P^{3}}{\mu^{2}}\right)\mathrm{Pr}\right]^{1/4}$$

$$= 0.27\frac{k}{P}\left(\frac{g\beta\rho^{2}}{\mu^{2}}\right)^{1/4}\mathrm{Pr}^{1/4}\,\Delta T^{1/4}\left(P^{3}\right)^{1/4}$$

$$= 0.27k\left(\frac{g\beta\rho^{2}}{\mu^{2}}\right)^{1/4}\mathrm{Pr}^{1/4}\,\Delta T^{1/4}\left(\frac{P^{3}}{P^{4}}\right)^{1/4}$$

$$= \alpha\left(\frac{\Delta T}{P}\right)^{0.25},\ \alpha = 0.27k\left(\frac{g\beta\rho^{2}}{\mu^{2}}\right)^{1/4}\mathrm{Pr}^{1/4}$$

T(C)	**Alpha**	**Ra (DT=1,H=1)**	**Ra (DT=5,H=2)**	**Ra (DT=100,H=2.5)**
0	0.0011	2260	$9x10^{4}$	$4x10^{6}$
25	0.0011	1629	$7x10^{4}$	$3x10^{6}$
50	0.0011	1173	$5x10^{4}$	$2x10^{6}$
75	0.0011	846	$3x10^{4}$	$1x10^{6}$
100	0.0010	609	$2x10^{4}$	$1x10^{6}$
125	0.0010	439	$2x10^{4}$	$7x10^{5}$
150	0.00098	316	$1x10^{4}$	$5x10^{5}$

Horizontal plate facing up, turbulent flow -

$$8x10^6 < Ra < 1.610^9$$

$$h = \frac{k}{H}(0.15)(Gr\,\Pr)^{1/3} = 0.15\frac{k}{H}\left[\left(\frac{g\beta\rho^2\Delta T H^3}{\mu^2}\right)\Pr\right]^{1/3}$$

$$= 0.15\frac{k}{H}\left(\frac{g\beta\rho^2}{\mu^2}\right)^{1/3}\Pr^{1/3}\Delta T^{1/3}\left(H^3\right)^{1/3}$$

$$= 0.15k\left(\frac{g\beta\rho^2}{\mu^2}\right)^{1/3}\Pr^{1/3}\Delta T^{1/3}\left(\frac{H^3}{H^3}\right)^{1/3}$$

$$= \alpha\Delta T^{0.33},\ \alpha = 0.15k\left(\frac{g\beta\rho^2}{\mu^2}\right)^{1/3}\Pr^{1/3}$$

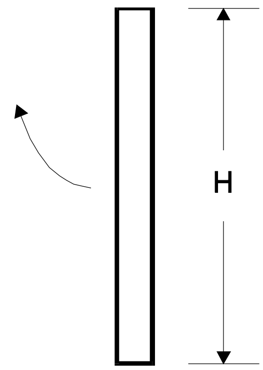
T(C)	**Alpha**
0	0.0012
25	0.0011
50	0.0011
75	0.0010
100	0.00097
125	0.00091
150	0.00086

Horizontal plate facing down, turbulent flow -

Nothing offered so only choice is to use $\alpha_{Down} = \frac{1}{2}\alpha_{Up}$

In summary, classical flat plate natural convection formulae may be simplified for an average *film* (average of surface and ambient temperatures) of *0 to 100 °C:*

Laminar flow ($10^4 < Ra_P < 10^9$)

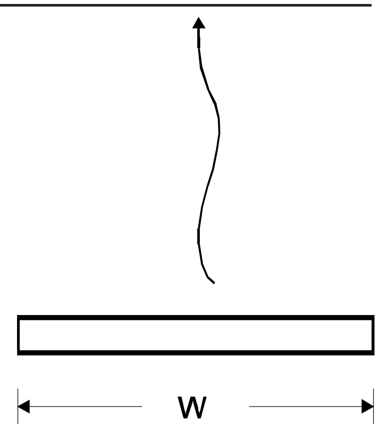


$$h_C = 0.0024\left(\frac{\Delta T}{P}\right)^{0.25}$$

Turbulent flow ($10^9 < Ra_P < 10^{12}$)

$$h_C = 0.0009\Delta T^{0.33}$$

Laminar flow ($2.2\text{x}10^4 < Ra_P < 810^6$)

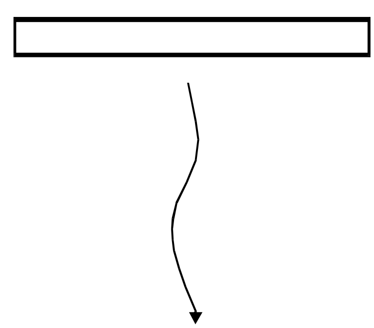


$$h_C = 0.0022\left(\frac{\Delta T}{P}\right)^{0.25}$$

Turbulent flow ($8\text{x}10^6 < Ra_P < 1.610^9$)

$$h_C = 0.0011\Delta T^{0.33}$$

Laminar flow ($3\text{x}10^5 < Ra_P < 3x10^{10}$)



$$h_C = 0.0011\left(\frac{\Delta T}{P}\right)^{0.25}$$

Turbulent flow ($3\text{x}10^{10} < Ra_P$)

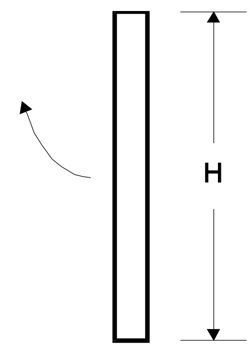
$$h_C = 0.0005\Delta T^{0.33}$$

Vertical plate: P = H (in.); Horizontal plate: P = WL/[2(W+L)]

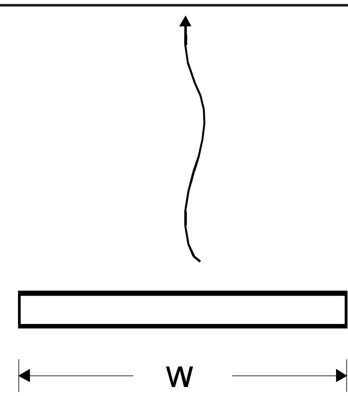
Flat Plates Simplified - Small Device (H,W<6 in.)

"Probably" laminar flow only

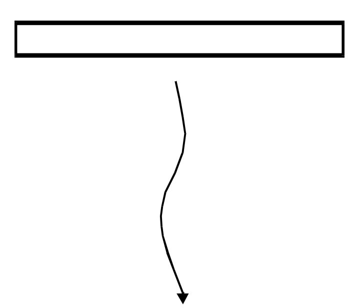
$$h_C = 0.0022\left(\frac{\Delta T}{P}\right)^{0.35}$$



$$h_C = 0.0018\left(\frac{\Delta T}{P}\right)^{0.33}$$



$$h_C = 0.0009\left(\frac{\Delta T}{P}\right)^{0.33}$$

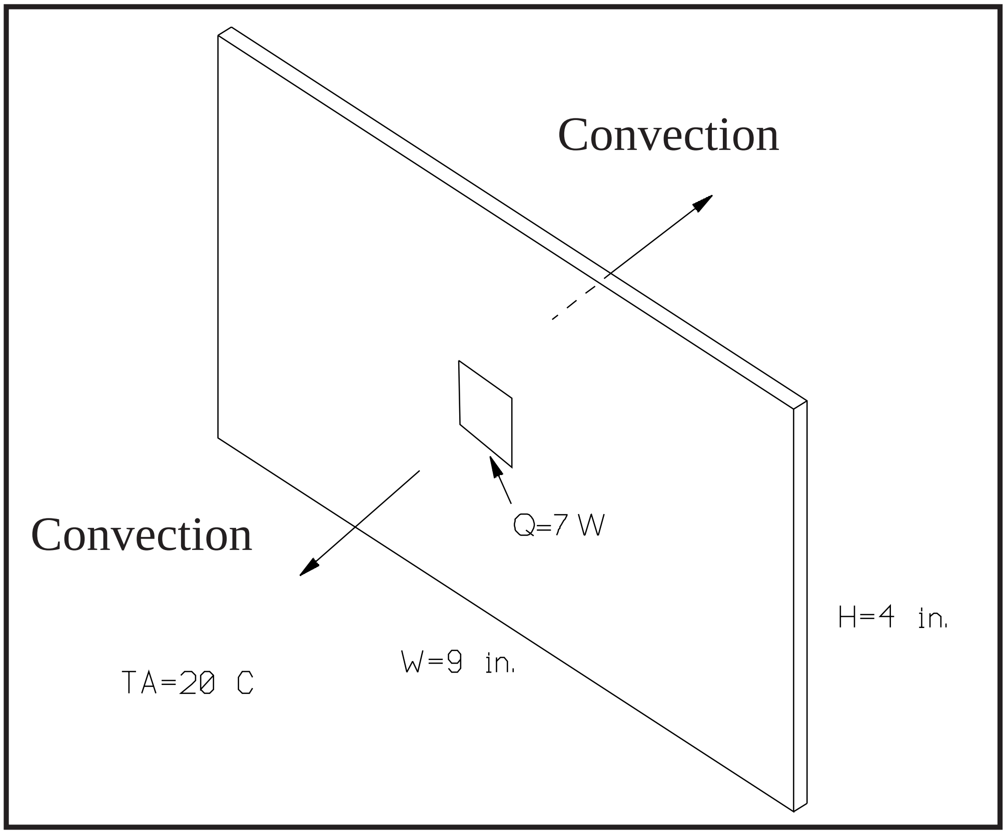


Vertical plate: P = H (in.)

Horizontal plate: P = WL/[2(W+L)]

Example

Average Plate Temperature and Convection Coefficient



Temperature Rise Calculation

$$Q_C = \bar{h}_C A_S (T_S - T_A)$$

$$= \bar{h}_C A_S \Delta T$$

$$= 0.0022 \left(\frac{\Delta T}{H} \right)^{0.35} A_S \Delta T$$

$$= 0.0022 \left(\frac{\Delta T}{4.0} \right)^{0.35} (2 x 9.0\ in.\ x 4.0\ in.) \Delta T$$

assuming laminar flow, but using "small device".

Solving for

$$\Delta T = 5.61 Q_c^{1/1.35} = 56.1(7.0)^{1/1.35} = 23.7\ ^oC$$

Convection Coefficient Calculation

$$\bar{h}_C = 0.0022\left(\frac{\Delta T}{H}\right)^{0.35}$$

$$= 0.0022\left(\frac{23.7}{4.0}\right)^{0.35}$$

$$= 0.0041\ W/in.^2 \cdot {}^oC$$

Convection Resistance Calculation

$$R_C = \frac{1}{\bar{h}_C A_S} = \frac{1}{(0.0041)(2x9.0\ in.\ x4.0\ in.)}$$

$$= 3.39\ ^oC / W$$

Check on Rayleigh Number *Ra* -

$$\bar{T}_{Film} = \frac{(QR + T_A) + T_A}{2} = \frac{\left[(7\,W)\left(3.39\ ^oC/W\right) + 20\ ^oC\right]}{2}$$

$$= 32\ ^oC$$

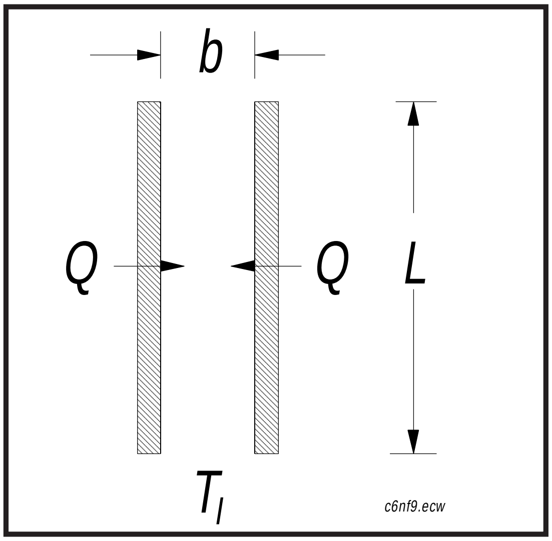
$$Ra = \gamma\left(\bar{T}_{Film} = 32\ ^oC\right)(\Delta T)H^3 = \left(1.5x10^3\right)(23.7)(4.0)^3$$

$= 2.3x10^6$, which is indicative of being very laminar

Natural Convection - Some General Correlations Applicable to Vertical Circuit Board Channels

Vertical Parallel Plate Correlations -
Application: Circuit Board Channels - Isothermal Surfaces, Air Only:

(Bar-Cohen & Rohsenow,
Teertstra, P., Culham, J.R., and Yovanovich, M.M, 1996)



$$\overline{Nu}_b = \left[\left(\frac{C}{Ra_b} \right)^2 + \left(\frac{1}{0.59 Ra_b^{1/4}} \right)^2 \right]^{-1/2} \quad \begin{cases} C = 24 : \text{Symmetrically heated walls.} \\ C = 12 : \text{One heated wall, one} \\ \text{unheated adiabaticwall.} \end{cases}$$

$Ra_b \equiv$ Modified Channel Rayleigh number for uniform wall T_W

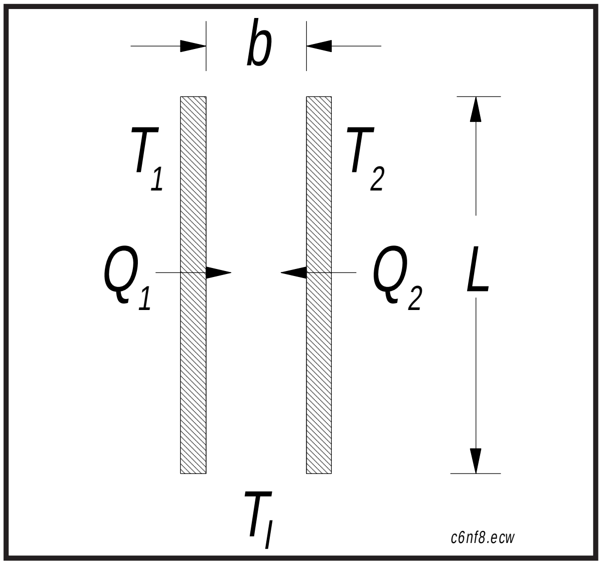
$$Ra_b = \frac{g\beta (T_W - T_I) b^4}{\upsilon^2 L} \cdot Pr = \left(\frac{g\beta}{\upsilon^2} \right) Pr \left(\frac{b^4}{L} \right) (T_W - T_I)$$

$$= \gamma \left(b^4 / L \right) (T_W - T_I)$$

$$b_{opt} = 2.714 P^{-1/4}, \; Nu_{opt} = 1.31, \; Ra_{opt} = 54.3, \; P = Ra_{opt} / b^4$$

Bar-Cohen b_{opt} based on $\frac{d}{db}\left[Q_{Total} / (T_W - T_I) \right] = 0$ for an *assembly* of cards.

Unequal, Isothermal Temperature Walls:
(Raithby & Hollands, 1985;
Teertstra, P., Culham, J.R., and Yovanovich, M.M., 1996)



$$\overline{Nu}_b = \left[\left(\frac{90(1+r_T)^2}{4r_T^2 + 7r_T + 4} \cdot \frac{1}{\overline{Ra}_b} \right)^{1.9} + \left(\frac{1}{0.62\overline{Ra}_b^{1/4}} \right)^{1.9} \right]^{-1/1.9}$$

$\overline{Nu}_b$ and $\overline{Ra}_b$ are based on the average wall temperature as defined in

$$\overline{Ra}_b \equiv \text{Channel Rayleigh number, } T_1 \neq T_2$$

$$\overline{Ra}_b = \frac{g\beta(1+r_T)(T_1 - T_I)b^4}{2\upsilon^2 L} \cdot Pr$$

$$r_T = \frac{T_2 - T_I}{T_1 - T_I}$$

Air properties are evaluated at the average film temperature $\left(\overline{T}_w + T_I\right)/2$ except in cases of small $\overline{Ra}_b$ where the average wall temperature, $\overline{T}_w$, should be used. In all cases β is based on the inlet temperature, i.e. $\beta = 1/T_I$ (using absolute temperature).

Symmetric, Isoflux Walls:
(Bar-Cohen & Rohsenow
Teerstra, P., Culham, J.R., and Yovanovich, M.M., 1996) -

$$Nu_L = \left[\frac{24C}{Ra_b^*} + \frac{2.51}{\left(Ra_b^*\right)^{0.4}}\right]^{-1/2}$$

$C = 2,$ Symmetrically heated walls

$C = 1,$ Asymmetrically heated walls (one heated, the other unheated, i.e. adiabatic)

$Ra_b^* \equiv$ Modified channel Rayleigh number for uniform heat flux (UHF)

$$= \left(\frac{g\beta}{\upsilon^2}\right)\Pr\left(\frac{b^5}{L}\right)\left(\frac{q}{k}\right) = \gamma\left(\frac{b^5}{L}\right)\left(\frac{q}{k}\right)$$

$q =$ Wall heat flux, i.e. $\frac{Q_{One\ Wall\ Side}}{LW}$

$L =$ Wall height

$W =$ Wall depth

Symmetric heating: $b_{opt} = 1.47R^{-1/5},\ Nu_{opt} = 0.351,\ Ra_{opt}^* = 6.9$

Asymmetric heating: $b_{opt} = 1.17R^{-1/5},\ Nu_{opt} = 0.464,\ Ra_{opt}^* = 6.9$

$$R = \frac{Ra_b^*}{b^5} = \left(\frac{g\beta}{\upsilon^2}\right)\Pr\left(\frac{q}{kL}\right) = \gamma\left(\frac{q}{kL}\right)$$

Isothermal Temperature, Vertical Flat Plate:

.

$$Ra_L \equiv \text{Vertical plate Rayleigh number}$$

$$= Gr_L \Pr = \frac{g\beta b\left(T_W - T_I\right)L^3}{\upsilon^2}\Pr$$

$10^4 < Ra_L < 10^9$, (McAdams, 1954) -

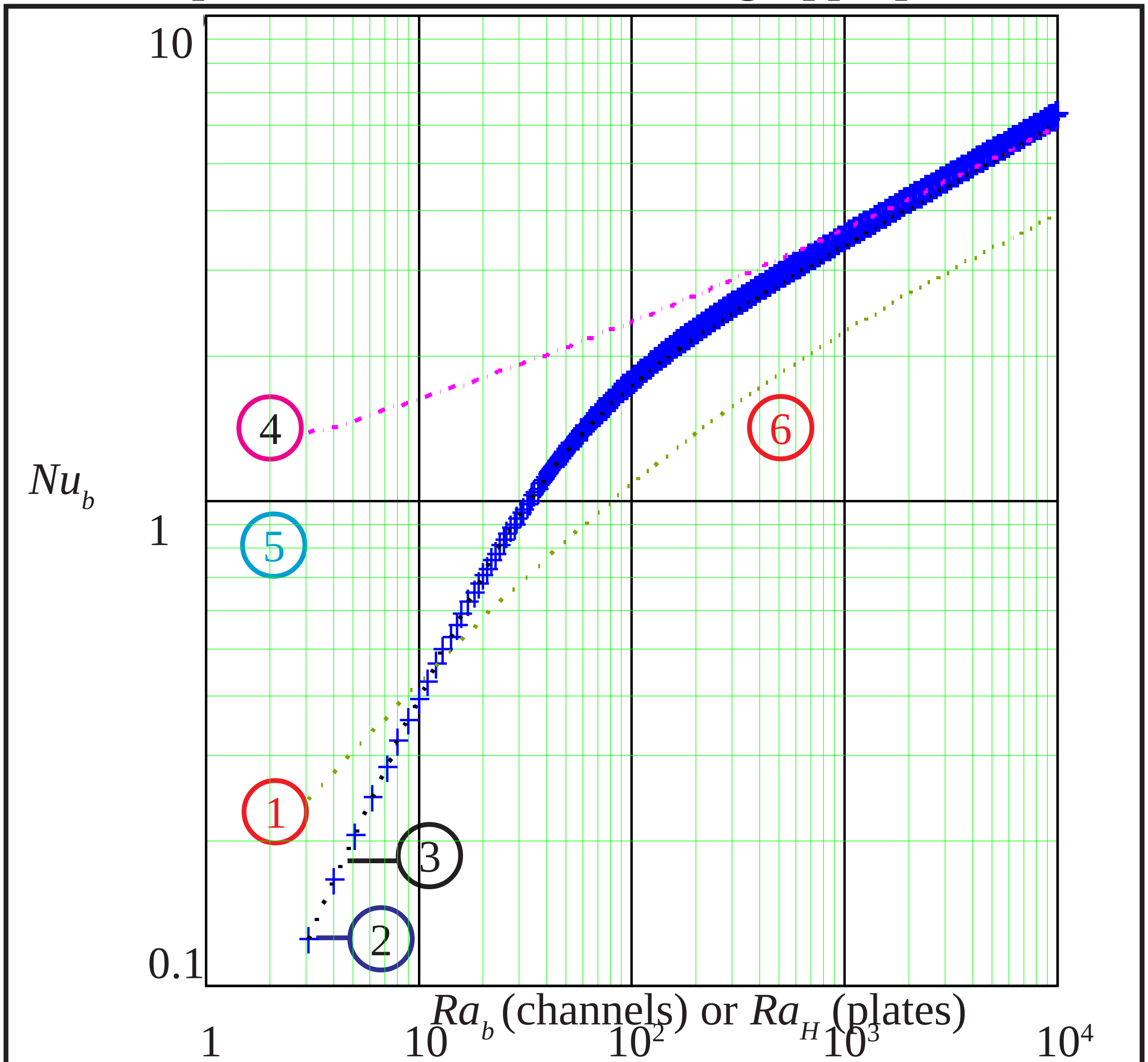
$$Nu_L = 0.59\,Ra_L^{1/4}$$

$Ra_L < 10^9$, (Church, S.W., and H.S. Chu, 1975) -

$$Nu_L = 0.68 + \frac{0.67\,Ra_L^{1/4}}{\left[1 + \left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{4/9}}$$

In all channel and single plate instances, air properties are evaluated at average film temperature, except Raithby and Hollands where in the case of small Ra_b, the average wall temperature should be used. In all cases, β, the thermal expansion coefficient is determined using the inlet temperature, i.e. $\beta = 1/T_I$ where T_I is in an absolute temperature scale.

Comparison of Correlations Using Appropriate *Ra* -



1 Bar-Cohen & Rohsenow - One heated wall - Isothermal

2 Raithby & Hollands - Symmetric - Isothermal

3 Bar-Cohen & Rohsenow - Symmetric - Isothermal

4 Churchill & Chu - Vertical plate ($Ra_L<10^9$) - Isothermal

5 McAdams - Vertical Plate ($10^4<Ra_L<10^9$) - Isothermal

6 Bar-Cohen & Rohsenow - Symmetric - Isoflux (uses Ra_b^*)

Optimum Circuit Board Spacing -

An optimum board spacing may be estimated using the plotted channel *Nusselt numbers*. Using the channel Rayleigh number, Ra_b, plotted as the abscissa, $(Ra_b) = Ra_{b-opt}$, a selected value,

$$Ra_b = (b/L)Gr_b \Pr = (b/L)\left[\left(g\rho^2\beta/\mu^2\right)b^3\right]\Delta T \Pr$$

$$Ra_{b-opt} = \left(b_{opt}/L\right)\left[\left(g\rho^2\beta/\mu^2\right)b_{opt}^3\right]\Delta T \Pr$$

$$b_{opt} = \left\{\left[\frac{L Ra_{b_{opt}}}{\left(\frac{g\rho^2\beta}{\mu^2}\right)\Delta T \Pr}\right]^{1/4}\right\},\quad Ra_{b-opt} = \begin{cases} 54, & \text{Symmetric} \\ 21.5, & \text{Asymmetric} \end{cases}$$

$$b_{opt} = Ra_{b_{opt}}{}^{1/4} P^{-1/4}$$

$$\text{where } P = \frac{Ra_b}{b^4} = \frac{g\beta\rho^2(T_w - T_I)}{\mu^2 L}\Pr = \gamma\frac{(T_W - T_A)}{L}$$

Referring to the summary containing the *Bar-Cohen & Rohsenow - Symmetric* correlations, use $Ra_{b_{opt}} = 54$, or the graph which appears to be the location, $Ra_{b_{opt}} = 54$, at which any further decrease in *Ra* would result in a significant decrease in *Nu*.

A safe upper limit to b_{opt} is obtained for a *reasonably* small value of $T_w - T_I$ at a *reasonably* high average of T_w, T_I, i.e.

$$T_w - T_I = 30\ {}^oC \text{ at } \frac{T_w + T_I}{2} = \frac{(30+50)+50}{2} = 65\ {}^oC$$

Then for L, b in *inches*, $P = \left[1.34x10^3(30)(0.72)\right]/L = 2.9x10^4/L$ and

$$\text{Symmetric, isothermal boards: } b_{opt} = 0.21L^{1/4};\ L, b \text{ in inches}$$

$$\text{Asymmetric, isothermal boards: } b_{opt} = 0.17L^{1/4};\ L, b \text{ in inches}$$

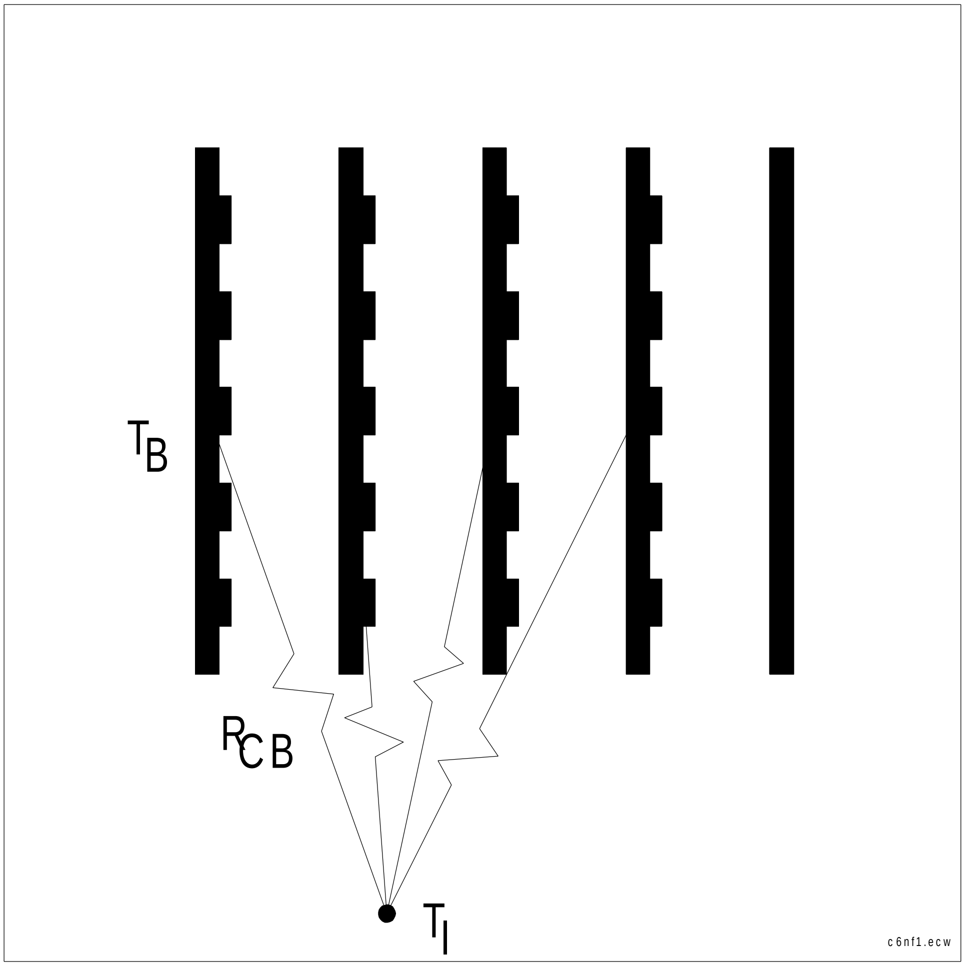
Application Example of Determining Optimum Board Spacing -

As an example, suppose that we have vertical board channels that are about *10 inches* high. Our optimum spacing is then

$$b_{opt} \cong 0.21L^{1/4} = 0.21(10\,inches)^{1/4} = 0.36\,in.$$

It is very important to remember that the board spacing b must refer to *component-surface to component-surface*.

Recommended Usage of Vertical Channel Correlations in Sealed Enclosure Models



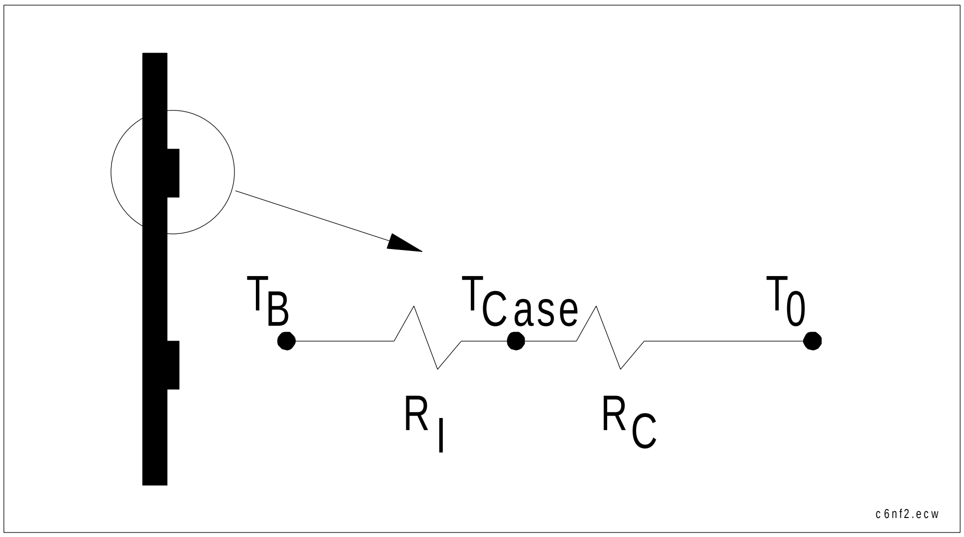
If only average board temperature required:

$$R_{CB} = \frac{1}{h_{CB} A_B}, \text{ where } h_{CB} \equiv \text{ channel } h \text{ for board}$$

$$T_I \cong T_{Air}, \text{ where } T_{Air} \equiv \text{ enclosure air computed for 6 sided box.}$$

$$T_{Air} \text{ is an } \textit{average maximum} \text{ (see } \textit{TCEE}, \text{ Chpt. 8)}$$

If circuit board analysis program such as *PTAMS* will be used to calculate component temperatures:



where

$$T_B \equiv \text{ board temperature}$$

$$T_{Case} \equiv \text{ component case temperature}$$

$$T_0 \equiv \text{ local ambient air temperature for all components}$$

$$= T_I = T_{Air}$$

$$R_I \equiv \text{ component case to board resistance}$$

$$R_C \equiv \text{ component case to local ambient resistance}$$

$$= \frac{1}{h_{CB} A_{Comp.}}$$

Recommended Usage of Vertical Channel Correlations in Vented Enclosure Models

Important Note: The following is suggested for manual calculations. Solution using a network computer program permits a variation of this (see the same problem in Section II).

The procedure is similar to sealed enclosures except the channel h uses an inlet temperature T_I in the range of

$$T_{Air} \geq T_I \geq \frac{T_{Air} + T_A}{2} \rightarrow T_A$$

$$\Uparrow \qquad\qquad\qquad \Uparrow$$

for $\quad Q_d \leq \frac{1}{2} Q_{Total} \qquad Q_d \geq \frac{1}{2} Q_{Total}$

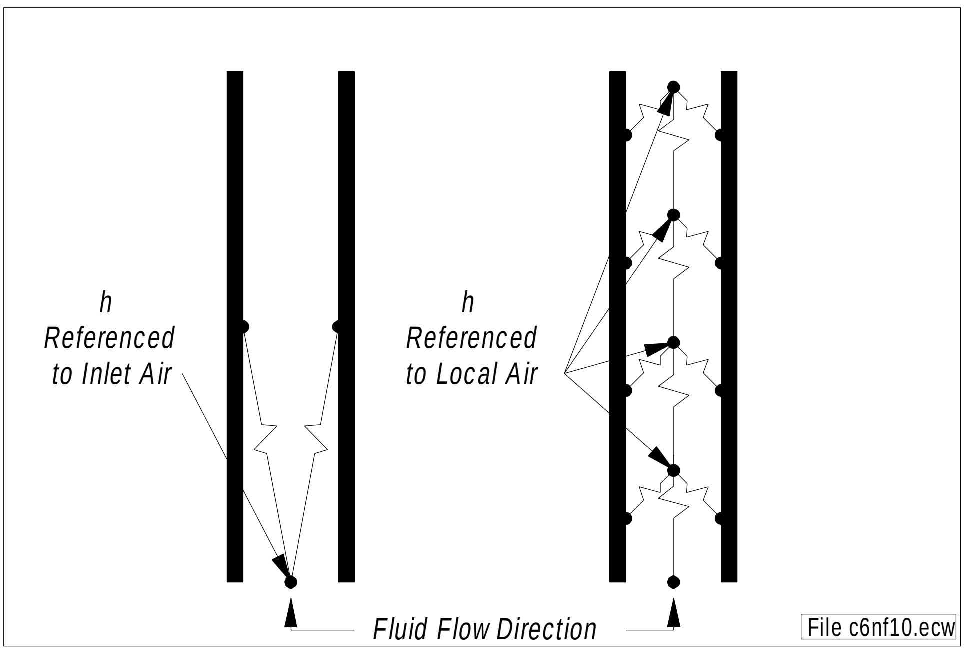
If circuit board analysis program such as *PTAMS* will be used to calculate component temperatures, then the preceding page applies, except that now we have

$$T_o = T_I = T_{Air} \qquad \text{or} \qquad T_o = T_I = \frac{T_{Air} + T_A}{2} \rightarrow T_A$$

$$\Uparrow \qquad\qquad\qquad \Uparrow$$

for $\quad Q_d \leq \frac{1}{2} Q_{Total} \qquad Q_d \geq \frac{1}{2} Q_{Total}$

Conversion of
Isothermal Plate Heat Transfer Coefficients,
Referenced to Inlet Air
to
Referenced to Local Air



The section entitled "Ducts and Finned Heat Sinks - A Model" should be reviewed by the student. In that section a derivation is given for a formula for the thermal resistance of an isothermal surface with a heat transfer coefficient referenced to the inlet.

The result is

$$\frac{C_I}{h_{Local}A_S} = \frac{1-e^{-\frac{h_{Local}A_S}{\dot{m}C_p}}}{\left(\dfrac{h_{Local}A_S}{\dot{m}C_p}\right)} \quad \text{or} \quad \frac{h_{Inlet}A_S}{h_{Local}A_S} = \frac{1-e^{-\frac{h_{Local}A_S}{\dot{m}C_p}}}{\left(\dfrac{h_{Local}A_S}{\dot{m}C_p}\right)}$$

where

$h_{Inlet} \equiv$ heat transfer coefficient referenced to inlet air temperature

$h_{Local} \equiv$ heat transfer coefficient referenced to local air temperature

$A_S \equiv$ total surface area of one interior surface of a channel

$\dot{m} \equiv$ mass flow rate of fluid $= \rho G$

$\rho \equiv$ fluid density, $G \equiv$ fluid volumetric flow rate

$C_P \equiv$ specific heat of fluid

It is straightforward to show that

$$h_{Local} = \frac{\dot{m}C_P}{A_S}\ln\left[\frac{1}{1-\dfrac{h_{Inlet}}{\left(\dot{m}C_P/A_S\right)}}\right] = \frac{\rho G C_P}{A_S}\ln\left[\frac{1}{1-\dfrac{h_{Inlet}}{\left(\rho G C_P/A_S\right)}}\right]$$

It is curious that even though the problem of interest is natural convection, we must know $\dot{m}$ to calculate h_{Loxal}.

Examination of the last result indicates that the value of h_{Local} blows up at $h_I / (\rho G C_P / A_S) = 1$ and h_{Local} is undefined for $h_I / (\rho G C_P / A_S) > 1$.

This can be explained physically by comparing, $\Delta T_f = T_E - T_I$, the temperature rise of the fluid from inlet air to exit air, with $\Delta T_W = T_W - T_I$, the wall (usually a circuit board) temperature rise rise above the inlet air temperature.

$$\Delta T_f = T_E - T_I = \frac{Q}{\rho G C_P}, \; \Delta T_W = T_W - T_I = \frac{Q}{h_{Inlet} A_A}$$

$$\frac{\Delta T_f}{\Delta T_W} = \frac{Q/(\rho G C_P)}{Q/(h_{Inlet} A_S)} = \frac{h_{Inlet}}{(\rho G C_P / A_S)}$$

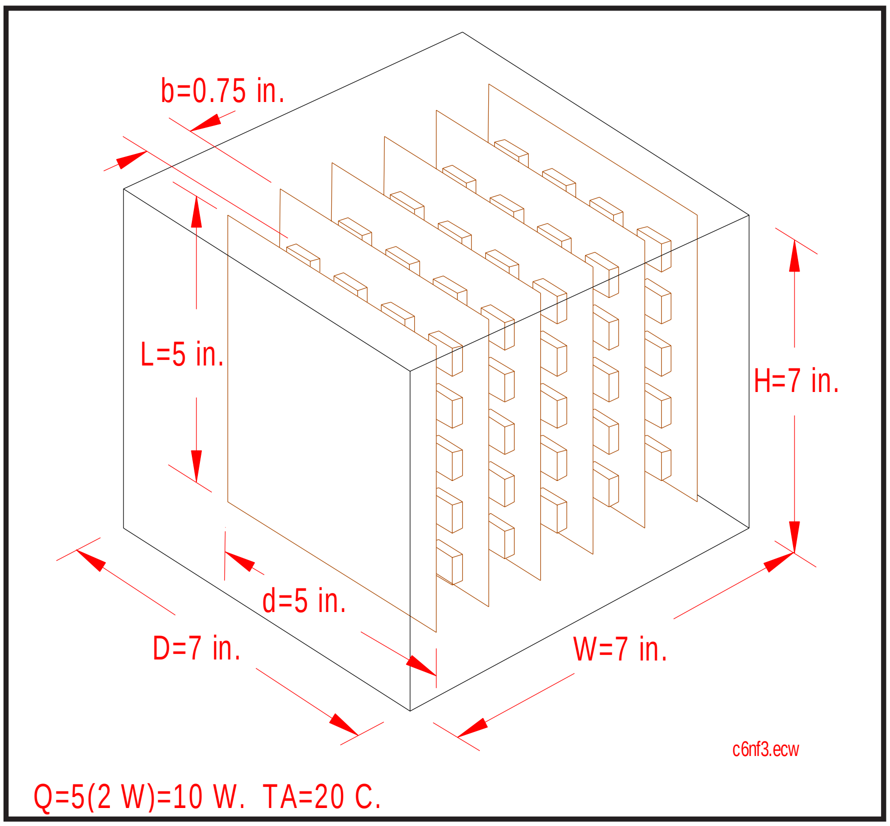
or

$$\frac{h_{Inlet}}{(\rho G C_P / A_S)} = \frac{\Delta T_f}{\Delta T_W}$$

If we realize that in the practical printed circuit board problem, the heat sources are on the board, and the board must therefore have a temperature rise above the inlet air that is greater than the temperature rise of the fluid from inlet to exit, i.e., $\Delta T_f / \Delta T_W < 1$, we should not have an ill-conditioned problem.

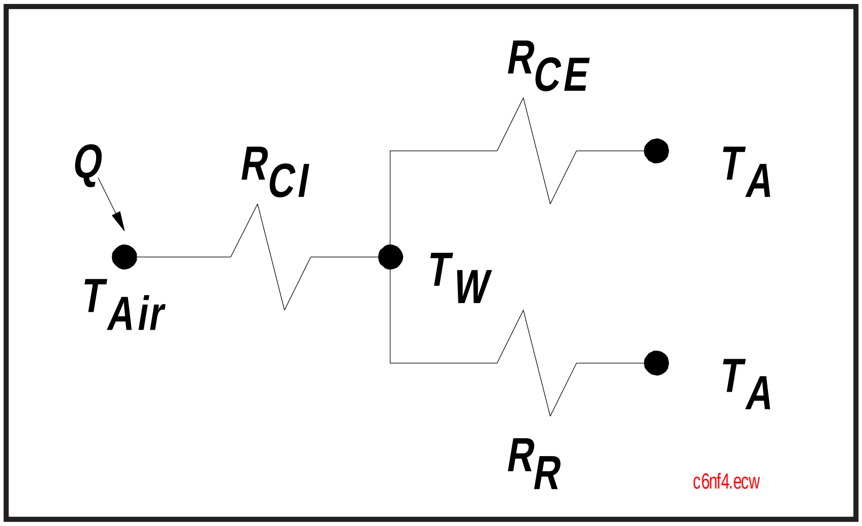
Application Example: Sealed Rectangular Enclosure - Thermal Network Model

Walls modeled as vertical and horizontal plates. Since all panels have the same dimensions, a generally not-recommended procedure of using the same convective heat transfer coefficent will be employed for each panel. This procedure should be used with caution and is used here for illustrative purposes (an iterative method usually requiring a computer program is the recommended procedure).



The problem is to calculate the average wall temperature and internal air temperature. Assume thin, highly conducting panels.

Thermal circuit for sample problem:



For now, assume that the surface emissivity is quite small, i.e., $\varepsilon = 0.1$ and radiation may be neglected (radiation will be covered later so that we will not have to neglect it if we don't wish to).

This package has about *2 watts* on each circuit board for a total dissipation of *Q=10 W.*

The convection resistances are

$$R_{CE} = \frac{1}{h_c A_S} = \frac{1}{0.0024\left(\frac{\Delta T}{H}\right)^{0.25}(6WH)}$$

$$R_{CI} = \frac{1}{h_c A_S} = \frac{1}{0.0024\left(\frac{\Delta T}{H}\right)^{0.25}(6WH)}$$

The equations for heat transfer are solved for the temperature rises:

$$T_W - T_A = \Delta T_{WA} = QR_{CE} = \frac{Q}{6WH(0.0024)\left(\frac{\Delta T_{WA}}{H}\right)^{0.25}}$$

$$\Delta T_{WA} = \left[\frac{QH^{0.25}}{6(0.0024)WH}\right]^{1/1.25} = \left[\frac{(10\,W)(7\,in.)^{0.25}}{6(0.0024)(7\,in.)(7\,in.)}\right]^{1/1.25}$$

$$= 12\ {}^{o}C$$

The internal air temperature rise above the wall temperature is obviously the same, i.e.

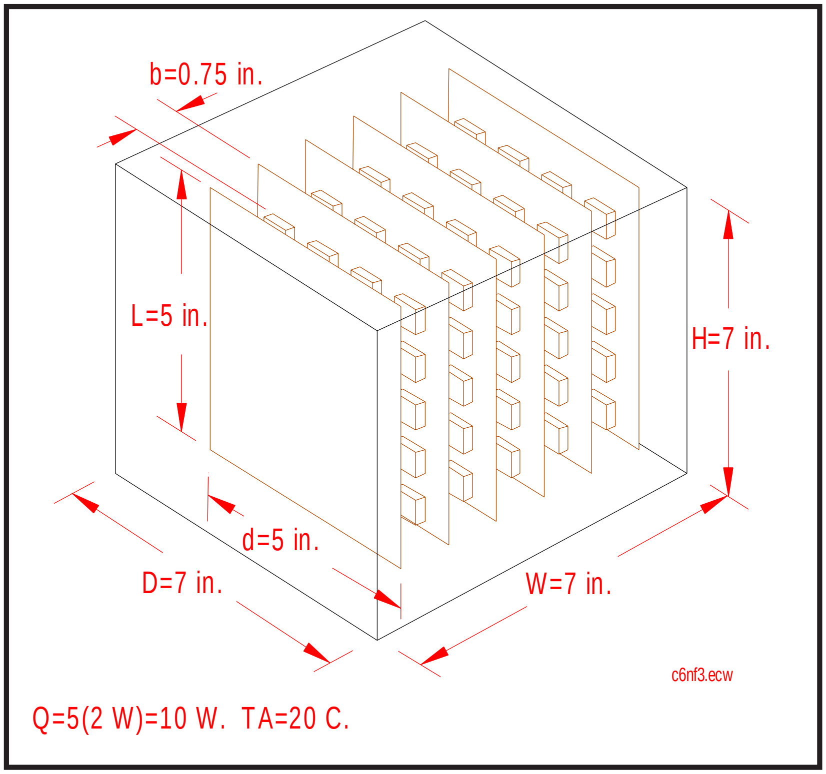
$$\Delta T_{AirW} = 12\ {}^{o}C$$

$$T_{Air} = \Delta T_{AirW} + \Delta T_{WA} + T_A = 12 + 12 + 20$$

$$\cong 44\ {}^{o}C$$

Application Example: Five Vertical Circuit Board Channels In a Sealed Enclosure

This is a continuation of a cube shaped enclosure for which the internal air temperature was previously calculated to be 44^oC and the average wall temperature was calculated to be 32 ^oC.



Even though the component surface to opposite board surface is given as *0.75 in.*, the optimum spacing should be calculated.

We see that the boards could be closer together.

$$b_{opt} = 0.21L^{1/4} = 0.21(5in.)^{1/4} = 0.31in.$$

Using the isothermal, vertical channel result from Bar-Cohen & Rohsenow for the circuit boards:

Using T_B for the surface temperature,

$$\bar{N}u_b = \left[\left(\frac{24}{Ra_b} \right)^2 + \left(\frac{1}{0.59 Ra_b^{1/4}} \right)^2 \right]^{-1/2}$$

$$Ra_b = \frac{g \beta \rho^2 (T_B - T_I) b^4}{\mu^2 L} \Pr = \left[\left(\frac{g \beta \rho^2}{\mu^2} \right) \Pr \right] \frac{(T_B - T_I) b^4}{L}$$

$$= \gamma \frac{(T_B - T_I) b^4}{L}$$ where γ is presented as a plot in the previous section, "Flat Plates Simplified".

We shall assume an average "film" temperature (average of T_B and T_I) of $50\ ^oC$ so that

$$\gamma = \frac{g \beta \rho^2}{\mu^2} = 1.3x10^3$$

An iterative procedure is required, starting with an estimate of $\bar{T}_B - T_I$ to calculate the actual $\bar{T}_B - T_I$. Only the last step of the iterations are shown here.

Starting with $\bar{T}_B - T_I = 12^o C$

$$Ra_b = \gamma \frac{\left(12^0 C\right)(0.75)^4}{5} = \frac{1.3x10^3 (12)(0.75)^4}{5} = 9.84x10^2$$

$$\overline{Nu}_b = \left\{ \left(\frac{24}{9.84x10^2} \right)^2 + \frac{1}{\left[0.59\left(9.84x10^2\right)^{1/4}\right]^2} \right\}^{-1/2} = 3.3$$

$$h_{CB} = \frac{k}{b}\overline{Nu}_b = \left(\frac{7x10^{-4}}{0.75} \right)(3.3) = 0.0031$$

Note: Comparing h_{CB} with a single vertical flat plate h_L produces a similar result:

$$Ra_L = Ra_b\left(\frac{L^4}{b^4}\right) = \left(9.84x10^2\right)\left(\frac{5.0}{0.75}\right)^4 = 1.94x10^6$$

$$Nu_L = 0.59Ra_L^{1/4} = 0.59\left(1.94x10^6\right)^{1/4} = 22.0$$

$$h_L = \left(\frac{k}{L}\right)Nu_L = \left(\frac{7x10^{-4}}{5.0}\right)(22.0) = 0.0031\, W/in.^2 \cdot {}^oC$$

The selection of the proper board area for the component side is not really obvious. However, if the IC packages are mostly low-profile surface mount devices (SMD), it is probably reasonable to use

$$A_S = dL = (5in.)(5in.) = 25in.^2$$

Q=2.0 W/2 (assumes board convects *1.0 W* from <u>each side</u>)

$$\overline{T}_B - T_I = \frac{Q}{h_{CB}A_s} = \frac{1.0}{(0.0031)(25)}$$

$$= 12\ {}^oC$$

If a realistic estimate of the board channel inlet air temperature is T_{Air}, then

$$\overline{T}_B = \left(\overline{T}_B - T_I\right) + T_{Air} = 12 + 44 = 56\ {}^oC$$

Using the isoflux, vertical channel result from Bar-Cohen & Rohsenow for the circuit boards:

The expected board temperature is not expected to be very different from the isothermal result, so all physical quantities shall be taken from the isothermal results.

$$Ra_b^* = \gamma\left(\frac{b^5}{L}\right)\left(\frac{q}{k}\right) = 1.3x10^3\left[\frac{(0.75)^5}{5.0}\right]\left(\frac{1.0/(5.0)^2}{6.5x10^{-4}}\right) = 3.80x10^3$$

$$\overline{N}u_b = \left[\frac{48}{Ra_b^*} + \frac{2.51}{Ra_b^{*0.4}}\right]^{-1/2}$$

$$= \left[\frac{48}{3.80x10^3} + \frac{2.51}{\left(3.80x10^3\right)^{0.4}}\right]^{-1/2} = 3.08$$

$$h = \left(\frac{k}{b}\right)\overline{N}u_b = \left(\frac{6.5x10^{-4}}{0.75}\right)(3.08) = 2.67x10^{-3}$$

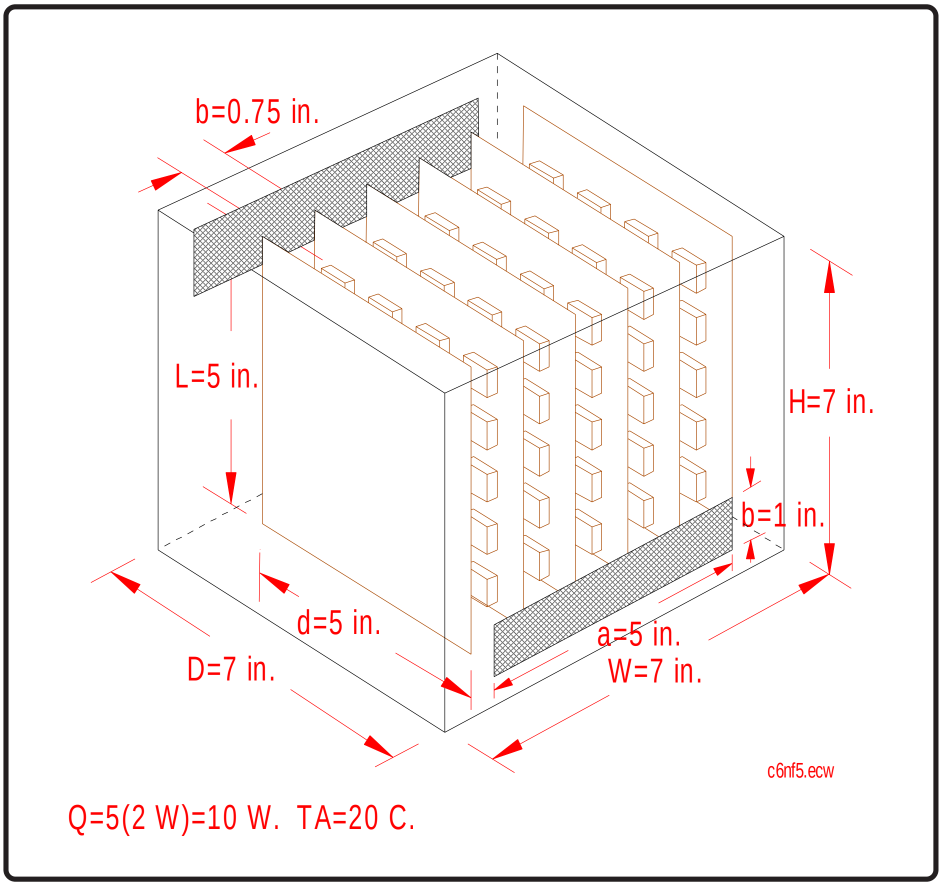
$$R = \frac{1}{hL^2} = \frac{1}{\left(2.67x10^{-3}\right)(5.0)^2} = 14.99$$

$$T_B - T_I = QR = (1.0)(14.99) = 15.0\ {}^oC$$

If a realistic estimate of the board channel inlet air temperature is T_{Air}, then

$$\bar{T}_B = \left(\bar{T}_B - T_I\right) + T_{Air} = 15 + 44 = 59\,^oC$$

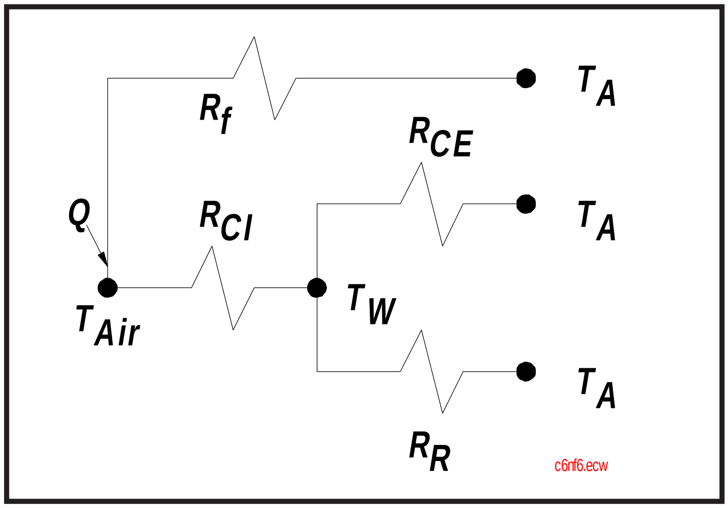
Application Example: Vented Rectangular Enclosure - Thermal Network Model



The illustrated enclosure is identical to the previous box analyzed except that two identical vents have been added in an effort to reduce the internal temperature. The vents are 35 % perforated with 0.188 in. diameter holes.

The calculation will begin with a repeat of the sealed version of the problem, followed by the vented version. It is important that you carefully follow the *<u>iterative method used for both versions</u>*.

The thermal circuit is



The first step is to set up the equations that will be needed. Beginning with the airdraft G,

$$G = 1.53x10^{-2}\left(\frac{Q_d d}{R_a}\right)^{1/3}$$

R_a is the total *airflow* resistance. In this instance we shall calculate this to be the series sum of the inlet, expansion from the inlet, card cage contraction/cards/expansion, contraction to the exit, and the exit elements, respectively.

$$R_{In}, R_{InEx}, R_C, R_{Cards}, R_E, R_{ExCon}, \text{and } R_{Ex}$$

$$R_{In} = \frac{2.0x10^{-3}}{A_{In}^2} = \frac{2.0x10^{-3}}{(5in.x1in.x0.35)^2} = 6.5x10^{-4}$$

$$R_{Ex} = R_{In} = 6.5x10^{-4}$$

The expansion from the inlet is

$$R_{InEx} = 1.29x10^{-3}\left[\frac{1}{A_1}(1-0.5)\right]^2$$

$$= 1.29x10^{-3}\left[\frac{1}{1.0\,in.\,x\,5.0\,in.}(1-0.5)\right]^2$$

$$= 1.29x10^{-5}$$

The contraction into the card cage is taken to be over the entire card cage base (foot print).

$$R_C = \frac{0.63x10^{-3}}{A_f^2} = \frac{0.63x10^{-3}}{(5.0\,in.\,x\,5.0\,in.\,0.75)^2} = 1.79x10^{-6}$$

The card cage resistance is

$$R_{CC} = \frac{1.95x10^{-3}nL}{A^2} = \frac{1.95x10^{-3}(1)(5.0\,in.)}{(5.0\,in.\,x5.0\,in.)^2} = 1.56x10^{-5}$$

The card cage expansion is taken to be from the entire card cage top.

$$R_E = 1.29x10^{-3}\left[\frac{1}{A_1}\left(1-\frac{A_1}{A_2}\right)\right]^2$$

$$= 1.29x10^{-3}\left[\frac{1}{5.0x5.0x0.75}(1-0.75)\right]^2 = 2.29x10^{-7}$$

The contraction into the exit is

$$R_{ExCon} = \frac{0.63x10^{-3}}{A^2} = \frac{0.63x10^{-3}}{(1.0\,in.\,x\,5.0\,in.)^2} = 1.01x10^{-6}$$

The total airflow resistance is then

$$\begin{aligned} R_a &= R_{In} + R_{InEx} + R_C + R_{Cards} + R_E + R_{ExCon} + R_{Ex} \\ &= 6.5x10^{-4} + 1.29x10^{-5} + 1.79x10^{-6} + 1.56x10^{-5} \\ &\quad +2.29x10^{-7} + 1.01x10^{-6} + 6.5x10^{-4} \\ &= 1.33x10^{-3} \end{aligned}$$

The required equation for calculating the airdraft is

$$G = 1.53x10^{-2}\left(\frac{Q_d d}{R_a}\right)^{1/3} = 1.53x10^{-2}\left[\frac{Q_d(5in.)}{1.33x10^{-3}}\right]^{1/3}$$

$$\boxed{G = 0.24Q_d^{1/3}}$$

The thermal fluid resistance R_f is readily calculated using the air temperature rise formula

$$\Delta T = \frac{1.76 Q_d}{G}, \qquad R_f = \frac{T_{Air} - T_0}{Q_d} = \frac{\Delta T}{Q_d}$$

$$\boxed{R_f = \frac{1.76}{G}}$$

As in the two preceding examples using this enclosure example, the external emissivity is assumed sufficiently small that the external radiation may be neglected.

It is easily shown that the thermal resistances in series and parallel add in the same manner as electrical resistances (the reader should prove this as an exercise). The total convection path thermal resistance is then

$$R_C = R_{CE} + R_{CI} = 2 R_{CE}$$

$$R_{CE} = \frac{1}{6 A_S h_{CE}} = \frac{1}{6 A_S 0.0024 \left(\frac{\Delta T_{W-A}}{H} \right)^{0.25}}$$

$$= \frac{1}{6(7\, in.\, x\, 7\, in.) 0.0024 \left(\frac{\Delta T_{W-A}}{7} \right)^{0.25}}$$

$$R_{CE} = \frac{2.27}{\Delta T^{0.25}}$$

$$R_C = 2R_{CE}$$

Then

$$\boxed{R_C = \frac{4.54}{\Delta T_{W-A}^{0.25}}}$$

The total resistance from the internal air to the external ambient is

$$\boxed{R_{Total} = \frac{R_f R_C}{R_f + R_C}}$$

and the overall temperature rise from ambient to the internal air is

$$T_{Air} - T_A = \Delta T_{Total}$$

$$\boxed{T_{Air} - T_A = R_{Total} Q}$$

In this problem

$$\boxed{T_W - T_A = (T_{Air} - T_A)/2}$$

The airdraft is calculated using

$$T_{Air} - T_A = 1.76\frac{Q_d}{G}$$

$$\boxed{Q_d = \left(\frac{T_{Air} - T_A}{1.76}\right)G}$$

Iteration 1: Q_d, G both 0.0 for sealed box by definition.
Guess 1st value of T_W-T_A=10 ^{0}C.

$$\text{Then } R_C = 4.54 / \Delta T_{W-A}^{0.25}$$

$$R_{Total} = R_f R_C / \left(R_f + R_C\right) \rightarrow R_C \text{ for } R_f = \infty$$

$$T_{Air} - T_A = R_{Total} Q$$

Iteration 2:

$$T_W - T_A = \left(T_{Air} - T_A\right)/2 \text{ using the most recently calculated } T_{Air} - T_A$$

.

Iteration 4: The results are finished for the sealed enclosure.

Iteration 5: Guess the 1st value of Q_d, e.g. $Q_d = 5.0$.

$$G = 0.24 Q_d^{1/3},\ R_f = 1.76/G$$

$$T_W - T_A = \left(T_{Air} - T_A\right)/2 \text{ using the most recent } T_{Air} - T_A.$$

$$R_C = 4.54 / \Delta T_{W-A}^{0.25}$$

$$R_{Total} = R_f R_C / \left(R_f + R_C\right)$$

$$T_{Air} - T_A = R_{Total} Q$$

Iteration 6: $Q_d = \left(\dfrac{T_{Air} - T_A}{1.76}\right) G, \quad G = 0.24 Q_d^{1/3}$

.

.

Results of each iteraton step:

Iter	$\boldsymbol{Q_d}$	***G***	$\boldsymbol{R_f}$	$\boldsymbol{T_W - T_A}$	$\boldsymbol{R_C}$	$\boldsymbol{R_{Total}}$	$\boldsymbol{T_{Air} - T_A}$
1	0	0.0	inf.	10.00	2.55	2.55	25.5
2	0	0.0	inf.	12.75	2.40	2.40	24.0
3	0	0.0	inf.	12.0	2.44	2.44	24.4
4	0	0.0	inf.	12.2	2.43	2.43	24.3
5	5.0	0.39	4.48	12.2	2.43	1.57	15.7
6	3.48	0.35	5.05	7.85	2.71	1.76	17.6
7	3.50	0.35	5.04	8.8	2.64	1.73	17.3
8	3.44	0.35	5.07	8.65	2.65	1.74	17.4

We shall now attempt to improve on the airflow resistance R_a by using the perforated plate resistance data from Adam, Fried & Idlechick.

$$\mathrm{Re}_D = \frac{V_d d}{\upsilon} = \frac{Gd}{5\upsilon A_f} = \frac{(0.344)(0.188\,in.)}{5(0.023)\left(\frac{5.0\,in.\,x\,1.0\,in.\,x\,0.35}{144}\right)} = 47$$

Referring to the Adam, Fried & Idelchick graphs,

$$K_d = K_a f^2 = (12)(0.35)^2 = 1.5$$

$$R_{In} = R_{Ex} = K_d\left(\frac{1.29x10^{-3}}{A_f^2}\right)$$

$$= 1.5\left[\frac{1.29x10^{-3}}{(5.0x1.0x0.35)^2}\right] = 6.3x10^{-4}$$

and

$$R_a = 6.03x10^{-4} + 1.79x10^{-6} + 1.56x10^{-5} + 2.29x10^{-7} + 6.03x10^{-4}$$
$$= 1.28x10^{-3}$$

$$G = 1.53x10^{-2}\left(\frac{Q_d d}{R_a}\right)^{1/3} = 1.53x10^{-2}\left[\frac{Q_d(5)}{1.28x10^{-3}}\right]^{1/3} = 0.24Q_d^{1/3}$$

which is exactly equal to that which was just used.

Application Example: Five Vertical Circuit Board Channels In a Vented Enclosure

The overall air temperature rise is seen to be 17 ^oC for the vented problem as opposed to 24 ^oC for the sealed version. This is not a tremendous improvement, but could be worthwhile in marginal designs.

As recommended in the section covering circuit board channels in natural convection, air drafts that are somewhat small (resulting in Q_d =3.4 W out of a total of 10 W) should result in a board channel inlet ambient of approximately

$$T_I \cong T_{Air} = 17 + 20 = 37\,^oC$$

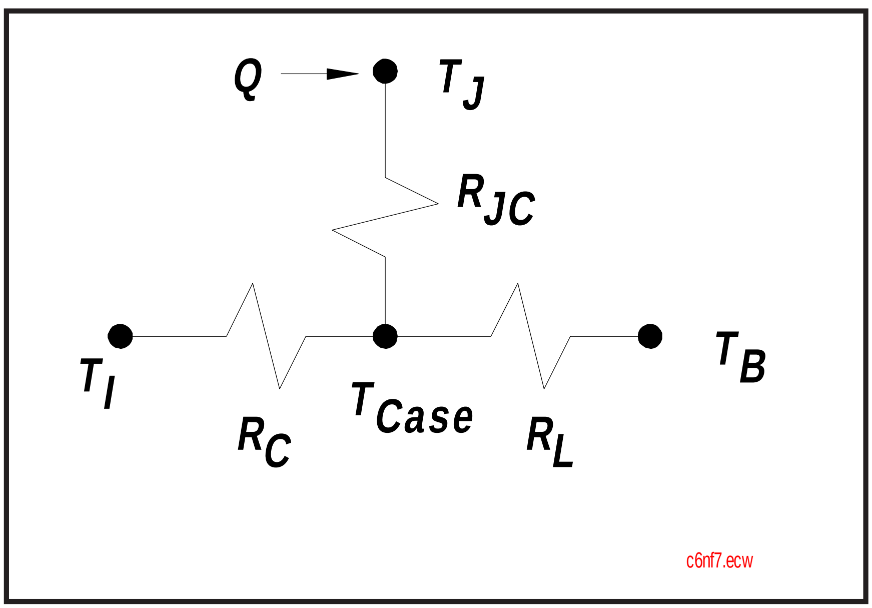
We can use the average board temperature rise of 12 ^oC computed for the sealed enclosure to get

$$\bar{T}_B = \left(\bar{T}_B - T_I\right) + \left(T_I - T_A\right) + T_A = 12 + \left(37 - 20\right) + 20 = 49\,^oC$$

The next step is the calculation of the temperature of a 20 pin, plastic SOL device on one of the circuit boards. This component is of particular concern because it dissipates 0.5W, a considerable fraction of the total board dissipation .

The following calculation is intended to illustrate the last step in a problem such as this - the calculation of a junction temperature.

As an example, consider a 20 pin, plastic SOL device from the vendor data. The component is thermally connected to the a board and air ambient in a manner that is shown in the following thermal circuit. The vendor data indicates a junction to case thermal resistance of $R_{JC} = 20\,{}^{o}C/W$.



We shall assume that the major heat conduction path to the circuit board is through the package leads. The total lead resistance is

$$R_L = \left(\frac{1}{20}\right)\frac{l}{kwt} = \left(\frac{1}{20}\right)\frac{(0.06\,in./2)}{\left(10\,W/in.^oC\right)(0.016\,in.)(0.006\,in.)}$$

$$= 1.55\,^oC/W$$

We can calculate the component convection resistance using the same heat transfer coefficient as we used for the circuit boards.

$$R_C = \frac{1}{hA} = \frac{1}{\left(0.0031\,W/in.^2\cdot{}^oC\right)(0.497\,in.)(0.217\,in.)}$$

$$= 2991\,^oC/W$$

The equations needed to calculate the junction temperature are set up as follows:

$$Q = Q_C + Q_L$$

$$= \frac{T_{Case} - T_I}{R_C} + \frac{T_{Case} - T_B}{R_L} = \frac{T_{Case}}{R_C} - \frac{T_I}{R_C} + \frac{T_{Case}}{R_L} - \frac{T_B}{R_L}$$

$$= T_{Case}\left(\frac{1}{R_C} + \frac{1}{R_L}\right) - \left(\frac{T_I}{R_C} + \frac{T_B}{R_L}\right)$$

The case temperature is calculated for the situation of the vented enclosure where both the internal air and average board temperatures have been calculated.

$$T_{Case} = \frac{Q + \left(\frac{T_I}{R_C} + \frac{T_B}{R_L}\right)}{\frac{1}{R_C} + \frac{1}{R_L}}$$

$$= \frac{0.5 + \left(\frac{37}{2991} + \frac{49}{1.55}\right)}{\frac{1}{2991} + \frac{1}{1.55}}$$

$$= 50\,^oC$$

It is not surprising that the case temperature is very nearly the board temperature when one compares the component convection resistance of 2991 $^oC/W$ with the lead conduction resistance of 1.55 ^oC.

The component junction temperature is calculated as

$$T_J = QR_{JC} + T_C$$

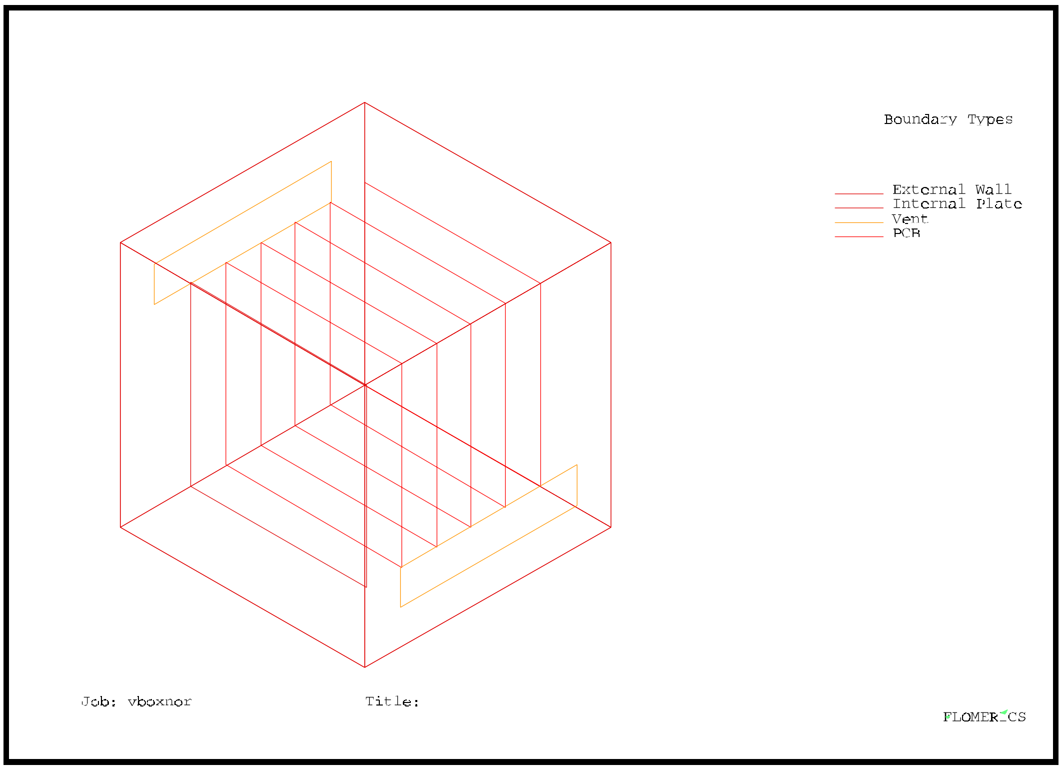
$$= (0.5W)\left(20\,^oC/W\right) + 50$$

$$= 60\,^oC$$

A junction temperature of $60\,^oC$ seems quite low, perhaps suggesting that the system is over designed. However, when one considers a more realistic design specification of an external ambient of $T_A = 50\ ^oC$, we need to consider then that the predicted junction temperature would be $30\,^oC$ greater, i.e. $T_J = 90\,^oC$, which is probably adequate and the design may not require the air vents.

Analysis of The Enclosure Problems Using Computational Fluid Dynamics (FLOTHERM™ software.) -

The problem geometry from the FLOTHERM screen is shown below. Although the drawing indicates air vents, these vents were temporarily deactivated for the sealed version of the problem.

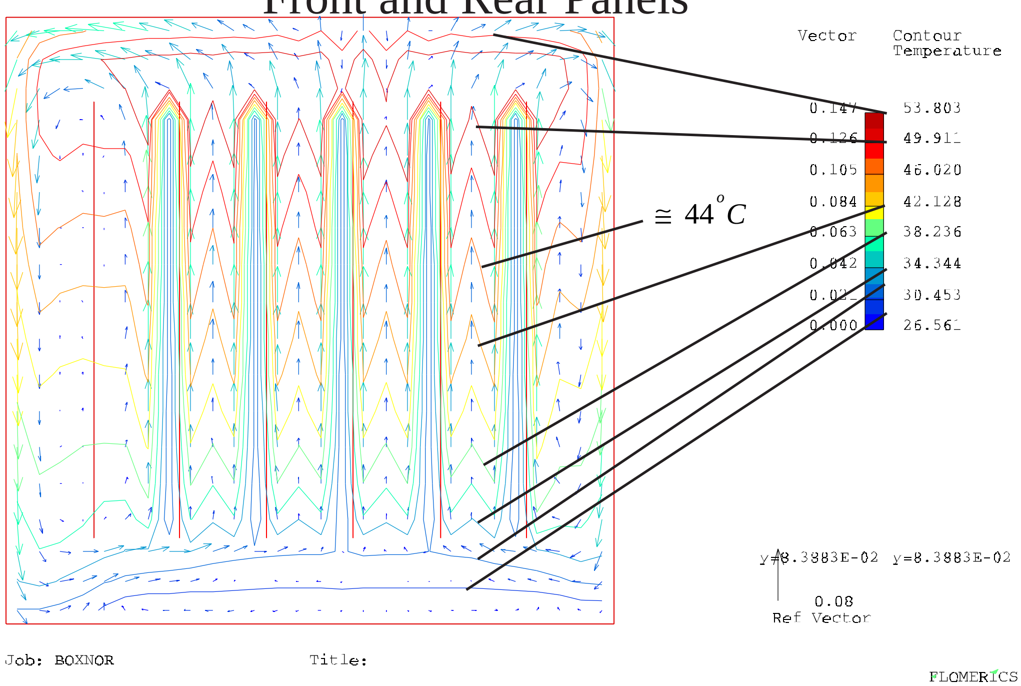


Summary of Both Preceding Manual Calculations and CFD Results for the Sealed Enclosure:

Location	Manual	FLOTHERM
T_A	*20*	*20*
T_W	*32*	*39 [(51+26+4*39)/6]*
T_{Air}	*44*	*51, 43, 34 (top, mid, bottom of air channel)*
T_B-T_I	*12*	*14 w/4 cells, 15 w/20 cells across channel*
T_B	*56 [T_I=T_{Air}]*	*54*

Thus the chosen method of calculating T_I (T_I=[T_{Air}+T_W]/2)for a sealed enclosure is validated.

Front View of Enclosure in Plane Halfway Between Front and Rear Panels

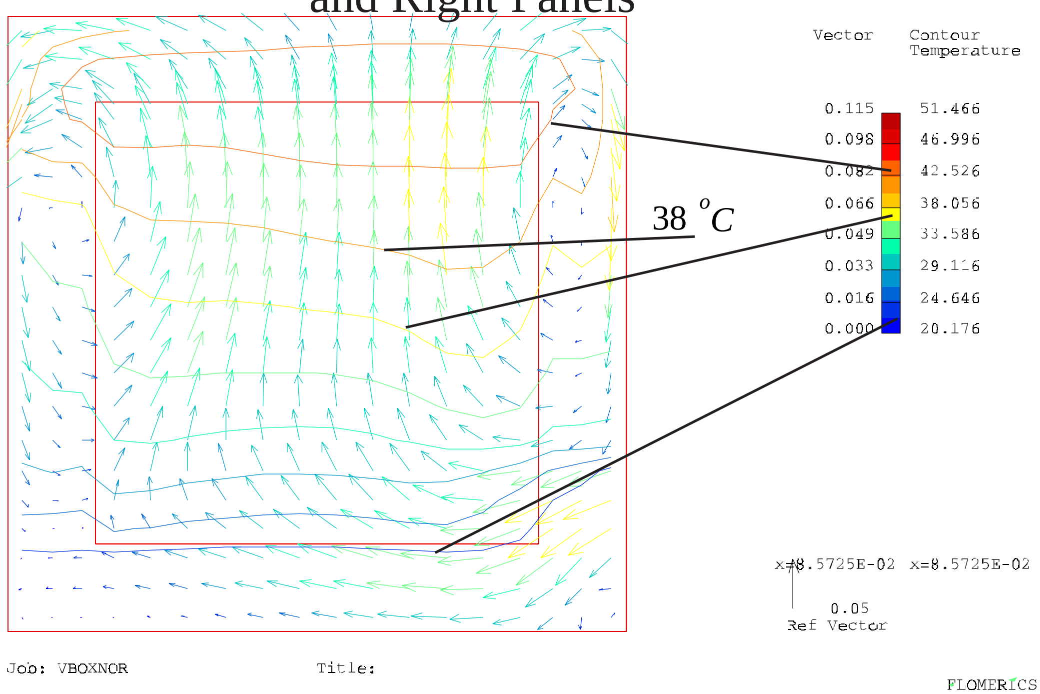


Summary of Both Preceding Manual Calculations and CFD Results for the Vented Enclosure:

Location	Manual	FLOTHERM
T_A	*20*	*20*
T_W	*29*	*32 [(45+21+4*32)/6]*
T_{Air}	*37*	*44, 35, 22 (top, mid, bottom of air channel)*
T_B-T_I	*12*	*13 w/4 cells, 17 w/20 cells across channel*
T_B	*49[T_I=37]*	*51 (no. of cells unknown)*

Thus the chosen method of calculating $T_I \triangleq (T_{Air} + T_{Wall})/2$ is adequate for this vented enclosure where Q_d=*3.4 W* out of the *10 W* total.

Side View of Enclosure in Plane Halfway Between Left and Right Panels



A Comment Concerning Mixed (Free and Forced Convection)

There may be situations for which both forced *and* free convection contributions are significant. Most general heat transfer texts indicate the following criteria:

Totally forced convection:	$Gr_L / Re_L^2 \ll 1$
Mixed convection:	$Gr_L / Re_L^2 \sim 1$
Totally free convection:	$1 \ll Gr_L / Re_L^2$

where L is the characteristic length.

The typical method of combined the two convection effects is to use

$$Nu_{Combined} = \left(Nu_{Forced}^3 \pm Nu_{Free}^3 \right)^{1/3}$$

where "+" is used for the forced and free convection in the same vertical direction and the "-" is used for opposing forced and free convection.

Moffat and Ortega (1988) examine adiabatic the Nusselt number Nu_a normalized on the forced convection value $Nu_{a\text{-}forced}$, i.e. (Nu_a /$Nu_{a\text{-}forced}$), for an array of cubical elements. The flow regime parameter used was a local Gr_{ad}/Re_B^2 where B is the component height (perpendicular to plane of board). The local adiabatic Gr_{ad} and Re_B are defined by Moffat and Ortega as

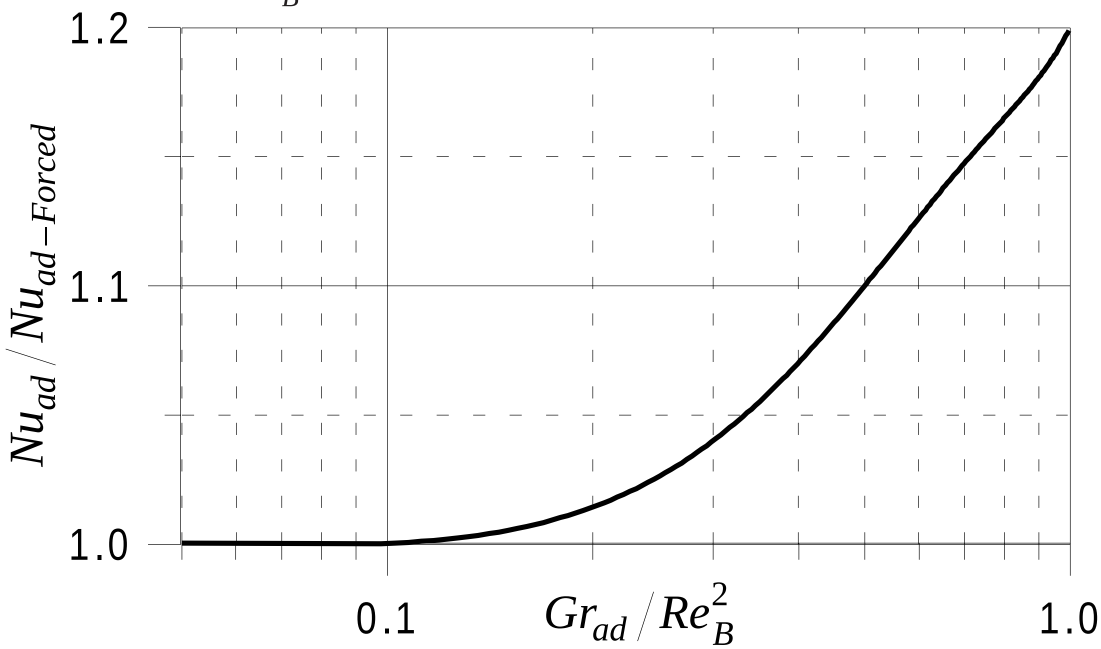
$$Gr_{ad} = \frac{g\left[\left(T - T_{ad}\right)/T_{ad}\right]B^3}{\upsilon^3}, \quad Re_B = \frac{V_{Approach}B}{\upsilon}$$

Totally forced convection: $Gr_L/Re_L^2 < 0.3$

Mixed convection: $0.3 < Gr_L/Re_L^2 < 10.0$

Totally free convection: $10.0 < Gr_L/Re_L^2$

Ortega and Moffet compared empirical measurements of Nu_{ad} to $Nu_{ad\text{-}forced}$ for cubical blocks with channel spacing to block rations of 1.5-4.0 and Re_B =121-124.

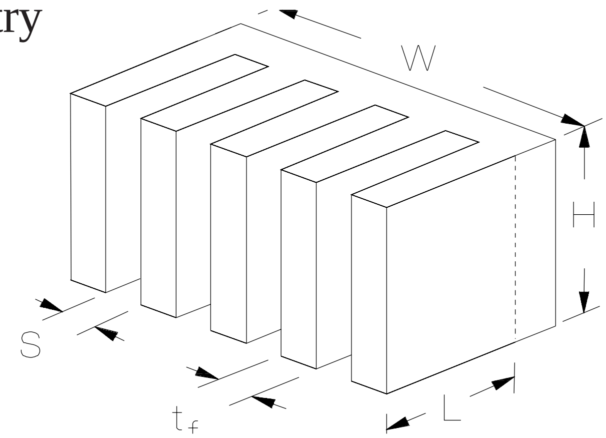


Adapted from Ortega and Moffat (1988).

Natural Convection - Finned Heat Sinks

Heat Sinks

Geometry



Exterior surface conductance: $C_{Ext} = h_H A_{Ext} \eta$

Exterior surface area: $A_{Ext} = 2H(L + t_b)$

Interior surface conductance: $C_{Int} = h_i A_i \eta$

Interior surface area: $A_{Int} = WH\left[1 + \frac{2(N-1)}{W} L\right]$

Note: this form of A_i includes fin tips and fin bases between fins. The use of $C_{Int} = h_c A_{Int} \eta$ is an approximation. Some people separate the conductance into unfinned and fined portions on the finned side. This is not necessary when the finned area is much greater than the unfinned area.

Convection Coefficients

Exterior surfaces

$$h_H = 0.0024\left(\frac{\Delta T}{H}\right)^{0.25}, \qquad \text{vertical flat plate}$$

Interior surfaces

$$h_C = \left(\frac{h_C}{h_H}\right)_U h_H, \qquad \text{vertical U - channel}$$

Van de Pol and Tierney U-Channel Correlation from Experimental Data

$$Nu_r = \frac{Ra^*}{\psi}\left\{1 - \exp\left[-\psi\left(\frac{0.5}{Ra^*}\right)^{3/4}\right]\right\}$$

$$\psi = \frac{24\left[1 - 0.483e^{(-0.17/a)}\right]}{\left\{\left[1 + \frac{a}{2}\right]\left[1 + \left(1 - e^{-0.83a}\right)\left(9.14a^{1/2}e^{VS} - 0.61\right)\right]\right\}^3}$$

$$Ra^* = \left(\frac{r}{H}\right)Gr_r\,\Pr,\; r = 2LS/(2L + S),\; a = S/L,\; V = -11.8\left(in.^{-1}\right)$$

All physical properties except β evaluated at the surface temperature. β is evaluated at the fluid temperature. Van de Pol and Tierney indicate that as $L/S \rightarrow 0,\ h_c/h_H \rightarrow 1.0$.

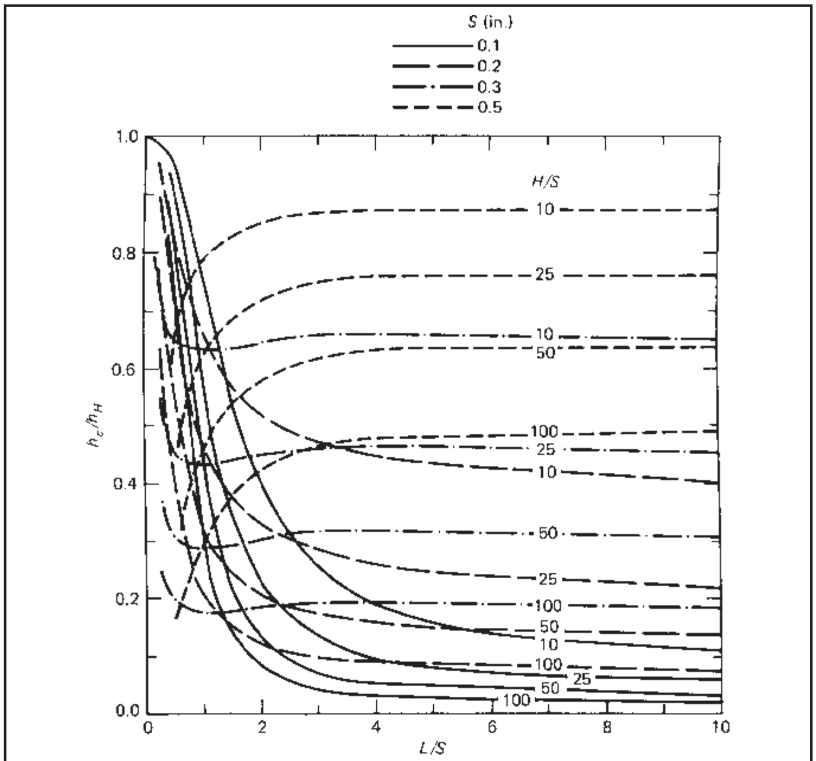


Fig. 5-14. Correction factor to vertical plate heat transfer coefficient for finned heat sink. $\Delta T = 10°C$.

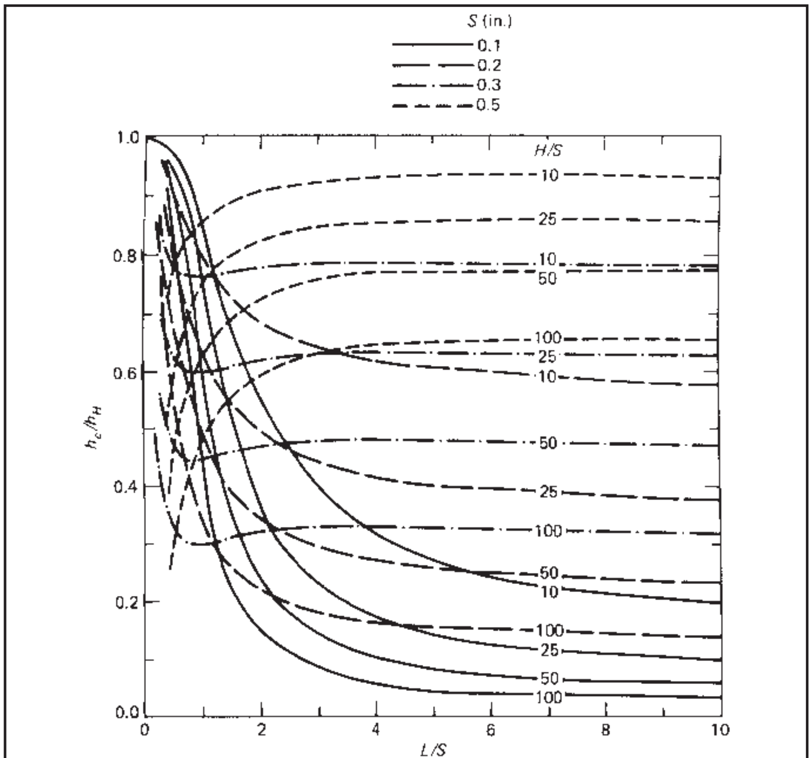


Fig. 5-15. Correction factor to vertical plate heat transfer coefficient for finned heat sink. $\Delta T = 25°C$.

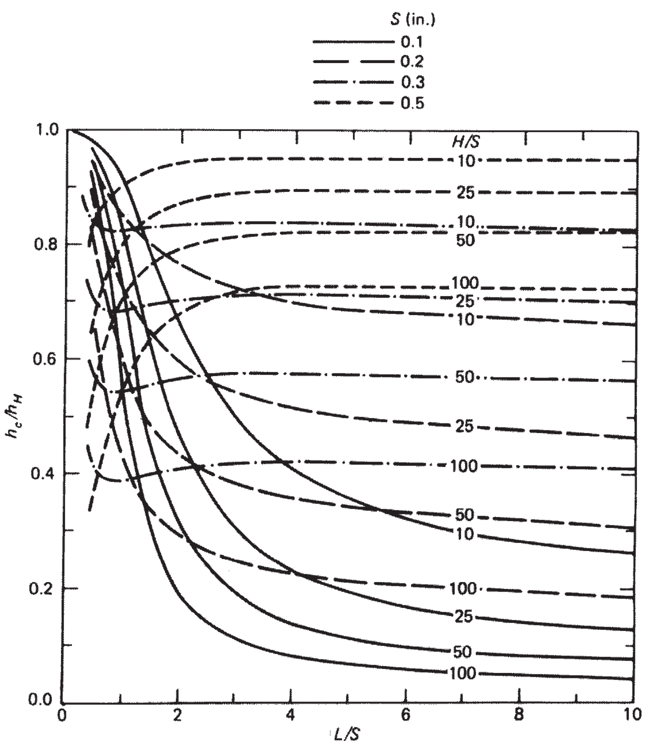


Fig. 5-16. Correction factor to vertical plate heat transfer coefficient for finned heat sink. $\Delta T = 50°C$.

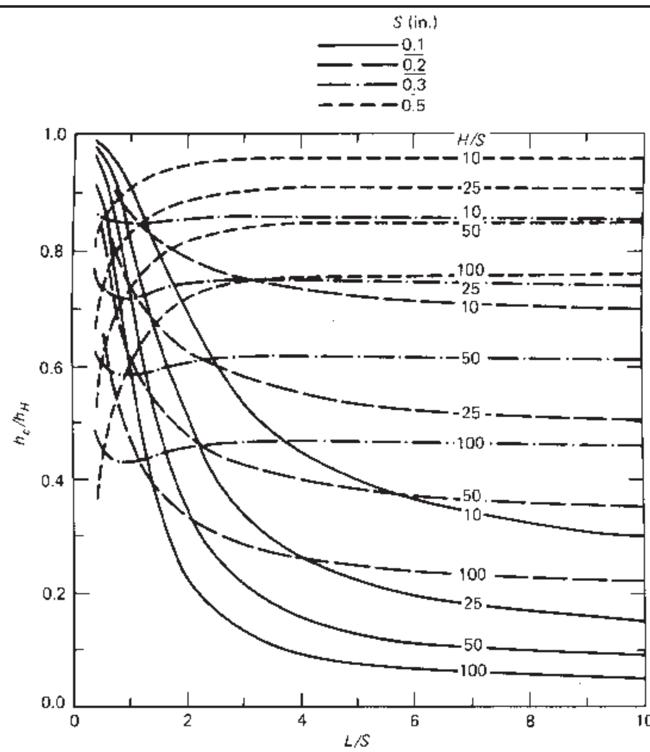
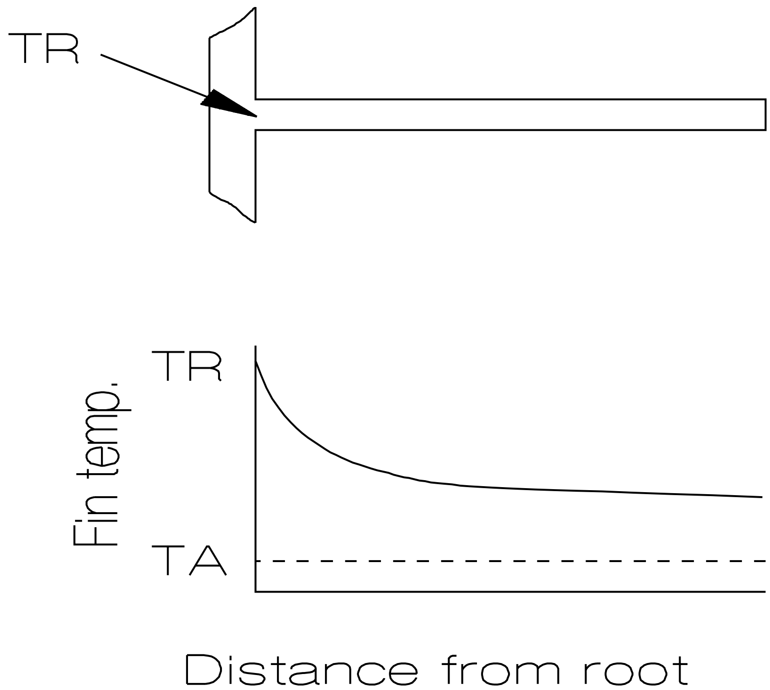
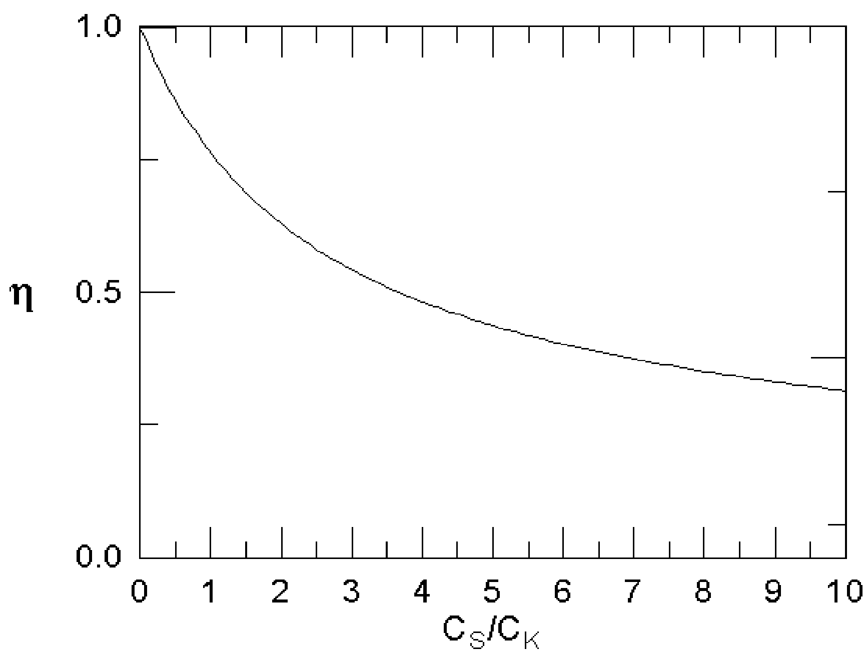


Fig. 5-17. Correction factor to vertical plate heat transfer coefficient for finned heat sink. $\Delta T = 100°C$.

Fin Efficiency





Efficiency of a rectangular fin

$$C_S = hA_{fin}$$

$$C_k = kA_k \ / \ L$$

More Detail on Fin Efficiency*

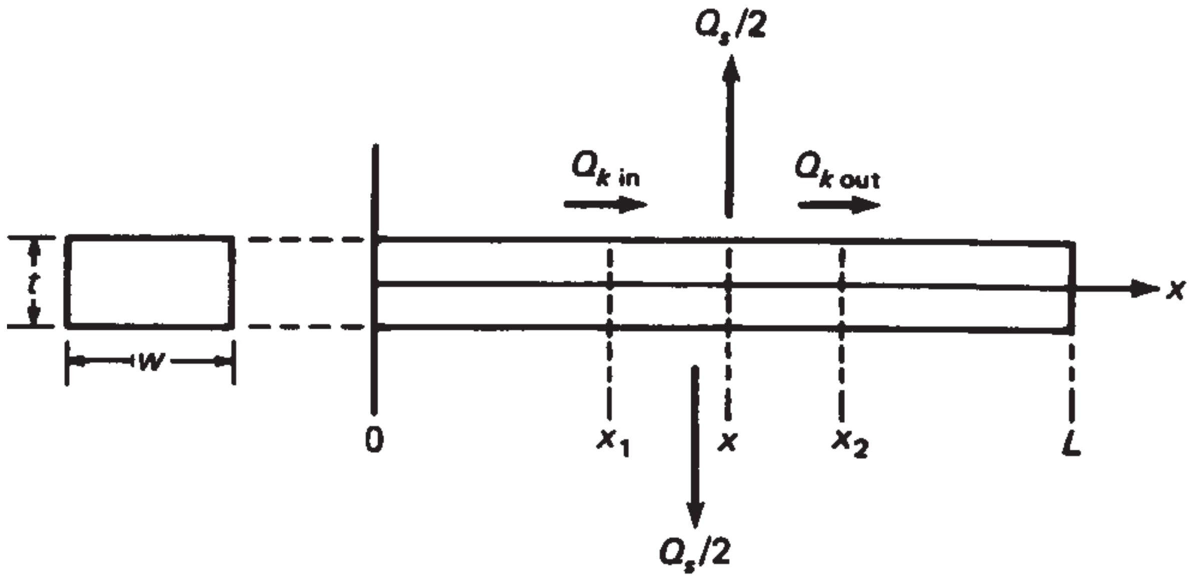


Fig. 4-1. Energy balance on an element $\Delta x = x_2 - x_1$.

Steady-state energy balance on element referenced to a zero temperature ambient:

$$\text{heat into } \Delta x \ - \ \text{heat out of } \Delta x = 0$$

$$\left[-kA_k \left.\frac{dT}{dx}\right|_{x_1} + Q_V \Delta x A_k\right] - \left[-kA_k \left.\frac{dT}{dx}\right|_{x_2} + 2h(w+t)\Delta x \left.T\right|_x\right] = 0$$

---

* The following pages up to the next example may be skipped on a first reading.

Dividing each term by $k\Delta x A_k$ and rearranging:

$$\frac{1}{\Delta x}\left[\left.\frac{dT}{dx}\right|_{x_2} - \left.\frac{dT}{dx}\right|_{x_1}\right] - \frac{2h(w+t)}{kA_k}\left.T\right|_x = -\frac{Q_V}{k}$$

Taking the limit of $\Delta x \to 0$:

$$\frac{d^2T}{dx^2} - \vartheta^2 T = -\frac{Q_V}{k} \qquad \text{TCEE E4.1}$$

where $\vartheta^2 = R_k \big/ \left(L^2 R_s\right), \quad R_k = L/(kA_k), R_s = 1/(hA_s),$
$A_k = wt, \quad A_s = 2(w+t)L$

The general solution is

$$T = c_1 \cosh \vartheta x + c_2 \sinh \vartheta x + \alpha / \vartheta^2, \quad \alpha = Q_V / k$$

The typical fin problem does not require an internal source, therefore the first step in finding solutions is

$$T = c_1 \cosh \vartheta x + c_2 \sinh \vartheta x$$

Case 1 - Very long (infinite) fin:

$$T = (c_1 + c_2)e^{\vartheta x} + (c_1 - c_2)e^{-\vartheta x}$$

or

$$T = c_3 e^{\vartheta x} + c_4 e^{-\vartheta x}$$

The condition that T remain finite as $x \to \infty$ indicates $c_3=0$.

Using a root temperature $T=T_0$,

$$T = T_0 e^{-\vartheta x}$$

The heat loss from the fin is

$$Q = -kA_k \frac{dT}{dx}\bigg|_{x=0} = kA_k T_0 \vartheta = kA_k T_0 \sqrt{\frac{R_k}{R_s}} \frac{1}{L} = \frac{T_0}{\sqrt{R_k Rs}}$$

The resistance from fin root to ambient is

$$R = \frac{T_0}{Q} = \sqrt{R_k R_s}$$

$$\frac{R}{R_s} = \sqrt{\frac{R_k}{R_s}}$$

Case 2 - Finite Length Fin, Insulated Tip:

$$T = c_1 \cosh \vartheta x + c_2 \sinh \vartheta x$$

$x = 0$: $\quad -kA_k \frac{dT}{dx} = Q, \quad Q = |Q|$

$$c_2 = -\frac{Q}{kA_k \vartheta} = -\frac{Q}{kA_k} L \sqrt{\frac{R_s}{R_k}} = -QR_k \sqrt{\frac{R_s}{R_k}} = -Q\sqrt{R_k R_s}$$

$$x = L: \qquad -kA_k \frac{dT}{dx} = 0$$

$$c_1 = -c_2 \coth \vartheta L = Q\sqrt{R_k R_s} \coth \vartheta L$$

The solution for the temperature is therefore

$$\begin{aligned} T &= Q\sqrt{R_k R_s}\left[\coth \vartheta L \cosh \vartheta x - \sinh \vartheta x\right] \\ &= Q\sqrt{R_k R_s}\left[\coth \sqrt{\frac{R_k}{R_s}} \cosh\left(\sqrt{\frac{R_k}{R_s}}\frac{x}{L}\right) - \sinh\left(\sqrt{\frac{R_k}{R_s}}\frac{x}{L}\right)\right] \end{aligned}$$

The root temperature is

$$T_0 = Q\sqrt{R_k R_s} \coth \sqrt{\frac{R_k}{R_s}}$$

so that the fin heat loss may be solved for as

$$Q = \frac{T_0}{\sqrt{R_k R_s}} \tanh \sqrt{\frac{R_k}{R_s}}$$

The fin resistance from root to ambient is

$$R = \frac{T_0}{Q} = \frac{T_0}{\dfrac{T_0}{\sqrt{R_k R_s}} \tanh \sqrt{\dfrac{R_k}{R_s}}} = \sqrt{R_k R_s} \coth \sqrt{\frac{R_k}{R_s}}$$

$$\frac{R}{R_s} = \sqrt{\frac{R_k}{R_s}} \coth \sqrt{\frac{R_k}{R_s}} \qquad \text{TCEE E4.5}$$

Case 3 - Finite Length Fin, Non-Insulated Tip:

$$T = c_1 \cosh \vartheta x + c_2 \sinh \vartheta x$$

$$x = 0: \qquad T = T_0, \quad c_1 = T_0$$

$$x = L: \qquad -kA_k \left.\frac{dT}{dx}\right|_{x=L} = hA_k \left.T\right|_{x=L}$$

$$-kA_k \vartheta (c_1 \sinh \vartheta L + c_2 \cosh \vartheta L) = hA_k (c_1 \cosh \vartheta L + c_2 \sinh \vartheta L)$$

$$-k\vartheta (T_0 \sinh \vartheta L + c_2 \cosh \vartheta L) = h (T_0 \cosh \vartheta L + c_2 \sinh \vartheta L)$$

$$c_2(h\sinh\vartheta L + k\vartheta\cosh\vartheta L) = -T_0(h\cosh\vartheta L + k\vartheta\sinh\vartheta L)$$

$$c_2 = -\frac{T_0(h\cosh\vartheta L + k\vartheta\sinh\vartheta L)}{k\vartheta\cosh\vartheta L + h\sinh\vartheta L}$$

$$= -\frac{T_0\left(\sinh\vartheta L + \frac{h}{k\vartheta}\cosh\vartheta L\right)}{\left(\cosh\vartheta L + \frac{h}{k\vartheta}\sinh\vartheta L\right)}$$

$$T = T_0\cosh\vartheta x - \frac{T_0\left(\sinh\vartheta L + \frac{h}{k\vartheta}\cosh\vartheta L\right)}{\left(\cosh\vartheta L + \frac{h}{k\vartheta}\sinh\vartheta L\right)}\sinh\vartheta x$$

$$= T_0\left[\cosh\vartheta x - \frac{\left(\sinh\vartheta L + \frac{h}{k\vartheta}\cosh\vartheta L\right)}{\left(\cosh\vartheta L + \frac{h}{k\vartheta}\sinh\vartheta L\right)}\sinh\vartheta x\right]$$

$$= T_0\left[\begin{array}{l}\cosh\left(\sqrt{\frac{R_k}{R_s}}\frac{x}{L}\right) - \frac{\left(\sinh\sqrt{\frac{R_k}{R_s}} + \frac{A_k}{A_s}\sqrt{\frac{R_k}{R_s}}\cosh\sqrt{\frac{R_k}{R_s}}\right)}{\left(\cosh\sqrt{\frac{R_k}{R_s}} + \frac{A_k}{A_s}\sqrt{\frac{R_k}{R_s}}\sinh\sqrt{\frac{R_k}{R_s}}\right)}\bullet \\ \sinh\left(\sqrt{\frac{R_k}{R_s}}\frac{x}{L}\right)\end{array}\right]$$

The fin heat loss is determined from the gradient of the temperature solution:

$$Q = \frac{kA_k}{L}\sqrt{\frac{R_k}{R_s}}T_0\left[\frac{\left(\sinh\sqrt{\frac{R_k}{R_s}} + \frac{A_k}{A_s}\sqrt{\frac{R_k}{R_s}}\cosh\sqrt{\frac{R_k}{R_s}}\right)}{\left(\cosh\sqrt{\frac{R_k}{R_s}} + \frac{A_k}{A_s}\sqrt{\frac{R_k}{R_s}}\sinh\sqrt{\frac{R_k}{R_s}}\right)}\right]$$

$$= \frac{T_0}{\sqrt{R_k R_s}}\left[\frac{\left(\sinh\sqrt{\frac{R_k}{R_s}} + \frac{A_k}{A_s}\sqrt{\frac{R_k}{R_s}}\cosh\sqrt{\frac{R_k}{R_s}}\right)}{\left(\cosh\sqrt{\frac{R_k}{R_s}} + \frac{A_k}{A_s}\sqrt{\frac{R_k}{R_s}}\sinh\sqrt{\frac{R_k}{R_s}}\right)}\right]$$

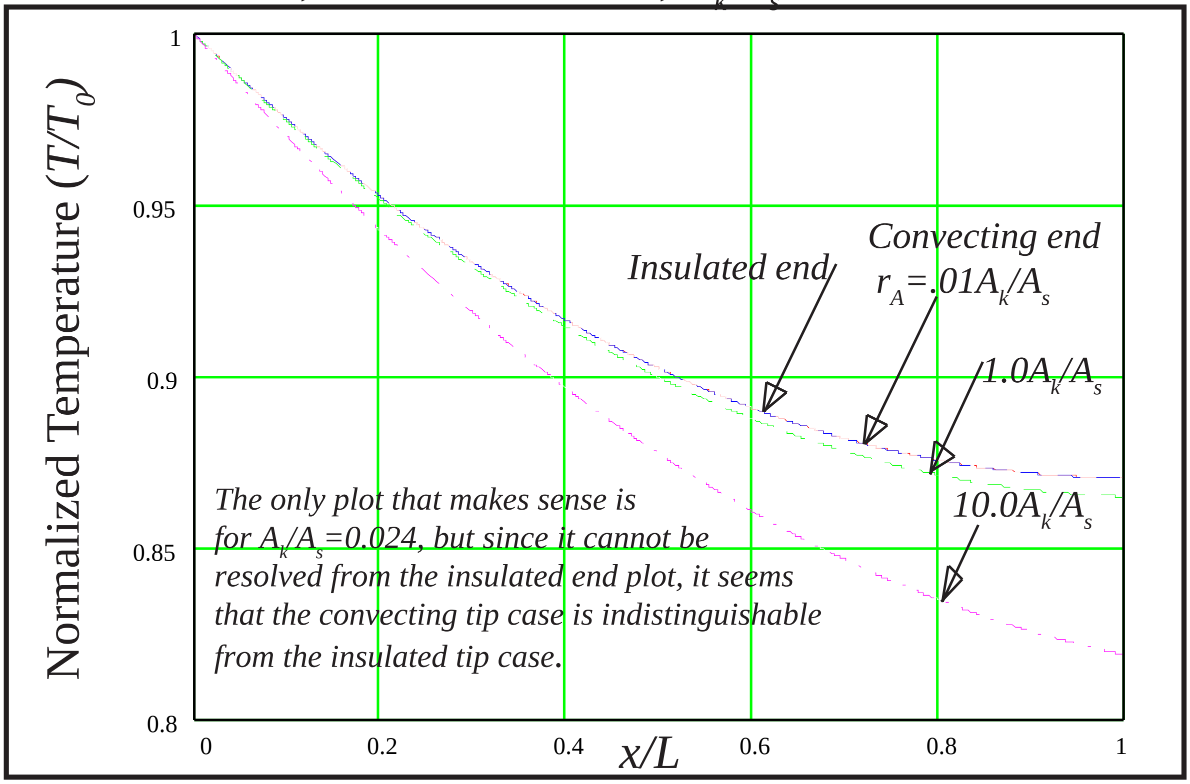
The fin resistance from root to ambient is

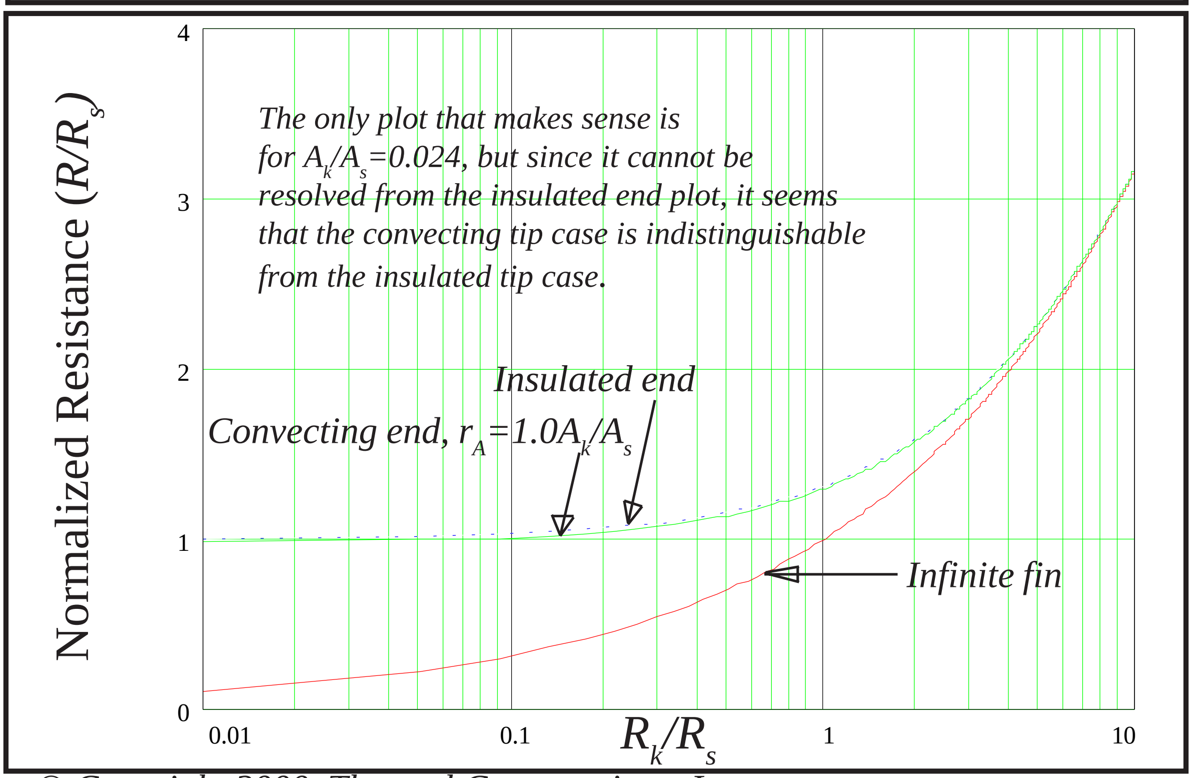
$$R = \frac{\sqrt{R_k R_s}\left(\cosh\sqrt{\frac{R_k}{R_s}} + \frac{A_k}{A_s}\sqrt{\frac{R_k}{R_s}}\sinh\sqrt{\frac{R_k}{R_s}}\right)}{\sinh\sqrt{\frac{R_k}{R_s}} + \frac{A_k}{A_s}\sqrt{\frac{R_k}{R_s}}\cosh\sqrt{\frac{R_k}{R_s}}}$$

$$\frac{R}{R_s} = \sqrt{\frac{R_k}{R_s}}\,\frac{\left(\cosh\sqrt{\frac{R_k}{R_s}} - \frac{A_k}{A_s}\sqrt{\frac{R_k}{R_s}}\sinh\sqrt{\frac{R_k}{R_s}}\right)}{\sinh\sqrt{\frac{R_k}{R_s}} + \frac{A_k}{A_s}\sqrt{\frac{R_k}{R_s}}\cosh\sqrt{\frac{R_k}{R_s}}}$$

Temperature and Resistance Plots

L=1.0 in., t=0.05 in., w=1.0 in., A_k/A_s=0.024,
k=5.0 W/in.•°C, h=0.01 W/in.^{2}•°C, R_k/R_s=0.29





Definition of fin efficiency:

$$\eta = \text{Fin efficiency}$$

$$= \frac{\text{actual heat dissipated by fin}}{\text{maximum heat dissipated by fin}}$$

$$= \frac{Q}{Q_{Max}} = \frac{Q}{hA_{fin}T_0},$$

assuming a zero ambient temperature.

Case 1 - Very Long (infinite) Fin: $A_{fin}=A_s$

$$\eta = R_s\left(\frac{Q}{T_0}\right) = R_s\frac{\left(\frac{T_0}{\sqrt{R_k R_s}}\right)}{T_0}$$

$$= \sqrt{\frac{R_s}{R_k}}$$

There is a problem with the "infinite fin -

$$\eta = \sqrt{\frac{R_s}{R_k}} = \sqrt{\frac{\left(\frac{1}{hA_s}\right)}{\left(\frac{L}{kA_k}\right)}} = \sqrt{\frac{kA_k}{hLA_s}} = \sqrt{\frac{kwt}{hL^2 2(w+t)}} \xrightarrow[L\to\infty]{} 0$$

This is not an unreasonable result. We just cannot use the fin efficiency concept for an infinite fin.

Case 2 - Finite Length Fin, Insulated Tip: $A_{fin}=A_s$

$$\eta = R_s\left(\frac{Q}{T_0}\right) = R_s\,\frac{\left(\dfrac{T_0}{\sqrt{R_k R_s}}\tanh\sqrt{\dfrac{R_k}{R_s}}\right)}{T_0}$$

$$= \sqrt{\frac{R_s}{R_k}}\tanh\sqrt{\frac{R_k}{R_s}}$$

or

$$\eta = \sqrt{\frac{C_k}{C_s}}\tanh\sqrt{\frac{C_s}{C_k}} \qquad \text{TCEE E5.5}$$

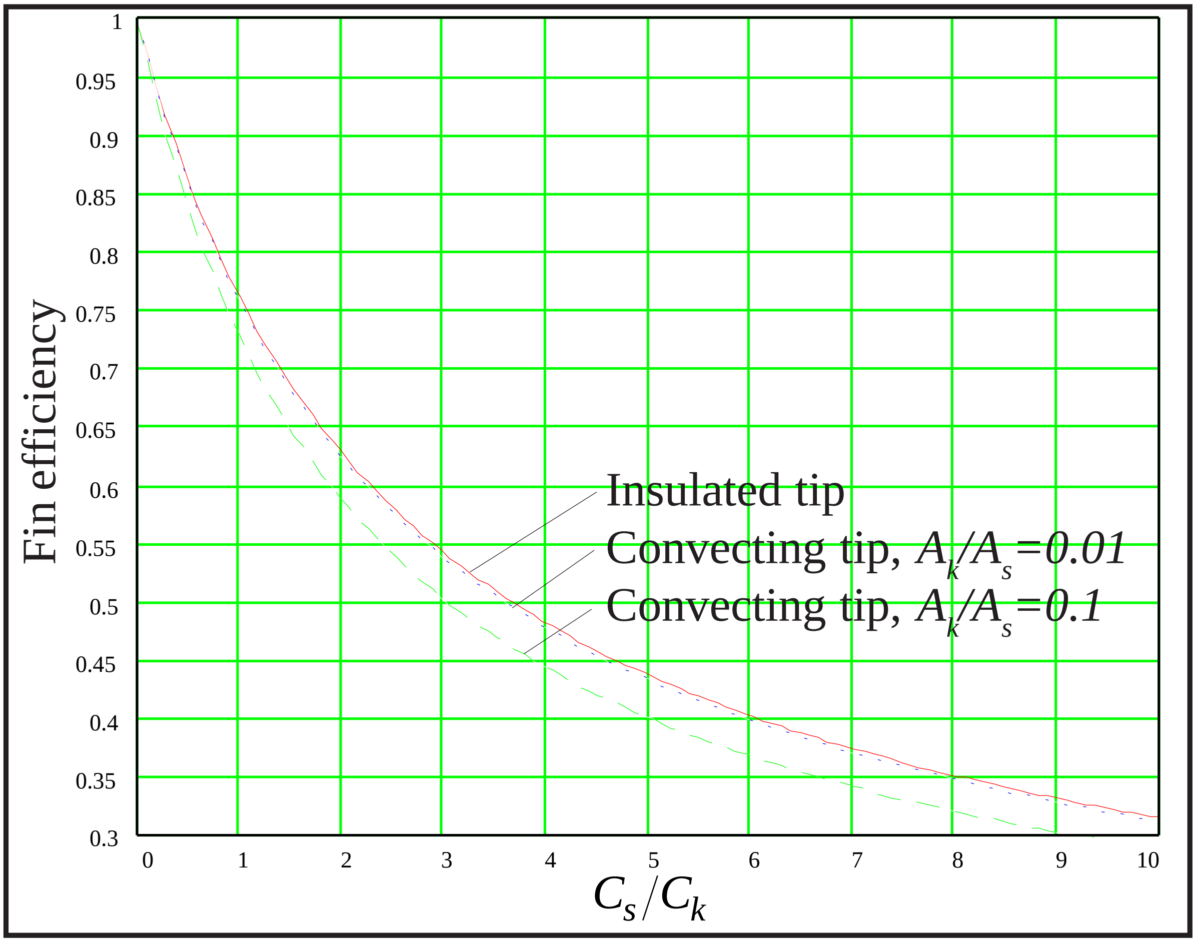
Case 3 - Finite Length Fin, Non-Insulated Tip: $A_{fin}=A_s+A_k$

$$\eta = \frac{1}{hA_{fin}}\left(\frac{Q}{T_0}\right) = \frac{1}{h(A_s + A_k)}\left(\frac{Q}{T_0}\right) = \frac{1}{hA_s\left(1+\dfrac{A_k}{A_s}\right)}\left(\frac{Q}{T_0}\right)$$

$$= \frac{1}{\left(1+\dfrac{A_k}{A_s}\right)}\left(\frac{R_s}{T_0}\right)\left\{\frac{T_0}{\sqrt{R_k R_s}}\left[\frac{\left(\sinh\sqrt{\dfrac{R_k}{R_s}} + \dfrac{A_k}{A_s}\sqrt{\dfrac{R_k}{R_s}}\cosh\sqrt{\dfrac{R_k}{R_s}}\right)}{\left(\cosh\sqrt{\dfrac{R_k}{R_s}} + \dfrac{A_k}{A_s}\sqrt{\dfrac{R_k}{R_s}}\sinh\sqrt{\dfrac{R_k}{R_s}}\right)}\right]\right\}$$

$$= \frac{1}{\left(1+\dfrac{A_k}{A_s}\right)}\sqrt{\frac{R_s}{R_k}}\left(\frac{\tanh\sqrt{\dfrac{R_k}{R_s}} + \dfrac{A_k}{A_s}\sqrt{\dfrac{R_k}{R_s}}}{1+\dfrac{A_k}{A_s}\sqrt{\dfrac{R_k}{R_s}}\tanh\sqrt{\dfrac{R_k}{R_s}}}\right)$$

Fin efficiency plots



Note:

$$\frac{A_k}{A_s} = \frac{wt}{2L(w+t)} \xrightarrow[t \text{ very small}]{} \frac{t}{2L}$$

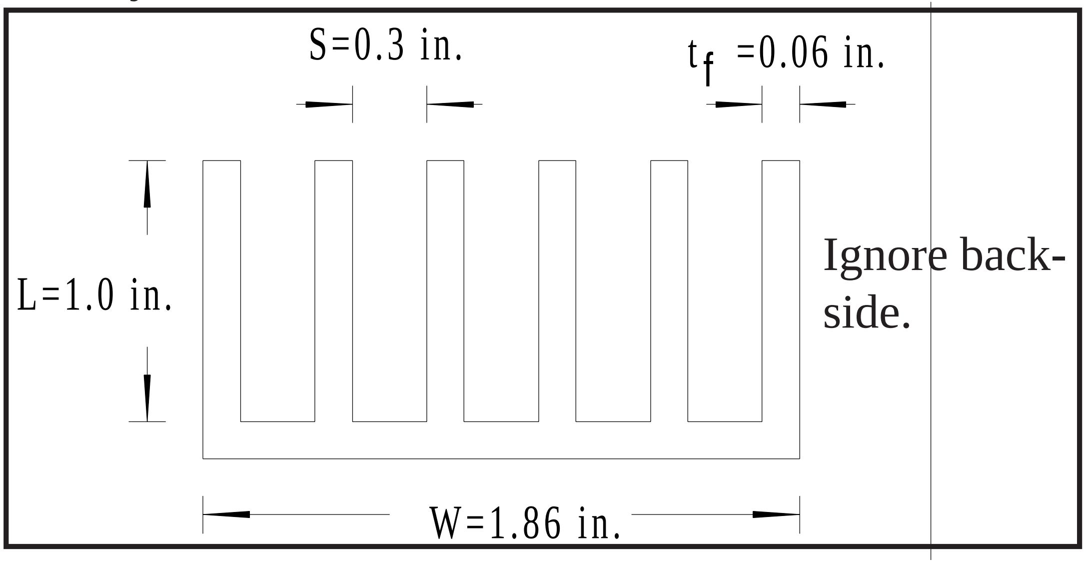
$$\frac{C_s}{C_k} = \frac{2hL^2(w+t)}{kwt} \xrightarrow[t \text{ very small}]{} \frac{2hL^2}{kt}$$

If $t=0.1$, $L=1.0$, then $A_k/A_s=0.05$. If t much larger than 0.1, the one-dimensional approximation probably becomes invalid. Common sense would also tell us that in the case of the one-dimensional fin problem, there can be no real difference between the insulated and convecting tip cases.

Example

Finned Heat Sink - Calculate Q for T_{Base}-T_A=50°C

Geometry



Convection Coefficients

$$S = 0.30\ in., \quad L = 1.0\ in., \quad H = 5.0\ in.$$

$$L/S = 1.0/3.0 = 3.33, \quad H/S = 5.0/0.30 = 16.7$$

From Fig. 5-16

$$H/S = 10, \quad h_C/h_H = 0.84$$

$$H/S = 25, \quad h_C/h_H = 0.71$$

$$H/S = 50, \quad h_C/h_H = 0.56$$

For $H/S = 16.7,$ use $h_C/h_H = 0.78$

$$h_H = 0.0024\left(\frac{\Delta T}{H}\right)^{0.25} = 0.0024\left(\frac{50}{5.0}\right)^{0.25}$$

$$= 0.0043\, W/in.^2 \cdot {}^oC$$

$$h_C = \left(\frac{h_C}{h_H}\right)h_H = 0.78(0.0043) = 0.0033\, W/in.^2 \cdot {}^oC$$

Fin Efficiency

$$C_S = hA_{fin} = 0.0033(1.0\ in.\ x 5.0\ in.\ x 2)$$

$$= 0.033\ W/{}^oC$$

$$C_k = k\,A_k/L = \left(5.0\ W/in\cdot{}^oC\right)(0.06\ in.\ x 5.0\ in.)/(1.0\ in.)$$

$$= 1.5\ W/{}^oC$$

$$C_S/C_k = 0.033/1.5 = 0.022$$

From Fig. 5-2, $\eta \cong 1.0$

Convection Conductance

Exterior surfaces:

$$
\begin{aligned}
C_{SExt} &= h_H A_{Ext} \eta \\
&= (0.0043)(1.0\ in.)(5.0\ in.) x 2 \text{ fins} x 1.0 \\
&= 0.022\ W / {}^oC
\end{aligned}
$$

Inner surfaces:

$$
\begin{aligned}
A_{Int} &= WH\left[1 + \frac{2(N-1)}{W} L\right] \\
&= (1.86)(5.0)\left[1 + \frac{2(6-1)}{1.86} 1.0\right] \\
&= 59.3\ in.^2
\end{aligned}
$$

$$
\begin{aligned}
C_{SInt} &= h_i A_i \eta \\
&= (0.0033)(59.3)(1.0) \\
&= 0.196
\end{aligned}
$$

Total:

$$C = C_{SExt} + C_{SInt} = 0.022 + 0.196$$
$$= 0.22 \; W/{}^{o}C$$

$$R = \text{resistance} = \frac{1}{C}$$
$$= \frac{1}{0.22} = 4.6 \; {}^{o}C/W$$

Heat Transfer Rate

$$\Delta T = RQ$$
$$Q = \Delta T/R = 50 \; {}^{o}C/\left(4.6 \; {}^{o}C/W\right)$$
$$= 10.9 \; W$$

Radiation

Simple Radiation

Blackbody Emission Spectrum

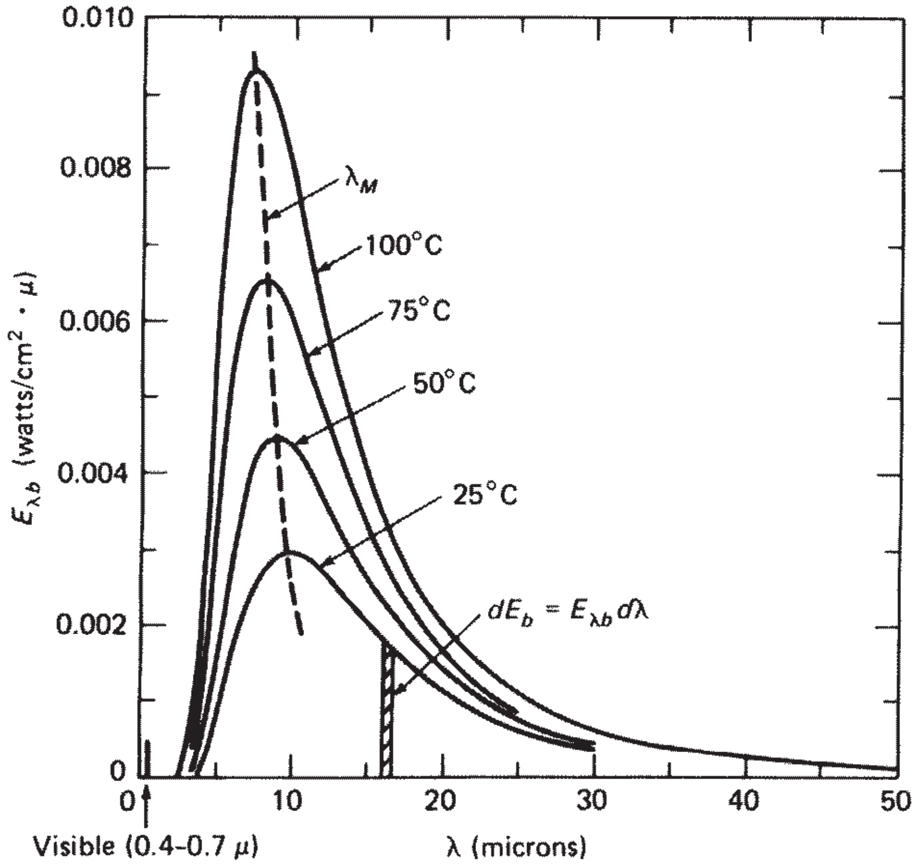


Fig. 3-2. Low temperature spectral distribution of monochromatic emissive power.

Monochromatic Emissive Power

$$E_{\lambda b} = \left(\frac{c_1}{\lambda^5}\right) \Big/ \left(e^{c_2/\lambda T'} - 1\right)$$

$$c_1 = 3.7403 x 10^4, \qquad c_2 = 1.4387 x 10^4$$

$$\lambda = \lambda[\mu], \qquad 1\mu \equiv 10^{-6} m = 10^{-4} cm$$

$$E_{\lambda b} = E_{\lambda b}\left[W/cm^2 \cdot \mu\right]$$

Total Blackbody Emissive Power

$$E_b = \int_0^\infty E_{\lambda b} d\lambda = \left(\frac{\pi^4}{15}\right)\left(\frac{c_1}{c_2^4}\right) T'^4$$

$$T'[K] = T\left[{}^oC\right] + 273.15$$

$$E_b = E_b\left[W/in.^2\right]$$

for $\sigma = 3.657\text{x}10^{-11}\ W/in.^2 \cdot K^4$

$\sigma = 5.668\text{x}10^{-12}\ W/cm^2 \cdot K^4$

$\sigma = 5.668\text{x}10^{-8}\ W/m^2 \cdot K^4$

Geometric Shape Effects for Black Body Surfaces

It can be shown that the net radiation exchanged between two surface area elements is

$$dQ_{NET} = (E_{b1} - E_{b2})\frac{\cos\phi_1 \cos\phi_2 dA_1 dA_2}{\pi r^2}$$

or

$$Q_{NET} = (E_{b1} - E_{b2})\int_1\int_2 \frac{\cos\phi_1 \cos\phi_2 dA_1 dA_2}{\pi r^2}$$

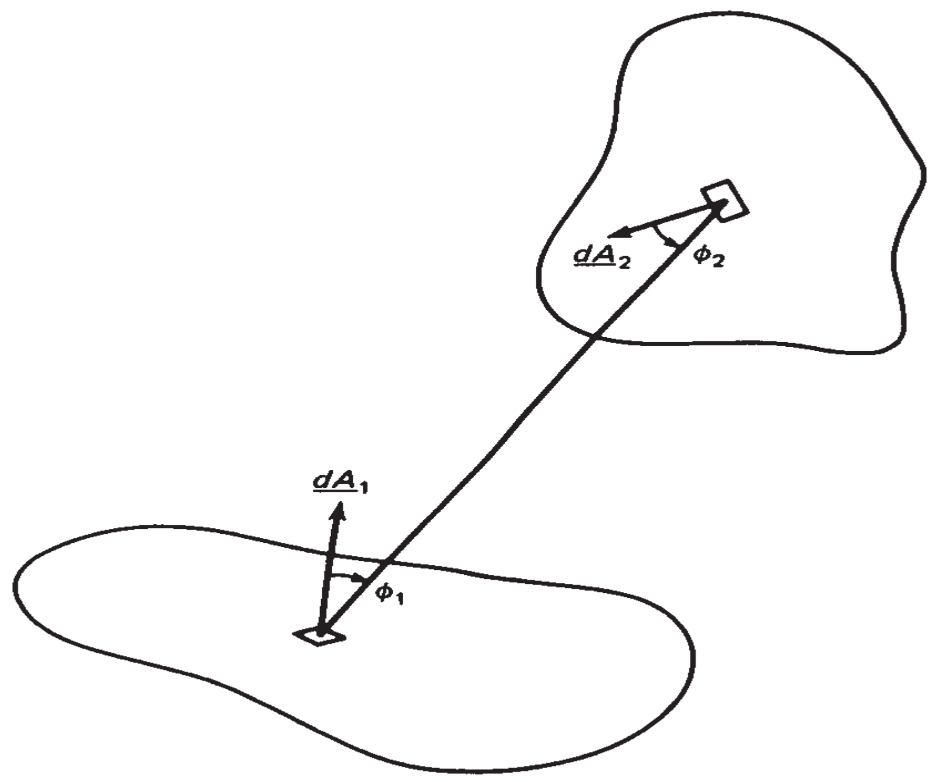


Fig. 3-5. Radiation exchange geometry.

The quantity within the double integral is written

$$A_1F_{1-2} = \int_1\int_2 \frac{\cos\phi_1 \cos\phi_2 dA_1 dA_2}{\pi r^2} \qquad \text{TCEE E3.5}$$

where $F_{1\text{-}2}$ is defined as the geometric shape or configuration factor and represents the fraction of radiation emitted by surface 1 and intercepted by surface 2.

The reciprocity relation is determined by inspection of E3.5

$$A_1F_{1-2} = A_2F_{2-1} \qquad \text{TCEE E3.6}$$

so that

$$Q_{NET} = (E_{b1} - E_{b2})F_{1-2}A_1$$

or

$$Q_{NET} = (E_{b1} - E_{b2})F_{2-1}A_2$$

Conservation of energy may be applied to N surfaces:

$$A_1F_{1-1} + A_1F_{1-2} + \cdots + A_1F_{1-N} = A_1$$

$$A_1(F_{1-1} + F_{1-2} + \cdots + F_{1-N}) = A_1$$

$$\sum_{j=1}^{N} F_{ij} = 1$$

Computation of radiation exchange between black diffuse surfaces is accomplished using

$$Q_{ij} = \sigma F_{ij} A_i \left(T_i'^4 - T_j'^4\right) \qquad \text{TCEE E3.7}$$

Parallel Plate Shape Factors

$$x = L/S, \qquad y = W/S$$

$$\left(\frac{\pi x y}{2}\right) F_{1-2} = \ln\left[\frac{\left(1+x^2\right)\left(1+y^2\right)}{1+x^2+y^2}\right]^{1/2}$$

$$+y\sqrt{1+x^2}\tan^{-1}\left(\frac{y}{\sqrt{1+x^2}}\right)$$

$$+x\sqrt{1+y^2}\tan^{-1}\left(\frac{x}{\sqrt{1+y^2}}\right)$$

$$-y\tan^{-1}y - x\tan^{-1}x$$

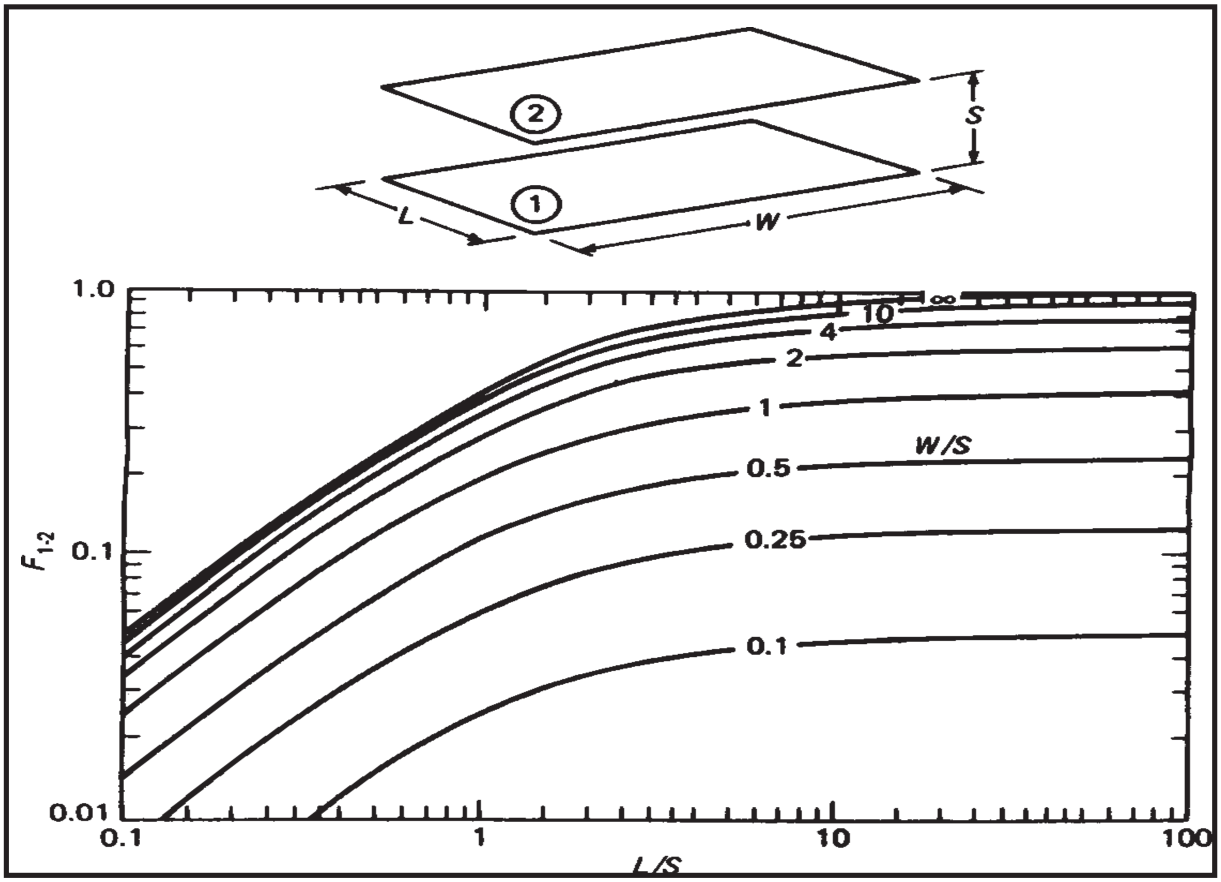


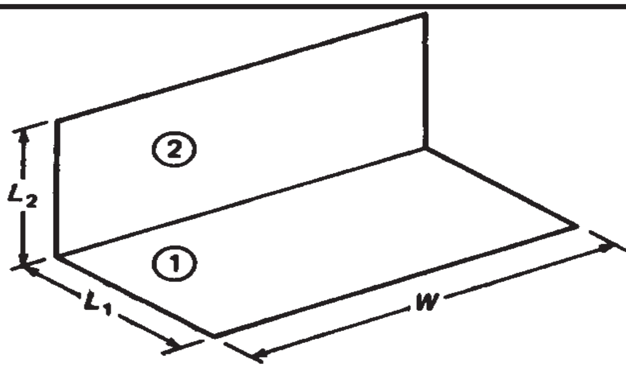
Fig. 3-6. Parallel plate shape factors.

Perpendicular Plate Shape Factors

$$x = L_2 / W, \quad y = L_1 / W, \qquad z = x^2 + y^2$$

$$(\pi y) F_{1-2} = \frac{1}{4} \left\{ \begin{array}{l} \ln\left[\dfrac{(1+x^2)(1+y^2)}{1+z}\right] + y^2 \ln\left[\dfrac{y^2(1+z)}{(1+y^2)z}\right] \\ + x^2 \ln\left[\dfrac{x^2(1+z)}{z(1+x^2)}\right] \end{array} \right\}$$

$$+ y \tan^{-1}(1/y) + x \tan^{-1}(1/x) - \sqrt{z} \tan^{-1}\left(1/\sqrt{z}\right)$$



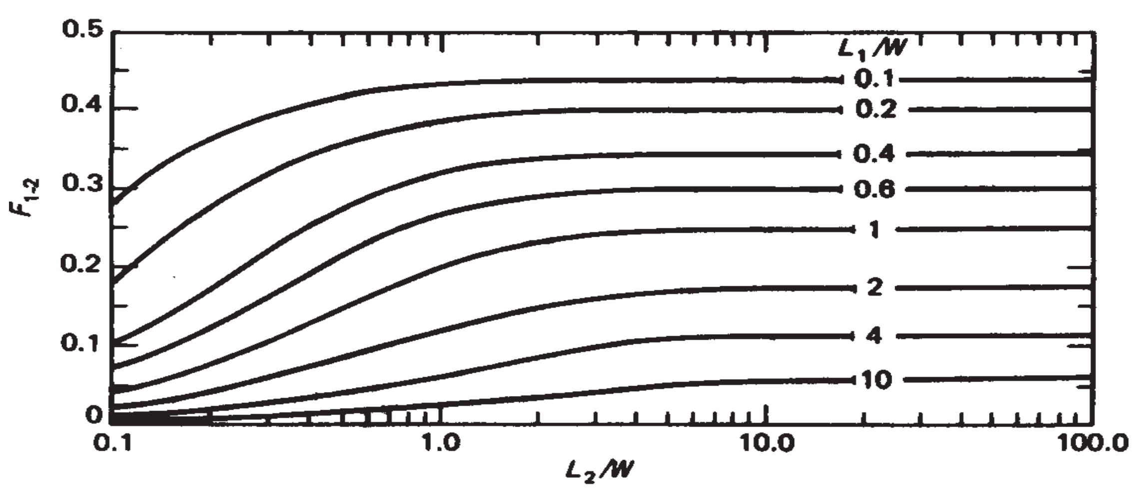


Fig. 3-7. Perpendicular plate shape factors.

Properties of Non-Black Surfaces

$E_\lambda \equiv$ Monochromatic gray - body emissive power

$\varepsilon_\lambda \equiv$ Monochromatic emissivity

$$\varepsilon_\lambda = \frac{E_\lambda}{E_{\lambda b}}$$

$\varepsilon \equiv$ emissivity (actually total emissivity)

$$\varepsilon = \frac{E}{E_b}, \quad E = \varepsilon E_b$$

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda b} d\lambda}{\int_0^\infty E_{\lambda b} d\lambda}$$

Example Material

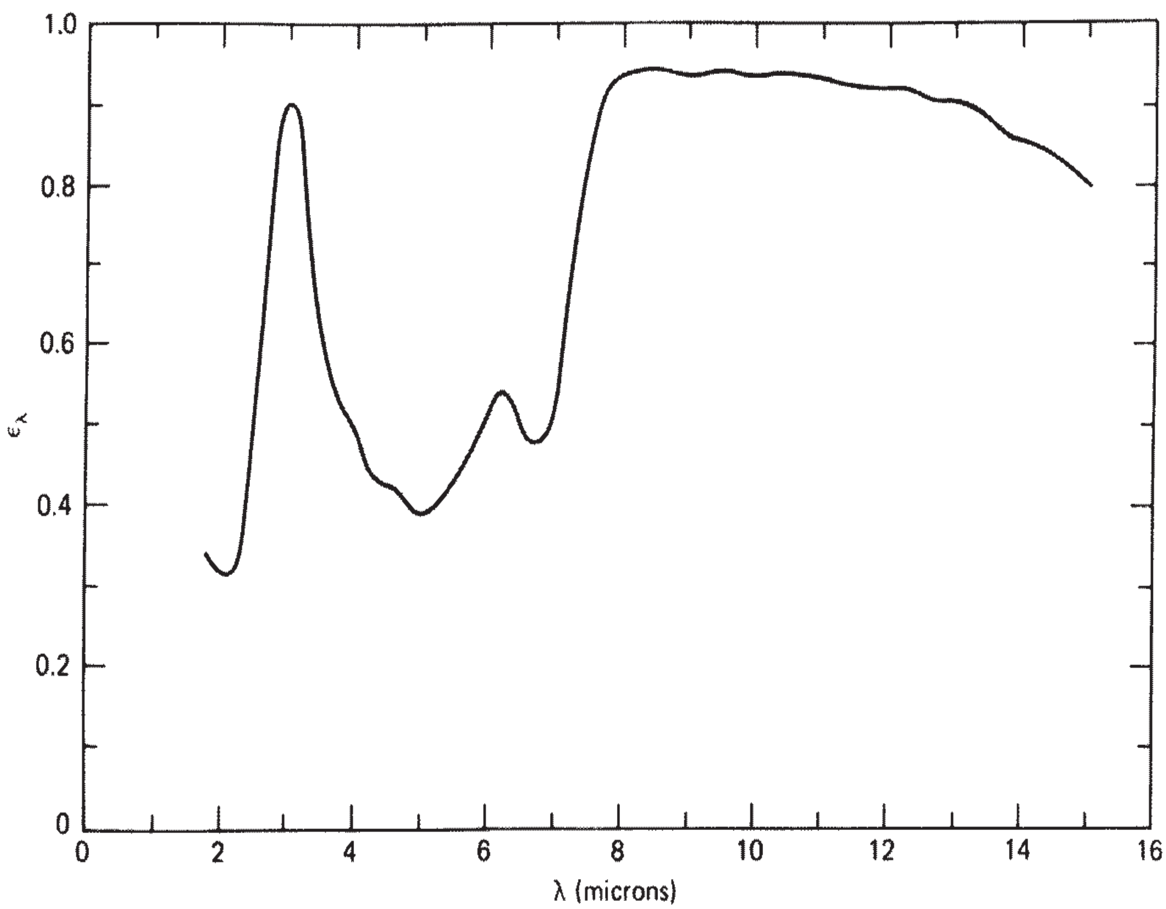


Fig. 3-11. Monochromatic emittance for anodized aluminum at 282°C. Plotted from [16], Table 2.

Radiation Heat Transfer Coefficient

For a small body (#1) surrounded by a large body (#2)

$$Q_r = \varepsilon_1 A_1 \sigma\left[(T_1 + 273.15)^4 - (T_2 + 273.15)^4\right]$$

$$= \varepsilon_1 A_1 \sigma \frac{\left(T_1'^4 - T_2'^4\right)}{(T_1 - T_2)}(T_1 - T_2)$$

$$= \varepsilon_1 A_1 h_r (T_1 - T_2)$$

$\varepsilon_1 =$ emissivity of surface 1

$A_1 =$ area of surface 1 ($in.^2$)

$h_r =$ radiative heat transfer coefficient

$$= \sigma \frac{\left(T_1'^4 - T_2'^4\right)}{T_1 - T_2} = \sigma\left(T_1'^3 + T_1'^2 T_2' + T_1' T_2'^2 + T_2'^3\right)$$

$$= 3.657x10^{-11}\left(T_1'^3 + T_1'^2 T_2' + T_1' T_2'^2 + T_2'^3\right),\ W/in.^2 \cdot {}^oC$$

$Q_r =$ net radiative heat between surfaces 1 and 2

Note: $R_r = \dfrac{1}{\varepsilon_1 A_1 h_r}$

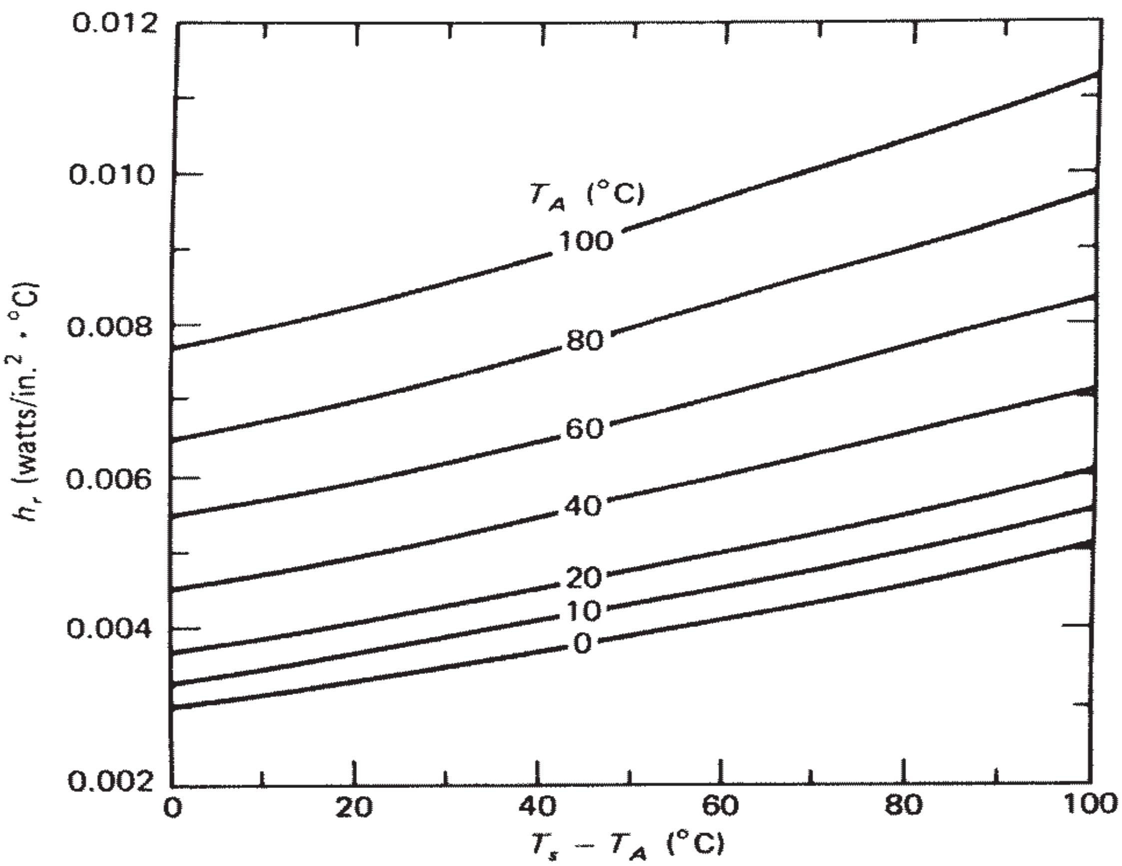
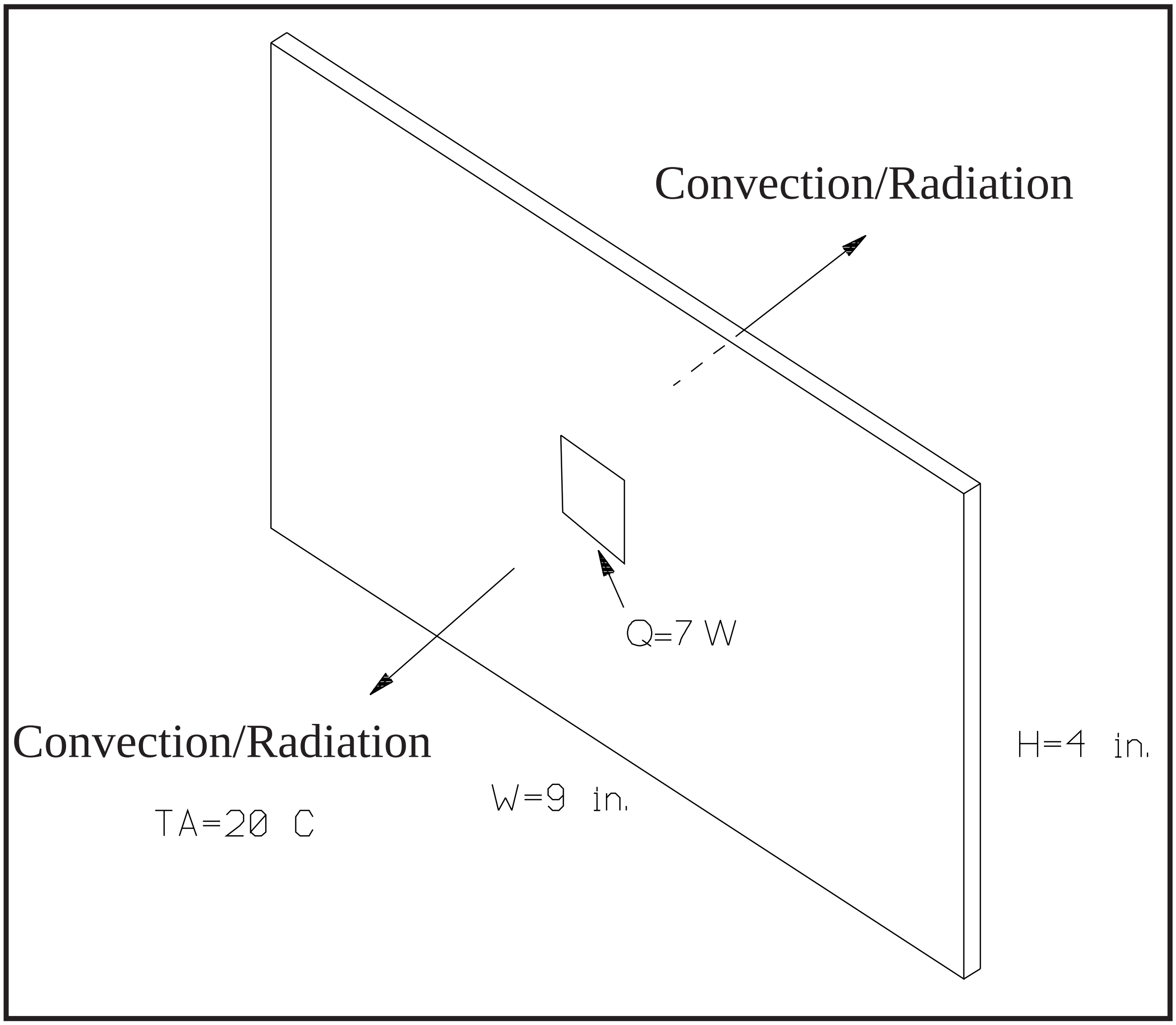


Fig. 3-16. Radiative heat transfer coefficient.

Example

Average Plate Temperature and Heat Transfer Coefficient
Previous natural convection example used here



$$Q = 7.0\ W \text{ total},\ \varepsilon = 0.6,\ T_A = 20\ ^oC$$

Temperature Rise Calculation

$$Q = Q_C + Q_r = h_C A_S (T_S - T_A) + \varepsilon h_r A_S (T_S - T_A)$$

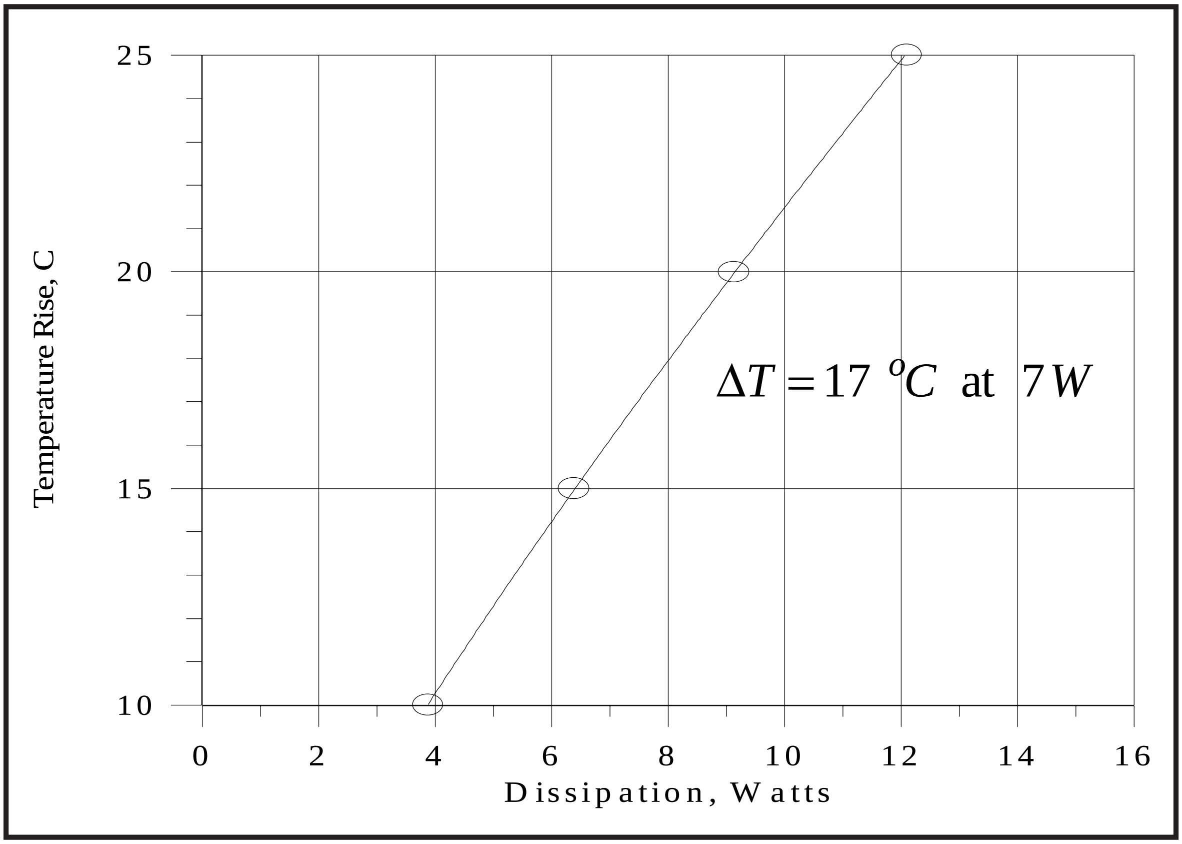
$$= 0.0022 \left(\frac{\Delta T}{H} \right)^{0.35} A_S \Delta T + \varepsilon h_r A_S \Delta T$$

$$= 0.0022 \left(\frac{\Delta T}{4.0} \right)^{0.35} (2 x 9.0\ in.\ x 4.0\ in.) \Delta T$$

$$+ 0.6 h_r (2 x 9.0\ in.\ x 4.0\ in.) \Delta T$$

$$Q = 0.098 \Delta T^{1.35} + 43.2 h_r \Delta T$$

$\Delta T\ (^oC)$	$Q_C\ (W)$	$h_r\ (W/in.^2 \cdot {}^oC)$	$Q_r\ (W)$	$Q\ (W)$
10	2.19	0.0039	1.69	3.88
15	3.79	0.0040	2.59	6.38
20	5.59	0.0041	3.54	9.13
25	7.56	0.0042	4.54	12.10



Total Heat Transfer Coefficient

$$h = h_c + \varepsilon h_r = 0.0022\left(\frac{\Delta T}{H}\right)^{0.35} + \varepsilon h_r$$

$$= 0.0022\left(\frac{17}{4.0}\right)^{0.35} + 0.6(0.0040) = 0.0037 + 0.0024$$

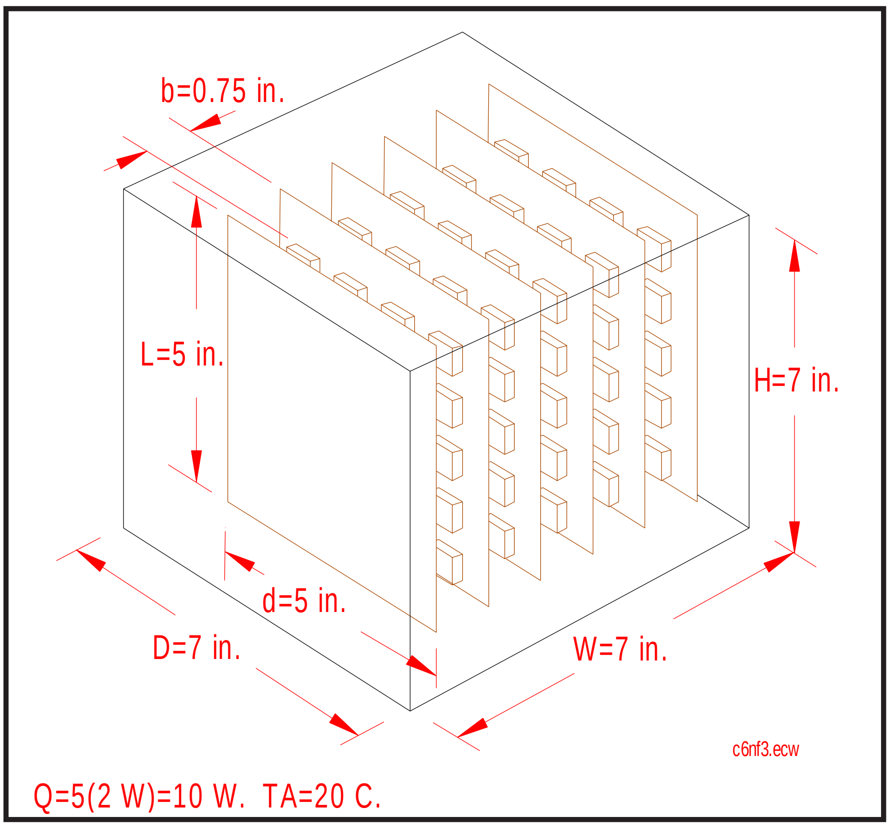
$$= 0.0062\ W\big/ in.^2 \cdot {}^oC \text{ for each side}$$

Thermal Resistance

$$R = \frac{\Delta T}{Q} = \frac{17}{7} = 2.32\ ^oC / W \text{ at } 7\ W$$

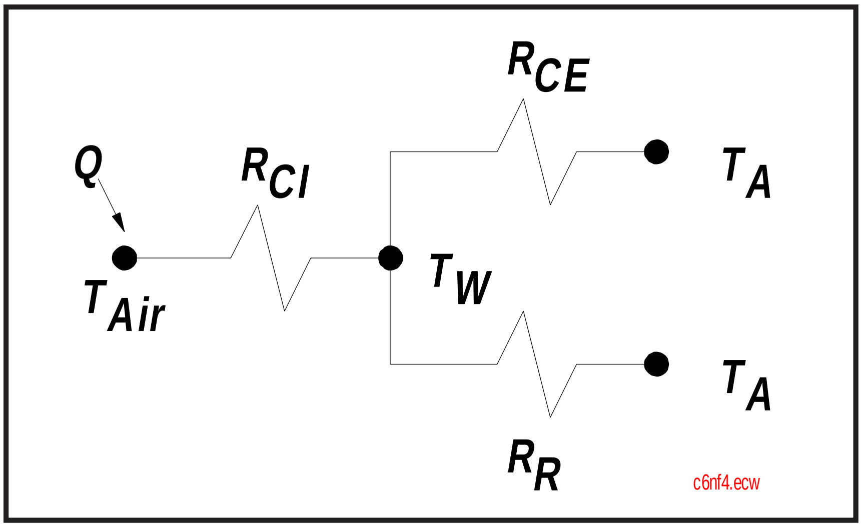
Application Example: Thermal Network Model for a Sealed Rectangular Enclosure

This is a continuation of a previous problem for an enclosure that was cooled externally by natural convection only by assuming a small emissivity. We shall now consider the effects when a non zero emissivity is used. The walls were modeled as vertical and horizontal plates. Since all panels have the same dimensions, a generally unrecommended procedure of using the same convective heat transfer coefficent will be employed for each panel. This procedure should be used with caution and is used here for illustrative purposes (an iterative method usually requiring a computer program is the recommended procedure.The problem



is to calculate the average wall temperature and internal air temperature. Assume thin, highly conducting panels.

Thermal circuit for sample problem:



A reasonable exterior surface emissivity is $\varepsilon = 0.8$.

This package has about *2 watts* on each circuit board for a total dissipation of *Q=10 W.*

The convection resistances are

$$R_{CE} = \frac{1}{h_c A_S} = \frac{1}{0.0024\left(\frac{\Delta T}{H}\right)^{0.25}(6WH)}$$

$$R_{CI} = \frac{1}{h_c A_S} = \frac{1}{0.0024\left(\frac{\Delta T}{H}\right)^{0.25}(6WH)}$$

The total external convection resistance is

$$R_{CE} = \frac{1}{6WH(0.0024)\left(\frac{\Delta T_{WA}}{H}\right)^{0.25}} = \frac{H^{0.25}}{6WH(0.0024)\Delta T_{WA}^{0.25}}$$

$$= \frac{(7\,in.)^{0.25}}{6(7\,in.)(7\,in.)(0.0024)\Delta T_{WA}^{0.25}} == \frac{1}{0.434\Delta T_{WA}^{0.25}}$$

The total external radiation resistance is

$$R_R = \frac{1}{\varepsilon h_r A_S} = \frac{1}{\varepsilon h_r 6WH} = R_R = \frac{1}{\varepsilon h_r A_S} = \frac{1}{\varepsilon h_r 6WH}$$

$$= 1 \Big/ \left\{ 0.8\sigma(6WH) \begin{bmatrix} (T_W + 273)^3 + (T_W + 273)^2 (T_A + 273) \\ +(T_W + 273)(T_A + 273)^2 + (T_A + 273)^3 \end{bmatrix} \right\}$$

$$= 1 \Big/ \left\{ \begin{matrix} 0.8\left(3.657x10^{-11}\right)(6x7x7) \\ \begin{bmatrix} (T_W + 273)^3 + (T_W + 273)^2 (T_A + 273) \\ +(T_W + 273)(T_A + 273)^2 + (T_A + 273)^3 \end{bmatrix} \end{matrix} \right\}$$

$$= 1 \Big/ \left\{ \begin{matrix} 8.60x10^{-9} \\ \begin{bmatrix} (T_W + 273)^3 + (T_W + 273)^2 (T_A + 273) \\ +(T_W + 273)(T_A + 273)^2 + (T_A + 273)^3 \end{bmatrix} \end{matrix} \right\}$$

Converting the convection and radiation resistances to conductances,

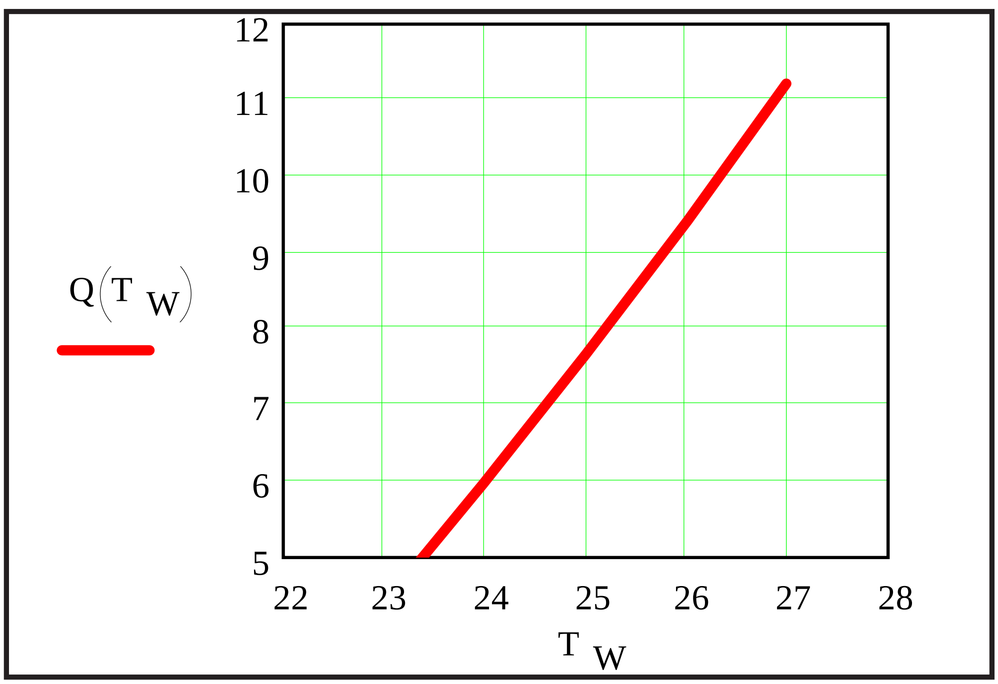
$$C_{CE} = 0.434\Delta T_{WA}^{0.25}$$

$$C_R = 8.60x10^{-9}\begin{bmatrix}(T_W + 273)^3 + (T_W + 273)^2(293) \\ +(T_W + 273)(293)^2 + (293)^3\end{bmatrix}$$

The wall to ambient temperature rise is calculated from

$$Q = (C_{CE} + C_R)(T_W - T_A)$$

and plotting.



The wall to ambient temperature rise is read from the plot as about $\Delta T_{WA} = 6\ ^oC$. This is about half of the prediction when radiation was nglected because of a small emissivity. The internal resistance is

$$R_I = \frac{1}{0.434 \Delta T_{IW}^{0.25}}$$

and

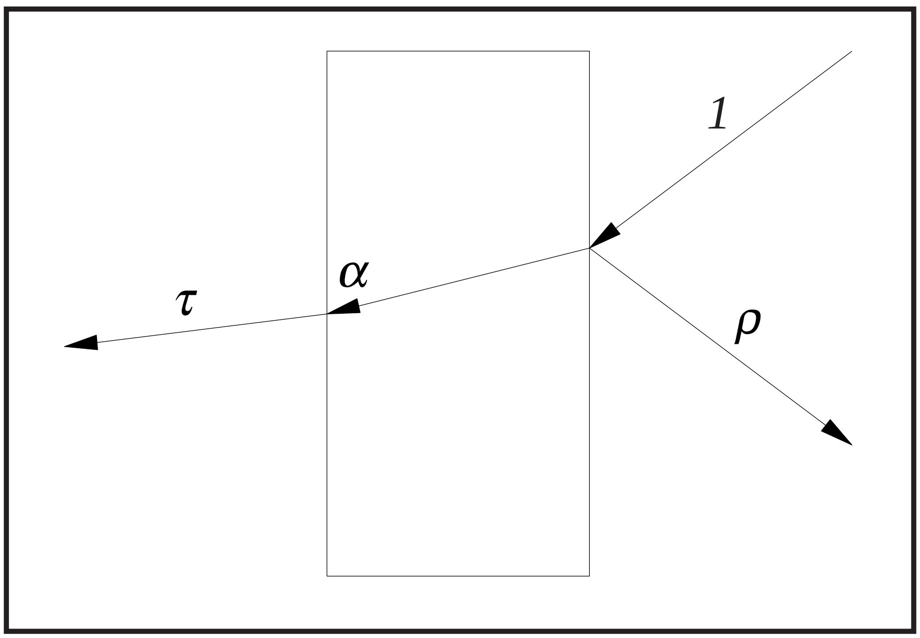
$$\Delta T_{IW} = \left(\frac{Q}{0.434} \right)^{1/1.25} = \left(\frac{10}{0.434} \right)^{1/1.25} = 12\ ^oC$$

so that

$$\Delta T_{IA} = \Delta T_{IW} + \Delta T_{WA} = 12 + 6 = 18\ ^oC$$

Radiation Exchange for Multiple Gray-Body Surfaces

<u>Reflection, Absorption, and Transmission</u>



$$1 = \rho + \alpha + \tau$$

$\rho \equiv$ reflectivity

$\alpha \equiv$ absorptivity

$\tau \equiv$ transmissivity

Most solid bodies have negligible transmission, i.e.

$$\tau = 0$$
$$1 = \rho + \alpha$$

Two Types of Reflection Phenomena

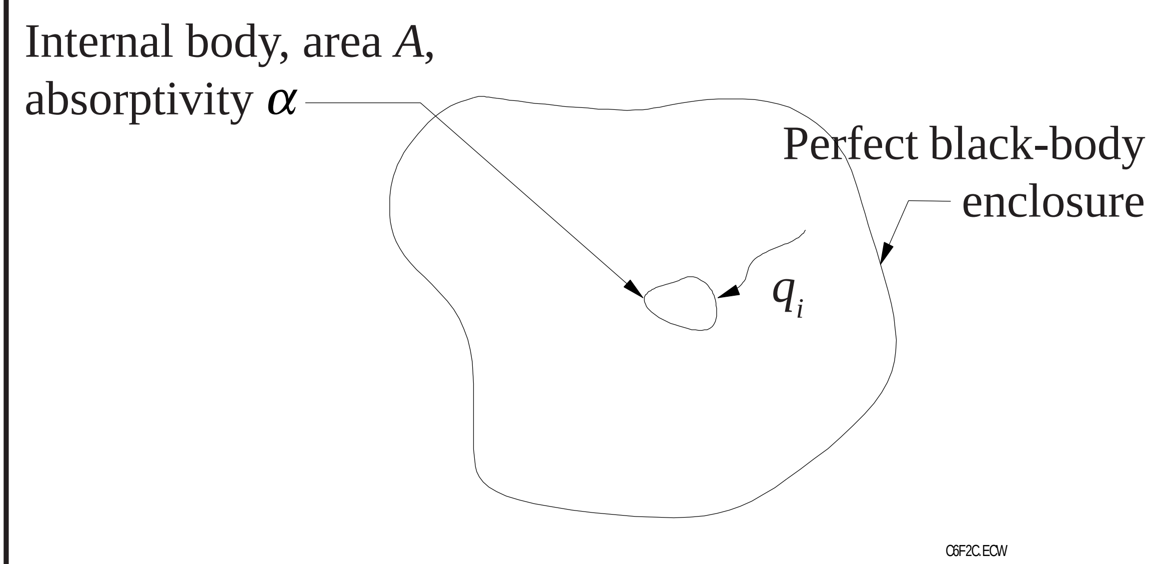
Specular: angle of incidence = angle of reflection.

Diffuse: incident beam is reflected uniformly in all directions.

Very rough surfaces reflect diffusely.

Assume surfaces are adequately rough to be mostly diffusely reflective for engineering calculations.

Kirchhoff's Identity



Enclosure radiates $q_i[W/area]$ onto inner body

Internal gray-body in equilibrium (steady-state) with enclosure:

Energy emitted by internal body =

Energy absorbed by internal body

$$EA = q_i A\alpha$$

Internal gray-body replaced by internal black body with identical geometry *and temperature*. At equilibrium:

$$E_b A = q_i A(1)$$

But

$$\varepsilon \equiv \frac{E}{E_b}$$

Therefore $\boxed{\varepsilon = \alpha}$ Kirchhoff's Identity for radiating surfaces in equilibrium

Gray-Body Radiation Exchange

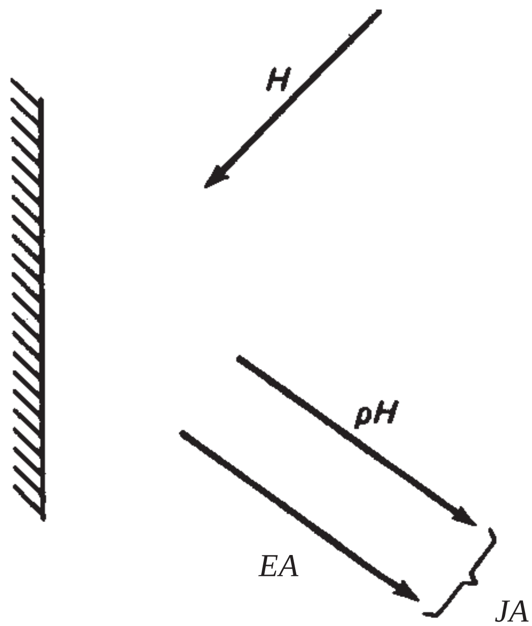


Fig. 3-13. Emission and reflection from an opaque surface.

If multiple surfaces are involved in radiative heat transfer processes and the reflectivity is not very nearly zero, the various surfaces and their reflections may need to be considered. The following paragraphs result in what is known as the "Hottel Script F" method.

Consider the surface shown in Fig. 3-13 from TCEE.

E = total emissive power from surface under consideration

$H \equiv$ total irradiance from all other surfaces (radiation / time arriving at surface)

$J \equiv$ radiosity (total radiation / time $\cdot$ area leaving surface)

ρH = reflected irradiance

$$JA = EA + \rho H$$

If the particular surface under consideration is denoted as surface 1 and is irradiated by only one other surface, e.g. surface 2, the irradiation of surface 1 is

$$H_1 = F_{2-1} A_2 J_2$$

and the total radiation leaving surface 1 is

$$\begin{aligned} J_1 A_1 &= E_1 A_1 + \rho_1 H_1 \\ &= \varepsilon_1 E_{b1} A_1 + \rho_1 F_{2-1} A_2 J_2 \end{aligned}$$

For a system of N surfaces, this is easily generalized to

$$J_i A_i = \varepsilon_i E_{bi} A_i + \sum_{j=1}^{N} \rho_i F_{ji} A_j J_j$$

Using the reciprocity relation

$$F_{ji} A_j = F_{ij} A_i$$

$$J_i A_i = \varepsilon_i E_{bi} A_i + \rho_i A_i \sum_{j=1}^{N} F_{ij} J_j$$

$$J_i = \varepsilon_i E_{bi} + \rho_i \sum_{j=1}^{N} F_{ij} J_j \qquad \text{TCEE E3.11}$$

$$J_i - \rho_i \sum_{j=1}^{N} F_{ij} J_j = \varepsilon_i E_{bi}$$

Multiply the preceding by A_i/ρ_i .

$$(A_i/\rho_i)\left(J_i - \rho_i \sum_{j=1}^{N} F_{ij} J_j \right) = (\varepsilon_i A_i/\rho_i) E_{bi}$$

$$\sum_{j=1}^{N} \Big[(A_i/\rho_i)\big(\delta_{ij} - \rho_i F_{ij}\big)\Big] J_j = \varepsilon_i A_i E_{bi}/\rho_i$$

where

$$\delta_{ij} \equiv \text{Kronecker delta} \;=\; \begin{cases} 0 \text{ for } i \neq j \\ 1 \text{ for } i = j \end{cases}$$

The [...] may be written as a matrix element

$$e_{ij} = \big[E_{ij}\big] = (A_i/\rho_i)\big(\delta_{ij} - \rho_i F_{ij}\big)$$

and $[J_i]$, $[T_i]$ are elements of the column vectors $[J]$ and $[T]$:

$$[T_i] = \varepsilon_i A_i E_{bi} / \rho_i$$

where $E_{bi} = \sigma T_i'^4$ and $T' = T[^oC] + 273.14$

Then

$$[E][J] = [T]$$

which may be solved for the radiosity

$$[J] = [E]^{-1}[T]$$

In simultaneous equation form the radiosity is

$$J_i = \sum_{j=1}^{N} e_{ij}^{-1} \varepsilon_j A_j E_{bj} / \rho_j \qquad (a)$$

where $e_{ij}^{-1} = [E_{ij}]^{-1}$

The total radiation leaving surface i is

$$\begin{aligned} J_i A_i &= A_i E_i + \rho_i H_i \\ &= A_i \varepsilon_i E_{bi} + \rho_i H_i \\ &= A_i \varepsilon_i \sigma T_i'^4 + \rho_i H_i \end{aligned}$$

Then

$$H_i = (A_i / \rho_i)\left(J_i - \varepsilon_i \sigma T_i'^4\right)$$

If surface i has an absorbtivity α_i, the net radiative heat transfer $Q_{i\,Net}$ from surface i is

$$\begin{aligned} Q_{i\,Net} &= \varepsilon_i A_i \sigma T_i'^4 - \alpha_i H_i \\ &= \varepsilon_i A_i \sigma T_i'^4 - \varepsilon_i H_i \end{aligned}$$

Using $\alpha_i = \varepsilon_i$ (Kirchhoff's Identity for radiating surfaces in equilibrium)

Then

$$\begin{aligned} Q_{i\,Net} &= -\varepsilon_i\left(H_i - A_i \sigma T_i'^4\right) \\ &= -\varepsilon_i\left[(A_i / \rho_i)\left(J_i - \varepsilon_i \sigma T_i'^4\right) - A_i \sigma T_i'^4\right] \\ &= -(A_i \varepsilon_i / \rho_i)\left[\left(J_i - \varepsilon_i \sigma T_i'^4\right) - \rho_i \sigma T_i'^4\right] \\ &= -(A_i \varepsilon_i / \rho_i)\left[J_i - (\rho_i + \varepsilon_i)\sigma T_i'^4\right] \end{aligned}$$

Substituting Equation (a) for J_i

$$Q_{i\,Net} = -(A_i\varepsilon_i/\rho_i)\left[\sum_{j=1}^{N} e_{ij}^{-1}\left(\varepsilon_j A_j/\rho_j\right)\sigma T_j'^4 - (\rho_i + \varepsilon_i)\sigma T_i'^4\right]$$

$$= -(A_i\varepsilon_i/\rho_i)\sum_{j\neq i}^{N}\left\{\begin{array}{l} e_{ij}^{-1}\left(\varepsilon_j A_j/\rho_j\right)\sigma T_j'^4 \\ -\left[\rho_i + \varepsilon_i - e_{ii}^{-1}(\varepsilon_i A_i/\rho_i)\right]\sigma T_i'^4 \end{array}\right\} \qquad \text{(b)}$$

At equilibrium for $T_j' = T_i'$ for all i,j, $Q_{i\,Net}=0$.

It can be concluded that

$$\rho_i + \varepsilon_i - e_{ii}^{-1}(\varepsilon_i A_i/\rho_i) = \sum_{j\neq i}^{N} e_{ij}^{-1}\,\varepsilon_j A_j/\rho_j \qquad \text{(c)}$$

which can be substituted back into Equation (b) to give

$$Q_{i\,Net} = -(\varepsilon_i A_i/\rho_i)\sum_{j\neq i}^{N} e_{ij}^{-1}\left(\varepsilon_j A_j/\rho_j\right)\sigma\left(T_j'^4 - T_i'^4\right)$$

$$= \sum_{j\neq i}^{N} e_{ij}^{-1}\left(\varepsilon_i\varepsilon_j A_i A_j/\rho_i\rho_j\right)\sigma\left(T_i'^4 - T_j'^4\right)$$

Defining F_{ij} as

$$F_{ij} = \left(\frac{\varepsilon_i \varepsilon_j}{\rho_i \rho_j}\right) A_j e_{ij}^{-1}, \qquad i \neq j$$

and also

$$F_{ij} = \left(\frac{\varepsilon_i \varepsilon_j}{\rho_i \rho_j}\right) A_j \left(e_{ij}^{-1} - \frac{\rho_i}{A_i}\right), \qquad j = i$$

(with the latter still to be proven)

and

$$Q_{i\,Net} = \sum_{j=1}^{N} F_{ij} A_i \sigma\left(T_i'^4 - T_j'^4\right)$$

Note then that since

$$Q_{ij} = F_{ij} A_i \sigma\left(T_i'^4 - T_j'^4\right)$$

it is also true that

$$Q_{ji} = F_{ji} A_j \sigma\left(T_j'^4 - T_i'^4\right)$$

But since $Q_{ji} = -Q_{ij}$

$$F_{ji} A_j \left(T_j'^4 - T_i'^4\right) = -F_{ij} A_i \left(T_i'^4 - T_j'^4\right)$$

$$= F_{ij} A_i \left(T_j'^4 - T_i'^4\right)$$

$$\boxed{F_{ji} A_j = F_{ij} A_i} \qquad \text{Reciprocity}$$

An important "theorem":

Also suppose that all $T_j' = 0$. Then

$$Q_{i\,Net} = \sum_{j=1}^{N} F_{ij} A_i \sigma\left(T_i'^4 - T_j'^4\right) = \sum_{j=1}^{N} F_{ij} A_i \sigma T_i'^4 = A_i \sigma T_i'^4 \sum_{j=1}^{N} F_{ij}$$

but also if all $T_j' = 0,$ surface i is only radiator and radiates by an amount

$$Q_{i\,Net} = \sigma \varepsilon_i A_i T_i'^4$$

$$\sigma A_i T_i'^4 \sum_{j=1}^{N} F_{ij} = \varepsilon_i \sigma A_i T_i'^4$$

$$\boxed{\sum_{j=1}^{N} F_{ij} = \varepsilon_i}$$

Derivation of F_{ij}, $i = j$:

Beginning with $\sum_{j=1}^{N} F_{ij} = \varepsilon_i$

$$\sum_{j=1}^{N} F_{ij} = \sum_{j \neq i}^{N} F_{ij} + F_{ii}$$

$$\sum_{j \neq i}^{N} F_{ij} + F_{ii} = \varepsilon_i$$

$$F_{ii} = \varepsilon_i - \sum_{j \neq i}^{N} F_{ij}$$

$$= \varepsilon_i - \sum_{j \neq i}^{N} \frac{\varepsilon_i \varepsilon_j}{\rho_i \rho_j} A_j e_{ij}^{-1} \qquad \text{from Equation (c)}$$

$$= \varepsilon_i - \frac{\varepsilon_i}{\rho_i} \sum_{j \neq i}^{N} \frac{\varepsilon_j}{\rho_j} A_j e_{ij}^{-1}$$

But we have from Equation (c)

$$\sum_{j \neq i}^{N} \frac{\varepsilon_j}{\rho_j} A_j e_{ij}^{-1} = \rho_i + \varepsilon_i - e_{ii}^{-1} \left(\frac{\varepsilon_i A_i}{\rho_i} \right)$$

Then

$$F_{ii} = \varepsilon_i - \left(\frac{\varepsilon_i}{\rho_i} \right) \left[\rho_i + \varepsilon_i - e_{ii}^{-1} \left(\frac{\varepsilon_i A_i}{\rho_i} \right) \right]$$

$$= \varepsilon_i - \left(\frac{\varepsilon_i}{\rho_i} \right) \rho_i - \left(\frac{\varepsilon_i}{\rho_i} \right) \left[\varepsilon_i - e_{ii}^{-1} \left(\frac{\varepsilon_i A_i}{\rho_i} \right) \right]$$

$$= \left(\frac{\varepsilon_i}{\rho_i} \right) \left[e_{ii}^{-1} \left(\frac{\varepsilon_i A_i}{\rho_i} \right) - \varepsilon_i \right]$$

$$= \left(\frac{\varepsilon_i}{\rho_i} \right) \left(\frac{\varepsilon_i A_i}{\rho_i} \right) \left(e_{ii}^{-1} - \frac{\rho_i}{A_i} \right)$$

$$F_{ij} = \left(\frac{\varepsilon_i}{\rho_i}\right)\left(\frac{\varepsilon_j A_j}{\rho_j}\right)\left(e_{ij}^{-1} - \frac{\rho_i}{A_i}\right), \qquad i = j$$

$$F_{ij} = \left(\frac{\varepsilon_i \varepsilon_j}{\rho_i \rho_j}\right) A_j \left(e_{ij}^{-1} - \frac{\rho_i}{A_i}\right), \qquad i = j$$

In summary,

$$F_{ij} = \left(\frac{\varepsilon_i \varepsilon_j}{\rho_i \rho_j}\right) A_j e_{ij}^{-1}, \qquad i \neq j$$

and also

$$F_{ij} = \left(\frac{\varepsilon_i \varepsilon_j}{\rho_i \rho_j}\right) A_j \left(e_{ij}^{-1} - \frac{\rho_i}{A_i}\right), \qquad j = i$$

PC - TNETFA uses $Q_{ij} = F_{ij} A_i \sigma\left(T_i'^4 - T_j'^4\right)$ using $F_{ij} A_i \equiv$ "conductance" for multi - surface radiation (CTYPE = -2).

Thermal Radiation Network Method

The rules developed for this method may be applied to some important cases. The "radiosity equation" was previously derived:

$$J_i = \varepsilon_i E_{bi} + \rho_i \sum_{j=1}^{N} F_{ij} J_j \qquad \text{TCEE E3.11}$$

Rearrange the preceding equation,

$$\rho_i \sum_{j=1}^{N} F_{ij} J_j + \varepsilon_i E_{bi} - J_i = 0$$

add and substract $\varepsilon_i J_i$ to get

$$\rho_i \sum_{j=1}^{N} F_{ij} J_j + \varepsilon_i E_{bi} - J_i + \varepsilon_i J_i - \varepsilon_i J_i = 0$$

$$\rho_i \sum_{j=1}^{N} F_{ij} J_j + \varepsilon_i E_{bi} - (1 - \varepsilon_i) J_i - \varepsilon_i J_i = 0$$

Using $\rho = 1 - \alpha$ and Kirchhoff''s identity, $\alpha = \varepsilon$

$$\rho_i A_i \sum_{j=1}^{N} F_{ij} J_j + \varepsilon_i A_i E_{bi} - \rho_i A_i J_i - \varepsilon_i A_i J_i = 0$$

after multiplying every term by by A_i.

Since $\sum_{j=1}^{N} F_{ij} = 1,$ the third term of the preceding equation may be multiplied by the summation term.

$$\rho_i A_i \sum_{j=1}^{N} F_{ij} J_j + \varepsilon_i A_i E_{bi} - \rho_i A_i J_i \sum_{j=1}^{N} F_{ij} - \varepsilon_i A_i J_i = 0$$

$$\rho_i A_i \sum_{j=1}^{N} F_{ij} \left(J_j - J_i \right) - \varepsilon_i A_i \left(J_i - E_{bi} \right) = 0$$

$$\frac{\left(J_i - E_{bi} \right)}{\left(\rho_i / \varepsilon_i A_i \right)} = \sum_{j=1}^{N} \frac{\left(J_j - J_i \right)}{\left(A_i F_{ij} \right)^{-1}}$$

The "trick" is to now recognize a form of Kirchhoff's law for radiative heat transfer (per unit area and time) "currents" where the current into a "node" equals the sum of the currents out of the node. The denominators for each of the two sides of the preceding equation are then identified as "surface" and "spatial" resistances.

$$\boxed{R_i = \frac{1 - \varepsilon_i}{\varepsilon_i A_i}} \qquad \text{TCEE E3.17}$$

$$\boxed{R_{ij} = \frac{1}{A_i F_{ij}}} \qquad \text{TCEE E3.16}$$

The thermal radiation equivalent of Kirchhoff's law is then

$$\boxed{\frac{(E_{bi} - J_i)}{R_i} = \sum_{j=1}^{N} \frac{\left(J_i - J_j\right)}{R_{ij}}} \qquad \text{TCEE E3.18}$$

It is now necessary to more completely identify the exact nature of the left and right sides of the preceding equation.

First consider the net radiative heat loss from surface *i*:

Net radiative heat loss from surface i =
total radiative heat out of i -
total heat rate into i

$$Q_{i\,Net} = J_i A_i - H_i$$

It was shown earlier that

$$J_i A_i = E_i A_i + \rho_i H_i$$
$$= \varepsilon_i E_{bi} A_i + \rho_i H_i$$

Solving for H_i

$$H_i = \frac{1}{\rho_i} J_i A_i - \frac{\varepsilon_i}{\rho_i} E_{bi} A_i$$

Substituting H_i into $Q_{i\,Net}$

$$Q_{iNet} = J_i A_i - \frac{1}{\rho_i} J_i A_i + \frac{\varepsilon_i}{\rho_i} E_{bi} A_i$$

$$= \left(1 - \frac{1}{\rho_i}\right) J_i A_i + \frac{\varepsilon_i}{\rho_i} E_{bi} A_i$$

$$= \left(\frac{\rho_i - 1}{\rho_i}\right) J_i A_i + \frac{\varepsilon_i}{\rho_i} E_{bi} A_i$$

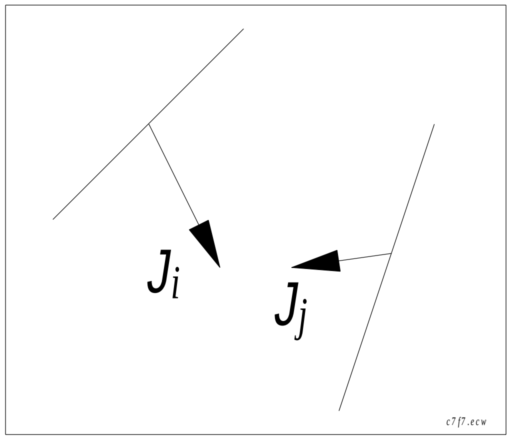
Since $\rho_i - 1 = -\varepsilon_i$ and $\rho_i = 1 - \varepsilon_i$

$$Q_{i\,Net} = -\left(\frac{\varepsilon_i}{1-\varepsilon_i}\right) J_i A_i + \left(\frac{\varepsilon_i}{1-\varepsilon_i}\right) E_{bi} A_i$$

$$\boxed{Q_{i\,Net} = \frac{(E_{bi} - J_i)}{R_i}} \qquad R_i = \frac{1-\varepsilon_i}{\varepsilon_i A_i}$$

TCEE E3.19

Now consider the net radiative heat exchange between surfaces *i* and *j*:



Net radiative heat rate exchange between surfaces i and j =

Radiative heat rate from surface i that is intercepted by surface j -

Radiative heat rate from surface j that is intercepted by surface i

$$Q_{ij} = F_{ij} J_i A_i - F_{ji} J_j A_j$$
$$= F_{ij} A_i J_i - F_{ji} A_j J_j$$

Using reciprocity $\quad F_{ji} A_j = F_{ij} A_i$

$$Q_{ij} = F_{ij} A_i J_i - F_{ij} A_i J_j = F_{ij} A_i \left(J_i - J_j \right)$$

$$\boxed{Q_{ij} = \frac{\left(J_i - J_j \right)}{R_{ij}}} \qquad R_{ij} = \frac{1}{F_{ij} A_i}$$

Example of how a circuit and elements are defined:

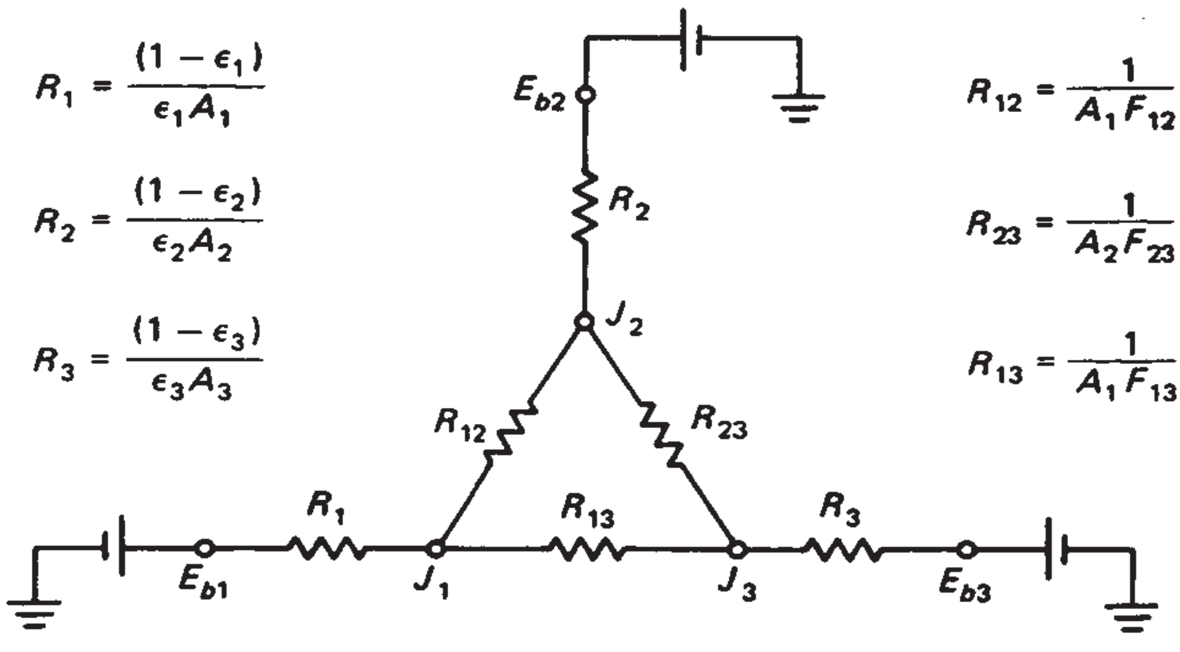


Fig. 3-14. Three-surface thermal radiation network.

Two surfaces - a simple, but important result.

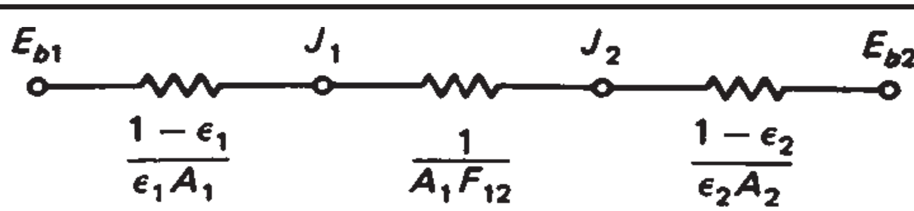


Fig. 3-15. Radiation circuit for two surfaces.

The resistance between E_{b1} and E_{b2} is

$$R = \frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{1-2}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}$$

The net radiative heat transfer is

$$Q_{Net} = \frac{(E_{b1} - E_{b2})}{R}$$
$$= \sigma \mathcal{F}_{1-2} A_1 \left(T_1'^4 - T_2'^4\right)$$

so that the identification is made for the script F:

$$R = \frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{1-2}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}$$

$$\boxed{\mathcal{F}_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)\left(\frac{A_1}{A_2}\right) + \frac{1}{F_{1-2}}}}$$ TCEE E3.21

Consider the special case of an electronic enclosure in a room.

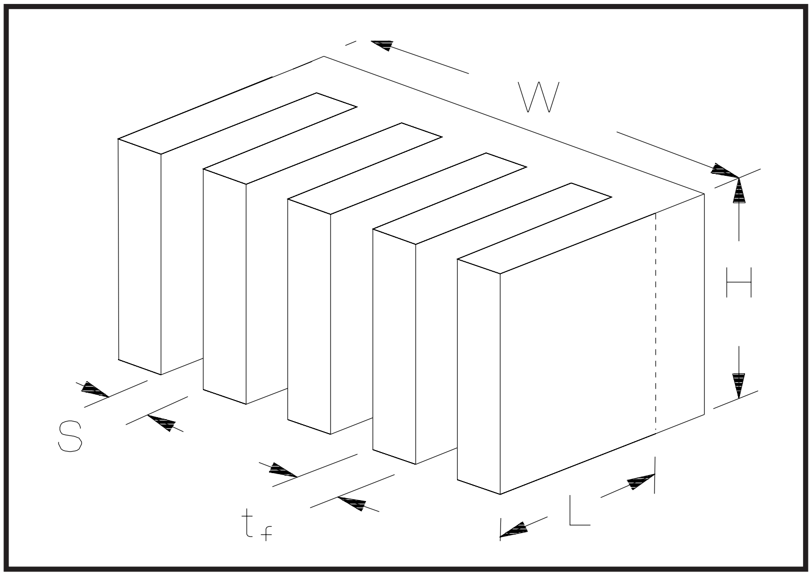
$$F_{1-2} = 1.0$$

$$A_2 >> A_1$$

Then

$$\boxed{\mathcal{F}_{1-2} = \varepsilon_1}$$ TCEE E3.22

Finned Surfaces (Shielding)



Unfinned surface

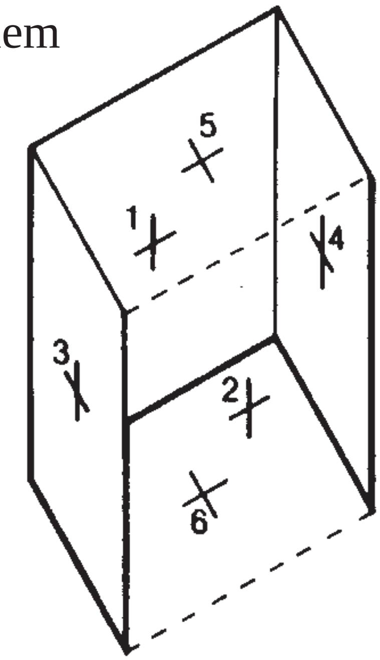
$$Q_r = \varepsilon h_r A_S (T_S - T_A)$$

Finned surface

$$Q_r = F\eta h_r A_S (T_S - T_A)$$

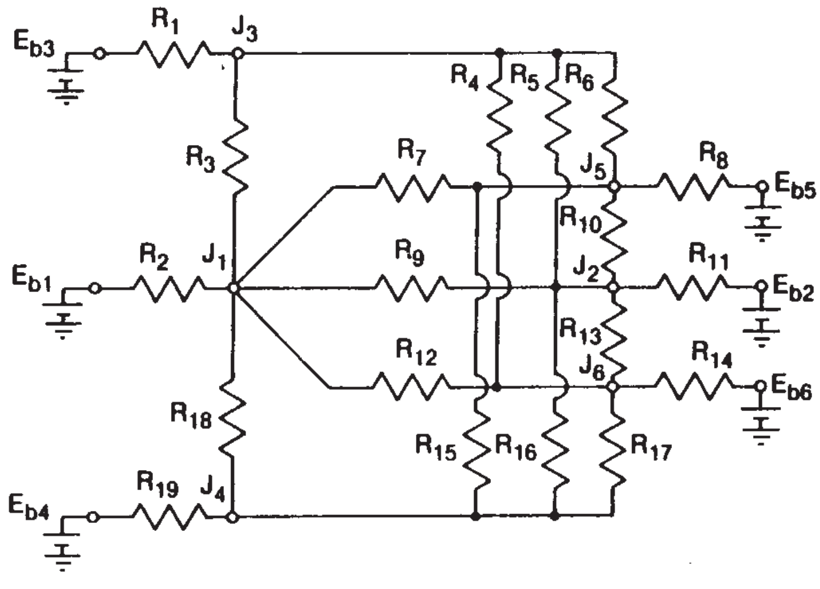
Note: $$R_r = \frac{1}{F\eta h_r A_S}$$

The Problem



Node and surface identification

The Complete Circuit



Equivalent circuit

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The Solution Summarized

$$F = 2C_{Net} \Big/ \left[H(S + 2L) \right]$$

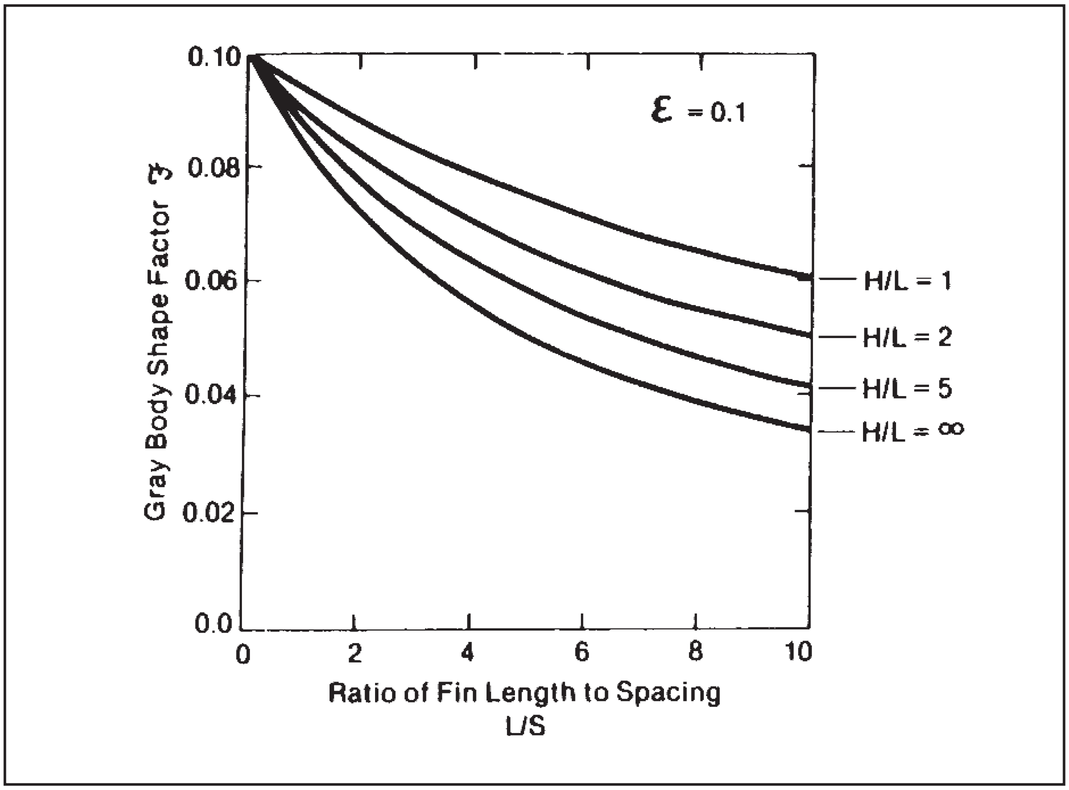
$$C_{Net} = \left[(R_a + R_b + R_e)(R_c + R_d + R_e) - R_e^2 \right] \Big/$$

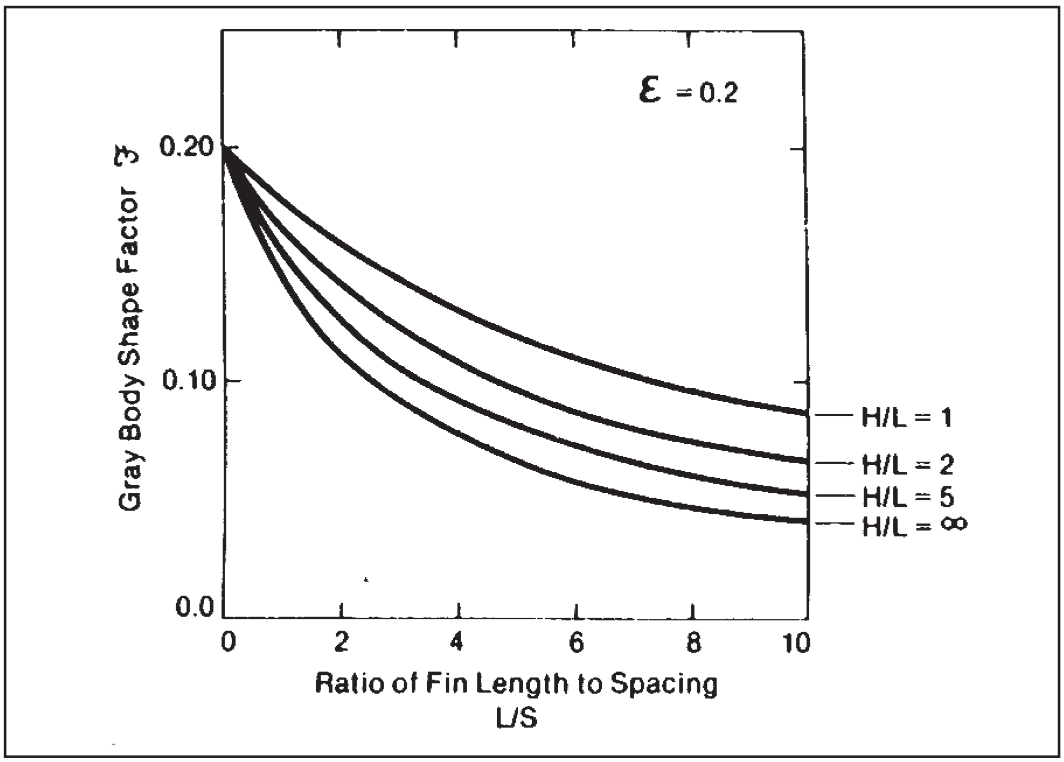
$$\left\{ \begin{array}{l} (R_b + R_d)\left[(R_a + R_b + R_e)(R_c + R_d + R_e) - R_e^2 \right] \\ -R_b\left[R_b(R_c + R_d + R_e) + R_e R_d \right] \\ -R_d\left[R_d(R_a + R_b + R_e) + R_b R_e \right] \end{array} \right\}$$

$$R_a = (1-\varepsilon) \big/ (\varepsilon A_3), \quad R_b = 2(1-\varepsilon) \big/ (\varepsilon A_1)$$

$$R_c = 1 \big/ (A_1 F_{1-3} + 2A_3 F_{3-5})$$

$$R_d = 2 \big/ (A_1 F_{1-2} + 2A_1 F_{1-5})$$





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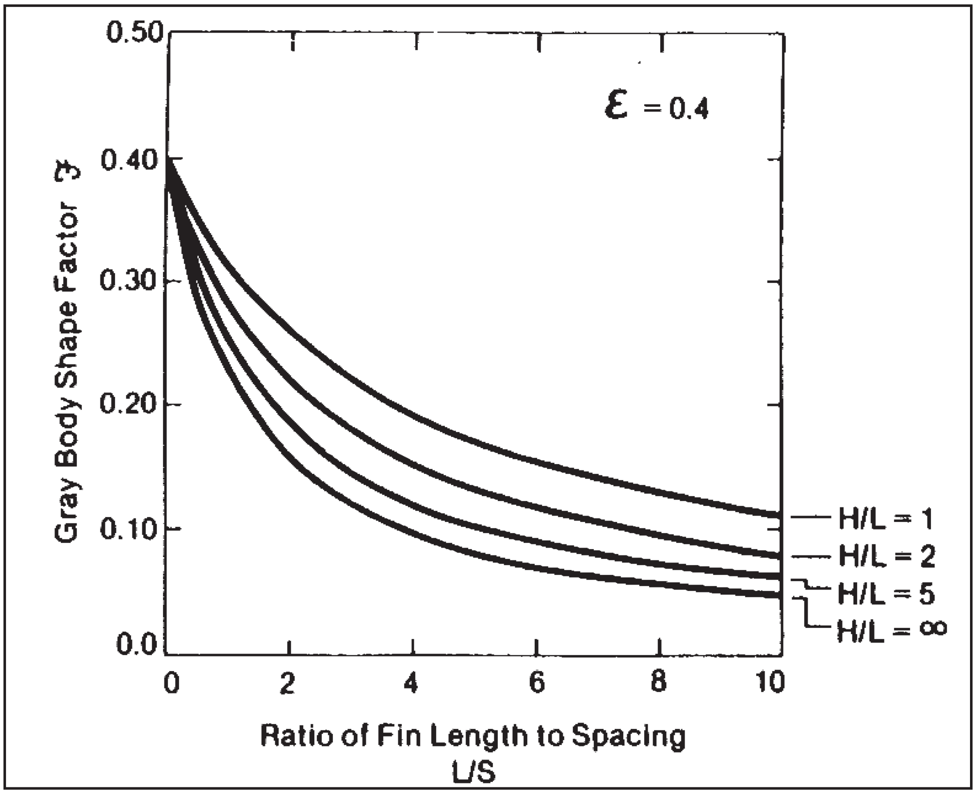

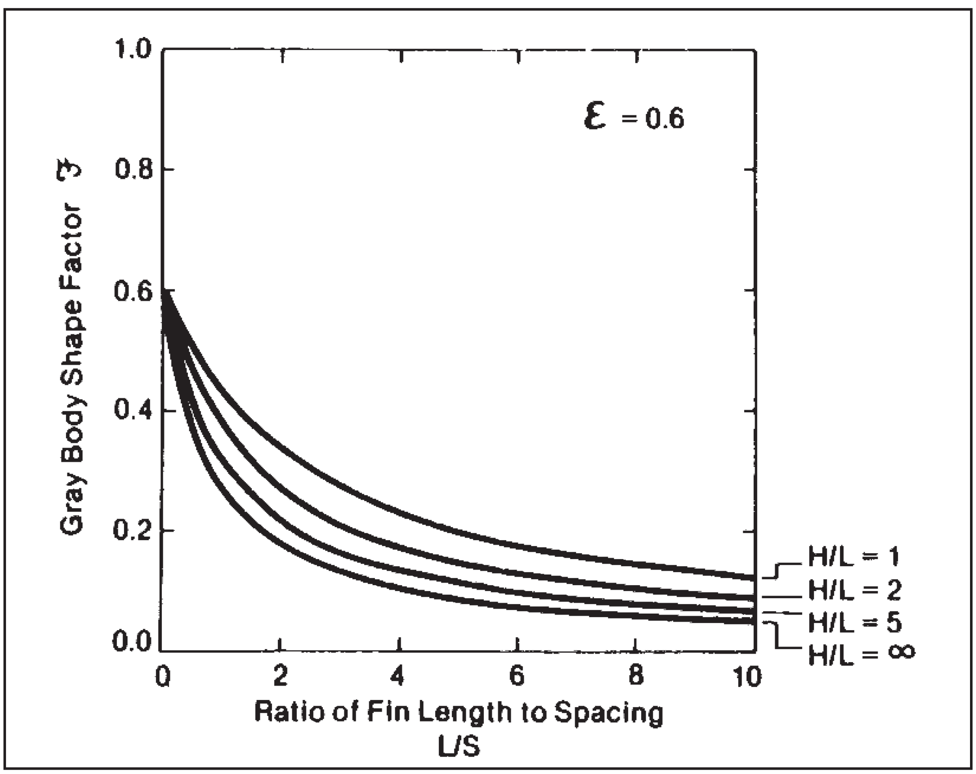

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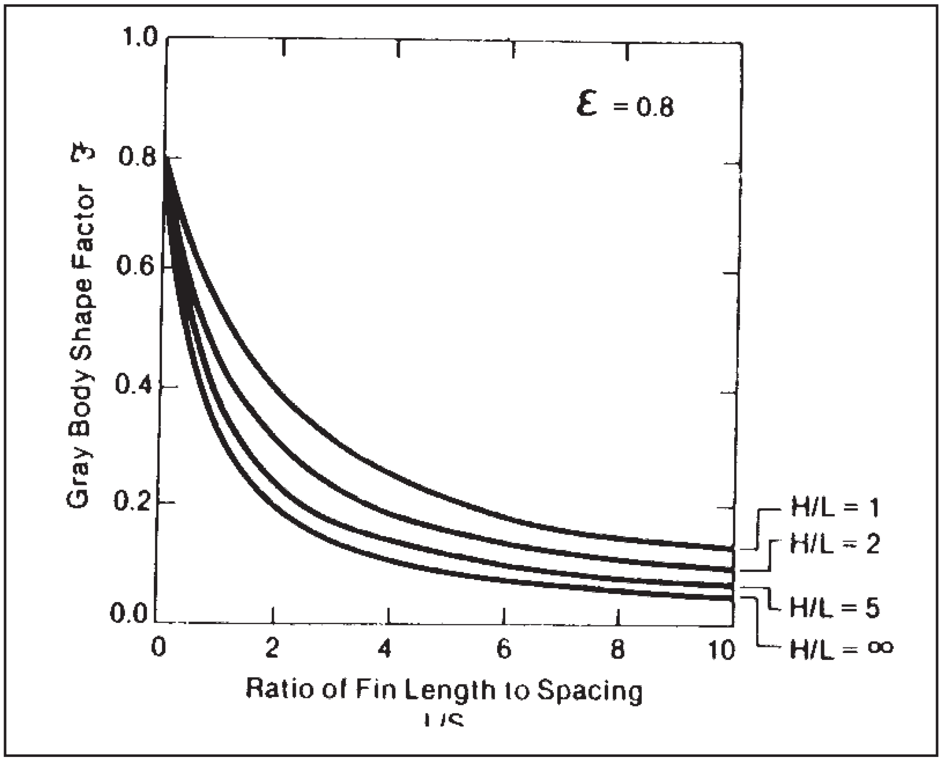

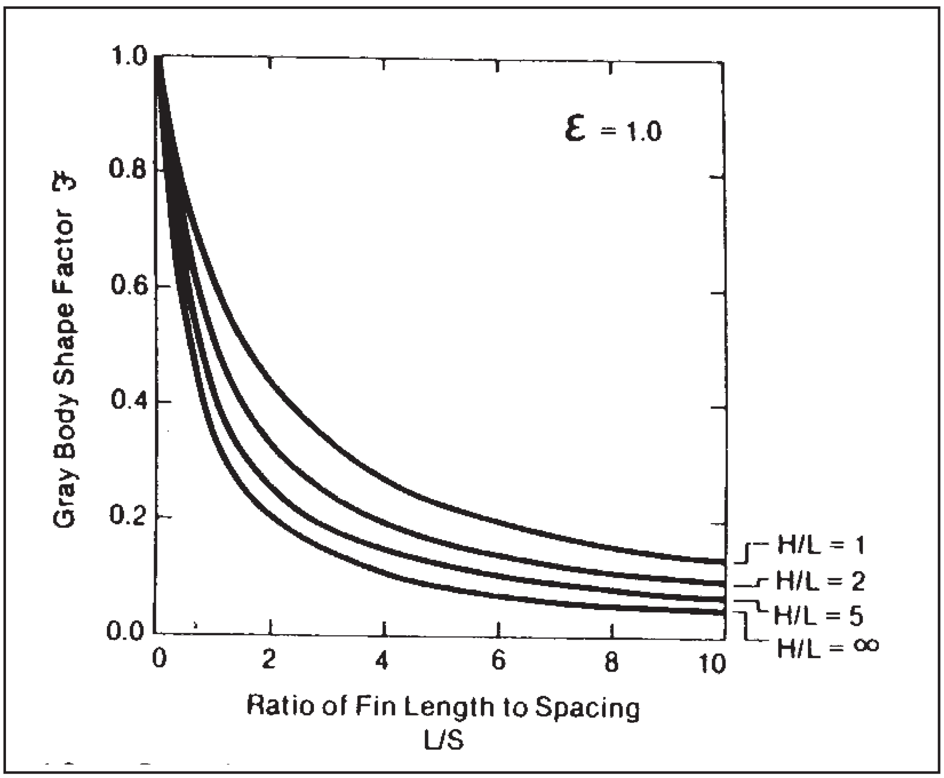

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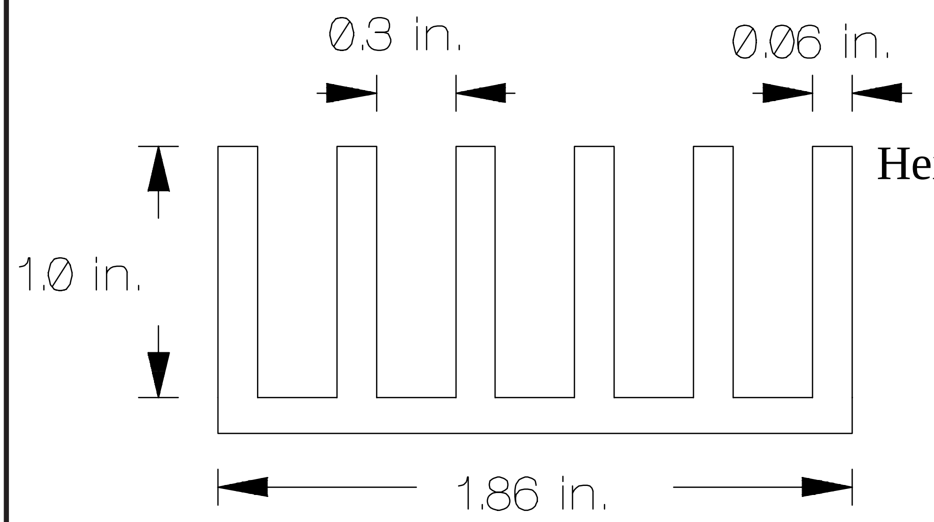
Example

Previous Finned Heat Sink

Geometry



$\text{Height} = 5.0\ in.$

$\Delta T = 50\ {}^oC$

$T_A = 20\ {}^oC$

$\varepsilon = 0.8$

Heat transfer coefficients previously calculated

$$h_c = 0.0033\ W/in.^2 \cdot {}^oC$$

For radiation

$$L/S = 1.0\ in./0.3\ in. = 3.33$$

$$H/L = 5.0\ in./1.0\ in. = 5.0$$

From Fig. 5-11, $F = 0.16$

From Fig. 3-16, $h_r = 0.0047\ W/in.^2 \cdot {}^oC$

Previously calculated convection conductance

$$C_C = 0.22\ W/^oC$$

Radiation conductance,

Outer surfaces

$$\begin{aligned} C_{rExt} &= \varepsilon h_r A_{Est} \eta \\ &= (0.8)\left(0.0047\ W/in.^{2\,o}C\right)(1.0\ in.)(5.0\ in.) \\ &\quad x2\ \ finsx1.0 \\ &= 0.038\ W/^oC \end{aligned}$$

Inner surfaces

Previously, $A_{Int} = 59.3\ in.^2$

$$\begin{aligned} C_{rInt} &= F\eta h_r A_{Int} \\ &= (0.16)(1.0)\left(0.0047\ W/in.^2 \cdot {}^oC\right)\left(59.3\ in.^2\right) \\ &= 0.045\ W/^oC \end{aligned}$$

Total radiation

$$C_r = C_{rExt} + C_{rInt} = 0.038 + 0.045$$
$$= 0.083\ W/{}^oC$$

Total conductance and resistance

$$C = C_C + C_r = 0.22 + 0.083 = 0.30\ W/{}^oC$$
$$R = 1/C = 3.3\ {}^oC/W$$

Heat transfer

$$\Delta T = RQ$$
$$Q = \Delta T/R = 50\ {}^oC/\left(3.3\ {}^oC/W\right)$$
$$Q = 15.1\ W$$

It was previously calculated that 10.9 *W* could be convected.

Check of fin efficiency

$$\text{Total } h = h_c + Fh_r$$
$$= 0.0033 + (0.16)(0.0047)$$
$$= 0.0033 + 0.00075$$
$$= 0.0041\ W/in.^2 \cdot {}^oC$$

Fin efficiency

$$C_S = hA_{fin}$$
$$= \left(0.0041\ W/in.^2 \cdot {}^oC\right)(1.0\ in.\ x\,5.0\ in.\ x\,2)$$
$$= 0.041\ W/{}^oC$$

$$C_k = kA_k/L$$
$$= \left(5.0\ W/in.\cdot {}^oC\right)(0.06\ in.\ x\,5.0\ in.)\ /\ 1.0\ in.$$
$$= 1.5\ W/{}^oC$$

$$C_S/C_k = 0.041/1.5$$
$$= 0.027$$

From Fig. 5-2, $\eta \cong 1.0$

Conduction

One-Dimensional Conduction

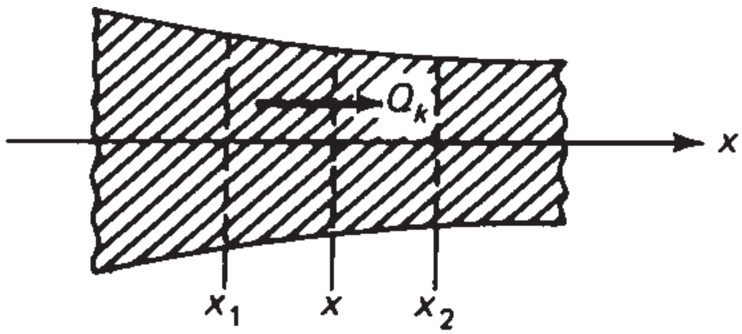


Fig. 1-1. Heat conduction in a one-dimensional solid element.

Fourier's Law:

$$Q_k = -kA_k \left.\frac{dT}{dx}\right|_x \qquad \text{TCEE E1.1}$$

$Q_k \equiv$ heat transferred, *watts*

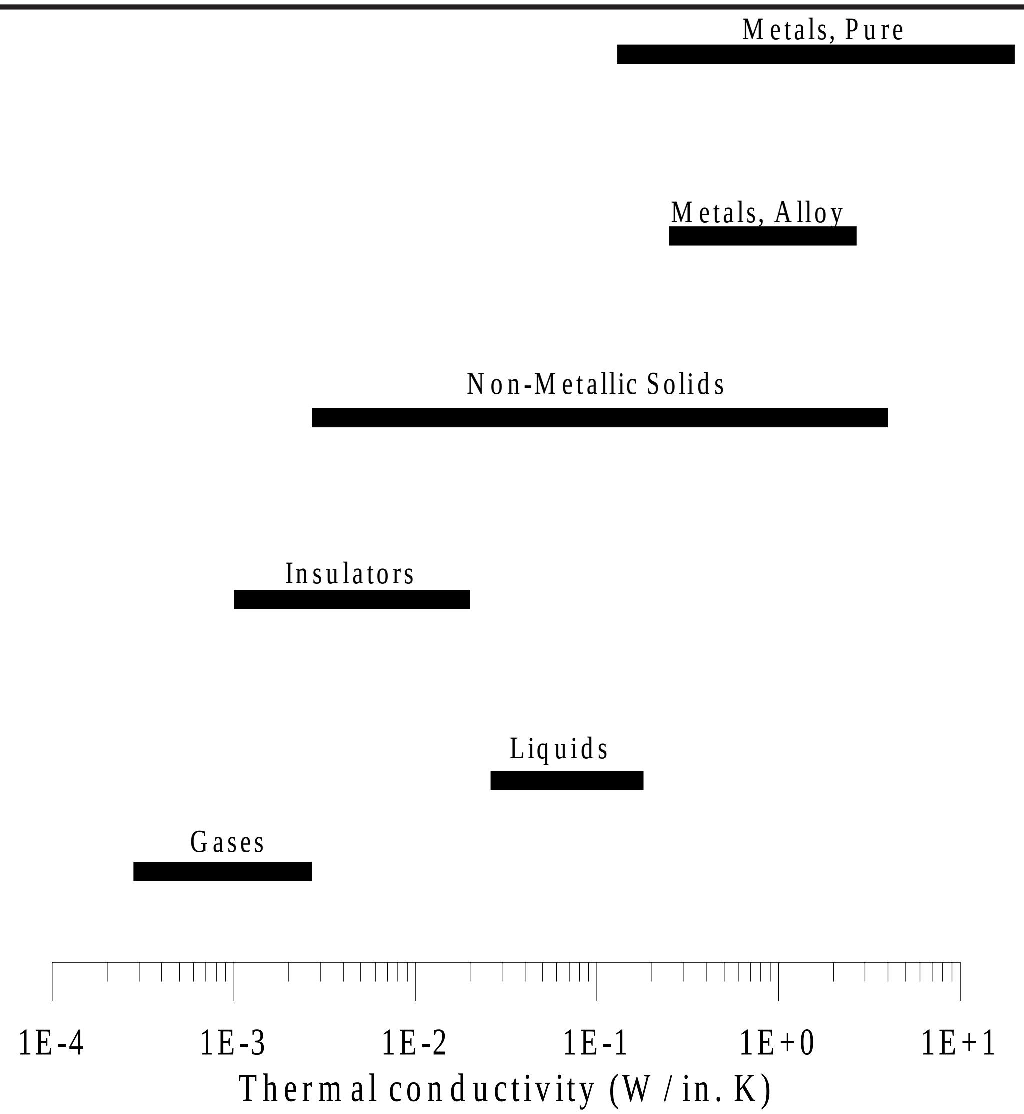
$A_k \equiv$ cross-sectional area of heat flow path, $cm^2, m^2,$ or $in.^2$

$\dfrac{dT}{dx} \equiv$ temperature gradient, ${}^oC/cm$, ${}^oC/m$, or ${}^oC/in.$

$k \equiv$ thermal conductivity, $watts/{}^oC \cdot cm$, $watts/{}^oC \cdot m$, or $watts/{}^oC \cdot in.$,

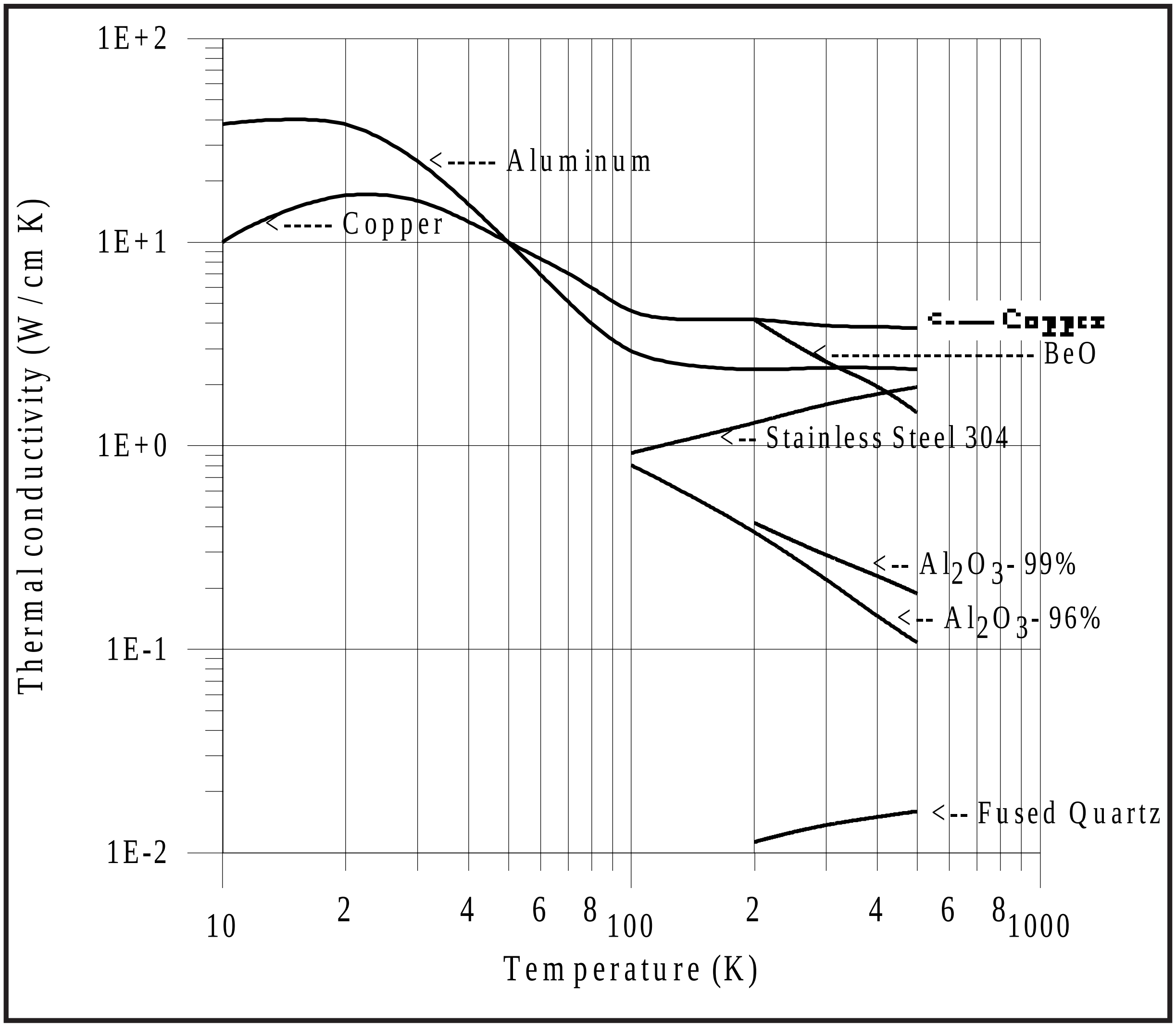
Thermal Conductivity of Materials

Comparison of Gas, Liquid, and Solid Phases



The indicated conductivity ranges are approximate estimates only.

Temperature Dependence of Some Common Solids



Data obtained from a variety of sources. Users should consult the literature for more exact values pertaining to their application.

Conductivity of Some Common Electronic Packaging Materials at Room Temperature. Values From Several Sources.

Material	k (W/in. °C)	Material	k (W/in. °C)
Diamond	20.0	Kovar	0.5
Silver	10.6	Epoxy resin, BeO filled	0.09
Copper	9.6	Quartz	0.05
Eutectic bond	7.5	SiO_2	0.04
Gold	7.5	Borosilicate glass	0.026
Aluminum	5.5	Glass frit	0.024
Beryllia	4-8	Conductive epoxy	0.02
Molybdenum	3.7	Sylgard resin	0.01
Nickel	2.3	Epoxy glass laminate	0.007
Silicon	2.1	Doryl cement	0.007
Steel	1.2	Epoxy resin, unfilled	0.004
Solder (60-40)	0.9	Silcone RTV, unfilled	0.004
Lead	0.8	Thermal compound, filled	0.02
Alumina (99%)	0.8	Mica	0.02
Alumina (96%)	0.6		

Physical Phenomena of Thermal Conduction in a Solid.

A solid is comprised of free electrons and atoms bound in a lattice in a periodic arrangement (usually).

The transport of heat in a solid is the combined effect of the free electrons (normally associated with a metal) and the solid lattice. This combined effect may be incorporated into the thermal conductivity of the solid as:

$$k = k_e + k_l$$

where $k_e \equiv$ electronic contribution to conductivity

$k_l \equiv$ lattice contribution to conductivity

For pure metals,

$$k \cong k_e$$

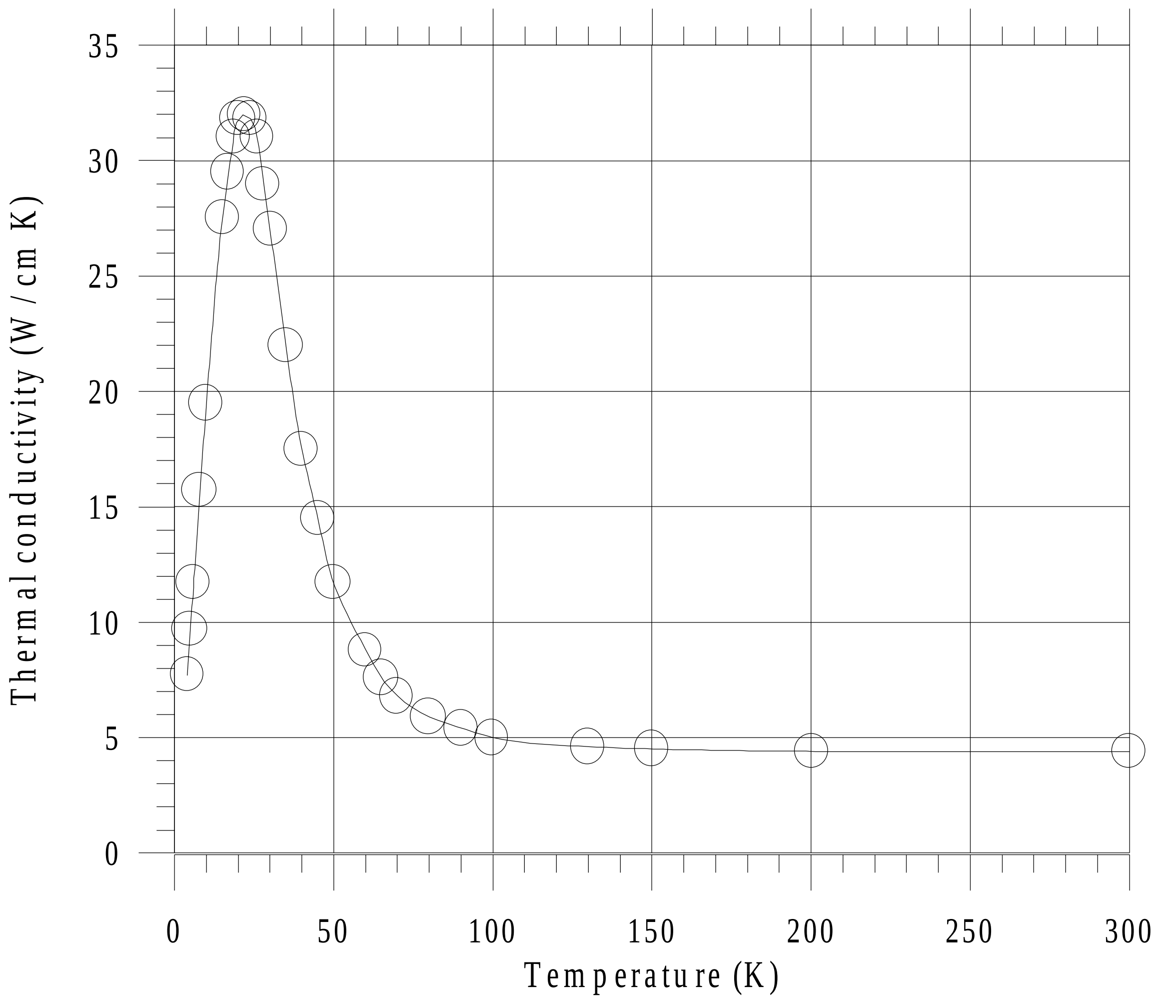
non-metallic solids,

$$k \cong k_l$$

semiconductors,

$$k \cong k_l$$

Thermal Conductivity of a Common Metal (Copper) with Imperfections at "Low Temperature".



Data from Childs, G.E., Ericks, L.J., Powell, R.L., Thermal Conductivity of Solids at Room Temperature and Below, U.S. Department of Commerce, 1973, page 507, B.Z.S.-2.

The thermal resistivity of a metal is defined as

$$\rho = \frac{1}{k}$$

Resistance to heat flow in a metal is found to be described by "Matthiessen's rule":

$$\rho = \rho_l + \rho_i$$

where

$\rho_l \equiv$ defect resistivity and

$\rho_i \equiv$ intrinsic resistivity

The defect resistivity is due to scattering of the electronic heat carriers from lattice defects and impurities and is typically described by

$$\rho_l = A / T$$

where A is a constant.

The intrinsic resistivity is due to scattering of the electronic heat carriers from the lattice and is typically described by

$$\rho_i = BT^2$$

where B is a constant.

The net thermal conductivity is therefore

$$k = \frac{1}{\rho} = \frac{1}{\frac{A}{T} + BT^2}$$

This equation is used to describe the portion of the thermal conductivity curve over the temperature range shown in the plot for copper from. The defect portion dominates conduction up to the vicinity of the conductivity maximum. The purer the metal , the greater the peak, i.e. the maximum conductivity decreases as defects and impurities are added to the metal.

The effects of defects and lattice scattering are equal at the temperature where the conductivity maximum occurs.

The lattice scattering term dominates at temperatures from the conductivity maximum and greater. The conductivity of copper changes very little from 100 to 300 *K*.

Sometimes it is possible to curve fit the conductivity data to the preceding equation for *k* so that the fit is quite good up to modest temperatures, e.g. 100 *K* for copper. Even when such a fit is possible, it is usually very difficult to model *k(T)* over a large temperature range, i.e. 4-300 *K*.

Thermal Conductivity of a Non-Metallic Solid

Heat conduction in a non-metallic, crystalline material is due to transfer of the lattice vibrational energy from a higher temperature to a lower temperature. Solid state physicists describe the vibrational nature in terms of a particle-like quantity called a phonon. Just as electromagnetic propagation can described by photons, vibrational energy in a crystal can be described by phonons.

The resistance to thermal energy transfer by phonons in the lattice is largely due to two mechanisms:

1. Geometrical scattering of the phonons from the crystal boundary and also from lattice imperfections.

2. Scattering of phonons by other phonons.

It can be shown that if the forces between atomic entities in the crystal are those of a harmonic oscillator, i.e. if

$$F = -a\Delta x$$

there is no mechanism for collisions between different phonons and the thermal resistivity is determined only by collisions of a phonon with a crystal boundary or imperfection.

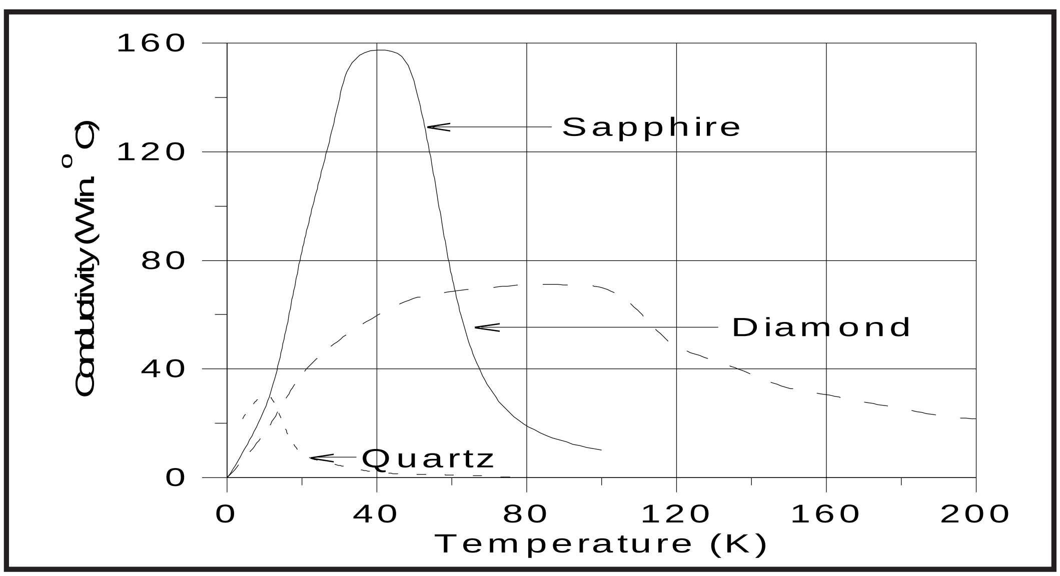
If the forces between atomic entities in the crystal are those of an anharmonic oscillator, e.g.

$$F = -a\Delta x - b\Delta x^2 - c\Delta x^3 - \cdots$$

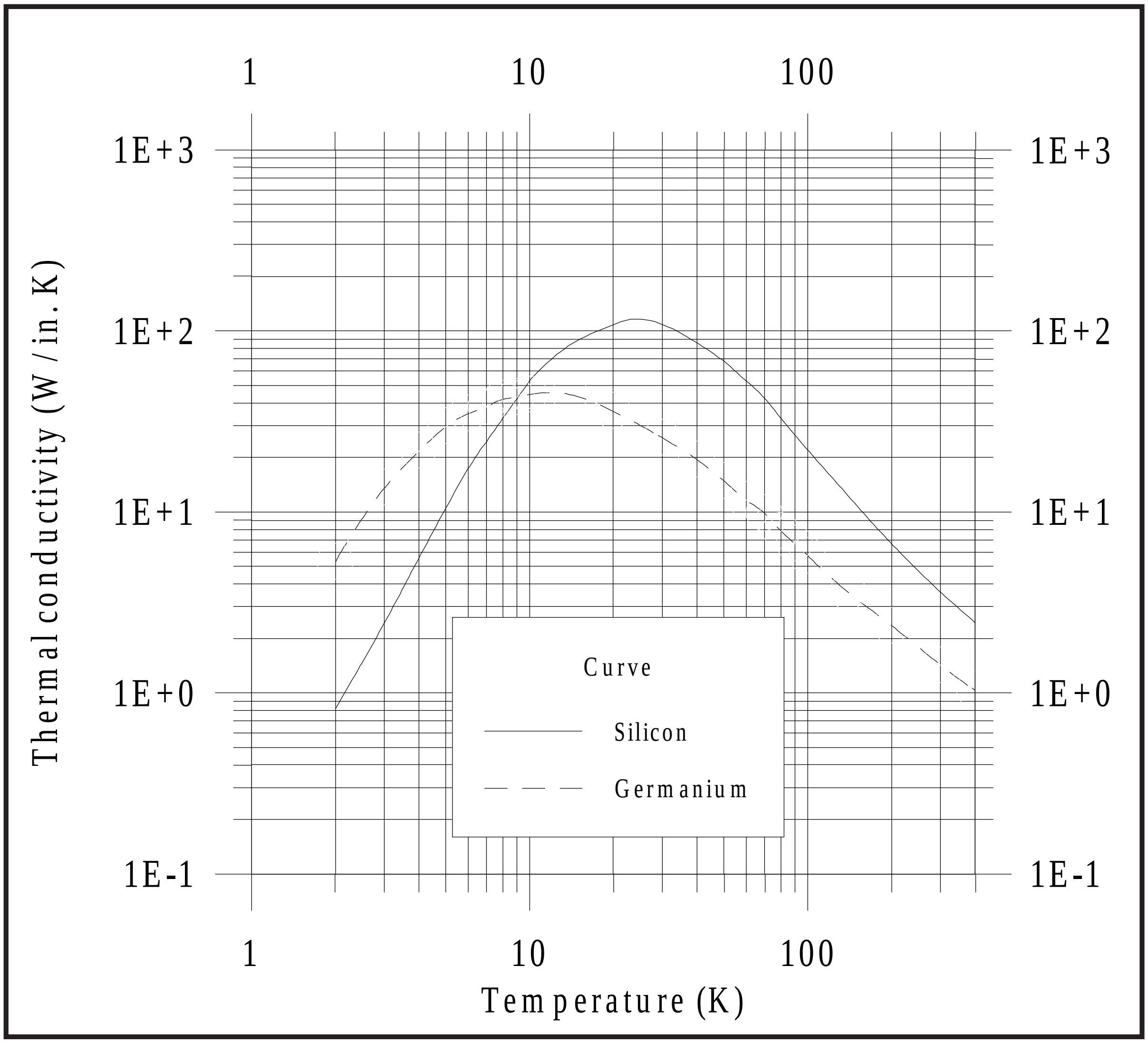
then there is a phonon-phonon interaction and a contribution to the thermal resistivity of the crystal. Mathematical formulation of this problem is very difficult for all but the simplest of structures. Never the less, the temperature dependence of non-metallic insulators follows something like

$$k = \frac{1}{\dfrac{\alpha}{T^3} + \beta T}$$ where α and β are constants.

The α/T^3 term is due to phonon scattering with the crystal boundary and lattice defects and is important in a rather low temperature region. The βT term is due to phonon - phonon scattering and is dominant in a higher temperature region, i.e. above the temperature where the thermal conductivity maxi - mum occurs.



Thermal Conductivity of Semiconductors



Ref.: Y.S. Touloukian, Y.S., R.W. Powell, C.Y. Ho, and P.G. Klemens, eds., Thermophysical Properties of Matter, TPRC Data Series, Vol. 1, 1970; undoped silicon, page 339, undoped germanium, page 131.

Thermal Resistance

Integrate Fourier's Law:

$$Q_k = -kA_k \, dT/dx$$

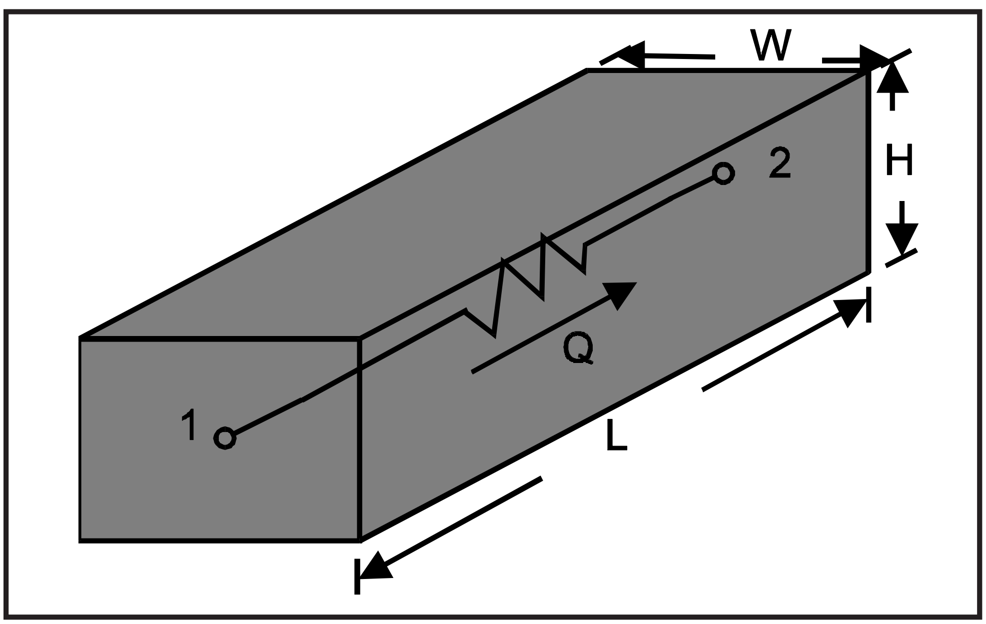
$$-\int_{T_1}^{T_2} dT = \int_{T_2}^{T_1} dT = Q_k \int_0^L \frac{dx}{kA_k}$$

$$T_1 - T_2 = R_k Q_k$$

$$\Delta T = R_k Q_k$$

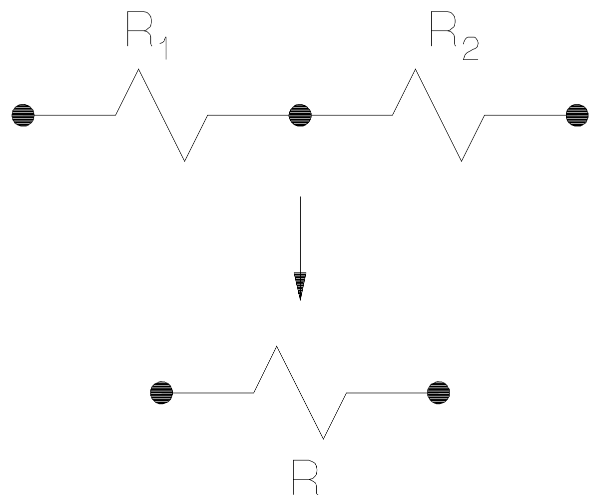
where the thermal resistance R_k is

$$R_k = \int_0^L \frac{dx}{kA_k}$$



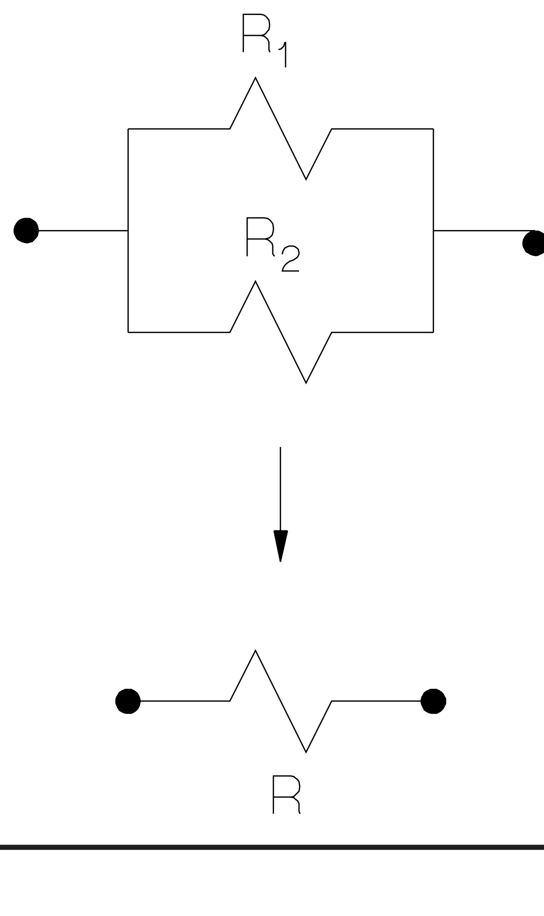
Addition of Resistances

Series



$$R = R_1 + R_2$$

Parallel



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Addition of Conductances

Conductance

$$C = \frac{1}{R}$$

Series Addition

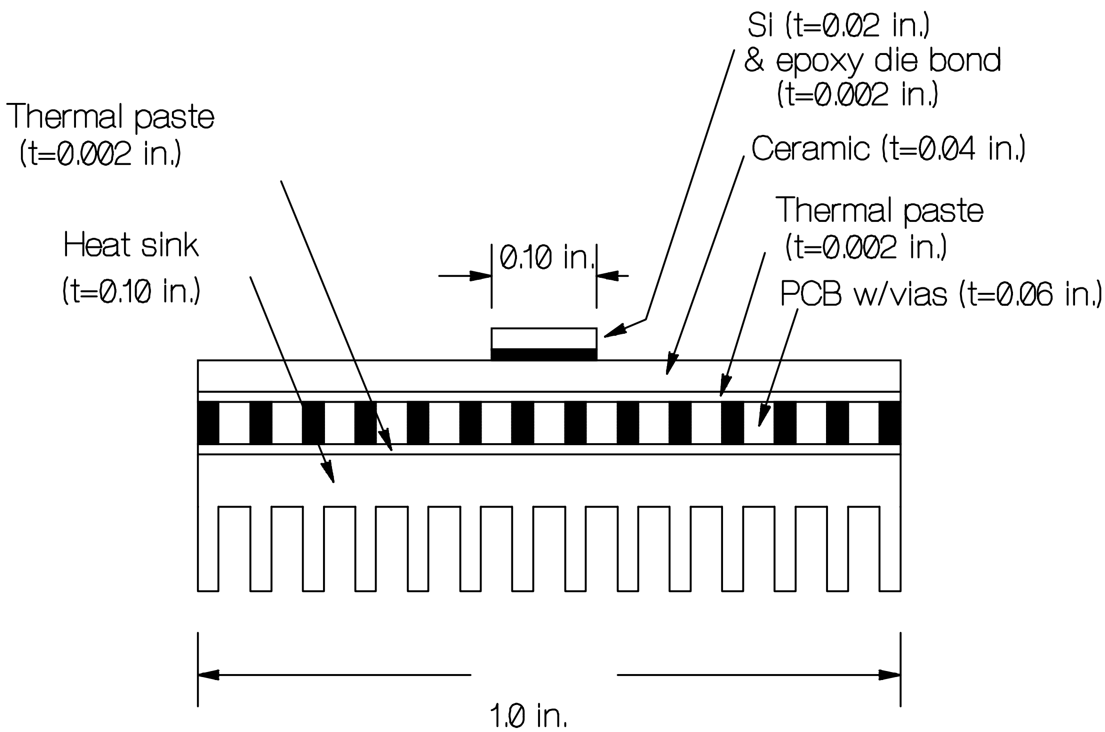
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Parallel Addition

$$C = C_1 + C_2$$

Example - Chip with Heat Sinking

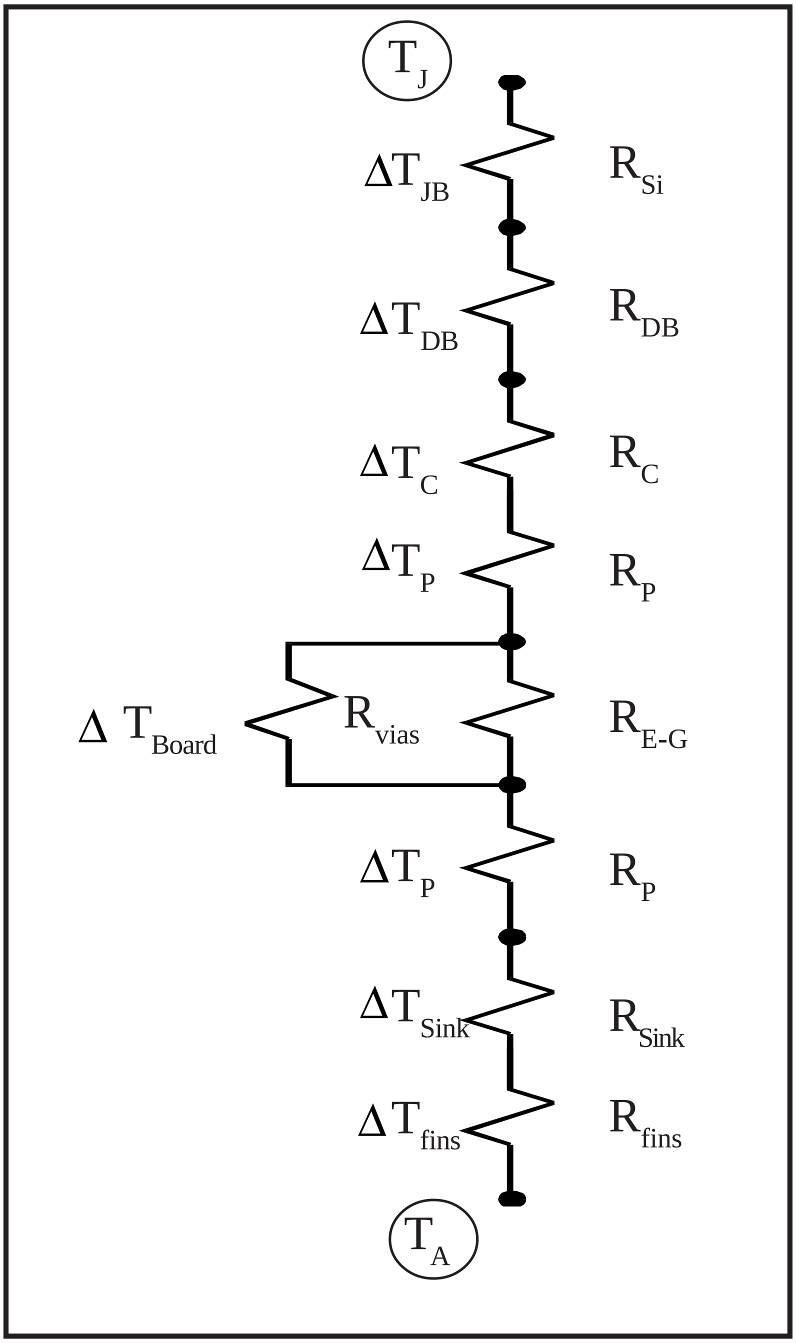
Geometry



Q = chip dissipation = 2.0 W

Twenty vias per inch

Thermal Circuit

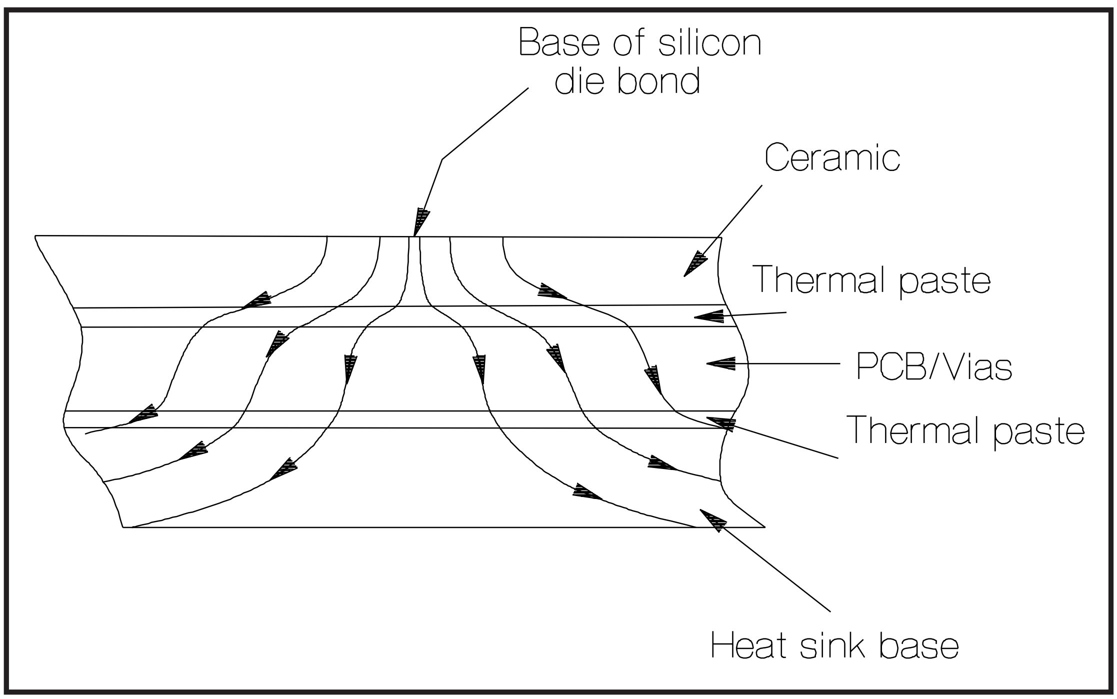


Calculations

$$R_{Si} = \frac{t}{kA} = \frac{0.02\ in.}{\left(2\ W/in.\,{}^{o}C\right)(0.1\ in.)^2} = 1.0\ {}^{o}C/W$$

$$R_{DB} = \frac{0.002}{(0.02)(0.1)^2} = 10.0\ {}^{o}C/W$$

Calculation of R through ceramic, paste, PCB w/vias, and into heat sink presents a problem because we don't know the cross-sectional area of the flow path.



An optimistic approximation -

$$A = (1.0\ in.)(1.0\ in.) = 1.0 in.^2$$

$$R_C = \frac{0.04\ in.}{(0.7\ W/in.\cdot{}^oC)(1.0\ in.^2)} = 0.06\ {}^oC/W$$

$$R_P = \frac{0.002\ in.}{(0.02\ W/in.\cdot{}^oC)(1.0\ in.^2)} = 0.10\ {}^oC/W$$

$$R_{vias} = \frac{0.06\ in.}{(20x20)(0.9\ W/in.\cdot{}^oC)\pi\left(\frac{0.01}{2}\ in.\right)^2} = 2.1\ {}^oC/W$$

$$R_{EG} = \frac{0.06\ in.}{\left(0.007\ W/in.\cdot{}^oC\right)\left[(1.0\ in.)^2 - (20x20)\pi\left(\frac{0.01\ in.}{2}\right)^2\right]}$$

$$= 8.6\ {}^oC/W$$

$$R_{board} = \frac{R_{vias} \cdot R_{EG}}{R_{vias} + R_{EG}}$$

$$= \frac{(2.1)(8.6)}{2.1 + 8.6} = 1.7$$

$$R_{Sink} = \frac{0.1\ in.}{5(1.0\ in.^2)} = 0.02\ {}^oC/W$$

A pessimistic approximation -

$$A = (0.1\,in.)(0.1\,in.) = 0.01\,in.^2$$

$$R_C = \frac{0.04\,in.}{\left(0.7\,W/in.\cdot{}^oC\right)\left(0.01\,in.^2\right)} = 5.7\,{}^oC/W$$

$$R_P = \frac{0.002\,in.}{\left(0.02\,W/in.\cdot{}^oC\right)\left(0.01\,in.^2\right)} = 10.0\,{}^oC/W$$

$$R_{vias} = \frac{0.06\,in.}{(2x2)\left(0.9\,W/in.\cdot{}^oC\right)\pi\left(\frac{0.01}{2}\,in.\right)^2} = 212.2\,{}^oC/W$$

$$R_{EG} = \frac{0.06\,in.}{\left(0.007\,W/in.\cdot{}^oC\right)\left[(0.1\,in.)^2 - 2x2\pi\left(\frac{0.01}{2}\,in.\right)^2\right]}$$

$$= 884.9\,{}^oC/W$$

$$R_{board} = \frac{R_{vias}\cdot R_{EG}}{R_{vias} + R_{EG}} = 171.2\,{}^oC/W$$

The heat sink can probably be expected to spread the heat quite well and is therefore assumed the same as before, i.e.

$$R_{Sink} = \frac{0.1\,in.}{\left(5.0\,W/{}^oC\cdot in.\right)\left(1.0\,in.^2\right)} = 0.02\,{}^oC/W$$

Mostly a topic for later discussion:

R_{fins}: assume $A_{fins} = 2$ x planar base area

$$= 2\ in.^2$$

An h for $v = 400\ ft./\text{min.}$ is $h \cong 0.02\ W/in.^2 \cdot {}^oC$

$$R_{fins} = 1/hA_{fins} = 1/(0.02)\left(2\ in.^2\right) = 25\ {}^oC/W$$

Results

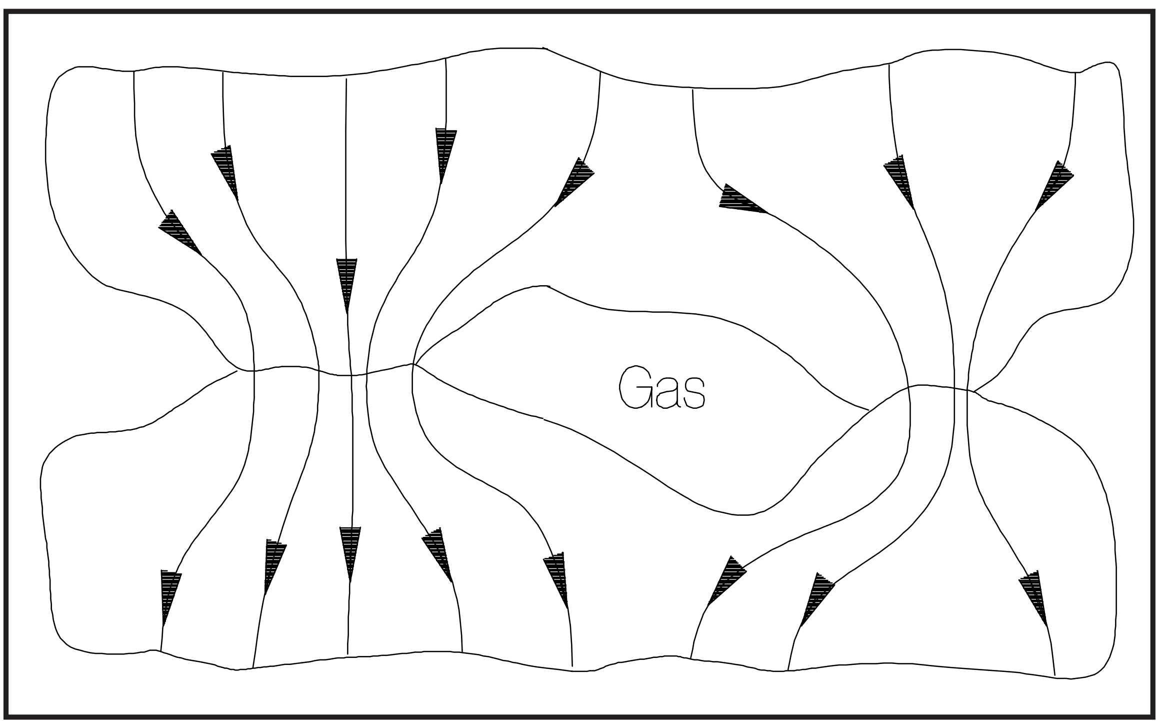
		OPTI	MISTIC	PESSI	MISTIC	ACTU	AL*
Material	**k (W/in.C)**	**R (C/W)**	**DT (C)**	**R (C/W)**	**DT (C)**	**R (C/W)**	**DT (C)**
Si	2.0	1.0	2.0	1.0	2.0	1.0	2.0
Die bond (epoxy)	0.02	10	20	10	20	10	20
Ceramic (Alumina)	0.7	0.06	0.1	5.7	11.4		
Paste	0.02	0.1	0.2	10	20		
E-G	0.007	8.6	---	884.9	---		
Vias (solder)	0.9	2.1	---	212.2	---		
PCB		1.7	3.4	171.2	342.3		
Paste	0.02	0.1	0.2	10	20		
Sink	5.0	0.02	0.04	0.02	0.04		
Fins	----	25	50	25.0	50.0	38.8	77.5
Total		38	76	232.9	465.8	49.8	99.5

* Reserved for later explanation.

Thermal Interface Resistance

Mechanical Joint

Conduction occurs through contact regions and also through gas gaps.



Some Definitions

Conductances:

$$h_c \equiv \text{contact conductance, } watts\,/\,area{\cdot}^{o}C$$

$$h_g \equiv \text{gap conductance, } watts\,/\,area{\cdot}^{o}C$$

$$A_a \equiv \text{apparent interface area, } area$$

$$C \equiv \text{interface conductance, } watts/^{o}C$$

$$C = (h_c + h_g)A_a$$

Resistances:

$$r_I \equiv \text{interface resistance, } area \cdot {}^oC / watt$$

$$R \equiv \text{interface resistance, } {}^oC / watt$$

$$r_I = \frac{1}{h_c + h_g}$$

$$R = \frac{1}{C}$$

Some Simple, but Useful Plots from TCEE:

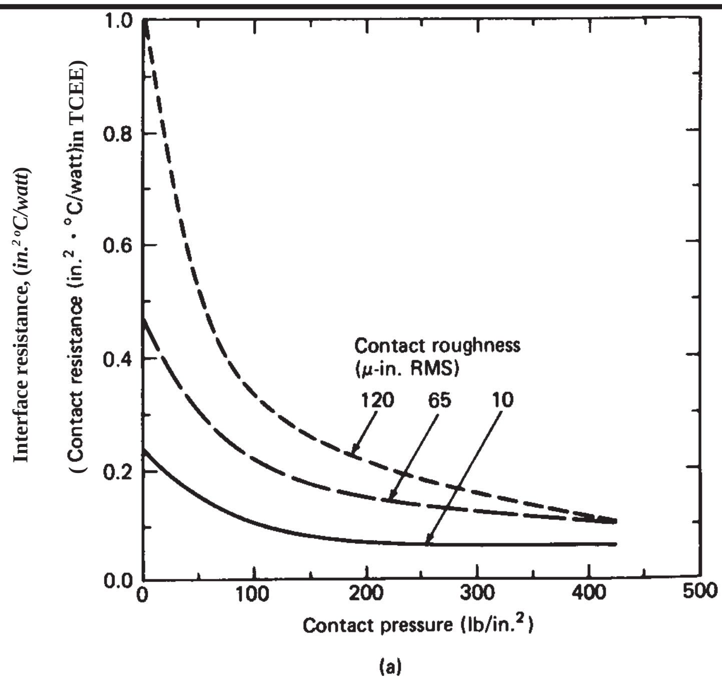


Fig. 1-4(a). ~~Contact~~ Interface resistance of 75S-T6 aluminum to aluminum joint with air interface. Mean temperature of joint = 93°C. Data plotted from [1].

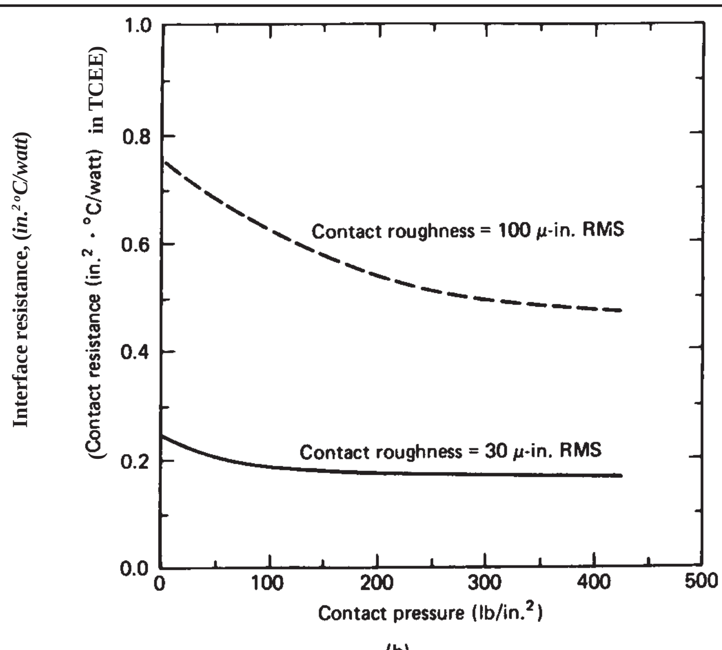


Fig. 1-4(b). ~~Contact~~ Interface resistance of stainless steel to stainless steel joint with air interface. Mean temperature of joint = 93°C. Data plotted from [1].

Application Example: TO-220 Power Transistor

TCEE Fig. 1-4(a) is applicable to a dry (no thermal paste) heat sink surface.

At least one major heat sink vendor indicates a contact roughness of about 30 $\mu \cdot inches$ for a power transistor base.

Then assuming a pressure of about *100 lb/in.*^{2}

$$r_I \cong 0.15 \left(in.^2 \cdot {}^oC\right) / watt$$

$$R_I = r_I / A_a = 0.15 / (0.5\, in. \cdot 0.5\, in.) = 0.6\ {}^oC / watt$$

The catalog of the same heat sink vendor indicates that a TO-220 transistor with a bare interface has a thermal resistance in the range of about *1* ^{o}*C/W.*

A More Complex Technique Following the Methods of Yovanovich and Antonetti

The total joint conductance is the sum of the contact and gap conductances.

$$C = C_c + C_g$$

The theory wherein Yovanovich et. al. develop a relation for C_c is complex and requires knowledge of numerous physical parameters*. Antonetti et. al. develop a relation that does not require the use of an *asperity slope* and is therefore more useful to engineers.** The developed contact conductance is

$$h_c = 4200 k_s R_a^{-0.257} \left(\frac{P}{H}\right)^{0.95} \qquad \left[\frac{watt}{m^2 \cdot K}\right]$$

where

k_s = harmonic mean thermal conductivity = $(2k_1 k_2)/(k_1 + k_2)$, $\left[\frac{watt}{m \cdot K}\right]$

R_a = combined average roughness = $\sqrt{R_{a_1}^2 + R_{a_2}^2}$, $[m]$

P = contact pressure, $[Pa]$

H = surface microhardness, $[Pa]$

---

* Bar-Cohen, A. and Kraus, A., editor, Advances in Thermal Modeling of Electronic Components and Systems, Vol. 1, Chpt.2, Hemisphere Publishing Co., New York, 1988.

** Antonetti, V.W., T.D. Whittle, and Simons, R.E., An Approximate Thermal Contact Conductance Correlation, Journal of Electronic Packaging, March 1993, Vol. 115, pp. 131-134.

Yovanovich et. al. have derived a gap conductance:

$$h_g = \frac{k_{g,\infty}}{1.184 R_a \left[-\ln\left(3.132 \frac{P}{H}\right)\right]^{0.547} + \alpha\beta\Lambda}$$

where α = accommodation parameter = $\dfrac{2 - \alpha_1}{\alpha_1} + \dfrac{2 - \alpha_2}{\alpha_2}$

for α_1 and α_2 are the accommodation coefficients at the two solid - gas interfaces.* Yovanovich recommends $\alpha_1 = 0.9$ for air at most clean metal surfaces.

β = fluid property parameter = $\dfrac{2\gamma}{[(\gamma + 1)\Pr]}$

for $\gamma \left[C_p / C_v\right]$ as the ratio of the specific heats, and Pr [dimensionless] is the Prandtl number.

$\gamma = 1.4$ and Pr = 0.7 for air in the vicinity of room temperature.

Λ = mean free path of the gas at some reference temperature $[T] = \Lambda_{g,\infty}\left(T_g / T_{g,\infty}\right)\left(P_{g,\infty} / P_g\right), [m]$

$\Lambda_{g,\infty} = 6.44 x 10^{-8}\ m$ at $T_{g,\infty} = 288\ K$.

* $\alpha \equiv$ ratio of actual mean-energy change of molecules colliding with a wall to the mean-energy change if molecules came into equilibrium with wall. See Present, R.D., Kinetic Theory of Gases, McGraw-Hill Book Co., 1958.

Application Example: TO-220 Power Transistor

Application of Yovanovich and Antonetti to Power Transistor on Aluminum: The following four pages are copies of Mathcad worksheets.

Input R1 in inches, P in psi, 10-26-94

Solid Properties: SI Units $\quad k_s := 157 \qquad H := 1180 \cdot 10^6$

Gas Properties: SI Units $\quad \alpha_1 := .9 \qquad \alpha_2 := .9 \qquad \gamma := 1.4$

$$Pr := .7 \qquad \Lambda := 6.44 \cdot 10^{-8} \cdot \left(\frac{373}{288}\right) \qquad k_g := 0.03$$

Contact Conductance Calculated in SI Units:

$$R_a(R_1) := \sqrt{2 \cdot \left(R_1 \cdot \frac{2.54}{100}\right)^2}$$

$$h_c(P, R_1) := 4200 \cdot k_s \cdot R_a(R_1)^{-0.257} \cdot \left[\frac{\left(\frac{P}{1.45 \cdot 10^{-4}}\right)}{H}\right]^{0.95}$$

Gap Conductance:

$$\alpha := \left(\frac{2-\alpha_1}{\alpha_1}\right) + \left(\frac{2-\alpha_2}{\alpha_2}\right) \qquad \beta := \frac{2\cdot\gamma}{(\gamma+1)\cdot Pr}$$

$$h_g(P,R_1) := \frac{k_g}{1.184 R_a(R_1)\cdot\left(-\ln\left(3.131\frac{P}{H}\right)\right)^{.547} + \alpha\cdot\beta\cdot\Lambda}$$

Total Interface Resistance in Units of sqin*C/watt:

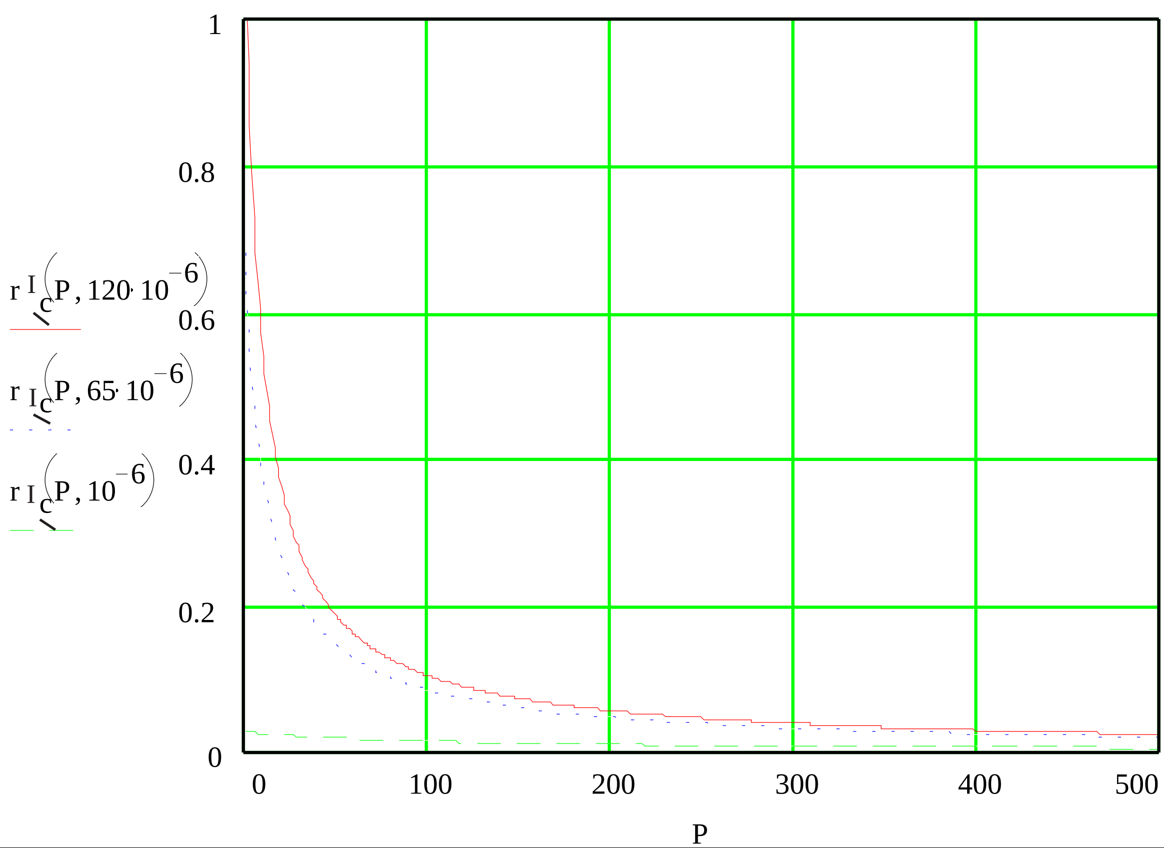
$$r_{c_I}(P,R_1) := \frac{1}{\left(h_c(P,R_1) + h_g(P,R_1)\right)\cdot\left(\frac{2.54}{100}\right)^2}$$

Total Resistance for a TO-220:

$$R(P,R_1) := \frac{r_{c_I}(P,R_1)}{0.5^2}$$

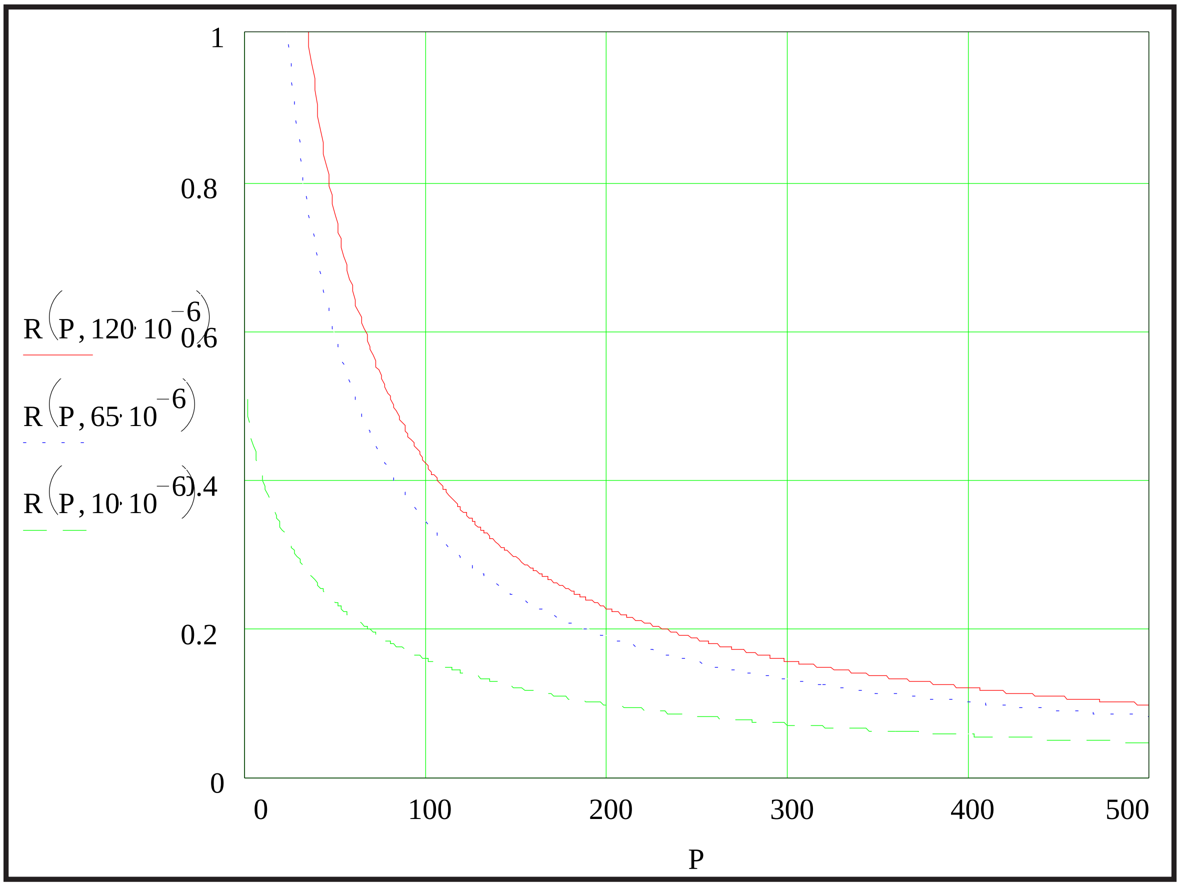
Plot of contact resistance for different roughnesses vs. pressure using Yovanovich and Antonetti:

$P := 1, 2..\ 500$



Comparison of above figure with TCEE Fig. 1-4(a) indicates the above prediction is considerably less than TCEE data.

Plot of thermal resistance for different roughnesses vs. pressure using Yovanovich and Antonetti:



Solutions to the One-Dimensional Heat Conduction Equation

The 1-D Heat Conduction Equation

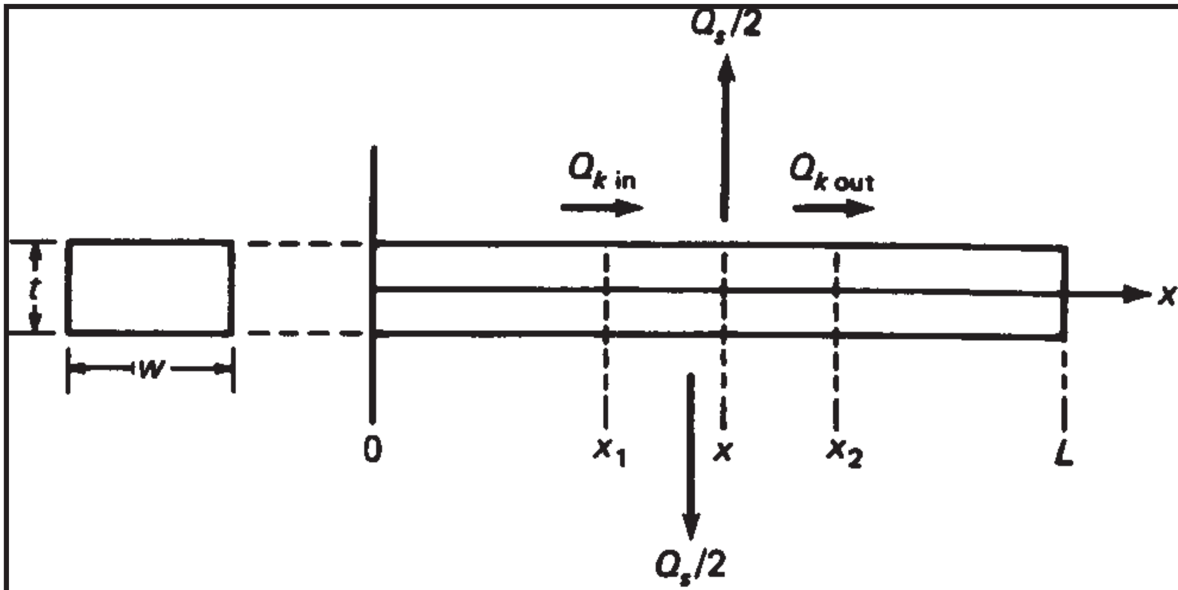


Fig. 4-1. Energy balance on an element $\Delta x = x_2 - x_1$.

Steady-state energy balance on element referenced to a zero temperature ambient:

$$\text{heat into } \Delta x \ - \ \text{heat out of } \Delta x = 0$$

$$\left[-kA_k \left.\frac{dT}{dx}\right|_{x_1} + Q_V \Delta x A_k \right] - \left[-kA_k \left.\frac{dT}{dx}\right|_{x_2} + 2h(w+t)\Delta x \left.T\right|_x \right] = 0$$

Dividing each term by $k\Delta x A_k$ and rearranging:

$$\frac{1}{\Delta x}\left[\left.\frac{dT}{dx}\right|_{x_2} - \left.\frac{dT}{dx}\right|_{x_1}\right] - \frac{2h(w+t)}{kA_k} T\Big|_x = -\frac{Q_V}{k}$$

Taking the limit of $\Delta x \to 0$:

$$\frac{d^2T}{dx^2} - \vartheta^2 T = -\frac{Q_V}{k} \qquad \text{TCEE E4.1}$$

where $\vartheta^2 = R_k \Big/ \left(L^2 R_S\right),\ R_k = L/(kA_k),\ R_S = 1\Big/\left(hA_S\right),$

$A_k = wt,\ A_S = 2(w+t)L$

The general solution is

$$T = c_1 \cosh \vartheta x + c_2 \sinh \vartheta x + \alpha / \vartheta^2,\ \alpha = Q_V / k$$

Uniform source - conduction:

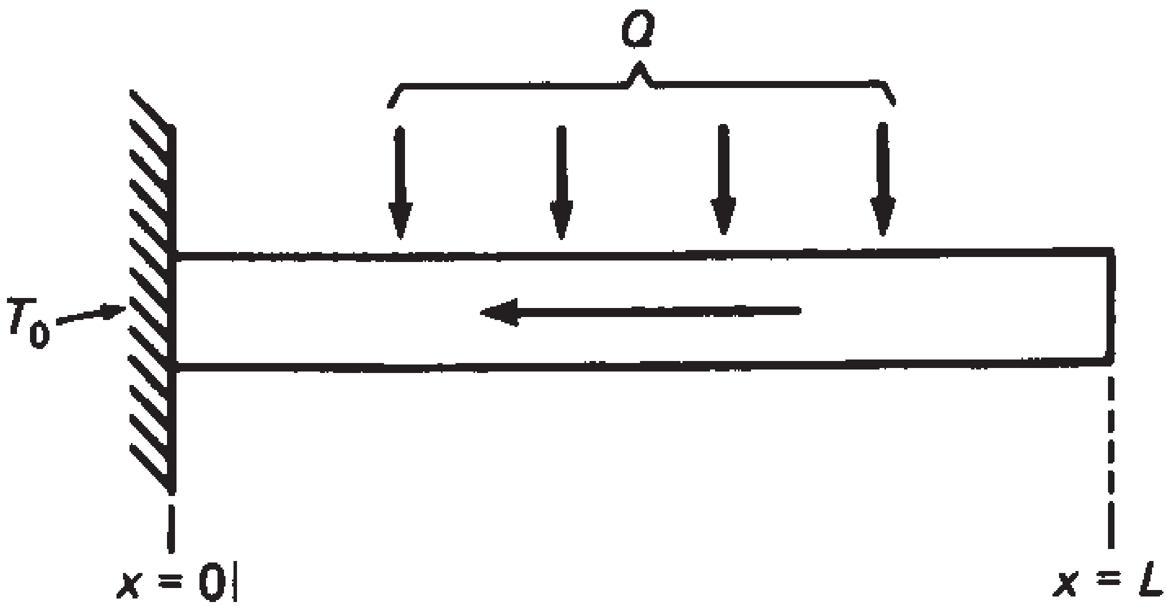


Fig. 4-2. Uniform source–conduction.

$$R = \frac{1}{2} R_k, \quad R_k = \frac{L}{kA_k}, \quad A_k = wt \qquad \text{TCEE E4.2}$$

Uniform source - conduction/convection-one end sinked:

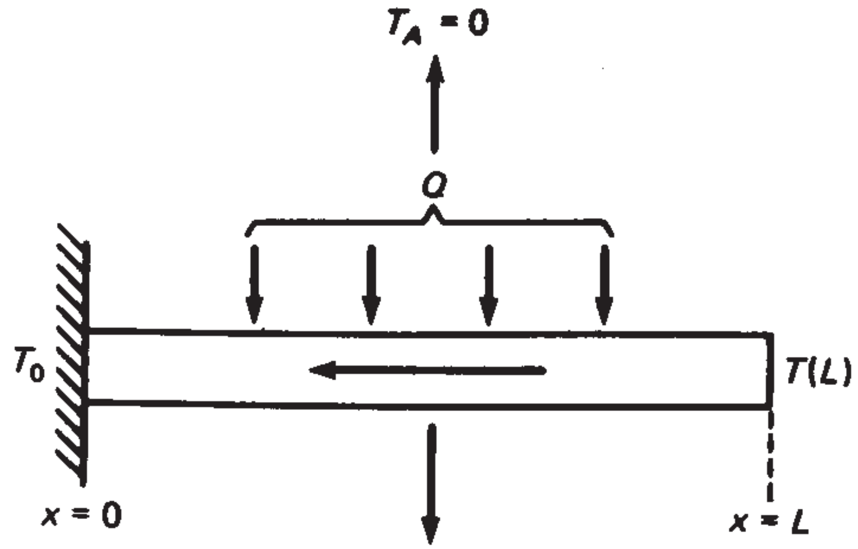


Fig. 4-4. Uniform source–conduction, convection–one end sinked.

$$R = \frac{\left(\dfrac{T_o}{Q} - R_S\right)}{\cosh\sqrt{\dfrac{R_k}{R_S}}} + R_S, \quad \frac{Q_o}{Q} = \left(\frac{T_o/Q}{R_S} - 1\right)\frac{\tanh\sqrt{\dfrac{R_k}{R_S}}}{\sqrt{\dfrac{R_k}{R_S}}}$$

TCEE E4.3, E4.4

$$R_k = \frac{L}{kA_k}, \quad R_S = \frac{1}{hA_S}, \quad A_k = wt, \quad A_S = 2wL$$

End source - conduction/convection:

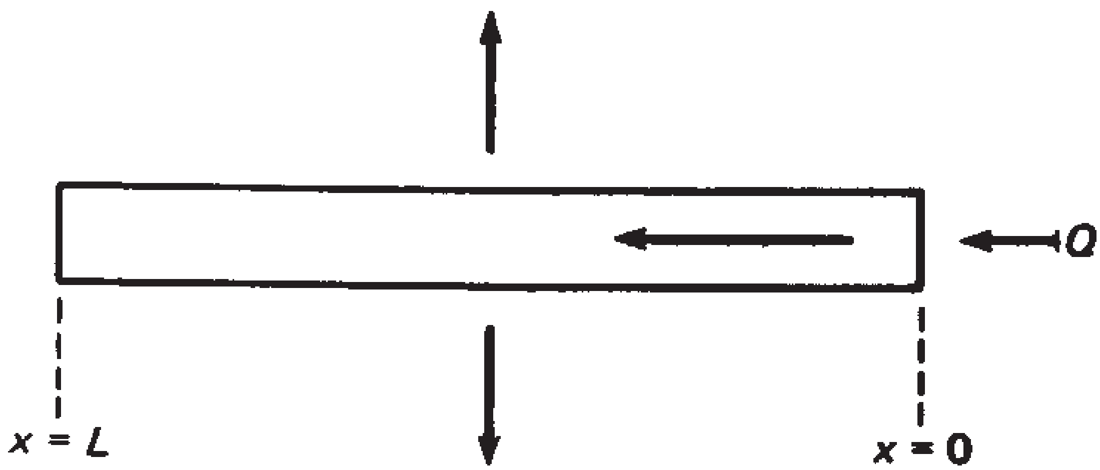


Fig. 4-7. End source–conduction, convection.

$$\frac{R}{R_S} = \sqrt{\frac{R_k}{R_S}} \coth \sqrt{\frac{R_k}{R_S}} \qquad \text{TCEE E4.5}$$

$$R_k = \frac{L}{kA_k}, \quad R_S = \frac{1}{hA_S}, \quad A_k = wt, \quad A_S = 2wL$$

End source - conduction/convection - opposite end sinked:

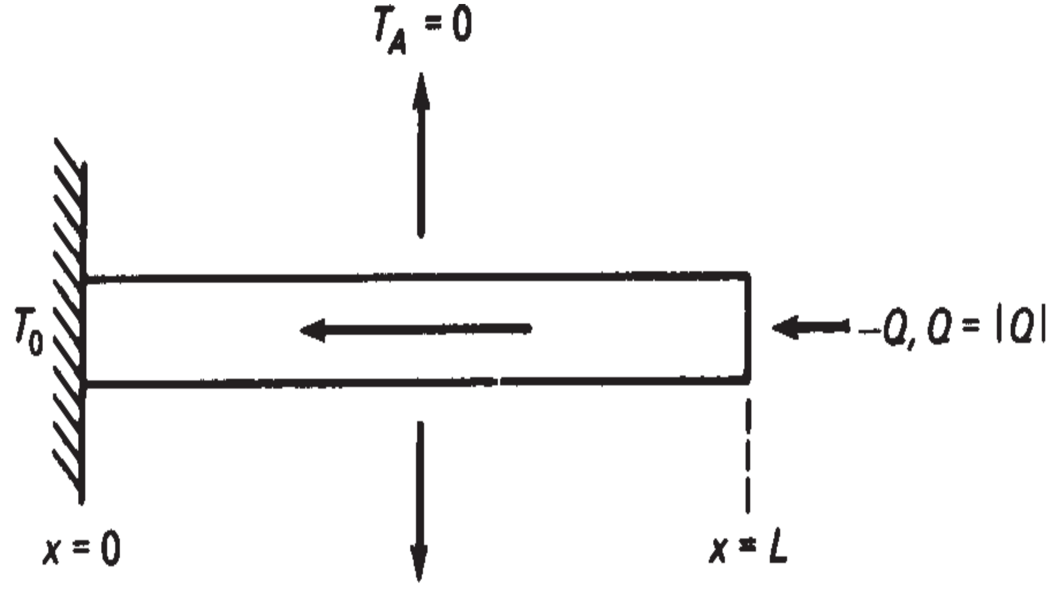


Fig. 4-10. End source–conduction, convection–opposite end sinked.

$$\frac{T(L)/Q}{R_S} = \frac{\left(\frac{T_o/Q}{R_S} + \sqrt{\frac{R_k}{R_S}} \sinh \sqrt{\frac{R_k}{R_S}}\right)}{\cosh \sqrt{\frac{R_k}{R_S}}}$$

TCEE E4.6, E4.7

$$\frac{Q_o}{Q} = \frac{\tanh \sqrt{\frac{R_k}{R_S}}}{\sqrt{\frac{R_k}{R_S}}} \left(\frac{T_o/Q}{R_S} - \frac{\sqrt{\frac{R_k}{R_S}}}{\sinh \sqrt{\frac{R_k}{R_S}}} \right)$$

$$R_k = \frac{L}{kA_k}, \quad R_S = \frac{1}{hA_S}, \quad A_k = wt, \quad A_S = 2wL$$

A couple of application scenarios:

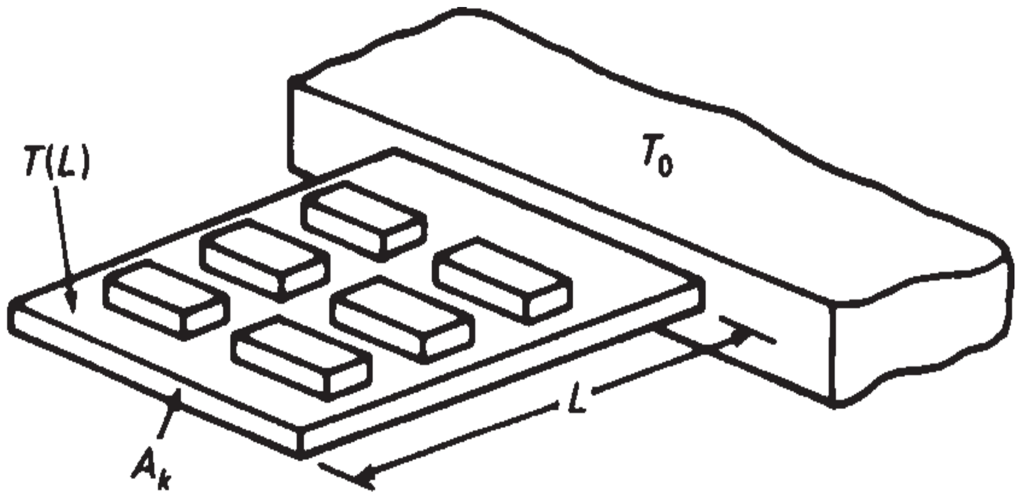


Fig. 4-3. Aluminum core board with negligible air cooling.

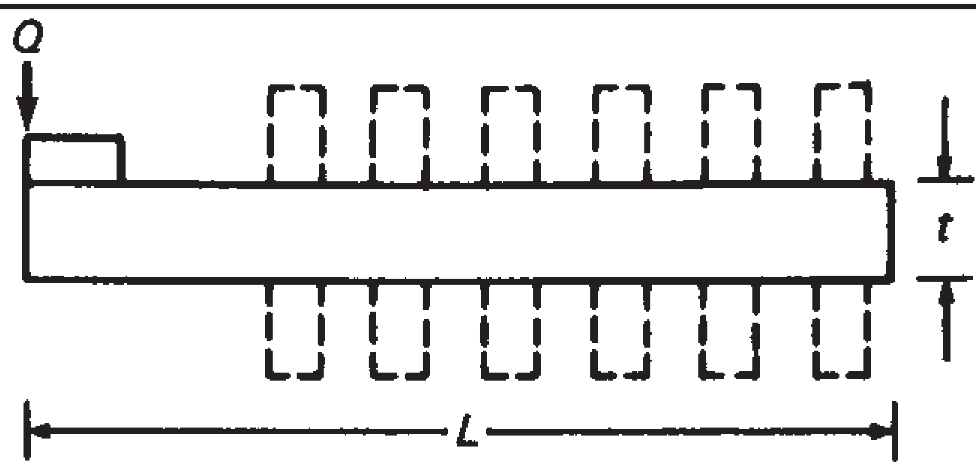


Fig. 4-9. Simple heat sink or chassis element.

Conducting/convecting disk or rectangle:

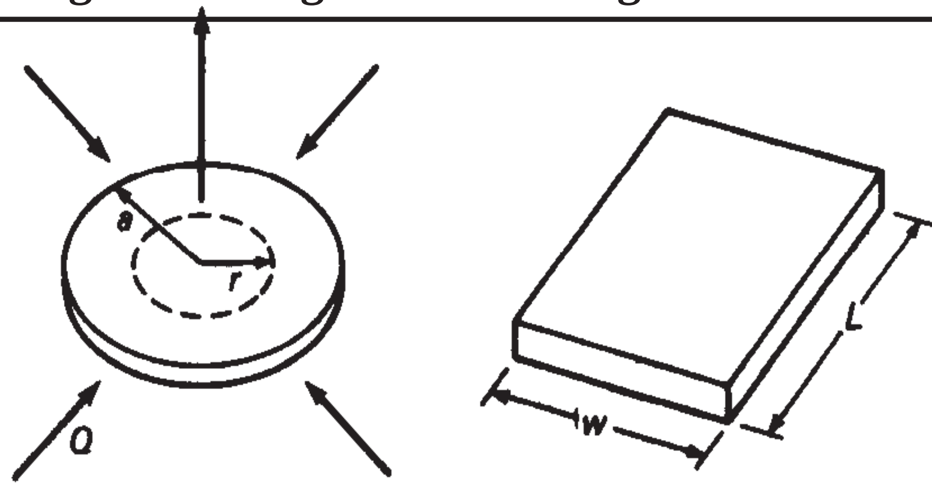


Fig. 4-13. Disk–conduction, convection.

$$\frac{R}{R_s} = \sqrt{\frac{R_{SQ}}{\pi R_S}}\,\frac{I_o\left(\sqrt{R_{SQ}/\pi R_S}\right)}{2I_1\left(\sqrt{R_{SQ}/\pi R_S}\right)}$$ TCEE E4.8

$$R_{SQ} = \frac{1}{kt}, \quad R_S = \frac{1}{hA_S}, \quad \text{use } A_S = wL$$

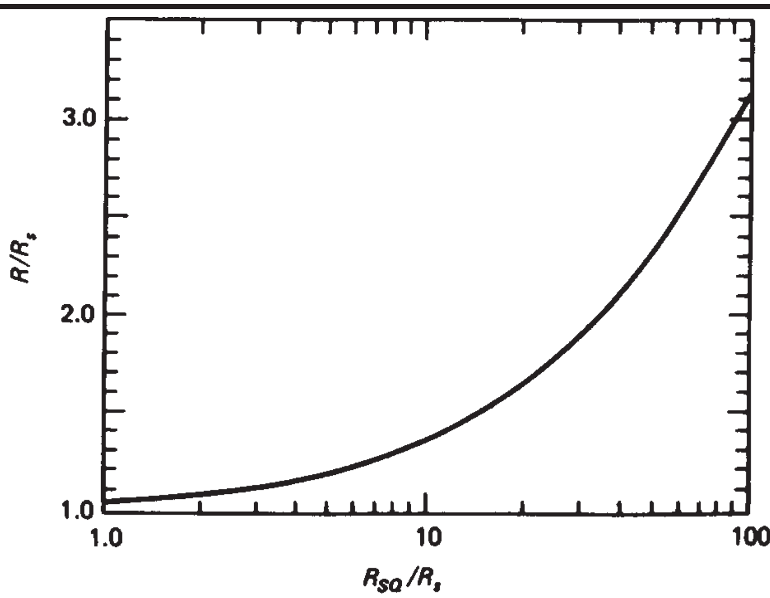
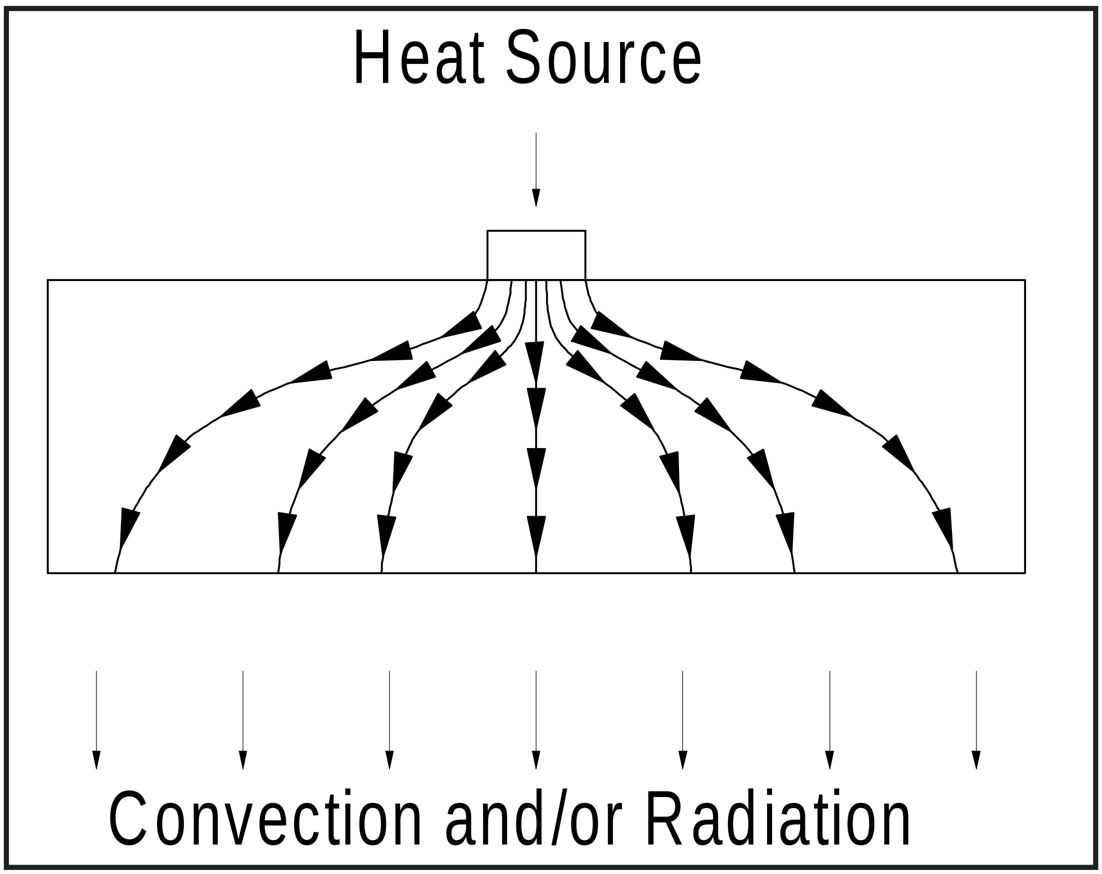


Fig. 4-14. Disk–conduction, convection, peripheral source. From [18], copyright © 1976 by the Institute of Electrical and Electronics Engineers, Inc. Reprinted by permission of the publisher.

Spreading/Constriction Resistance

The Problem



Some Terminology

The non-spreading contributions are usually a uniform conduction term (from source plane to base of geometry) plus a uniform Newtonian cooling term (from base of geometry to reference temperature). The Newtonian cooling term is usually due to convection and radiation.

$$R = R_U + R_{Sp}$$

$R \equiv$ Total thermal resistance from source to reference

$R_U \equiv$ Non-spreading contributions

$R_{Sp} \equiv$ Spreading or constriction contribution

$$\Delta T = \left(R_U + R_{Sp}\right)Q = \Delta T_U + \Delta T_{Sp}$$

$\Delta T \equiv$ Total temperature drop from source to reference

$\Delta T_U \equiv$ Non-spreading contributions

$\Delta T_{Sp} \equiv$ Spreading or constriction contribution

The uniform conduction resistance term is of course $R_k = \int dx / \left(kA_C\right) = L / \left(kA_C\right)$ for a thickness L and a cross-sectional area A_C. The Newtonian cooling term, if any, is $R_S = 1/\left(hA_S\right)$ for a total heat transfer coefficient h and a surface area A_S. If both contributions are relevant, then

$$R_U = R_k + R_S$$

Some Miscellaneous Values

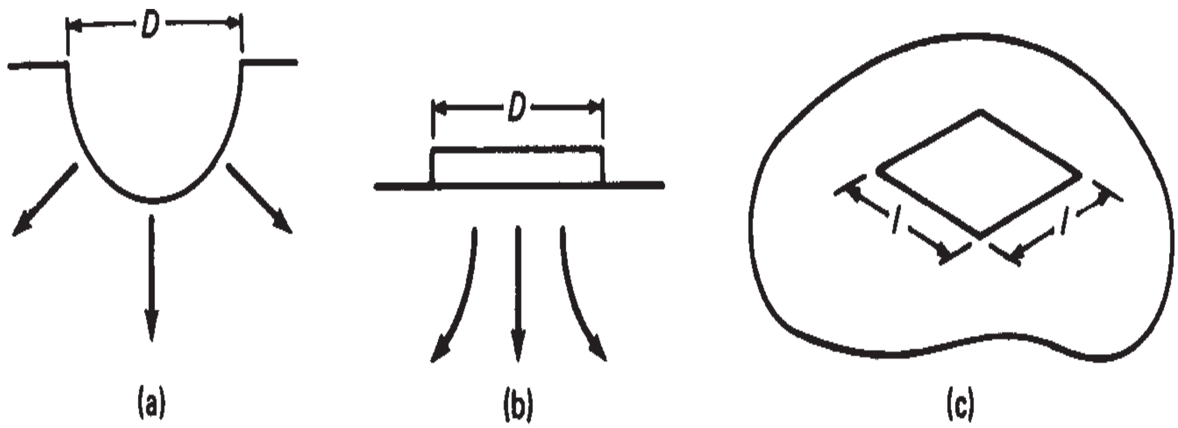


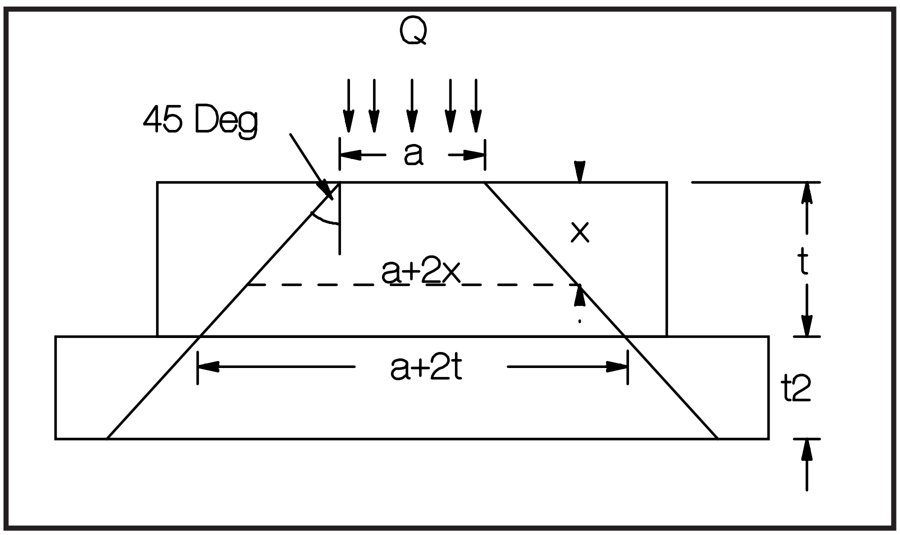
Fig. 4-15. Common geometry for spreading resistance in semi-infinite media.

Fig. 4-15	R_{sp}	Remark
(a)	$1/\pi Dk$	
(b)	$1/2Dk$	Ref. 19 uniform T
	$16/3\pi^2 Dk$	Ref. 19 uniform Q, ave. T
(c)	$1.1/2lk$	Ref. 20

Case (a) is included only to caution the reader that it should not be used to simulate planar sources. Case (c) may be extended to non-square sources by consulting the original references.

The Fixed Spreading Angle Method

This method is not generally recommended, but is included here because it is seen quite often in the literature and it is therefore important to understand its limitations. The relavant geometry (shown for two layers) is:



There are two general variations of the basic fixed angle method, (1) the average angle method, and (2) the integrated method:

The Average Area Method

The defining formula for thermal resistance is

$$R_k = \int \frac{dx}{kA}$$

If only the first thickness *t* is considered for a square source and an average area $\bar{A}$ is used, the resistance is then

$$R = \int \frac{dx}{k\bar{A}} = \frac{t}{k\bar{A}}$$

and since the average area is

$$\bar{A} = \frac{1}{t}\int_0^t (a + 2x\tan\alpha)dx = \left(a^2 + 2at\tan\alpha + \frac{4}{3}t^2\tan^2\alpha\right)$$

$$R = \frac{t}{k\left(a^2 + 2at\tan\alpha + \frac{4}{3}t^2\tan^2\alpha\right)}$$

A normalized, dimensionless resistance may be defined to be

$$kaR = \frac{1}{\left(\frac{a}{t} + 2\tan\alpha + \frac{4}{3a}t\tan^2\alpha\right)}$$

If

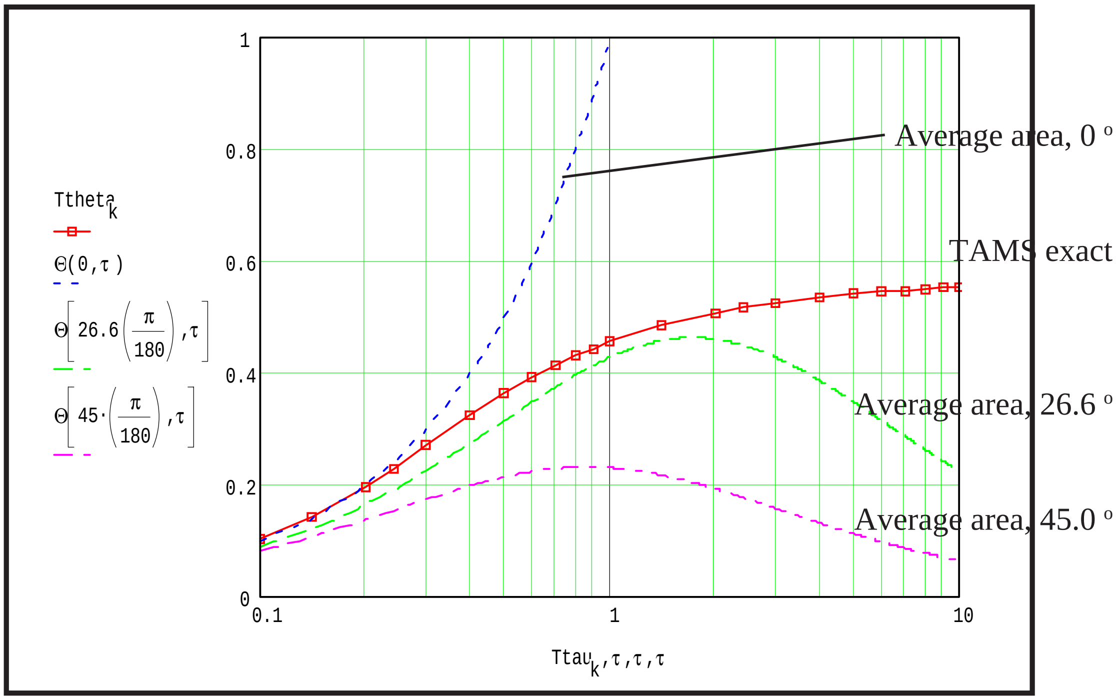
$$\Theta_{Ave.\,Area} = kaR, \qquad \tau = t/a$$

$$\Theta_{Ave.\,Area} = \frac{1}{\left(\frac{1}{\tau} + 2\tan\alpha + \frac{4}{3}\tau\tan^2\alpha\right)}$$

The following values of theta were calculated "exactly" using the TAMS program for a large substrate (compared to source size):

$$\text{Ttau} := \begin{bmatrix} .1 \\ .14 \\ .2 \\ .24 \\ .3 \\ .4 \\ .5 \\ .6 \\ .7 \\ .8 \\ .9 \\ 1.0 \\ 1.4 \\ 2.0 \\ 2.4 \\ 3.0 \\ 4.0 \\ 5.0 \\ 6.0 \\ 7.0 \\ 8.0 \\ 9.0 \\ 10.0 \end{bmatrix} \qquad \text{Ttheta} := \begin{bmatrix} 0.102 \\ 0.142 \\ 0.196 \\ 0.228 \\ 0.270 \\ 0.324 \\ 0.364 \\ 0.392 \\ 0.414 \\ 0.432 \\ 0.444 \\ 0.456 \\ 0.486 \\ 0.508 \\ 0.518 \\ 0.526 \\ 0.536 \\ 0.542 \\ 0.546 \\ 0.548 \\ 0.55 \\ 0.552 \\ 0.552 \end{bmatrix}$$

Plot of theta for different fixed angles vs. the exact theta$_k$



The "common lore" is that 45º is the most reasonable spreading angle, but R.F. David (TCEE Reference 39) suggested that 26.6º is the optimum angle.

The 26.6º angle method is pretty good for $\tau = t / a \leq 1$.

The problem is that the fixed angle method suggests that the resistance $\Theta_{Ave.\ Area} \xrightarrow[\tau \to \infty]{} 0$, which is incorrect theory.

The Integrated Resistance Method

Again, if only a single layer is considered, the resistance for a square source is

$$R_k = \frac{1}{k}\int_0^t \frac{dx}{A} = \frac{1}{k}\int_0^t \frac{dx}{(a+2x\tan\alpha)^2}$$

$$= -\frac{1}{2k}\left[\frac{1}{(a+2x\tan\alpha)}\right]_0^t = -\frac{1}{2k}\left[\frac{1}{2(a+2t\tan\alpha)} - \frac{1}{a}\right]$$

$$= \frac{t}{ka(a+2t\tan\alpha)}$$

A normalized, dimensionless resistance is $\Theta = kaR,\ \tau = t/a$, which can be statistically best fit to the "exact" TAMS program data to find the best angle.

$$\Theta = \frac{1}{\frac{1}{\tau} + 2\tan\alpha}$$

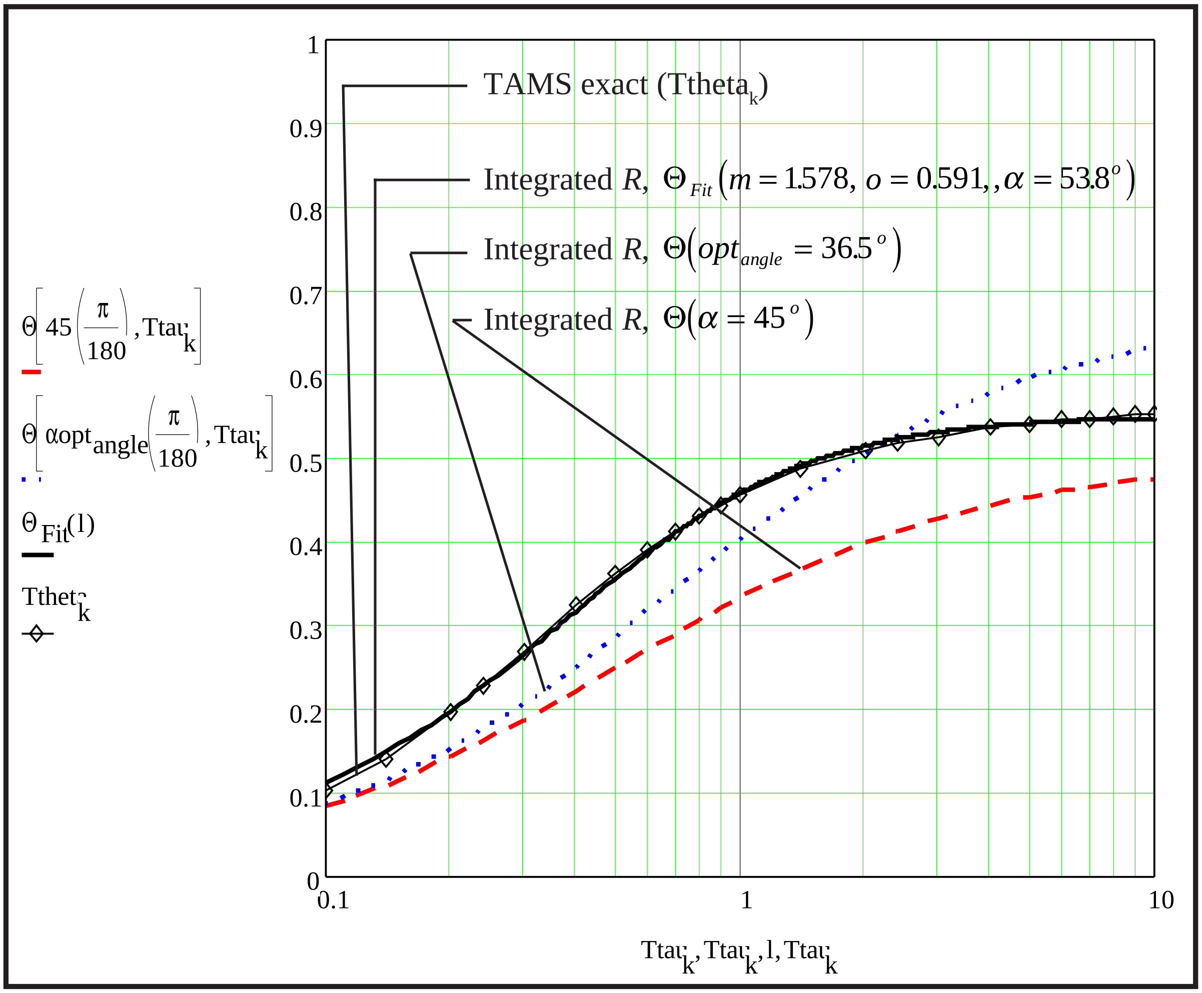
Another form of theta that can be statistically fit to the "exact" TAMS program data is:

$$\Theta_{Fit.} = \frac{1}{\left[\left(\frac{1}{\tau}\right)^m + 2\tan\alpha\right]^o}$$

Plot of Θ, Θ_{Fit}:

For Θ, the best α (opt. angle) = 36.5°

For Θ_{Fit}, the best m=1.578, o=0.591, $\alpha = 53.8^o$



The 36.5° fixed angle fit is very poor over most of the range evaluated, but the popular 45° fixed angle fit is even worse. The fit that optimizes the exponents m, o, as well as the angle is a very good fit, but is no longer a fixed angle method that can be applied to multi-layer situations.

45° Angle Model Considered in More Detail Due to Popularity

The square source, 45 degree angle formula is easily stated using the fixed angle result previously derived:

$$R_k = \left.\frac{t}{ka(a+2t\tan\alpha)}\right|_{\alpha=45^o} = \frac{t}{ka(a+2t)}$$

If only the first thickness *t* is considered for a rectangular source with dimensions *axb*,

$$R_k = \frac{1}{k}\int_0^t \frac{dx}{A} = \frac{1}{k}\int_0^t \frac{dx}{k(a+2x)(b+2x)}$$

$$= \frac{1}{2k}\left[\frac{\ln(b+2x)-\ln(a+2x)}{(a-b)}\right]_0^t$$

$$= \frac{1}{2k}\left[\frac{\ln(b+2t)-\ln(a+2t)-\ln(b)+\ln(a)}{(a-b)}\right]$$

$$= \frac{1}{2k}\left[\frac{\ln\left(\frac{b+2t}{a+2t}\right)+\ln\left(\frac{a}{b}\right)}{(a-b)}\right]$$

$$= \frac{1}{2k(a-b)}\ln\left[\left(\frac{a}{b}\right)\left(\frac{b+2t}{a+2t}\right)\right]$$

Multilayer problems are managed by successively applying the appropriate formula to each layer.

Spreading Resistance Following Yovanovich and Antonetti*:

Yovanovich and Antonetti discuss thermal spreading resistance R_{Sp} in terms of a dimensionless resistance Ψ_{Sp} such that

$$\Psi_{\infty} = k\sqrt{A_s}R_{Sp}$$

$$R_{Sp} = \frac{\Psi_{\infty}}{k\sqrt{A_s}}$$

where the subscript ∞ indicates a source area A_s on a semi-infinite body. The resistance is defined as the temperature difference between average source surface temperature and some reference temperature, divided by the total heat transfer from the source.

Yovanovich and Antonetti claim that the resistance is a weak function of the source shape and that, after examining several different source shapes,

$$\Psi_{\infty} = 0.467 \pm 5\%$$

* Yovanovich, M. M. and Antonetti, V.W., Advances in Thermal Modeling of Electronic Components and Systems, Vol. 1, Chpt. 2, Hemisphere Publishing Co., A. Bar-Cohen and A.D. Kraus, editors, New York, 1988.

Spreading Resistance - One and Two Dimensional Solutions

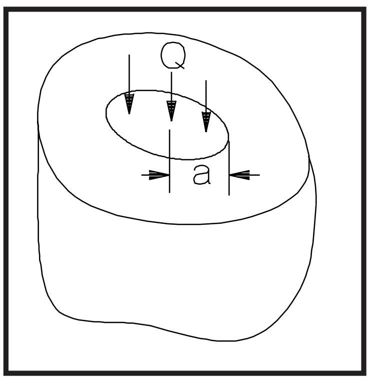
$$\Delta T = \Delta T_U + \Delta T_{Sp}$$

$$\Delta T \equiv \text{surface temperature rise above ambient at source center}$$

$$\Delta T_U \equiv \text{non-spreading contributions}$$

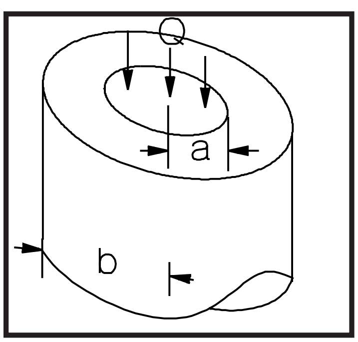
$$\Delta T_{Sp} \equiv \text{spreading resistance contribution}$$

Mikic, B.B., 1966, "Thermal Contact Resistance," Sc.D. Thesis, Dept. of Mech. Eng, MIT, Cambridge, Mass.:



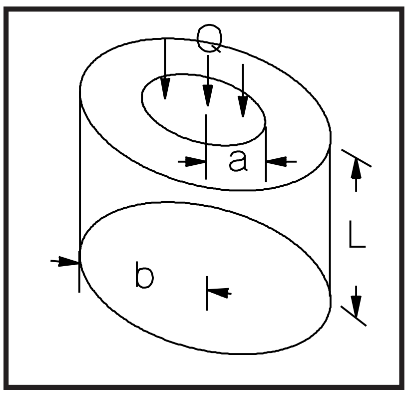
$$\Delta T_{Sp-Ave} = \frac{Q}{2\sqrt{\pi} a k}$$

Cooper, M.G., Mikic, B.B., and Yovanovich, M.M., 1969, "Thermal Contact Conductance," *Int. J. Heat Mass Transfer,* Vol. 12, pp. 279-300:



$$\Delta T_{Sp-Ave} = \frac{Q}{2\sqrt{\pi} a k}\left(1 - \frac{a}{b}\right)^{3/2}$$

Kennedy, D.P., 1960, "Spreading Resistance in Cylindrical Semiconductor Devices," *Journal of Applied Physics*, Vol. 31, pp. 1490-1497:



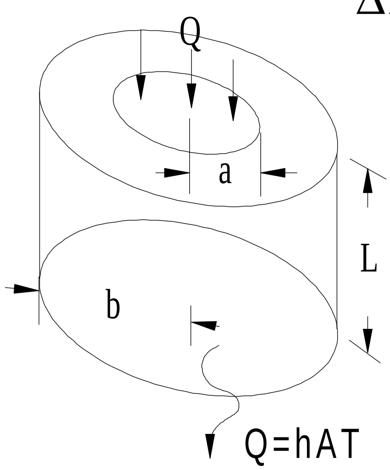
$$\Delta T_{Sp-Ave} = \left(\frac{4Q}{\pi a k}\right)\left(\frac{b}{a}\right)\sum_{m=0}^{\infty} \tanh\left(\lambda_m \frac{L}{b}\right) \bullet \frac{J_1^2\left[\lambda_m\left(\frac{a}{b}\right)\right]}{\lambda_m^3 J_o^2(\lambda_m)}$$

$$\Delta T_U \equiv QL \big/ \left(k\pi b^2\right)$$

and λ_n is such that the Bessel function $J_1(\lambda_n) = 0$. (*a/b=1* implies at *r=b*).

The cylinder has an isothermal base boundary condition

Lee, S., Song, V.A.S., Moran, K.P. "Constriction/Spreading Resistance Model for Electronics Packaging," *ASME/JSME Thermal Engineering Conference*, Vol. 4, ASME 1995, pp. 199-206:



$$\Delta T_{Sp-Ave} = \left(\frac{4Q}{\pi a k}\right)\left(\frac{b}{a}\right) \cdot$$

$$\sum_{m=0}^{\infty} \left\{ \frac{J_1^2\left[\lambda_m\left(\frac{a}{b}\right)\right]}{\lambda_m^3 J_o^2(\lambda_m)} \right\} \left\{ \frac{\tanh\left(\lambda_m \frac{L}{b}\right) + \frac{\lambda_m}{Bi}}{1 + \frac{\lambda_m}{Bi}\tanh\left(\lambda_m \frac{L}{b}\right)} \right\}$$

$$Bi \equiv \text{Biot number} \equiv \frac{hb}{k}$$

and λ_n is such that the Bessel function $J_1(\lambda_n) = 0$.

Also, $\Delta T_U = \dfrac{L}{k\pi b^2} + \dfrac{1}{h\pi b^2}$.

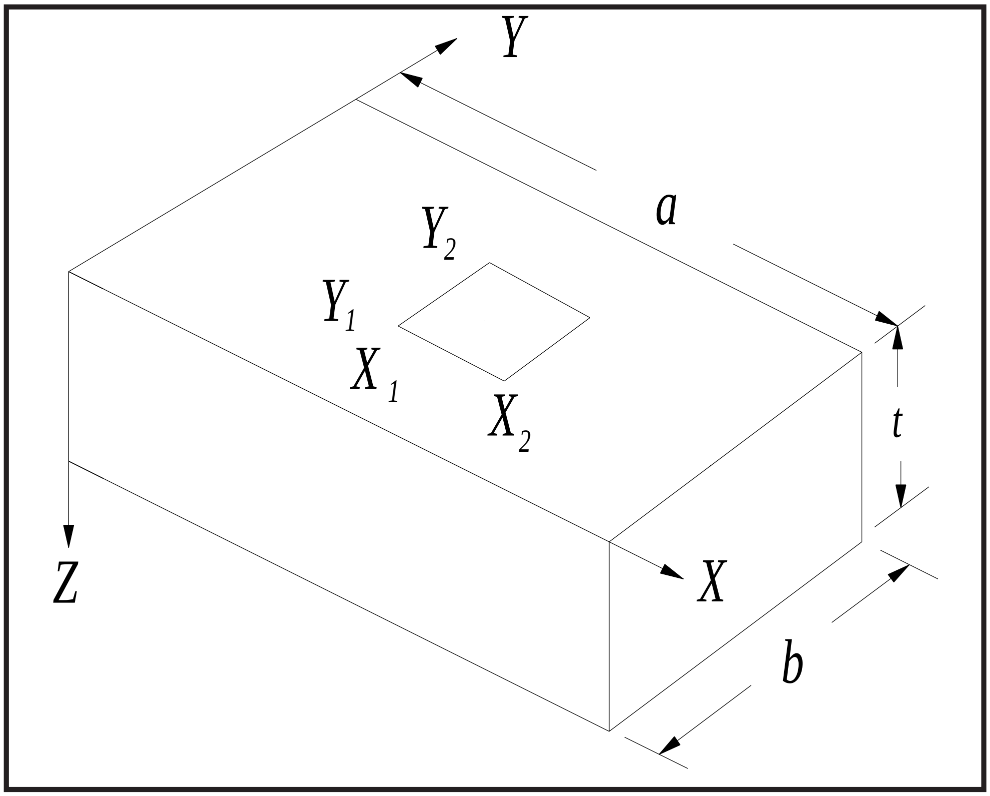
Lee, et. al. also claim (but have not published the method of obtaining) simple approximations:

$$\Delta T_{Sp-Ave} = \left(\frac{Q}{2ka\sqrt{\pi}}\right)\left(1 - \frac{a}{b}\right)^{3/2} \Phi_c$$

$$\Delta T_{Sp-Max} = \left(\frac{Q}{ka\pi}\right)\left(1 - \frac{a}{b}\right)\Phi_c$$

$$\text{where } \Phi_c = \frac{\tanh\left(\lambda_c \frac{t}{b}\right) + \frac{\lambda_c}{Bi}}{1 + \frac{\lambda_c}{Bi}\tanh\left(\lambda_c \frac{t}{b}\right)} \text{ with } \lambda_c = \pi + \frac{1}{\sqrt{\pi}\left(\frac{a}{b}\right)}$$

Ellison, Unpublished Theory:



Model assumes insulated top surface (*Z=0*) and Newtonian cooling at lower surface (*Z=t*).

The differential equation to be solved is the three-dimensional heat conduction equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = -\frac{Q_V}{k}$$

Boundary Conditions(Insulated) at Edges:

$$k\frac{\partial T}{\partial x} = 0;\ x = 0, a$$

$$k\frac{\partial T}{\partial y} = 0;\ y = 0, b$$

Boundary Conditions (Radiation/Convection) at Bottom Surface:

$$k\frac{\partial T}{\partial z} = 0;\ \ z = 0$$

$$k\frac{\partial T}{\partial z} + hT = 0; \qquad z = t$$

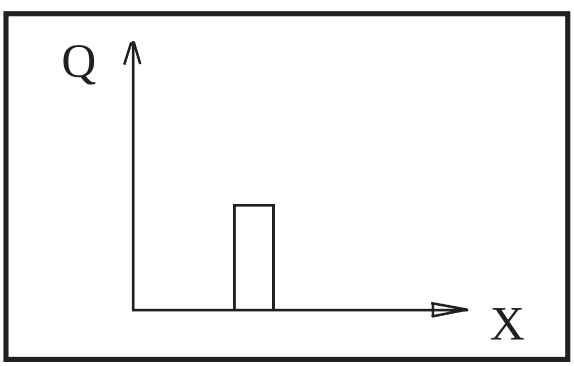
Temperature Representation:

$$T(r) = \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \varepsilon_l \varepsilon_m \Psi_{lm}(z) \cos\left(\frac{l\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right)$$

$$\varepsilon_l = \begin{bmatrix} 1/2, & l = 0 \\ 1, & l \neq 0 \end{bmatrix} \qquad l = 0,1,2\ldots$$

$$\varepsilon_m = \begin{bmatrix} 1/2, & m = 0 \\ 1, & m \neq 0 \end{bmatrix} \qquad m = 0,1,2\ldots$$

Source Representation;



$$Q_V(r) = \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \varepsilon_l \varepsilon_m \varphi_{lm}(z) \cos\left(\frac{l\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right)$$

The heat source is presumed to be uniform over the source from x to $x+\Delta x$ and y to $y+\Delta y$. The Fourier coefficients ϕ_{lm} are determined by using the orthogonal properties of the cosine functions. Multiply both sides of Q_V by

$$\cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{m'\pi y}{b}\right)$$

for integers l', m' and integrate over substrate dimensions a,b.

$$\int_{x=0}^{x=a}\int_{y=0}^{y=b} Q_V \cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{m'\pi y}{b}\right) dxdy =$$

$$\frac{ab}{4}\left(\begin{array}{l}\phi_{00}\delta_{l'0}\delta_{m'0} + \sum_{m=1}^{\infty}\phi_{0m}\delta_{l'0}\delta_{m'm} + \sum_{l=1}^{\infty}\phi_{l0}\delta_{m'0}\delta_{l'l} + \\ \\ \sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\phi_{lm}\delta_{l'l}\delta_{m'm}\end{array}\right)$$

where $\delta_{l'l}$ and $\delta_{m'm}$ are the Kronecker delta functions. The source function is modeled as having zero thickness and may therefore be represented by a Dirac delta function.

$$Q_V = q(r)\delta(z)$$

The various Fourier coefficients for the source expansion are found by respectively setting

$$l' = 0,\ m' = 0;\ l' \neq 0,\ m' = 0;\ l' = 0,\ m' \neq 0;\ l' \neq 0,\ m' \neq 0.$$

The following are obtained:

$$\phi_{00} = \frac{4}{ab} \int_{x=0}^{x=a} \int_{b=0}^{b=a} Q_V dxdy = \frac{4}{ab} q(r)\delta(z)(x_2 - x_1)(y_2 - y_1)$$

$$\phi_{l0} = \frac{4}{ab} \int_{x=0}^{x=a} \int_{y=0}^{y=b} Q_V \cos\left(\frac{l\pi x}{a}\right) dxdy = \frac{4}{\pi lb} q(r)\delta(z)(y_2 - y_1) \bullet$$

$$\left[\sin\left(\frac{l\pi x_2}{a}\right) - \sin\left(\frac{l\pi x_1}{a}\right)\right]$$

Similarly

$$\phi_{0m} = \frac{4}{\pi ma} q(r)\delta(z)(x_2 - x_1)\left[\sin\left(\frac{m\pi y_2}{b}\right) - \sin\left(\frac{m\pi y_1}{b}\right)\right]$$

$$\phi_{lm} = \frac{4}{\pi^2 lm} q(r)\delta(z)\left[\sin\left(\frac{l\pi x_2}{a}\right) - \sin\left(\frac{l\pi x_1}{a}\right)\right] \bullet$$

$$\left[\sin\left(\frac{m\pi y_2}{b}\right) - \sin\left(\frac{m\pi y_1}{b}\right)\right]$$

Substitution of both the source and temperature functions into the partial differential equation results in

$$\sum_{l=0}^{\infty}\sum_{m=0}^{\infty} k\left\{-\left[\left(\frac{l\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2\right]\Psi_{lm}+\frac{d^2\Psi_{lm}}{dz^2}\right\}\bullet$$

$$\varepsilon_l\varepsilon_m\cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)$$

$$=-\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\phi_{lm}\varepsilon_l\varepsilon_m\cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)$$

Setting the coefficients of like terms

$$\varepsilon_l\varepsilon_m\cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)$$

equal, a one-dimensional differential equation in z is obtained.

$$\frac{d^2\Psi_{lm}}{dz^2}-\gamma_{lm}^2\Psi_{lm}=-\frac{1}{k}\phi_{lm}$$

$$\gamma_{lm}^2=\left(\frac{l\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2$$

Solution of one-dimensional equation in z using a Green's function method:

The problem is now to find the Fourier coefficients for the temperature function from the one-dimensional equation.

The boundary conditions on T in the z-direction are easily shown to apply to ψ_{lm} where an ambient reference temperature of zero is assumed.

$$k\frac{d\Psi_{lm}}{dz}\bigg|_{z=0} = 0, \qquad k\frac{d\Psi_{lm}}{dz}\bigg|_{z=t} = -h\Psi_{lm}\big|_{z=t}$$

The properties (what the Green's function actually is) of $G(z|z')$ are determined next. Change the independent variable z to z'. Multiply both sides by the Green's function $G(z|z')$, where the notation $z|z'$ will become apparent later. Next integrate from $z' = 0$ to $t \cdot$

$$\int_{z'=0}^{z'=t} G(z|z')\left[\frac{d^2\Psi_{lm}}{dz'^2} - \gamma_{lm}^2\Psi_{lm}\right]dz' = -\frac{1}{k}\int_{z'=0}^{z'=t}\phi_{lm}G(z|z')dz'$$

$$\int_{z'=0}^{z'=t} G(z|z') \left[\frac{d^2\Psi_{lm}}{dz'^2} - \gamma_{lm}^2 \Psi_{lm} \right] dz' = -\frac{1}{k} \int_{z'=0}^{z'=t} \phi_{lm} G(z|z') dz'$$

Take special care when integrating in the vicinity of the field point $z' = z$,

$$\int_{z'=0}^{z'=z-\varepsilon} G(z|z') \left[\frac{d^2\Psi_{lm}}{dz'^2} - \gamma_{lm}^2 \Psi_{lm} \right] dz' +$$

$$\int_{z'=z+\varepsilon}^{z'=t} G(z|z') \left[\frac{d^2\Psi_{lm}}{dz'^2} - \gamma_{lm}^2 \Psi_{lm} \right] dz' =$$

$$-\frac{1}{k} \int_{z'=0}^{z'=z-\varepsilon} G(z|z') \phi_{lm} dz' - \frac{1}{k} \int_{z'=z+\varepsilon}^{z'=t} G(z|z') \phi_{lm} dz'$$

where we think of taking the limit as $\varepsilon \rightarrow 0$.

Examining the first term in the preceding equation and integrating the portion containing $G(z|z')\dfrac{d^2\Psi_{lm}}{dz'^2}$ by parts.

$$\int_{z'=0}^{z'=z-\varepsilon} G(z|z')\frac{d^2\Psi_{lm}}{dz'^2}dz' - \int_{z'=0}^{z'=z-\varepsilon}\gamma_{lm}^2 G\Psi_{lm}dz' =$$

$$G(z|z')\frac{d\Psi_{lm}}{dz'}\Bigg|_{z'=0}^{z'=z-\varepsilon} - \int_{z'=0}^{z'=z-\varepsilon}\left(\frac{dG(z|z')}{dz'}\right)\left(\frac{d\Psi_{lm}}{dz'}\right)dz'$$

$$-\int_{z'=0}^{z'=z-\varepsilon}\gamma_{lm}^2 G(z|z')\psi_{lm}dz'$$

Next, integrate the second integral on the right side of the equal sign by parts. The result is

$$\int_{z'=0}^{z'=z-\varepsilon} G(z|z')\frac{d^2\Psi_{lm}}{dz'^2}dz' - \int_{z'=0}^{z'=z-\varepsilon}\gamma_{lm}^2 G\Psi_{lm}dz' =$$

$$\left[G(z|z')\frac{d\Psi_{lm}}{dz'} - \Psi_{lm}\frac{dG(z|z')}{dz'}\right]_{z'=0}^{z'=z-\varepsilon} +$$

$$\int_{z'=0}^{z'=z-\varepsilon}\Psi_{lm}\left[\frac{d^2G(z|z')}{dz'^2} - \gamma_{lm}^2\right]dz'$$

Perform the same operations on the portion at $z' = z + \varepsilon$ to get the complete result of

$$\left[G(z|z')\frac{d\Psi_{lm}}{dz'} - \Psi_{lm}\frac{dG(z|z')}{dz'}\right]_{z'=0}^{z'=z-\varepsilon} -$$

$$\int_{z'=0}^{z'=z-\varepsilon} \Psi_{lm}\left[\frac{d^2G(z|z')}{dz'^2} - \gamma_{lm}^2 G(z|z')\right]dz' +$$

$$\left[G(z|z')\frac{d\Psi_{lm}}{dz'} - \Psi_{lm}\frac{dG(z|z')}{dz'}\right]_{z'=z+\varepsilon}^{z'=t} -$$

$$\int_{z'=z+\varepsilon}^{z'=t} \Psi_{lm}\left[\frac{d^2G(z|z')}{dz'^2} - \gamma_{lm}^2 G(z|z')\right]dz' =$$

$$-\frac{1}{k}\int_{z'=0}^{z'=z-\varepsilon}\phi_{lm}G(z|z')dz' - \frac{1}{k}\int_{z'=z+\varepsilon}^{z'=t}\phi_{lm}G(z|z')dz'$$

The properties of, or our definition of the Green's function are now determined by working with the preceding result. We bascially want most of the terms in the preceding equation to disappear. The reason for this will become clear.

First, we shall require

$$\frac{d^2G(z|z')}{dz'^2} - \gamma_{lm}^2 G(z|z') = 0 \text{ at } z' \neq z$$

This leaves us with

$$-\frac{1}{k}\int_{z'=0}^{z'=z-\varepsilon} \phi_{lm} G(z|z')dz' - \frac{1}{k}\int_{z'=z+\varepsilon}^{z'=t} \phi_{lm} G(z|z')dz' =$$

$$-\left[G(z|z')\frac{d\Psi_{lm}(z')}{dz'} - \Psi_{lm}(z')\frac{dG(z|z')}{dz'}\right]_{z'=0} +$$

$$\left[G(z|z')\frac{d\Psi_{lm}(z')}{dz'} - \Psi_{lm}(z')\frac{dG(z|z')}{dz'}\right]_{z'=t} +$$

$$\left[G(z|z')\frac{d\Psi_{lm}(z')}{dz'} - \Psi_{lm}(z')\frac{dG(z|z')}{dz'}\right]_{z'=z-\varepsilon} +$$

$$\left[\Psi_{lm}(z')\frac{dG(z|z')}{dz'} - G(z|z')\frac{d\Psi_{lm}(z')}{dz'}\right]_{z'=z+\varepsilon}$$

Rearranging the terms in the third and fourth terms on the RHS of the equal sign,

$$-\frac{1}{k}\int_{z'=0}^{z'=z-\varepsilon}\phi_{lm}G(z|z')dz' - \frac{1}{k}\int_{z'=z+\varepsilon}^{z'=t}\phi_{lm}G(z|z')dz' =$$

$$-\left[G(z|z')\frac{d\Psi_{lm}(z')}{dz'} - \Psi_{lm}(z')\frac{dG(z|z')}{dz'}\right]_{z'=0} +$$

$$\left[G(z|z')\frac{d\Psi_{lm}(z')}{dz'} - \Psi_{lm}(z')\frac{dG(z|z')}{dz'}\right]_{z'=t} +$$

$$\left[G(z|z')\frac{d\Psi_{lm}(z')}{dz'}\Bigg|_{z'=z-\varepsilon} - G(z|z')\frac{d\Psi_{lm}(z')}{dz'}\Bigg|_{z'=z+\varepsilon}\right] +$$

$$\left[\Psi_{lm}(z')\frac{dG(z|z')}{dz'}\Bigg|_{z'=z+\varepsilon} - \Psi_{lm}(z')\frac{dG(z|z')}{dz'}\Bigg|_{z'=z-\varepsilon}\right]$$

If we apply the adiabatic boundary condition at $z' = 0$ to both $G(z|z')$ and $\Psi_{lm}(z')$, the first RHS term vanishes. If we apply the Newtonian cooling boundary condition $z' = t$ to both $G(z|z')$ and $\Psi_{lm}(z')$, the second RHS term also vanishes.

We are then left with

$$-\frac{1}{k}\int_{z'=0}^{z'=-\varepsilon}\phi_{lm}G(z|z')dz'-\frac{1}{k}\int_{z'=z+\varepsilon}^{z'=t}\phi_{lm}G(z|z')dz'=$$

$$\left[G(z|z')\frac{d\Psi_{lm}(z')}{dz'}\bigg|_{z'=z-\varepsilon}-G(z|z')\frac{d\Psi_{lm}(z')}{dz'}\bigg|_{z'=z+\varepsilon}\right]+$$

$$\left[\Psi_{lm}(z')\frac{dG(z|z')}{dz'}\bigg|_{z'=z+\varepsilon}-\Psi_{lm}(z')\frac{dG(z|z')}{dz'}\bigg|_{z'=z-\varepsilon}\right]$$

The temperature and the z' portion $(\Psi(z'))$ of the temperature are continous in the z' direction. The heat flux is also continous in the z' direction, hence (for continuous k) $d\Psi_{lm}(z')/dz'$ is continous in the z' direction. If we REQUIRE that $G(z|z')$ is also continous in z', then the two terms on the LHS of the equation may be combined into one term and the first set of brackets on the RHSabove vanishes as we take the limit as $\varepsilon \to 0$.

The remaining RHS set of terms is accommodated by factoring out $\Psi_{lm}(z')$ and imposing a "jump" condition on the Green's function, i.e.

$$\frac{dG(z|z')}{dz'}\bigg|_{z'=z+\varepsilon}-\frac{dG(z|z')}{dz'}\bigg|_{z'=z-\varepsilon}=-1$$

With the properties of the Green's function now determined, we are left with the desired result:

$$\Psi_{lm}(z) = \frac{1}{k}\int_{z'=0}^{z'=t} \phi_{lm} G(z|z')dz'$$

where

$$\frac{d^2 G(z|z')}{dz'^2} - \gamma_{lm}^2 G(z|z') = 0, \quad z' \neq z$$

$$\frac{dG(z|z')}{dz'} = 0 \;\text{ at }\; z' = 0$$

$$\frac{dG(z|z')}{dz'} = -\frac{h}{k} G(z|z') \;\text{ at }\; z' = t$$

$G(z|z')$ is continuous at $z' = z$

$$\left.\frac{dG(z|z')}{dz'}\right|_{z'=z+\varepsilon} - \left.\frac{dG(z|z')}{dz'}\right|_{z'=z-\varepsilon} = -1, \quad \varepsilon \to 0$$

Solution of the Green's function problem:

The differential equation to be solved is

$$\frac{d^2G(z|z')}{dz'^2} - \gamma_{lm}^2 G(z|z') = 0 \text{ at } z' \neq z$$

for both $\gamma_{lm} = 0$ and $\gamma_{lm} \neq 0$. We shall solve for $\gamma_{lm} = 0$ first. The general solution is

$$G(z|z') = Az' + B \qquad 0 \leq z' < z$$

$$G(z|z') = Cz' + D \qquad z < z' \leq t$$

Applying $\left.\frac{dG(z|z')}{dz'}\right|_{z'=0} = 0 \quad \rightarrow \quad A = 0$

$$\left.\frac{dG(z|z')}{dz'}\right|_{z'=t} = -\frac{h}{k}G(z|z')\Big|_{z'=t} \quad \rightarrow \quad D = -\left(\frac{k}{h} + t\right)C$$

$G(z|z') =$ continuous at $z' = z$

Applying $G(z|z') = B \qquad 0 < z' < z$

$$G(z|z') = Cz' - C\left(\frac{k}{h} + t\right) = C\left(z' - t - \frac{k}{h}\right) \text{ at } z < z' \le t$$

Then at z′ = z (actually limit as $\varepsilon \to 0$)

$$B = C\left(z - t - \frac{k}{h}\right)$$

and

$$G(z|z') = C\left(z - t - \frac{k}{h}\right) \text{ at } 0 \le z' < z$$

$$G(z|z') = Cz' - C\left(\frac{k}{h} + t\right) = C\left(z' - \frac{k}{h} - t\right) \text{ at } 0 < z' \le t$$

Applying $\left.\frac{dG(z|z')}{dz'}\right|_{z'=z+\varepsilon} - \left.\frac{dG(z|z')}{dz'}\right|_{z'=z-\varepsilon} = -1$

$$\frac{d}{dz'}\left[C\left(z' - \frac{k}{h} - t\right)\right]_{z'=z+\varepsilon} - \frac{d}{dz'}\left[C\left(z - \frac{k}{h} - t\right)\right]_{z'=z-\varepsilon} = -1$$

$$C - 0 = -1$$

$$C = -1$$

Putting it all together,

$$G(z|z') = -\left(z - t - \frac{k}{h}\right),\ 0 \le z' < z$$

$$G(z|z') = -\left(z' - t - \frac{k}{h}\right),\ z < z' \le t$$

or

$$G(z_>|z_<) = -\left(z_> - t - \frac{k}{h}\right), \qquad \gamma_{lm} = 0$$

Now we shall solve

$$\frac{d^2G(z|z')}{dz'^2} - \gamma_{lm}^2 G(z|z') = 0 \ \text{ at } \ z' \ne z$$

The general solution is

$$G = A\sinh\gamma z' + B\cosh h\gamma z', \qquad 0 \le z' < z$$

$$G = C\sinh\gamma z' + D\cosh h\gamma z', \qquad z < z' \le t$$

where $G \equiv G(z|z')$ and $\gamma \equiv \gamma_{lm}$ is used for convenience.

Applying

$$\left.\frac{dG}{dz'}\right|_{z'=0} = 0$$

$$\gamma A\cosh\gamma z'\big|_{z'=0} + \gamma B\sinh\gamma z'\big|_{z'=0} = 0 \qquad \longrightarrow \qquad A = 0$$

Applying

$$\left.\frac{dG}{dz'}\right|_{z'=t} = -\frac{h}{k}G\big|_{z'=t},$$

$$\gamma C\cosh\gamma z'\big|_{z'=t} + \gamma D\sinh\gamma z'\big|_{z'=t} = -\frac{h}{k}\left.(C\sinh\gamma z' + D\cosh\gamma z')\right|_{z'=t}$$

$$\rightarrow \quad (C\cosh\gamma t + D\sinh\gamma t) = -\frac{h}{\gamma k}(C\sinh\gamma t + D\cosh\gamma t)$$

$$\text{or} \quad C\left(\cosh\gamma t + \frac{h}{\gamma k}\sinh\gamma t\right) = D\left(\sinh\gamma t + \frac{h}{\gamma k}\cosh\gamma t\right)$$

Applying $\quad G = \text{continuous at } z' = z$

$$B\cosh\gamma z = C\sinh\gamma z + D\cosh\gamma z$$

Applying $\quad \left.\frac{dG}{dz'}\right|_{z'=z+\varepsilon} - \left.\frac{dG}{dz'}\right|_{z'=z-\varepsilon} = -1$

$$C\cosh\gamma z + D\sinh\gamma z - B\sinh\gamma z = -1/\gamma$$

Summarizing up to this point,

$$A = 0$$

$$B = C\frac{\sinh\gamma t}{\cosh\gamma t} + D$$

$$C = B\frac{\sinh\gamma z}{\cosh\gamma z} - D\frac{\sinh\gamma z}{\cosh\gamma z} - \frac{1}{\gamma\cosh\gamma z}$$

$$D = -C\frac{\left(\cosh\gamma t + \frac{h}{\gamma k}\sinh\gamma t\right)}{\left(\sinh\gamma t + \frac{h}{\gamma k}\cosh\gamma t\right)}$$

Performing the necessary algebra,

$$A = 0$$

$$B = -\frac{\sinh \gamma z}{\gamma} + \left(\frac{\cosh \gamma t + \frac{h}{\gamma k} \sinh \gamma t}{\sinh \gamma t + \frac{h}{\gamma k} \cosh \gamma t} \right) \frac{\cosh \gamma z}{\gamma}$$

$$C = -\frac{1}{\gamma} \cosh \gamma z$$

$$D = \left(\frac{\cosh \gamma t + \frac{h}{\gamma k} \sinh \gamma t}{\sinh \gamma t + \frac{h}{\gamma k} \cosh \gamma t} \right) \frac{\cosh \gamma z}{\gamma}$$

Putting it all together,

$$0 \le z' < z$$

$$G = -\frac{1}{\gamma} \sinh \gamma z \cosh \gamma z' + \left(\frac{\cosh \gamma t + \frac{h}{\gamma k} \sinh \gamma t}{\sinh \gamma t + \frac{h}{\gamma k} \cosh \gamma t} \right) \frac{\cosh \gamma z \cosh \gamma z'}{\gamma}$$

$$z < z' \le t$$

$$G = -\frac{1}{\gamma} \sinh \gamma z' \cosh \gamma z + \left(\frac{\cosh \gamma t + \frac{h}{\gamma k} \sinh \gamma t}{\sinh \gamma t + \frac{h}{\gamma k} \cosh \gamma t} \right) \frac{\cosh \gamma z \cosh \gamma z'}{\gamma}$$

Recalling that the z dependent factor of the temperature function is calculated from

$$\Psi_{lm}(z) = \frac{1}{k}\int_{z'=0}^{z'=t} \phi_{lm} G(z|z')dz';$$

all of the ϕ_{lm} have a Dirac delta function, $\delta(z)$ factor;
the Dirac delta function factor has the property

$$f(0) = \int \delta(z) f(z) dz,$$

the ψ_{lm} are readily determined.

$$\Psi_{00} = \frac{4}{kab} Q\left(t + \frac{k}{h} - z\right)$$

$$\Psi_{l0} = \frac{8Qa}{\pi^2 b l^2 k \Delta x}\sin\left[\frac{l\pi}{2a}(x_2 - x_1)\right]\cos\left[\frac{l\pi}{2a}(x_2 + x_1)\right]\cdot$$

$$\left(\frac{\cosh\left[\frac{l\pi}{a}(z-t)\right] - \frac{ha}{l\pi k}\sinh\left[\frac{l\pi}{a}(z-t)\right]}{\sinh\left(\frac{l\pi t}{a}\right) + \frac{ha}{l\pi k}\cosh\left(\frac{l\pi t}{a}\right)}\right)$$

$$\Psi_{0m} = \frac{8Qb}{\pi^2 am^2 k\Delta y}\sin\left[\frac{m\pi}{2b}(y_2 - y_1)\right]\cos\left[\frac{m\pi}{2b}(y_2 + y_1)\right]\cdot$$

$$\left(\frac{\cosh\left[\frac{m\pi}{b}(z-t)\right] - \frac{hb}{m\pi k}\sinh\left[\frac{m\pi}{m}(z-t)\right]}{\sinh\left(\frac{m\pi t}{b}\right) + \frac{hb}{m\pi k}\cosh\left(\frac{m\pi t}{b}\right)}\right)$$

$$\Psi_{lm} = \frac{16Q}{k\pi^2 lm\Delta x\Delta y\sqrt{\left(\frac{l\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}} \cdot$$

$$\sin\left[\frac{l\pi}{a}(x_2 - x_1)\right]\cos\left[\frac{l\pi}{a}(x_2 + x_1)\right]\sin\left[\frac{m\pi}{b}(y_2 - y_1)\right]\cos\left[\frac{m\pi}{b}(y_2 + y_1)\right] \cdot$$

$$\left(\frac{\cosh\left(\sqrt{\left(\frac{l\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}(z - t)\right) - \frac{h}{k\sqrt{\left(\frac{l\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}}\sinh\left(\sqrt{\left(\frac{l\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}(z - t)\right)}{\sqrt{\left(\frac{l\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}\left(\sinh\left(\sqrt{\left(\frac{l\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}\, t\right) + \frac{h}{k\sqrt{\left(\frac{l\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}}\cosh\left(\sqrt{\left(\frac{l\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2} \right. \right.} \right.$$

The temperature for a source at $z=0$ and a field point x,y,z is then

$$T = \frac{1}{4}\Psi_{00} + \frac{1}{2}\sum_{l=1}^{\infty}\Psi_{l0}\cos\frac{l\pi x}{a} + \frac{1}{2}\sum_{m=1}^{\infty}\Psi_{0m}\cos\frac{m\pi y}{b}$$

$$+\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\Psi_{lm}\cos\frac{l\pi x}{a}\cos\frac{m\pi y}{b}$$

Remembering that the ambient reference temperature was previously assumed to be zero, we next write the temperature T, for a source at $z=0$,by inserting the formulae for Ψ_{00}, Ψ_{l0},(l not 0), and Ψ_{lm} (l and m not 0).

Using $\Delta x = x_2 - x_1, \Delta y = y_2 - y_1, x_c = (a/2), y_c = (b/2)$: $\quad T(x,y,z) = \frac{Q}{kab}\left[(t-z)+\frac{k}{h}\right] +$

$$\frac{4}{k\pi^2}\left(\frac{a}{b}\right)\frac{Q}{\Delta x}\sum_{l=1}^{\infty}\frac{1}{l^2}\sin\left(\frac{l\pi}{2}\frac{\Delta x}{a}\right)\cos\left(l\pi\frac{x_c}{a}\right)\cos\left(l\pi\frac{x}{a}\right)\bullet\left\{\frac{\cosh\left[l\pi\frac{(z-t)}{a}\right]-\left[\frac{(ha/k)}{l\pi}\right]\sinh\left[l\pi\frac{(z-t)}{a}\right]}{\sinh\left(l\pi\frac{t}{a}\right)+\left[\frac{(ha/k)}{l\pi}\right]\cosh\left(l\pi\frac{t}{a}\right)}\right\}+$$

$$\frac{4}{k\pi^2}\left(\frac{b}{a}\right)\frac{Q}{\Delta y}\sum_{m=1}^{\infty}\frac{1}{m^2}\sin\left(\frac{m\pi}{2}\frac{\Delta y}{b}\right)\cos\left(m\pi\frac{y_c}{b}\right)\cos\left(m\pi\frac{y}{b}\right)\bullet\left\{\frac{\cosh\left[m\pi\frac{(z-t)}{b}\right]-\left[\frac{(hb/k)}{m\pi}\right]\sinh\left[m\pi\frac{(z-t)}{b}\right]}{\sinh\left(m\pi\frac{t}{b}\right)+\left[\frac{(hb/k)}{m\pi}\right]\cosh\left(m\pi\frac{t}{b}\right)}\right\}+$$

$$\frac{16Q}{k\pi^2\Delta x\Delta y}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{lm}\sin\left(\frac{l\pi}{2}\frac{\Delta x}{a}\right)\sin\left(\frac{m\pi}{2}\frac{\Delta y}{b}\right)\bullet\cos\left(l\pi\frac{x_c}{a}\right)\cos\left(m\pi\frac{y_c}{b}\right)\cos\left(l\pi\frac{x}{a}\right)\cos\left(m\pi\frac{y}{b}\right)\bullet$$

$$\left\{\frac{\cosh\left[\pi\sqrt{l^2+m^2\left(\frac{a}{b}\right)^2}\left(\frac{z-t}{a}\right)\right]-\left(\frac{ha/k}{\pi\sqrt{l^2+m^2\left(\frac{a}{b}\right)^2}}\right)\sinh\left[\pi\sqrt{l^2+m^2\left(\frac{a}{b}\right)^2}\left(\frac{z-t}{a}\right)\right]}{\left(\frac{1}{a}\right)\pi\sqrt{l^2+m^2\left(\frac{a}{b}\right)^2}\left\{\sinh\left[\pi\sqrt{l^2+m^2\left(\frac{a}{b}\right)^2}\left(\frac{t}{a}\right)\right]+\left(\frac{ha/k}{\pi\sqrt{l^2+m^2\left(\frac{a}{b}\right)^2}}\right)\cosh\left[\pi\sqrt{l^2+m^2\left(\frac{a}{b}\right)^2}\left(\frac{t}{a}\right)\right]\right\}}\right\}$$

Noting that "$R(x,y,z)$" $= \dfrac{T}{Q}$, $\quad Rk\Delta x = \left[\left(\dfrac{t-z}{b}\right) + \dfrac{k}{hb}\right]\left(\dfrac{\Delta x}{a}\right) +$

$$\frac{4}{\pi^2}\left(\frac{a}{b}\right)\sum_{l=1}^{\infty}\frac{1}{l^2}\sin\left(\frac{l\pi}{2}\frac{\Delta x}{a}\right)\cos\left(l\pi\frac{x_c}{a}\right)\cos\left(l\pi\frac{x}{a}\right)\bullet\left\{\frac{\cosh\left[l\pi\dfrac{(z-t)}{a}\right]-\left[\dfrac{(ha/k)}{l\pi}\right]\sinh\left[l\pi\dfrac{(z-t)}{a}\right]}{\sinh\left(l\pi\dfrac{t}{a}\right)+\left[\dfrac{(ha/k)}{l\pi}\right]\cosh\left(l\pi\dfrac{t}{a}\right)}\right\}+$$

$$\frac{4}{\pi^2}\left(\frac{\Delta x}{a}\right)\left(\frac{b}{\Delta y}\right)\sum_{m=1}^{\infty}\frac{1}{m^2}\sin\left(\frac{m\pi}{2}\frac{\Delta y}{b}\right)\cos\left(m\pi\frac{y_c}{b}\right)\cos\left(m\pi\frac{y}{b}\right)\bullet\left\{\frac{\cosh\left[m\pi\dfrac{(z-t)}{b}\right]-\left[\dfrac{(hb/k)}{m\pi}\right]\sinh\left[m\pi\dfrac{(z-t)}{b}\right]}{\sinh\left(m\pi\dfrac{t}{b}\right)+\left[\dfrac{(hb/k)}{m\pi}\right]\cosh\left(m\pi\dfrac{t}{b}\right)}\right\}+$$

$$\frac{16}{\pi^2}\left(\frac{a}{\Delta y}\right)\sum_{l=1,\,m=1}^{\infty}\frac{1}{lm}\sin\left(\frac{l\pi}{2}\frac{\Delta x}{a}\right)\sin\left(\frac{m\pi}{2}\frac{\Delta y}{b}\right)\bullet\cos\left(l\pi\frac{x_c}{a}\right)\cos\left(m\pi\frac{y_c}{b}\right)\cos\left(l\pi\frac{x}{a}\right)\cos\left(m\pi\frac{y}{b}\right)\bullet$$

$$\left\{\frac{\cosh\left[\pi\sqrt{l^2+m^2\left(\dfrac{a}{b}\right)^2}\left(\dfrac{z-t}{a}\right)\right]-\left(\dfrac{ha/k}{\pi\sqrt{l^2+m^2\left(\dfrac{a}{b}\right)^2}}\right)\sinh\left[\pi\sqrt{l^2+m^2\left(\dfrac{a}{b}\right)^2}\left(\dfrac{z-t}{a}\right)\right]}{\left(\dfrac{1}{a}\right)\pi\sqrt{l^2+m^2\left(\dfrac{a}{b}\right)^2}\left\{\sinh\left[\pi\sqrt{l^2+m^2\left(\dfrac{a}{b}\right)^2}\left(\dfrac{t}{a}\right)\right]+\left(\dfrac{ha/k}{\pi\sqrt{l^2+m^2\left(\dfrac{a}{b}\right)^2}}\right)\cosh\left[\pi\sqrt{l^2+m^2\left(\dfrac{a}{b}\right)^2}\left(\dfrac{t}{a}\right)\right]\right\}}\right\}$$

Using $\frac{x_c}{a} = \frac{\left(\frac{1}{2}a\right)}{a} = \frac{1}{2}, \frac{y_c}{b} = \frac{\left(\frac{1}{2}b\right)}{b} = \frac{1}{2}, y = \frac{b}{2}$ and dimensionless variables:

$$\psi = Rk\Delta x, \quad \psi = \psi_{Unif} + \psi_{Sp};$$

$$\rho = \frac{a}{b}, \alpha = \frac{\Delta x}{a}, \beta = \frac{\Delta y}{a}, \mu = \frac{x}{a}, \xi = \frac{z}{a}, \ \tau = \frac{t}{a}, Bi = \frac{ha}{k}, Bi \cdot \tau = \left(\frac{ha}{k}\right)\left(\frac{t}{a}\right) = \frac{ht}{k}$$

Plotting Contours in the XZ Plane at y=b/2,

$$Rk\Delta x = \alpha\rho\left(\tau - \xi + \frac{1}{Bi}\right) +$$

$$\frac{4}{\pi^2}\rho\sum_{l=1}^{\infty}\frac{1}{l^2}\sin\left(\frac{l\pi\alpha}{2}\right)\cos\left(\frac{l\pi}{2}\right)\cos(l\pi\mu)\left\{\frac{\cosh\left[l\pi(\xi-\tau)\right]-\left(\frac{Bi\tau}{l\pi\tau}\right)\sinh\left[l\pi(\xi-\tau)\right]}{\sinh(l\pi\tau)+\left(\frac{Bi\tau}{l\pi\tau}\right)\cosh(l\pi\tau)}\right\}+$$

$$\frac{4}{\pi^2}\left(\frac{1}{\rho}\right)\left(\frac{\alpha}{\beta}\right)\sum_{m=1}^{\infty}\frac{1}{m^2}\sin\left(\frac{m\pi\beta\rho}{2}\right)\cos^2\left(\frac{m\pi}{2}\right)\bullet\left\{\frac{\cosh\left[m\pi(\xi-\tau)\rho\right]-\left(\frac{Bi\tau}{m\pi\tau\rho}\right)\sinh\left[m\pi\rho(\xi-\tau\right.}{\sinh(m\pi\rho\tau)+\left[\frac{Bi\tau}{m\pi\tau\rho}\right]\cosh(m\pi\rho\tau)}\right.$$

$$\frac{16}{\pi^2}\left(\frac{1}{\beta}\right)\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{lm}\sin\left(\frac{l\pi}{2}\alpha\right)\sin\left(\frac{m\pi}{2}\beta\rho\right)\bullet\cos\left(\frac{l\pi}{2}\right)\cos^2\left(\frac{m\pi}{2}\right)\cos(l\pi\mu)\bullet$$

$$\left\{\frac{\cosh\left[\pi\sqrt{l^2+m^2\rho^2}(\xi-\tau)\right]-\left(\frac{Bi\tau}{\pi\tau\sqrt{l^2+m^2\rho^2}}\right)\sinh\left[\pi\sqrt{l^2+m^2\rho^2}(\xi-\tau)\right]}{\pi\sqrt{l^2+m^2\rho^2}\left[\sinh\left(\pi\tau\sqrt{l^2+m^2\rho^2}\right)+\left(\frac{Bi\tau}{\pi\tau\sqrt{l^2+m^2\rho^2}}\right)\cosh\left(\pi\tau\sqrt{l^2+m^2\rho^2}\right)\right]}\right\}$$

Calculating or Plotting Maximum Spreading Resistance at $z = 0\ (\xi = 0), x = a/2\ (\mu = 1/2)$ (and $y = b/2$)

$$\psi = \psi_{Uniform} + \psi_{Sp}, \quad \psi_{Uniform} = \alpha\rho\left(\tau - \xi + \frac{1}{Bi}\right)$$

$$\psi_{Sp} = \frac{4\rho}{\pi^2}\sum_{l=1}^{\infty}\frac{1}{l^2}\sin\left(\frac{l\pi\alpha}{2}\right)\cos^2\left(\frac{l\pi}{2}\right)\left[\frac{\cos(l\pi\tau) + \left(\frac{Bi\tau}{l\pi\tau}\right)\sinh(l\pi\tau)}{\sinh(l\pi\tau) + \left(\frac{Bi\tau}{l\pi\tau}\right)\cos(l\pi\tau)}\right] +$$

$$\frac{4}{\pi^2\rho}\left(\frac{\alpha}{\beta}\right)\sum_{m=1}^{\infty}\frac{1}{m^2}\sin\left(\frac{m\pi\beta\rho}{2}\right)\cos^2\left(\frac{m\pi}{2}\right)\left[\frac{\cosh(m\pi\rho\tau) + \left(\frac{Bi\tau}{m\pi\rho\tau}\right)\sinh(m\pi\rho\tau)}{\sinh(m\pi\rho\tau) + \left(\frac{Bi\tau}{m\pi\rho\tau}\right)\cosh(m\pi\rho\tau)}\right] +$$

$$\frac{16}{\pi^2\beta}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{lm}\cos^2\left(\frac{l\pi}{2}\right)\cos^2\left(\frac{m\pi}{2}\right)\sin\left(\frac{l\pi\alpha}{2}\right)\sin\left(\frac{m\pi\beta\rho}{2}\right)\bullet$$

$$\left\{\frac{\cosh\left(\pi\sqrt{l^2 + m^2\rho^2}\,\tau\right) + \left(\frac{Bi\tau}{\pi\tau\sqrt{l^2 + m^2\rho^2}}\right)\sinh\left(\pi\sqrt{l^2 + m^2\rho^2}\,\tau\right)}{\pi\sqrt{l^2 + m^2\rho^2}\left[\sinh\left(\pi\sqrt{l^2 + m^2\rho^2}\,\tau\right) + \left(\frac{Bi\tau}{\pi\tau\sqrt{l^2 + m^2\rho^2}}\right)\cosh\left(\pi\sqrt{l^2 + m^2\rho^2}\,\tau\right)\right]}\right\}$$

The contour computation time can be reduced by noting that

$$\cos\left(\frac{l\pi}{2}\right)=\begin{cases}0 & l=odd\\ -1 & l=2,6,10\ldots\\ +1 & l=4,8,12\ldots\end{cases}$$

$$\cos\left(\frac{m\pi}{2}\right)=\begin{cases}0 & m=odd\\ -1 & m=2,6,10\ldots\\ +1 & m=4,8,12\ldots\end{cases}$$

Then set

$$l\rightarrow 2l,\ l=1,2\ldots$$

$$\cos\left(\frac{l\pi}{2}\right)\rightarrow\cos(l\pi)=(-1)^{l},\ l=1,2,3\ldots$$

$$m\rightarrow 2m,\ m=1,2\ldots$$

$$\cos\left(\frac{m\pi}{2}\right)\rightarrow\cos(m\pi)=(-1)^{m},\ m=1,2,3\ldots$$

The maximum spreading computation time can be similarly reduced by using

$$l\rightarrow 2l,\quad \cos^{2}\left(\frac{l\pi}{2}\right)\rightarrow 1,\qquad l=1,2,3\ldots$$

$$m\rightarrow 2m,\ \cos^{2}\left(\frac{m\pi}{2}\right)\rightarrow 1,\qquad m=1,2,3\ldots$$

and dividing ψ_{Sp} numerators and denominators by cosh( ) to get tanh( ), which eliminates overflow in cosh( ) and sinh( ) for large arguments.

Finally, for Contour Plotting

$$\mu = x/a\,,\xi = z/a\,,\rho = a/b\,,\alpha = \Delta x/a\,,\beta = \Delta y/a\,,\tau = t/a\,,Bi\tau = ht/k$$

$$\psi = \psi_{Uniform} + \psi_{Sp}, \qquad \psi_{Uniform} = \rho\alpha\left(\tau - \xi + \frac{\tau}{Bi\tau}\right)$$

$$\psi_{Sp} = \frac{\rho}{\pi^2}\sum_{l=1}^{\infty}\frac{(-1)^l}{l^2}\sin(l\pi\alpha)\cos(2l\pi\mu)\left\{\frac{\cosh[2l\pi(\xi-\tau)] - \left(\frac{Bi\tau}{2l\pi\tau}\right)\cosh[2l\pi(\xi-\tau)]}{\sinh(2l\pi\tau) + \left(\frac{Bi\tau}{2l\pi\tau}\right)\cosh(2l\pi\tau)}\right\} +$$

$$\left(\frac{1}{\pi^2}\right)\left(\frac{1}{\rho}\right)\left(\frac{\alpha}{\beta}\right)\sum_{m=1}^{\infty}\frac{1}{m^2}\sin(m\pi\beta\rho)\left\{\frac{\cosh[2m\pi(\xi-\tau)\rho] - \left(\frac{Bi\tau}{2m\pi\tau\rho}\right)\sinh[2m\pi(\xi-\tau)\rho]}{\sinh(2m\pi\tau\rho) + \left(\frac{Bi\tau}{2m\pi\tau\rho}\right)\cosh(2m\pi\tau\rho)}\right\} +$$

$$\left(\frac{4}{\pi^2}\right)\left(\frac{1}{\beta}\right)\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^l}{lm}\cos(2l\pi\mu)\sin(l\pi\alpha)\sin(m\pi\beta\rho)\,\bullet$$

$$\left\{\frac{\cosh\left[2\pi\sqrt{l^2+m^2\rho^2}(\xi-\tau)\right] - \left(\frac{Bi\tau}{2\pi\tau\sqrt{l^2+m^2\rho^2}}\right)\sinh\left[2\pi\sqrt{l^2+m^2\rho^2}(\xi-\tau)\right]}{2\pi\sqrt{l^2+m^2\rho^2}\left[\sinh\left(2\pi\sqrt{l^2+m^2\rho^2}\,\tau\right) + \left(\frac{Bi\tau}{2\pi\tau\sqrt{l^2+m^2\rho^2}}\right)\cosh\left(2\pi\sqrt{l^2+m^2\rho^2}\,\tau\right)\right]}\right\}$$

Finally, for Maximum Spreading

$$\rho = a/b,\ \alpha = \Delta x/a,\ \beta = \Delta y/a,\ \tau = t/a,\ Bi\tau = ht/k$$

$$\psi = \psi_{Uniform} + \psi_{Sp}, \qquad \psi_{Uniform} = \rho\alpha\tau\left(1 + \frac{1}{Bi\tau}\right)$$

$$\psi_{Sp} = \frac{\rho}{\pi^2}\sum_{l=1}^{\infty}\frac{1}{l^2}\sin(l\pi\alpha)\left[\frac{1 + \left(\frac{Bi\tau}{2l\pi\tau}\right)\tanh(2l\pi\tau)}{\left(\frac{Bi\tau}{2l\pi\tau}\right) + \tanh(2l\pi\tau)}\right] + \left(\frac{1}{\pi^2}\right)\left(\frac{1}{\rho}\right)\left(\frac{\alpha}{\beta}\right)\sum_{m=1}^{\infty}\frac{1}{m^2}\sin(m\pi\beta\rho)\left[\frac{1 + \left(\frac{Bi\tau}{2m\pi\rho\tau}\right)\tanh(2m\ldots}{\left(\frac{Bi\tau}{2m\pi\rho\tau}\right) + \tanh(2m\pi\ldots}\right.$$

$$\left(\frac{4}{\pi^2}\right)\left(\frac{1}{\beta}\right)\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{lm}\sin(l\pi\alpha)\sin(m\pi\beta\rho)\bullet\left\{\frac{1 + \left(\frac{Bi\tau}{2\pi\sqrt{l^2 + m^2\rho^2}}\right)\tanh\left(2\pi\sqrt{l^2 + m^2\rho^2}\,\tau\right)}{2\pi\sqrt{l^2 + m^2\rho^2}\left[\left(\frac{Bi\tau}{2\pi\sqrt{l^2 + m^2\rho^2}}\right) + \tanh\left(2\pi\sqrt{l^2 + m^2\rho^2}\,\tau\right)\right]}\right\}$$

Spreading Resistance Design Curves Generated Using Fourier Series Solution -

The following curves are plotted in a dimensionless form

$\psi_{Sp} = k\Delta x R_{sp}$ vs. $\tau = t / a$

where

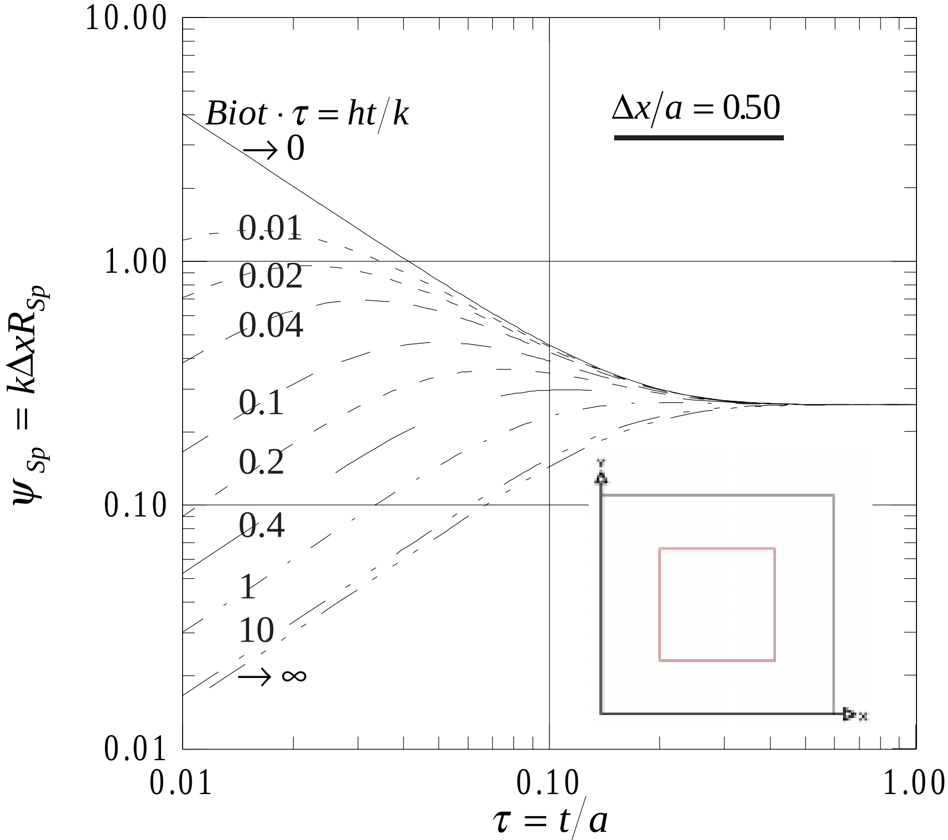
t = substrate thickness
a = substrate dimension in x-direction
k = substrate thermal conductivity

The *Biot* parameter on each of the graphs is defined as

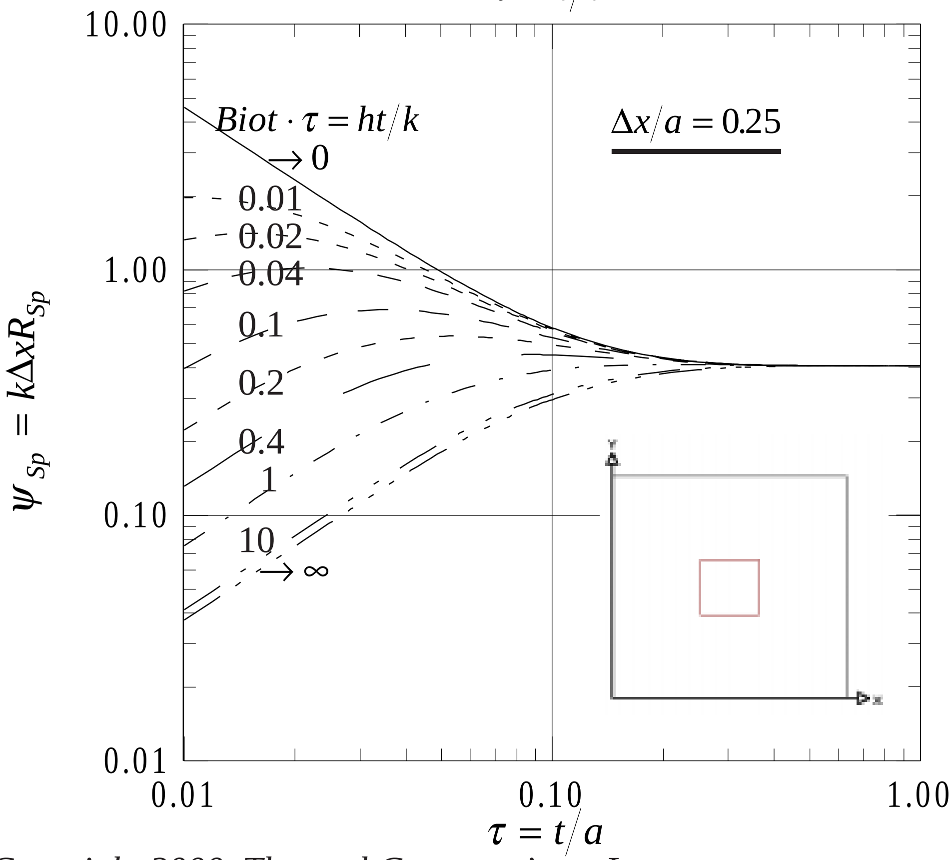
Biot = *ha/k*, therefore

$Biot \cdot \tau = ht / k$, which is a more standard form of the Biot number

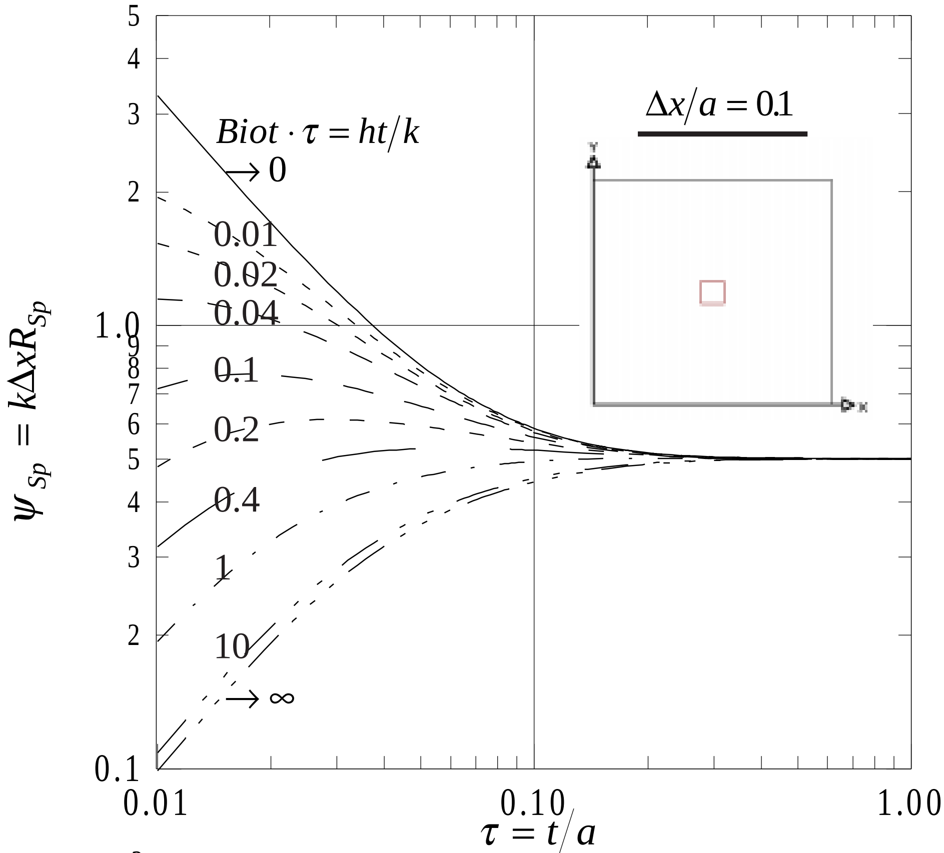
Note: the Spreading Resistance, ψ_{Sp}, is not to be confused with the Fourier coefficients Ψ_{lm}.



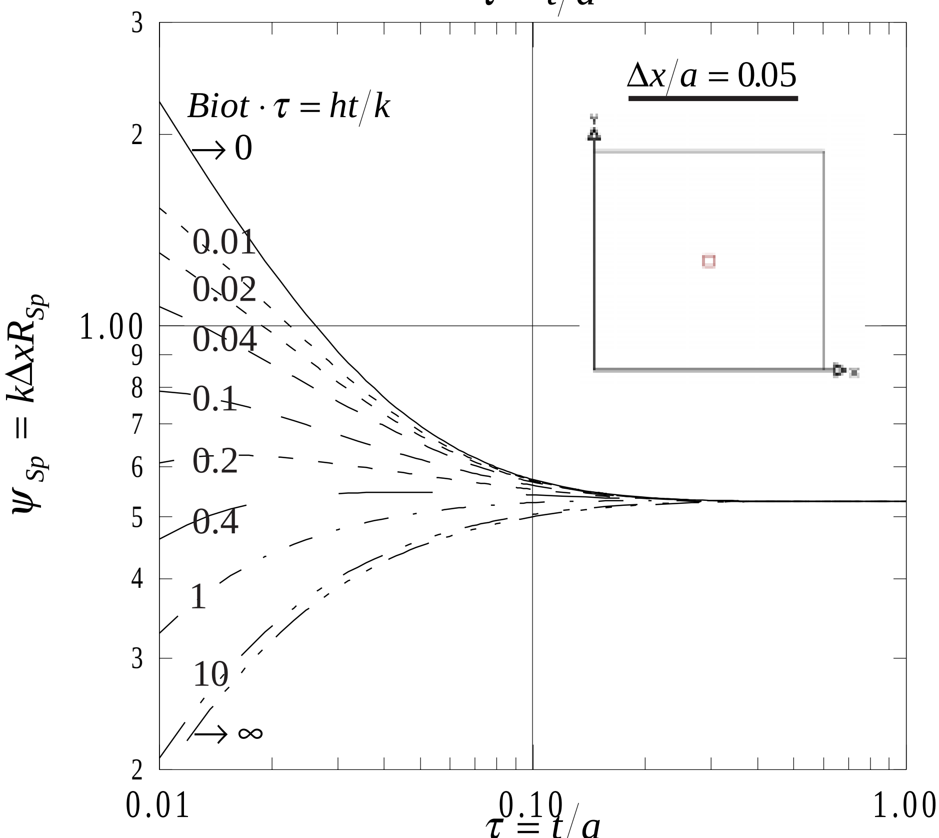
10.00
1.00
0.10
0.01
0.01
0.10
1.00
$Biot \cdot \tau = ht/k$
$\rightarrow 0$
0.01
0.02
0.04
0.1
0.2
0.4
1
10
$\rightarrow \infty$
$\Delta x/a = 0.50$
$\psi_{Sp} = k\Delta x R_{Sp}$
$\tau = t/a$



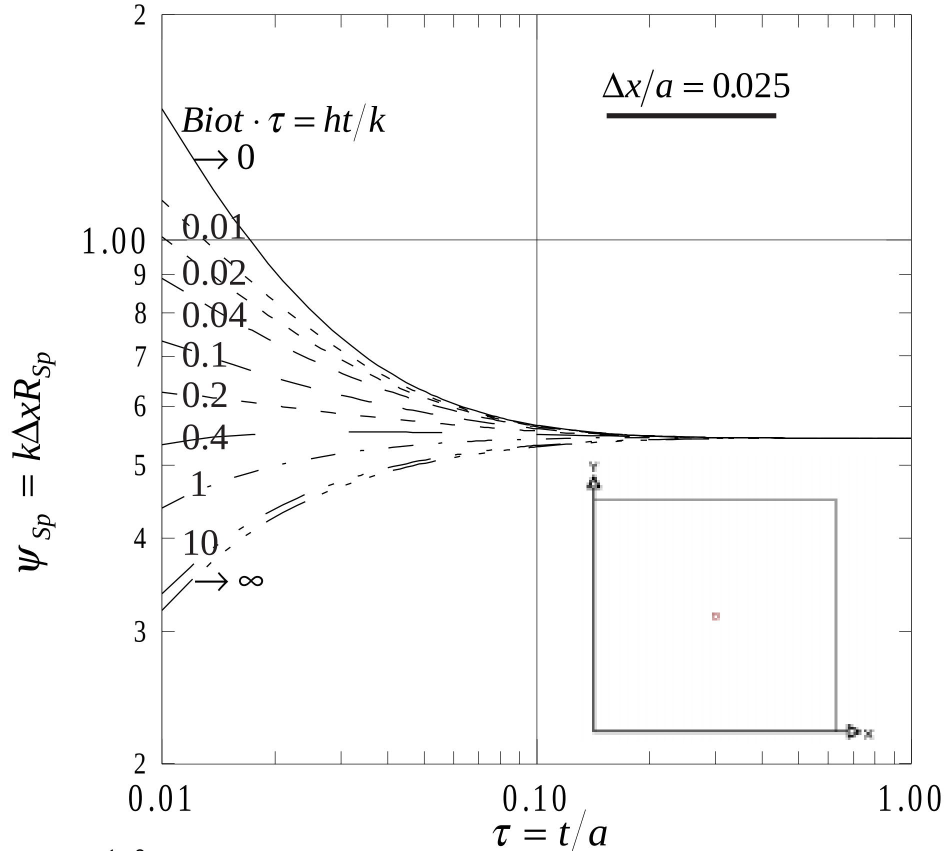
10.00
1.00
0.10
0.01
0.01
0.10
1.00
$Biot \cdot \tau = ht/k$
$\rightarrow 0$
0.01
0.02
0.04
0.1
0.2
0.4
1
10
$\rightarrow \infty$
$\Delta x/a = 0.25$
$\psi_{Sp} = k\Delta x R_{Sp}$
$\tau = t/a$

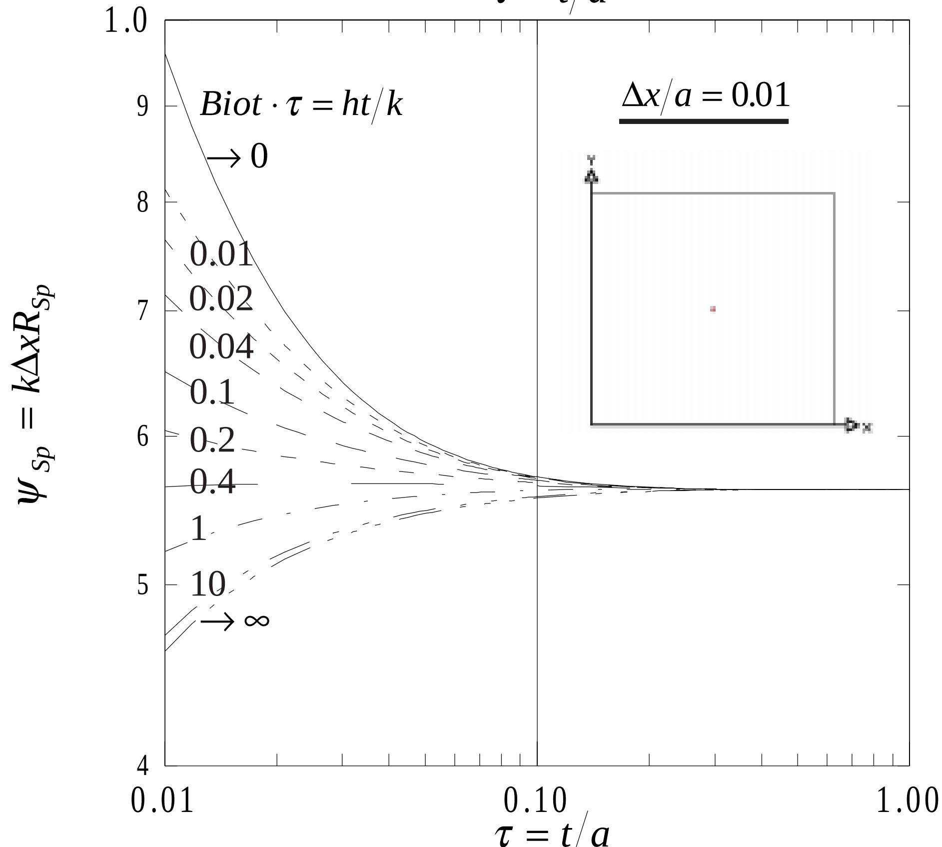


$\Delta x/a = 0.1$
$Biot \cdot \tau = ht/k$
$\to 0$
0.01
0.02
0.04
0.1
0.2
0.4
1
10
$\to \infty$
$\psi_{Sp} = k\Delta x R_{Sp}$
$\tau = t/a$
0.01
0.10
1.00



$\Delta x/a = 0.05$
$Biot \cdot \tau = ht/k$
$\to 0$
0.01
0.02
0.04
0.1
0.2
0.4
1
10
$\to \infty$
$\psi_{Sp} = k\Delta x R_{Sp}$
$\tau = t/a$
0.01
0.10
1.00


$\Delta x/a = 0.025$
$Biot \cdot \tau = ht/k$
$\rightarrow 0$
0.01
0.02
0.04
0.1
0.2
0.4
1
10
$\rightarrow \infty$
$\psi_{Sp} = k\Delta x R_{Sp}$
$\tau = t/a$
2
1.00
9
8
7
6
5
4
3
2
0.01
0.10
1.00


$\Delta x/a = 0.01$
$Biot \cdot \tau = ht/k$
$\rightarrow 0$
0.01
0.02
0.04
0.1
0.2
0.4
1
10
$\rightarrow \infty$
$\psi_{Sp} = k\Delta x R_{Sp}$
$\tau = t/a$
1.0
9
8
7
6
5
4
0.01
0.10
1.00

Application Example: IC Chip on Alumina Ceramic.

$$k = 1.0\,W/in.\cdot{}^oC,\ a = 1.0\,in.,\ t = 0.04\,in.$$

Then $\Delta x/a = 0.05,\ \tau = t/a = 0.04$

Case 1 - $h = 0.005\,W/in.^2\cdot{}^oC$

$$Bi\cdot\tau = ht/k = (0.005)(0.04)/1.0 = 2x10^{-4} \approx 0$$

$$R = R_U + R_{Sp} = R_{Unif.Conv.} + R_{Unif.Cond.} + R_{Sp}$$

$$= \frac{1}{hab} + \frac{t}{kab} + \frac{\psi_{Sp}}{k\Delta x}$$

Finding $\psi_{Sp} = 0.78$ from the graphs,

$$R = \frac{1}{\left(0.005\,W/in.^2\cdot{}^oC\right)(1.0\,in.)^2} + \frac{0.04\,in.}{\left(1.0\,W/in.\cdot{}^oC\right)(1.0\,in.)^2} +$$

$$\frac{0.78}{\left(1.0\,W/in.\cdot{}^oC\right)(0.05\,in.)}$$

$$= 200\,{}^oC/W + 0.04\,{}^oC/W + 15.6\,{}^oC/W = 215.64\,{}^oC/W$$

Case 2 - $h = \infty$.

$$Bi \cdot \tau \approx \infty$$

$$R = R_U + R_{Sp} = R_{Unif.Conv.} + R_{Unif.Cond.} + R_{Sp}$$
$$= \frac{1}{hab} + \frac{t}{kab} + \frac{\psi_{Sp}}{k\Delta x}$$

Find $\psi_{Sp} = 0.42$ from the graphs ,

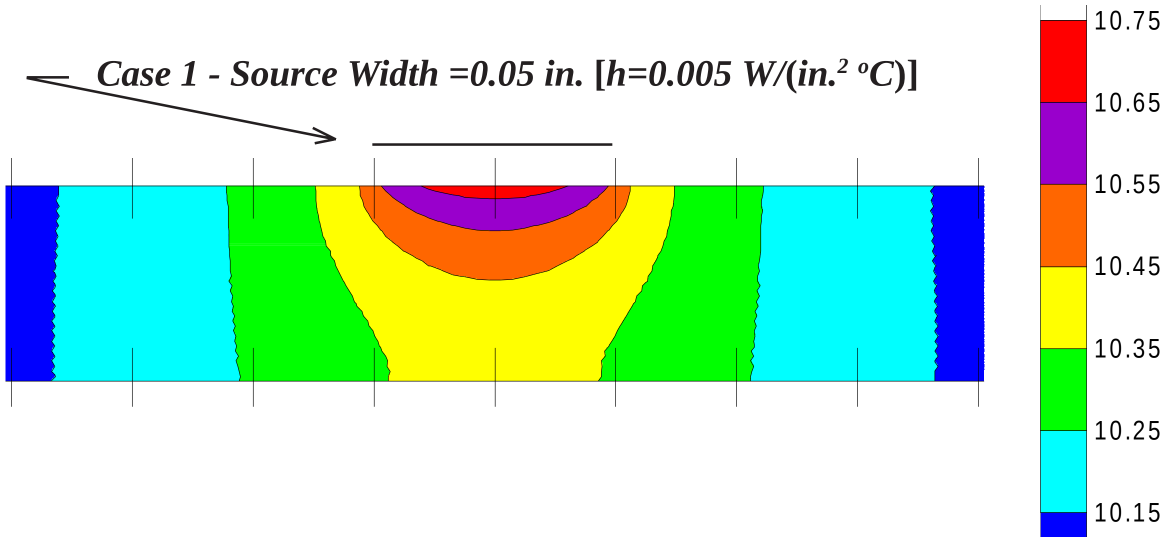
$$R = 0 + \frac{0.04\ in.}{\left(1.0\ W/in.\cdot{}^oC\right)(1.0\,in.)^2} + \frac{0.42}{\left(1.0 W/in.\cdot{}^oC\right)(0.05 in.)}$$
$$= 0 + 0.04\ {}^oC/W + 8.40\ {}^oC/W = 8.44\ {}^oC/W$$

Temperature contours for Cases 1 and 2 are next plotted in a plane taken through the center of the source.

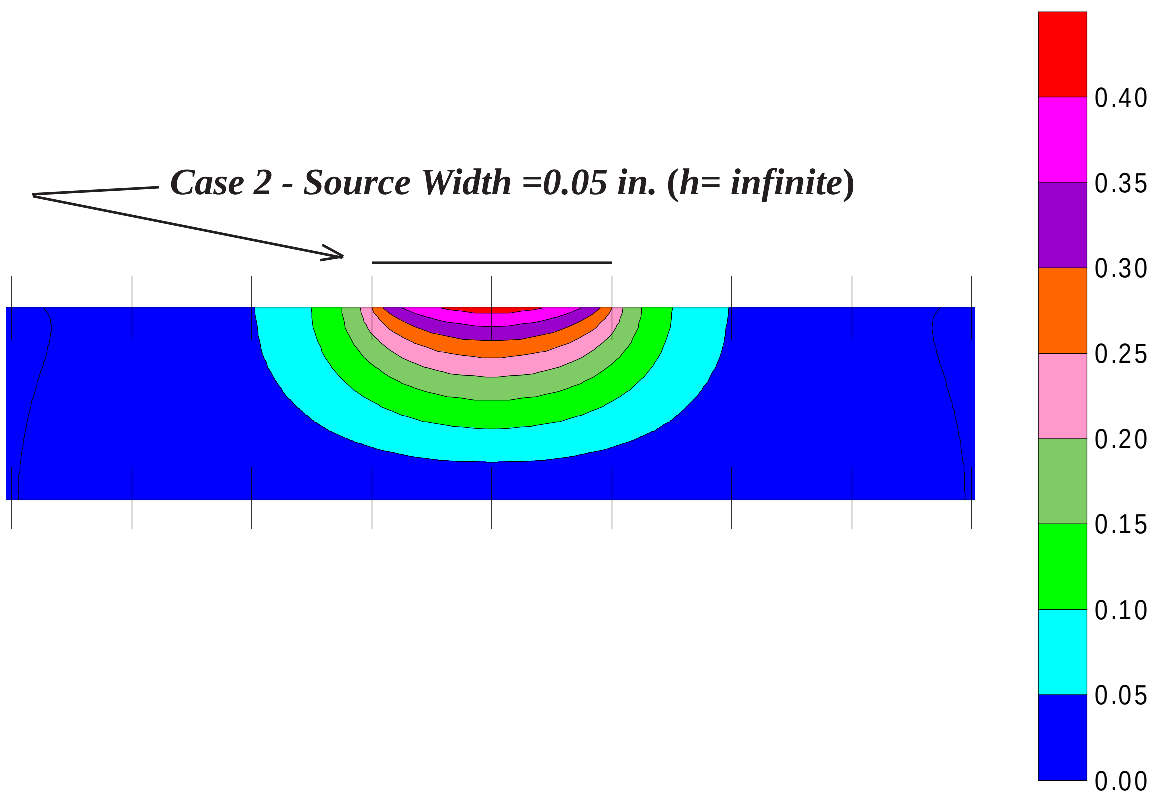
Contours of Total "Resistance"

$$k\Delta x R = k\Delta x\left(R_U + R_{Sp}\right) = k\Delta x\left[T(x, y=0, z) - T_{Ambient}\right]/Q$$

Case 1 - Source Width =0.05 in. [h=0.005 W/(in.² °C)]



Case 2 - Source Width =0.05 in. (h= infinite)



Mathematical Methods Useful In Thermal Analysis of Electronic Systems and Components

ThermalNetworks

Theory - Steady State Heat Flow in a Thermal Network

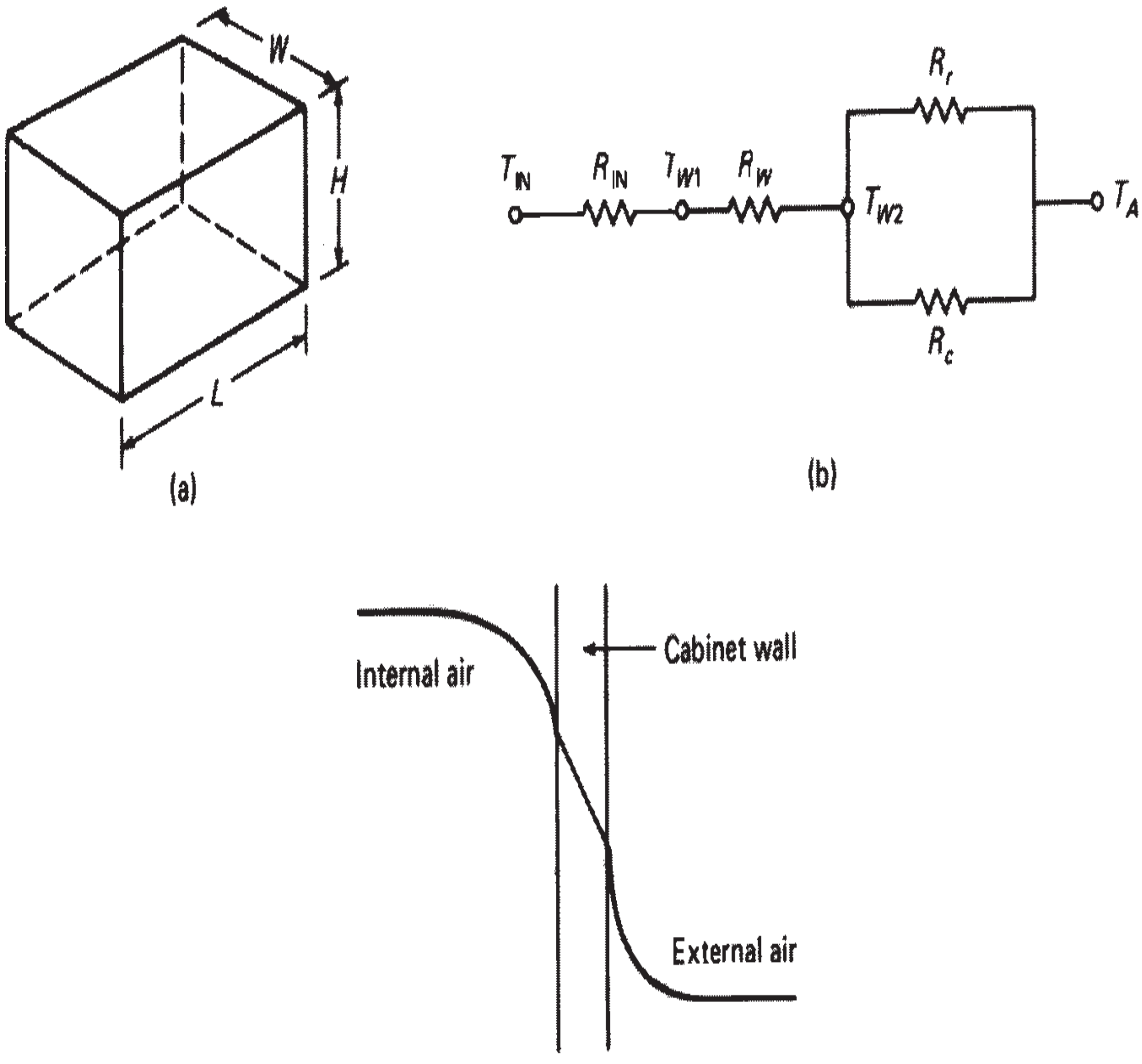


Fig. 1-11. Heat dissipation from a plastic cabinet. (a) Cabinet. (b) Thermal circuit. (c) Temperature gradient.

Heat Flow Balance

At node 2,

Heat in = Heat out

Suppose that there is a source Q_2 at node 2.

Then

$$Q_2 = C_w(T_{w2} - T_{w1}) + C_r(T_{w2} - T_A) + C_C(T_{w2} - T_A)$$

$$T_{w2} = \frac{\left[\left(C_w T_{w1} + C_r T_A + C_C T_A\right) + Q_2\right]}{\left(C_w + C_r + C_C\right)}$$

In general, at any non-fixed temperature node i,

$$T_i = \frac{\sum\limits_{j \neq i} C_{ij} T_j + Q_i}{\sum\limits_{j \neq i} C_{ij}} \qquad \text{E1.15}$$

for a Gauss-Seidel iteration scheme, which is applied when the *MODE* parameter in TNETFA is set equal to 1 (*MODE = 1*).

A "relaxed" temperature T_{Ri} may be computed from the unrelaxed temperature T_i.

$$T_{Ri} = T_{oi} + \beta(T_i - T_{oi}), \qquad \beta < 2.0 \qquad\qquad \text{E1.16, 1.17}$$

where T_{oi} is computed for the iteration prior to that for T_i.

An energy balance check may be made after any desired number of iterations. The residual r_i for node i is

$$r_i = \sum_{j \neq i} C_{ij}\left(T_i - T_j\right) - Q_i$$

A <u>system</u> energy balance is

$$E.B. = \sum_{i=1}^{\#nodes} r_i$$

with all fixed temperature nodes excluded.

Note: PC-TNETFA V1.0 and all later versions use

$$E.B. = \left[\frac{\sum_{i=1}^{\#nodes} |r_i|}{\sum_{i=1}^{\#nodes} Q_i}\right] x100.0$$

and is therefore expressed as a percent.

Generalization of

$$Q_2 = C_w(T_{w2} - T_{w1}) + C_r(T_{w2} - T_A) + C_C(T_{w2} - T_A)$$

for any node *i* is

$$\left(\sum_j C_{ij}\right)T_i + \sum_j \left(-C_{ij}\right)T_j = Q_i$$

where the summation over *j* includes only those nodes that are connected to node *i*.

The preceding equation is solvable by numerous schemes. A straight forward application of the Gauss-Jordan scheme is used in TNETFA when the *MODE* parameter is set equal to 11 (*MODE = 11*).

Strictly speaking, the Gauss-Jordan method is considered a linear equation solver. However, non-linear problems where, for example, one or more conductances are temperature dependent, are easily accommodated by repeated application of the solver to the set of algebraic equations. Prior to each solution attempt, all temperature dependent conductances are updated to correspond to the most recently computed set of temperatures.

A completely linear problem (no temperature dependent thermal conductances) would therefore require only one iteration.

The relaxation control described on the preceding page does not apply to the Gauss-Jordan method, but the energy balance computation is applicable.

Theory - Time Dependent Heat Flow in a Thermal Network

Forward Finite Difference in Time - An Explicit Method
(As Used in TNETFA):

Referring back to Fig. 1-11 and examining node 2 in a non-steady-state mode,

Heat in - Heat Out = Stored Energy

$$Q_2 - \left[C_w\left(T_{w2} - T_{w1}\right) + C_r\left(T_{w2} - T_A\right) + C_C\left(T_{w2} - T_A\right)\right]$$

$$= \left(\rho C_p \Delta V\right)_2 \left(T_{w2}^{\Delta} - T_{w2}\right) / \Delta t$$

where

T_{w2} = temperature of node 2 at time t

T_{w2}^{Δ} = temperature of node 2 at time $t + \Delta t$

T_{w1} = temperature of node 1 at time t

ρ = density of node 2 (outer wall 1/2)

ΔV = volume of node 2

C_p = specific heat of node 2

Generalizing for any non-fixed temperature node i

$$Q_i - \sum_{j \neq i} C_{ij}\left(T_i - T_j\right) = \frac{\left(\rho C_p \Delta V\right)_i \left(T_i^{\Delta} - T_i\right)}{\Delta t}$$

Solving for T_i^{Δ}

$$T_i^{\Delta} = T_i\left(1 - STAB_i\right) + \frac{\Delta t}{CAP_i}\left(Q_i + \sum_{j \neq i} C_{ij} T_j\right) \qquad \text{E 1.18}$$

where $$CAP_i = \left(\rho C_p \Delta V\right)_i \qquad \text{E 1.19}$$

$$STAB_i = \frac{\Delta t}{CAP_i} \sum_{j \neq i} C_{ij} \qquad \text{E 1.20}$$

Required for numerical stability,

$$STAB_i \leq 1.0$$

$$\Delta t \leq \frac{CAP_i}{\sum_{j \neq i} C_{ij}} \qquad \text{E 1.21}$$

Notice that Eqn E 1.18 indicates that a nodal temperature at any given time step is calculated from temperatures at a previous time. This is why the method is call *explicit.*

Forward Finite Difference in Time - An Explicit Method
(As Recommended by Holman, 1990):

$$Q_i - \sum_{j \neq i} C_{ij}\left(T_i - T_j\right) = \frac{\left(\rho C_p \Delta V\right)_i \left(T_i^{\Delta} - T_i\right)}{\Delta t}$$

Solving for T_i^{Δ}

$$T_i^{\Delta} = \frac{\Delta t}{CAP_i}\left[Q_i + \sum_{j \neq i} C_{ij}\left(T_j - T_i\right)\right] + T_i$$

where $\quad CAP_i = \left(\rho C_p \Delta V\right)_i \qquad$ E 1.19

Holman contends that this formulation can result in fewer roundoff errors with large C_{ij} or small R_{ij}. The stability of the solution must still be considered as determined by *E 2.21*.

Backward Difference in Time - An Implicit Method
Set Up for Gauss Seidel Iterative Solution Method:

Heat into node i - Heat out of node i =
Thermal energy stored

$$Q_i^{\Delta} - \sum_j C_{ij}\left(T_i^{\Delta} - T_j^{\Delta}\right) = \frac{\left(\rho C_p \Delta V\right)_i}{\Delta t}\left(T_i^{\Delta} - T_i\right)$$

Solving for T_i^{Δ}

$$Q_i^{\Delta} - T_i^{\Delta} \sum_{j \neq i} C_{ij} + \sum_{j \neq i} C_{ij} T_j^{\Delta} = \frac{\left(\rho C_p \Delta V\right)_i}{\Delta t} T_i^{\Delta} - \frac{\left(\rho C_p \Delta V\right)_i}{\Delta t} T_i$$

$$T_i^{\Delta} = \frac{\sum_{j \neq i} C_{ij} T_j^{\Delta} + \frac{\left(\rho C_p \Delta V\right)_i}{\Delta t} T_i + Q_i^{\Delta}}{\sum_{j \neq i} C_{ij} + \frac{\left(\rho C_p \Delta V\right)_i}{\Delta t}}$$

Backward Difference in Time - An Implicit Method
Set Up for Simultaneous Equation Solution Methods:

Heat into node *i* - Heat out of node *i* = Thermal energy stored

$$Q_i^{\Delta} - \sum_j C_{ij}\left(T_i^{\Delta} - T_j^{\Delta}\right) = \frac{\left(\rho C_p \Delta V\right)_i}{\Delta t}\left(T_i^{\Delta} - T_i\right)$$

$$Q_i^{\Delta} - \sum_j C_{ij}\left(T_i^{\Delta} - T_j^{\Delta}\right) = \frac{CAP_i}{\Delta t}\left(T_i^{\Delta} - T_i\right)$$

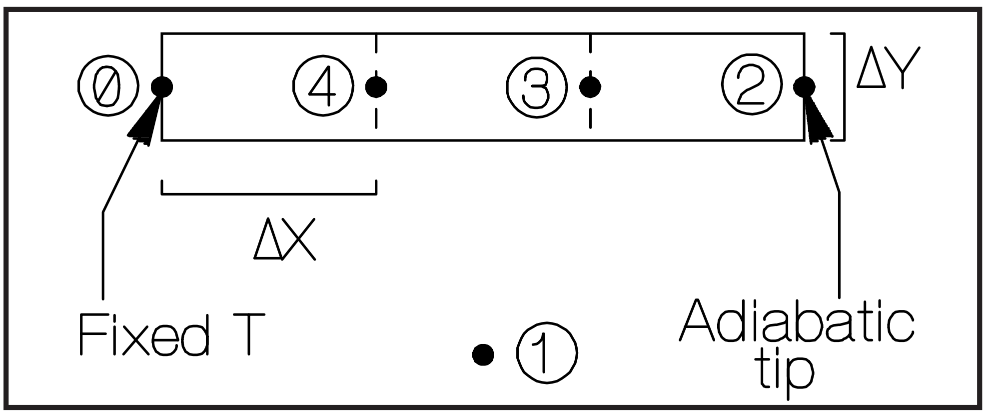
$$\left(\frac{CAP_i}{\Delta t} + \sum_j C_{ij}\right)T_i^{\Delta} - \sum_j C_{ij} T_j^{\Delta} = \frac{CAP_i}{\Delta t} T_i + Q_i^{\Delta}$$

Notice that this equation cannot be solved explicitly for T_i^{Δ} at time $t + \Delta t$ in terms of $T_i,\ T_j$ at time t. This is why the equations must be solved by an *implicit* method. This method *has not been implemented in TNETFA.*

An example that demonstrates the implicit method set up for a simultaneous solution method:

A four node fin problem.

Node 1: air
Node 2: fin section with adiabatic tip
Node 3: fin center section
Node 4: fin center section
"Node 0": fixed temperature base



$$C_{12} = 2h\left(\frac{\Delta x}{2}\right)\Delta z; \quad C_{13} = 2h\Delta x\Delta z; \; C_{14} = 2h\Delta x\Delta z$$

$$C_{23} = k\frac{\Delta y\Delta z}{\Delta x}; \quad C_{21} = C_{12}$$

$$C_{32} = k\frac{\Delta y\Delta z}{\Delta x}; \quad C_{34} = k\frac{\Delta y\Delta z}{\Delta x}; \quad C_{31} = C_{13}$$

$$C_{43} = C_{34}; \quad C_{40} = k\frac{\Delta y\Delta z}{\Delta x}; \; C_{41} = C_{14};$$

$$C_{40} = k\frac{\Delta y\Delta z}{\Delta x}$$

Write system of equations from generalized equations:

At node 1 -

$$\left(\frac{CAP_1}{\Delta t} + C_{10} + C_{12} + C_{13} + C_{14}\right)T_1^\Delta - C_{10}T_0^\Delta - C_{12}T_2^\Delta - C_{13}T_3^\Delta - C_{14}T_4^\Delta = \frac{CAP_1}{\Delta t}T_1 + Q_1^\Delta$$

At node 2 -

$$\left(\frac{CAP_2}{\Delta t} + C_{21} + C_{23}\right)T_2^\Delta - C_{21}T_1^\Delta - C_{23}T_3^\Delta = \frac{CAP_2}{\Delta t}T_2 + Q_2^\Delta$$

At node 3 -

$$\left(\frac{CAP_3}{\Delta t} + C_{31} + C_{32} + C_{34}\right)T_3^\Delta - C_{31}T_1^\Delta - C_{32}T_2^\Delta - C_{34}T_4^\Delta = \frac{CAP_3}{\Delta t}T_3 + Q_3^\Delta$$

At node 4 -

$$\left(\frac{CAP_4}{\Delta t} + C_{40} + C_{41} + C_{43}\right)T_4^\Delta - C_{40}T_0^\Delta - C_{41}T_1^\Delta - C_{43}T_3^\Delta = \frac{CAP_4}{\Delta t}T_4 + Q_4^\Delta$$

Re-writing equations to line up identical temperatures in the same columns and noting $T_0^\Delta \equiv T_0$:

At node 1 -

$$\left(\frac{CAP_1}{\Delta t} + C_{10} + C_{12} + C_{13} + C_{14}\right)T_1^\Delta - C_{12}T_2^\Delta -$$

$$C_{13}T_3^\Delta - C_{14}T_4^\Delta = C_{10}T_0^\Delta + \frac{CAP_1}{\Delta t}T_1 + Q_1^\Delta$$

At node 2 -

$$-C_{21}T_1^\Delta + \left(\frac{CAP_2}{\Delta t} + C_{21} + C_{23}\right)T_2^\Delta - C_{23}T_3^\Delta - 0 \cdot T_4^\Delta =$$

$$\frac{CAP_2}{\Delta t}T_2 + Q_2^\Delta$$

At node 3 -

$$-C_{31}T_1^\Delta - C_{32}T_2^\Delta + \left(\frac{CAP_3}{\Delta t} + C_{31} + C_{32} + C_{34}\right)T_3^\Delta - C_{34}T_4^\Delta =$$

$$\frac{CAP_3}{\Delta t}T_3 + Q_3^\Delta$$

At node 4 -

$$-C_{41}T_1^\Delta - 0 \cdot T_2^\Delta - C_{43}T_3^\Delta + \left(\frac{CAP_4}{\Delta t} + C_{40} + C_{41} + C_{43}\right)T_4^\Delta =$$

$$C_{40}T_0 + \frac{CAP_4}{\Delta t}T_4 + Q_4^\Delta$$

A program (BASIC, C, FORTRAN, MAPLE, or even Mathcad) could now be written using "canned" simultaneous equation solver that iteratively applied to the matrix problem

$$\mathbf{CT} = \mathbf{Q}$$

where $\mathbf{C}$ is the 4x4 matrix representing the coefficients of each T_i^{Δ}, **T** is the column vector representing the T_i^{Δ}, and **Q** is the column vector representing all of the terms on the right hand side of the system of equations.

The main aspects of the algorithm are:

1. Store initial conditions in **T** and **Q**.
2. Solve matrix problem for **T**.
3. Store solved temperatures from **T** in **Q**.
4. Repeat steps 2 - 3 for as many steps as desired.

Note: the time step must be kept reasonably small.

Fourier Series Methods

Theoretical Basis - Geometry

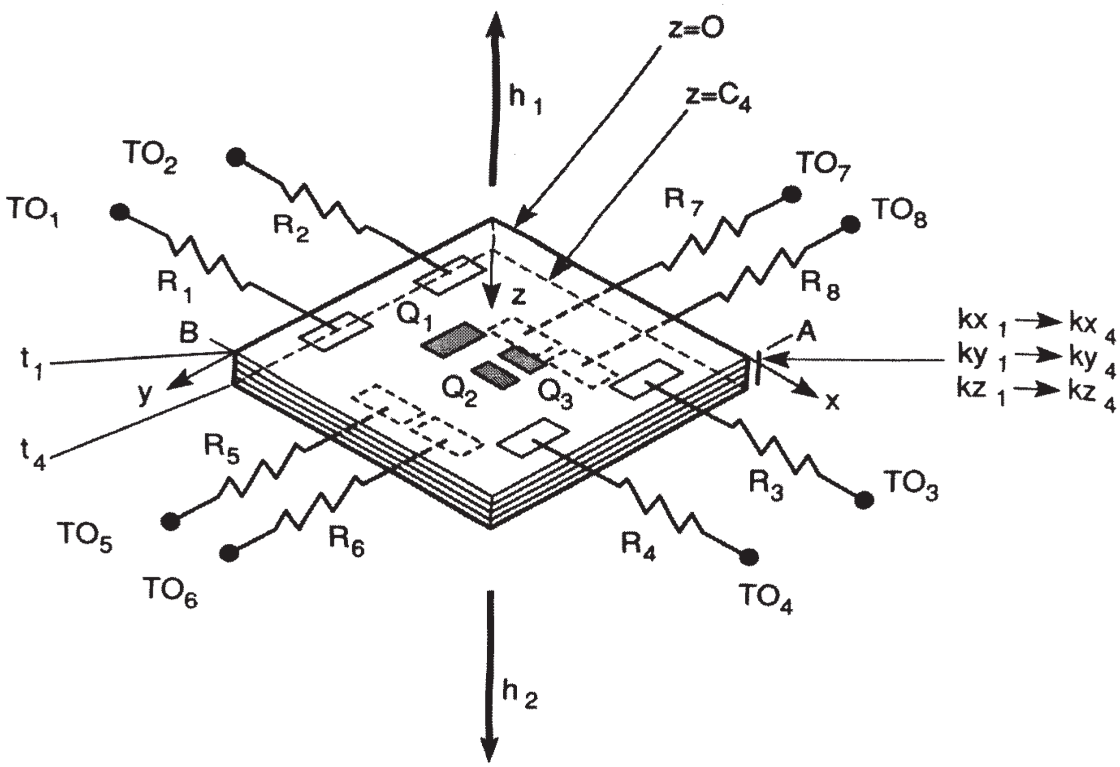


Figure 3.13 Geometry and revelant heat transfer quantities for heat sources and lumped parameter thermal resistances on a multilayer substrate [Ellison (1984)] [Reprinted with premission of the International Society for Hybrid Microelectronics, Reston, VA].

Theoretical Basis - Summary

Heat Conduction Equation:

$$k_{xi}\frac{\partial^2 T}{\partial x^2}+k_{yi}\frac{\partial^2 T}{\partial y^2}+k_{zi}\frac{\partial^2 T}{\partial z^2}=-Q_V$$

Boundary Conditions(Insulated) at Edges:

$$\left.\begin{aligned} k_{xi}\frac{\partial T}{\partial x}&=0; \qquad x=o,A \\ k_{yi}\frac{\partial T}{\partial y}&=0; \qquad y=0,B \end{aligned}\right] \qquad EII.1$$

Boundary Conditions (Radiation/Convection) at Surfaces:

$$\left.\begin{aligned} k_{z1}\frac{\partial T}{\partial z}-h_1T&=0; \qquad z=0 \\ k_{z4}\frac{\partial T}{\partial z}+h_2T&=0; \qquad z=c_4 \end{aligned}\right] \qquad EII.2$$

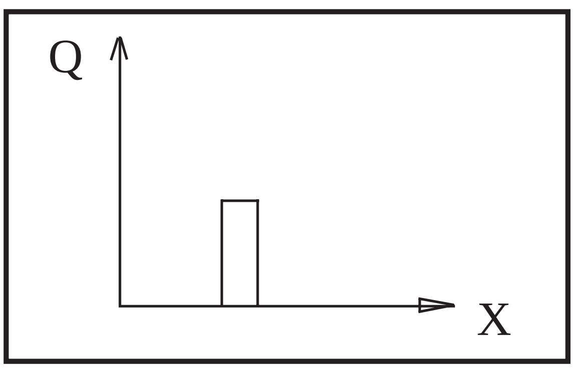
Temperature Representation:

$$T(r) = \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \varepsilon_l \varepsilon_m \psi_{lm}(z) \cos\left(\frac{l\pi x}{A}\right) \cos\left(\frac{m\pi y}{B}\right)$$

$$\varepsilon_l = \begin{bmatrix} 1/2, & l = 0 \\ 1, & l \neq 0 \end{bmatrix} \qquad l = 0,1,2\ldots$$

$$\varepsilon_m = \begin{bmatrix} 1/2, & m = 0 \\ 1, & m \neq 0 \end{bmatrix} \qquad m = 0,1,2\ldots$$

Source Representation;



$$Q_V(r) = \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \varepsilon_l \varepsilon_m \varphi_{lm}(z) \cos\left(\frac{l\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right)$$

Solution:

Substitute T,Q_V into heat conduction equation to get:

$$\sum_l \sum_m \left[\frac{d^2\psi_{lm}}{dz^2} - \gamma_{lm}^2 \psi_{lm} \right] \cos\left(\frac{l\pi x}{A}\right) \cos\left(\frac{m\pi y}{B}\right)$$

$$= \sum_l \sum_m \left[-\frac{1}{k_z} \phi_{lm} \right] \cos\left(\frac{l\pi x}{A}\right) \cos\left(\frac{m\pi y}{B}\right)$$

Set coefficients of cos product for left, right sides equal.

$$\frac{d^2\psi_{lm}}{dz^2} - \gamma_{lm}^2 \psi_{lm} = -\frac{1}{k_z} \phi_{lm}$$

$$\gamma_{lm}^2 = \left(\frac{l\pi}{A}\right)^2 \left(\frac{k_x}{k_z}\right) + \left(\frac{m\pi}{B}\right)^2 \left(\frac{k_y}{k_z}\right)$$

Solution is of the form

$$\Psi_{lm} = A_{lm} \cosh \gamma_{lm} z + B_{lm} \sinh \gamma_{lm} z$$

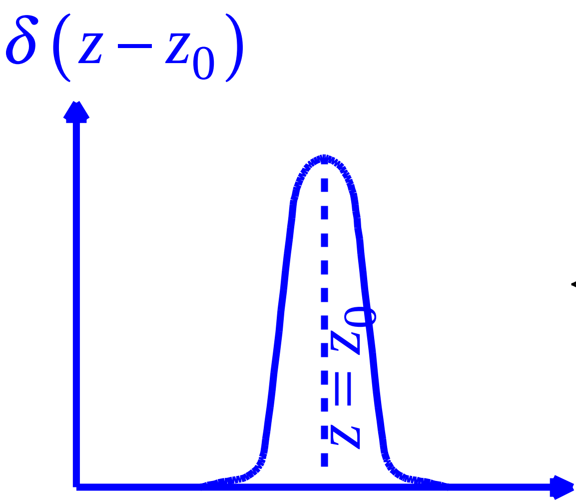
where the A_{lm}, B_{lm} are determined by satisfying the boundary conditions at each layer surface.

Theoretical Basis - More Detailed, But First An Aside

The Kronecker Delta:

$$\delta_{lm} = \begin{cases} 0, & l \neq m \\ 1, & l = m \end{cases}$$

The Dirac Delta Function:



Definition

$$\begin{cases} \delta(z - z_0) = 0, \quad z \neq z_0 \\ \displaystyle\int_{\text{Including } z = z_0} \delta(z - z_0)\, dz = 1 \end{cases}$$

Some Properties,

$$\delta(z) = \delta(-z), \quad z\delta'(z) = -\delta(z)$$

$$\delta'(z) = -\delta'(-z), \delta(az) = \frac{1}{a}\delta(z)$$

$$z\delta(z) = 0, \quad f(z)\delta(z - z_0) = f(z_0)\delta(z - z_0)$$

Theoretical Basis - More Detailed

Review of how Fourier coefficients are calculated:

Multiply both sides of source function by

$$\cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{m'\pi y}{b}\right)$$

and integrate, i.e.

$$\int_0^a\int_0^b Q_V \cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{m'\pi y}{b}\right)dxdy =$$

$$\int_0^a\int_0^b \sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \varepsilon_l \varepsilon_m \Phi_{lm}(z)\cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)$$

$$\bullet\cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{m'\pi y}{b}\right)dxdy$$

$$\int_0^a \int_0^b Q_V \cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{m'\pi y}{b}\right) dxdy =$$

$$\int_0^a \int_0^b \left(\frac{1}{2}\right)^2 \Phi_{00} \cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{m'\pi y}{b}\right) dxdy +$$

$$\int_0^a \int_0^b \sum_{l=1}^{\infty} \left(\frac{1}{2}\right)\Phi_{l0} \cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{m'\pi y}{b}\right) dxdy +$$

$$\int_0^a \int_0^b \sum_{m=1}^{\infty} \left(\frac{1}{2}\right)\Phi_{0m} \cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\cos\left(\frac{m'\pi y}{b}\right) dxdy +$$

$$\int_0^a \int_0^b \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \Phi_{lm} \cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)\cos\left(\frac{m'\pi y}{b}\right) dxdy$$

The integrations are straightforward (with a little care). For example:

$$\int_0^a \cos\left(\frac{l'\pi x}{a}\right)dx = \left(\frac{a}{l'\pi}\right)\int_0^{l'\pi} \cos u\,du = \left(\frac{a}{l'\pi}\right)\sin u\Big|_0^{l'\pi}$$

$$= \frac{a}{l'\pi}\sin\left(l'\pi\right) = \begin{cases} a, & l' = 0 \\ 0, & l' \neq 0 \end{cases} = a\delta_{l'0}$$

where $\delta_{l'0}$ is the Kronecker delta.

Similarly,

$$\int_0^b \cos\left(\frac{m'\pi y}{b}\right)dy = b\delta_{m'0}$$

The next integrals are:

$$\int_0^a \cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{l\pi x}{a}\right)dx = \frac{a}{\pi}\int_0^\pi \cos(l'u)\cos(lu)\,du$$

$$= \frac{a}{\pi}\left\{\frac{\sin[(l'-l)u]}{2(l'-l)} + \frac{\sin[(l'+l)u]}{2(l'+l)}\right\}\Bigg|_0^\pi =$$

$$\frac{a}{\pi}\begin{cases}\frac{\pi}{2},\ l'=l\\ 0,\ \ l'\neq l\end{cases} = \frac{a}{2}\delta_{l'l};\ l',l\neq 0$$

Similarly:

$$\int_0^b \cos\left(\frac{m'\pi y}{b}\right)\cos\left(\frac{m\pi y}{b}\right)dy == \frac{b}{2}\delta_{m'm};\ \ m',\ m\neq 0$$

In a shorthand form:

$$\int_0^a\int_0^b Q_V \cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{l'\pi x}{a}\right)dxdy =$$

$$\frac{1}{4}\Phi_{00}ab\delta_{l'0}\delta_{m'0} + \frac{1}{2}\sum_{l=1}^{\infty}\Phi_{l0}\left(\frac{a}{2}\right)\delta_{l'l}b\delta_{m'0}$$

$$+\frac{1}{2}\sum_{m=1}^{\infty}\Phi_{0m}a\delta_{l'0}\left(\frac{b}{2}\right)\delta_{m'm} +$$

$$\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\Phi_{lm}\left(\frac{a}{2}\right)\delta_{l'l}\left(\frac{b}{2}\right)\delta_{m'm}$$

or

$$\left(\frac{4}{ab}\right)\int_0^a\int_0^b Q_V \cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{m'\pi y}{b}\right)dxdy$$

$$= \Phi_{00}\delta_{l'0}\delta_{m'0}$$

$$+\sum_{l=1}^{\infty}\Phi_{l0}\delta_{l'l}\delta_{m'0} + \sum_{m=1}^{\infty}\Phi_{0m}\delta_{l'0}\delta_{m'm} +$$

$$\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\Phi_{lm}\delta_{l'l}\delta_{m'm}$$

We now turn our attention to any single source Q_V at $z=z_0$:

$$Q_V = q\left(x, y\right)\delta\left(z - z_0\right)$$

where δ is the Dirac delta function. Then

$$Q = \int Q_V dxdydz = \int_z \delta(z - z_0)dz \int_{\Delta x}\int_{\Delta y} q(x, y)dxdy$$

For the typical case of $z_0=0$,

$$Q = \left(\int_z \delta(z)dz\right)\left(\int_{\Delta x}\int_{\Delta y} q(x, y)dxdy\right)$$

and for q(x,y) = q uniform over $\Delta x,\ \Delta y$,

$$Q = (1)\left(q\Delta x\Delta y\right) \Rightarrow q \equiv flux\left[W/area\right]$$

Then for any z_0:

$$\left(\frac{4}{ab}\right)\int_0^a\int_0^b q\delta\left(z-z_0\right)\cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{m'\pi y}{b}\right)dxdy =$$

$$\Phi_{00}\delta_{l'0}\delta_{m'0} + \sum_{l=1}^{\infty}\Phi_{l0}\delta_{l'l}\delta_{m'0}$$

$$+\sum_{m=1}^{\infty}\Phi_{0m}\delta_{l'0}\delta_{m'm} + \sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\Phi_{lm}\delta_{l'l}\delta_{m'm}$$

We find the individual Fourier coefficients for the source by selecting various $l', l, m', and\ m\ values$. Then selecting $l' = 0,\ m' = 0$:

$$\Phi_{00} = \frac{4}{ab}\int_0^a\int_0^b q\delta\left(z-z_0\right)\cos\left(\frac{l'\pi x}{a}\right)\cos\left(\frac{m'\pi y}{b}\right)dxdy$$

$$= \frac{4q\delta\left(z-z_0\right)\left(x_2-x_1\right)\left(y_2-y_1\right)}{ab}$$

Selecting $l' = l \neq 0,\ m' = 0$,

$$\Phi_{l0} = \frac{4q\delta\left(z - z_0\right)\left(y_2 - y_1\right)}{ab} \int_{x_1}^{x_2} \cos\left(\frac{l\pi x}{a}\right) dx =$$

$$\frac{4q\delta\left(z - z_0\right)\left(y_2 - y_1\right)}{\pi lb}\left[\sin\left(\frac{l\pi x_2}{a}\right) - \sin\left(\frac{l\pi x_1}{a}\right)\right]$$

Similarly,

$$\Phi_{0m} = \frac{4q\delta\left(z - z_0\right)\left(x_2 - x_1\right)}{\pi ma} \bullet \left[\sin\left(\frac{m\pi y_2}{b}\right) - \sin\left(\frac{m\pi y_1}{b}\right)\right]$$

$$\Phi_{lm} = \frac{4q\delta\left(z - z_0\right)}{\pi^2 lm}\left[\sin\left(\frac{l\pi x_2}{a}\right) - \sin\left(\frac{l\pi x_1}{a}\right)\right] \bullet \left[\sin\left(\frac{m\pi y_2}{b}\right) - \sin\left(\frac{m\pi y_1}{b}\right)\right]$$

Temperature representation:

A representation of temperature as

$$T(r) = \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \varepsilon_l \varepsilon_m \psi_{lm}(z) \cos\left(\frac{l\pi x}{A}\right) \cos\left(\frac{m\pi y}{B}\right)$$

$$\varepsilon_l = \begin{bmatrix} 1/2, & l = 0 \\ 1, & l \neq 0 \end{bmatrix} \qquad l = 0,1,2\ldots$$

$$\varepsilon_m = \begin{bmatrix} 1/2, & m = 0 \\ 1, & m \neq 0 \end{bmatrix} \qquad m = 0,1,2\ldots$$

satisfies the edge boundary conditions

$$\left.\begin{array}{ll} k_{xi} \dfrac{\partial T}{\partial x} = 0; & x = o, A \\ & \\ k_{yi} \dfrac{\partial T}{\partial y} = 0; & y = 0, B \end{array}\right] \qquad EII.1$$

Substitution of both the source and temperature functions into the partial differential equation results in

$$\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\left\{-\left[k_{x_i}\left(\frac{l\pi}{a}\right)^2+k_{y_i}\left(\frac{m\pi}{b}\right)^2\right]\psi_{lm}+k_{z_i}\frac{d^2\psi_{lm}}{dz^2}\right\}\bullet$$

$$\varepsilon_l\varepsilon_m\cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)$$

$$=-\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\phi_{lm}\varepsilon_l\varepsilon_m\cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)$$

Setting the coefficients of like terms

$$\varepsilon_l\varepsilon_m\cos\left(\frac{l\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right)$$

equal, a one-dimensional differential equation in z is obtained.

$$\frac{d^2\psi_{lm}}{dz^2}-\gamma_{lm}^2\psi_{lm}=-\frac{1}{k_{z_i}}\phi_{lm}$$

$$\gamma_{lm}^2=\alpha_l^2+\beta_m^2$$

$$\alpha_l^2=\left(\frac{k_{x_i}}{k_{z_i}}\right)\left(\frac{l\pi}{a}\right)^2,\quad \beta_m^2=\left(\frac{k_{y_i}}{k_{z_i}}\right)\left(\frac{m\pi}{b}\right)^2$$

Solution of one-dimensional equation in z:

The problem is now to find the Fourier coefficients for the temperature function from the one-dimensional equation.

The boundary conditions on T in the z-direction are easily shown to apply to ψ_{lm}.

$$k_{z_i} \left.\frac{d\psi_{lm}}{dz}\right|_{z=0} = h_1 \psi_{lm}\big|_{z=0}, \quad k_{z_i} \left.\frac{d\psi_{lm}}{dz}\right|_{z=c_4} = -h_2 \psi_{lm}\big|_{z=c_4}$$

$$\psi_{lm}\big|_{z=c_i^-} = \psi_{lm}\big|_{z=c_i^+}, \qquad i = 1,2,3$$

$$k_{z_i} \left.\frac{d\psi_{lm}}{dz}\right|_{z=c_i^-} = k_{z_i} \left.\frac{d\psi_{lm}}{dz}\right|_{z=c_i^+}, \quad i = 1,2,3$$

where c_i^-, c_i^+ and $z = c_i - \varepsilon, z = c_i + \varepsilon$, respectively for $\varepsilon \to 0$.

Use of the Green's function method to find ψ_{lm} -

$$\frac{d^2\psi_{lm}(z)}{dz^2} - \gamma_{lm}^2 \psi_{lm}(z) = -\frac{1}{k_{z_i}} \phi_{lm}(z)$$

$$\psi_{lm} = \int_0^{c_2} \frac{1}{k_{z_i}} G(z|z')\phi_{lm}(z')dz'$$

where $G(z|z')$ is the desired Green's function.

The subscripts l,m are dropped. Since z is an independent variable, it is changed from z to z' and the dependence of $\psi, \phi,$ and γ on z' is understood. Multiply both sides of

$$\frac{d^2\psi}{dz'} - \gamma^2\psi = -\frac{1}{k_{z_i}}\phi$$

by $G(z|z')$ and integrate over the full plate thickness.

$$\int_0^{c_2} G(z|z')\left[\frac{d^2\psi}{dz'^2} - \gamma^2\psi\right]dz' = -\int_0^{c_2}\frac{1}{k_{z_i}}\phi G(z|z')dz'$$

The integrals are broken into pieces so that

$$\int_0^{c_2} = \int_0^{z_1-\varepsilon} + \int_{z_1+\varepsilon}^{c_1-\varepsilon} + \int_{c_1+\varepsilon}^{z_2-\varepsilon} + \int_{z_2+\varepsilon}^{c_2}$$

and taking the limit $\varepsilon \to 0$. The quantities z_1 and $z_{2\ 2}$ are field points in the regions $0 \le z_1 \le c_1$ and $c_1 \le z_2 \le c_2$, respectively.

Each integral with a term $G(z|z')\dfrac{d^2\psi}{dz^2}$ as an integrand is integrated twice by parts. Only the first integral is derived here.

$$\int_0^{c_2} G(z|z')\left[\frac{d^2\psi}{dz'^2} - \gamma^2\psi\right]dz' = \int_0^{z_1-\varepsilon} G(z|z')\frac{d^2\psi}{dz'^2}dz' - \int_0^{z_1-\varepsilon} \gamma^2 G(z|z')\psi dz'$$

$$= \left[G(z|z')\frac{d\psi}{dz'}\right]_{z'=0}^{z'=z_1-\varepsilon} - \int_0^{z_1-\varepsilon}\left(\frac{dG(z|z')}{dz'}\right)\left(\frac{d\psi}{dz'}\right)dz' - \int_0^{z_1-\varepsilon} \gamma^2 G(z|z')\psi dz'$$

$$= \left[G(z|z')\frac{d\psi}{dz'} - \psi\frac{dG(z|z')}{dz'}\right]_{z'=0}^{z'=z_1-\varepsilon} - \int_0^{z_1-\varepsilon}\psi\left[\frac{d^2G(z|z')}{dz'^2} - \gamma^2 G(z|z')\right]dz'$$

The first property of the Green's function is established by

$$\frac{d^2G(z|z')}{dz'^2} - \gamma^2 G(z|z') = 0, \qquad z' \neq z_1$$

and the remaining integrations are similarly completed and satisfy

$$\frac{d^2G(z|z')}{dz'^2} - \gamma^2 G(z|z') = 0, \qquad z' \neq z_2, c_1$$

After rearrangement of terms, the result is

$$
\begin{aligned}
-\int_0^{c_2} \frac{1}{k_{z_i}} G(z|z')\phi dz' = &-\left[G(z|0)\psi'(0) - G'(z|0)\psi(0)\right] \\
&+\left[G(z|c_2)\psi'(c_2) - G'(z|c_2)\psi(c_2)\right] \\
&+\left[\psi\left(z_1^+\right)G'\left(z|z_1^+\right) - \psi\left(z_1^-\right)G'\left(z|z_1^-\right)\right] \\
&+\left[\psi\left(z_2^+\right)G'\left(z|z_2^+\right) - \psi\left(z_2^-\right)G'\left(z|z_2^-\right)\right] \\
&+\left[G\left(z|z_1^-\right)\psi'\left(z_1^-\right) - G\left(z|z_1^+\right)\psi'\left(z_1^+\right)\right] \\
&+\left[G\left(z|z_2^-\right)\psi'\left(z_2^-\right) - G\left(z|z_2^+\right)\psi'\left(z_2^+\right)\right] \\
&+\left[\psi\left(c_1^+\right)G'\left(z|c_1^+\right) - \psi\left(c_1^-\right)G'\left(z|c_1^-\right)\right] \\
&+\left[G(z|)\psi'\left(c_1^-\right) - G\left(z|c_1^+\right)\psi'\left(c_1^+\right)\right]
\end{aligned}
$$

where

$z_1^+, z_1^-,$ are abbreviations for $z_1 + \varepsilon,\ z_1 - \varepsilon,$ etc., respectively.

Simplification to

$$\psi(z) = \int_0^{c_2} \frac{1}{k_{z_i}} G(z|z')\phi(z')dz'$$

requires implementation of the continuity of temperature and temperature gradient at $z' = z_1, z_2$ plus the additional Green's function properties:

$$k_{z_i} \frac{dG(z|z')}{dz'} - h_1 G(z|z') = 0, \qquad z' = 0$$

$$k_{z_i} \frac{dG(z|z')}{dz'} + h_2 G(z|z') = 0, \qquad z' = c_2$$

Also

$$\frac{dG\left(z|c_1^-\right)}{dz'} = \frac{dG\left(z|c_1^+\right)}{dz'}$$

$$k_2 G\left(z|c_1^-\right) = k_1 G\left(z|c_1^+\right)$$

and

$$G(z|z') = \text{continuous}, \qquad z' = z_1, z_2$$

If the temperature is calculated in the first layer

$$\frac{dG\left(z|z'^{+}\right)}{dz'} - \frac{dG\left(z|z'^{-}\right)}{dz'} = -1, \qquad z' = z_1$$

$$\frac{dG\left(z|z'^{+}\right)}{dz'} = \frac{dG\left(z|z'^{-}\right)}{dz'}, \qquad z' = z_2$$

If the temperature is calculated in the second layer

$$\frac{dG\left(z|z'^{+}\right)}{dz'} - \frac{dG\left(z|z'^{-}\right)}{dz'} = -1, \qquad z' = z_2$$

$$\frac{dG\left(z|z'^{+}\right)}{dz'} = \frac{dG\left(z|z'^{-}\right)}{dz'}, \qquad z' = z_1$$

Solution form of Green's function -

For the four layer problem,

$$
\begin{aligned}
\gamma = 0: \quad & G = Az' + B & & z' < z \\
& = Cz' + D & & z < z' < c_1 \\
& = E_i z' + F_i & & c_1 < z', \quad i = 2-4 \\
\gamma \neq 0: \quad & G = G\sinh\gamma z' + H\cosh\gamma z' & & z' < z \\
& = I\sinh\gamma z' + J\cosh\gamma z' & & z < z' < c_1 \\
& = K\sinh\gamma z' + L\cosh\gamma z' & & c_1 < z', \quad i = 2-4
\end{aligned}
$$

The various Green's functions are then integrated as required. The use of the Dirac delta function for the z-dependence of the source function simplifies the integration:

$$\psi = \int \frac{1}{k_{z_i}} G(z|z')\phi(z')dz'$$

The multi-source problem:

For a single source

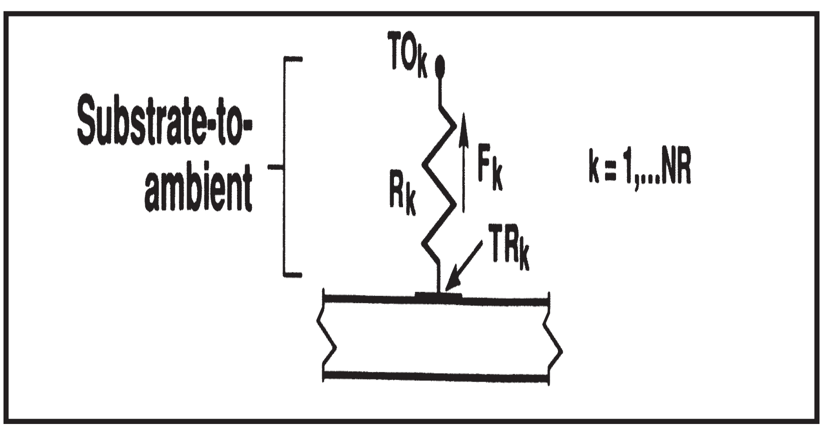
$$T = \theta Q_S + T_A$$

where θ is the Fourier series solution for a single source with a dissipation Q_S. θ may also be referred to as an "influence coefficient".

For NS sources,

$$T = \sum_{j=1}^{NS} \theta_j Q_{Sj} + T_A$$

Heat removal by multiple lumped resistances



Referring to the preceding illustration,

$$T_{R_l} - T_{O_l} = R_l F_l$$

When extended to multiple resistances and applying the multi-source methodology,

$$T - T_A = \sum_{j=1}^{NS} \theta_j Q_{Sj} - \sum_{k=1}^{NR} \theta_k F_k$$

Applying this equation for temperature T to the specific case of the temperature T_{Rl} on the substrate surface at the *lth* resistance site, substracting T_{Ol} from both sides and moving T_A to the right side,

$$T_{Rl} - T_{Ol} = \sum_{j=1}^{NS} \theta_{lj} Q_{Sj} - \sum_{k=1}^{NR} \theta_{lk} F_k + (T_A - T_{Ol})$$

Setting the right-hand sides of this equation and that at the top of the page equal to one another

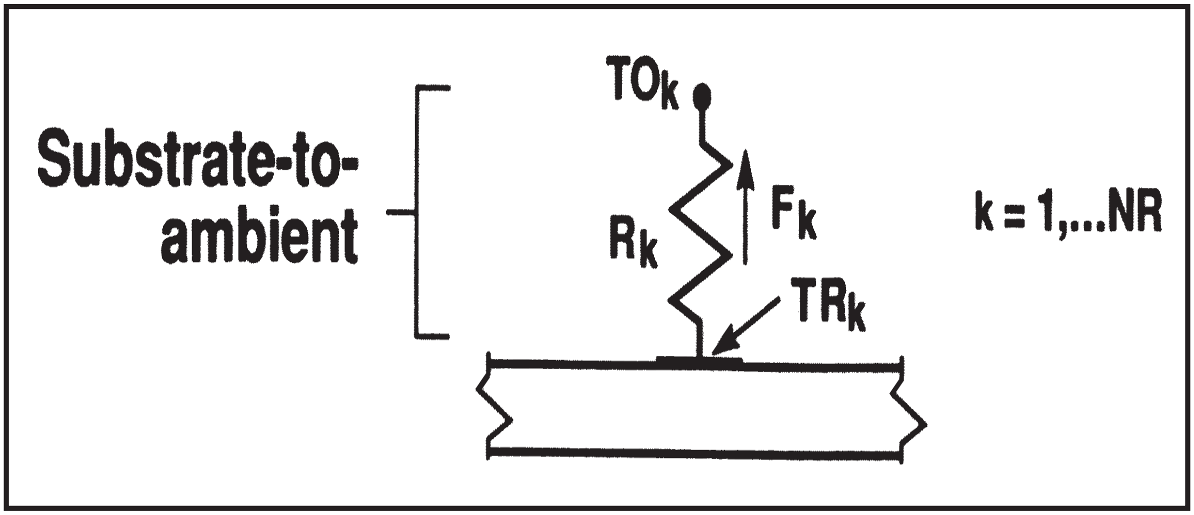
$$R_l F_l = \sum_{j=1}^{NS} \theta_{lj} Q_{Sj} - \sum_{k=1}^{NR} \theta_{lk} F_k + \left(T_A - T_{Ol}\right)$$

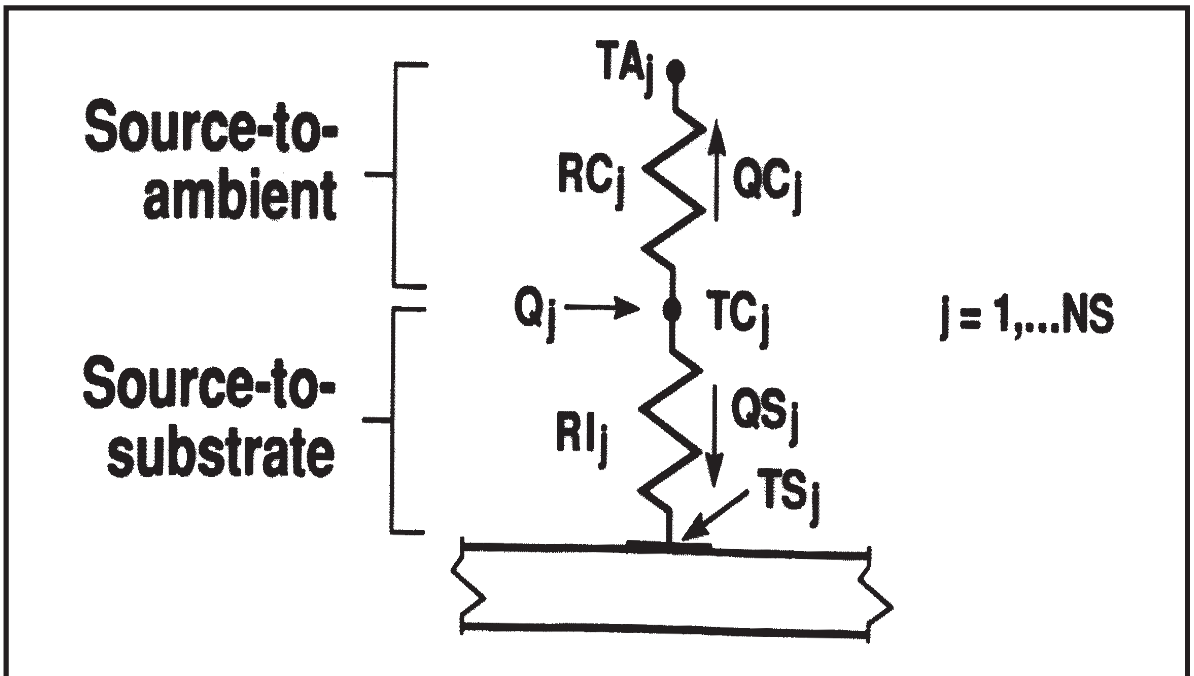
$$\sum_{j=1}^{NS}\theta_{lj}Q_{Sj} - \left(\sum_{k=1}^{NR}\theta_{lk}F_k + R_l F_l\right) = \left(T_{Ol} - T_A\right)$$

$$\sum_{j=1}^{NS}\theta_{lj}Q_{Sj} - \sum_{k=1}^{NR}(\theta_{lk} + R_k\delta_{lk})F_k = \left(T_{Ol} - T_A\right);\ l = 1, NR$$

The TAMS program uses this equation to calculate the heat transfer F_k in each of the NR resistances

Fourier method applied to similar substrate problem, but with resistances between source and substrate and between source and local ambient -





The TAMS theory associated with the substrate-to-ambient thermal resistances (including upper illustration on preceding page) applies to this program.

The method is similarly applied to the lower illustration on the preceding page associated with the sources.

The temperature drop from source-to-substrate is

$$T_{Ci} - T_{Si} = R_{Ii} Q_{Si}$$

and the the temperature drop from the source to local ambient is

$$T_{Ci} - T_{Ai} = R_{Ci} Q_{Ci}$$

The temperature drop from the substrate to the local ambient is

$$\begin{aligned} T_{Si} - T_{Ai} &= R_{Ci} Q_{Ci} - R_{Ii} Q_{Si} \\ &= R_{Ci}\left(Q_i - Q_{Si}\right) - R_{Ii} Q_{Si} \\ T_{Si} - T_{Ai} &= R_{Ci} Q_i - \left(R_{Ci} + R_{Ii}\right) Q_{Si} \end{aligned}$$

The term Q_i is the actual source dissipation and Q_{Si} is the portion of the source dissipation that conducts directly into the substrate.

The temperature at any *ith* source, with all possible resistance terms included is

$$T_{Si} - T_{Ai} = \sum_{j=1}^{NS} \theta_{ij} Q_{Sj} - \sum_{k=1}^{NR} \theta_{ik} F_k + (T_A - T_{Ai})$$

The right hand side of this equation is set equal to the right hand side of the last equation on the preceding page.

$$R_{Ci} Q_i - \left(R_{Ci} + R_{Ii}\right) Q_{Si} = \sum_{j=1}^{NS} \theta_{ij} Q_{Sj} - \sum_{k=1}^{NR} \theta_{ik} F_k + (T_A - T_{Ai})$$

$$\sum_{j=1}^{NS} \theta_{ij} Q_{Sj} + \left(R_{Ci} + R_{Ii}\right) Q_{Si} - \sum_{k=1}^{NR} \theta_{ik} F_k = R_{Ci} Q_i + (T_{Ai} - T_A)$$

$$\sum_{j=1}^{NS} \left[\theta_{ij} + \left(R_{Cj} + R_{Ij}\right)\delta_{ij}\right] Q_{Sj} - \sum_{k=1}^{NR} \theta_{ik} F_k = R_{Ci} Q_i + (T_{Ai} - T_A);\ i = 1, NS$$

$$\sum_{j=1}^{NS} \theta_{lj} Q_{Sj} - \sum_{k=1}^{NR} (\theta_{lk} + R_k \delta_{lk}) F_k = \left(T_{Ol} - T_A\right);\ l = 1, NR$$

The last equation is used with the similar TAMS equation to determine (in the PTAMS program) the heat conducted from the source-to-substrate and from the substrate-to-local ambient.

Examples Using the

Electronics Thermal Analysis Package

on Personal Computers

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Software Packages Supplied

NPRO/TNETFA - General Purpose Modeling

Description: TNETFA calculates temperatures for any problem that may be described by a thermal network. Thermal problem may be steady-state or time-dependent. Also solves airflow/ pressure circuits.

Typical Problems: sealed enclosures, vented enclosures, fan cooled enclosures, hybrids, and chip packages

Problem Size: TNETFA V5 is a 32 bit MS Windows program. A circuit may contain up to 5000 'nodes' and approximately 25,000 conductors.

Input Format: ASCII data file. May be prepared with text editor, word processor, or supplied NPRO V5 pre/post-processor. The current version of NPRO is also a 32 bit MS Windows 95 program.

Output Format: formatted file containing temperatures listed by node number, inter-node heat flow, etc; formatted file listing only node numbers and temperatures - directly readable into NPRO for display of temperature vs. node number or time.

TPRO/TAMS - Component Modeling

Description: TAMS is an analytical solution to the three-dimensional heat conduction equation for rectangular structures with up to four layers of different thickness and thermal conductivity. Unique feature admits lumped parameter thermal resistances to remove heat from substrate.

Typical Problems: ideal for hybrids and multi-chip modules. Also used for ceramic IC packages, heat sinking (for power transistors) chassis panels, hot spot analysis of finned extrusions, and IC chip analysis.

Problem Size: TAMS V5 is a 32 bit MS Windows 95 program that uses dynamic memory allocation. The total number of Fourier series terms, heat sources, resistors, and grid points (locations where temperatures calculated) is therefore largely lilmited only by your hardware.

Input Format: ASCII data file. May be prepared with text editor, word processor, or supplied TPRO V5 pre/post processor. TPRO is an MS Windows 95 program. It is limited to 100 sources on each substrate surface, 100 resistors on each surface, and a nearly unlimited number of grid points.

Output Format: formatted file containing temperatures. Line profile data (T vs. x or y) directly readable into TPRO for display of a temperature profile. TPRO also displays temperatures superimposed onto a geometric drawing. More elegant temperature contours and maps are possible using commercial plotting programs. TPRO also displays Fourier series convergence analysis to ensure correct series truncation during solution.

PPRO/PTAMS - Circuit Board Modeling

Description: PTAMS is similar to TAMS in that it uses an analytical solution to the three-dimensional heat conduction equation for rectangular structures with up to four layers of different thickness and thermal conductivity. Lumped parameter thermal resistances may also be used here to remove heat from substrate. Principal deviation from TAMS is that PTAMS permits a thermal resistance between substrate and heat source and also between local ambient temperature and heat source. PTAMS is a 32 bit MS Windows 95 program.

Typical Problems: main use is as a circuit board thermal analyzer. Intended to give a package/hybrid designer an estimate of printed circuit board conduction effects.

Problem Size: Both PTAMS and PPRO permit up to 200*x* 200 Fourier series, 100 sources on each board surface, and 100 resistors on each board surface.

Input Format: ASCII data file. May be prepared with text editor, word processor, or supplied PPRO V5, MS Windows 95 preprocessor.

Output Format: formatted file containing temperatures. PPRO may be used to superimpose board and case temperatures on a geometry drawing. Line profile data (T vs. x or y) may be created by reading PTAMS output and input files into TPRO to create the appropriate 'grid', then executing TAMS. TPRO, TAMS, and a commercial plotting program may also be used to create board surface temperature contours and maps.. PPRO also displays Fourier series convergence analysis to ensure correct series truncation during solution.

NSINK - Heat Sink Extrusion Designing

Description: use this program for designing and analyzing extruded heat sinks cooled by natural convection and radiation. Variables include fin depth, thickness, spacing, number of, etc.

Typical Problems: power supply heat sink extrusion analysis. Also useful for flat panel heat sinks.

Problem Size: N/A.

Input Format: enter data within NSINK menu system or read TPRO/TAMS file containing basic heat sink and source geometry.

Output Format: screen display of calculated thermal resistance, temperature and effective total heat transfer coefficient. May also update TPRO/TAMS file with calculated effective heat transfer coefficient. May also create a file containing heat sink thermal conductance vs. temperature for incorporation into large TNETFA system model.

TNETFA - A General Purpose Thermal Network Analyzer

NPRO/TNETFA: Rules

Appendix iv. NPRO/TCEE - Data Set Cross Reference Table

The purpose of the following table is to assist you in identifying how Data Sets from TCEE, Tables 10-1 and 10-2 are associated with the various major options in NPRO.

TCEE Data Set	NPRO Option
1	*Edit - Title Line 1*
	Edit - Title Line 2
2	*Edit - Solution Type*
	Edit - Units
3	*Automatic by NPRO*
4	*B.C./Start Temps*
	Sources - Steady
5	*Sources - t,Q Pairs*
6	*Capacitance - Single*
	Capacitance - String of
7	*Conductors - String of*
8	*Conductors - Single*
9	*Multi-Surf-Rad - Single Input*
	Multi-Surf-Rad - Multiple Input
10	*T, K Arrays*
11	*Natural-Conv*
12	*Forced-Conv*
13	*Forced-Conv*
14	*Solution-Cntrl - Steady State*
	Soution-Cntrl - Time Dependent

TNETFA Input Format

Table 10-1. Dcscription of data sets and variables.

Data Set Number	Data Sets and Variables
1	TITLE LINES Use for problem identification. Two lines required.
2	PROBLEM TYPE IDENTIFIER A single line specifying noniterate or iterate, units, parameter run requests. MODE: 0-suppress iteration, print node connections based on input. l-attempt steady state solution, Gauss-Seidel Method (5000 nodes or less). 2-attempt velocity potential solution. 3-attempt time-dependent thermal solution. 11-attempt steady-state solution, Gauss-Jordan Method (100 nodes or less. UNITS: 0-X, L, D-ft V-ft/sec Q-Btu/hr T-deg F C-Btu/(hr · deg F) G-ft^3/min. ρCp-Btu/(ft^3 · deg F) CAP-Btu/deg F (El.19) Time-hr UNITS: 1-X, L, D-cm V-cm/sec Q-watts T-deg C C-watts/deg C G-cm^3/sec ρCp-cal(cm^3 · deg C) CAP-joules/deg C Time-sec UNITS: 2-X, L, D-in. V-ft/min. Q-watts T-deg C C-watts/deg C C-ft^3/min. ρCp-cal(cm^3 · deg C) CAP-joules/sec Time-sec

Table 10-1 . (Continuod)

Data Set Number	Data Sets and Variables
	ICSE: Number of additional problem runs using repeats of Data Sets 9-14 with parameter variations within those sets. Additional sets (9-14) are added to input following last line of set 14 for first run. Set ICSE = O for no parameter variations
3	BASIC INPUT QUANTITIES A single line specifying number of nodes, and number of lines required for most data sets Exceptions, etc., are appropriate^b identified as required. NN: total number of nodes. NCT: number of fixed-temperature nodes specified in Data Set 4, e.g., an ambient temperature node. NZS: number of nodes requiring individual starting temperature and power input in Data Set 4. NQCRV: number of time-dependent curves in Data Set 5 specifying time dependence of heat rate (power) dissipation of any heat source node. NCBLC: number of lines in Data Set 7 for automatic construction of conductor strings NCS: number of lines in Data Set 8 specifying single conductor input. NCRV: number of curves in Data Set 10 used to specify temper-ature-dependent thermal conductivity or any multiplying factor of variable C in Data Sets 7, 8. NNCNV: number of lines in Data Set 11 specifying natural convec-tion parameters. NFCNV: number of lines in Data Set 12 specifying forced convecffon parameters
4	TEMPERATURE, POWER INPUT A multiple-line data set specifying starting temperatures for steady state and time-dependent problems, fixed-temperature nodes, and constant heat rates TSET, QSET: starting temperature and heat rate for all nodes. Ex-ceptions accounted for in remainder of this data set. One input line used. NC, TC: node number and temperature for fixed-temperature node, e.g., ambient. Number of lines indicated by NCT in Data Set 3. A minimum of one data line re-quired for all problems N, T, Q: node number, starting temperature, and constant heat rate. Number of lines indicated by NZS in Data Set 3. No minimum required number of data lines

Table 10-1. (Continued)

Data Set Number	Data Sets and Variables
5	TIME-DEPENDENT HEAT RATE CURVES A multiple-line data set speclfying time-dependent heat rate dissipation for any non-fixed-temperature node. N: node number NQPRS: number of time, heat rate data pairs to specify complete Q(t) dependence. TIME, Q: time, heat rate values. A minimum of two pairs per curve required. Multiple data pairs per line acceptable. Data pairs must be input in order of increasing time. Heat rates at times outside limits specified by frst and last data pair are set at frst and last specified heat rates, respectively.
6	CAPACITANCE INPUT A mu1tlple-line data set specifying nodal thermal capacitance for time-dependent problems. These data may be input for a steady state problem in anticipation of later use in a time dependent analysis of the same network model. SINCAP, STRCAP: a header line required by all problems. Both values are zero if no additional data in this set. SINCAP: number of input lines specifying single node capacitance input via data line N, CAP, CURVE. Use zero if no input lines of this type. N, CAP, CURVE: node number, capacitance, and cune number (in Data Set 10) for temperature-dependent capacltance. If no curve required, input zero for CURVE. Actual capacitance determined by product of CAP in Data Set 6 and k in Data Set 10; therefore any combinatlon of CAP, k that gives correct value of CAP*k may be used. Value of k returned from curve in Data Set 10 determined by temperature of node N. STRCAP: number of input lines specifying identical values of CAP, CURVE for consecutive nodes NA through NB in data line NA, NB, CAP, CURVE. Use zero if no input iines of this type. NA, NB, CAP, CURVE: consecutive node numbers from and including NA through NB for nodes with capacitance C~ and using curve CURVE. All preceding comments regarding CAP, CURVE in this data set apply.

Table 10-1. (Continued)

Data Set Number	Data Sets and Variables

7 AUTOMATIC CONDUCTOR STRING GENERATOR

A data set used to simultaneously specify several conductances with identical values of C, CTYPE (see definition to follow), and the appropriate node connections. The number of input lines is indicated by NCBLC in Data Set 3. Conductor input not accommodated by the string generator is specified in Data Set 8.

NBLD: number of conductors generated by this line of data.
NAI: first node number in string.
NAS: node NAI incrementor or decrementor-may be set at zero if required.
NB 1: second node number in string.
NBS: node NB1 incrementor or decrementor-may be set at zero if required.
C: element value u specified by Table 10-3.
CTYPE: term that specifia element type corresponding to C (see Table 10-3).

8 SINGLE CONDUCTOR INPUT

A data set used to specify single conductances and the appropriate node connectlons that are not accommodated by Data Set 7. The number of input lines is indicated by NCS in Data Set 3.

NA: first node number.
NB: second node number.
C: element value as specified by Table 10-3.
CTYPE: term that specifies element type corresponding to C (see Table 10-3).

9 AREA, EMISSIVITY INPUT FOR MULTI-SURFACE RADIATION EXCHANGE

A multip1e-line data set specifying the uea and emissivity of every surface participating in multi-surface radiation exchange. This set is required when and only when a CTYPE value of "-2" is specified in any input line in Data Sets 7 and/or 8. Data Set 3 does not specify any input relevant here.

SINAE, STRAE: a header line required when this data set is used. Either value, but not both values may be zero.
SINAE: number of input lines specifying a singule surface area and emissivity input via data line N, AR, EM. Use zero if no input lines of this type.
N, AR, EM: node number, area, and emissivity.

Table 10-1. (Continued)

Data Set Number	Data Sets and Variables

STRAE: number of input lines specifying identical values of AR, EM for a string of consecutive node numbers NA through NB in data line NA, NB, AR, EM. Use zero if no input lines of this type.

NA, NB AR, EM: consecutive node numbers from and including NA through NB for nodes with area AR and emissivity EM.

10 TEMPERATURE-DEPENDENT MULTIPLICATIVE FACTORS

A multiline data set used to provide a temperature-dependent multiplying factor to the C values specified in Data Sets 7, 8 with associated CTYPE values of 1-100. This may be used to input temperature-dependent quantities such as thermal conductivity, heat transfer coefficients not within the TNETFA library of elements, etc.

Data Set 6 temperature-dependent nodal capacitance will also refer to this data set via the value CURVE in Set 6.

Conductance or capacitance common to several nodes may be varied by altermg a curve value here rather than several input lines in Data Sets 6, 7, or 8.

The number of curves is indicated by NCRV in Data Set 3.

CURVE, NPAIRS: a header line preceding each curve set.

CURVE: an integer corresponding directly to the value indicated by CTYPE in Data Sets 7, 8. CURVE values must start with the value "1" and be numbered consecutively.

NPAIRS: The number of T, k data pairs for the respective CURVE. NPAIRS must specify at least two data pairs.

T, k: The X, Y coordinates appropriately specifying the curve. A curve indicating a capacitance factor for Data Set 6 linearly interpolated will return a k value based on the temperature T of the node. CAP in Data Set 6 is multiplied by k.

A CTYPE indicating a CURVE for Data Sets 7, 8 will return a linearly interpolated k value based on a T equal to an average temperature of the two nodes interconnecting the conductances. C in Data Sets 7, 8 is multiplied by k.

Table 10-1. (Continued)

Data Set Number	Data Sets and Variables

11 NATURAL CONVECTION PARAMETERS

Natural convectionTNETFA library elements are indicated by CTYPE = 101-200 in Data Sets 7, 8, for which the appropriate C input is nodal surface area. Data Set 11 is used to input both the type (ATYPE) of natural convection and the significant dimensional parameter (AA 1).

The input sequence in which ATYPE, AAl appear is precisely CTYPE - 100; e.g., if CTYPE = 109, the corresponding ATYPE, AAl is the ninth data line in Data Set 11.

ATYPE: term that specifies device orientation and direction of heat transfer.

AAl: significant dimensional parameter: height or WL/(2W+2L)(see Table 10-4 for details).

AA2: channel spacing (surface to surface).

AA3: volumetric flow rate when surface temperature referenced to local air.

12 FORCED CONVECTION PARAMETERS

Forced convection TNETFA library elements are indicated by CYTPE = 201-300 in Data Sets 7, 8, for which the appropriate C input is nodal surface area Data Set 12 is used to input the type (BTYPE) of forced convection dimensional parameters, airflow rate, etc.

BTYPE: specifies type of forced airflow.

BBl-BB4: required parameters (see Table 1~5).

13 VELOCITY POTENTIAL

This is a one-line data set used only when MODE = 2, the velocity potential case.

VO: inlet air velocity.

14 RUN CONTROL STATEMENTS

This three-line data set controls iterations (steady state), time stepS (time dependent), and print intervals,

NLOOP: maximum number of steady s~ate iterations.

BETA: steady state overrelaxation constant; 0.0<BETA < 2.0 (El.17).

ALDT: maximum allowed temperature change between any two successive steady state iterations. Iteration is terminated by a maximum temperature change per iteration, MAXDT* < ALDT.

LOOPEN: number of steady state iterations between printouts of total system energy balance.

DELT: time step for time-dependent computations (El.21).

MAXT: time limit on time-dependent mode.

Table 10-1. (Continued)

Data Set Number	Data Sets and Variables
	TPRINT: number of steady state iterations or time steps (TPRINT = time printout interval/DELT) between temperature printouts
	NPRINT: indicator used to wppress (NPRINT = 0) or print (NPRINT = 1) details of node connections.

*Ignore MAXDT output value for zero iterations (LOOPCT = 0).

Table 10-2. Arrangement of variables in order of input.

Data Set	Input Variables									Require (1)
1	TITLE LINE 1									X
	TITLE LINE 2									X
2	MODE^I	UNITS^I	ICSE^I							X
3	NN^I	NCT^I	NZS^I	NQCRV^I	NCBLC^I	NCS^I	NCRV^I	NNCNV^I	NFCNV^I	X
4	TSET	QSET								X
	NC^I	TC								X
	N^I	T	Q							
5	N^I	NQPRS^I								
	TIME	Q	TIME	Q . . .						
6	SINCAP^I	STRCAP^I								X
	N^I	CAP	CURVE^I							
	NA^I	NB^I	CAP	CURVE^I						
7	NBLD^I	NA1^I	NAS^I	NB1^I	NBS^I	C	CTYPE^I			
8	NA^I	NB^I	C	CTYPE^I						
9	SINAE^I	STRAE^I								
	N^I	AR	EM							
	NA^I	NB^I	AR	EM						
10	CURVE^I	NPAIRS^I								
	T_1 k_1	T_2	k_2	. . .						
11	ATYPE^I	AA1	AA2	AA3						
12	BTYPE^I	BB1	BB2	BB3	BB4^I					
13	VO									
14	NLOOP^I	BETA	ALDT	LOOPEN^I						X
	DELT	MAXT								X
	TPRINT^I	NPRINT^I								X

(1) Identifies required input for *all* problems.

^{I}Indicates integer input.

Partial Description of ...

Table 10-3. Element descriptions for Data Sets 7, 8

MODE	CTYPE	Description
O, 1, 3,11	-1	Simple radiation library element.
	-2	Multi-surface radiation library element.
	O	Nonspecific. C used as conductance.
	1-100	Temperature-dependent multiplicative factor.
	101-200	Natural convection library element.
	201-300	Forced convection library element.
	301	Volumetric airflow rate.
	311-400	Volumetric non-airflow rate. ρC_p must be input in Data Set 10.
	401	Airflow resistance, laminar.
	402	Airflow resistance, turbulent.
2	0	Velocity potential problem, x-direction.
	1	Velocity potential problem, y-direction.

Partial Description of ...

Table 10-4. Additional detail on Data Set 11-natural convection.

Mode of Heat Transfer	ATYPE	AAl	AA2	AA3
Vertical plate or cylinder	1	Plate height H, cylinder height H		
Horizontal rectangular plate Convection from upper surface to air ($T_s > T_{AIR}$) or convection from air to lower surface ($T_{AIR} > T_s$)	2	WL/[(W + L)2]		
Horizontal rectangular plate Convection from air to upper surface ($T_{AIR} > T_s$) or convection from lower surface to air ($T_s > T_{AIR}$)	3	WL /[(W + L) 2]		
Horizontal air space Heat transfer in upward direction. b = air space thickness. One node required at midspace.	4	b		
Vertical air space Heat transfer in horizontal direction. b = air space thickness. One node required at midspace.	5	b		
Small rectangular plate Vertical orientation, H < 6 in. Heat transfer to or from either surface.	6	H		
Horizontal orientation W, L < 6 in. Convection from upper surface to air or convection from air to lower surface.	7	WL/[2(W + L)]		

Table 10-4 Continued.

Mode of Heat Transfer	ATYPE	AAl	AA2	AA3
Convection from air to upper surface or convection from lower surface to air.	8	WL/[2(W + L)]		
Vertical Channel, *h* referenced to channel inlet air.	9	H	b	
Vertical Channel, *h* referenced to local air.	10	H	b	G

Table 10-5. Additional detail on Data Set 12-forced convection.

Mode of Heat Transfer	BTYPE	BB1	BB2	BB3	BB4	Reference
Duct, laminar flow (Re_D ~ 2100)	1	V	DH	L	A	E2-34
Duct, turbulent flow (Re_D > 10,000)	2	V	DH	L	A	E2.37, E2.38, E2.39
Flat plate, laminar flow ave h (Re_L < 5 X 105)	3	V	A	L	A	E2.20, E2.21, E2.22
Flat plate, turbulent flow ave h (Re_L > 5 X 105)	4	V	A	L	A	[2], Eqn. 6-67
Flat plate, laminar flow local h (Re_x < 5 X 1 05)	5	V	A	ΔX	I	E2.20
Flat plate, turbulent flow local h (Re_x > 5 X 105)	6	V	A	ΔX	I	[2], Eqn. 6-66

A: Arbitrary, but required numeric input.

V: Flow velocity.

ΔX: Length of node in string of equal length nodes.

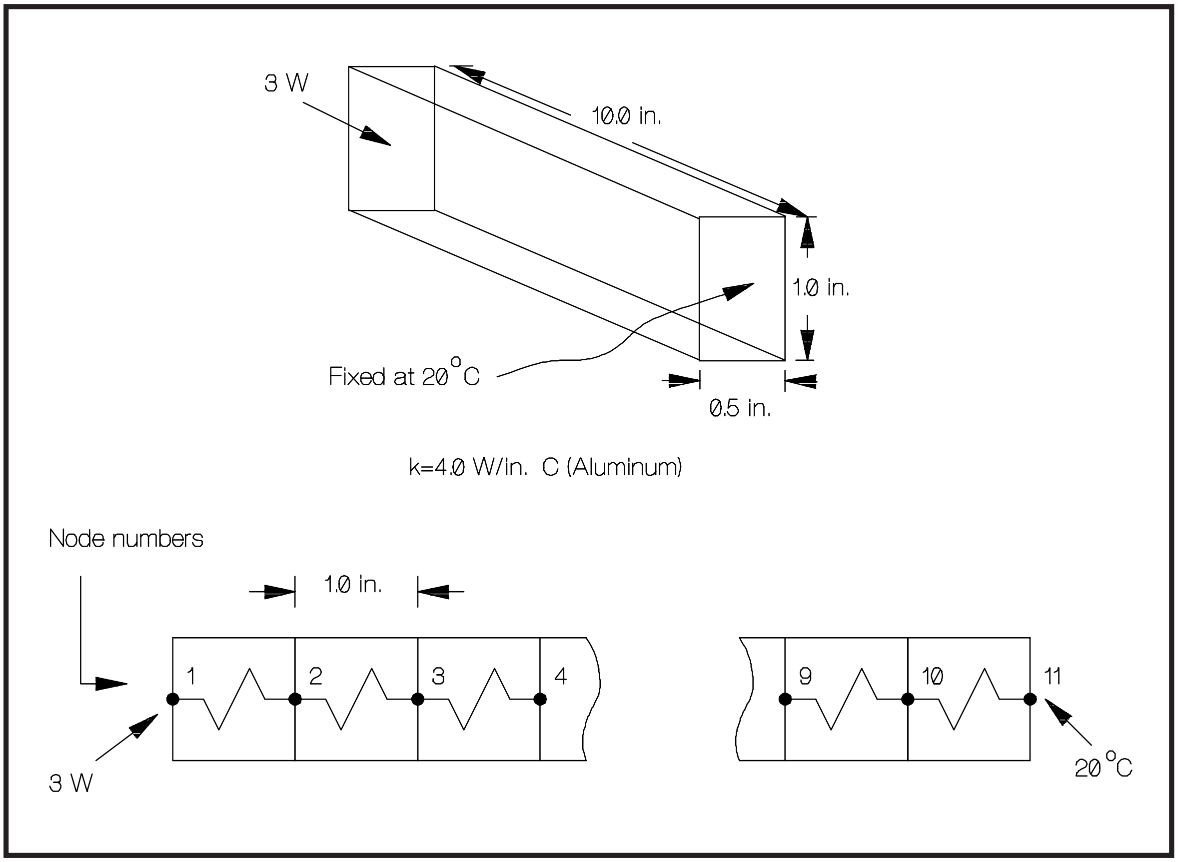
D, DH: Hydraulic diameter.

I: number of node at leading edge of string. Nodes must be numbered in ascending order starting with node 1. Missing node numbers not allowed in any given string. Use integer format.

L: Length of plate or duct.

Example

One-Dimensional Conducting Bar



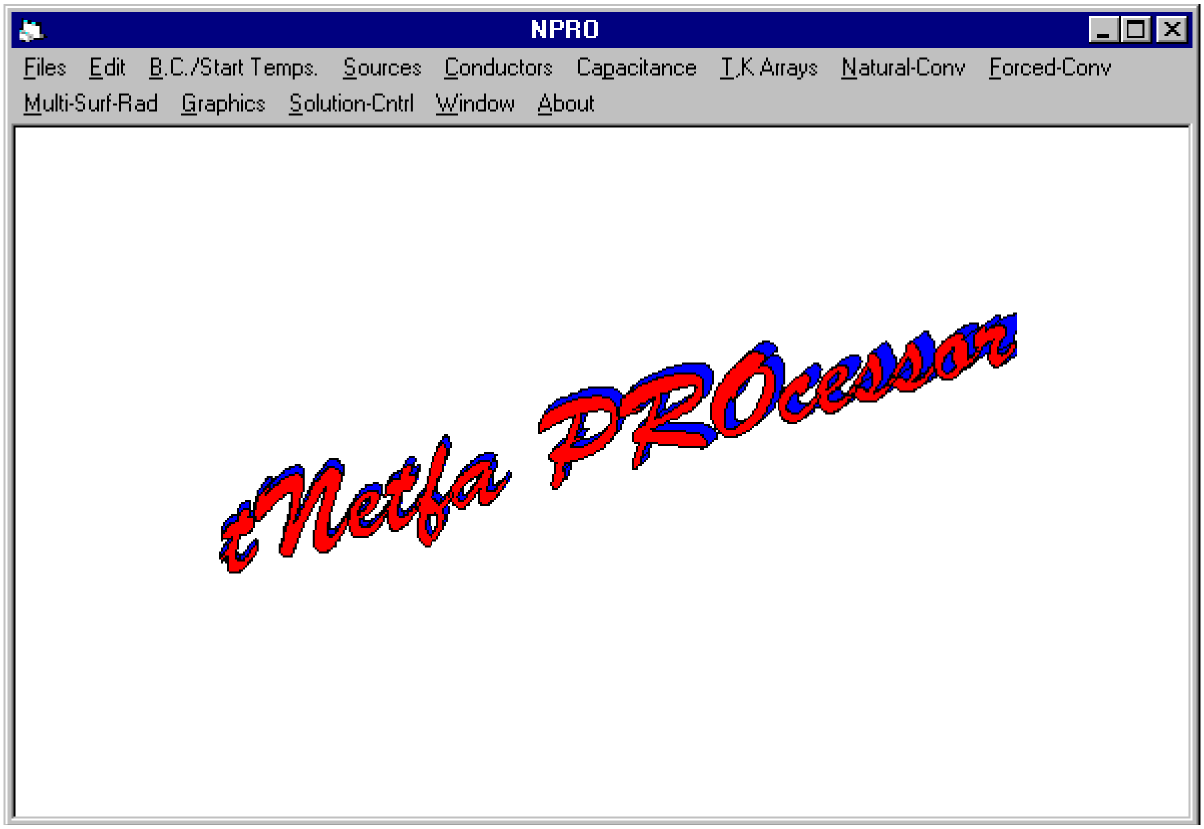
Conduction conductance $C = 1/R$

$$C = \frac{kA_k}{l} = \frac{(4.0\ W/in.\cdot{}^oC)(1.0\ in.)(0.5\ in.)}{(1.0\ in.)}$$

$$= 2.0\ W/{}^oC$$

NPRO OPTION	DATA SET	TNETFA INPUT							
Edit - Title Line 1	1	SAMPLE TNETFA PROBLEM							
Edit - Title Line 2	1	1-D BAR							
Edit - Solution Type	2	11	0	0					
	3	11	1	1	0	1	0	0	0
B.C./Start Temps	4	20.0	0.0						
B.C./Start Temps	4	11	20.0						
Sources - Steady	4	1	20.0	3.0					
Capacitance	6	0	0						
Conductors - String of	7	10	1	1	2	1	2.0	0	
Solution Cntrl - Steady State	14	20	1.0	0.01	10				
Solution Cntrl - Steady State	14	0	0						
Solution Cntrl - Steady State	14	10	0						

See Tables 10-2, 10-3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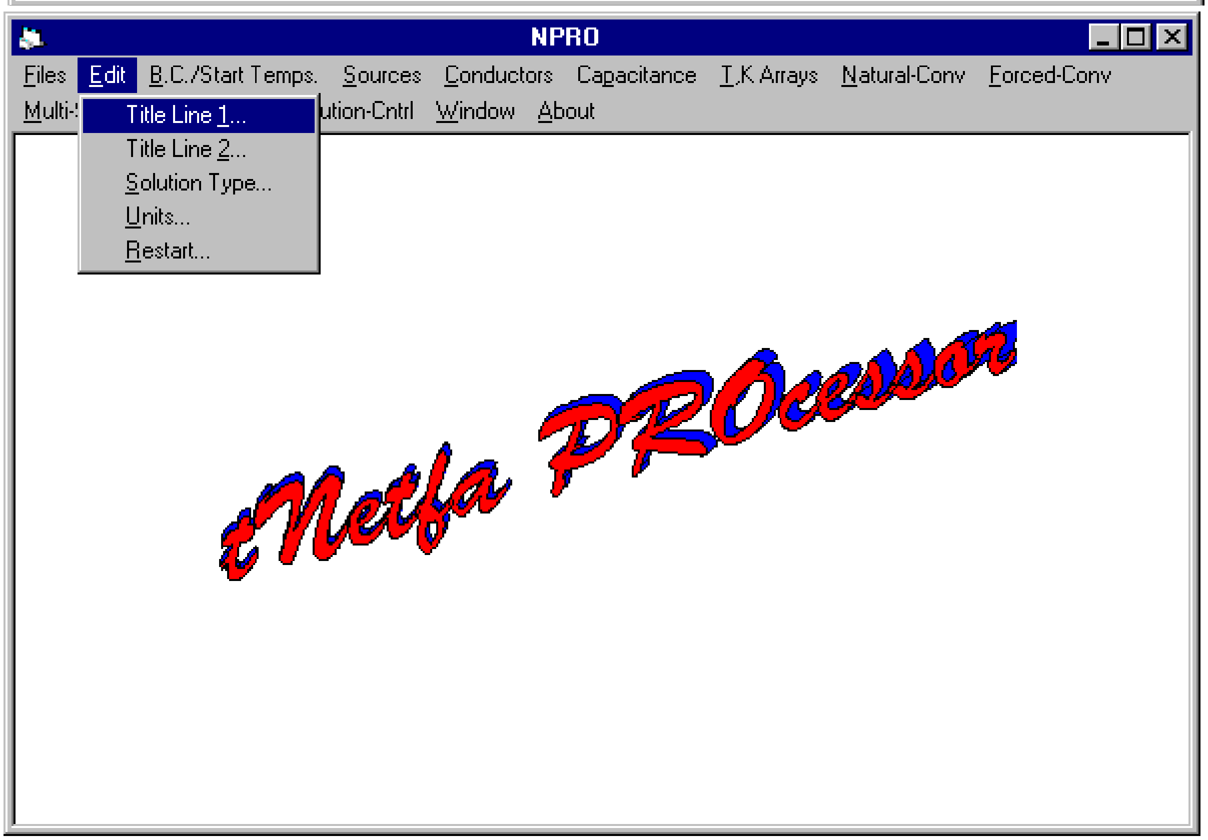
Multi-Surf-Rad Graphics Solution-Cntrl Window About
tNetfa PROcessor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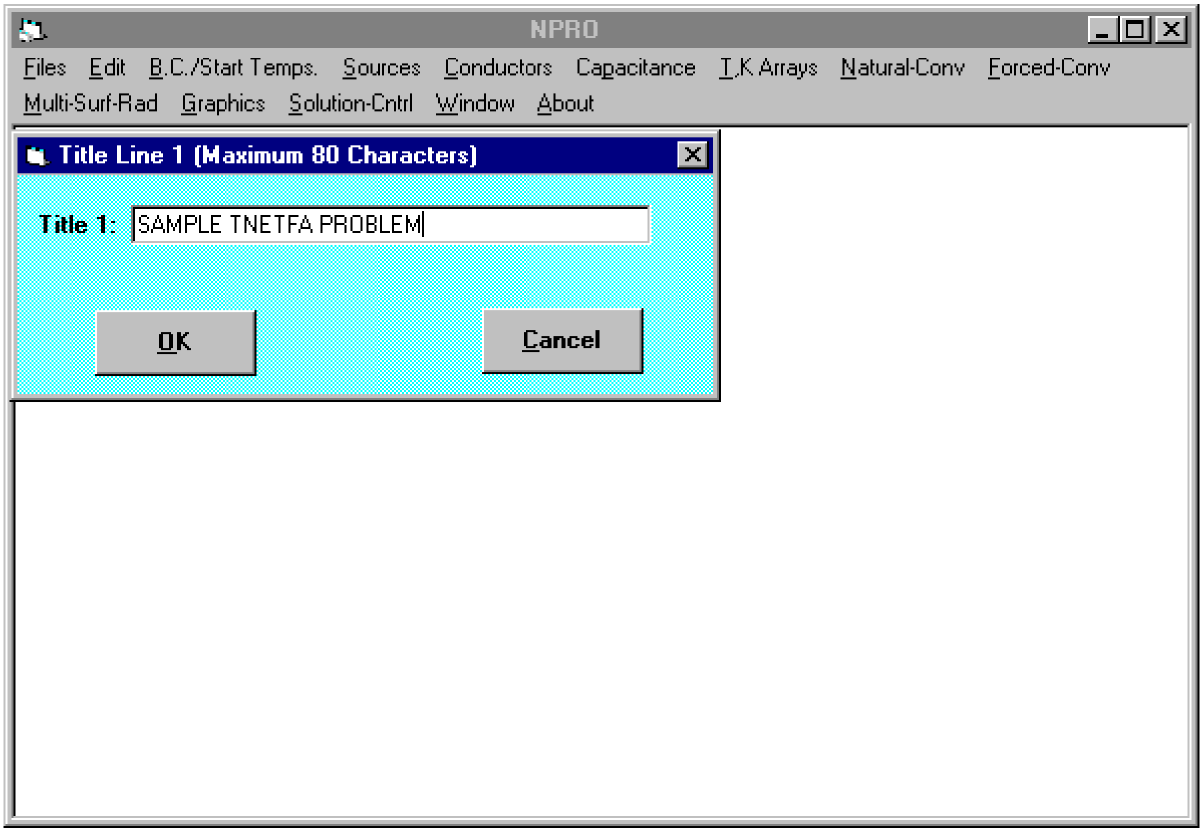
Files Edit B.C./Start Temps. Sources Conductors Capacitance T.K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Title Line 1...
Title Line 2...
Solution Type...
Units...
Restart...
tNetfa PROcessor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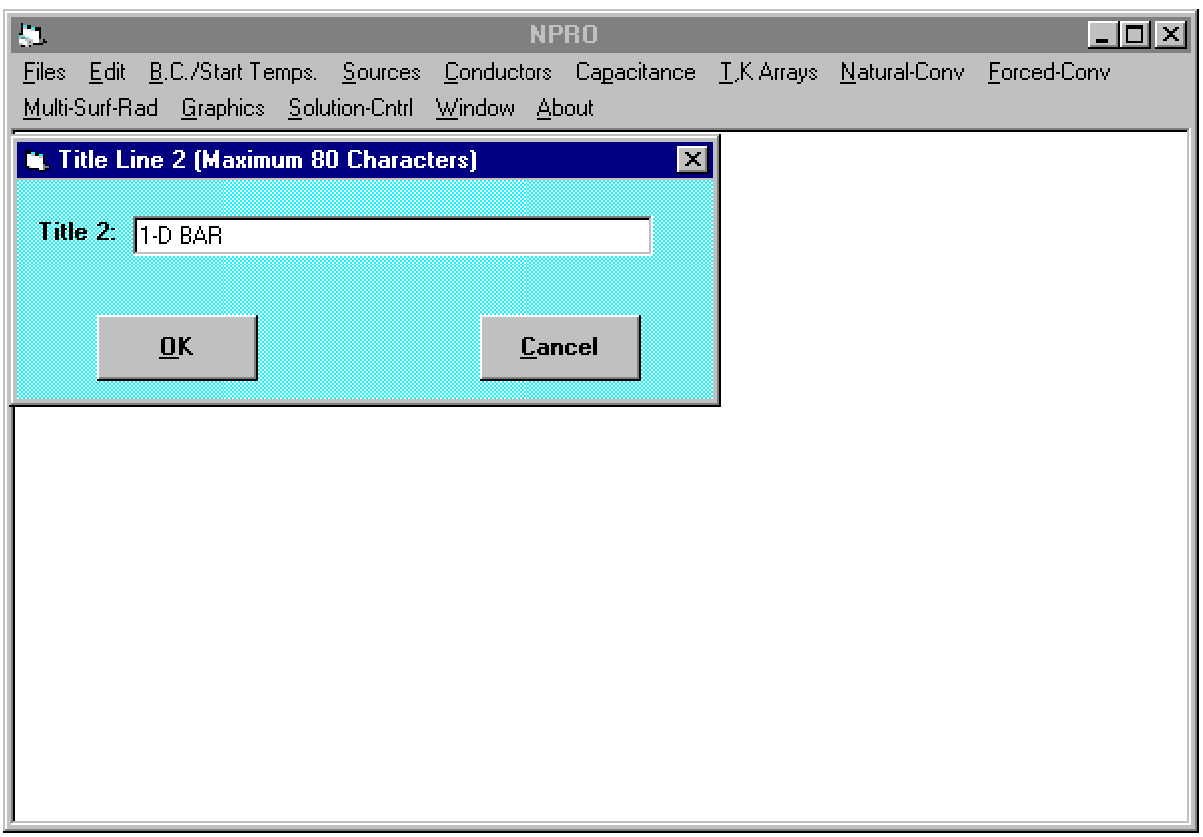
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Title Line 1 (Maximum 80 Characters)
Title 1:
SAMPLE TNETFA PROBLEM
OK
Cancel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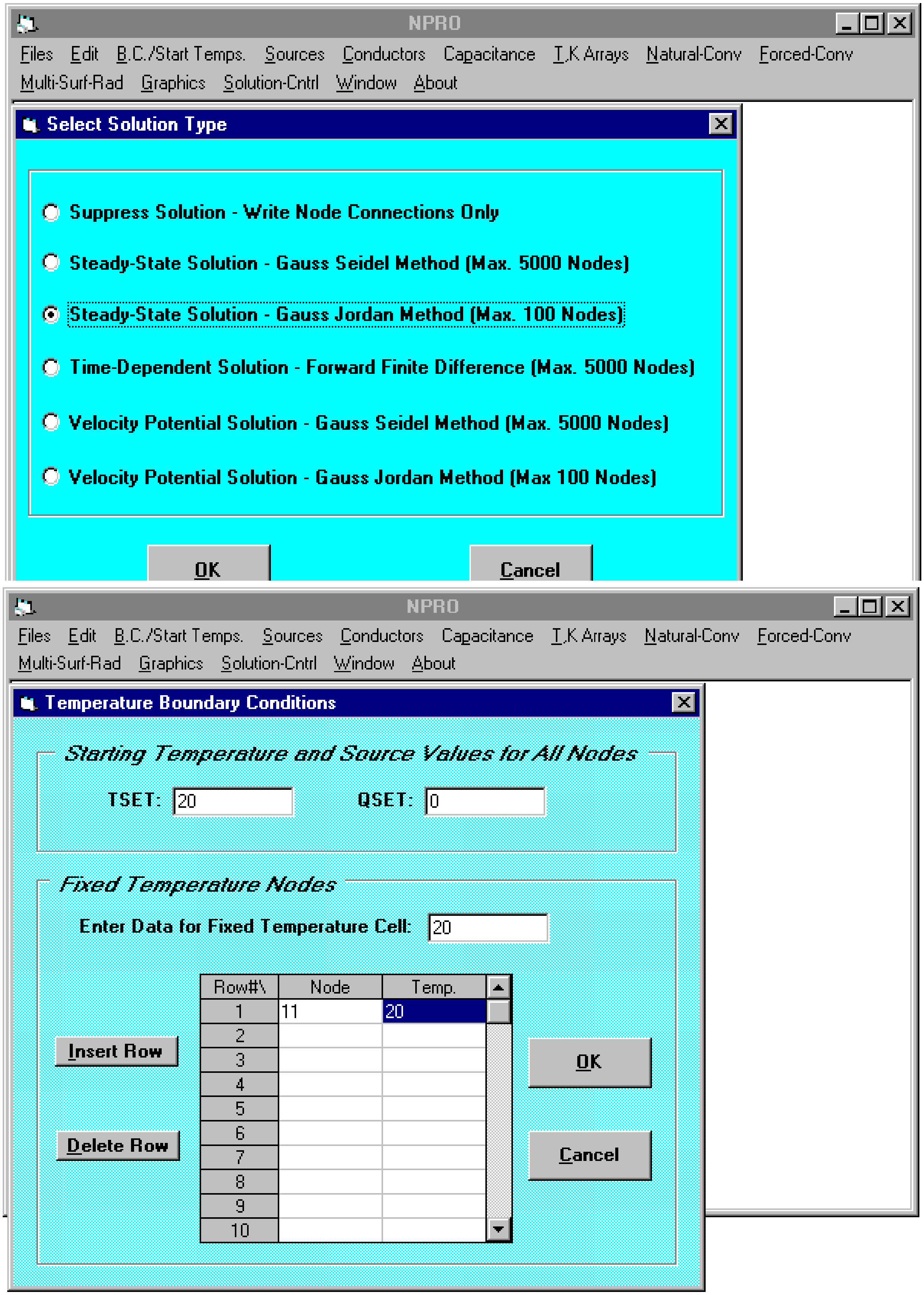
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Title Line 2 (Maximum 80 Characters)
Title 2:
1-D BAR
OK
Cancel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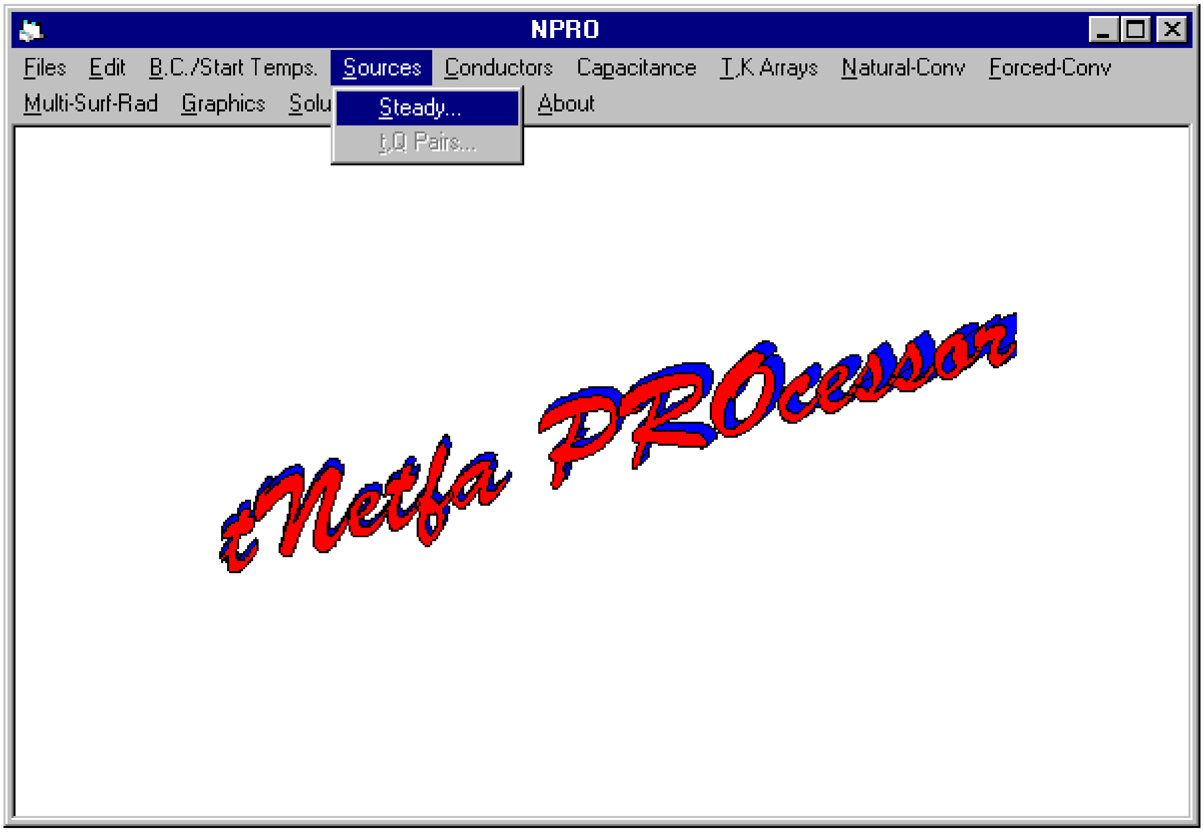
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Select Solution Type
Suppress Solution - Write Node Connections Only
Steady-State Solution - Gauss Seidel Method (Max. 5000 Nodes)
Steady-State Solution - Gauss Jordan Method (Max. 100 Nodes)
Time-Dependent Solution - Forward Finite Difference (Max. 5000 Nodes)
Velocity Potential Solution - Gauss Seidel Method (Max. 5000 Nodes)
Velocity Potential Solution - Gauss Jordan Method (Max 100 Nodes)
OK
Cancel
NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Temperature Boundary Conditions
Starting Temperature and Source Values for All Nodes
TSET: 20
QSET: 0
Fixed Temperature Nodes
Enter Data for Fixed Temperature Cell: 20
Row#\ Node Temp.
1 11 20
2
3
4
5
6
7
8
9
10
Insert Row
Delete Row
OK
Cancel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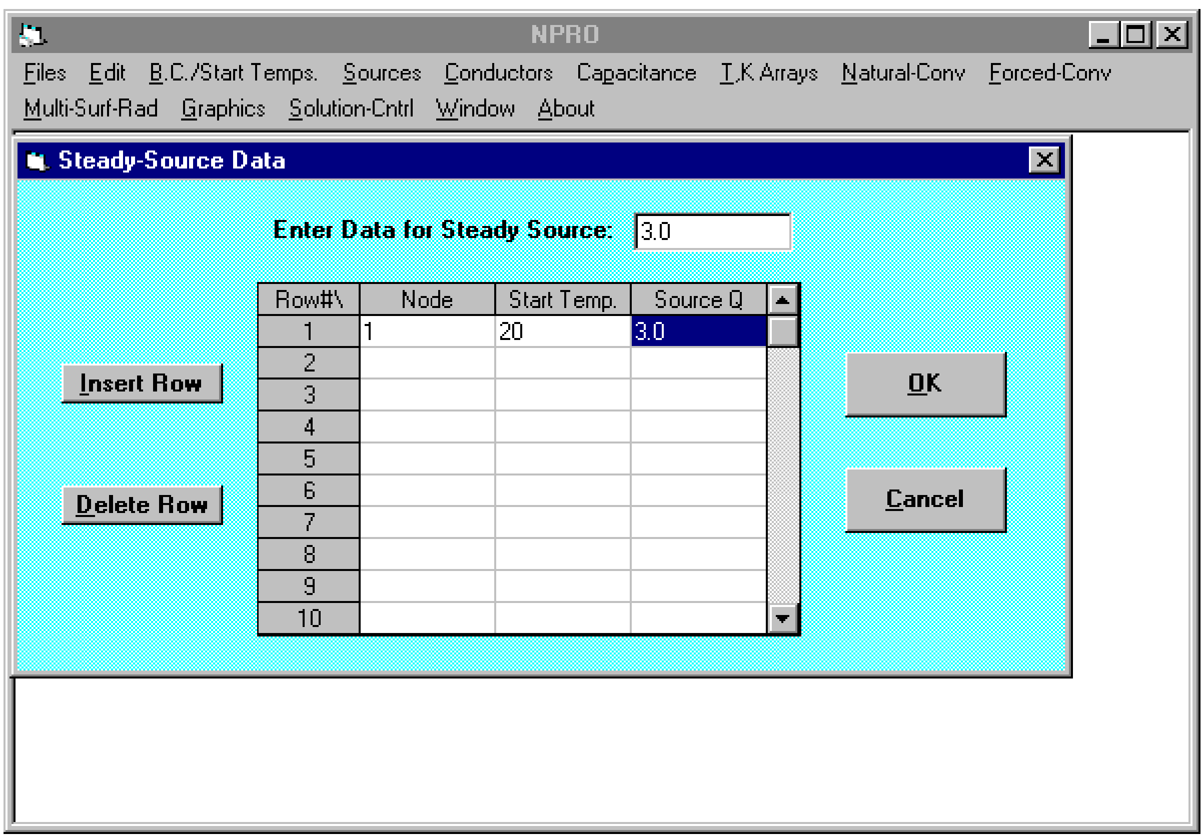
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solu About
Steady...
t,Q Pairs...
tNetfa PROcessor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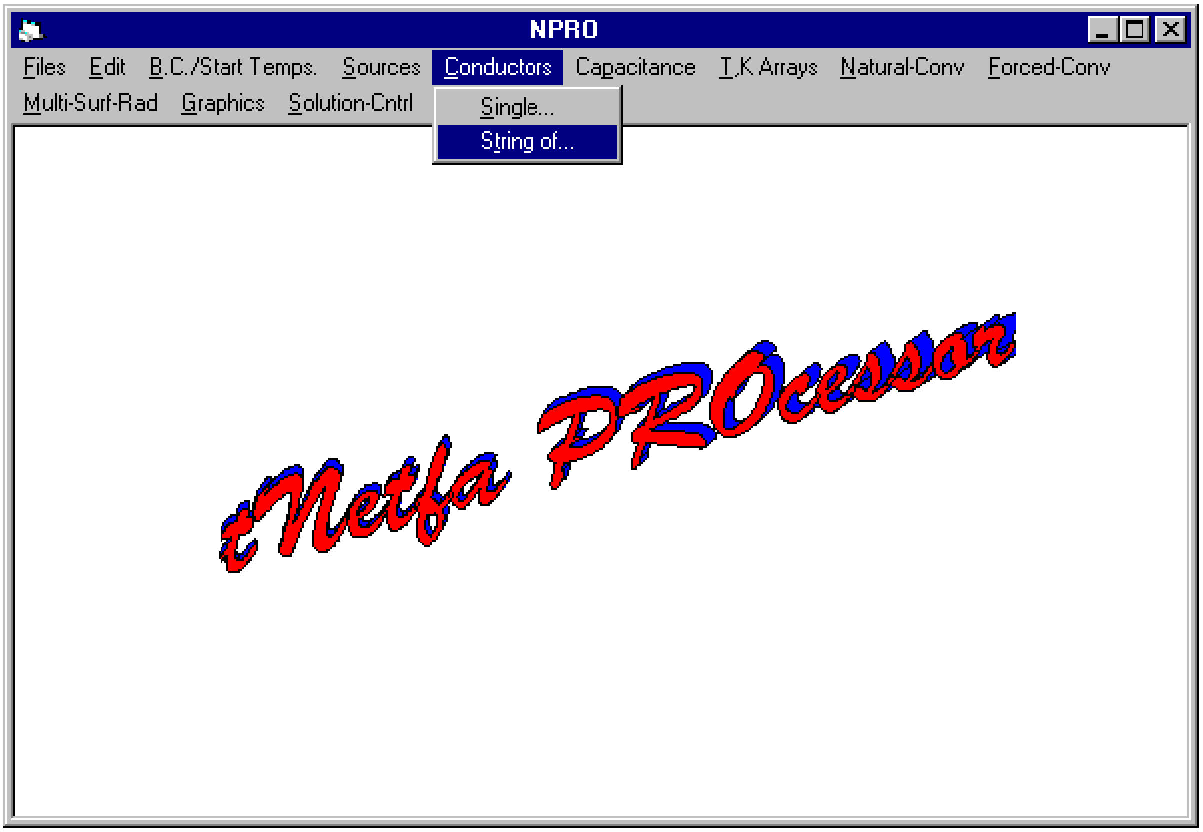
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Steady-Source Data
Enter Data for Steady Source:
3.0
Row#\ Node Start Temp. Source Q
1 1 20 3.0
2
3
4
5
6
7
8
9
10
Insert Row
Delete Row
OK
Cancel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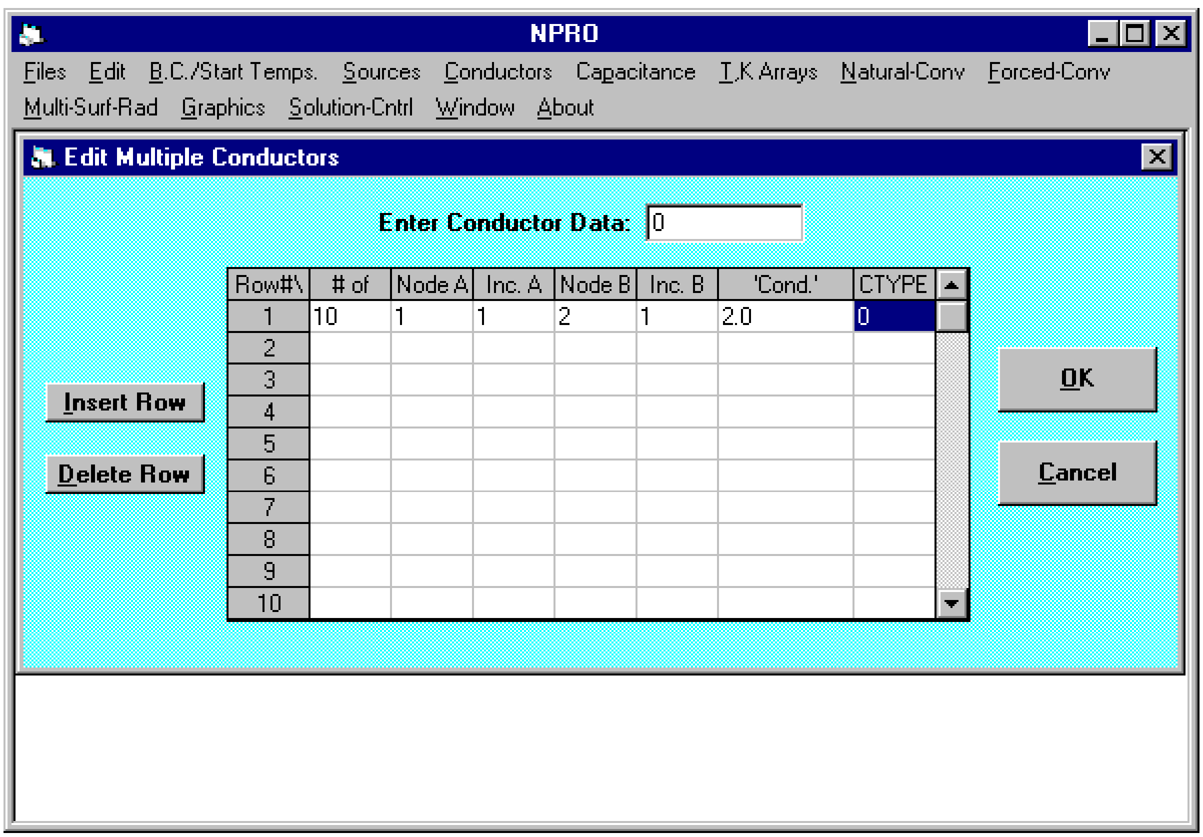
Multi-Surf-Rad Graphics Solution-Cntrl
Single...
String of...
tNetfa PROcessor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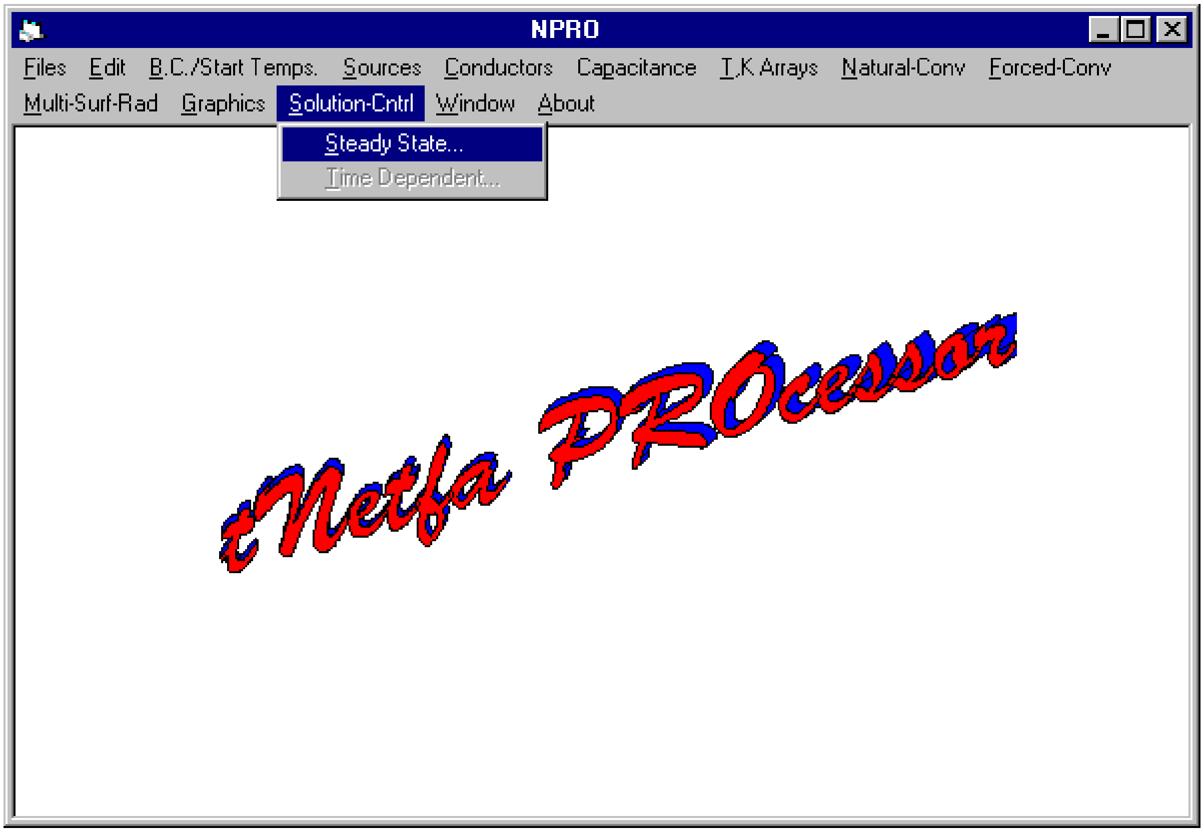
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Edit Multiple Conductors
Enter Conductor Data: 0
Row#\ # of Node A Inc. A Node B Inc. B 'Cond.' CTYPE
1 10 1 1 2 1 2.0 0
2
3
4
5
6
7
8
9
10
Insert Row
Delete Row
OK
Cancel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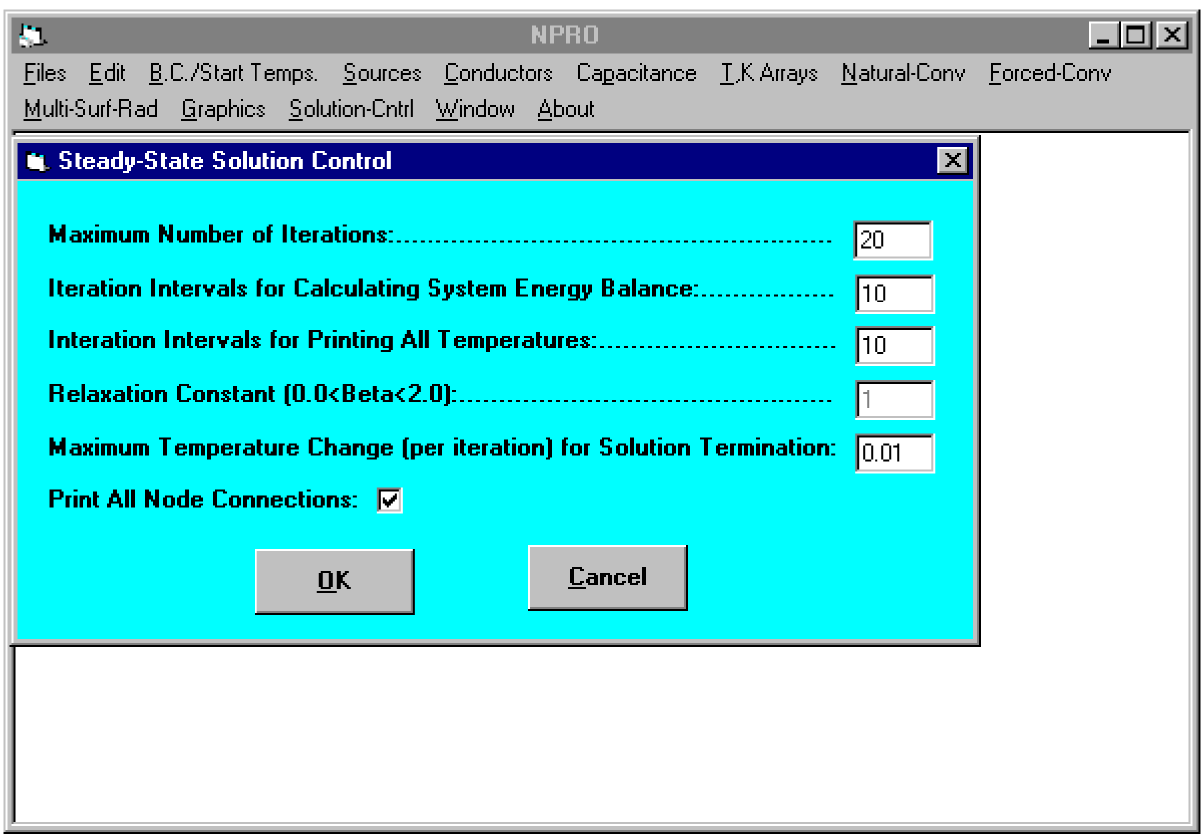
Files Edit B.C./Start Temps. Sources Conductors Capacitance T.K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Steady State...
Time Dependent...
tNetfa PROcessor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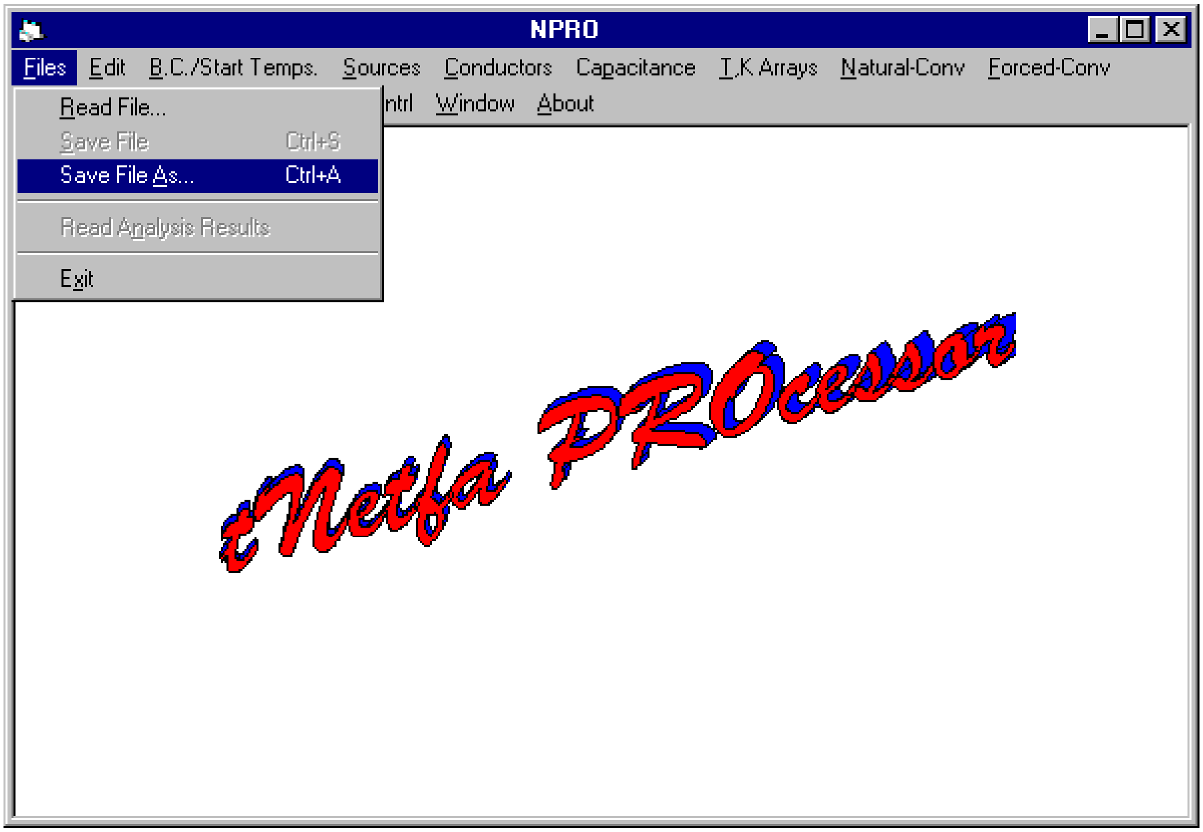
Files Edit B.C./Start Temps. Sources Conductors Capacitance T.K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Steady-State Solution Control
Maximum Number of Iterations:.. 20
Iteration Intervals for Calculating System Energy Balance:.................. 10
Interation Intervals for Printing All Temperatures:............................... 10
Relaxation Constant (0.0<Beta<2.0):.. 1
Maximum Temperature Change (per iteration) for Solution Termination: 0.01
Print All Node Connections:
OK
Cancel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Edit
B.C./Start Temps.
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Conductors
Capacitance
T.K Arrays
Natural-Conv
Forced-Conv
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Window
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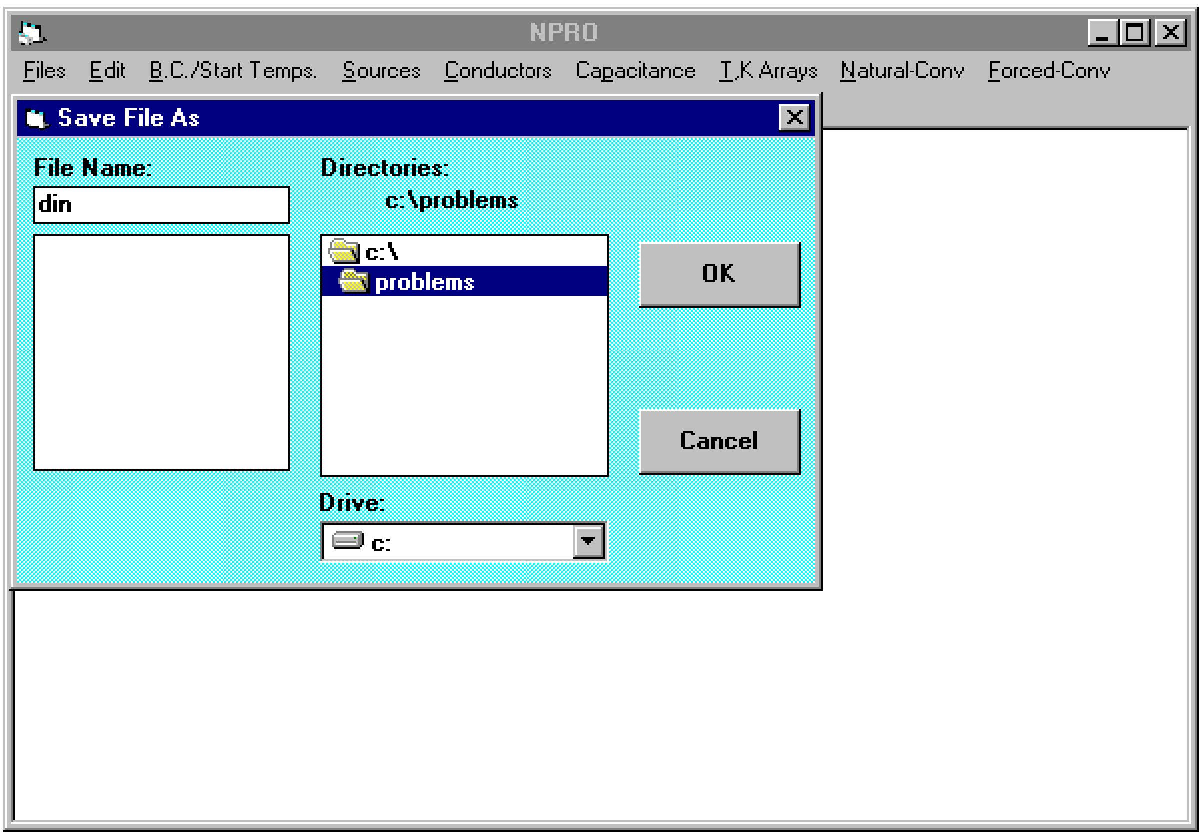
Save File As...
Ctrl+A
Read Analysis Results
Exit
tNetfa PROcessor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B.C./Start Temps.
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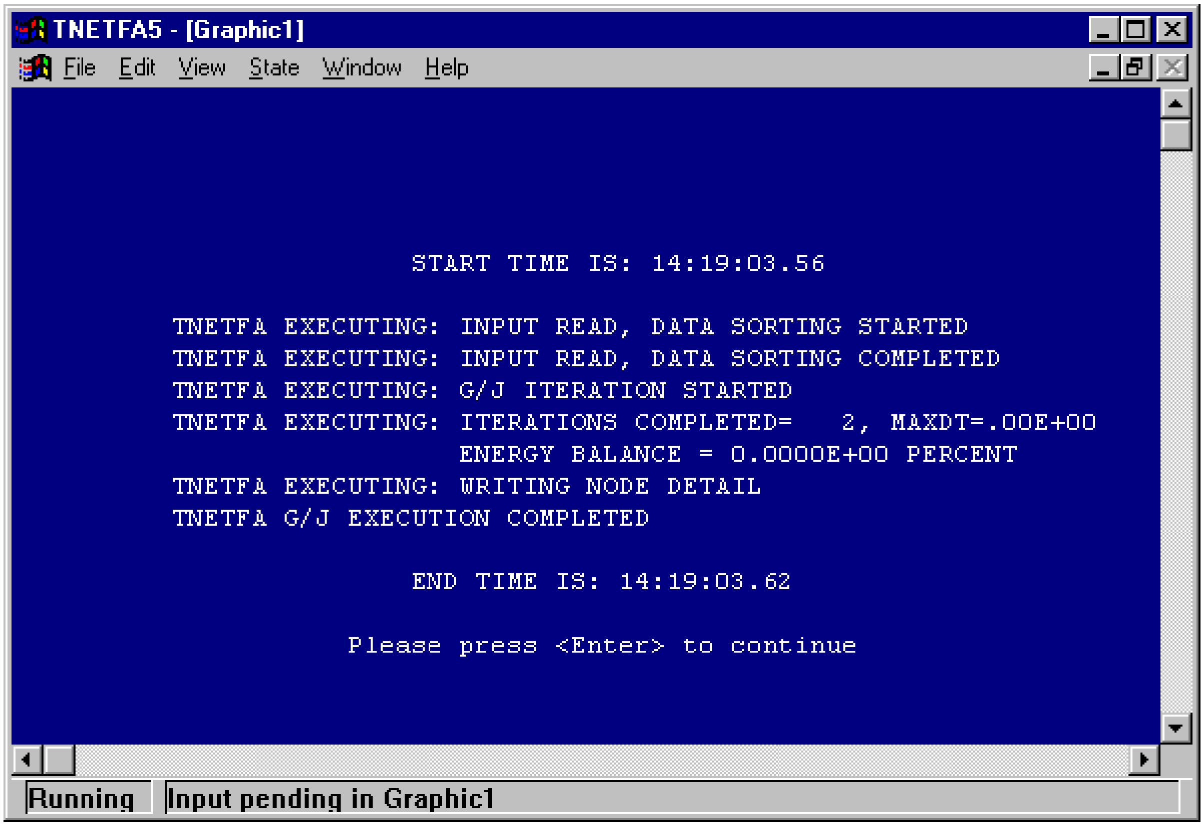
Conductors
Capacitance
T.K Arrays
Natural-Conv
Forced-Conv
Save File As
File Name:
din
Directories:
c:\problems
c:\
problems
OK
Cancel
Drive:
c: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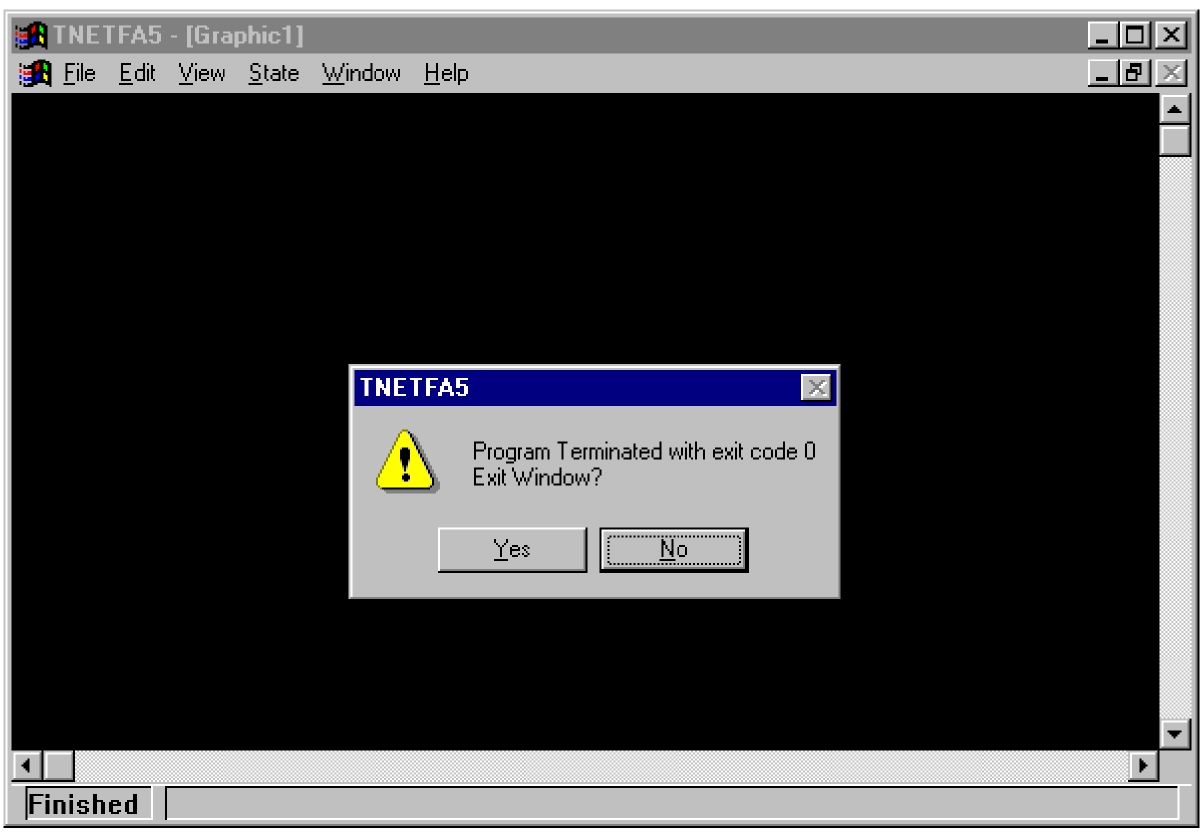
File Edit View State Window Help
START TIME IS: 14:19:03.56
TNETFA EXECUTING: INPUT READ, DATA SORTING STARTED
TNETFA EXECUTING: INPUT READ, DATA SORTING COMPLETED
TNETFA EXECUTING: G/J ITERATION STARTED
TNETFA EXECUTING: ITERATIONS COMPLETED= 2, MAXDT=.00E+00
ENERGY BALANCE = 0.0000E+00 PERCENT
TNETFA EXECUTING: WRITING NODE DETAIL
TNETFA G/J EXECUTION COMPLETED
END TIME IS: 14:19:03.62
Please press <Enter> to continue
Running
Input pending in Graphic1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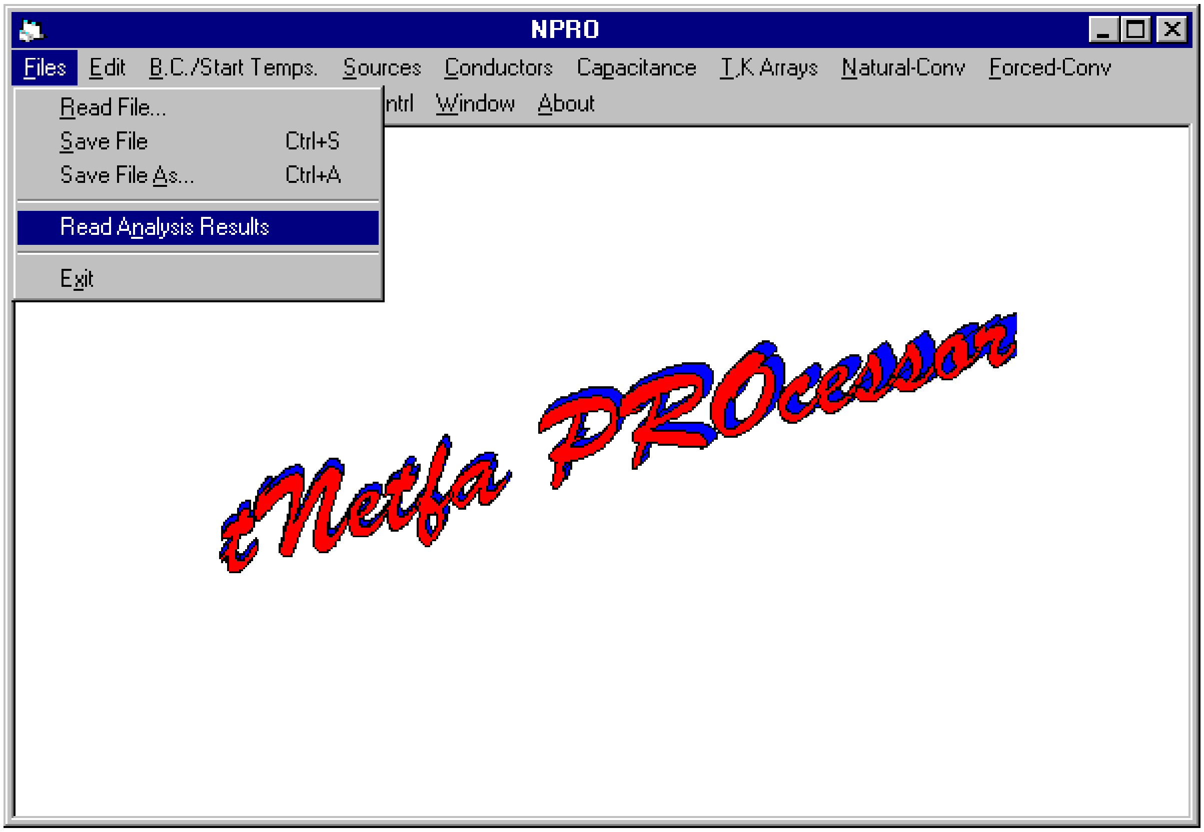
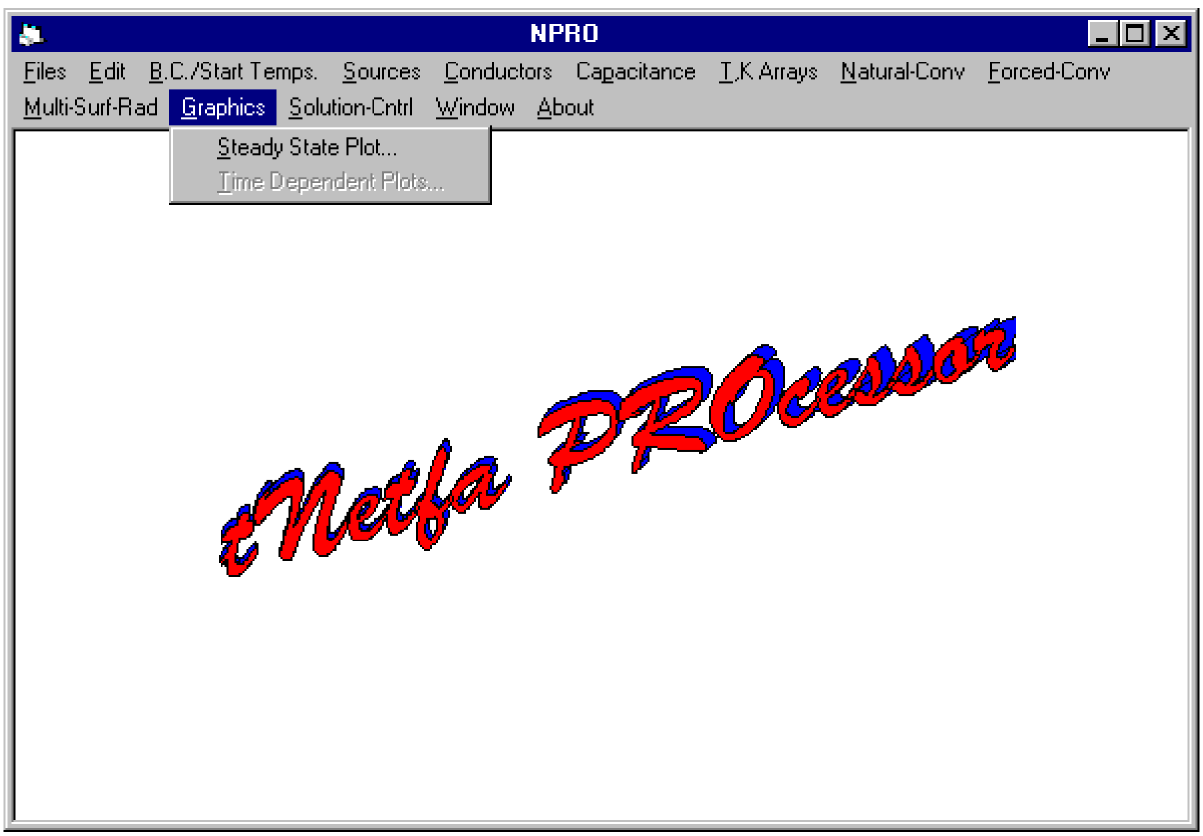
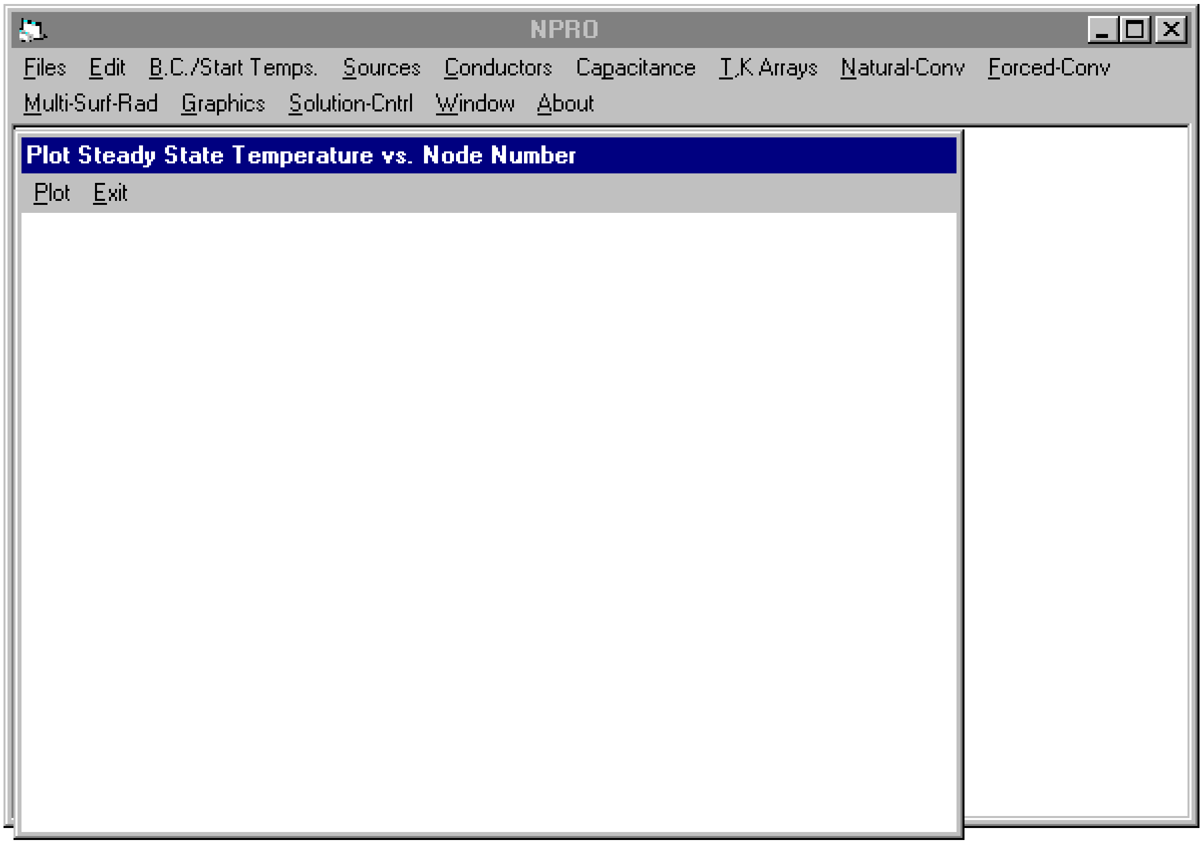
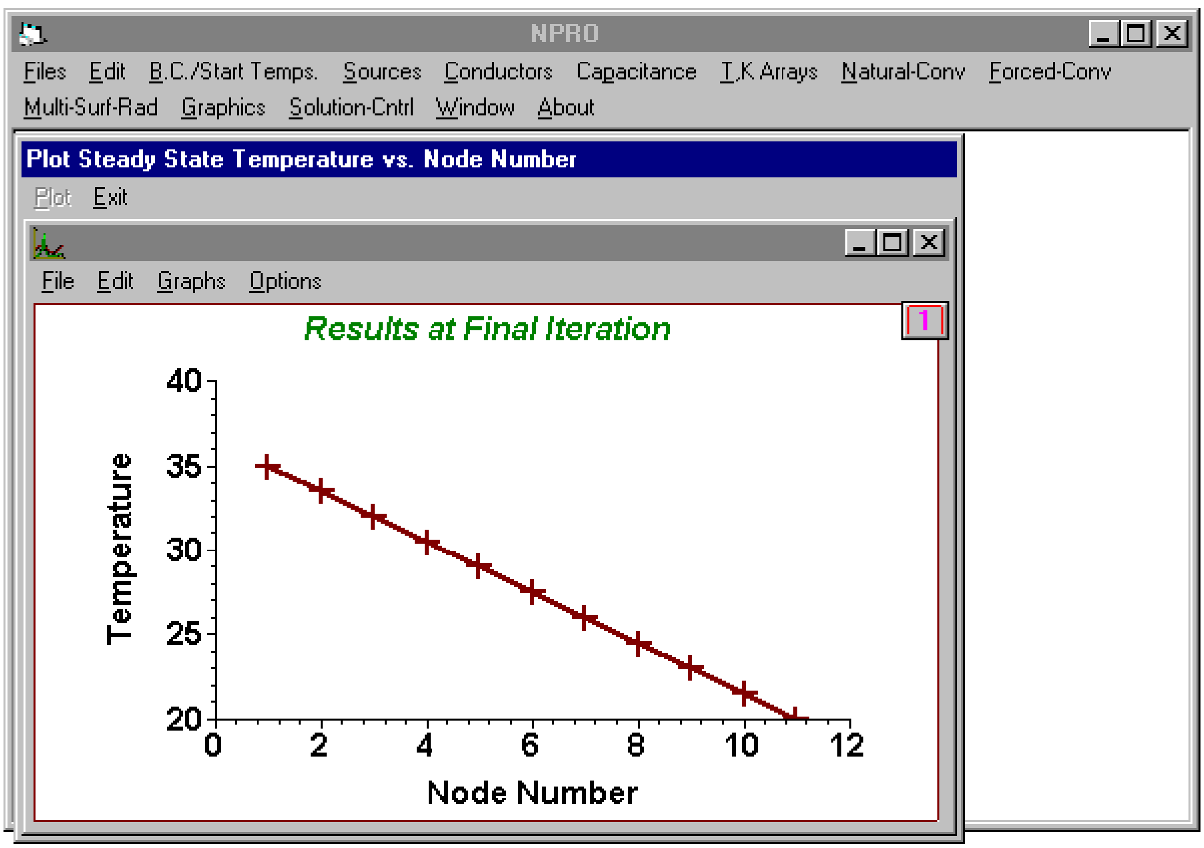
File Edit View State Window Help
TNETFA5
Program Terminated with exit code 0
Exit Window?
Yes
No
Finished


NPRO
Files
Edit
B.C./Start Temps.
Sources
Conductors
Capacitance
T.K Arrays
Natural-Conv
Forced-Conv
ntrl
Window
About
Read File...
Save File Ctrl+S
Save File As... Ctrl+A
Read Analysis Results
Exit
tNetfa PROcessor


NPRO
Files
Edit
B.C./Start Temps.
Sources
Conductors
Capacitance
T.K Arrays
Natural-Conv
Forced-Conv
Multi-Surf-Rad
Graphics
Solution-Cntrl
Window
About
Steady State Plot...
Time Dependent Plots...
tNetfa PROcessor


NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Plot Steady State Temperature vs. Node Number
Plot Exit


NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Plot Steady State Temperature vs. Node Number
Plot Exit
File Edit Graphs Options
Results at Final Iteration
1
Temperature
40
35
30
25
20
0
2
4
6
8
10
12
Node Number

```
********************************************************************************
 ****         Electronics Thermal Analysis Package - PC TNETFA V5.0        ****
 ****            (C) Copyright 1996 by Thermal Computations, Inc.          ****
 ****                          Newberg, Oregon                             ****
********************************************************************************

SAMPLE TNETFA PROBLEM
1-D BAR

 UNITS=0
 NUMBER OF NODES=  11     NUMBER OF CONDUCTORS=  20
 NLOOP =   20  TPRINT=   10      NPRINT=    1
 LOOPEN=   10    ALDT= .1000E-01   BETA= 1.00

                                  LOOPCT=    0
                                  TEMPERATURES

  T(  1)= .2000E+02   T(  2)= .2000E+02   T(  3)= .2000E+02   T(  4)= .2000E+02
  T(  5)= .2000E+02   T(  6)= .2000E+02   T(  7)= .2000E+02   T(  8)= .2000E+02
  T(  9)= .2000E+02   T( 10)= .2000E+02   T( 11)= .2000E+02

                                  LOOPCT=    2
                                  TEMPERATURES

  T(  1)= .3500E+02   T(  2)= .3350E+02   T(  3)= .3200E+02   T(  4)= .3050E+02
  T(  5)= .2900E+02   T(  6)= .2750E+02   T(  7)= .2600E+02   T(  8)= .2450E+02
  T(  9)= .2300E+02   T( 10)= .2150E+02   T( 11)= .2000E+02
                                  MAXDT          = .0000E+00
                                  ENERGY BALANCE = 0.0000E+00 PERCENT


    DETAIL OF NODE   1     TEMPERATURE= .3500E+02             POWER= .3000E+01
                           STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
    NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
       2     0          .2000E+01     .2000E+01     .3000E+01
                                        NET TOTAL = .3000E+01

    DETAIL OF NODE   2     TEMPERATURE= .3350E+02             POWER= .0000E+00
                           STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
    NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
       3     0          .2000E+01     .2000E+01     .3000E+01
       1     0          .2000E+01     .2000E+01    -.3000E+01
                                        NET TOTAL = .0000E+00

    DETAIL OF NODE   3     TEMPERATURE= .3200E+02             POWER= .0000E+00
                           STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
    NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
       4     0          .2000E+01     .2000E+01     .3000E+01
       2     0          .2000E+01     .2000E+01    -.3000E+01
                                        NET TOTAL = .0000E+00
    DETAIL OF NODE   4     TEMPERATURE= .3050E+02             POWER= .0000E+00
                           STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
    NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
```

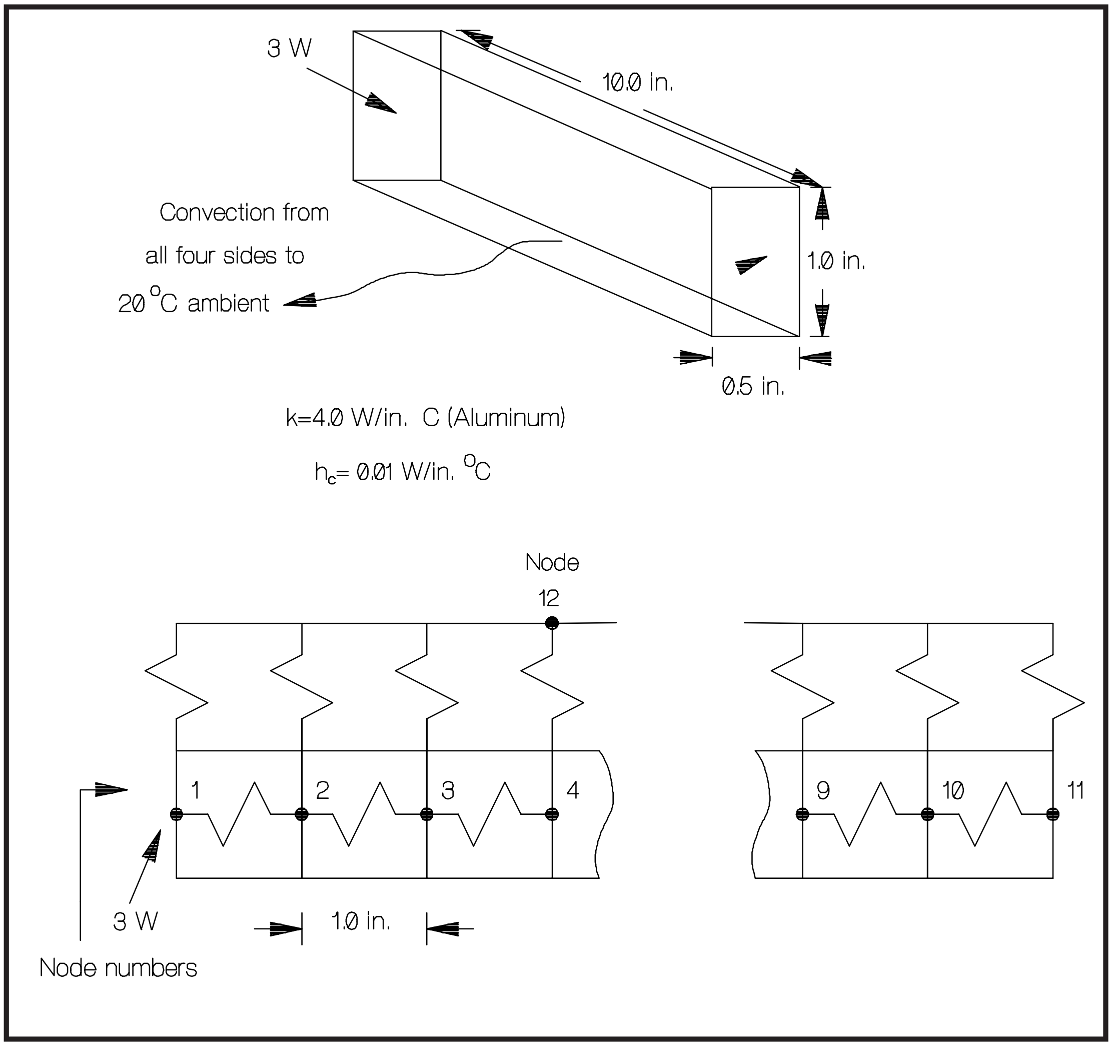
```
    5     0          .2000E+01     .2000E+01     .3000E+01
    3     0          .2000E+01     .2000E+01    -.3000E+01
                                       NET TOTAL = .0000E+00

  DETAIL OF NODE   5     TEMPERATURE= .2900E+02             POWER= .0000E+00
                         STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
 NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
    6     0          .2000E+01     .2000E+01     .3000E+01
    4     0          .2000E+01     .2000E+01    -.3000E+01
                                       NET TOTAL = .0000E+00

  DETAIL OF NODE   6     TEMPERATURE= .2750E+02             POWER= .0000E+00
                         STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
 NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
    7     0          .2000E+01     .2000E+01     .3000E+01
    5     0          .2000E+01     .2000E+01    -.3000E+01
                                       NET TOTAL = .0000E+00

  DETAIL OF NODE   7     TEMPERATURE= .2600E+02             POWER= .0000E+00
                         STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
 NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
    8     0          .2000E+01     .2000E+01     .3000E+01
    6     0          .2000E+01     .2000E+01    -.3000E+01
                                       NET TOTAL = .0000E+00

  DETAIL OF NODE   8     TEMPERATURE= .2450E+02             POWER= .0000E+00
                         STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
 NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
    9     0          .2000E+01     .2000E+01     .3000E+01
    7     0          .2000E+01     .2000E+01    -.3000E+01
                                       NET TOTAL = .0000E+00

  DETAIL OF NODE   9     TEMPERATURE= .2300E+02             POWER= .0000E+00
                         STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
 NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   10     0          .2000E+01     .2000E+01     .3000E+01
    8     0          .2000E+01     .2000E+01    -.3000E+01
                                       NET TOTAL = .0000E+00

  DETAIL OF NODE  10     TEMPERATURE= .2150E+02             POWER= .0000E+00
                         STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
 NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   11     0          .2000E+01     .2000E+01     .3000E+01
    9     0          .2000E+01     .2000E+01    -.3000E+01
                                       NET TOTAL = .0000E+00

  DETAIL OF NODE -11     TEMPERATURE= .2000E+02             POWER= .0000E+00
                         STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
                         THIS IS A CONSTANT TEMPERATURE NODE
 NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   10     0          .2000E+01     .2000E+01    -.3000E+01
                                       NET TOTAL =-.3000E+01
```

TNETFA Input File (DIN)

```
SAMPLE TNETFA PROBLEM
1-D BAR
 11 0   0
 11 1   1   0   1   0   0   0   0
 20 0
11  2.0000E+01
1   2.0000E+01    3.0000E+00
 0  0
10  1   1   2   1   2.0000E+00    0
 20 1   0.01    10
 0  0
 10 1
```

Example

OneDimensional Conducting/Convecting Bar



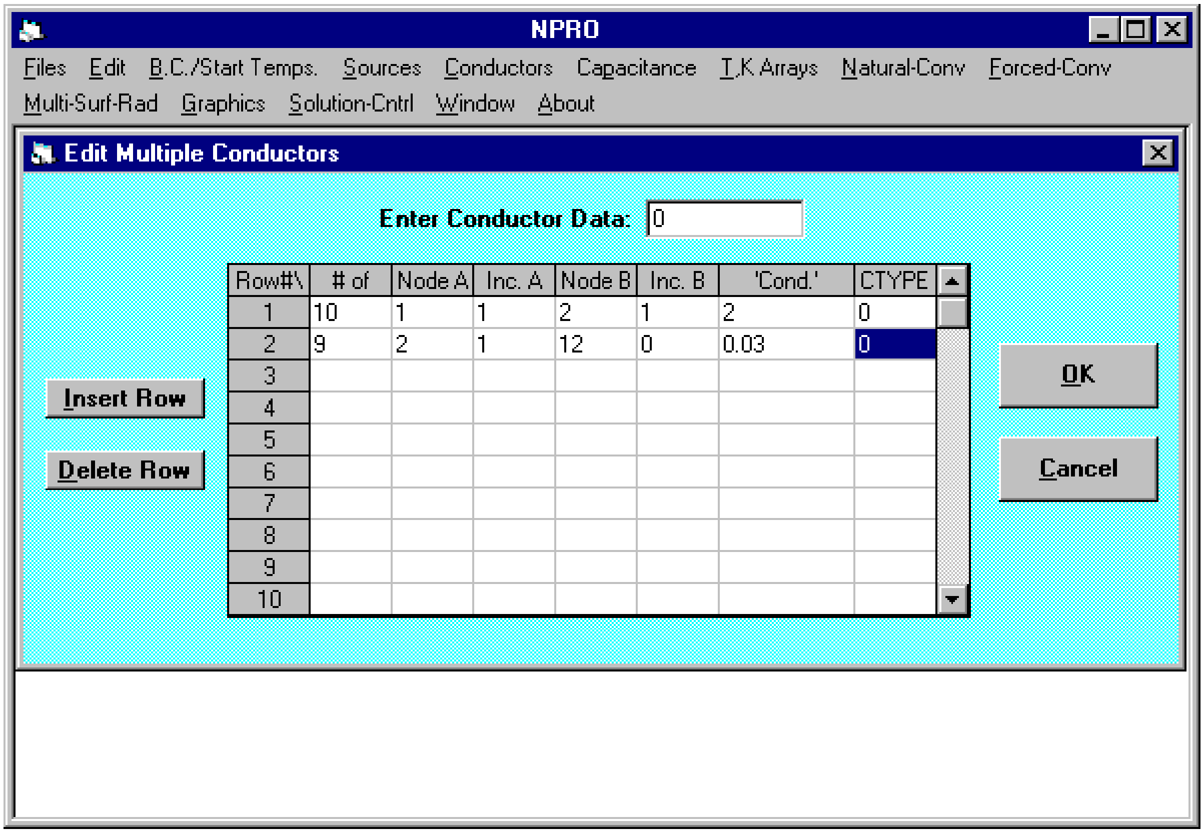
Convection conductance $C_c = 1/R_c$

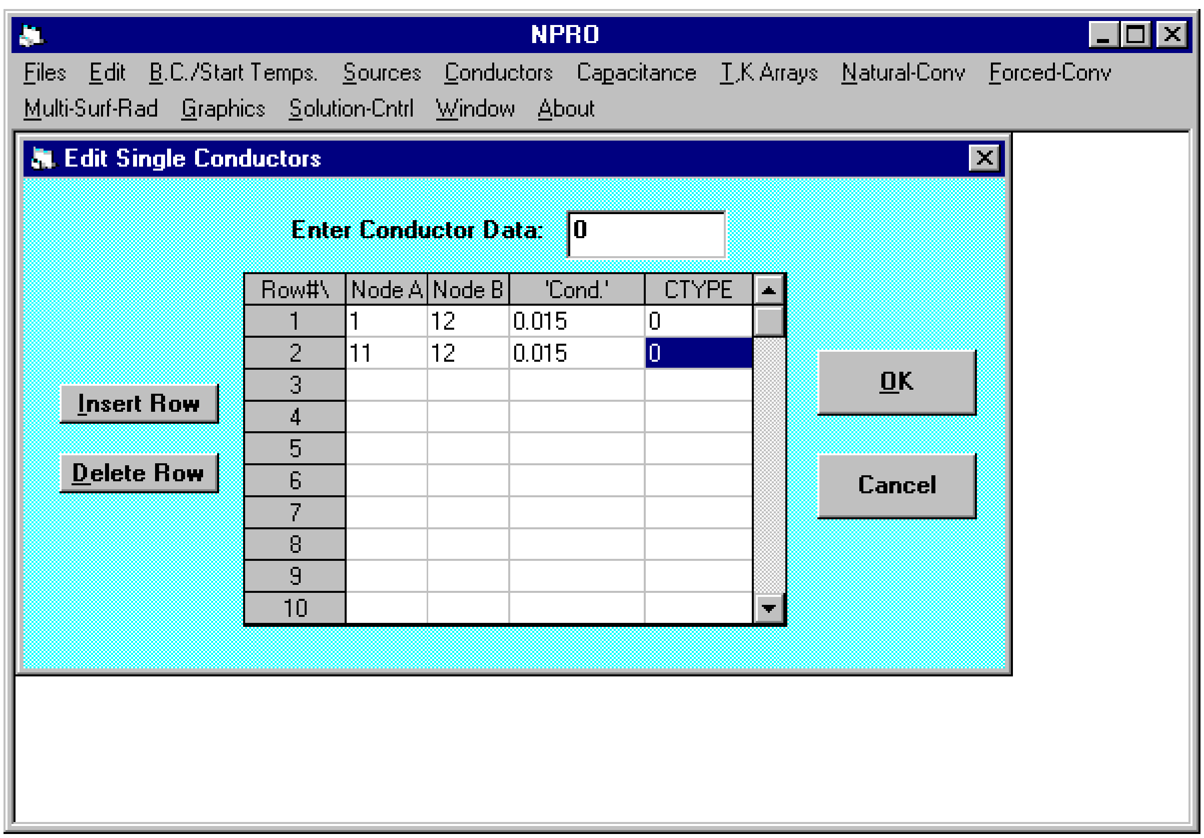
Ends: $C_c = h_c A_s = \left(0.01\, W/in.^2 \cdot {}^oC\right)\left(1.5\, in.^2\right)$

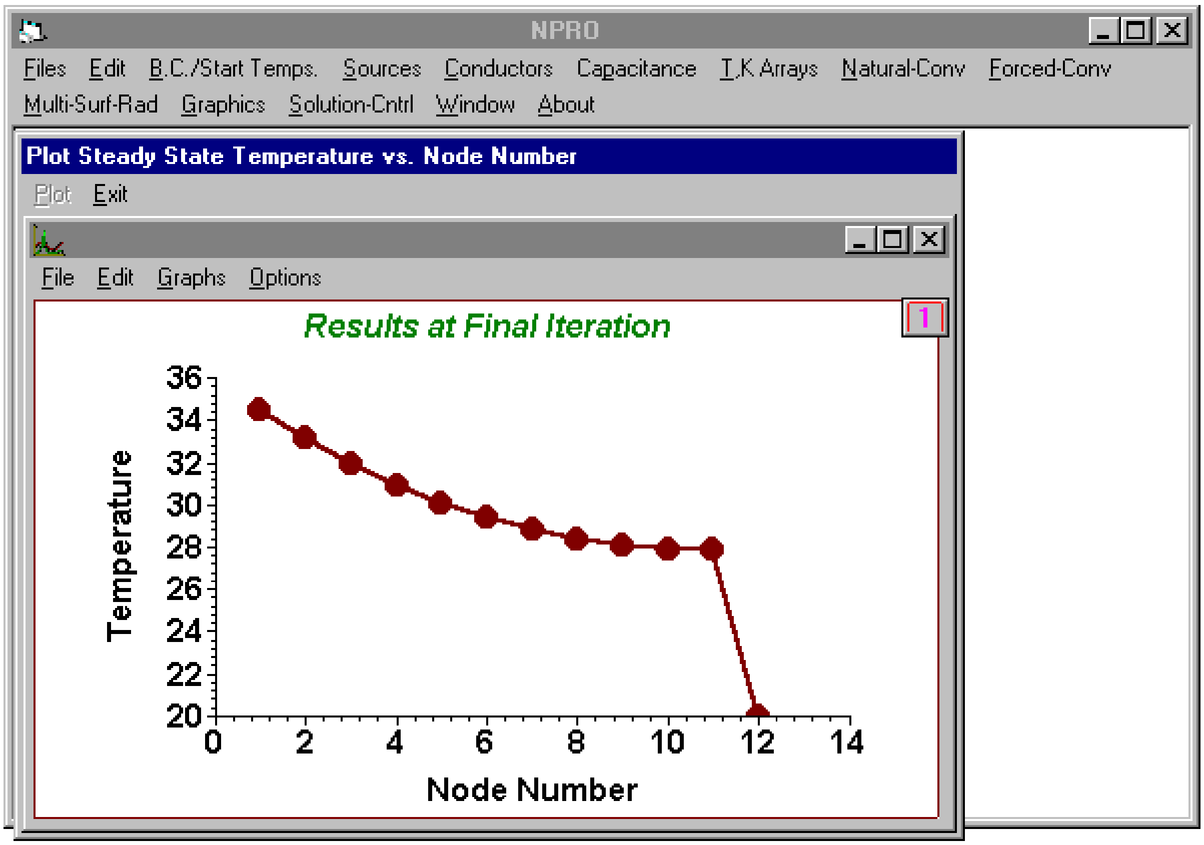
$= 0.015\, W/{}^oC$

Other: $C_c = (0.01)(3.0) = 0.030\, W/{}^oC$

NPRO OPTION	DATA SET	TNETFA INPUT								
Edit - Title Line 1	1	SAMPLE TNETFA PROBLEM								
Edit - Title Line 2	1	1-D BAR WITH CONVECTION								
Edit - Solution Type	2	11	0	0						
	3	12	1	1	0	2	2	0	0	0
B.C./Start Temps	4	20.0	0.0							
B.C./Start Temps	4	12	20.0							
Sources - Steady	4	1	20.0	3.0						
Capacitance	6	0	0							
Conductors - String of	7	10	1	1	2	1	2.0		0	
Conductors - String of	7	9	2	1	12	0	0.03		0	
Conductors - Single	8	1	12	0.015		0				
Conductors - Single	8	11	12	0.015		0				
Solution Cntrl - Steady State	14	20	1.0	0.01	10					
Solution Cntrl - Steady State	14	0	0							
Solution Cntrl - Steady State	14	10	0							


NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T.K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Edit Multiple Conductors
Enter Conductor Data: 0
Row#\ # of Node A Inc. A Node B Inc. B 'Cond.' CTYPE
1 10 1 1 2 1 2 0
2 9 2 1 12 0 0.03 0
3
4
5
6
7
8
9
10
Insert Row
Delete Row
OK
Cancel


NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T.K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Edit Single Conductors
Enter Conductor Data: 0
Row#\ Node A Node B 'Cond.' CTYPE
1 1 12 0.015 0
2 11 12 0.015 0
3
4
5
6
7
8
9
10
Insert Row
Delete Row
OK
Cancel


NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Plot Steady State Temperature vs. Node Number
Plot Exit
File Edit Graphs Options
Results at Final Iteration
1
Temperature
36
34
32
30
28
26
24
22
20
0
2
4
6
8
10
12
14
Node Number

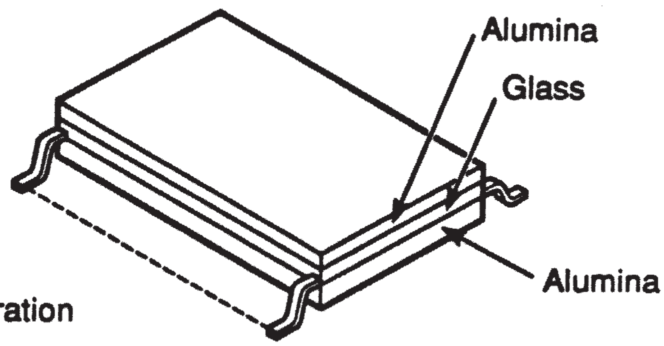
TNETFA Output File (DOUT)

```
********************************************************************************
 ****        Electronics Thermal Analysis Package - PC TNETFA V5.0         ****
 ****            (C) Copyright 1996 by Thermal Computations, Inc.          ****
 ****                           Newberg, Oregon                            ****
********************************************************************************

SAMPLE TNETFA PROBLEM
1-D BAR WITH CONVECTION

 UNITS=0
 NUMBER OF NODES=  12     NUMBER OF CONDUCTORS=  42
 NLOOP =   20  TPRINT=   10      NPRINT=    0
 LOOPEN=   10    ALDT= .1000E-01   BETA= 1.00

                                 LOOPCT=    0
                                 TEMPERATURES

  T(  1)= .2000E+02   T(  2)= .2000E+02   T(  3)= .2000E+02   T(  4)= .2000E+02
  T(  5)= .2000E+02   T(  6)= .2000E+02   T(  7)= .2000E+02   T(  8)= .2000E+02
  T(  9)= .2000E+02   T( 10)= .2000E+02   T( 11)= .2000E+02   T( 12)= .2000E+02

                                 LOOPCT=    2
                                 TEMPERATURES

  T(  1)= .3454E+02   T(  2)= .3315E+02   T(  3)= .3195E+02   T(  4)= .3094E+02
  T(  5)= .3009E+02   T(  6)= .2939E+02   T(  7)= .2883E+02   T(  8)= .2841E+02
  T(  9)= .2811E+02   T( 10)= .2793E+02   T( 11)= .2787E+02   T( 12)= .2000E+02
                                 MAXDT          = .0000E+00
                                 ENERGY BALANCE = 8.5913E-12 PERCENT
```

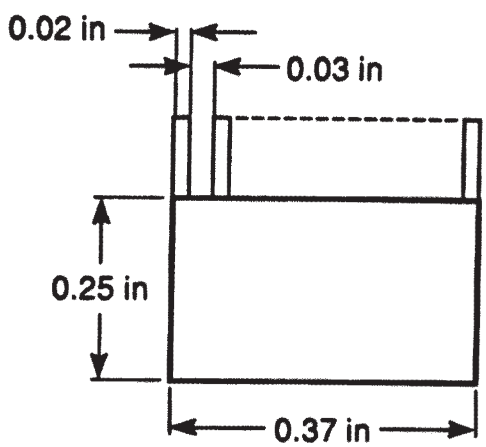
Example

Flatpack

(a) General Configuration



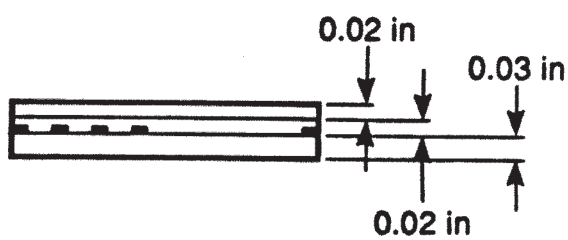
(b) Top View



Package is on 4 in.x5 in. horizontal board facing upward.

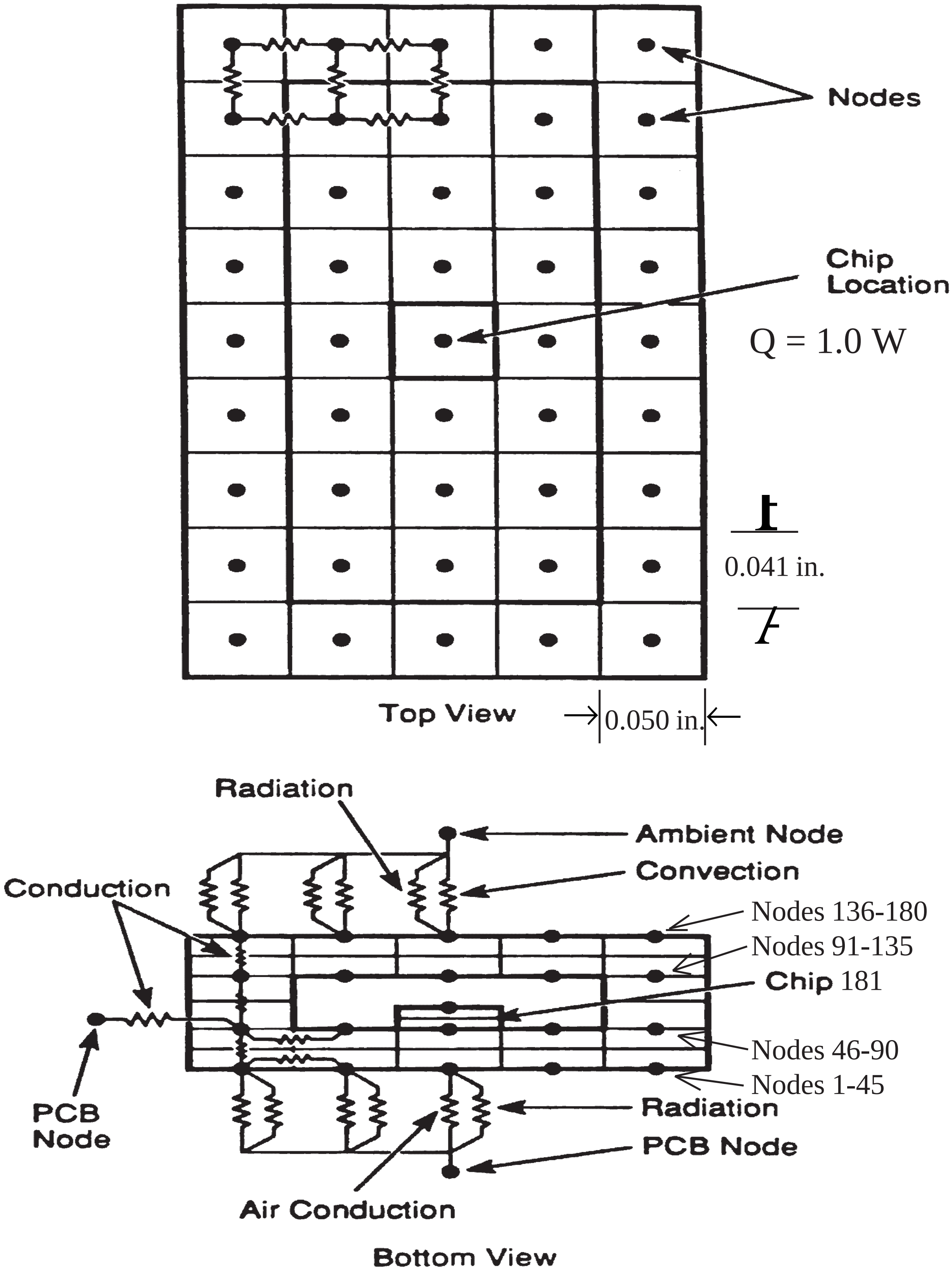
Package-to-board gap = 0.002 in.

(c) Side View



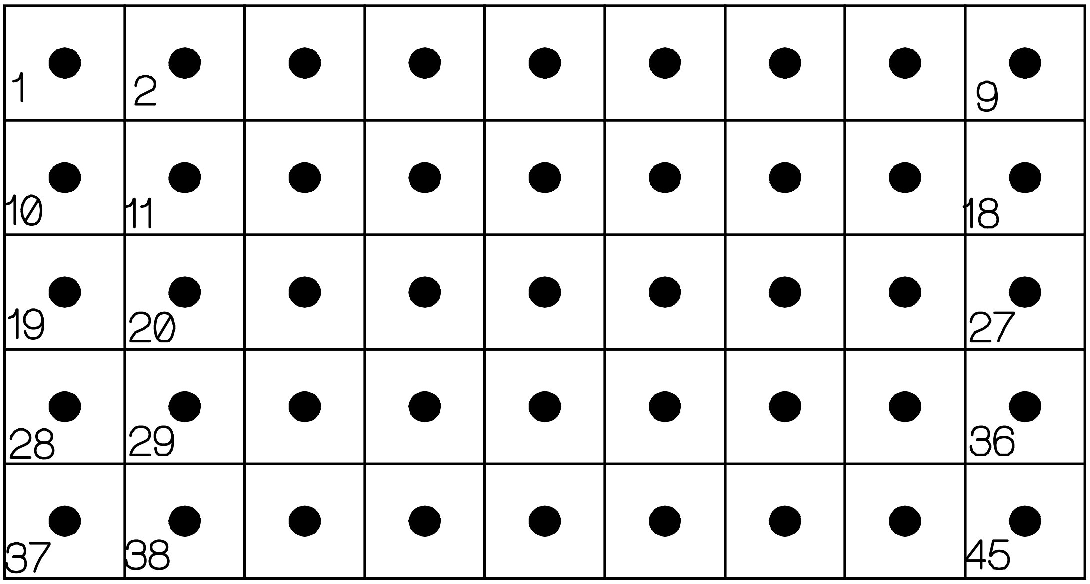
Board T= 50 ^{o}C.

Ambient for convection and radiation is T= 30 ^{o}C.

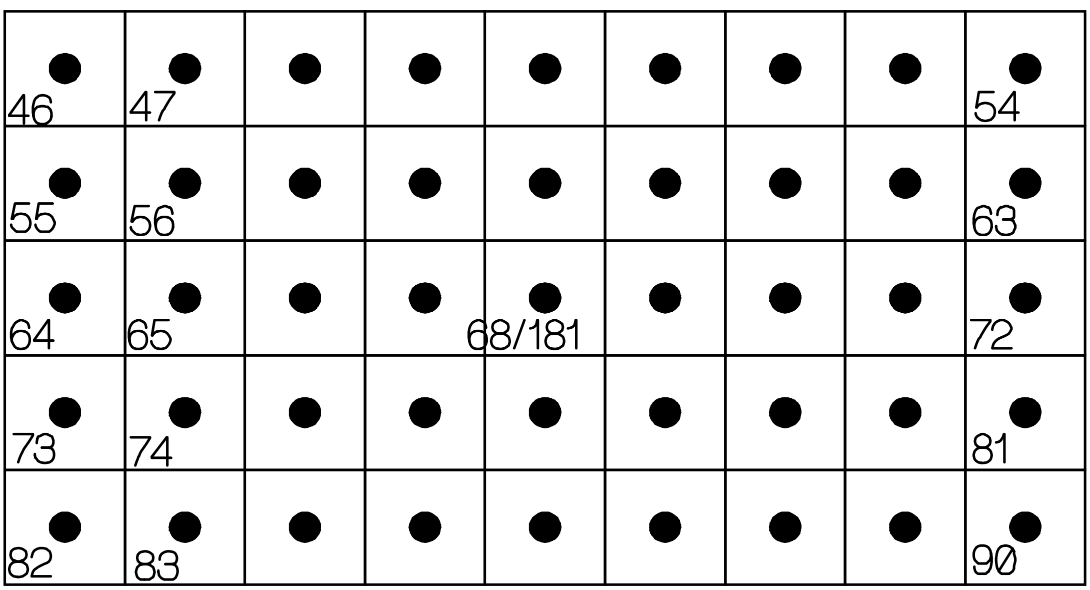


Nodes
Chip Location
Q = 1.0 W
t
0.041 in.
Top View
0.050 in.
Radiation
Ambient Node
Convection
Conduction
Nodes 136-180
Nodes 91-135
Chip 181
Nodes 46-90
Nodes 1-45
PCB Node
Radiation
PCB Node
Air Conduction
Bottom View

Node Map

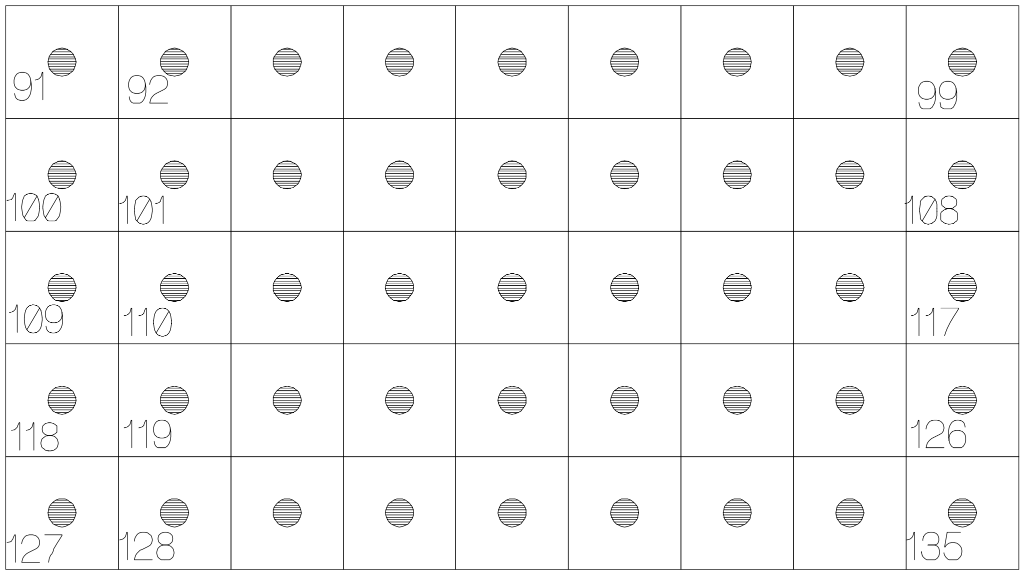


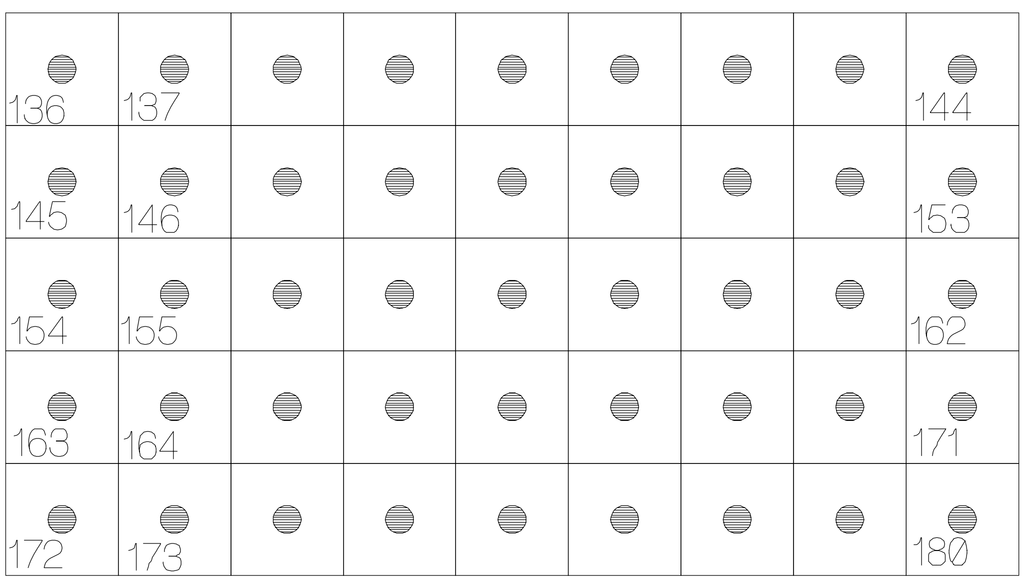
Air conduction to board node no. 182.
Radiation to board node no. 183.



Lead conduction to board node no. 186
from nodes 47-53, 83-89.

Node Map Continued





Convection to ambient air node no. 184.
Radiation to ambient node no. 185.

Typical computer program input

Conduction $C_k = k \frac{A_k}{L}$

$k \equiv$ thermal conductivity of material
$A_k \equiv$ cross-sectional area of conduction path
$L \equiv$ center-to-center node distance

Convection $C_c = h_c A_s$

$h_c \equiv$ convective heat transfer coefficient
$A_s \equiv$ convecting surface area of node

Radiation (simple)

$\varepsilon \equiv$ surface emissivity
$A_s \equiv$ radiating nodal surface area

Other

Node dissipations (for heat sources)
Fixed temperatures (ambients)
Solution and output control

Typical input calculations

Ceramic base conduction:

Put $k = 1.0\ W/in.\cdot{}^oC$ in array 1.

$$\text{Long dir. } C = k\frac{wt}{L} = \frac{(1.0)(0.05)(0.015)}{(0.041)}$$
$$= (1.0)(0.0183)$$

$$\text{Short dir. } C = \frac{(1.0)(0.041)(0.015)}{(0.05)}$$
$$= (1.0)(0.0123)$$

Glass:

Put $k = 0.03$ in array 2

Ceramic top:

Put $k = 1.0$ in array 3

Air gap conduction:

Put T, k pairs in array 4

Package-to-board radiation:

If $\varepsilon_1 = \varepsilon_2 = 0.5, \ \mathcal{F} = 0.33$

$$\mathcal{F}A = (0.33)(0.041)(0.05)$$

$$= 6.77x10^{-4} \quad \text{for each node conductance input.}$$

Package-to-board lead conduction:

From pkg nodes $47 \rightarrow 53$ and $83 \rightarrow 89$ to board node 186, divide amongst 14 conductors.

$$R_{total} = \frac{1}{16 \text{ leads}}\left(\frac{l}{kA_k}\right)$$

$$= \frac{1}{16}\left[\frac{(0.05\,in.)}{(10)(0.02)(0.008)}\right]$$

$$= 1.953 \quad \text{for copper}$$

From 14 nodes,

$$r = 14R_{total} = 273$$

$$c = 1/r = 0.0366\ W/{}^oC$$

Convection to ambient

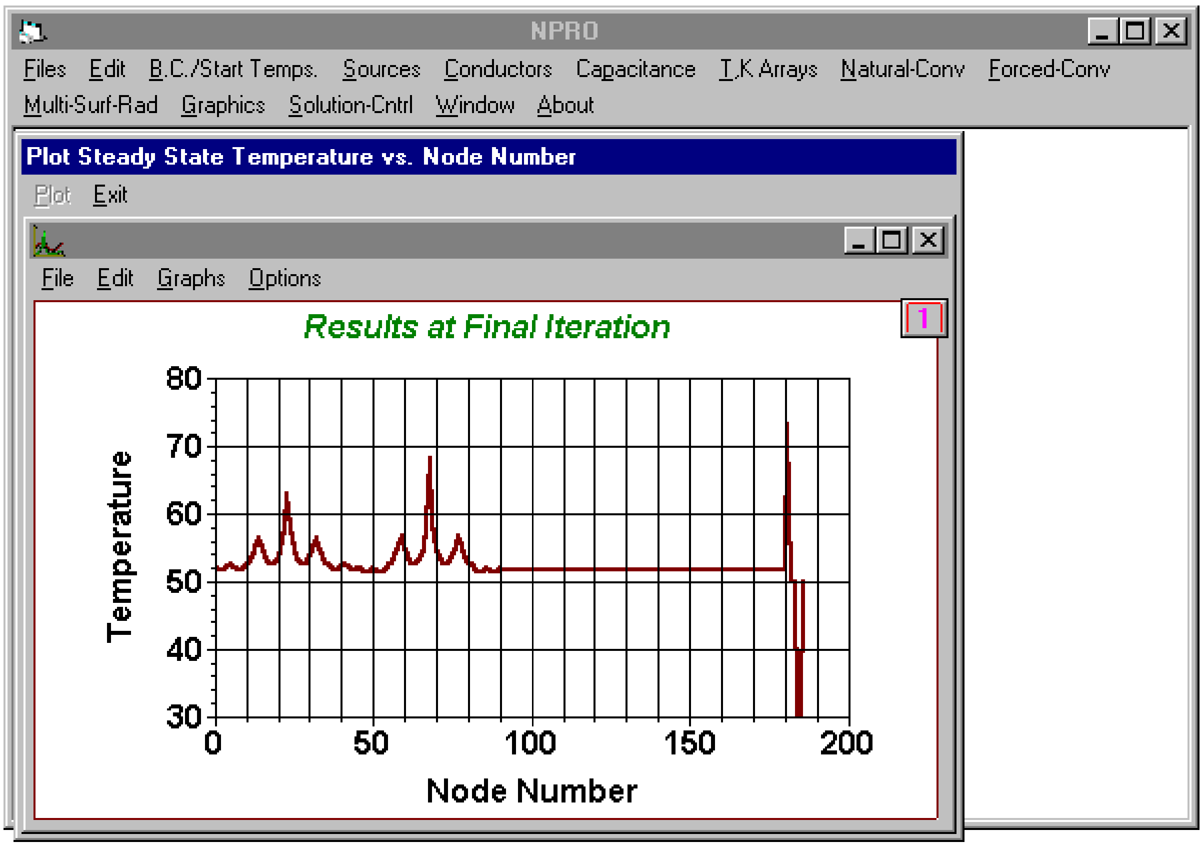
Use natural convection *h* for entire board.

ATYPE = 7 for small device.

$$P = \frac{WL}{2(W+L)} = \frac{4x5}{2(4+5)} = 1.11\,in.$$

Radiation to ambient:

$$\varepsilon A_s = (0.5)(0.041\,in.)(0.05\,in.)$$
$$= 0.001025\,in.^2$$



NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T.K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Plot Steady State Temperature vs. Node Number
Plot Exit
File Edit Graphs Options
Results at Final Iteration
1
Temperature
80
70
60
50
40
30
0
50
100
150
200
Node Number

TNETFA Partial Output File (DOUT)

```
********************************************************************************
 ****        Electronics Thermal Analysis Package - PC TNETFA V5.0          ****
 ****           (C) Copyright 1996 by Thermal Computations, Inc.            ****
 ****                         Newberg, Oregon                               ****
********************************************************************************

Flatpack on 4 in. x 5 in. horizontal board.
0.05 in. x 0.04 in. chip.

 UNITS=2
 NUMBER OF NODES= 186     NUMBER OF CONDUCTORS=1322
 NLOOP =  500  TPRINT=  500      NPRINT=    0
 LOOPEN=  500    ALDT= .1000E-02   BETA= 1.50

 ARRAY DATA
 ARRAY  1       2  X-Y PAIRS
  .2000E+02,   .1000E+01     .1000E+03,   .1000E+01
 ARRAY  2       2  X-Y PAIRS
  .2000E+02,   .3000E-01     .1000E+03,   .3000E-01
 ARRAY  3       2  X-Y PAIRS
  .2000E+02,   .1000E+01     .1000E+03,   .1000E+01
 ARRAY  4       4  X-Y PAIRS
  .0000E+00,   .6153E-03     .3800E+02,   .6769E-03
  .9300E+02,   .7684E-03     .1490E+03,   .8483E-03

 NATURAL CONVECTION PARAMETER
   1    SMALL SURFACE, HORIZONTAL,
        HEATED SIDE FACING UP OR COOLED SIDE FACING DOWN:     P= .1110E+01



  T(185)= .3000E+02   T(186)= .5000E+02

                                 LOOPCT=  162
                                 TEMPERATURES

  T(  1)= .5195E+02   T(  2)= .5172E+02   T(  3)= .5191E+02   T(  4)= .5234E+02
  T(  5)= .5266E+02   T(  6)= .5234E+02   T(  7)= .5191E+02   T(  8)= .5172E+02
  T(  9)= .5195E+02   T( 10)= .5256E+02   T( 11)= .5288E+02   T( 12)= .5372E+02
  T( 13)= .5513E+02   T( 14)= .5652E+02   T( 15)= .5513E+02   T( 16)= .5372E+02
  T( 17)= .5288E+02   T( 18)= .5256E+02   T( 19)= .5289E+02   T( 20)= .5347E+02
  T( 21)= .5493E+02   T( 22)= .5793E+02   T( 23)= .6306E+02   T( 24)= .5793E+02
  T( 25)= .5493E+02   T( 26)= .5347E+02   T( 27)= .5289E+02   T( 28)= .5256E+02
  T( 29)= .5288E+02   T( 30)= .5372E+02   T( 31)= .5513E+02   T( 32)= .5652E+02
  T( 33)= .5513E+02   T( 34)= .5372E+02   T( 35)= .5288E+02   T( 36)= .5256E+02
  T( 37)= .5195E+02   T( 38)= .5172E+02   T( 39)= .5191E+02   T( 40)= .5234E+02
  T( 41)= .5266E+02   T( 42)= .5234E+02   T( 43)= .5191E+02   T( 44)= .5172E+02
```

```
T( 45)= .5195E+02   T( 46)= .5192E+02   T( 47)= .5141E+02   T( 48)= .5154E+02
T( 49)= .5189E+02   T( 50)= .5217E+02   T( 51)= .5189E+02   T( 52)= .5154E+02
T( 53)= .5141E+02   T( 54)= .5192E+02   T( 55)= .5256E+02   T( 56)= .5287E+02
T( 57)= .5372E+02   T( 58)= .5518E+02   T( 59)= .5684E+02   T( 60)= .5518E+02
T( 61)= .5372E+02   T( 62)= .5287E+02   T( 63)= .5256E+02   T( 64)= .5289E+02
T( 65)= .5349E+02   T( 66)= .5499E+02   T( 67)= .5846E+02   T( 68)= .6830E+02
T( 69)= .5846E+02   T( 70)= .5499E+02   T( 71)= .5349E+02   T( 72)= .5289E+02
T( 73)= .5256E+02   T( 74)= .5287E+02   T( 75)= .5372E+02   T( 76)= .5518E+02
T( 77)= .5684E+02   T( 78)= .5518E+02   T( 79)= .5372E+02   T( 80)= .5287E+02
T( 81)= .5256E+02   T( 82)= .5192E+02   T( 83)= .5141E+02   T( 84)= .5154E+02
T( 85)= .5189E+02   T( 86)= .5217E+02   T( 87)= .5189E+02   T( 88)= .5154E+02
T( 89)= .5141E+02   T( 90)= .5192E+02   T( 91)= .5176E+02   T( 92)= .5173E+02
T( 93)= .5172E+02   T( 94)= .5173E+02   T( 95)= .5174E+02   T( 96)= .5173E+02
T( 97)= .5172E+02   T( 98)= .5173E+02   T( 99)= .5176E+02   T(100)= .5180E+02
T(101)= .5176E+02   T(102)= .5173E+02   T(103)= .5173E+02   T(104)= .5173E+02
T(105)= .5173E+02   T(106)= .5173E+02   T(107)= .5176E+02   T(108)= .5181E+02
T(109)= .5182E+02   T(110)= .5177E+02   T(111)= .5174E+02   T(112)= .5173E+02
T(113)= .5172E+02   T(114)= .5173E+02   T(115)= .5174E+02   T(116)= .5177E+02
T(117)= .5183E+02   T(118)= .5180E+02   T(119)= .5176E+02   T(120)= .5173E+02
T(121)= .5173E+02   T(122)= .5173E+02   T(123)= .5173E+02   T(124)= .5174E+02
T(125)= .5176E+02   T(126)= .5181E+02   T(127)= .5176E+02   T(128)= .5173E+02
T(129)= .5172E+02   T(130)= .5173E+02   T(131)= .5174E+02   T(132)= .5173E+02
T(133)= .5172E+02   T(134)= .5173E+02   T(135)= .5176E+02   T(136)= .5176E+02
T(137)= .5173E+02   T(138)= .5172E+02   T(139)= .5173E+02   T(140)= .5174E+02
T(141)= .5173E+02   T(142)= .5172E+02   T(143)= .5173E+02   T(144)= .5176E+02
T(145)= .5179E+02   T(146)= .5175E+02   T(147)= .5173E+02   T(148)= .5172E+02
T(149)= .5172E+02   T(150)= .5173E+02   T(151)= .5173E+02   T(152)= .5176E+02
T(153)= .5179E+02   T(154)= .5181E+02   T(155)= .5176E+02   T(156)= .5174E+02
T(157)= .5172E+02   T(158)= .5172E+02   T(159)= .5173E+02   T(160)= .5174E+02
T(161)= .5177E+02   T(162)= .5181E+02   T(163)= .5179E+02   T(164)= .5175E+02
T(165)= .5173E+02   T(166)= .5173E+02   T(167)= .5172E+02   T(168)= .5173E+02
T(169)= .5173E+02   T(170)= .5176E+02   T(171)= .5179E+02   T(172)= .5176E+02
T(173)= .5173E+02   T(174)= .5172E+02   T(175)= .5173E+02   T(176)= .5174E+02
T(177)= .5173E+02   T(178)= .5173E+02   T(179)= .5174E+02   T(180)= .5176E+02
T(181)= .7318E+02   T(182)= .5000E+02   T(183)= .5000E+02   T(184)= .3000E+02
T(185)= .3000E+02   T(186)= .5000E+02
                                  MAXDT          = .9755E-03
                                  ENERGY BALANCE = 4.5735E-01 PERCENT
```

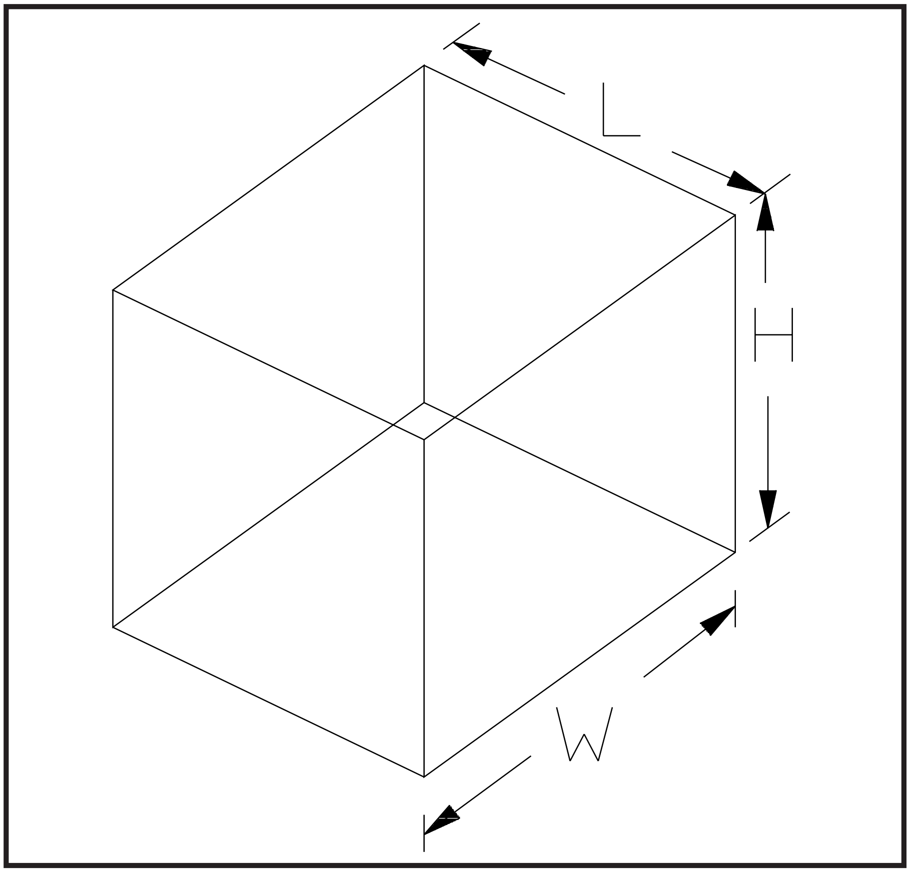
TNETFA Input File (DIN)

```
Flatpack on 4 in. x 5 in. horizontal board.
0.05 in. x 0.04 in. chip.
 1         2           0
 186       5           1           0           45          0           4           1           0
 50            0
182           5.0000E+01
183           5.0000E+01
184           3.0000E+01
185           3.0000E+01
186           5.0000E+01
181           6.0000E+01    1.0000E+00
 0             0
8             1             1             2             1             1.8300E-02    1
8             10            1             11            1             1.8300E-02    1
8             19            1             20            1             1.8300E-02    1
8             28            1             29            1             1.8300E-02    1
8             37            1             38            1             1.8300E-02    1
36            1             1             10            1             1.2300E-02    1
8             46            1             47            1             1.8300E-02    1
8             55            1             56            1             1.8300E-02    1
8             64            1             65            1             1.8300E-02    1
8             73            1             74            1             1.8300E-02    1
8             82            1             83            1             1.8300E-02    1
36            46            1             55            1             1.2300E-02    1
45            1             1             46            1             6.8300E-02    1
8             46            1             47            1             1.2200E-02    2
8             82            1             83            1             1.2200E-02    2
4             46            9             55            9             8.2000E-03    2
4             54            9             63            9             8.2000E-03    2
8             91            1             92            1             1.8300E-02    3
8             100           1             101           1             1.8300E-02    3
8             109           1             110           1             1.8300E-02    3
8             118           1             119           1             1.8300E-02    3
8             127           1             128           1             1.8300E-02    3
36            91            1             100           1             1.2300E-02    3
8             136           1             137           1             1.8300E-02    3
8             145           1             146           1             1.8300E-02    3
8             154           1             155           1             1.8300E-02    3
8             163           1             164           1             1.8300E-02    3
8             172           1             173           1             1.8300E-02    3
36            136           1             145           1             1.2300E-02    3
45            91            1             136           1             1.0250E-01    3
8             91            1             92            1             1.2200E-02    2
8             127           1             128           1             1.2200E-02    2
4             91            9             100           9             8.2000E-03    2
4             99            9             108           9             8.2000E-03    2
9             46            1             91            1             1.0250E-01    2
9             82            1             127           1             1.0250E-01    2
3             55            9             100           9             1.0250E-01    2
3             63            9             108           9             1.0250E-01    2
45            1             1             182           0             1.0250E+00    4
45            1             1             183           0             6.7700E-04    -1
45            136            1             184           0             2.0500E-03    101
1             181           0             68            0             2.0500E-01    0
7             47            1             186           0             3.6600E-02    0
7             83            1             186           0             3.6600E-02    0
45            136           1             185           0             1.0250E-03    -1
```

```
1      2
2.0000E+01     1.0000E+00     1.0000E+02     1.0000E+00
2      2
2.0000E+01     3.0000E-02     1.0000E+02     3.0000E-02
3      2
2.0000E+01     1.0000E+00     1.0000E+02     1.0000E+00
4      4
0.0000E+00     6.1530E-04     3.8000E+01     6.7690E-04
9.3000E+01     7.6840E-04     1.4900E+02     8.4830E-04
7             1.1100E+00
 500           1.5           0.001         500
 0             0
 500           0
```

Example

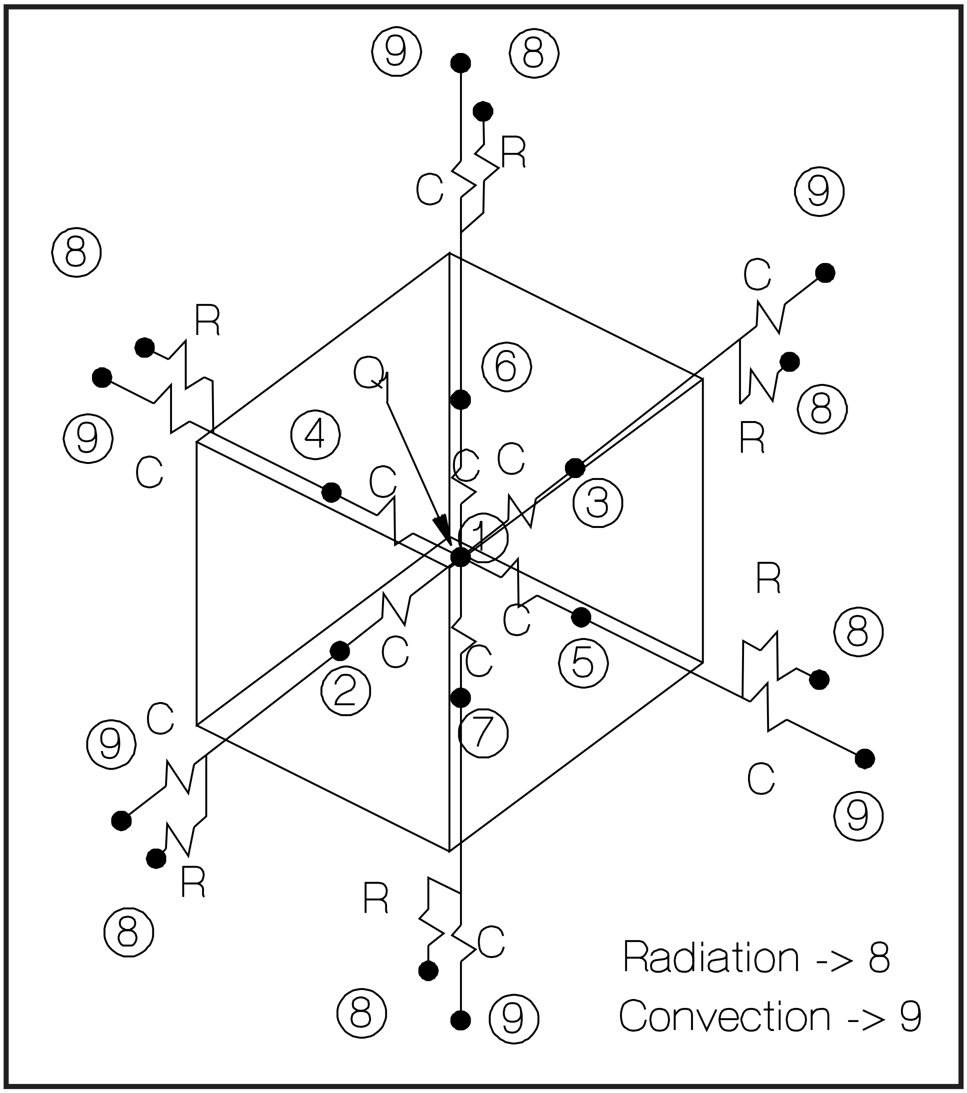
Sealed Enclosure



Geometry:

$$L = \text{length}$$
$$W = \text{width}$$
$$H = \text{height}$$
$$t_w = \text{wall thickness} \ll L, W, H$$
$$k = \text{wall thermal conductivity}$$
$$\varepsilon = \text{emissivity of enclosure exterior}$$

Thermal circuit

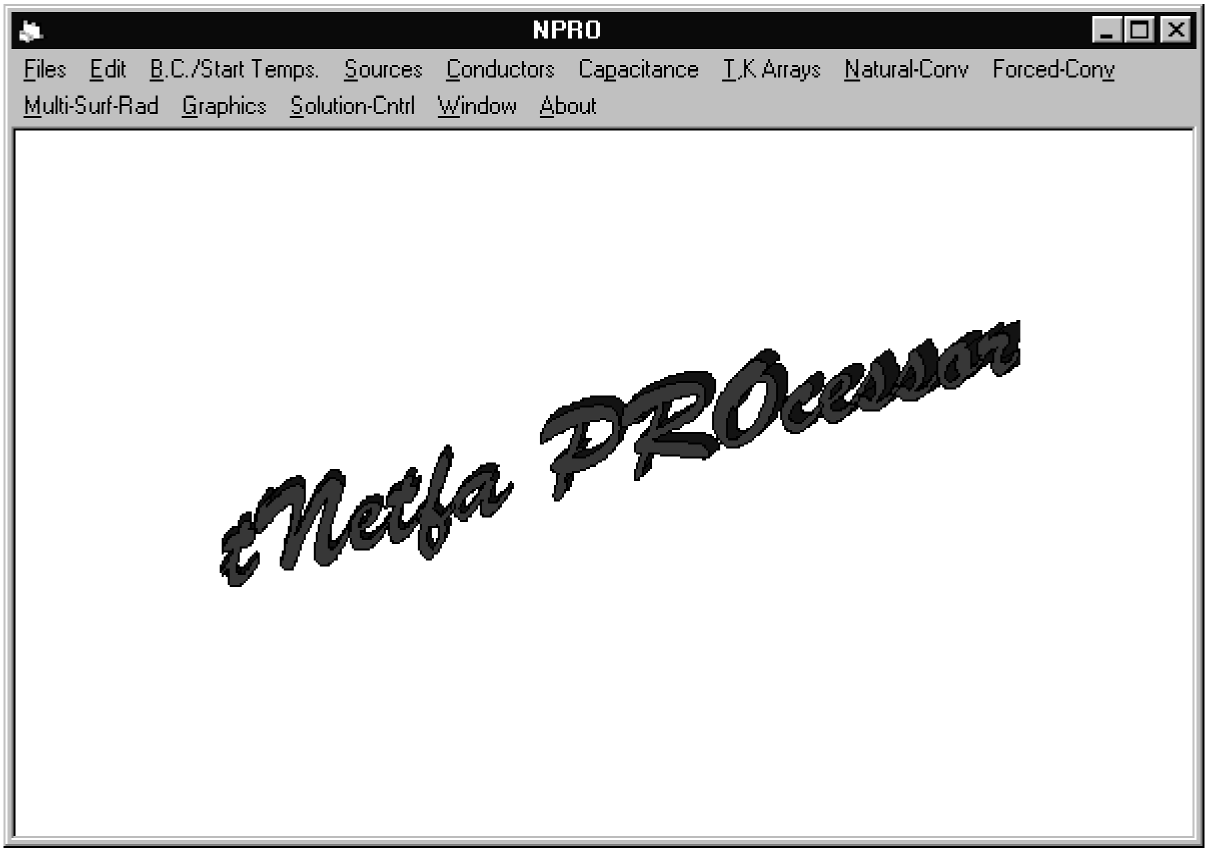


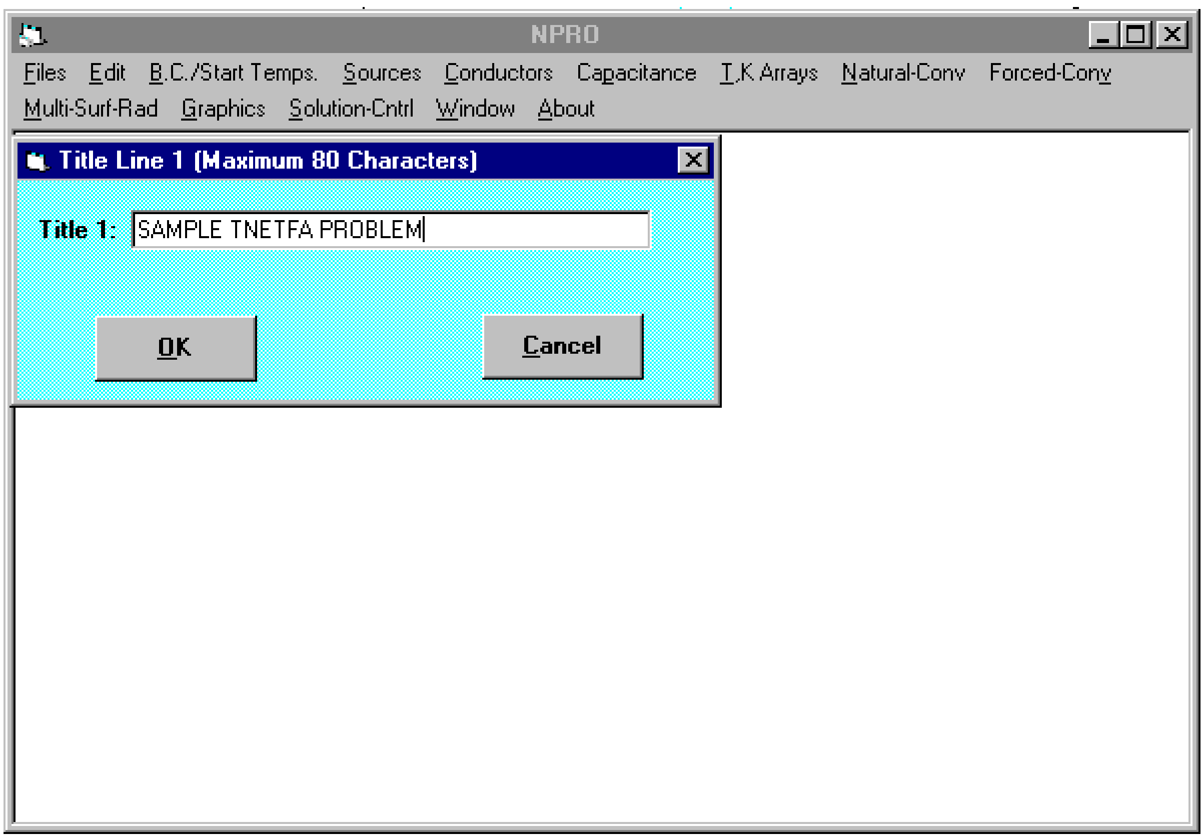
$$H = W = L = 10.0\ in.$$

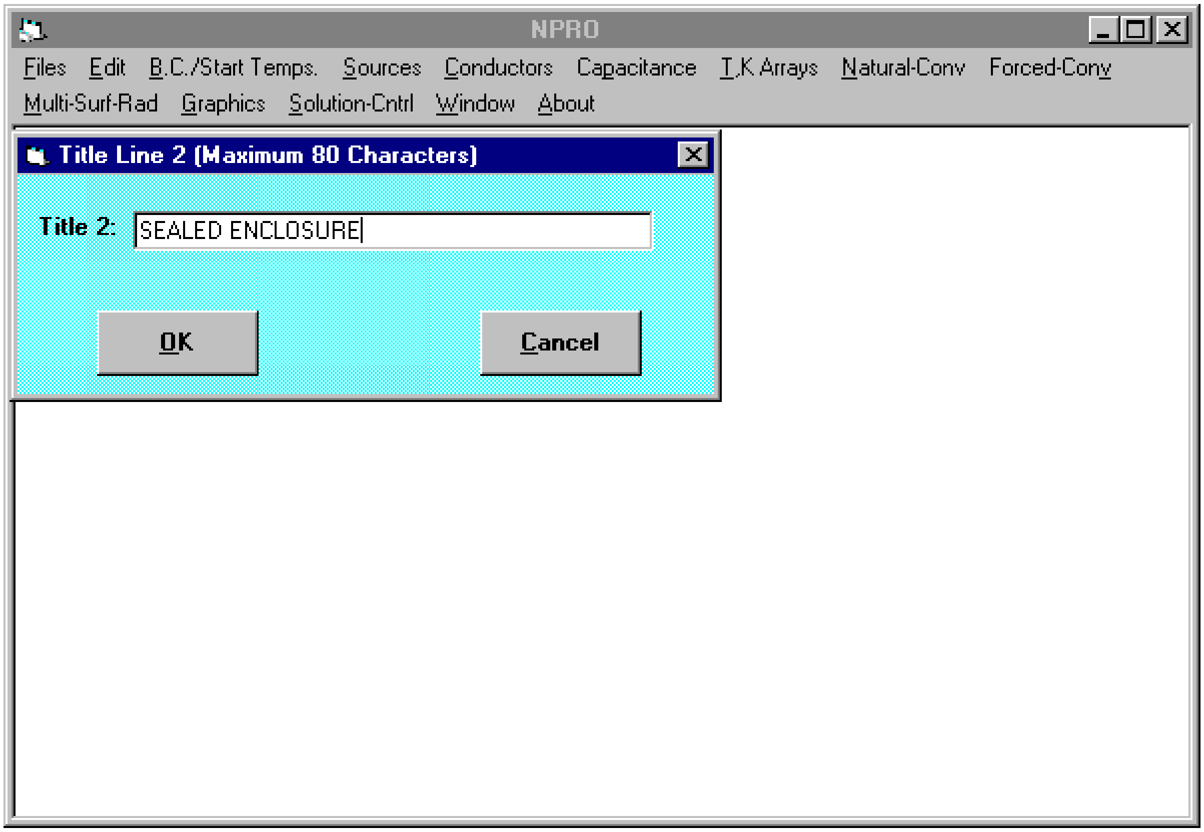
$$\varepsilon = 0.9,\ \text{exterior}$$

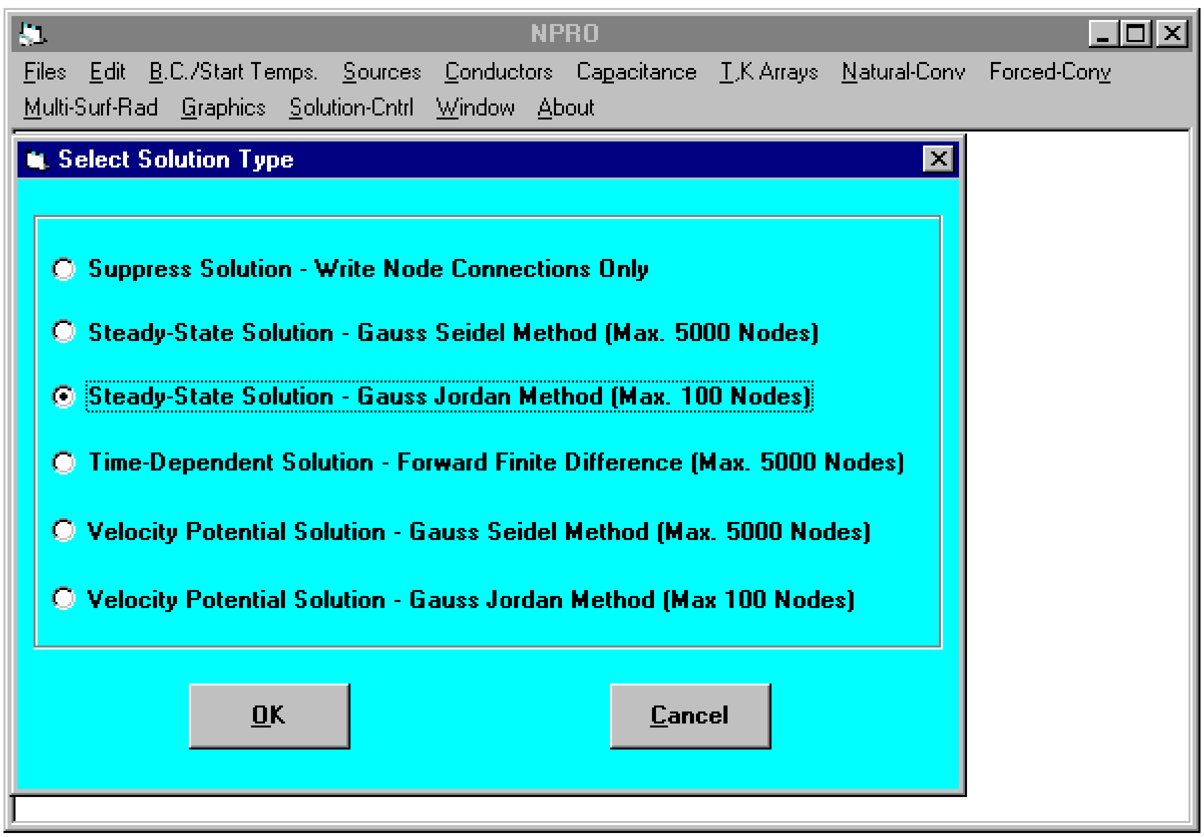
$$Q1 = 12\ \text{W}$$

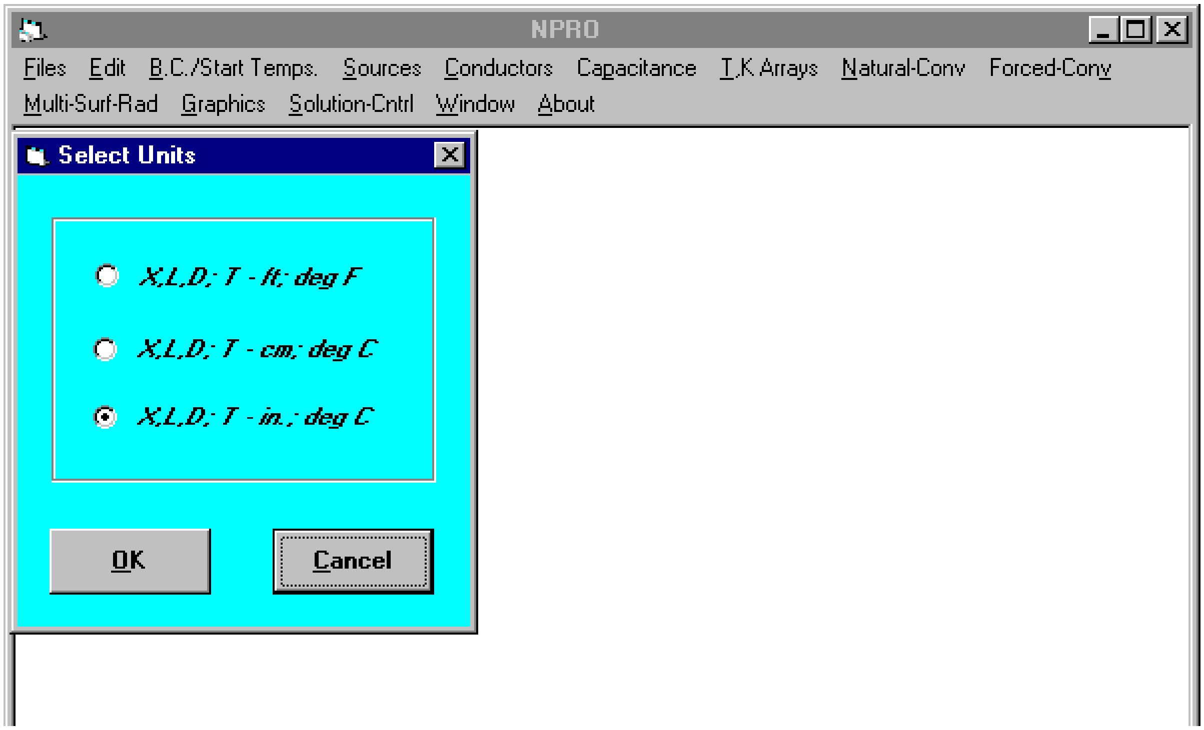
NPRO OPTION	DATA SET	TNETFA INPUT								
Edit - Title Line 1	1	SAMPLE TNETFA PROBLEM								
Edit - Title Line 2	1	SEALED ENCLOSURE								
Edit - Solution Type	2	11	2	0						
	3	9	2	1	0	7	0	0	3	0
B.C./Start Temps	4	20.0	0.0							
B.C./Start Temps	4	8	20.0							
B.C./Start Temps	4	9	20.0							
Sources - Steady	4	1	20.0	12.0						
Capacitance	6	0	0							
Conductors - String of	7	4	1	0	2	1	100.0	101		
Conductors - String of	7	1	1	0	6	0	100.0	102		
Conductors - String of	7	1	1	0	7	0	100.0	103		
Conductors - String of	7	4	2	1	9	0	100.0	101		
Conductors - String of	7	1	6	0	9	0	100.0	102		
Conductors - String of	7	1	7	0	9	0	100.0	103		
Conductors - String of	7	6	2	1	8	0	90.0	-1		
Natural Convection	11	1	10.0							
Natural Convection	11	2	2.5							
Natural Convection	11	3	2.5							
Solution-Cntrl - Steady State	14	25	1.0	0.01	5					
Solution-Cntrl - Steady State	14	0	0							
Solution-Cntrl - Steady State	14	5	0							

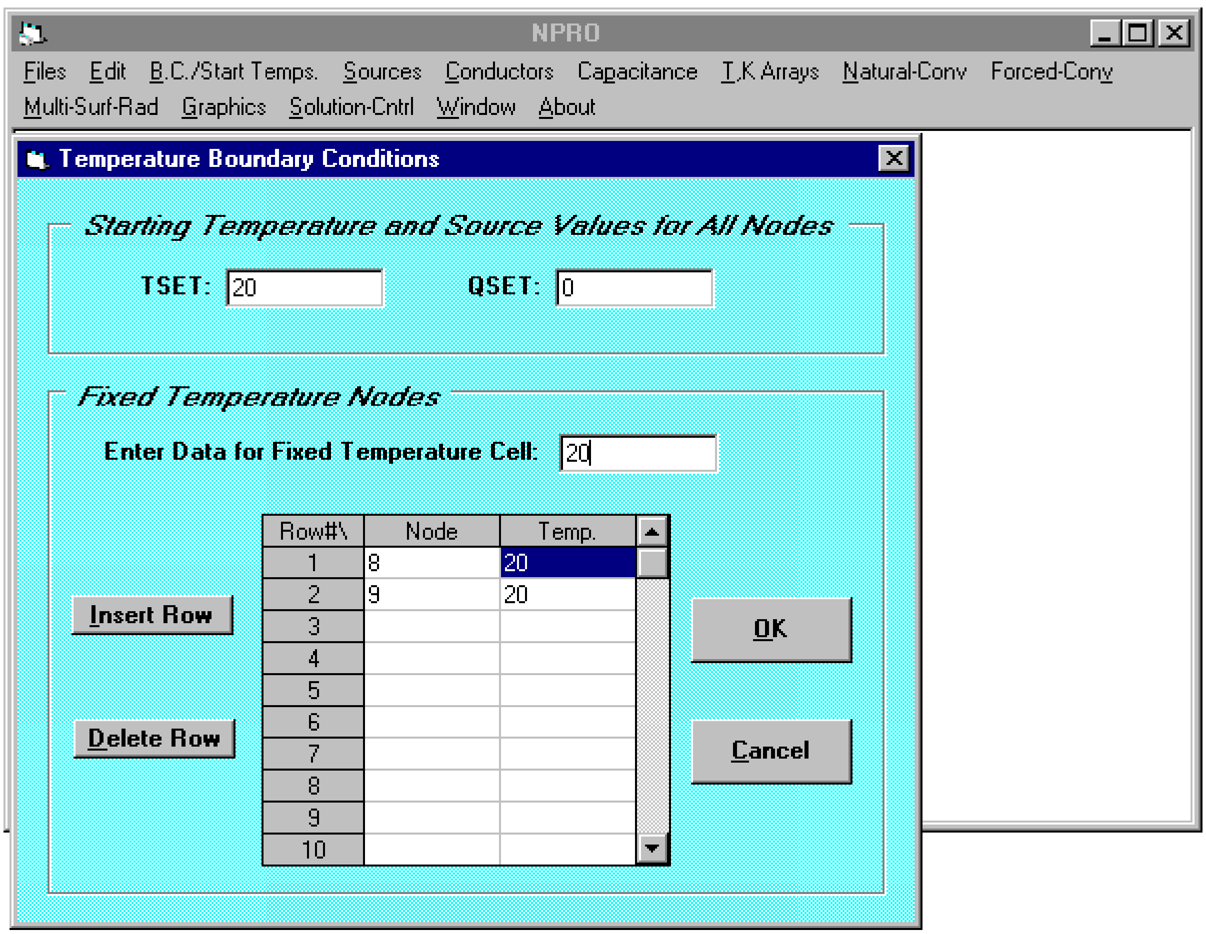

NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
tNetfa PROcessor

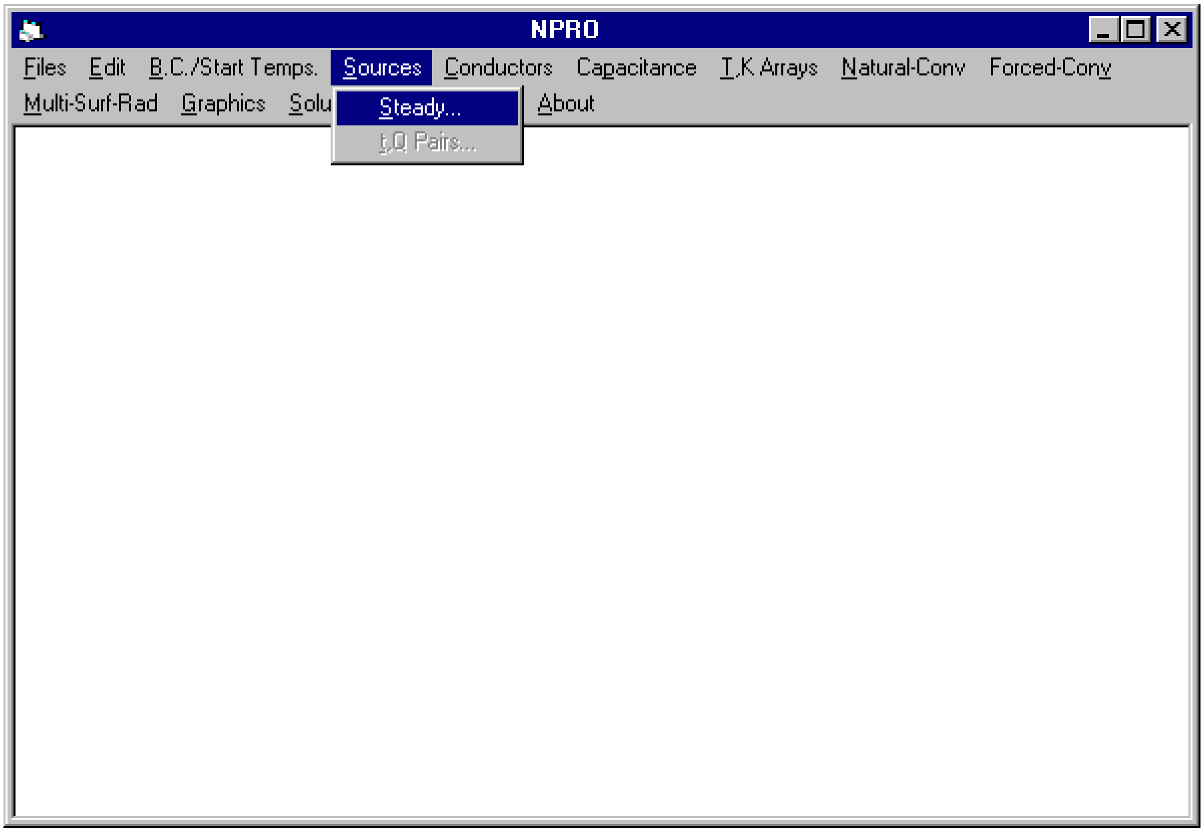

NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Title Line 1 (Maximum 80 Characters)
Title 1:
SAMPLE TNETFA PROBLEM
OK
Cancel

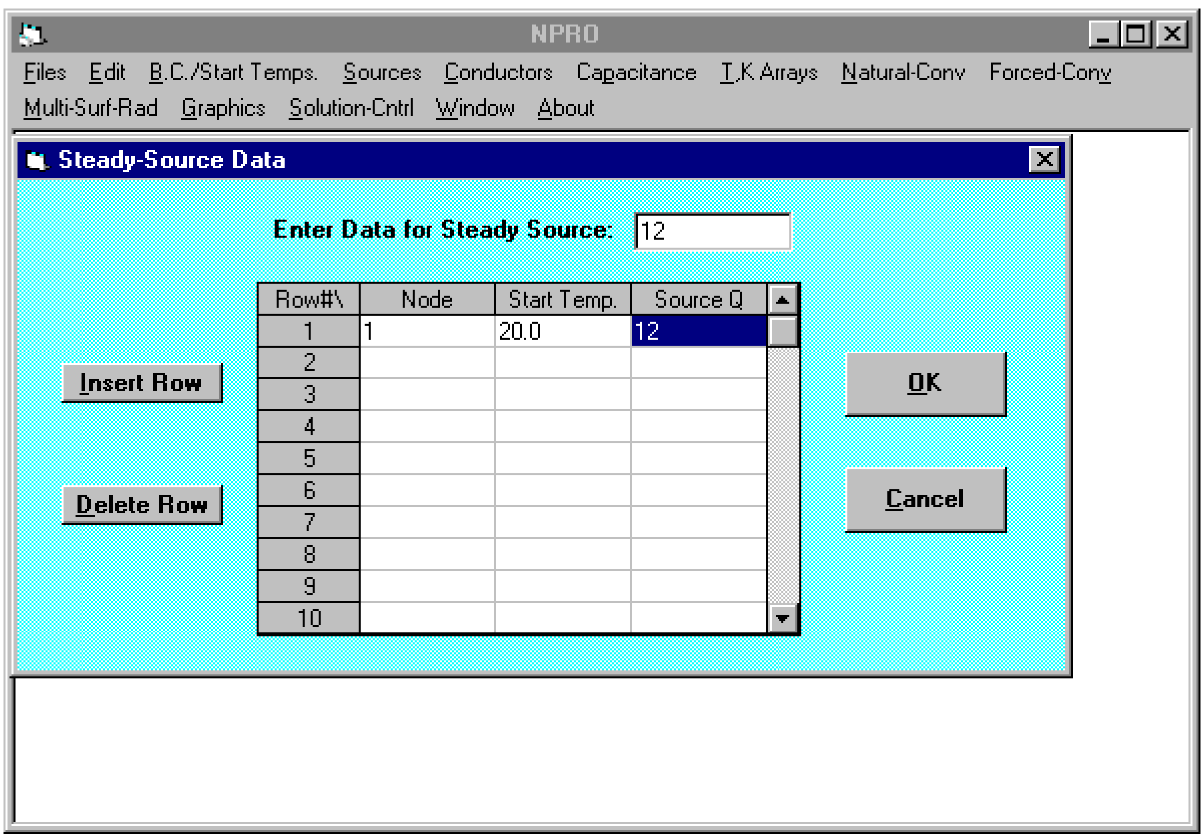

NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Title Line 2 (Maximum 80 Characters)
Title 2: SEALED ENCLOSURE
OK
Cancel

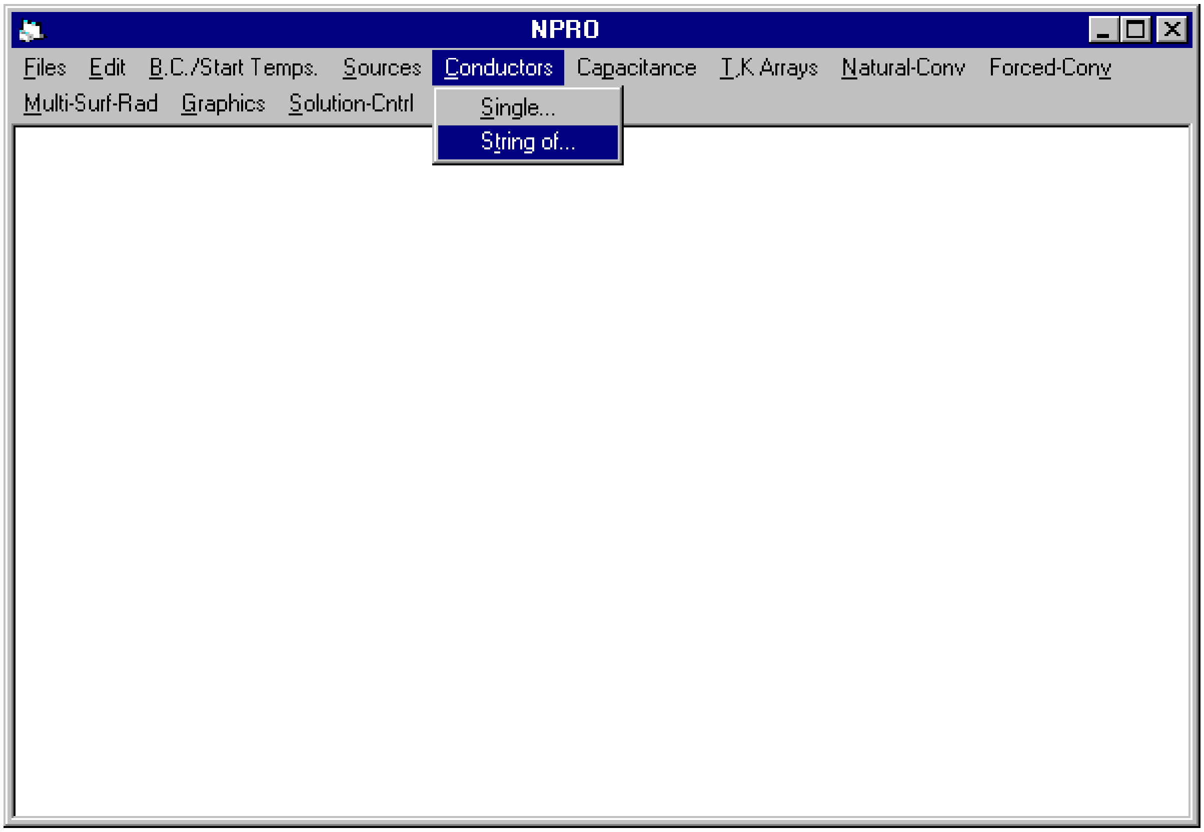

NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Select Solution Type
Suppress Solution - Write Node Connections Only
Steady-State Solution - Gauss Seidel Method (Max. 5000 Nodes)
Steady-State Solution - Gauss Jordan Method (Max. 100 Nodes)
Time-Dependent Solution - Forward Finite Difference (Max. 5000 Nodes)
Velocity Potential Solution - Gauss Seidel Method (Max. 5000 Nodes)
Velocity Potential Solution - Gauss Jordan Method (Max 100 Nodes)
OK
Cancel

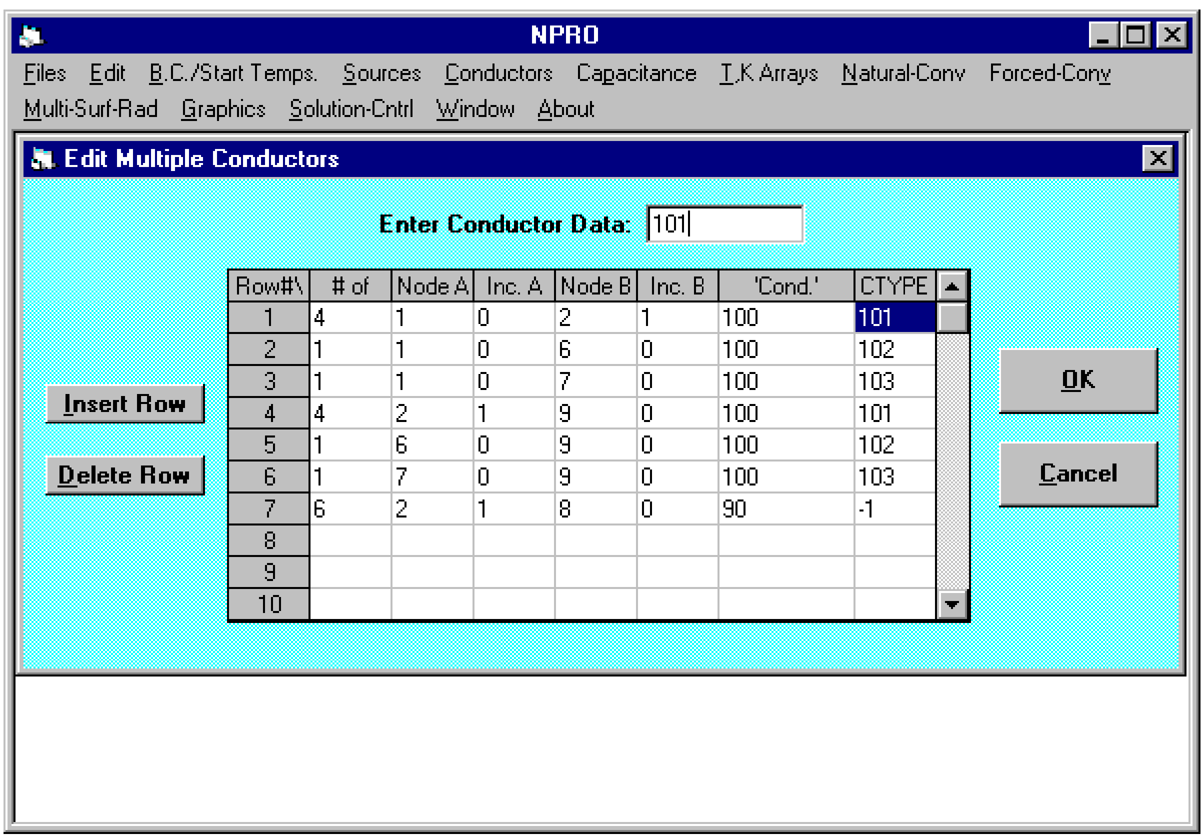

NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Select Units
X,L,D; T - ft; deg F
X,L,D; T - cm; deg C
X,L,D; T - in.; deg C
OK
Cancel


NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Temperature Boundary Conditions
Starting Temperature and Source Values for All Nodes
TSET: 20
QSET: 0
Fixed Temperature Nodes
Enter Data for Fixed Temperature Cell: 20
Row#\ Node Temp.
1 8 20
2 9 20
3
4
5
6
7
8
9
10
Insert Row
Delete Row
OK
Cancel


NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solu About
Steady...
t,Q Pairs...


NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Steady-Source Data
Enter Data for Steady Source: 12
Row#\ Node Start Temp. Source Q
1 1 20.0 12
2
3
4
5
6
7
8
9
10
Insert Row
Delete Row
OK
Cancel

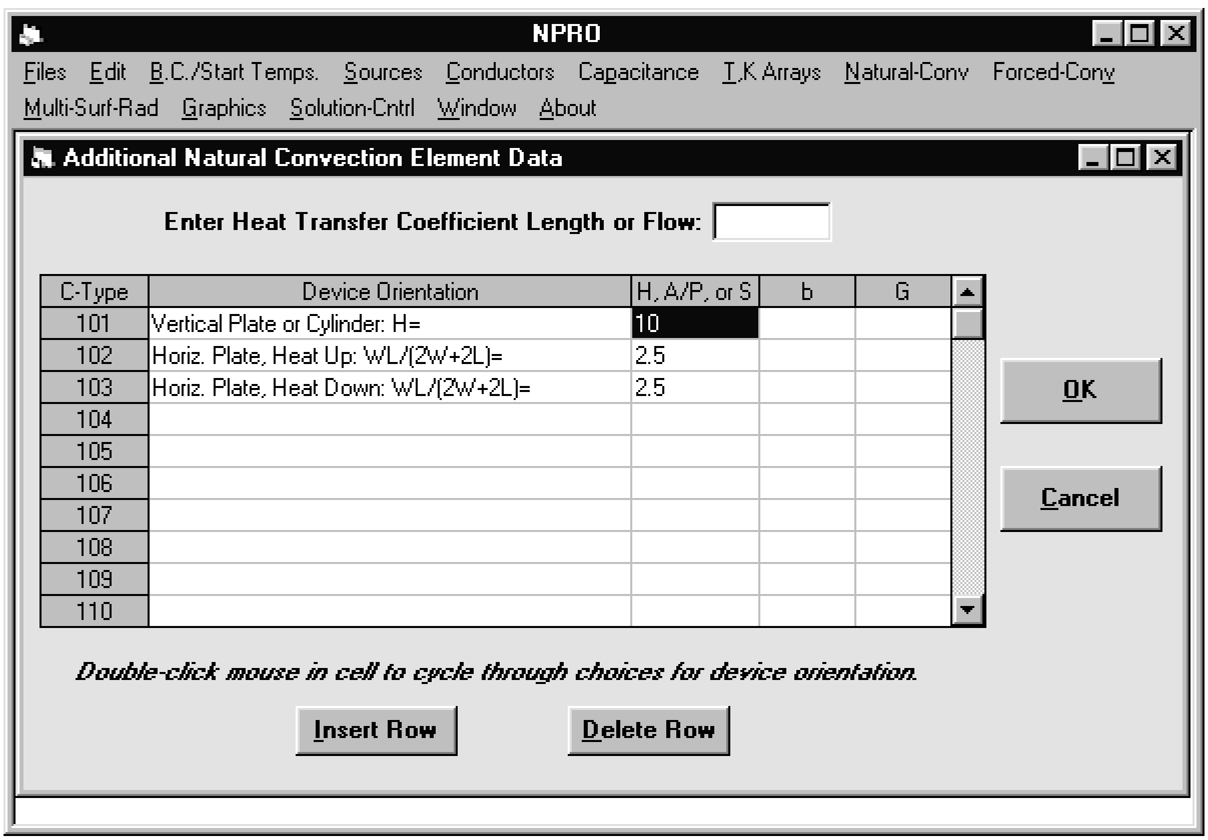

NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl
Single...
String of...

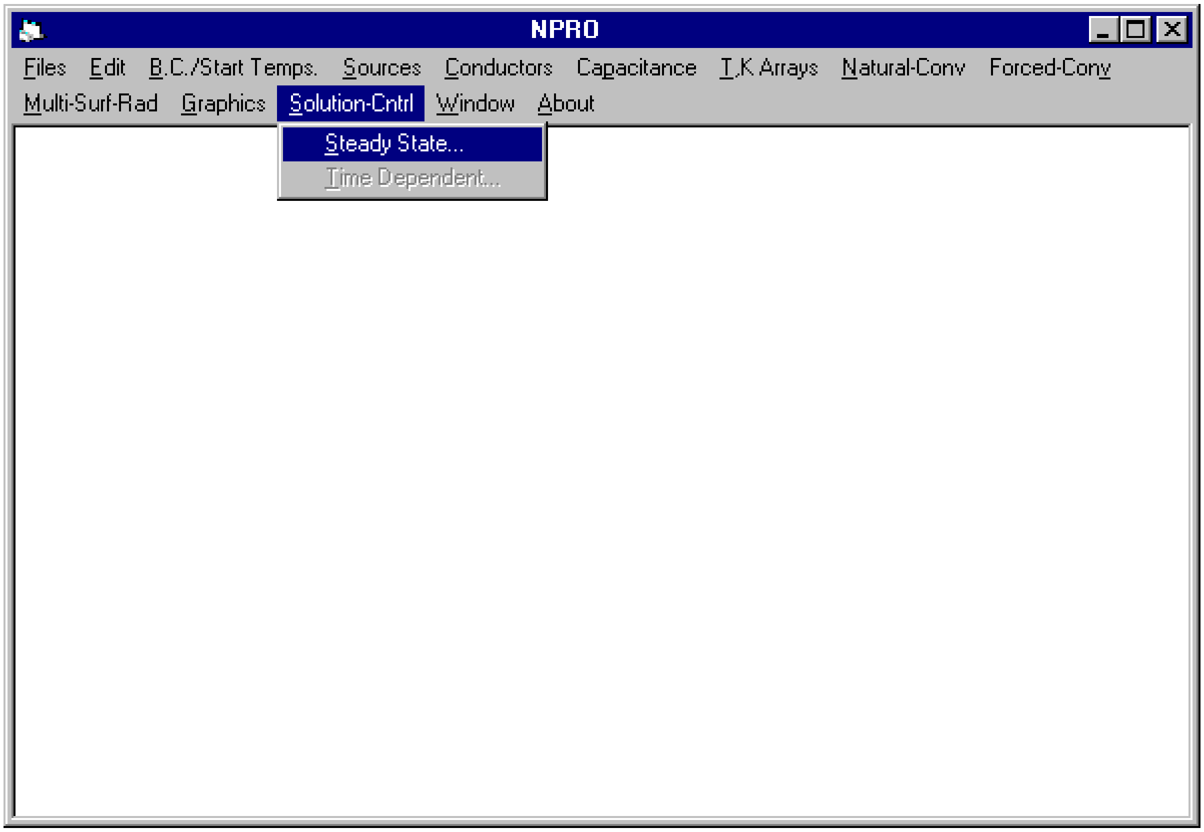

NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T,K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Edit Multiple Conductors
Enter Conductor Data: 101
Row#\ # of Node A Inc. A Node B Inc. B 'Cond.' CTYPE
1 4 1 0 2 1 100 101
2 1 1 0 6 0 100 102
3 1 1 0 7 0 100 103
4 4 2 1 9 0 100 101
5 1 6 0 9 0 100 102
6 1 7 0 9 0 100 103
7 6 2 1 8 0 90 -1
8
9
10
Insert Row
Delete Row
OK
Cancel

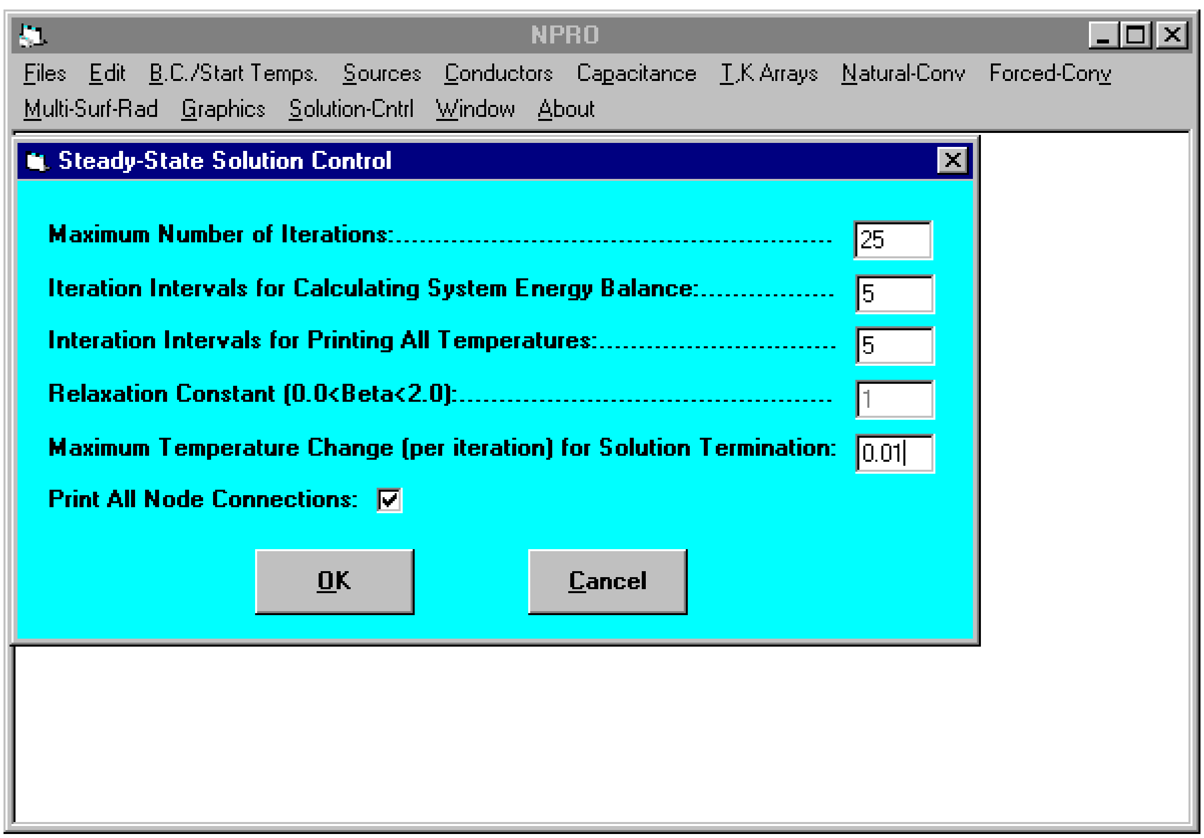
The characteristic lengths for the enclosure are:

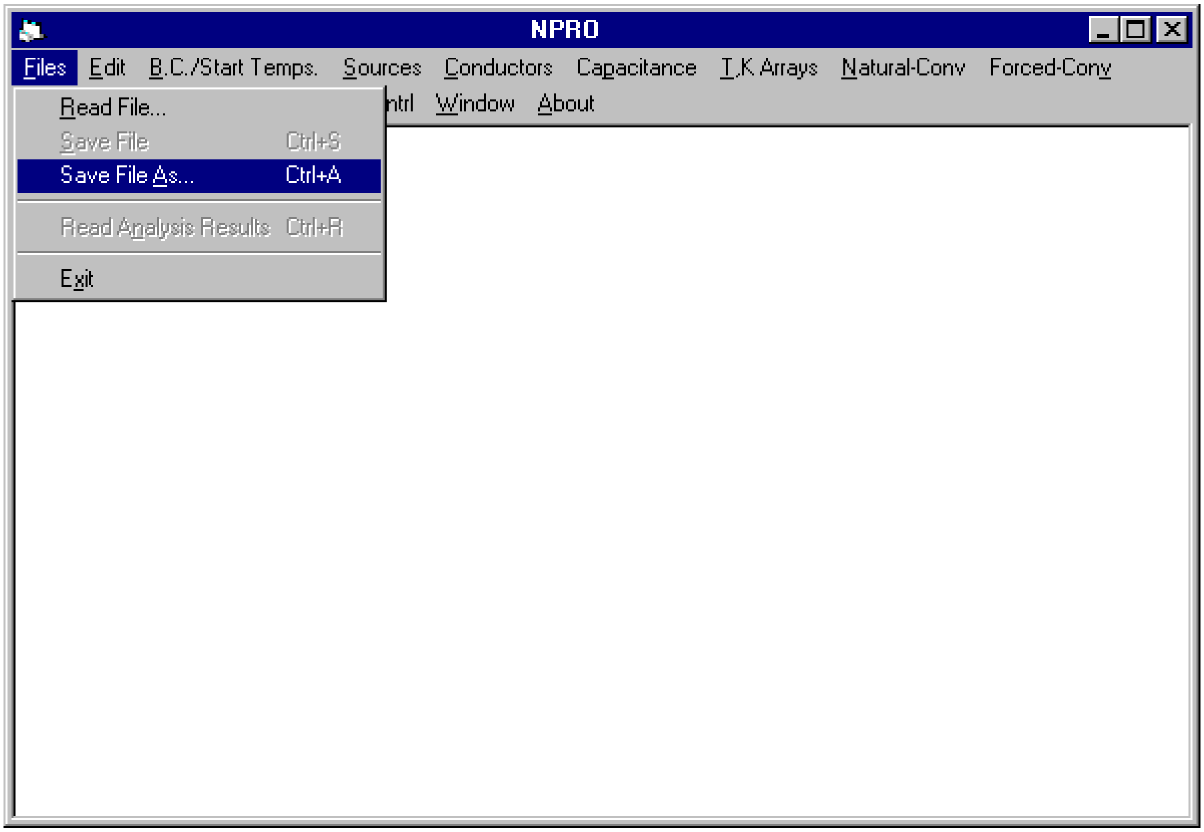
Vertical panels: $P = H = 10.0\,in.$

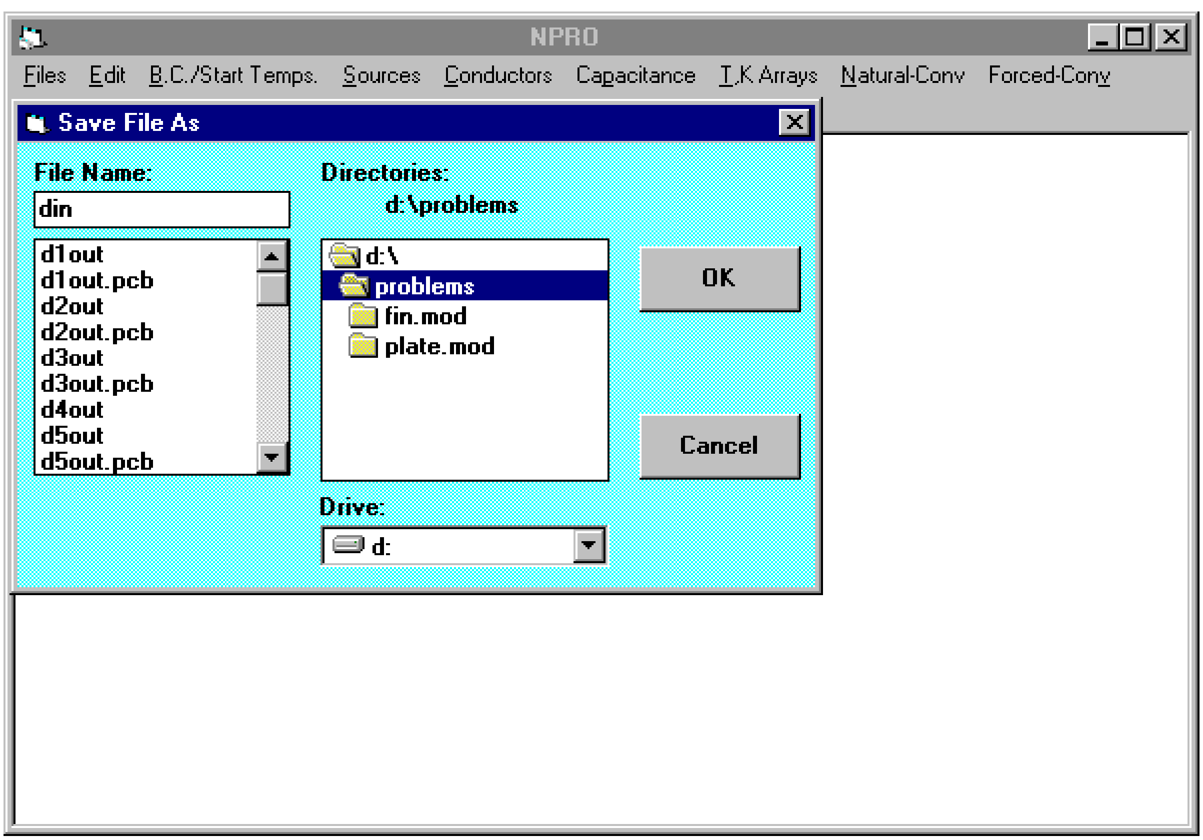
Horizontal panels:
$$P = \frac{Area}{Perimeter} = \frac{WL}{2(W+L)}$$
$$= \frac{(10\,in.)(10\,in.)}{2(10\,in. + 10\,in.)} = 2.5\,in.$$

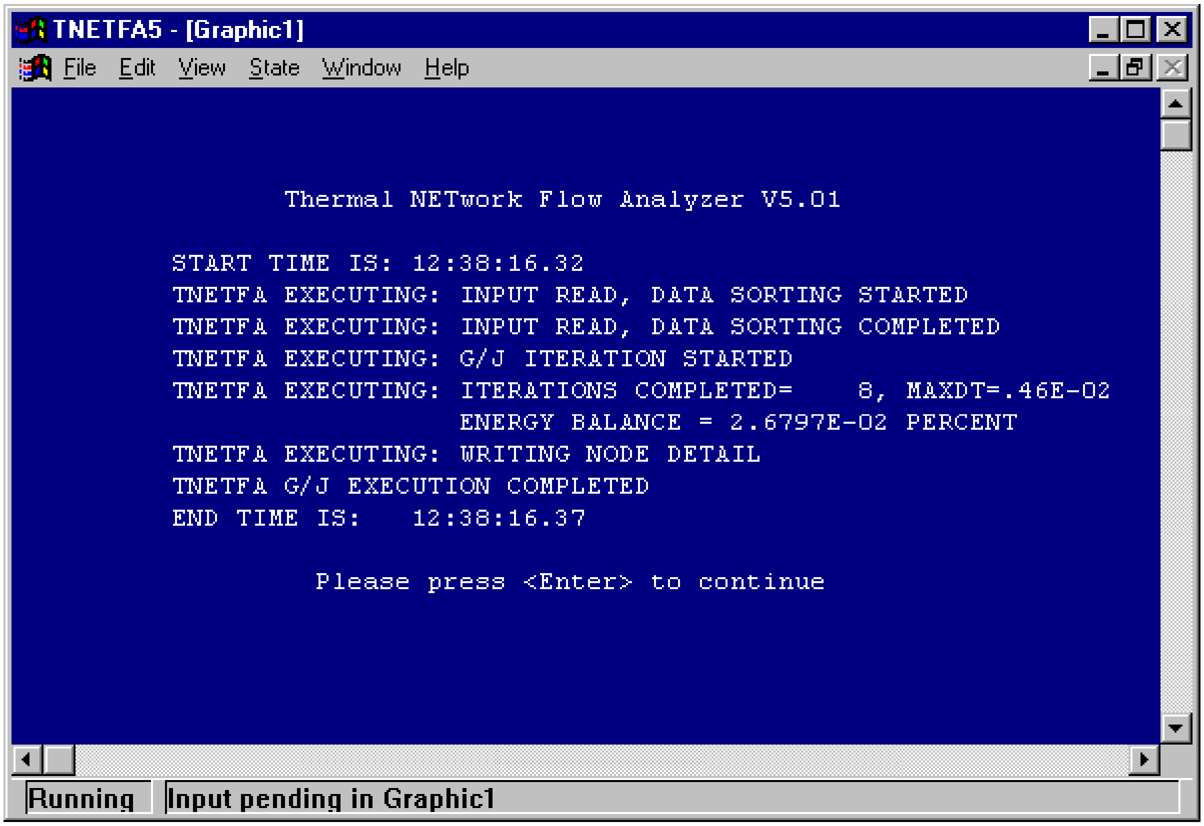


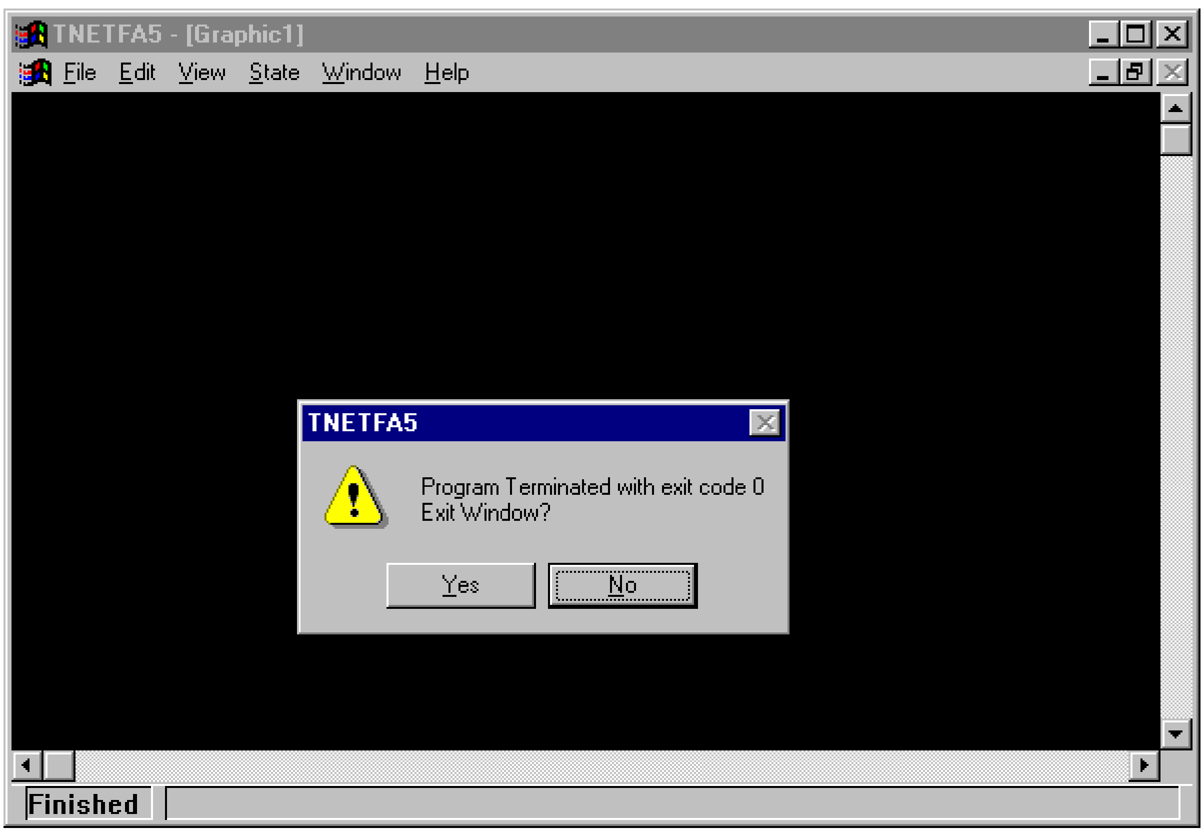

NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T.K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Steady State...
Time Dependent...

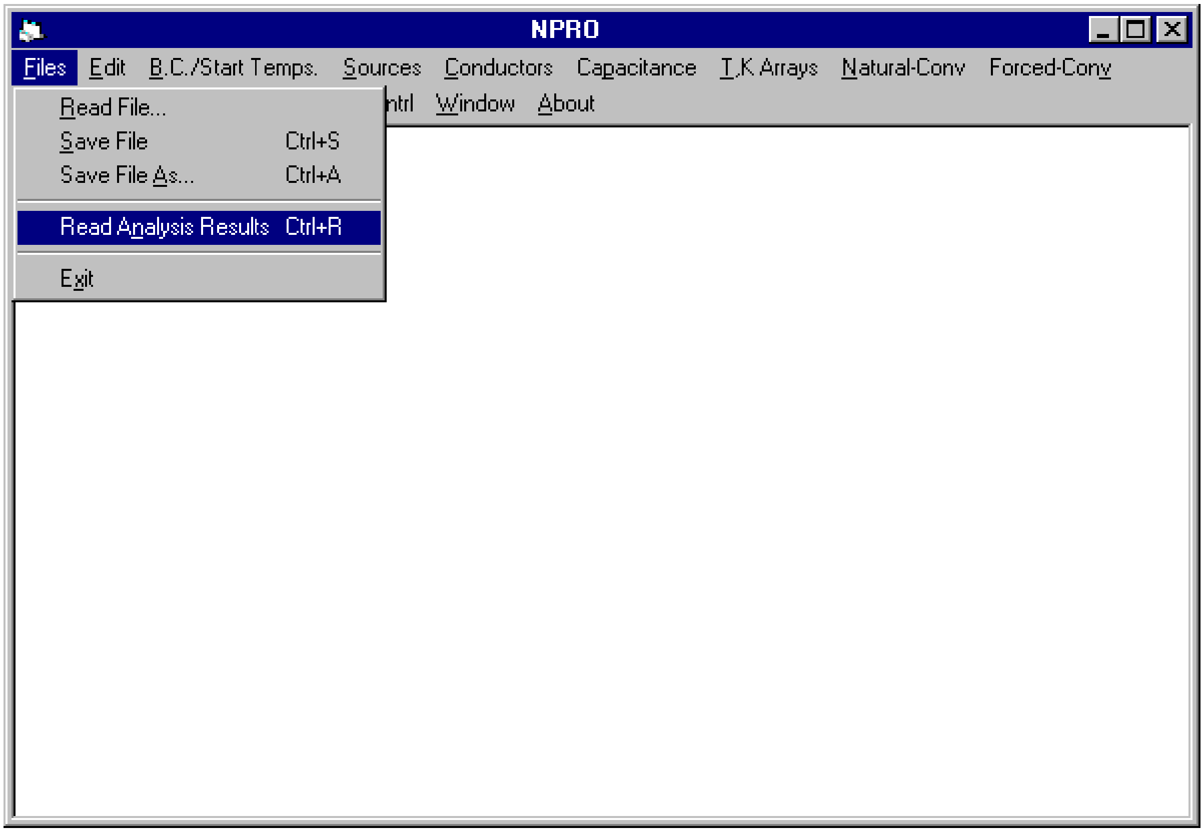

NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T.K Arrays Natural-Conv Forced-Conv
Multi-Surf-Rad Graphics Solution-Cntrl Window About
Steady-State Solution Control
Maximum Number of Iterations:
25
Iteration Intervals for Calculating System Energy Balance:
5
Interation Intervals for Printing All Temperatures:
5
Relaxation Constant (0.0<Beta<2.0):
1
Maximum Temperature Change (per iteration) for Solution Termination:
0.01
Print All Node Connections:
OK
Cancel

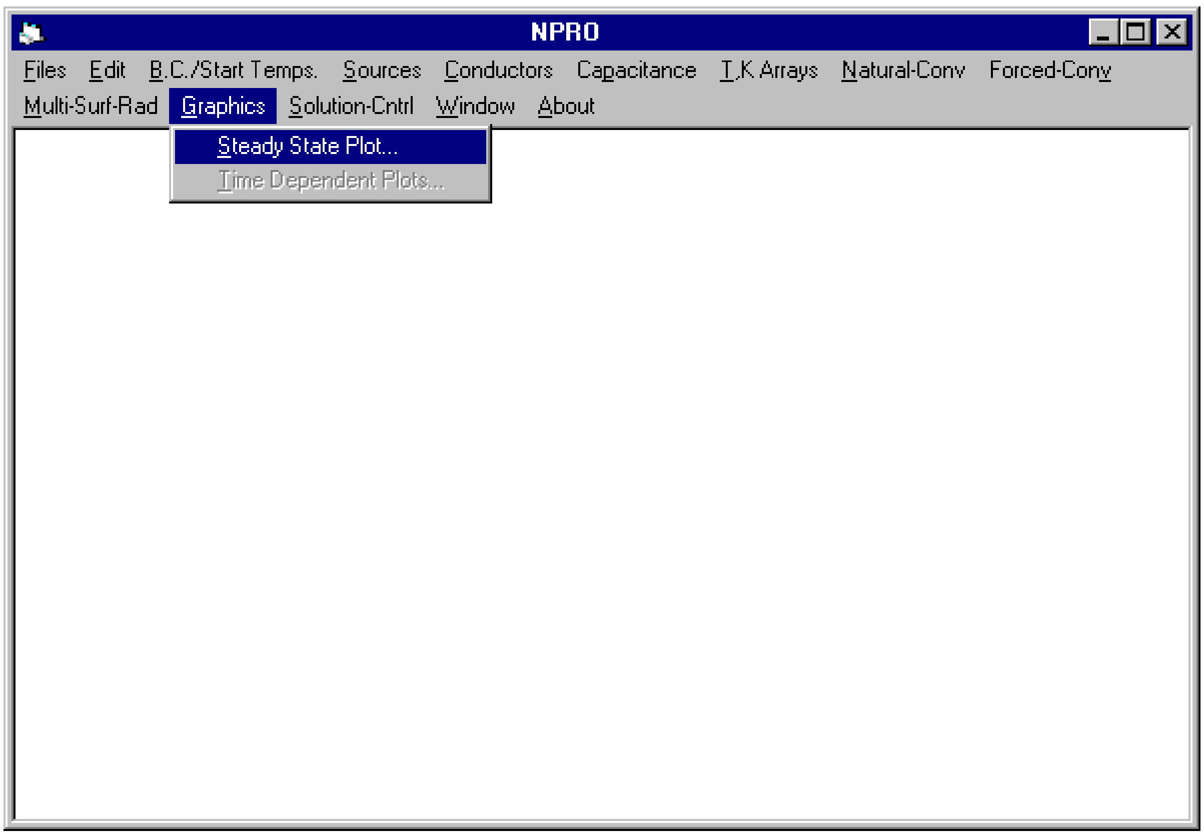

NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T.K Arrays Natural-Conv Forced-Conv
ntrl Window About
Read File...
Save File Ctrl+S
Save File As... Ctrl+A
Read Analysis Results Ctrl+R
Exit

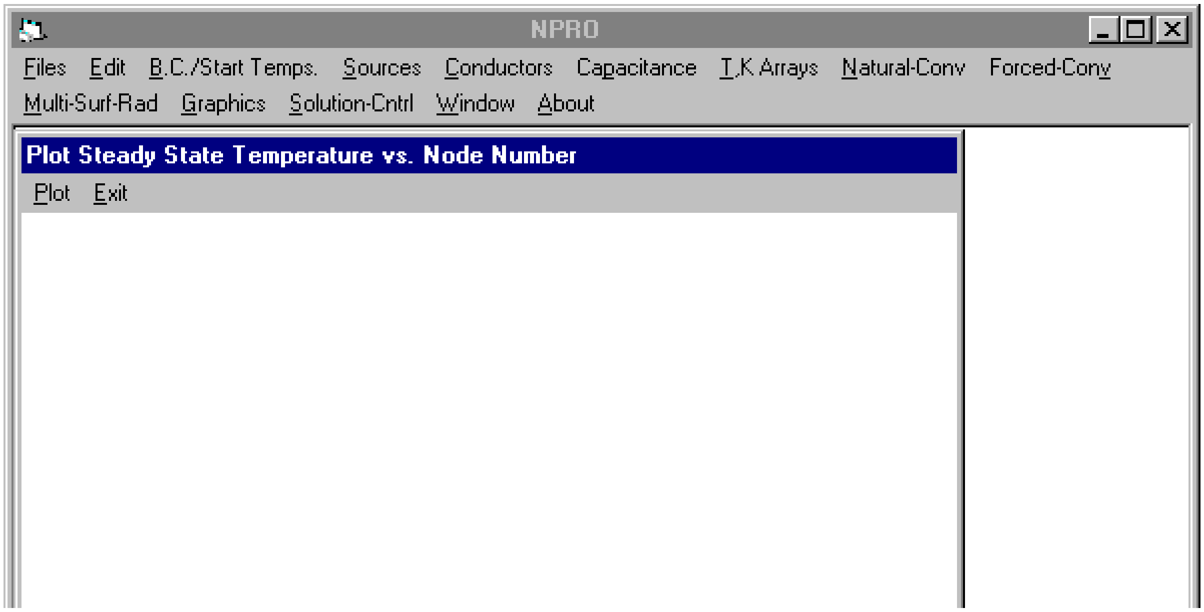

NPRO
Files Edit B.C./Start Temps. Sources Conductors Capacitance T.K Arrays Natural-Conv Forced-Conv
Save File As
File Name:
din
d1out
d1out.pcb
d2out
d2out.pcb
d3out
d3out.pcb
d4out
d5out
d5out.pcb
Directories:
d:\problems
d:\
problems
fin.mod
plate.mod
OK
Cancel
Drive:
d:


TNETFA5 - [Graphic1]
File Edit View State Window Help
Thermal NETwork Flow Analyzer V5.01
START TIME IS: 12:38:16.32
TNETFA EXECUTING: INPUT READ, DATA SORTING STARTED
TNETFA EXECUTING: INPUT READ, DATA SORTING COMPLETED
TNETFA EXECUTING: G/J ITERATION STARTED
TNETFA EXECUTING: ITERATIONS COMPLETED= 8, MAXDT=.46E-02
ENERGY BALANCE = 2.6797E-02 PERCENT
TNETFA EXECUTING: WRITING NODE DETAIL
TNETFA G/J EXECUTION COMPLETED
END TIME IS: 12:38:16.37
Please press <Enter> to continue
Running
Input pending in Graphic1

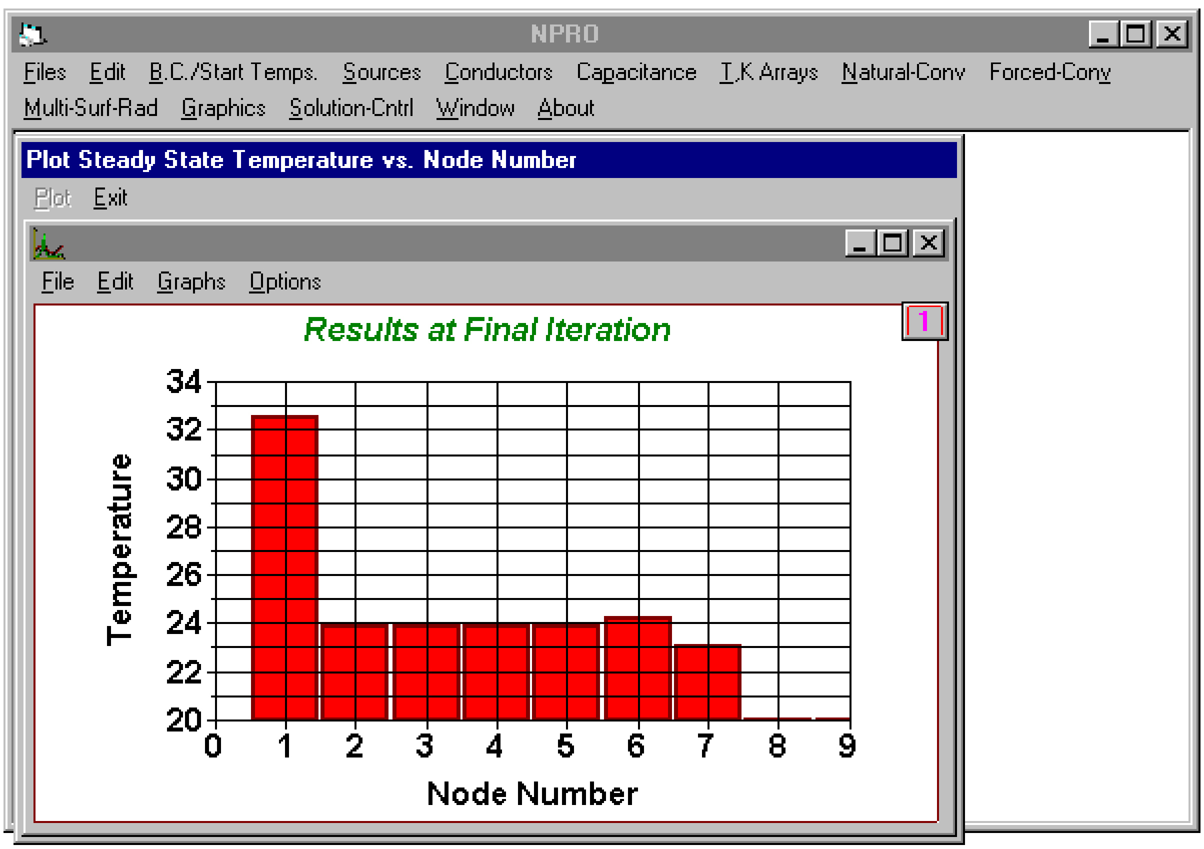

TNETFA5 - [Graphic1]
File Edit View State Window Help
TNETFA5
Program Terminated with exit code 0
Exit Window?
Yes
No
Finished


NPRO
Files
Edit
B.C./Start Temps.
Sources
Conductors
Capacitance
T.K Arrays
Natural-Conv
Forced-Conv
ntrl
Window
About
Read File...
Save File Ctrl+S
Save File As... Ctrl+A
Read Analysis Results Ctrl+R
Exit


NPRO
Files
Edit
B.C./Start Temps.
Sources
Conductors
Capacitance
T.K Arrays
Natural-Conv
Forced-Conv
Multi-Surf-Rad
Graphics
Solution-Cntrl
Window
About
Steady State Plot...
Time Dependent Plots...



The following has been modified from the default graph by double clicking on the data and changing to bar-style, etc.



TNETFA Output File (DOUT)

```
*******************************************************************************
 ****        Electronics Thermal Analysis Package - PC TNETFA V5.0        ****
 ****            (C) Copyright 1996 by Thermal Computations, Inc.         ****
 ****                           Newberg, Oregon                           ****
*******************************************************************************

SAMPLE TNETFA PROBLEM
SEALED ENCLOSURE

 UNITS=2
 NUMBER OF NODES=   9     NUMBER OF CONDUCTORS=  36
 NLOOP =   25  TPRINT=    5      NPRINT=    1
 LOOPEN=    5    ALDT= .1000E-01   BETA= 1.00

 NATURAL CONVECTION PARAMETER
   1    VERTICAL FLAT PLATE OR CYLINDER:                          P= .1000E+02
   2    HORIZONTAL FLAT PLATE OR CYLINDER,
        HEATEDSIDE FACING UP OR COOLED SIDE FACING DOWN:         P= .2500E+01
   3    HORIZONTAL FLAT PLATE OR CYLINDER,
        HEATEDSIDE FACING DOWN OR COOLED SIDE FACING UP:         P= .2500E+01

                                 LOOPCT=    0
                                 TEMPERATURES

  T(  1)= .2000E+02   T(  2)= .2000E+02   T(  3)= .2000E+02   T(  4)= .2000E+02
  T(  5)= .2000E+02   T(  6)= .2000E+02   T(  7)= .2000E+02   T(  8)= .2000E+02
  T(  9)= .2000E+02

                                 LOOPCT=    5
                                 TEMPERATURES

  T(  1)= .3244E+02   T(  2)= .2381E+02   T(  3)= .2381E+02   T(  4)= .2381E+02
  T(  5)= .2381E+02   T(  6)= .2417E+02   T(  7)= .2315E+02   T(  8)= .2000E+02
  T(  9)= .2000E+02
                                 MAXDT          = .2285E+00
                                 ENERGY BALANCE = 1.3014E+00 PERCENT


                                 LOOPCT=    8
                                 TEMPERATURES

  T(  1)= .3240E+02   T(  2)= .2381E+02   T(  3)= .2381E+02   T(  4)= .2381E+02
  T(  5)= .2381E+02   T(  6)= .2417E+02   T(  7)= .2315E+02   T(  8)= .2000E+02
  T(  9)= .2000E+02
                                 MAXDT          = .3353E-02
                                 ENERGY BALANCE = 1.9090E-02 PERCENT
```

```
DETAIL OF NODE   1     TEMPERATURE= .3240E+02            POWER= .1200E+02
                       STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   4   101     1    .1000E+03     .2353E+00     .2021E+01     .2353E-02
   2   101     1    .1000E+03     .2353E+00     .2021E+01     .2353E-02
   7   103     3    .1000E+03     .1552E+00     .1435E+01     .1552E-02
   6   102     2    .1000E+03     .3012E+00     .2478E+01     .3012E-02
   3   101     1    .1000E+03     .2353E+00     .2021E+01     .2353E-02
   5   101     1    .1000E+03     .2353E+00     .2021E+01     .2353E-02
                                    NET TOTAL = .1200E+02

DETAIL OF NODE   2     TEMPERATURE= .2381E+02            POWER= .0000E+00
                       STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   8    -1          .9000E+02     .3377E+00     .1285E+01     .3752E-02
   1   101     1    .1000E+03     .2353E+00    -.2021E+01     .2353E-02
   9   101     1    .1000E+03     .1934E+00     .7363E+00     .1934E-02
                                    NET TOTAL = .1930E-03

DETAIL OF NODE   3     TEMPERATURE= .2381E+02            POWER= .0000E+00
                       STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   8    -1          .9000E+02     .3377E+00     .1285E+01     .3752E-02
   1   101     1    .1000E+03     .2353E+00    -.2021E+01     .2353E-02
   9   101     1    .1000E+03     .1934E+00     .7363E+00     .1934E-02
                                    NET TOTAL = .1930E-03

DETAIL OF NODE   4     TEMPERATURE= .2381E+02            POWER= .0000E+00
                       STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   8    -1          .9000E+02     .3377E+00     .1285E+01     .3752E-02
   1   101     1    .1000E+03     .2353E+00    -.2021E+01     .2353E-02
   9   101     1    .1000E+03     .1934E+00     .7363E+00     .1934E-02
                                    NET TOTAL = .1930E-03

DETAIL OF NODE   5     TEMPERATURE= .2381E+02            POWER= .0000E+00
                       STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   1   101     1    .1000E+03     .2353E+00    -.2021E+01     .2353E-02
   9   101     1    .1000E+03     .1934E+00     .7363E+00     .1934E-02
   8    -1          .9000E+02     .3377E+00     .1285E+01     .3752E-02
                                    NET TOTAL = .1930E-03

DETAIL OF NODE   6     TEMPERATURE= .2417E+02            POWER= .0000E+00
                       STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   9   102     2    .1000E+03     .2561E+00     .1068E+01     .2561E-02
   1   102     2    .1000E+03     .3012E+00    -.2478E+01     .3012E-02
   8    -1          .9000E+02     .3383E+00     .1411E+01     .3759E-02
                                    NET TOTAL = .2454E-03
```

```
 DETAIL OF NODE   7     TEMPERATURE= .2315E+02             POWER= .0000E+00
                        STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE        C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   1   103     3    .1000E+03     .1552E+00    -.1435E+01     .1552E-02
   9   103     3    .1000E+03     .1194E+00     .3760E+00     .1194E-02
   8    -1          .9000E+02     .3365E+00     .1059E+01     .3739E-02
                                       NET TOTAL = .1273E-03

 DETAIL OF NODE  -8     TEMPERATURE= .2000E+02             POWER= .0000E+00
                        STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
                        THIS IS A CONSTANT TEMPERATURE NODE
NODE  CTYPE  CMODE        C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   4    -1          .9000E+02     .3377E+00    -.1285E+01     .3752E-02
   3    -1          .9000E+02     .3377E+00    -.1285E+01     .3752E-02
   2    -1          .9000E+02     .3377E+00    -.1285E+01     .3752E-02
   7    -1          .9000E+02     .3365E+00    -.1059E+01     .3739E-02
   6    -1          .9000E+02     .3383E+00    -.1411E+01     .3759E-02
   5    -1          .9000E+02     .3377E+00    -.1285E+01     .3752E-02
                                       NET TOTAL =-.7611E+01

 DETAIL OF NODE  -9     TEMPERATURE= .2000E+02             POWER= .0000E+00
                        STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
                        THIS IS A CONSTANT TEMPERATURE NODE
NODE  CTYPE  CMODE        C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   2   101     1    .1000E+03     .1934E+00    -.7363E+00     .1934E-02
   7   103     3    .1000E+03     .1194E+00    -.3760E+00     .1194E-02
   6   102     2    .1000E+03     .2561E+00    -.1068E+01     .2561E-02
   5   101     1    .1000E+03     .1934E+00    -.7363E+00     .1934E-02
   4   101     1    .1000E+03     .1934E+00    -.7363E+00     .1934E-02
   3   101     1    .1000E+03     .1934E+00    -.7363E+00     .1934E-02
                                       NET TOTAL =-.4389E+01
```

TNETFA Input File (DIN)

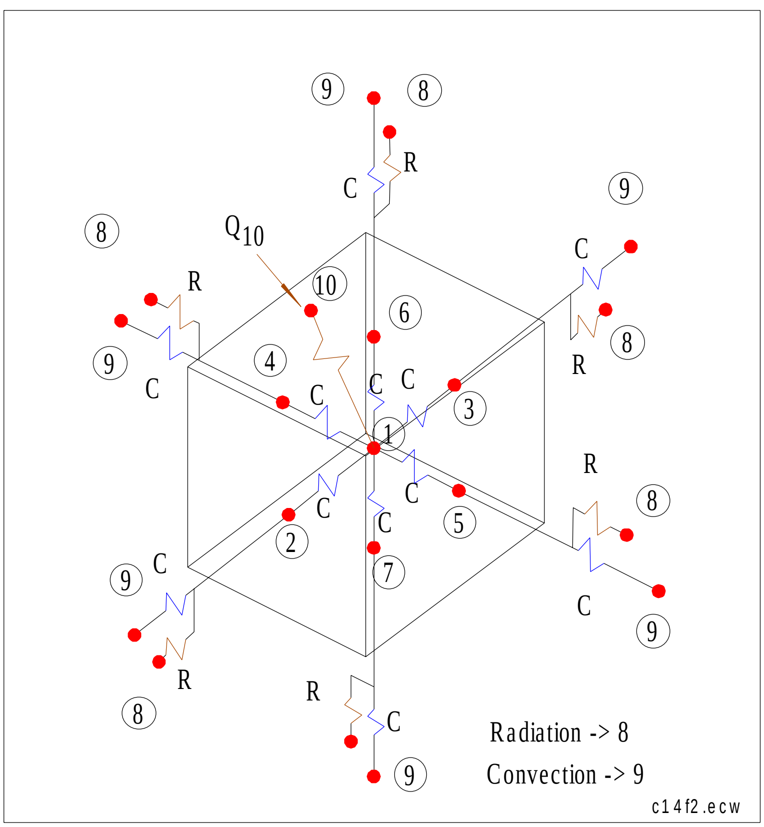
```
SAMPLE TNETFA PROBLEM
SEALED ENCLOSURE
 11      2      0
 9       2      1      0      7      0      0      3      0
 20      0
8       2.0000E+01
9       2.0000E+01
1       2.0000E+01  1.2000E+01
 0       0
4       1      0      2      1      1.0000E+02  101
1       1      0      6      0      1.0000E+02  102
1       1      0      7      0      1.0000E+02  103
4       2      1      9      0      1.0000E+02  101
1       6      0      9      0      1.0000E+02  102
1       7      0      9      0      1.0000E+02  103
6       2      1      8      0      9.0000E+01  -1
1       1.0000E+01
2       2.5000E+00
3       2.5000E+00
 25      1      0.01    5
 0       0
 5       1
```

Example
Sealed Enclosure Enclosure with Circuit Board Model

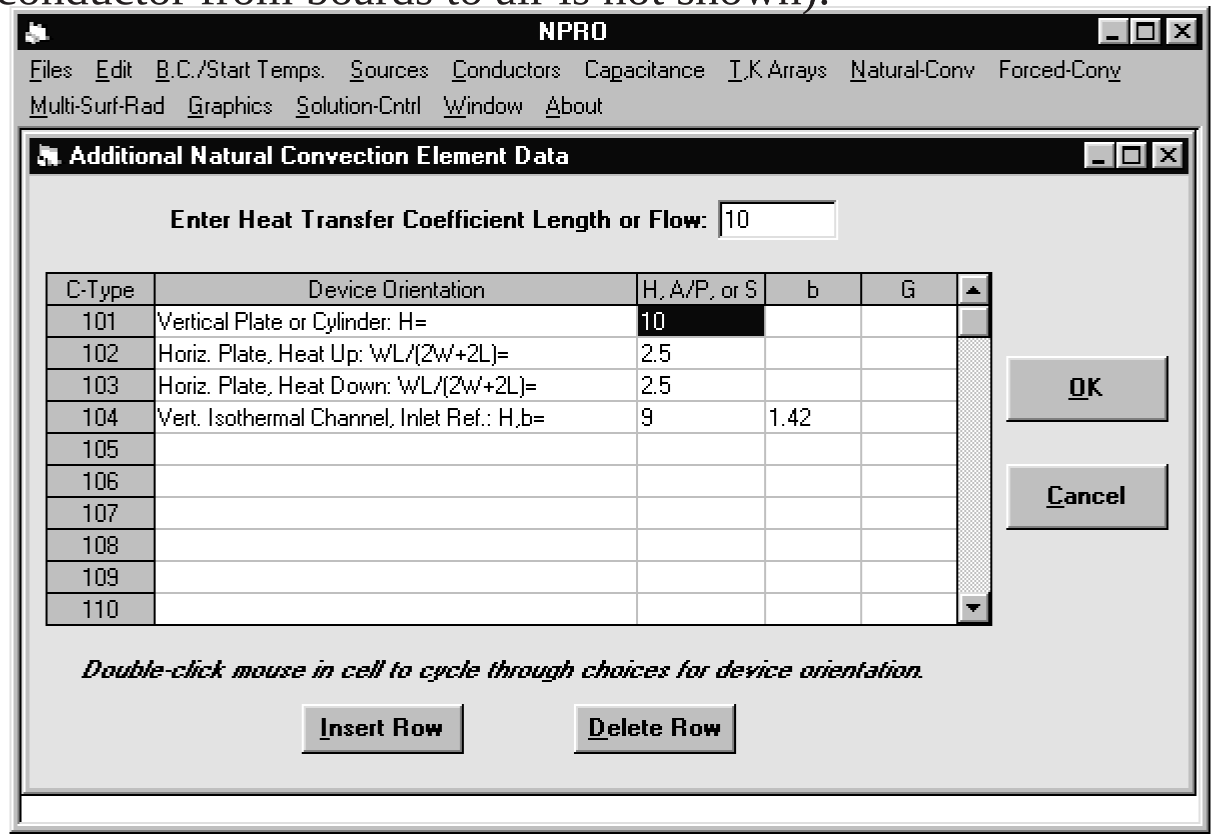
The preceding sealed enclosure is extended slightly to include the modeling of six vertical circuit boards that convect heat equally from each side of the boards. If we use component heights of 0.25 in., then the effective component to board spacing is $b=\dfrac{10\,in.-6(0.25\,in.)}{6}=1.42\,in.$ Node *10* is added as a heat source of 12 *W* that has a total convective surface area of

$$(2\,sides/board)(number\ of\ boards)(area\ of\ one\ side)=$$

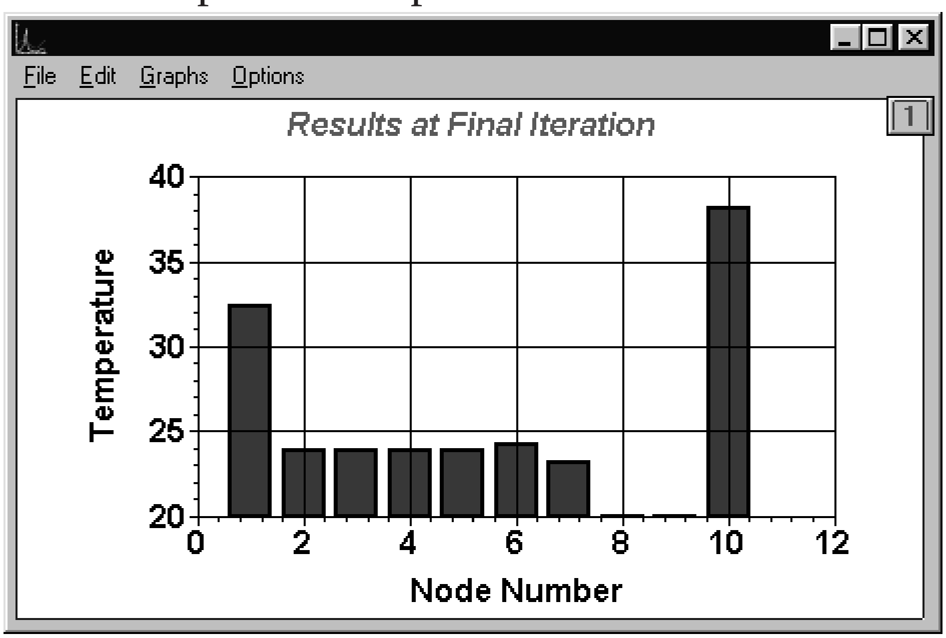
$$(2)(6)(9\,in.\,x\,9\,in.)=972\,in.^2$$



The changed natural convection screen is (input of an additional conductor from boards to air is not shown):



The air temperature (node 1) is not changed but we see that the average board temperature is predicted to be about 38 ^oC.

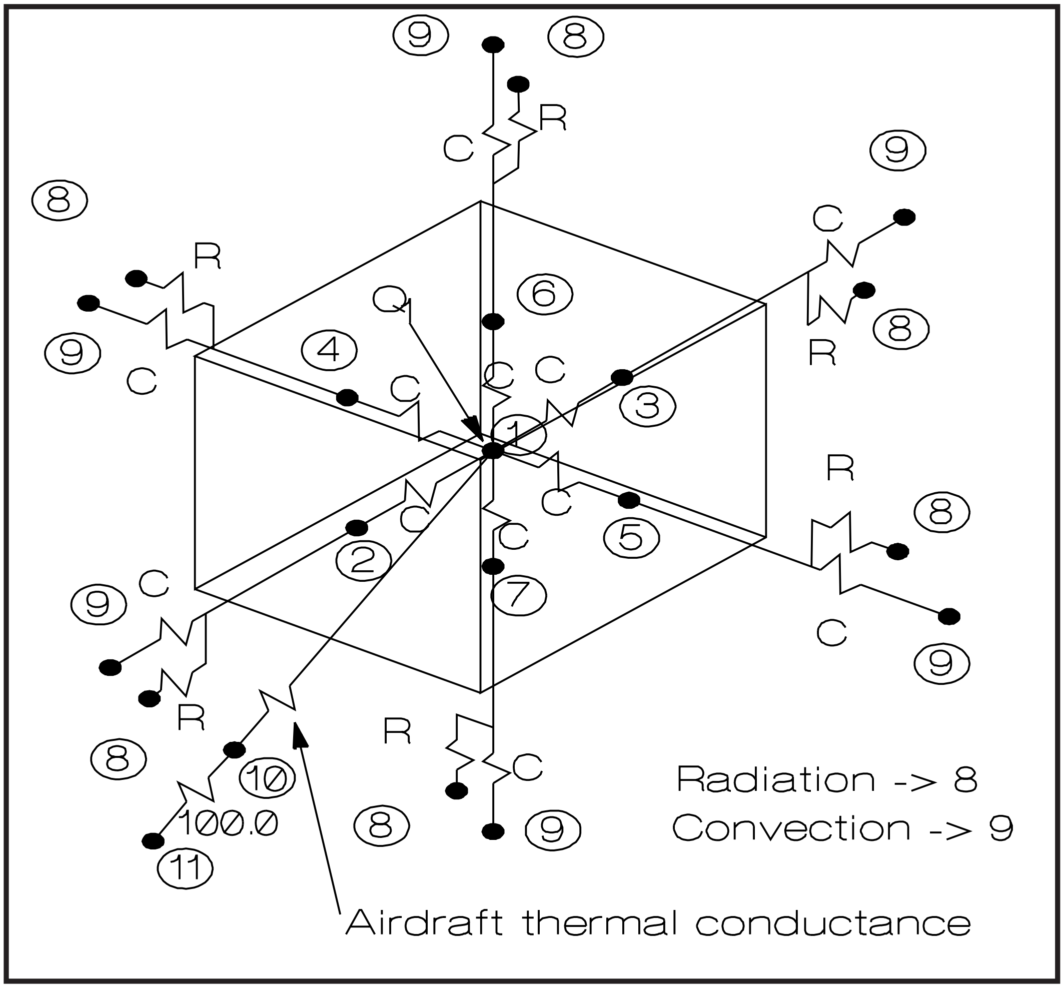


TNETFA Input File (DIN) for Model with Circuit Boards Added

```
SAMPLE TNETFA PROBLEM
SEALED ENCLOSURE WITH CIRCUIT BOARD MODEL
 1 2   0
 10 2   1   0   8   0   0   4   0
 20 0
8   2.0000E+01
9   2.0000E+01
10  2.0000E+01    1.2000E+01
 0  0
4   1   0   2   1   1.0000E+02    101
1   1   0   6   0   1.0000E+02    102
1   1   0   7   0   1.0000E+02    103
4   2   1   9   0   1.0000E+02    101
1   6   0   9   0   1.0000E+02    102
1   7   0   9   0   1.0000E+02    103
6   2   1   8   0   9.0000E+01    -1
1   10  0   1   0   9.7200E+02    104
1   1.0000E+01
2   2.5000E+00
3   2.5000E+00
9   9.0000E+00
1.4200E+00
 100   1   0   100
 0  0
 100   1
```

Example

Vented Enclosure



$$H = W = L = 10.0 \ in.$$

$$\varepsilon = 0.9, \text{ exterior}$$

$$Q1 = 12 \text{ W}$$

$$\text{Metal walls} \Rightarrow \text{neglect } R_w$$

$$A_{in} = 4 \ in.^2, \ A_{ex} = 4 \ in.^2, \ d = 8 \ in.$$

Airdraft resistance:

Using T.C.E.E., Fig. 6-9 and series resistance addition*,

$$R_a = 2.0x10^{-3}\left(\frac{1}{A_{in}^2} + \frac{1}{A_{ex}^2}\right) = 2.0x10^{-3}\left(\frac{1}{(4)^2} + \frac{1}{(4)^2}\right)$$

$$= 2.5x10^{-4}\ in.\ H_2O/(cfm)^2$$

Fluid draft thermal resistance:
From seminar notes

$$G = 1.53x10^{-2}(Qd/R_a)^{1/3}$$

$$= 1.53x10^{-2}\left(Qx8\ in./2.5x10^{-4}\right)^{1/3}$$

$$= 4.86x10^{-1}Q^{1/3}$$

Two elements are added to the sealed enclosure problem file:

(1) the C=100.0, CTYPE=0 and (2) C="G", CTYPE=301.

TNETFA is "iterated" manually, replacing a newly calculated G each time until G no longer changes.

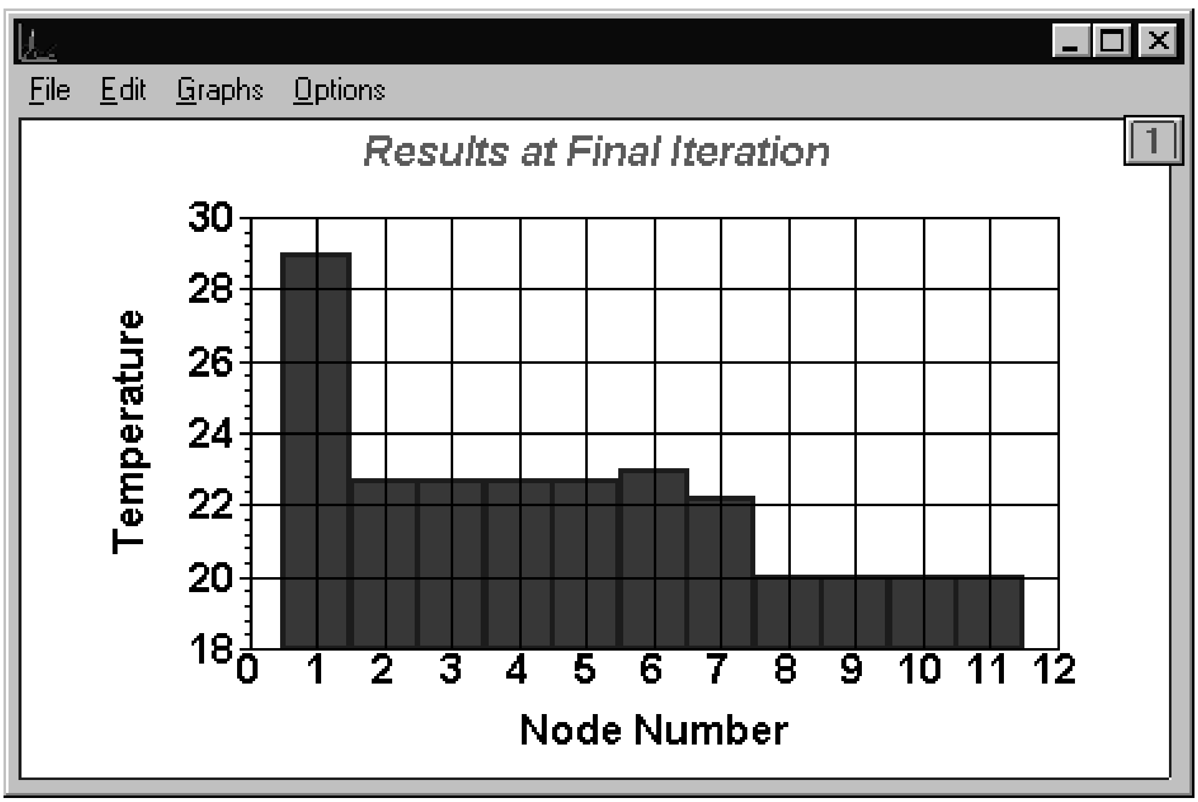
* Important Note: In the interest of time, we shall use the indicated formula for perforated plate resistance, but the reader is advised that the problem should be iterated further using resistances from the Idlechick and Fried data.

```
0                  0
100                1
```

Airdraft Iteration

Iteration No.	G(*cfm*)	Q_{1-10}
1	1.0	3.65
2	0.748	3.81
3	0.759	3.86
4	0.762	3.87
5	0.763	3.87

TNETFA Results Plotted by NPRO



```
******************************************************************************
****        Electronics Thermal Analysis Package - PC TNETFA V5.0        ****
****           (C) Copyright 1997 by Thermal Computations, Inc.          ****
****                          Newberg, Oregon                            ****
******************************************************************************

SAMPLE TNETFA PROBLEM
VENTED ENCLOSURE

UNITS=2
NUMBER OF NODES=  11     NUMBER OF CONDUCTORS=  40
NLOOP =   25  TPRINT=    5      NPRINT=    1
LOOPEN=    5    ALDT=0.1000E-01   BETA= 1.00

NATURAL CONVECTION PARAMETER
  1    VERTICAL FLAT PLATE OR CYLINDER:                        P=0.1000E+02
  2    HORIZONTAL FLAT PLATE OR CYLINDER,
       HEATEDSIDE FACING UP OR COOLED SIDE FACING DOWN:     P=0.2500E+01
  3    HORIZONTAL FLAT PLATE OR CYLINDER,
       HEATEDSIDE FACING DOWN OR COOLED SIDE FACING UP:     P=0.2500E+01

                                 LOOPCT=    0
                                 TEMPERATURES

 T(  1)=0.2000E+02   T(  2)=0.2000E+02   T(  3)=0.2000E+02   T(  4)=0.2000E+02
 T(  5)=0.2000E+02   T(  6)=0.2000E+02   T(  7)=0.2000E+02   T(  8)=0.2000E+02
 T(  9)=0.2000E+02   T( 10)=0.2000E+02   T( 11)=0.2000E+02

                                 LOOPCT=    5
                                 TEMPERATURES

 T(  1)=0.2896E+02   T(  2)=0.2267E+02   T(  3)=0.2267E+02   T(  4)=0.2267E+02
 T(  5)=0.2267E+02   T(  6)=0.2294E+02   T(  7)=0.2219E+02   T(  8)=0.2000E+02
 T(  9)=0.2000E+02   T( 10)=0.2000E+02   T( 11)=0.2000E+02
                                 MAXDT          =0.3580E-01
                                 ENERGY BALANCE = 2.7137E-01 PERCENT


                                 LOOPCT=    6
                                 TEMPERATURES

 T(  1)=0.2895E+02   T(  2)=0.2267E+02   T(  3)=0.2267E+02   T(  4)=0.2267E+02
 T(  5)=0.2267E+02   T(  6)=0.2294E+02   T(  7)=0.2219E+02   T(  8)=0.2000E+02
 T(  9)=0.2000E+02   T( 10)=0.2000E+02   T( 11)=0.2000E+02
                                 MAXDT          =0.6936E-02
                                 ENERGY BALANCE = 5.2838E-02 PERCENT
```

```
DETAIL OF NODE   1     TEMPERATURE=0.2895E+02            POWER=0.1200E+02
                       STABILITY CONSTANT =  0.00E+00     CAP=0.1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   5   101     1   0.1000E+03    0.2182E+00    0.1369E+01    0.2182E-02
  10   301         0.7630E+00    0.4324E+00    0.3869E+01
   3   101     1   0.1000E+03    0.2182E+00    0.1369E+01    0.2182E-02
   2   101     1   0.1000E+03    0.2182E+00    0.1369E+01    0.2182E-02
   7   103     3   0.1000E+03    0.1439E+00    0.9722E+00    0.1439E-02
   4   101     1   0.1000E+03    0.2182E+00    0.1369E+01    0.2182E-02
   6   102     2   0.1000E+03    0.2792E+00    0.1678E+01    0.2792E-02
                                    NET TOTAL =0.1200E+02

DETAIL OF NODE   2     TEMPERATURE=0.2267E+02            POWER=0.0000E+00
                       STABILITY CONSTANT =  0.00E+00     CAP=0.1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   8    -1         0.9000E+02    0.3357E+00    0.8967E+00    0.3730E-02
   9   101     1   0.1000E+03    0.1772E+00    0.4732E+00    0.1772E-02
   1   101     1   0.1000E+03    0.2182E+00    -.1369E+01    0.2182E-02
                                    NET TOTAL =0.5847E-03

DETAIL OF NODE   3     TEMPERATURE=0.2267E+02            POWER=0.0000E+00
                       STABILITY CONSTANT =  0.00E+00     CAP=0.1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   1   101     1   0.1000E+03    0.2182E+00    -.1369E+01    0.2182E-02
   8    -1         0.9000E+02    0.3357E+00    0.8967E+00    0.3730E-02
   9   101     1   0.1000E+03    0.1772E+00    0.4732E+00    0.1772E-02
                                    NET TOTAL =0.5847E-03

DETAIL OF NODE   4     TEMPERATURE=0.2267E+02            POWER=0.0000E+00
                       STABILITY CONSTANT =  0.00E+00     CAP=0.1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   8    -1         0.9000E+02    0.3357E+00    0.8967E+00    0.3730E-02
   1   101     1   0.1000E+03    0.2182E+00    -.1369E+01    0.2182E-02
   9   101     1   0.1000E+03    0.1772E+00    0.4732E+00    0.1772E-02
                                    NET TOTAL =0.5847E-03

DETAIL OF NODE   5     TEMPERATURE=0.2267E+02            POWER=0.0000E+00
                       STABILITY CONSTANT =  0.00E+00     CAP=0.1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   9   101     1   0.1000E+03    0.1772E+00    0.4732E+00    0.1772E-02
   1   101     1   0.1000E+03    0.2182E+00    -.1369E+01    0.2182E-02
   8    -1         0.9000E+02    0.3357E+00    0.8967E+00    0.3730E-02
                                    NET TOTAL =0.5847E-03

DETAIL OF NODE   6     TEMPERATURE=0.2294E+02            POWER=0.0000E+00
                       STABILITY CONSTANT =  0.00E+00     CAP=0.1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   9   102     2   0.1000E+03    0.2348E+00    0.6904E+00    0.2348E-02
   1   102     2   0.1000E+03    0.2792E+00    -.1678E+01    0.2792E-02
   8    -1         0.9000E+02    0.3362E+00    0.9883E+00    0.3735E-02
                                    NET TOTAL =0.8217E-03
```

```
DETAIL OF NODE   7     TEMPERATURE=0.2219E+02            POWER=0.0000E+00
                       STABILITY CONSTANT =  0.00E+00     CAP=0.1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   1   103     3   0.1000E+03    0.1439E+00    -.9722E+00    0.1439E-02
   9   103     3   0.1000E+03    0.1091E+00    0.2390E+00    0.1091E-02
   8    -1         0.9000E+02    0.3349E+00    0.7335E+00    0.3721E-02
                                     NET TOTAL =0.3300E-03

DETAIL OF NODE  -8     TEMPERATURE=0.2000E+02            POWER=0.0000E+00
                       STABILITY CONSTANT =  0.00E+00     CAP=0.1000E-19
                       THIS IS A CONSTANT TEMPERATURE NODE
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   4    -1         0.9000E+02    0.3357E+00    -.8967E+00    0.3730E-02
   3    -1         0.9000E+02    0.3357E+00    -.8967E+00    0.3730E-02
   2    -1         0.9000E+02    0.3357E+00    -.8967E+00    0.3730E-02
   7    -1         0.9000E+02    0.3349E+00    -.7335E+00    0.3721E-02
   6    -1         0.9000E+02    0.3362E+00    -.9883E+00    0.3735E-02
   5    -1         0.9000E+02    0.3357E+00    -.8967E+00    0.3730E-02
                                     NET TOTAL =-.5309E+01

DETAIL OF NODE  -9     TEMPERATURE=0.2000E+02            POWER=0.0000E+00
                       STABILITY CONSTANT =  0.00E+00     CAP=0.1000E-19
                       THIS IS A CONSTANT TEMPERATURE NODE
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
   2   101     1   0.1000E+03    0.1772E+00    -.4732E+00    0.1772E-02
   7   103     3   0.1000E+03    0.1091E+00    -.2390E+00    0.1091E-02
   6   102     2   0.1000E+03    0.2348E+00    -.6904E+00    0.2348E-02
   5   101     1   0.1000E+03    0.1772E+00    -.4732E+00    0.1772E-02
   4   101     1   0.1000E+03    0.1772E+00    -.4732E+00    0.1772E-02
   3   101     1   0.1000E+03    0.1772E+00    -.4732E+00    0.1772E-02
                                     NET TOTAL =-.2822E+01

DETAIL OF NODE  10     TEMPERATURE=0.2000E+02            POWER=0.0000E+00
                       STABILITY CONSTANT =  0.00E+00     CAP=0.1000E-19
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
  11     0         0.1000E+03    0.1000E+03    0.0000E+00
                                     NET TOTAL =0.0000E+00

DETAIL OF NODE -11     TEMPERATURE=0.2000E+02            POWER=0.0000E+00
                       STABILITY CONSTANT =  0.00E+00     CAP=0.1000E-19
                       THIS IS A CONSTANT TEMPERATURE NODE
NODE  CTYPE  CMODE       C       CONDUCTANCE      FLUX    HT TRANS COEF/SFA
  10     0         0.1000E+03    0.1000E+03    0.0000E+00
                                     NET TOTAL =0.0000E+00
```

TNETFA Input File (DIN)

```
SAMPLE TNETFA PROBLEM
VENTED ENCLOSURE
 11        2        0
 11        3        1        0        7        2        0        3        0
 20        0
8         2.0000E+01
9         2.0000E+01
11         2.0000E+01
1         2.0000E+01   1.2000E+01
 0         0
4         1         0         2         1         1.0000E+02   101
1         1         0         6         0         1.0000E+02   102
1         1         0         7         0         1.0000E+02   103
4         2         1         9         0         1.0000E+02   101
1         6         0         9         0         1.0000E+02   102
1         7         0         9         0         1.0000E+02   103
6         2         1         8         0         9.0000E+01   -1
1         10         7.6300E-01   301
10         11        1.0000E+02   0
1         1.0000E+01
2         2.5000E+00
3         2.5000E+00
 25        1        0.01      5
 0         0
 5         1
```

Example

TNETFA Solution
of Forced Air Cooled Enclosure

Reference: 1. Circuit, Figure 6-14, TCEE; 2. Element Vaules, Table 6-2, TCEE

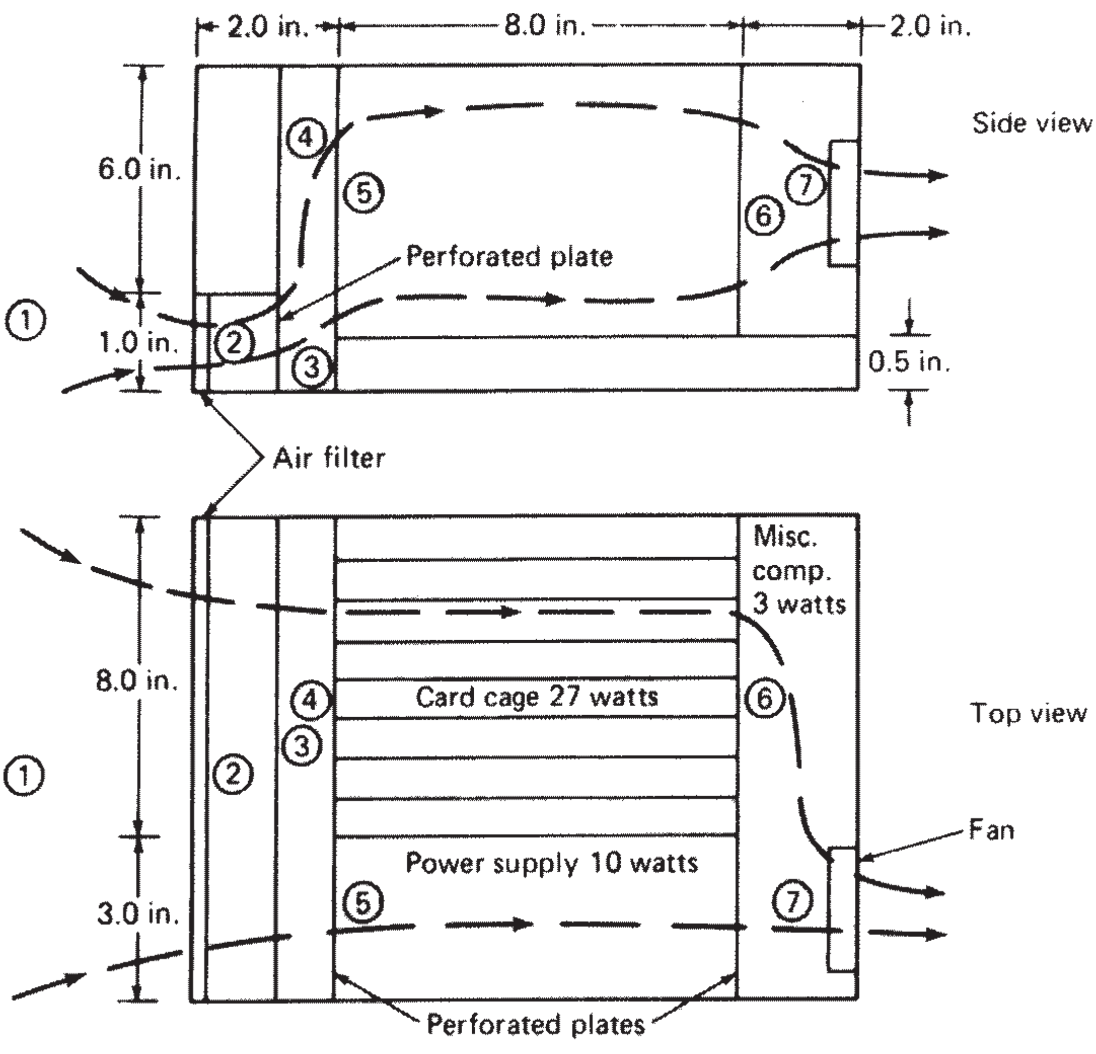


Fig. 6-13. Sample problem. Forced air cooled enclosure.

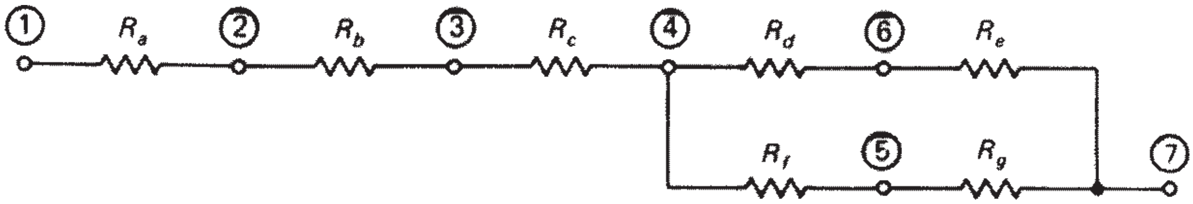


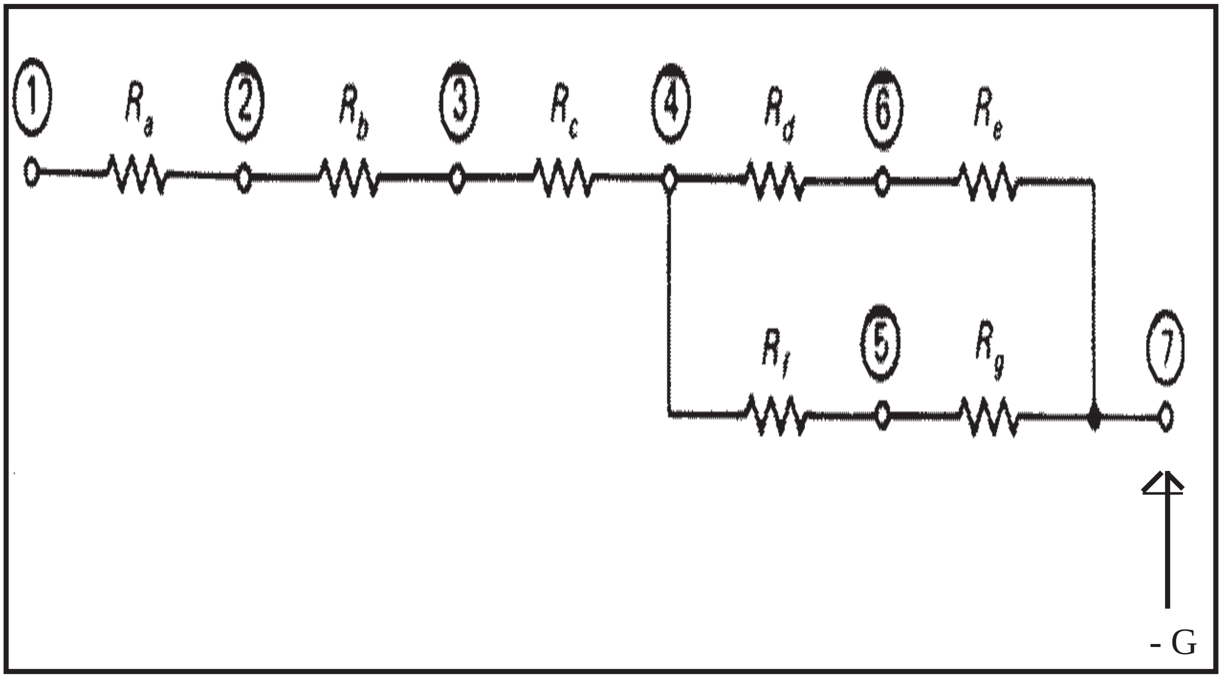
Fig. 6-14. Forced air flow circuit for cabinet illustrated in Fig. 6-13.

Table 6-2. Summary of airflow elements for sample problem illustrated in Fig. 6-13.

Node I	to Node J	Function	Value
1	2	Filter	$R_a = 1 \times 10^{-3}$, mfgs. data
2	3	Perforated plate	$R_b = 2.4 \times 10^{-3}/[(11)(1)(0.35)]^2 = 1.6 \times 10^{-4}$
3	4	Expansion	$R_c = 1.29 \times 10^{-3}\left\{\frac{1}{(1)(11)}\left[1 - \frac{(1)(11)}{(6.5)(8)}\right]\right\}^2 = 6.6 \times 10^{-6}$
4	6	Card cage	$R_d = \frac{3.08(1)(8)(10^{-4})}{[(8)(6)]^2} = 1.1 \times 10^{-6}$
4	5	Perforated plate	$R_f = 2.4 \times 10^{-3}/[(6.5)(3.0)(0.35)]^2 = 5.1 \times 10^{-5}$
5	7	Perforated plate	$R_g = 5.1 \times 10^{-5}$
6	7	Contraction	$R_e = 0.63 \times 10^{-3}/[(2)(6)]^2 = 4.4 \times 10^{-6}$

Note: A better perforated plate resistance would be $R=2.0x10^{-3}/A^2$.

Airflow circuit with airflow resistances



R_a = 1.0x10^{-3}

R_b = 1.6x10^{-4}

R_c = 6.6x10^{-6}

R_d = 1.1x10^{-6}

R_e = 4.4x10^{-6}

R_f= R_g= 5.1x10^{-5}

NPRO OPTION	DATA SET	TNETFA INPUT
Edit - Title Line 1	1	TNETFA Example
Edit - Title Line 2	1	Forced Air Flow
Edit - Solution Type	2	11 0 0
3	7	1 1 0 0 7 0 0 0
B.C./Start Temps	4	1 0.0
B.C./Start Temps	4	7 0.0 -5.0
Capacitance	6	0 0
Conductors - Single	8	1 2 1.0E-3 402
Conductors - Single	8	2 3 1.6E-4 402
Conductors - Single	8	3 4 6.6E-6 402
Conductors - Single	8	4 6 1.1E-6 402
Conductors - Single	8	4 5 5.1E-5 402
Conductors - Single	8	5 7 5.1E-5 402
Conductors - Single	8	6 7 4.4E-6 402
Solution-Cntrl - Steady	14	20 1.0 0.0001 5
Solution-Cntrl - Steady	14	0.0 0.0
Solution-Cntrl - Steady	14	5 1

TNETFA Results at Node 7

CFM	P (in. H_2O)
-5.0	-0.0292
-10.0	-0.11

Plot of P vs. CFM as in Figure 6-15:

$$
\begin{aligned}
P_{sys} &= RG^2 \\
&= \left[\frac{0.0292}{(5.0)^2}\right] G^2 \\
&= 1.168x10^{-3} G^2
\end{aligned}
$$

Intersection of Psys and fan curve --> 6.0 cfm.

Re-run TNETFA for fan at 6.0 cfm to get flow, pressure distribution in cabinet.

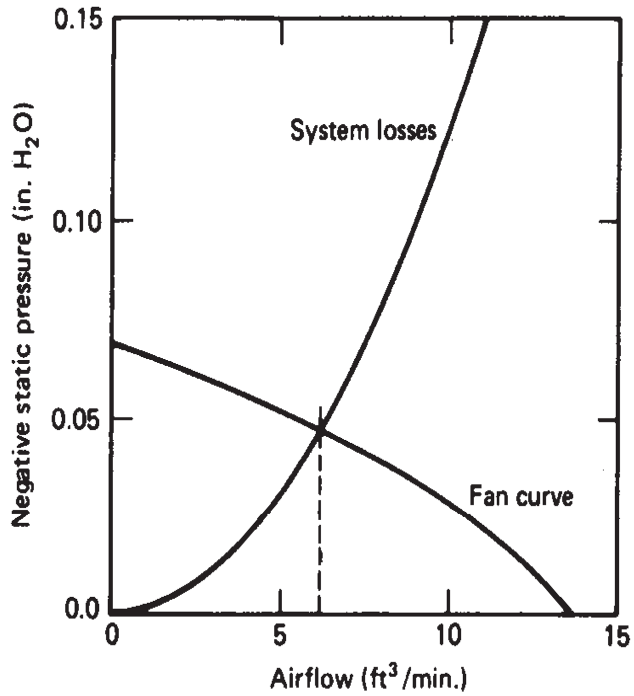


Fig. 6-15. Computed system airflow losses and fan curve for forced air flow system illustrated in Fig. 6-13.

Flow in the card cage path is computed from:

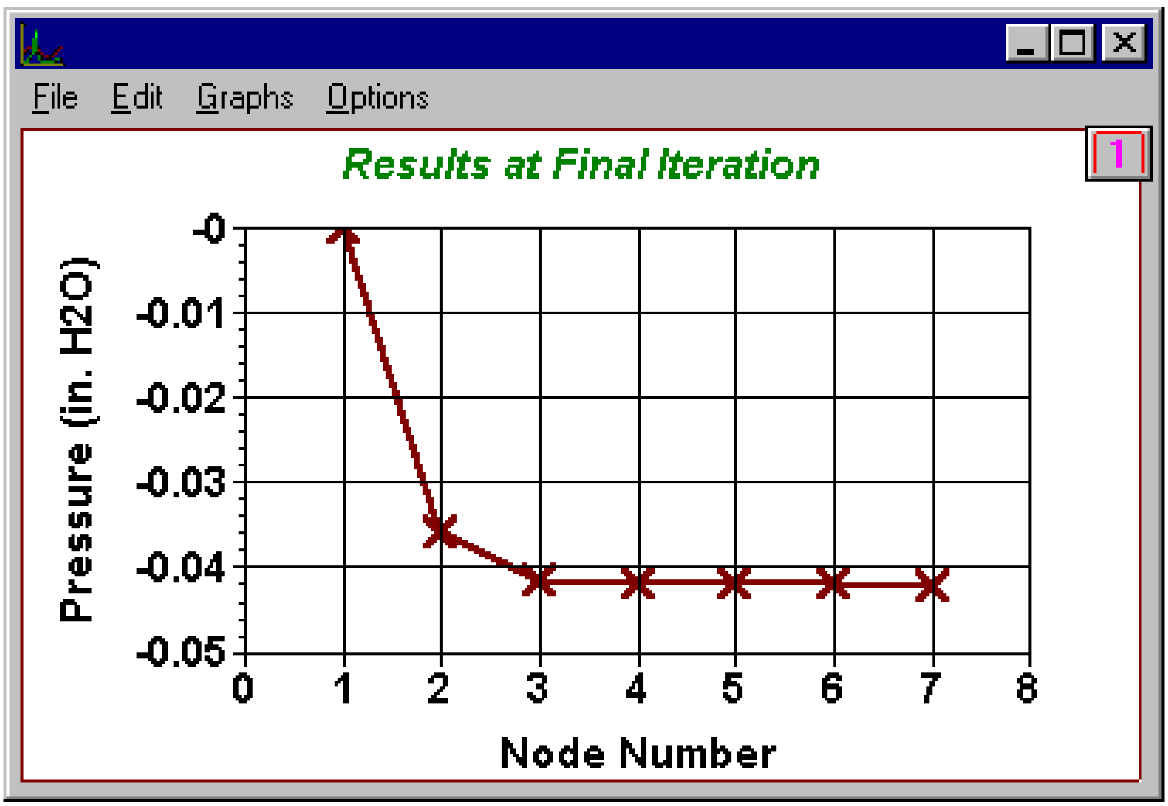
$$G_{cc} = \frac{G}{1+\sqrt{\frac{R_d + R_e}{R_f + R_g}}} = \frac{6}{1+\sqrt{\frac{5.5x10^{-6}}{1.0x10^{-4}}}}$$

$$= (0.81)(6) = 4.9 \ ft^3/\text{min.}$$

Flow in the power supply is, of course

$$G_{ps} = G - G_{cc} = 6.0 - 4.9 = 1.1 \ ft^3/\text{min.}$$

NPRO Plot of Pressure vs. Node Number

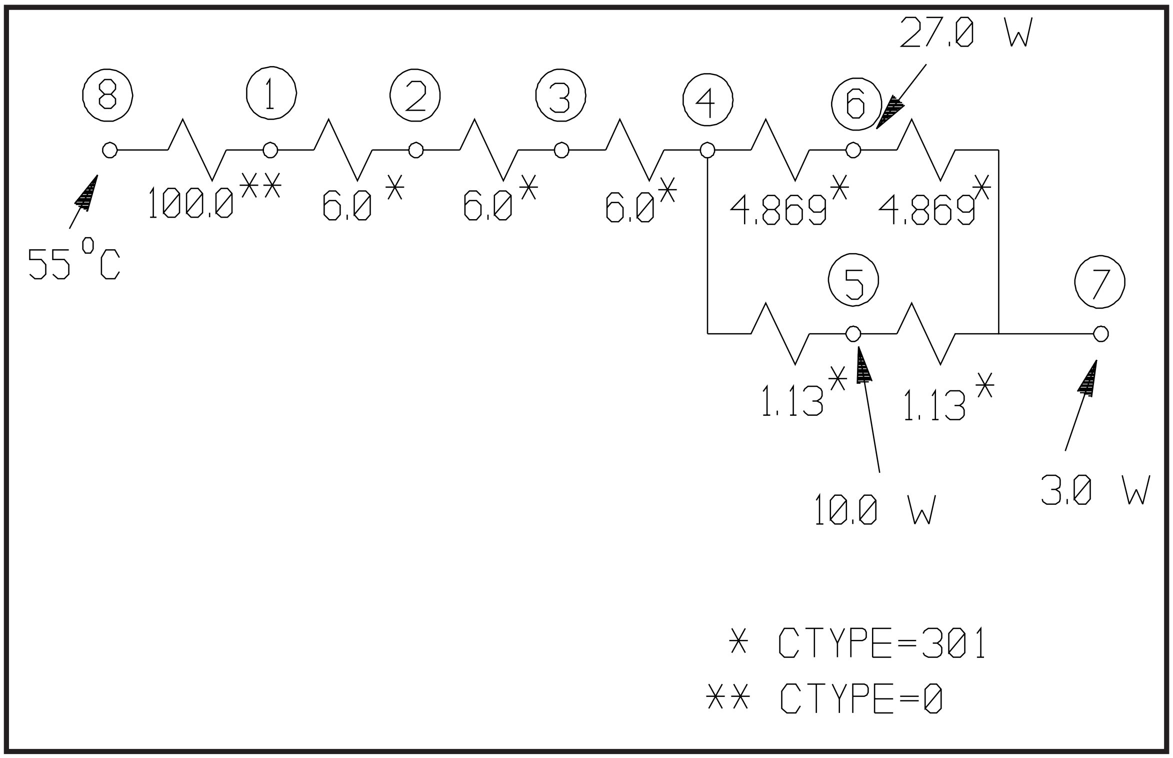


TNETFA Output File (DOUT)

```
 *********************************************************************************
****        Electronics Thermal Analysis Package - PC TNETFA V5.0        ****
****            (C) Copyright 1996 by Thermal Computations, Inc.          ****
****                        Newberg, Oregon                         ****
 *********************************************************************************

TNETFA  Example
Forced Air Flow

 UNITS=0
 NUMBER OF NODES=   7     NUMBER OF CONDUCTORS=  14
 NLOOP =   20  TPRINT=    5      NPRINT=    1
 LOOPEN=    5    ALDT= .1000E-03   BETA= 1.00

                        LOOPCT=    0
                        TEMPERATURES

  T(  1)= .0000E+00   T(  2)= .0000E+00   T(  3)= .0000E+00   T(  4)= .0000E+00
  T(  5)= .0000E+00   T(  6)= .0000E+00   T(  7)= .0000E+00

                        LOOPCT=    5
                        TEMPERATURES

  T(  1)= .0000E+00   T(  2)=-.2787E-01   T(  3)=-.3259E-01   T(  4)=-.3281E-01
  T(  5)=-.3287E-01   T(  6)=-.3283E-01   T(  7)=-.3293E-01
                        MAXDT          = .7171E-02
                        ENERGY BALANCE = 1.5438E+01 PERCENT


                        LOOPCT=   10
                        TEMPERATURES

  T(  1)= .0000E+00   T(  2)=-.3571E-01   T(  3)=-.4144E-01   T(  4)=-.4167E-01
  T(  5)=-.4174E-01   T(  6)=-.4170E-01   T(  7)=-.4180E-01
                        MAXDT          = .3209E-03
                        ENERGY BALANCE = 5.0882E-01 PERCENT


                        LOOPCT=   12
                        TEMPERATURES

  T(  1)= .0000E+00   T(  2)=-.3593E-01   T(  3)=-.4168E-01   T(  4)=-.4192E-01
  T(  5)=-.4198E-01   T(  6)=-.4194E-01   T(  7)=-.4205E-01
                        MAXDT          = .8092E-04
                        ENERGY BALANCE = 1.2737E-01 PERCENT
```

```
DETAIL OF NODE  -1     TEMPERATURE= .0000E+00            POWER= .0000E+00
                     STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
                     THIS IS A CONSTANT TEMPERATURE NODE
NODE  CTYPE  CMODE      C       CONDUCTANCE     FLUX    HT TRANS COEF/SFA
   2   402          .1000E-02     .1668E+03     .5994E+01
                                 NET TOTAL = .5994E+01

DETAIL OF NODE   2     TEMPERATURE=-.3593E-01            POWER= .0000E+00
                     STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE      C       CONDUCTANCE     FLUX    HT TRANS COEF/SFA
   3   402          .1600E-03     .1042E+04     .5995E+01
   1   402          .1000E-02     .1668E+03    -.5994E+01
                                 NET TOTAL = .1341E-02

DETAIL OF NODE   3     TEMPERATURE=-.4168E-01            POWER= .0000E+00
                     STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE      C       CONDUCTANCE     FLUX    HT TRANS COEF/SFA
   4   402          .6600E-05     .2526E+05     .5998E+01
   2   402          .1600E-03     .1042E+04    -.5995E+01
                                 NET TOTAL = .2334E-02

DETAIL OF NODE   4     TEMPERATURE=-.4192E-01            POWER= .0000E+00
                     STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE      C       CONDUCTANCE     FLUX    HT TRANS COEF/SFA
   6   402          .1100E-05     .1867E+06     .4869E+01
   3   402          .6600E-05     .2526E+05    -.5998E+01
   5   402          .5100E-04     .1735E+05     .1130E+01
                                 NET TOTAL = .1491E-02

DETAIL OF NODE   5     TEMPERATURE=-.4198E-01            POWER= .0000E+00
                     STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE      C       CONDUCTANCE     FLUX    HT TRANS COEF/SFA
   7   402          .5100E-04     .1735E+05     .1130E+01
   4   402          .5100E-04     .1735E+05    -.1130E+01
                                 NET TOTAL = .6017E-13

DETAIL OF NODE   6     TEMPERATURE=-.4194E-01            POWER= .0000E+00
                     STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE      C       CONDUCTANCE     FLUX    HT TRANS COEF/SFA
   7   402          .4400E-05     .4669E+05     .4868E+01
   4   402          .1100E-05     .1867E+06    -.4869E+01
                                 NET TOTAL =-.8238E-03

DETAIL OF NODE   7     TEMPERATURE=-.4205E-01            POWER=-.6000E+01
                     STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
NODE  CTYPE  CMODE      C       CONDUCTANCE     FLUX    HT TRANS COEF/SFA
   6   402          .4400E-05     .4669E+05    -.4868E+01
   5   402          .5100E-04     .1735E+05    -.1130E+01
                                 NET TOTAL =-.5998E+01
```

Thermal circuit with TNETFA computed airflow values

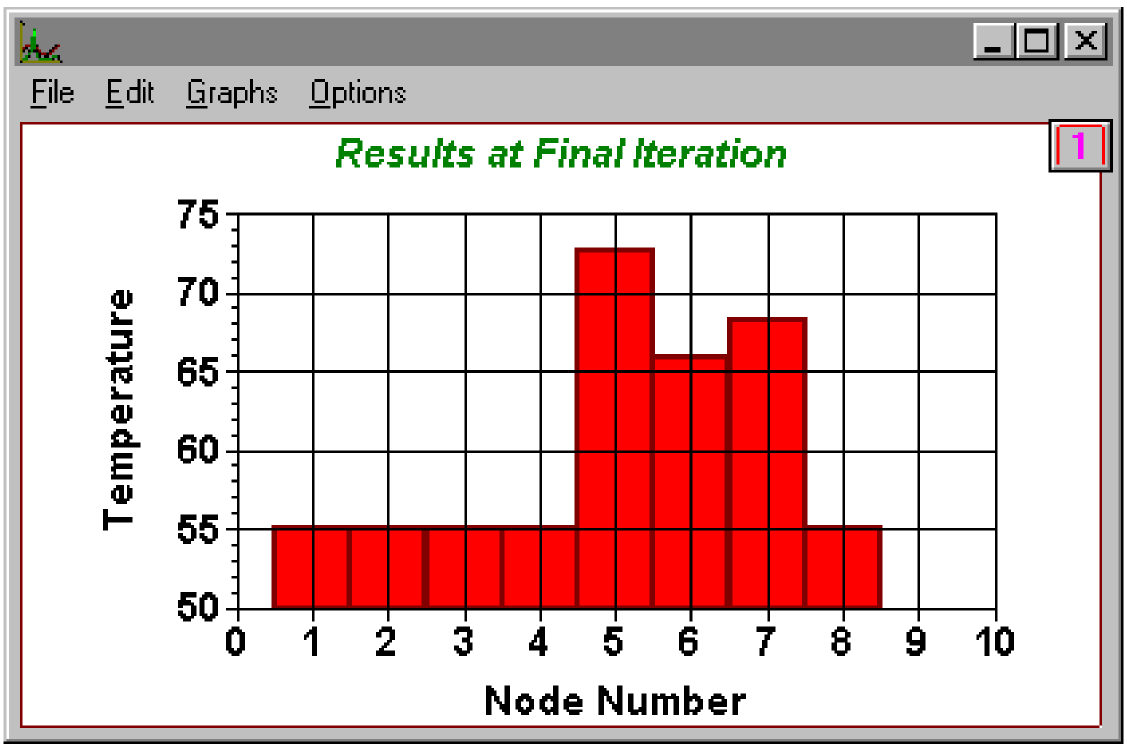


NPRO OPTION	DATA SET	TNETFA INPUT							
Edit - Title Line 1	1	TNETFA Example							
Edit - Title Line 2	1	Forced Air Flow - Thermal Circuit							
Edit - Solution Type	2	11	2	0					
	3	8	1 3	0	0	8	0	0	0
B.C./Start Temps	4	55.0	0.0						
B.C./Start Temps	4	8	55.0						
Sources - Steady	4	6	55.0	27.0					
Sources - Steady	4	5	55.0	10.0					
Sources - Steady	4	7	55.0	3.0					
Capacitance	6	0	0						
Conductors - Single	8	2	1	6.0	301				
Conductors - Single	8	3	2	6.0	301				
Conductors - Single	8	4	3	6.0	301				
Conductors - Single	8	6	4	4.869	301				
Conductors - Single	8	7	6	4.869	301				
Conductors - Single	8	5	4	1.13	301				
Conductors - Single	8	7	5	1.13	301				
Conductors - Single	8	1	8	100.0	0				
Solution-Cntrl - Steady	14	20	1.0	0.01	5				
Solution-Cntrl - Steady	14	0.0	0.0						
Solution-Cntrl - Steady	14	5	0						

TNETFA Output File (DOUT)

```
*******************************************************************************
****          Electronics Thermal Analysis Package - PC TNETFA V5.0          ****
****          (C) Copyright 1996 by Thermal Computations, Inc.               ****
****                          Newberg, Oregon                                ****
*******************************************************************************

TNETFA Example
Forced Air Flow - Thermal Circuit

 UNITS=2
 NUMBER OF NODES=   8     NUMBER OF CONDUCTORS=  16
 NLOOP =   20  TPRINT=    5      NPRINT=    0
 LOOPEN=    5    ALDT= .1000E-01   BETA= 1.00

                       LOOPCT=    0
                       TEMPERATURES

  T(  1)= .5500E+02   T(  2)= .5500E+02   T(  3)= .5500E+02   T(  4)= .5500E+02
  T(  5)= .5500E+02   T(  6)= .5500E+02   T(  7)= .5500E+02   T(  8)= .5500E+02

                       LOOPCT=    4
                       TEMPERATURES

  T(  1)= .5500E+02   T(  2)= .5500E+02   T(  3)= .5500E+02   T(  4)= .5500E+02
  T(  5)= .7259E+02   T(  6)= .6591E+02   T(  7)= .6816E+02   T(  8)= .5500E+02
                       MAXDT          = .2941E-03
                       ENERGY BALANCE = 1.6002E-05 PERCENT
```

NPRO Plot of Temperature vs. Node Number



TNETFA Example
Forced Air Flow - Thermal Circuit

```
 11        2        0
 8        1        3        0        0        8        0        0        0
 55        0
8        5.5000E+01
6        5.5000E+01    2.7000E+01
5        5.5000E+01    1.0000E+01
7        5.5000E+01    3.0000E+00
 0        0
2        1        6.0000E+00    301
3        2        6.0000E+00    301
4        3        6.0000E+00    301
6        4        4.8690E+00    301
7        6        4.8690E+00    301
5        4        1.1300E+00    301
7        5        1.1300E+00    301
1        8        1.0000E+02    0
 20        1        0.01        5
 0        0
 5        0
```

TAMS - A <u>T</u>hermal <u>A</u>nalyzer for <u>M</u>ultilayer <u>S</u>tructures

Some Program Features

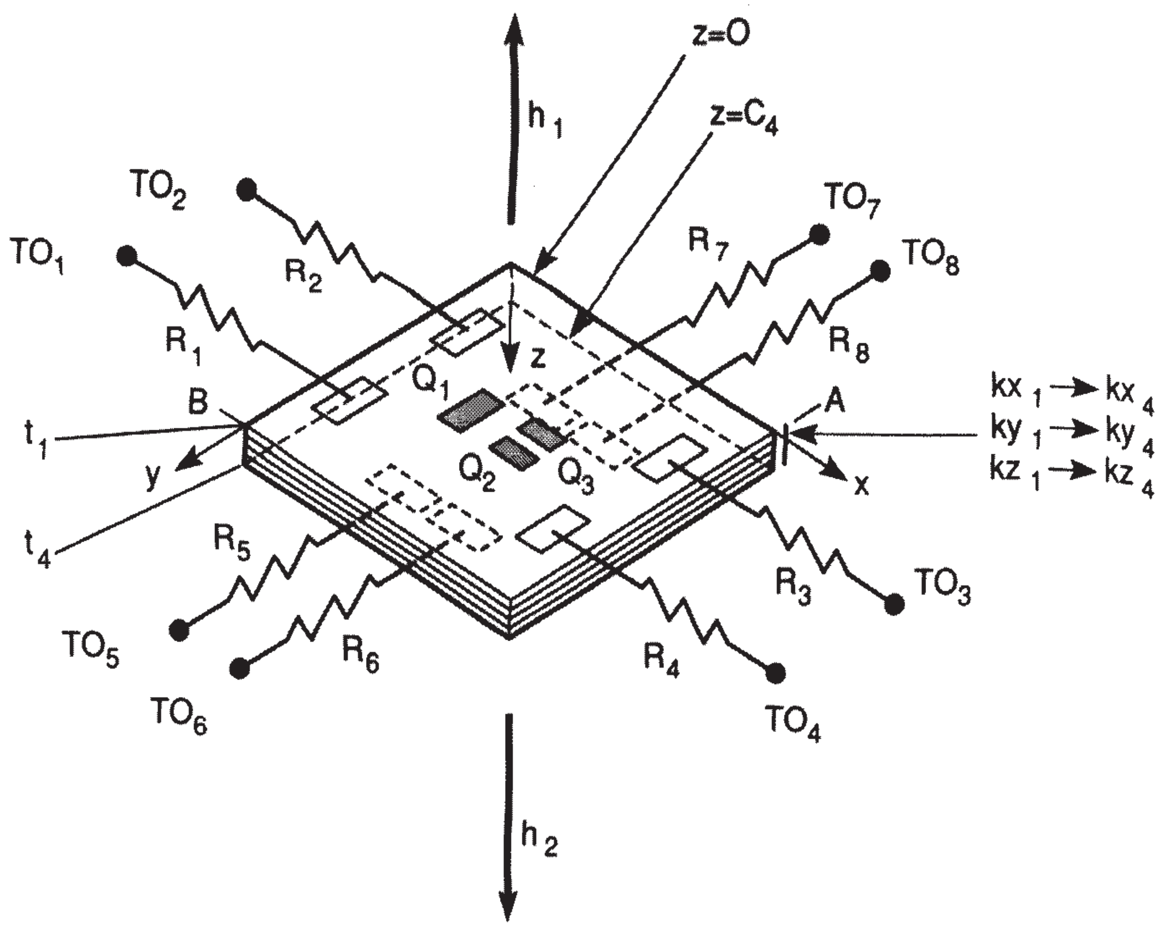


Figure 3.13 Geometry and revelant heat transfer quantities for heat sources and lumped parameter thermal resistances on a multilayer substrate [Ellison (1984)] [Reprinted with premission of the International Society for Hybrid Microelectronics, Reston, VA].

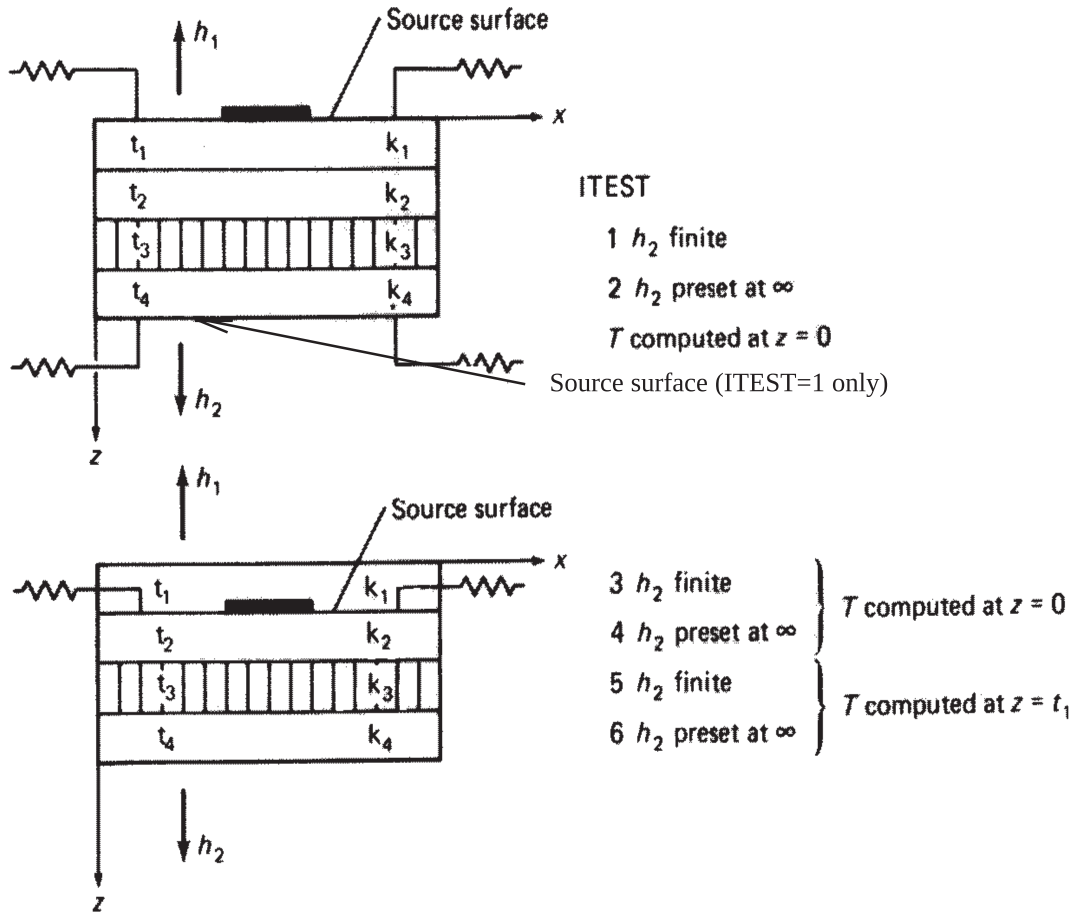


Fig. 9-2. Problem cases in TAMS. Reprinted from the Proceedings of the 1978 International Microelectronics Symposium, [37], by permission of the International Society for Hybrid Microelectronics, P.O. Box 3255, Montgomery, AL 36109.

TPRO/TAMS: Rules

Appendix iii: TPRO/TCEE - Data Set Cross Reference Table

The purpose of the following table is to assist you in identifying how Data Sets from TCEE, Tables 9-1, 9-2 are associated with the various major options in TPRO.

TCEE Data Set: Variable		**TPRO Option**
1:	Title Line	Edit - Title Line
2:	ITEST	Edit - Boundary Conditions
	MODE	Edit - Conductivities
3:	LMAX, MMAX	Fourier Terms
4:	A	Edit - Dimensions
	B	Edit - Dimensions
	t_1	Edit - Dimensions
	t_2	Edit - Dimensions
	t_3	Edit - Dimensions
	t_4	Edit - Dimensions
5:	h_1	Edit - Boundary Conditions
	h_2	Edit - Boundary Conditions
	T_A	Edit - Boundary Conditions

6:	k_1	Edit - Conductivities
	k_2	Edit - Conductivities
	k_3	Edit - Conductivities
	k_4	Edit - Conductivities
	k_{1x}	Edit - Conductivities
	k_{1y}	Edit - Conductivities
	k_{1z}	Edit - Conductivities
	k_{2x}	Edit - Conductivities
	k_{2y}	Edit - Conductivities
	k_{2z}	Edit - Conductivities
	k_{3x}	Edit - Conductivities
	k_{3y}	Edit - Conductivities
	k_{3z}	Edit - Conductivities
	k_{4x}	Edit - Conductivities
	k_{4y}	Edit - Conductivities
	k_{4z}	Edit - Conductivities
7:	NS1, NS2, NR1, NR2	Automatic by TPRO
8:	Source geometry	Edit Sources
9:	Toggles for source calc.	Edit Sources
10:	Resistor Geometry	Edit Resistors

TAMS Input Format

Table 9-1, Updated version. Definition of variables in data sets.

Data-Set	Variable Name	Variable Description
1		Title line
2	ITEST	Determines type of problem-see Fig. 9-2
	MODE	Determines run to be for isotropic k (MODE =0) or run to be for anisotropic k (MODE = 1)
	LMAX	Maximum number terms (Fourier series) for X component
	MMAX	Maximum number terms (Fourier series) for Y component
	A	X dimension of device
	B	Y dimension of device
	t_l	Thickness of layer 1
	t_2	Thickness of layer 2
	t_3	Thickness of layer 3
	t_4	Thickness of layer 4
	h_1	Heat transfer coefficient for Z = O surface
	h_2	Heat transter coefficient for Z = C_4 surface (C_4=t_l+t_2+t_3+t_4)
	T_A	Ambient temperature seen by surfaces Z = 0, C_4
	k_1	Thermal conducthity of layer 1 ~ MODE = O
	k_2	Thermal conductivity of layer 2
	k_3	Thermal conductivity of layer 3
	k_4	Thermal conductivity of layer 4
	k_{lx}	Thermal conductivity of layer 1 in X direction~ MODE = 1
	k_{1y}	Thermal conductivity of layer 1 in Y direction
	k_{lz}	Thermal conductivity of layer 1 in Z direction
	k_{2x}	Thermal conductivity of layer 2 in X direction
	k_{2y}	Thermal conductivity of layer 2 in Y direction
	k_{2z}	Thermal conductivity of layer 2 in Z direction
	k_{3x}	Thermal conductivity of layer 3 in X direction
	k_{3y}	Thermal conductivity of layer 3 in Y direction
	k_{3z}	Thermal conductivity of layer 3 in Z direction
	k_{4x}	Thermal conductivity of layer 4 in X direction
	k_{4y}	Thermal conductivity of layer 4 in Y direction
	k_{4z}	Thermal conductivity of layer 4 in Z direction

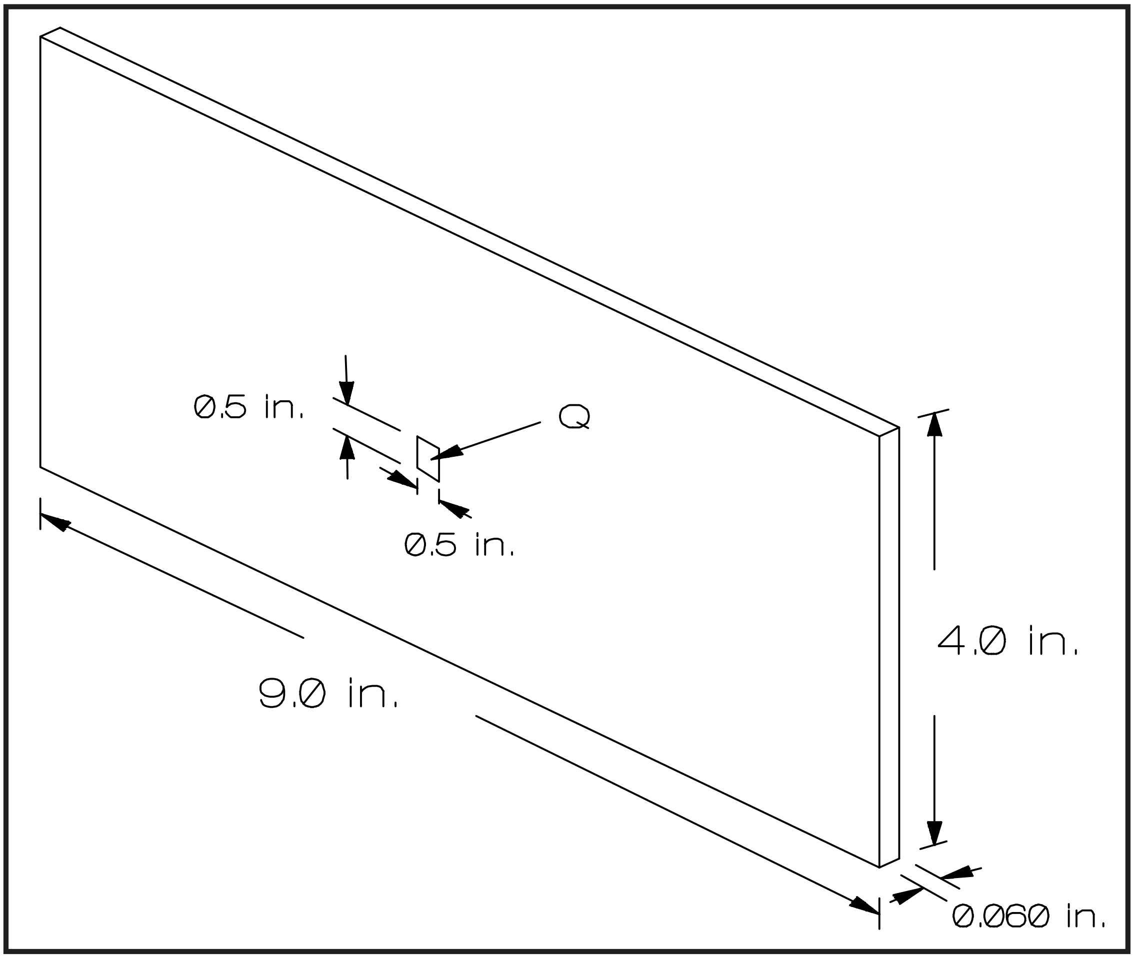
7	NS1	Total number of sources at Z=0 or t_1 plane.
	NS2	Total number of sources ar Z=C_4 plane.
	NR1	Number of leads or thermal resistors at Z = O
	NR2	Number of leads or thermal resistors at Z = C_4 (NR2 must be zero for all problems except ITEST = 1)
8	XS	Minimum X coordinate of a source
	DXS	Source dimension in X direction
	YS	Minimum Y coordinate of a source
	DYS	Source dimension in Y direction
	Q	Source power
9	Every Ith source that requires a temp. calculation requires the nonzero integer I. Input zero for all others.	
10	XR	Minimum X coordinate of a resistor pad
	DXR	Resistor pad dimension in X-direction
	YR	Minimum Y coordinate of a resistor pad
	DYR	Resistor pad dimension in Y-direction
	R	Thermal resistance
	T_o	Sink temperature of resistor at resistor end opposite substrate end

Table 9-2, updated. Data set input requirements-list directed input. An I superscript indicates integer input format required.

Data Set	Variables						
1	Title line-all 80 columns may be used						
2	$ITEST^I$ $MODE^I$						
3	$LMAX^I$ $MMAX^I$						
4	A	B	t_1	t_2	t_3	t_4	
5	h_1	h_2	T_A				
6	k_1	k_2	k_3	k_4	~	If MODE = O	
6	k1x	kly	klz		~	If MODE = 1	
6	k2x	k2y	k2z				
6	k3x	k3y	k3z				
6	k4x	k4y	k4z				
7	$NS1^I$	$NS2^I$	$NR1^I$	$NR2^I$			
8	XS(I)	DXS(I)	YS(I)		DYS(I)	Q(l)	
	.						
	.						
	.						
8	XS(NS)	DXS(NS)	YS(NS)		DYS(NS)	Q(NS)	
9	1^I	2^I	3^I				
10	XR(I)	DXR(I)	YR(I)	DYR(I)	R(l)	T_o(l)	
	.						
	.						
	.						
10	XR(NRI)	DXR(NRI)	YR(NRl)	DYR(NRI)	R(NRI)	T_o(NRl)	
	.						
	.						
	.						
10	XR(NR)	DXR(NR)	YR(NR)	DYR(NR)	R(NR)	T_o(NR)	

Example

Flat Panel with One Transistor



$$\varepsilon = 0.1, \quad \text{i.e. neglect radiation}$$

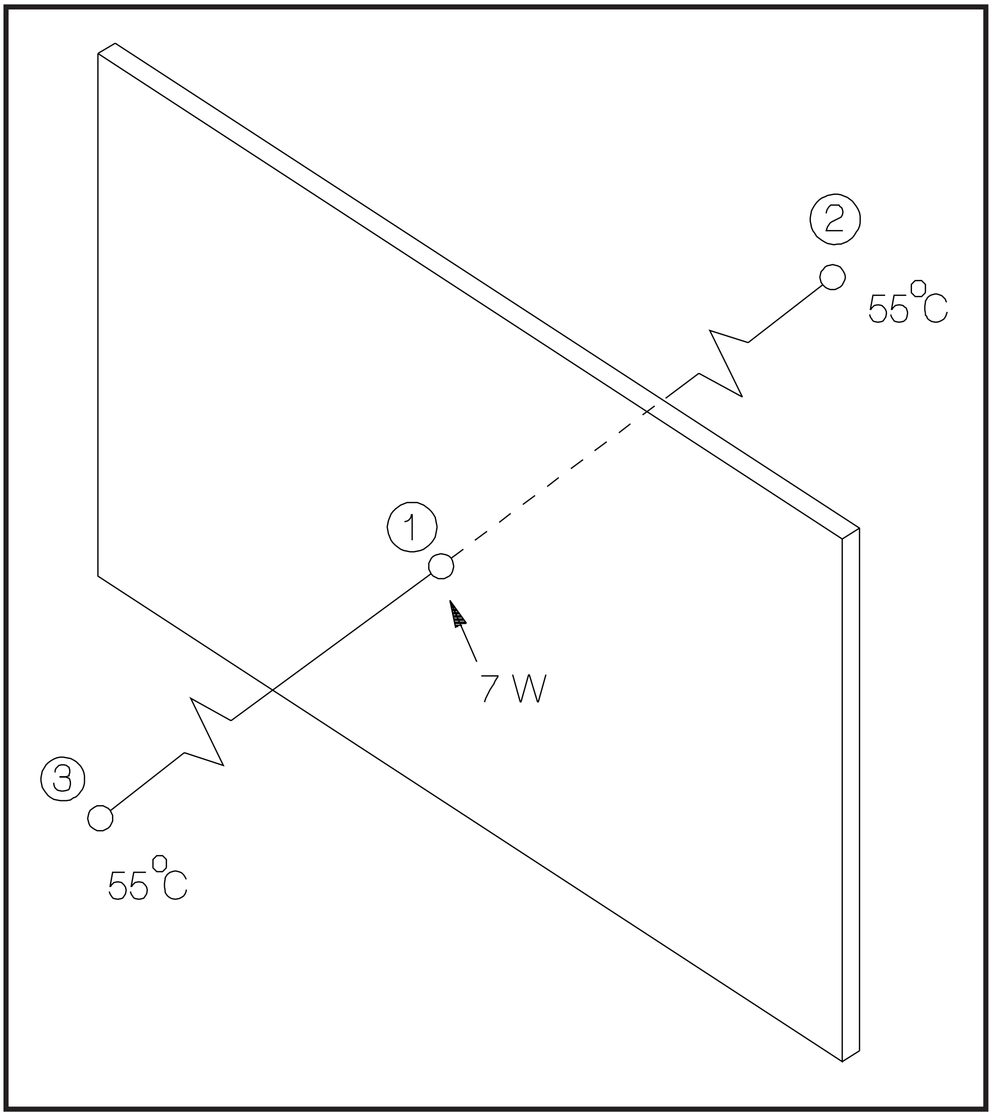
$$T_A = 55^oC$$

$$k = 4.0\ W/in.\cdot^oC$$

$$Q = 7.0\ W$$

Cooled by natural convection

TNETFA Solution of Average Thermal Resistance and h_1, h_2



NPRO OPTION	DATA SET	TNETFA INPUT								
Edit - Title 1	1	9x4 Flat Plate								
Edit - Title 2	1	Convection Only								
Edit - Solution Type and Edit - Units		2	11	2	0					
	3	3	2	1	0	1	0	0	1	0
B.C./Start Temps	4	55.0		0.0						
B.C./Start Temps	4	2	55.0							
B.C./Start Temps	4	3	55.0							
Sources - Steady	4	1	55.0	7.0						
	6	0	0							
Conductors - String of	7	2	1	0	2	1	36.0	101		
Natural Convection		11	1	4.0						
Solution-Cntrl - Steady	14	10	1.0	0.01	10					
Solution-Cntrl - Steady	14	0	0							
Solution-Cntrl - Steady	14	1								

Actual TNETFA File Used

```
9X4 Flat Plate
Convection Only
  11      2       0
   3      2      1      0      1      0      0      1      0
 .5500E+02    .0000E+00
   2     .5500E+02
   3     .5500E+02
   1     .5500E+02    .7000E+01
   0       0
   2       1       0       2       1     .3600E+02     101
   1     .4000E+01
  10     .1000E+01    .1000E-01      10
 .0000E+00    .0000E+00
  10       1
```

TNETFA Output File (DOUT)

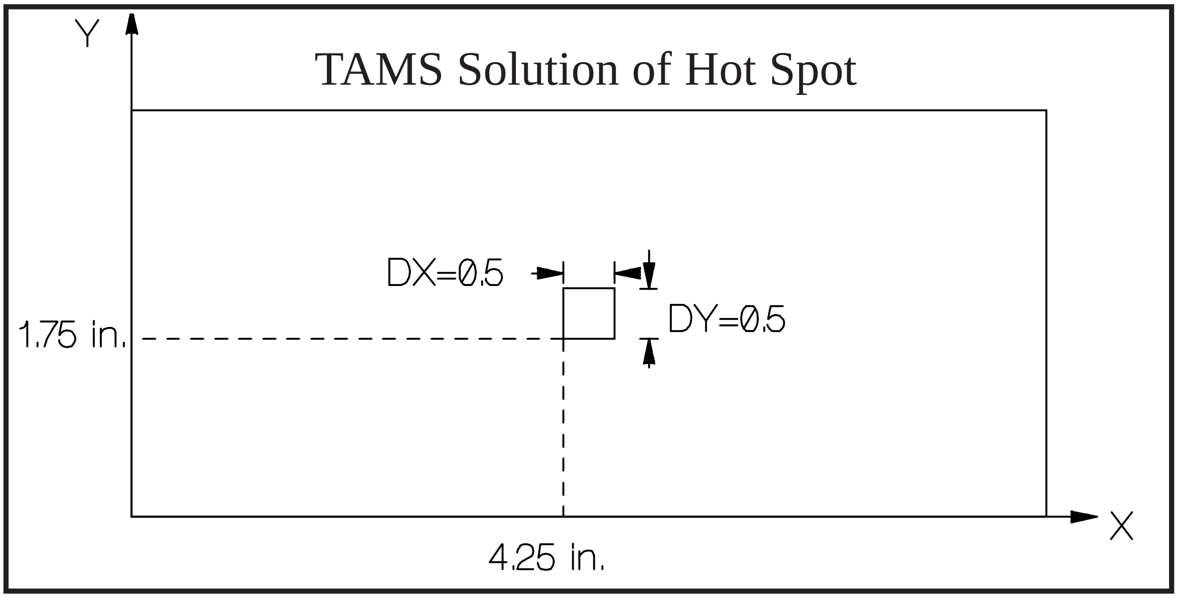
```
*****************************************************************************
****        Electronics Thermal Analysis Package - PC TNETFA V3.2       ****
****           (C) Copyright 1993 by Thermal Computations, Inc.         ****
****                        Hillsboro, Oregon                           ****
*****************************************************************************

9X4 Flat Plate
Convection Only

 UNITS=2
 NUMBER OF NODES=   3     NUMBER OF CONDUCTORS=   4
 NLOOP =   10  TPRINT=   10      NPRINT=    1
 LOOPEN=   10    ALDT= .1000E-01   BETA= 1.00

 NATURAL CONVECTION PARAMETER
   1    VERTICAL FLAT PLATE OR CYLINDER:                      P= .4000E+01

                                   LOOPCT=    0
                                   TEMPERATURES

  T(  1)= .5500E+02   T(  2)= .5500E+02   T(  3)= .5500E+02

                                   LOOPCT=    8
                                   TEMPERATURES

  T(  1)= .8105E+02   T(  2)= .5500E+02   T(  3)= .5500E+02
                                   MAXDT          = .8007E-02
                                   ENERGY BALANCE = 7.2570E-03 PERCENT


    DETAIL OF NODE   1     TEMPERATURE= .8105E+02            POWER= .7000E+01
                            STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
    NODE  CTYPE  CMODE       C      CONDUCTANCE      FLUX    HT TRANS COEF/SFA
       2   101     1    .3600E+02     .1343E+00     .3500E+01     .3731E-02
       3   101     1    .3600E+02     .1343E+00     .3500E+01     .3731E-02
                                         NET TOTAL = .6999E+01

    DETAIL OF NODE  -2     TEMPERATURE= .5500E+02            POWER= .0000E+00
                            STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
                            THIS IS A CONSTANT TEMPERATURE NODE
    NODE  CTYPE  CMODE       C      CONDUCTANCE      FLUX    HT TRANS COEF/SFA
       1   101     1    .3600E+02     .1343E+00    -.3500E+01     .3731E-02
                                         NET TOTAL =-.3500E+01

    DETAIL OF NODE  -3     TEMPERATURE= .5500E+02            POWER= .0000E+00
                            STABILITY CONSTANT =   .00E+00     CAP= .1000E-19
                            THIS IS A CONSTANT TEMPERATURE NODE
    NODE  CTYPE  CMODE       C      CONDUCTANCE      FLUX    HT TRANS COEF/SFA
       1   101     1    .3600E+02     .1343E+00    -.3500E+01     .3731E-02
                                         NET TOTAL =-.3500E+01
```

Heat transfer coefficient for each side of plate taken from TNETFA output file:

For each side of the plate,

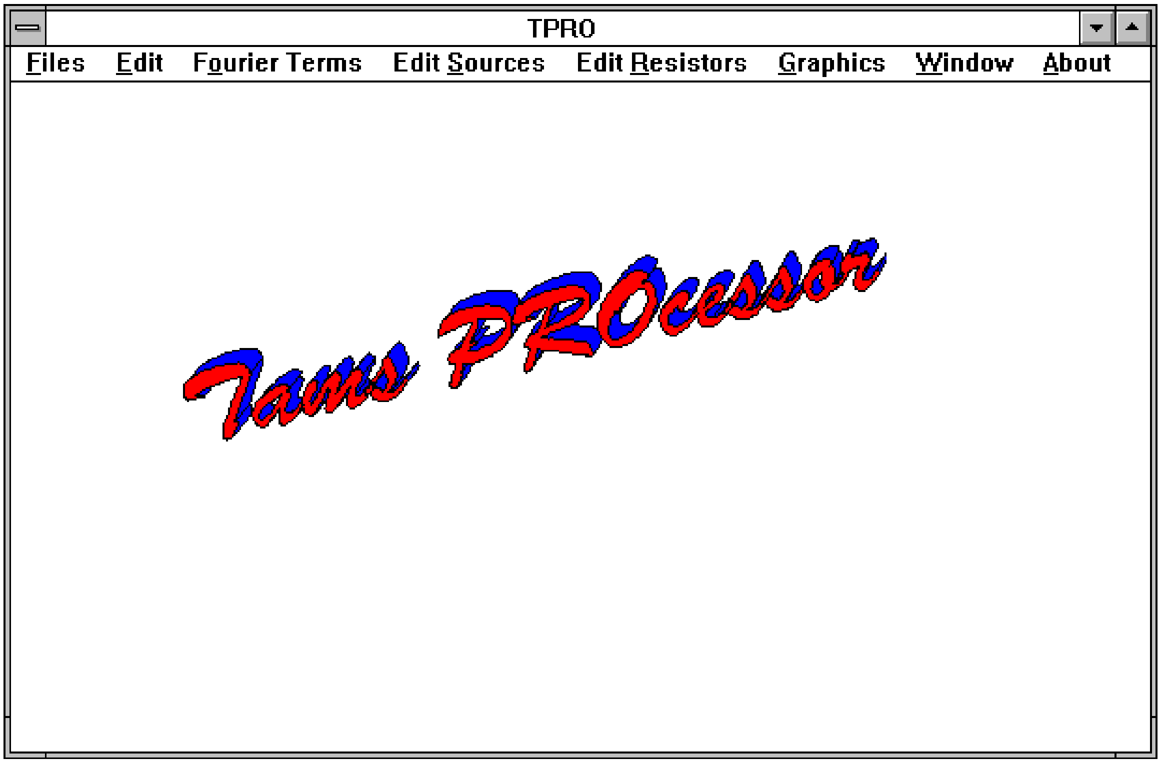
$$h_1 = 0.0037\ W/in.^2 \cdot {}^oC$$

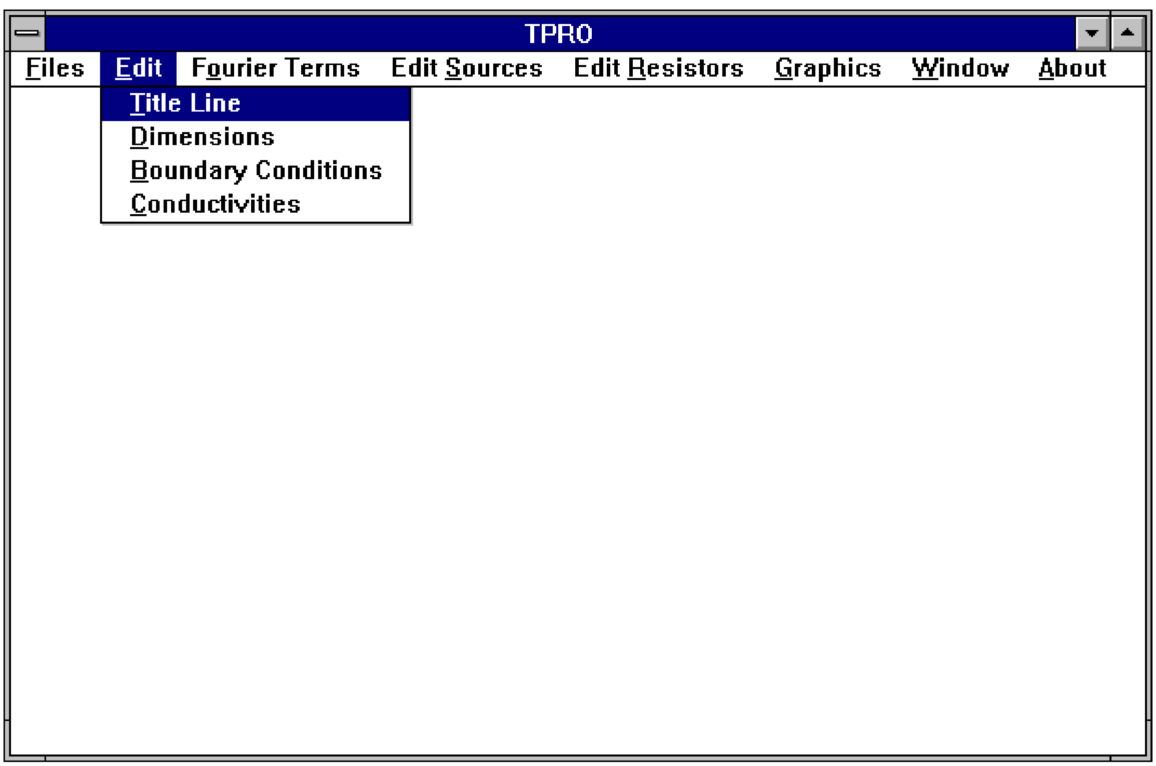
$$h_2 = 0.0037\ W/in.^2 \cdot {}^oC$$

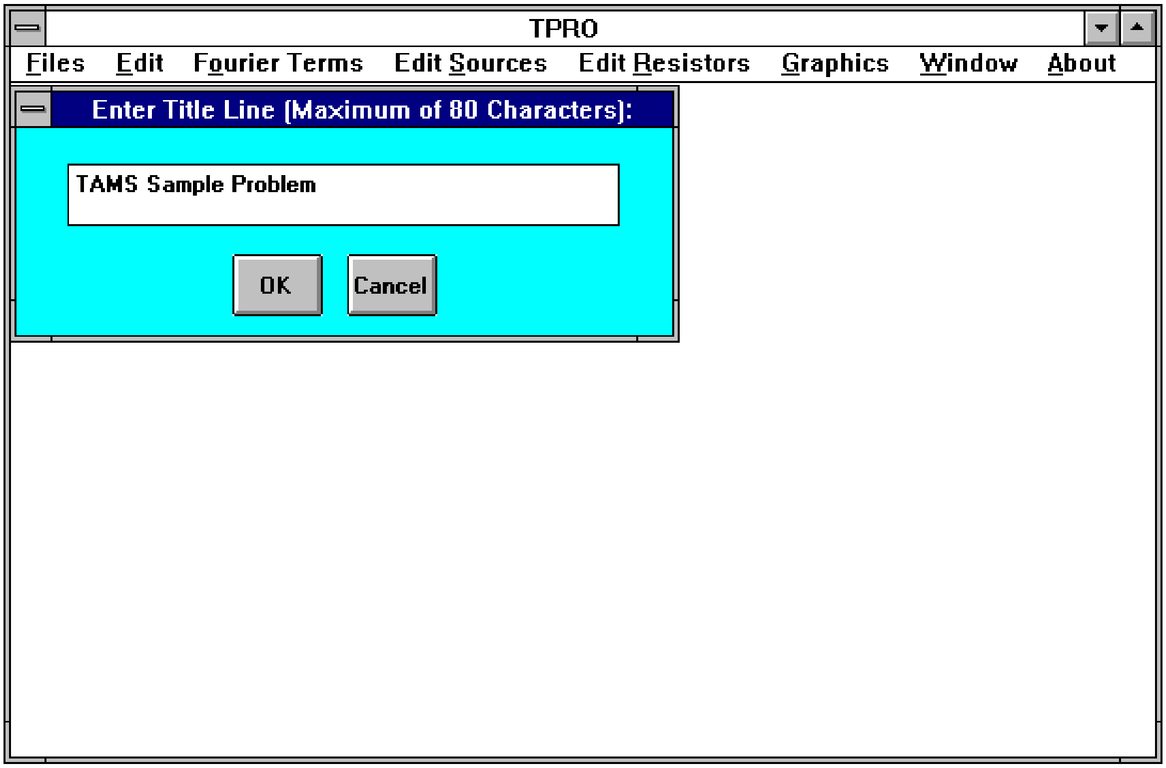


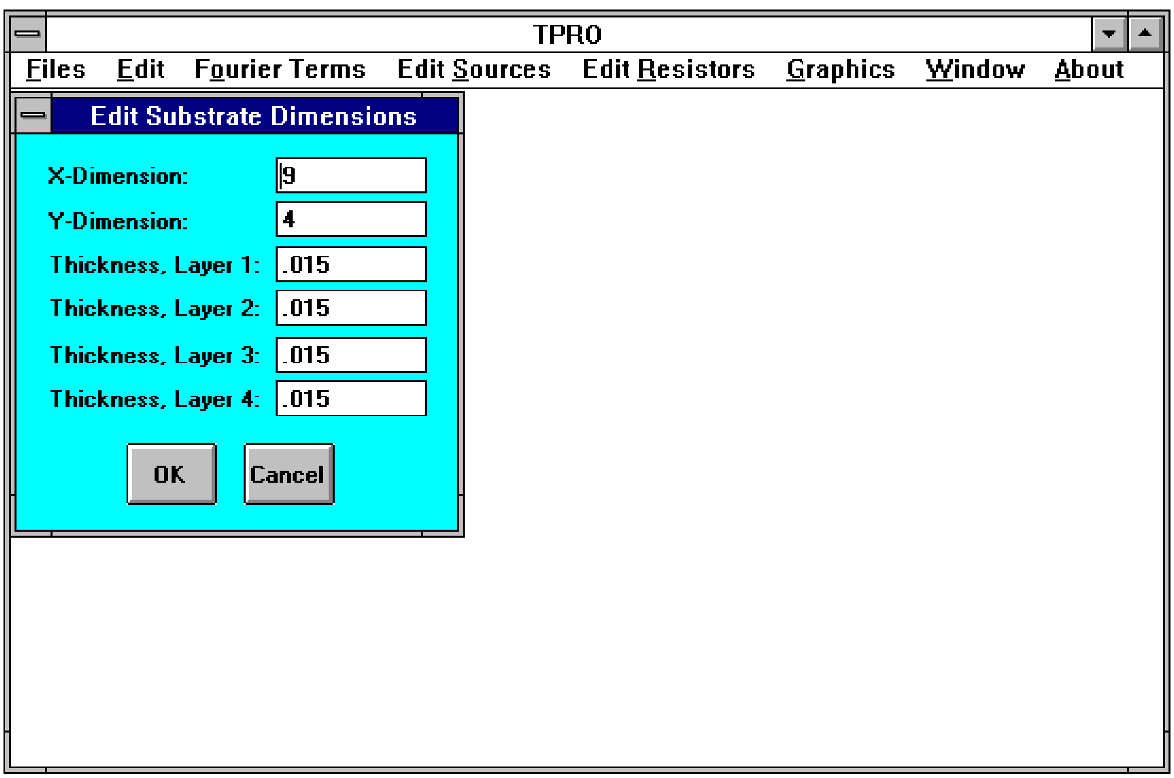
Minimum TAMS Input File to Solve This Problem -

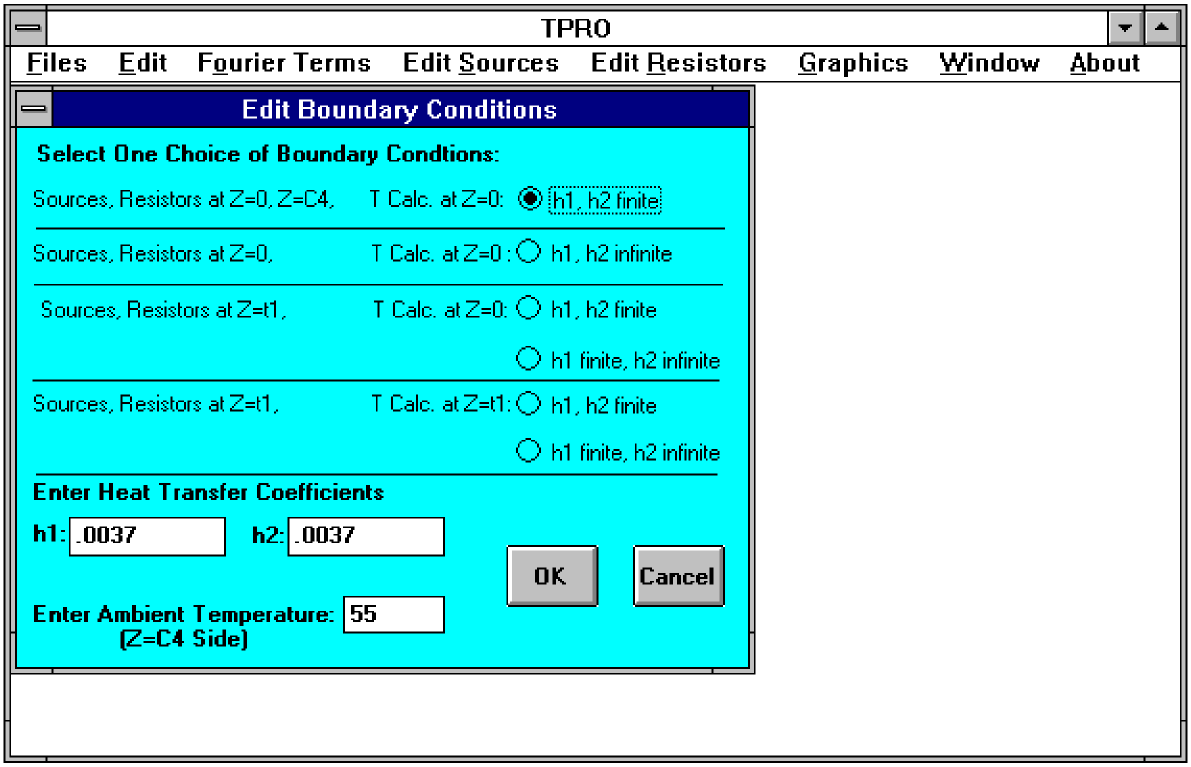
TPRO OPTION	DATA SET	TAMS INPUT DATA
Edit-Title Line	1	TAMS Sample Problem
Edit-Boundary Conditions and Conductivities	2	1 0
Fourier Terms	3	40 40
Edit-Dimensions	4	9.0 4.0 0.015 0.015 0.015 0.015
Edit-Boundary Conditions	5	0.0037 0.0037 55.0
Edit-Conduc-tivities	6	4.0 4.0 4.0 4.0 4.0
Edit Sources	7	1 0 0 0 (auto by TPRO)
Edit Sources	8	4.25 0.5 1.75 0.5 7.0
Edit Sources	9	1 (auto by TPRO if "toggles" on)

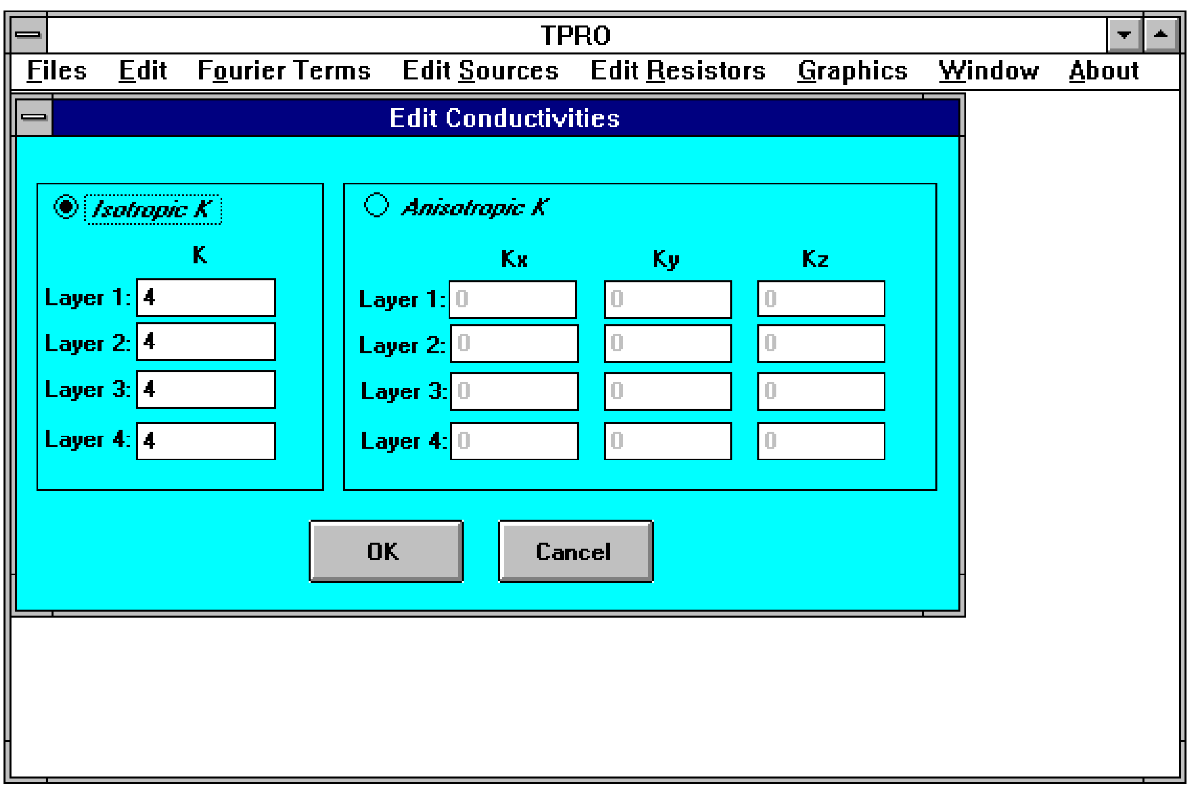

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Tams PROcessor

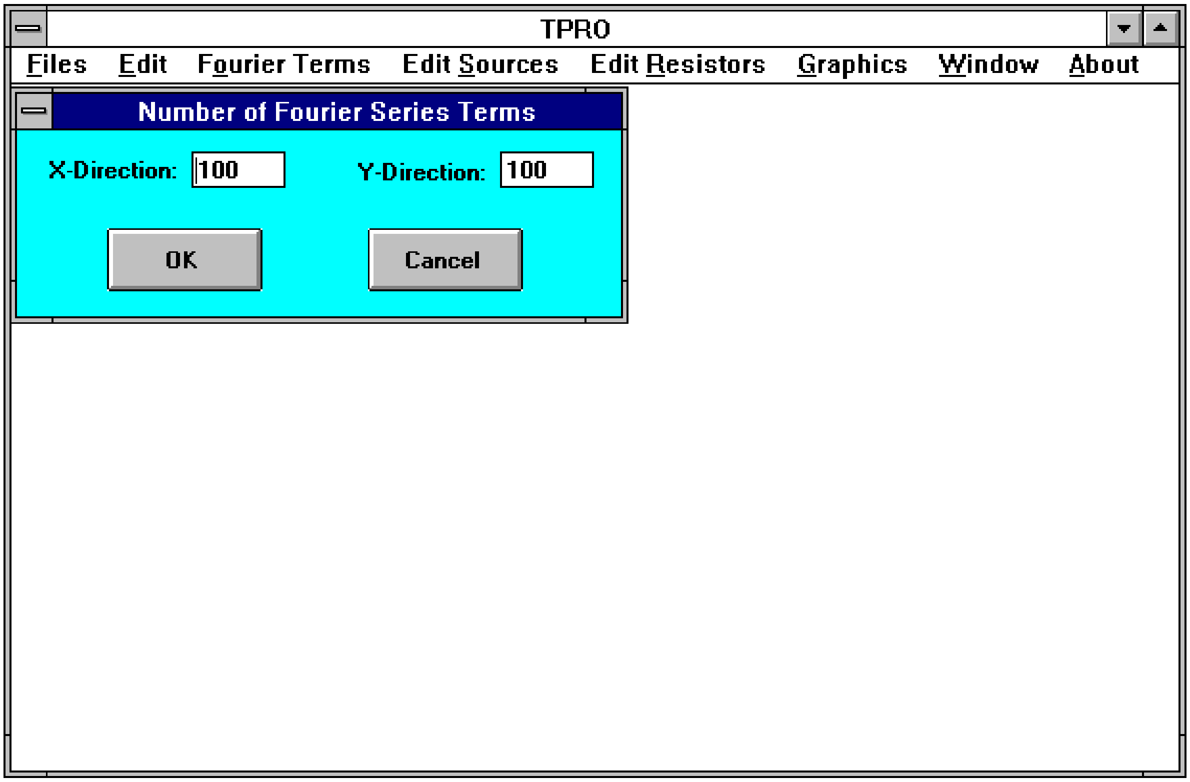

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Title Line
Dimensions
Boundary Conditions
Conductivities

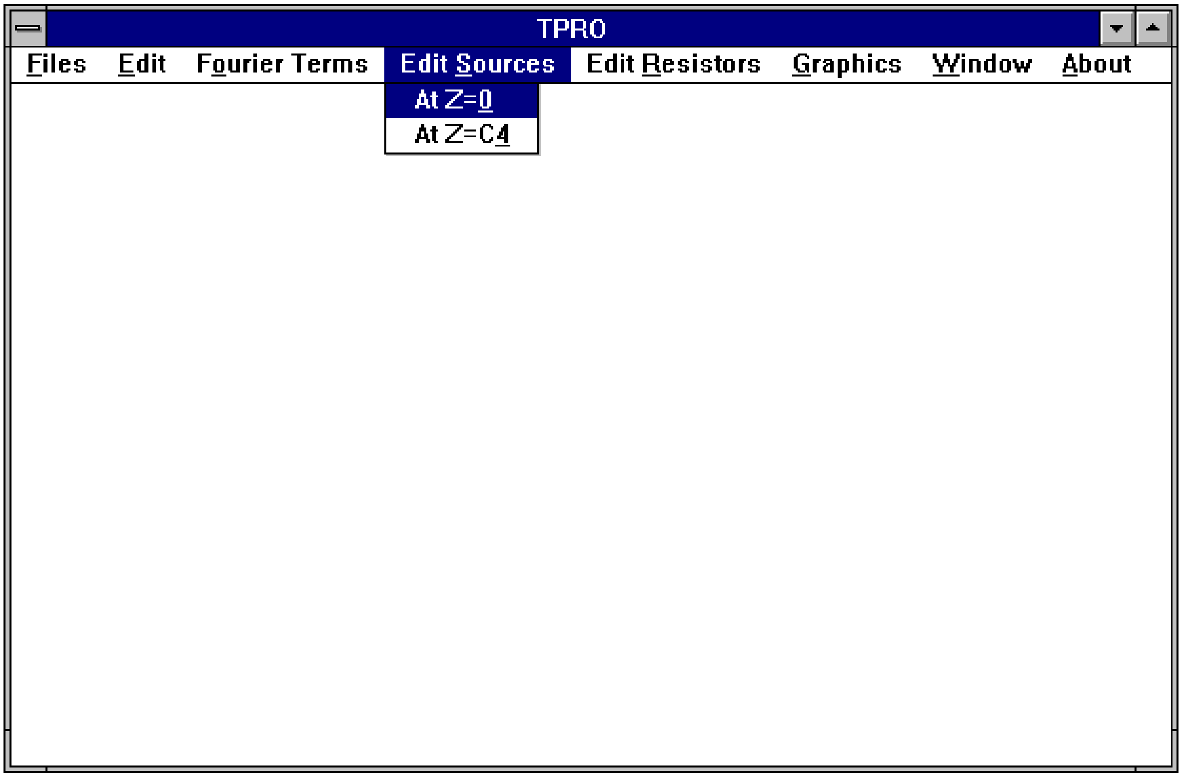

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Enter Title Line (Maximum of 80 Characters):
TAMS Sample Problem
OK
Cancel

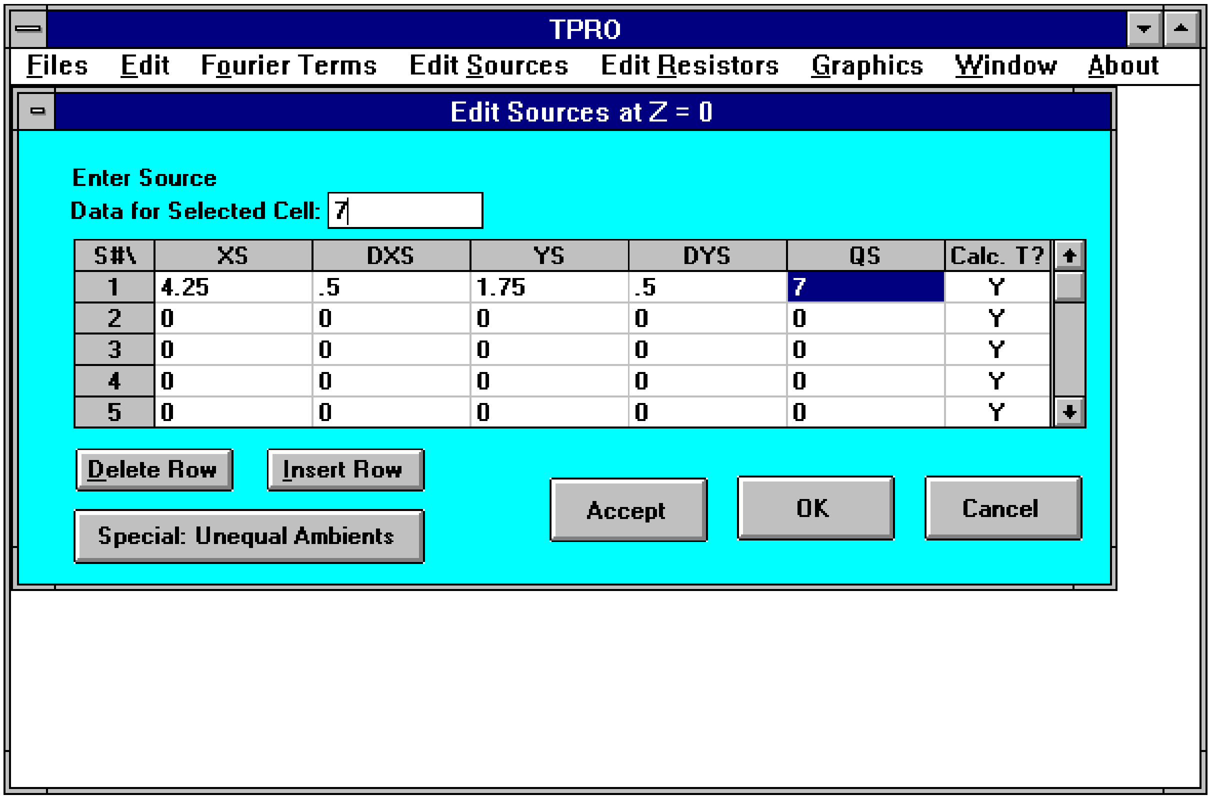

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Edit Substrate Dimensions
X-Dimension: 9
Y-Dimension: 4
Thickness, Layer 1: .015
Thickness, Layer 2: .015
Thickness, Layer 3: .015
Thickness, Layer 4: .015
OK
Cancel

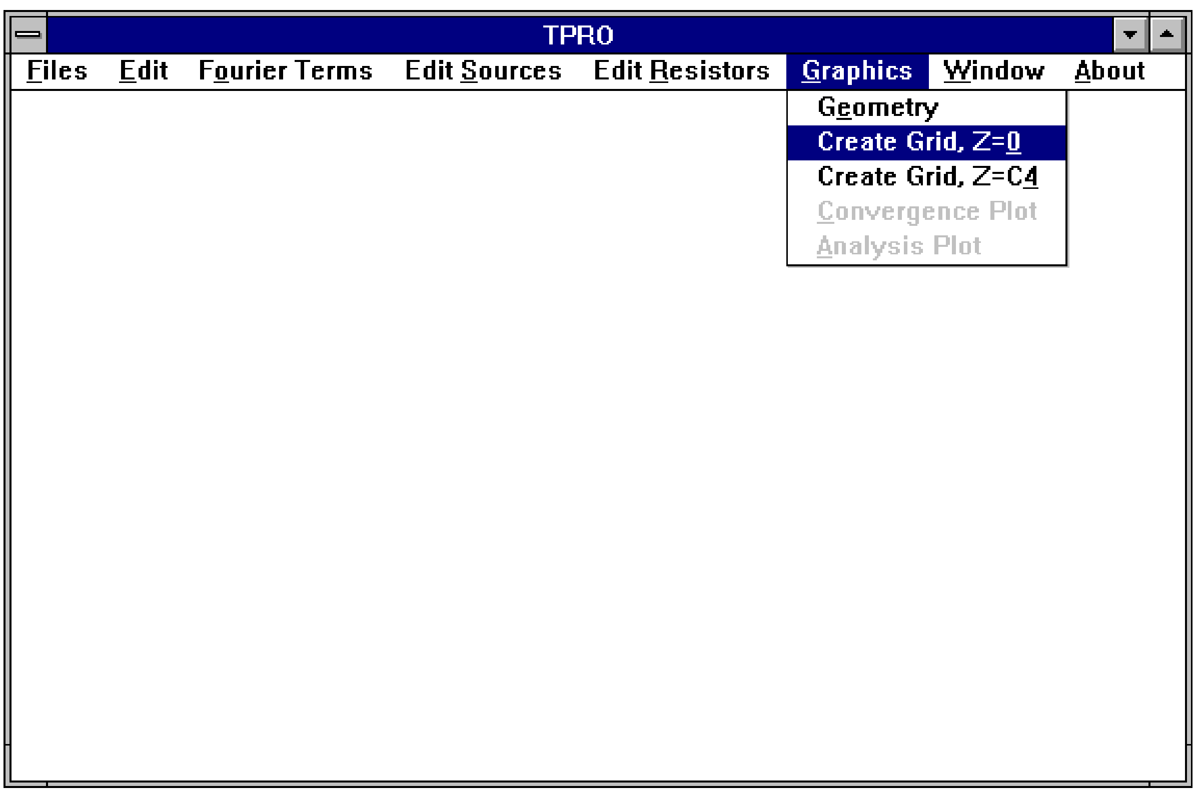

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Edit Boundary Conditions
Select One Choice of Boundary Condtions:
Sources, Resistors at Z=0, Z=C4, T Calc. at Z=0: h1, h2 finite
Sources, Resistors at Z=0, T Calc. at Z=0 : h1, h2 infinite
Sources, Resistors at Z=t1, T Calc. at Z=0: h1, h2 finite
h1 finite, h2 infinite
Sources, Resistors at Z=t1, T Calc. at Z=t1: h1, h2 finite
h1 finite, h2 infinite
Enter Heat Transfer Coefficients
h1: .0037 h2: .0037
OK Cancel
Enter Ambient Temperature: 55
(Z=C4 Side)

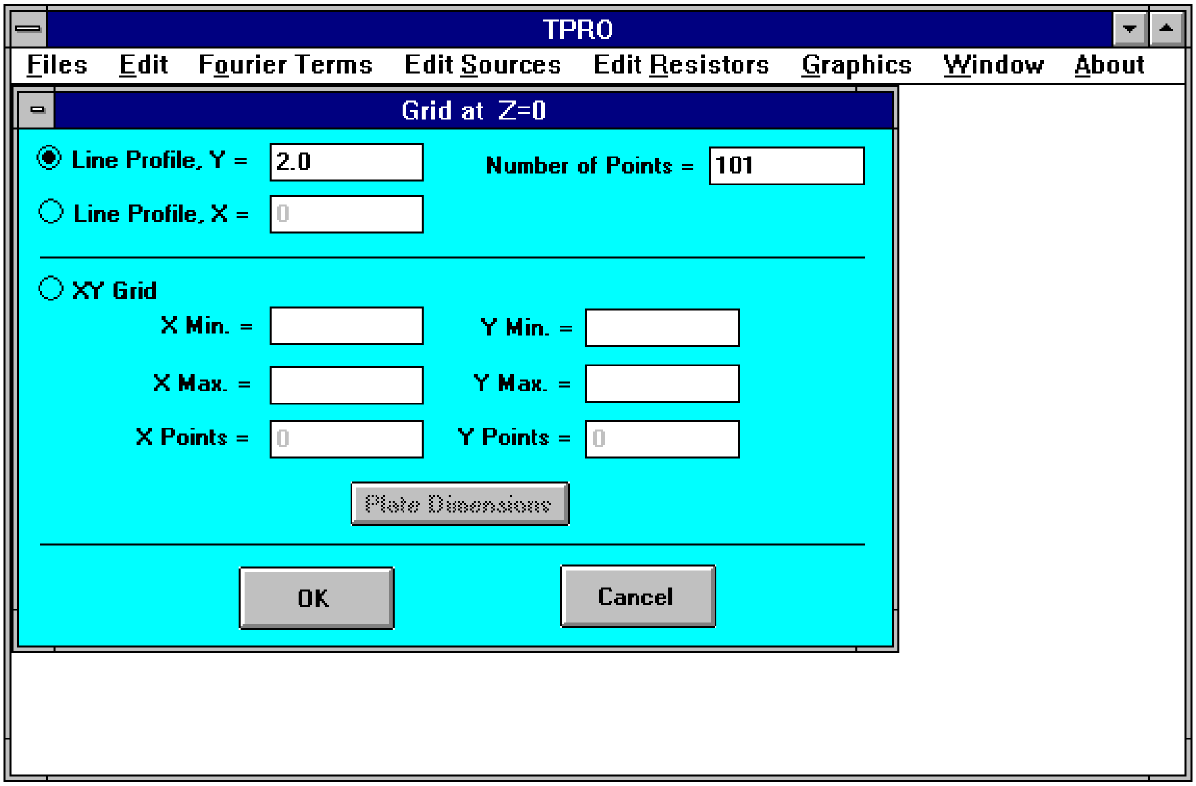

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Edit Conductivities
Isotropic K
K
Layer 1: 4
Layer 2: 4
Layer 3: 4
Layer 4: 4
Anisotropic K
Kx Ky Kz
Layer 1: 0 0 0
Layer 2: 0 0 0
Layer 3: 0 0 0
Layer 4: 0 0 0
OK Cancel

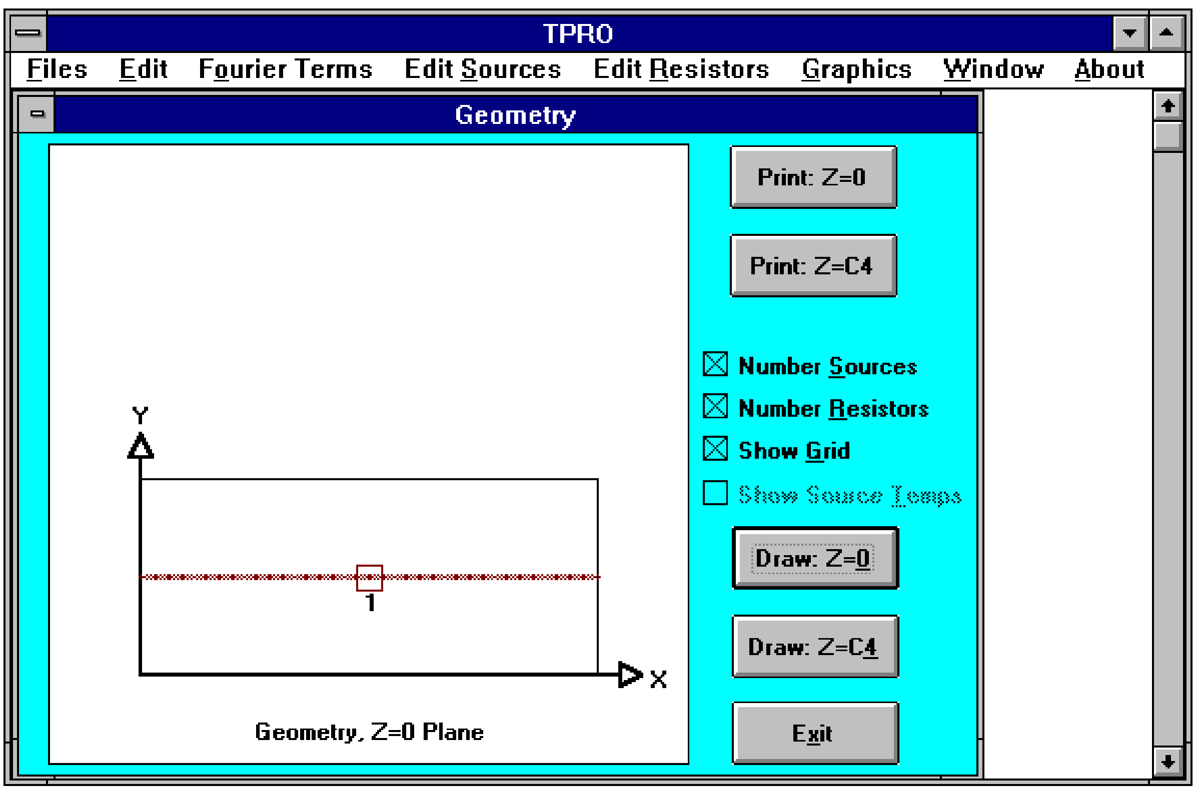

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Number of Fourier Series Terms
X-Direction: 100
Y-Direction: 100
OK
Cancel

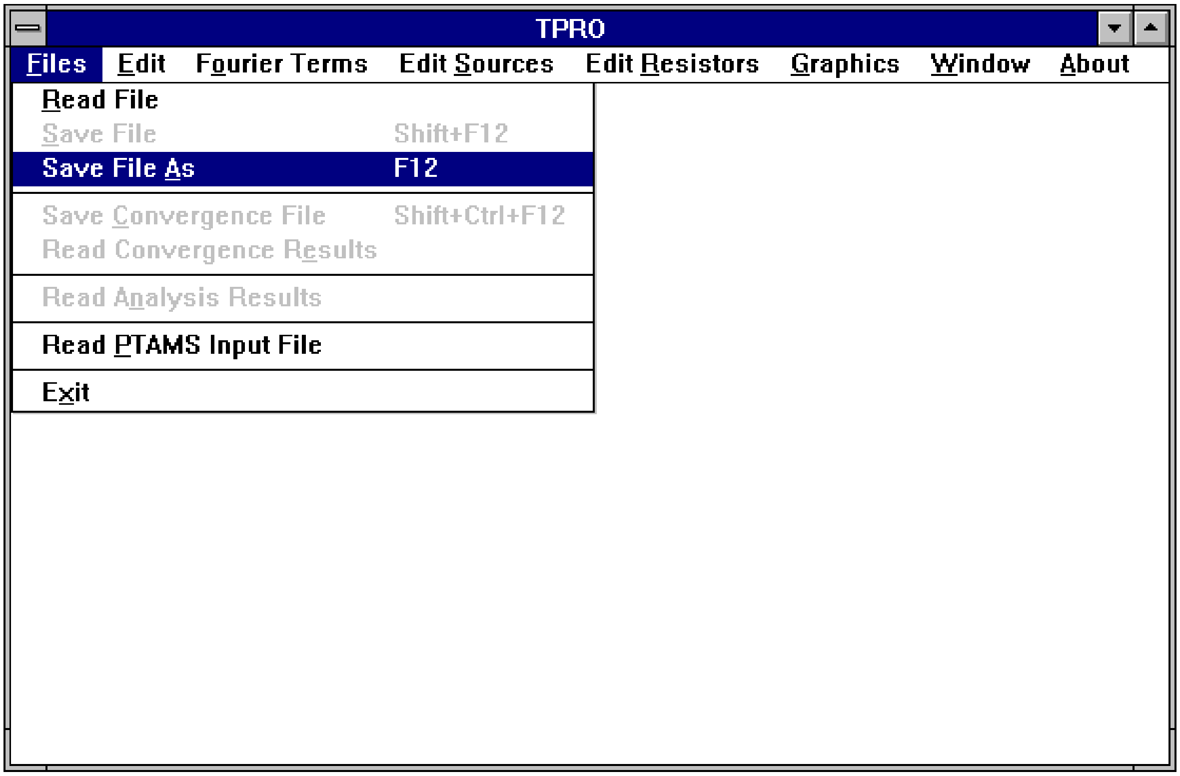

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
At Z=0
At Z=C4

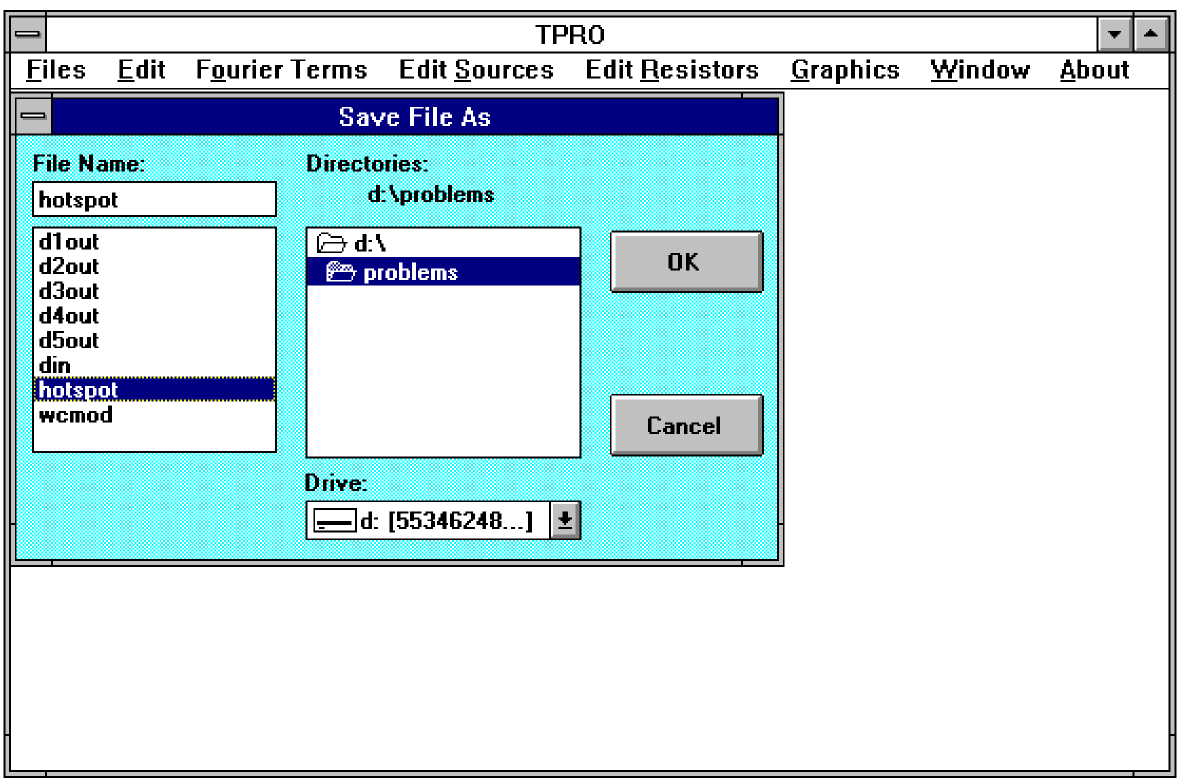

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Edit Sources at Z = 0
Enter Source
Data for Selected Cell: 7
S#\ XS DXS YS DYS QS Calc. T?
1 4.25 .5 1.75 .5 7 Y
2 0 0 0 0 0 Y
3 0 0 0 0 0 Y
4 0 0 0 0 0 Y
5 0 0 0 0 0 Y
Delete Row
Insert Row
Special: Unequal Ambients
Accept
OK
Cancel

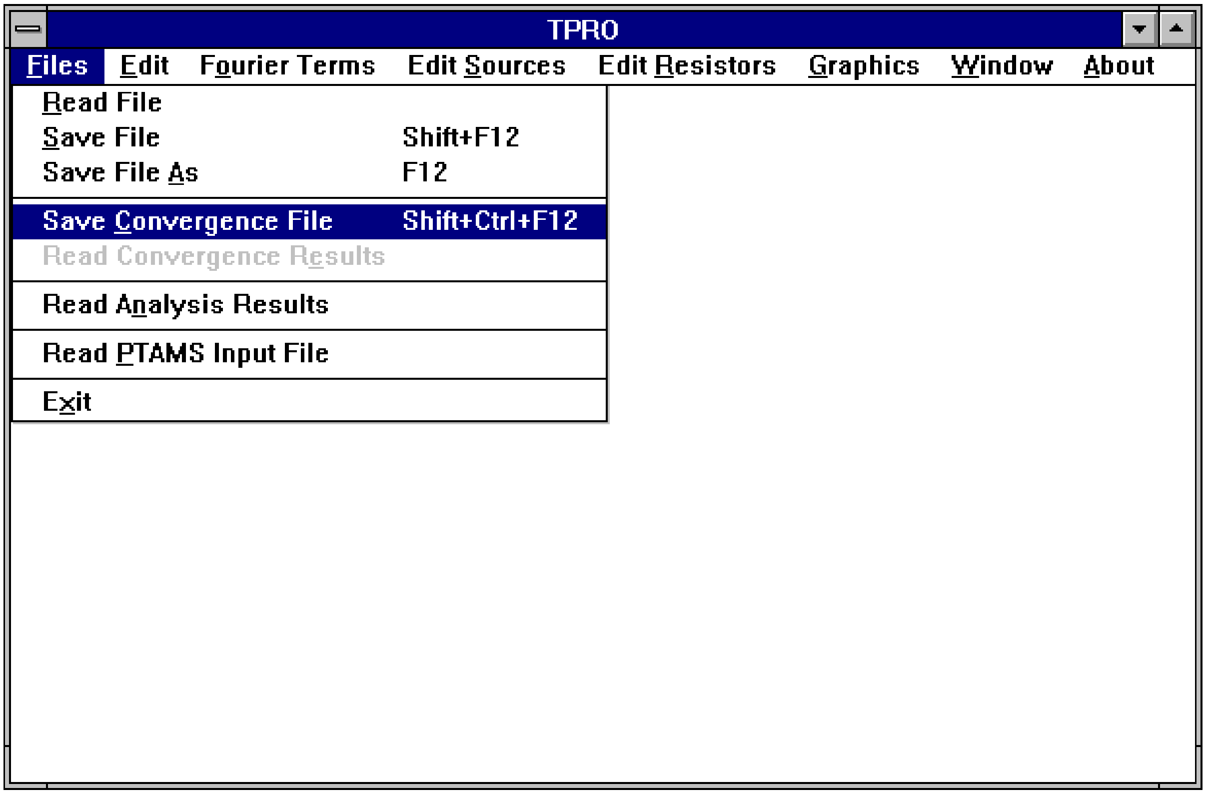

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Geometry
Create Grid, Z=0
Create Grid, Z=C4
Convergence Plot
Analysis Plot


TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Grid at Z=0
Line Profile, Y = 2.0
Number of Points = 101
Line Profile, X = 0
XY Grid
X Min. =
Y Min. =
X Max. =
Y Max. =
X Points = 0
Y Points = 0
Plate Dimensions
OK
Cancel

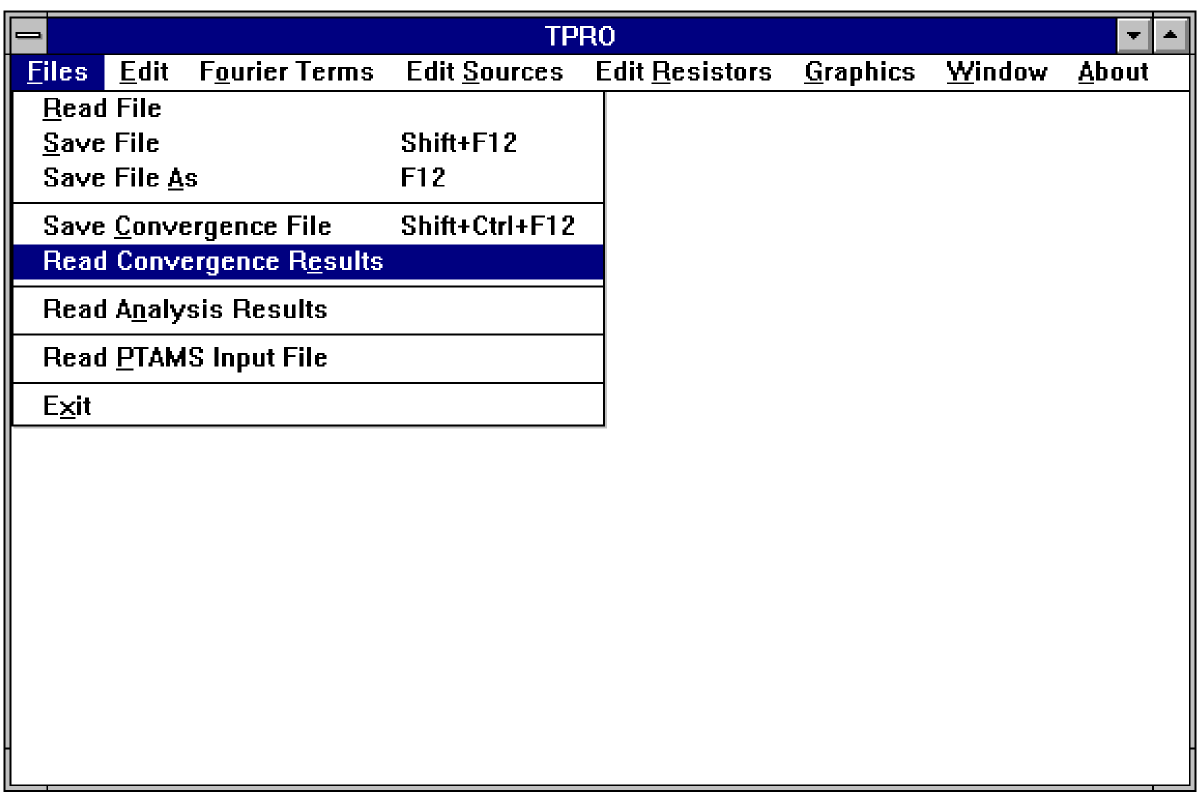

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Geometry
Print: Z=0
Print: Z=C4
Number Sources
Number Resistors
Show Grid
Show Source Temps
Draw: Z=0
Draw: Z=C4
Exit
Y
X
1
Geometry, Z=0 Plane

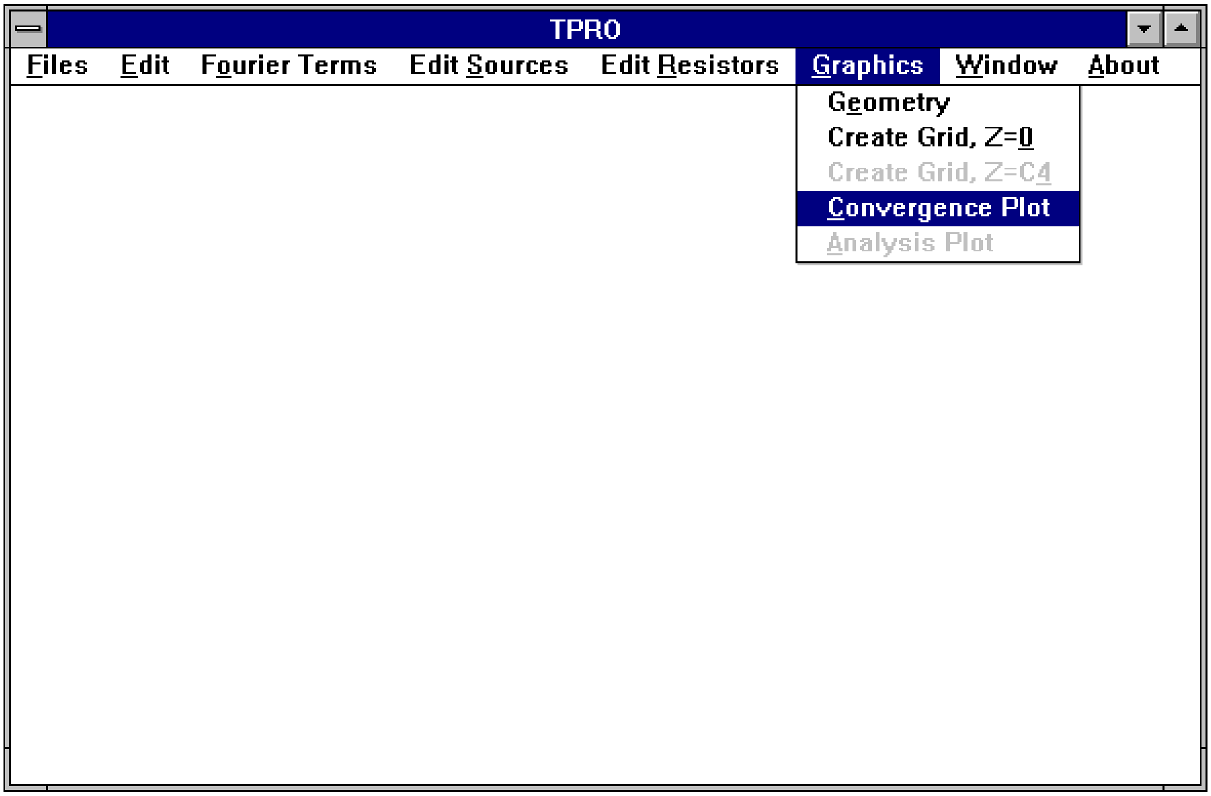

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Read File
Save File Shift+F12
Save File As F12
Save Convergence File Shift+Ctrl+F12
Read Convergence Results
Read Analysis Results
Read PTAMS Input File
Exit

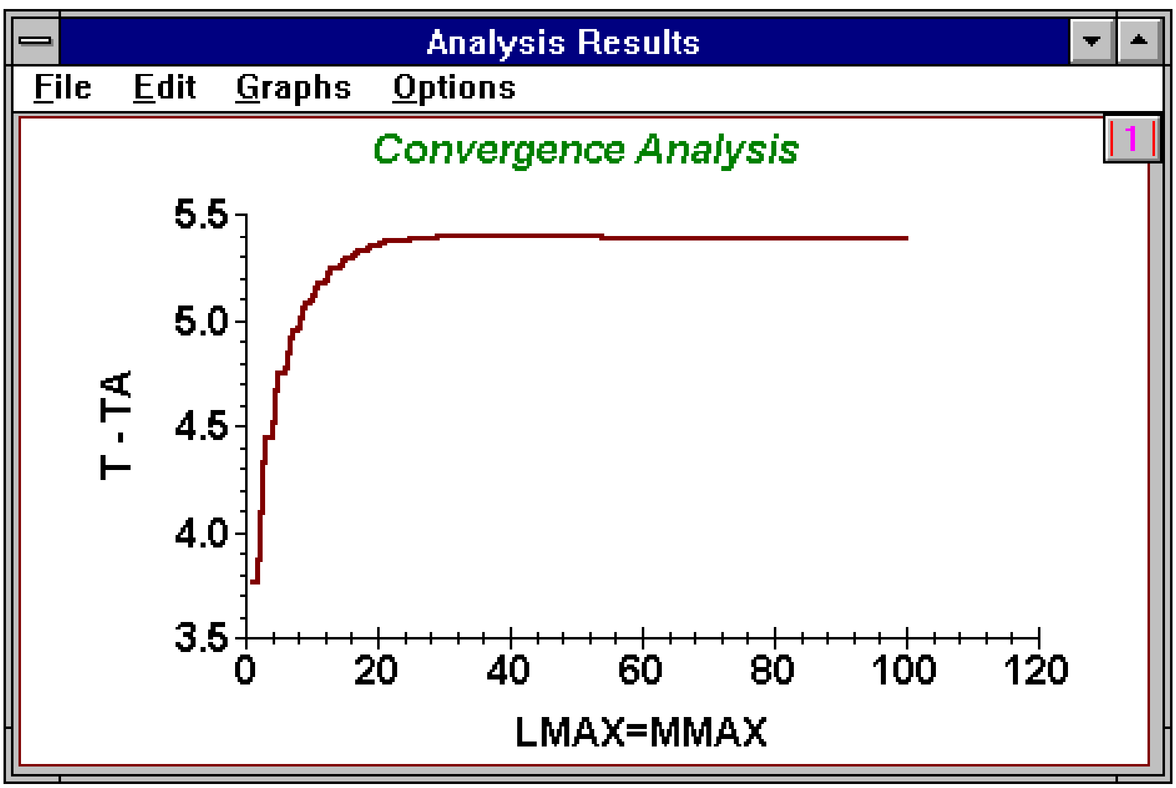

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Save File As
File Name:
hotspot
d1out
d2out
d3out
d4out
d5out
din
hotspot
wcmod
Directories:
d:\problems
d:\
problems
OK
Cancel
Drive:
d: [55346248...]

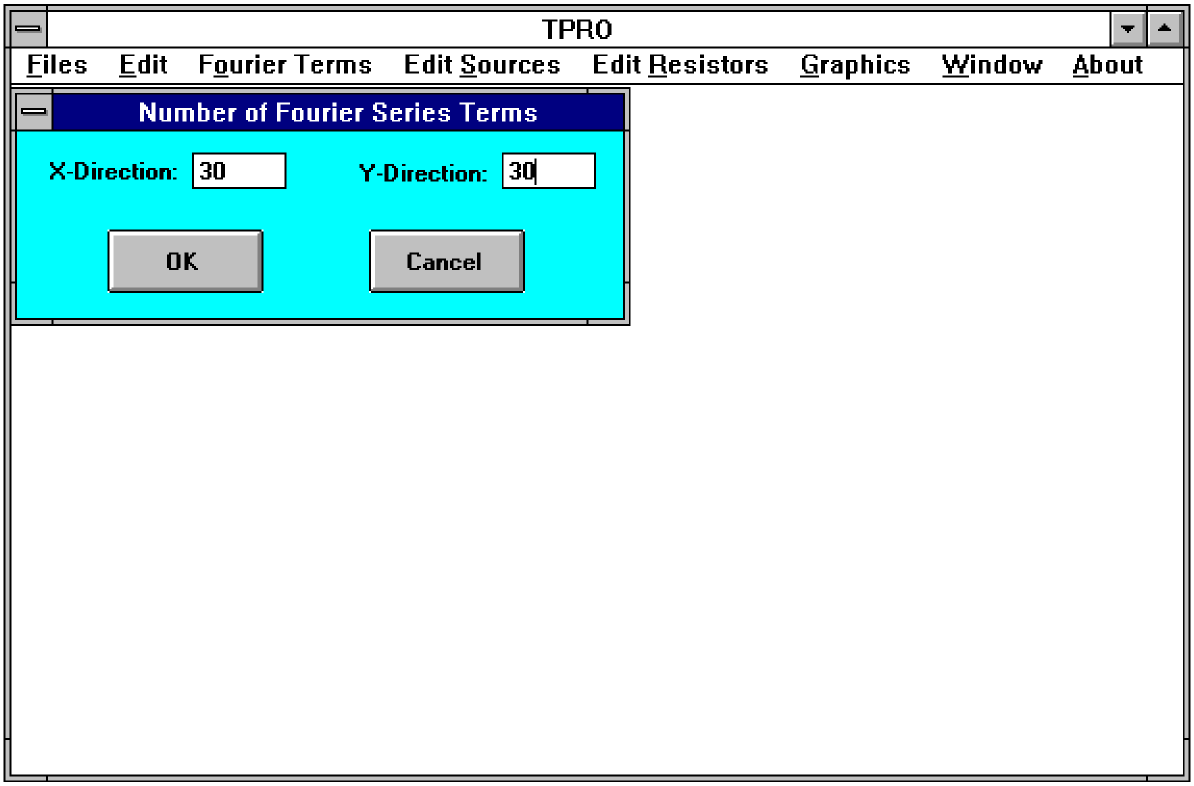


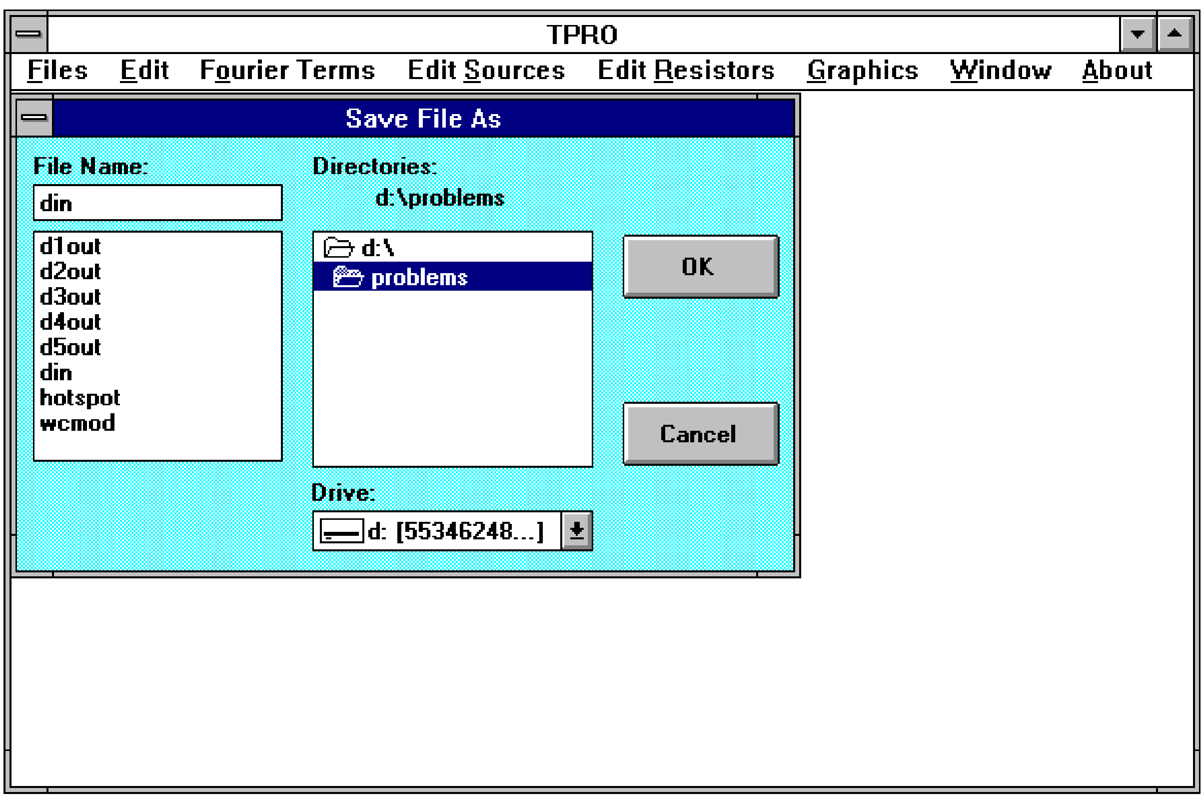
Run TAMS4 by doubling clicking on Icon



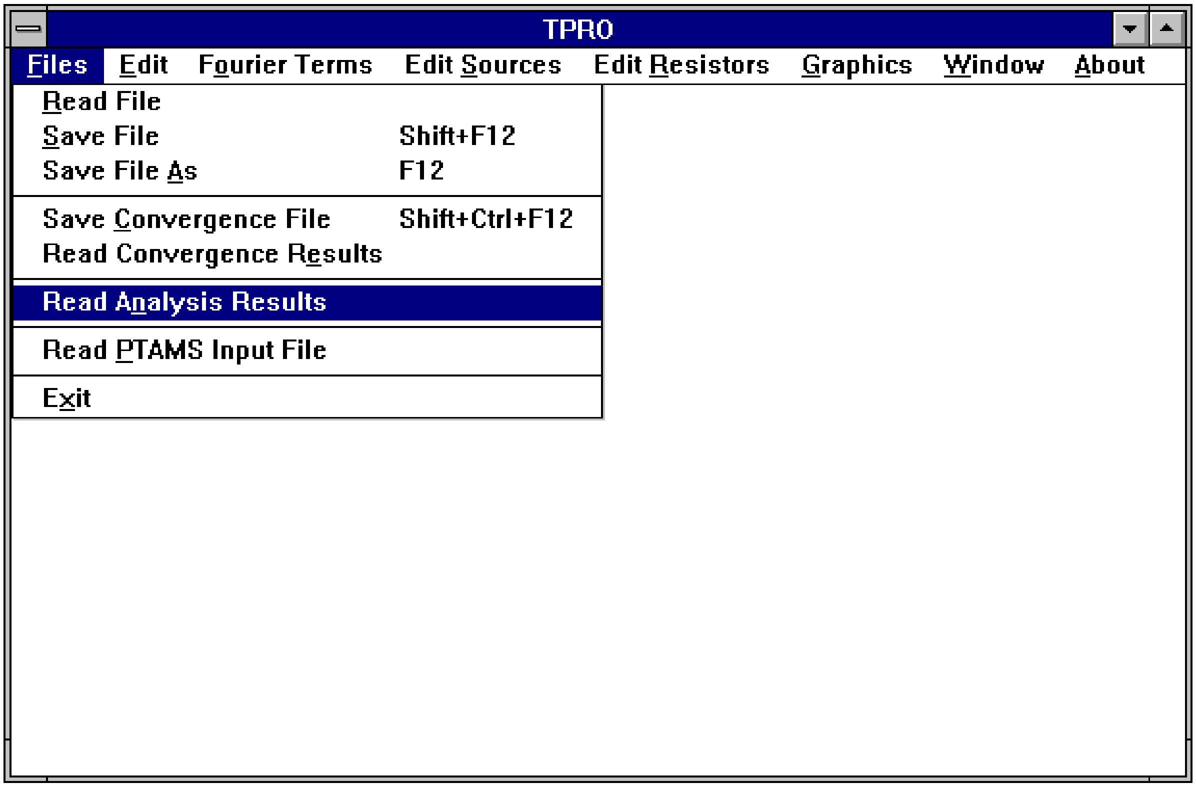

TPRO
Files
Edit
Fourier Terms
Edit Sources
Edit Resistors
Graphics
Window
About
Geometry
Create Grid, Z=0
Create Grid, Z=C4
Convergence Plot
Analysis Plot

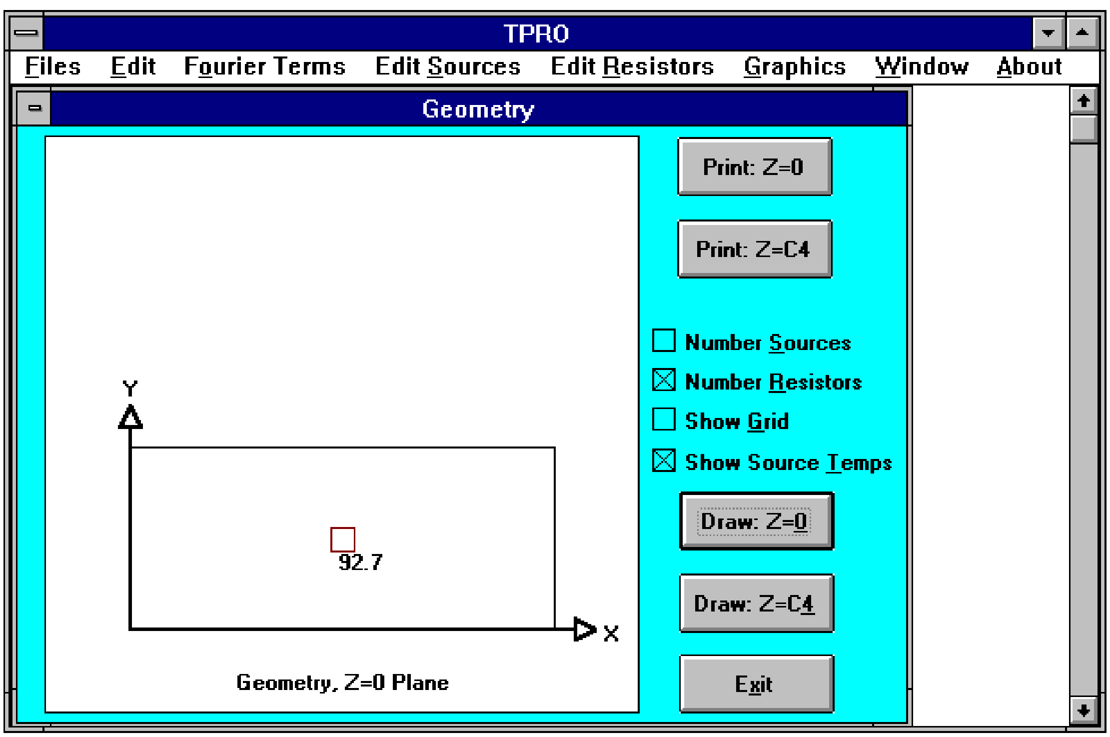

Analysis Results
File
Edit
Graphs
Options
Convergence Analysis
T - TA
5.5
5.0
4.5
4.0
3.5
0
20
40
60
80
100
120
LMAX=MMAX

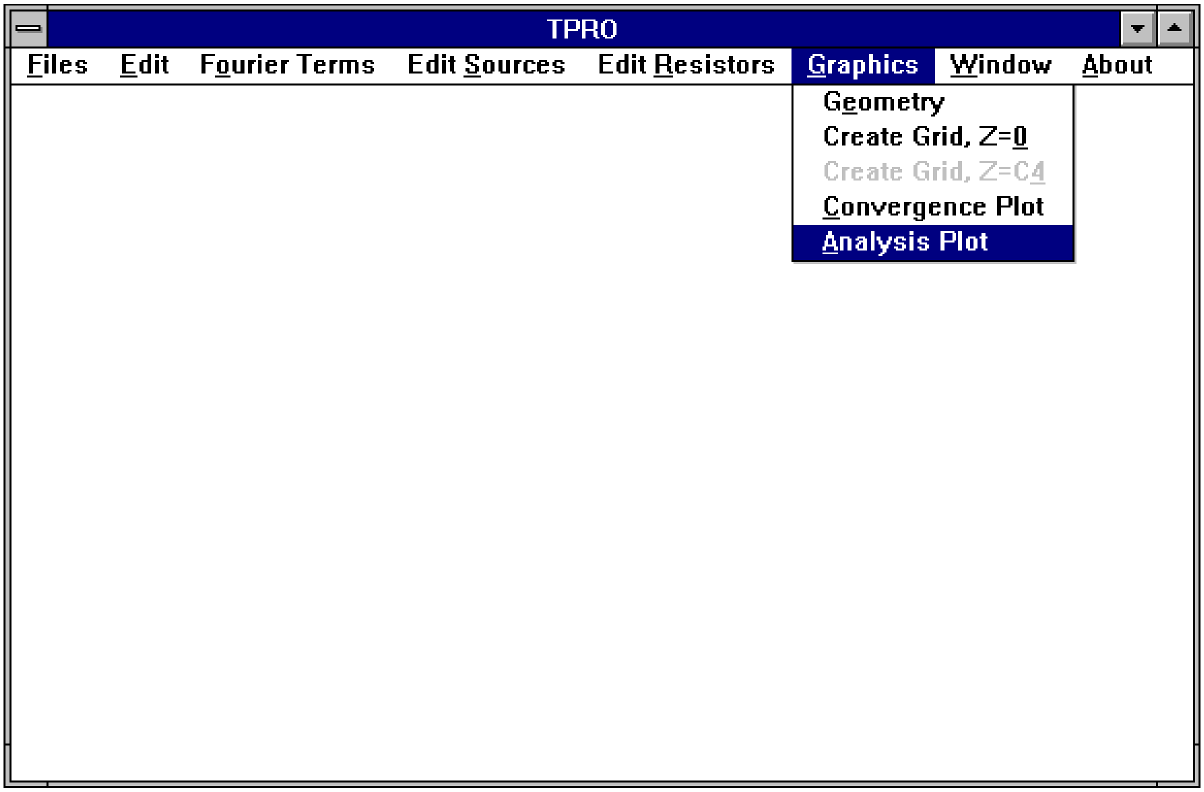

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Number of Fourier Series Terms
X-Direction: 30
Y-Direction: 30
OK
Cancel

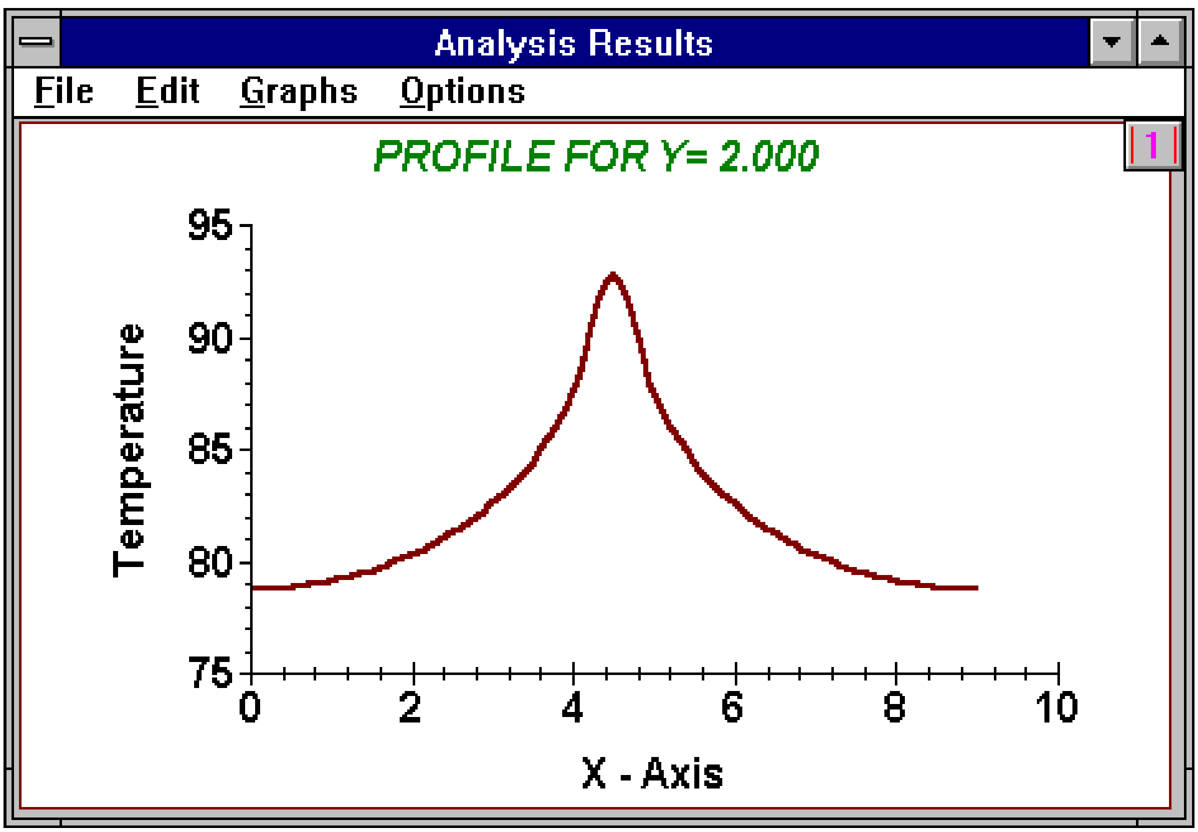

TPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Save File As
File Name:
din
d1out
d2out
d3out
d4out
d5out
din
hotspot
wcmod
Directories:
d:\problems
d:\
problems
OK
Cancel
Drive:
d: [55346248...]

Run TAMS4 by doubling clicking on Icon






TPRO
Files
Edit
Fourier Terms
Edit Sources
Edit Resistors
Graphics
Window
About
Geometry
Create Grid, Z=0
Create Grid, Z=C4
Convergence Plot
Analysis Plot


Analysis Results
File
Edit
Graphs
Options
PROFILE FOR Y= 2.000
1
Temperature
95
90
85
80
75
0
2
4
6
8
10
X - Axis

TAMS Input File Without Gridding

```
TAMS Sample Problem
 1             0
 30            30
9.0000E+00    4.0000E+00    1.5000E-02    1.5000E-02    1.5000E-02    1.5000E-02
3.7000E-03    3.7000E-03    5.5000E+01
4.0000E+00    4.0000E+00    4.0000E+00    4.0000E+00
 1             0             0             0
4.2500E+00    5.0000E-01    1.7500E+00    5.0000E-01    7.0000E+00
  1
```

TAMS Output File Without Gridding

```
*****************************************************************************
****         Electronics Thermal Analysis Package - PC TAMS V4.0        ****
****           (C) Copyright 1996 by Thermal Computations, Inc.         ****
****                        Hillsboro, Oregon                           ****
*****************************************************************************


 TAMS Sample Problem

   THERMAL ANALYSIS FOR NEWTON'S LAW COOLING AT Z=0 AND C4.
   SOURCES AND LEADS AT Z=0,C4.

   SUBSTRATE DIMENSIONS AND PHYSICAL CONSTANTS
      A = .9000E+01  B = .4000E+01
      T1= .1500E-01  T2= .1500E-01  T3= .1500E-01  T4= .1500E-01
      H1= .3700E-02  H2= .3700E-02
      K1= .4000E+01  K2= .4000E+01  K3= .4000E+01  K4= .4000E+01
      TA=  55.0

      NUMBER OF SOURCES=  1  NS1=  1  NS2=  0
      NUMBER OF RES.   =  0  NR1=  0  NR2=  0
      LMAX=  30  MMAX=  30

   SOURCE DATA
   SOURCE NO. I     XS(I)     DXS(I)      YS(I)     DYS(I)     Q(I)
         1         4.2500      .5000     1.7500      .5000    7.000
                                                 TOTAL Q=  7.000

   TEMPERATURES CALCULATED AT SOURCE CENTERS
   SOURCE NO. I       TS(I) WITH SOURCES ONLY      TS(I) WITH SOURCES AND RES.
         1                     92.7
```

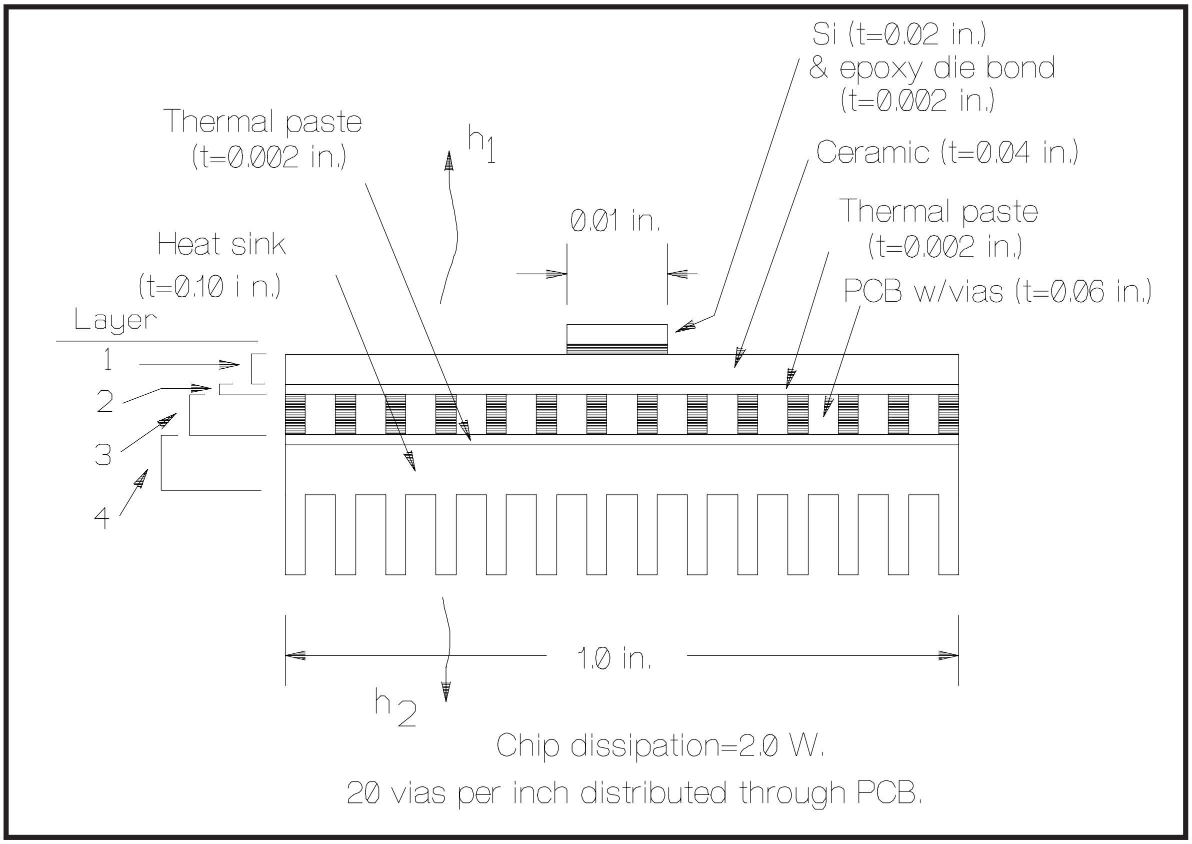
Comments About Results:

1. TNETFA predicted "average" plate temperature = 81.1 deg.

2. TAMS predicted peak temperature = 92.7 deg.

Example

Chip with Heat Sinking



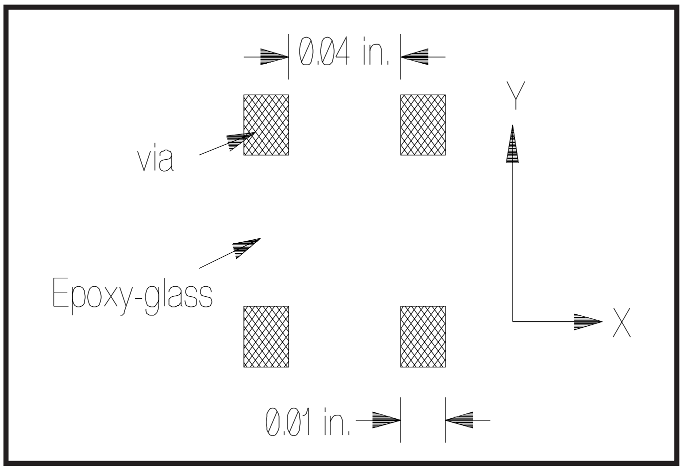
Calculation of Required Input Data

Use anisotropic k, i.e. MODE = 1.

Layer 1: $t = 0.04\ in.,\ k_x = k_y = k_z = 0.7\ W/in.\cdot{}^oC$

Layer 2: $t = 0.002\ in.,\ k_x = k_y = k_z = 0.02\ W/in.\cdot{}^oC$

Layer 3:



R_x, R_y clearly dominated by epoxy - glass

$\therefore\ k_x = k_y \cong 0.007\ W/in.\cdot{}^oC.$

For z - direction, use $R_{board} = 1.7\ {}^oC/W$
from manual calculation earlier in course.

$$R_{board} = \frac{t}{k_z A}, \qquad k_z = \frac{t}{R_{board} A}$$

$$k_z = \frac{0.06\ in.}{\left(1.7\ {}^oC / W\right)\left(1.0\ in.^2\right)} = 0.035\ W/in.\cdot{}^oC$$

Layer 4: combine paste and aluminum.

x, y directions -

$$k_x \frac{wt}{l} = \frac{(kt)_p}{l} + \frac{(kt)_{Al}}{l}$$

$$k_x = \frac{1}{t}\left[(kt)_p + (kt)_{Al}\right]$$

$$= \frac{1}{0.102}\left[(0.02)(0.002) + (5)(0.1)\right]$$

$$= 4.9$$

$$k_y = 4.9$$

z direction -

$$\frac{t}{k_z A} = \left(\frac{t}{k}\right)_p \frac{1}{A} + \left(\frac{t}{k}\right)_{Al} \frac{1}{A}$$

$$\frac{0.102}{k_z} = \frac{0.002}{0.07} + \frac{0.1}{5}$$

$$k_z = 2.1$$

$h_1, \quad h_2 \quad -$

Use $h_1 = 1.0x10^{-10}$

$R_{fins} = \dfrac{1}{h_2 WL}$ will allow for compensation due to fin area if W, L are width, length of substrate.

$$h_2 = \frac{1}{R_{fins}WL} = \frac{1}{(25)(1.0\ in.)(1.0\ in.)}$$

$$= 0.04\ W/in.^2 \cdot {}^oC$$

TAMS INPUT FILE

TCEE DATA SET	DATA					
1	Example - Chip with Heat Sinking					
2	1	1				
3	60	60				
4	1.0	1.0	0.04	0.002	0.06	0.102
5	1.0E-10	0.04	0.0			
6	0.7	0.7	0.7			
6	0.02	0.02	0.02			
6	0.007	0.007	0.035			
6	4.9	4.9	2.1			
7	1	0	0	0		
8	0.45	0.1	0.45	0.1	2.0	
9	1					

Actual TAMS Input File

```
Example - Chip with Heat Sinking
  1       1
 60      60
 .1000E+01   .1000E+01   .4000E-01   .2000E-02   .6000E-01   .1020E+00
 .1000E-09   .4000E-01   .0000E+00
 .7000E+00   .7000E+00   .7000E+00
 .2000E-01   .2000E-01   .2000E-01
 .7000E-02   .7000E-02   .3500E-01
 .4900E+01   .4900E+01   .2100E+01
  1       0       0       0
 .4500E+00   .1000E+00   .4500E+00   .1000E+00   .2000E+01
  1
```

TAMS Output File

```
*****************************************************************************
****         Electronics Thermal Analysis Package - PC TAMS V4.0         ****
****           (C) Copyright 1996 by Thermal Computations, Inc.          ****
****                          Hillsboro, Oregon                          ****
*****************************************************************************


Example - Chip with Heat Sinking

  THERMAL ANALYSIS FOR NEWTON'S LAW COOLING AT Z=0 AND C4.
  SOURCES AND LEADS AT Z=0,C4.

  SUBSTRATE DIMENSIONS AND PHYSICAL CONSTANTS
     A =  .1000E+01  B =  .1000E+01
     T1=  .4000E-01  T2=  .2000E-02  T3=  .6000E-01  T4=  .1020E+00
     H1=  .1000E-09  H2=  .4000E-01
     K1X= .7000E+00  K2X= .2000E-01  K3X= .7000E-02  K4X= .4900E+01
     K1Y= .7000E+00  K2Y= .2000E-01  K3Y= .7000E-02  K4Y= .4900E+01
     K1Z= .7000E+00  K2Z= .2000E-01  K3Z= .3500E-01  K4Z= .2100E+01
     TA=    .0

     NUMBER OF SOURCES=  1  NS1=  1  NS2=  0
     NUMBER OF RES.   =  0  NR1=  0  NR2=  0
     LMAX=  60  MMAX=  60

  SOURCE DATA
  SOURCE NO. I     XS(I)     DXS(I)      YS(I)     DYS(I)     Q(I)
        1          .4500      .1000      .4500      .1000    2.000
                                                   TOTAL Q=  2.000

  TEMPERATURES CALCULATED AT SOURCE CENTERS
  SOURCE NO. I       TS(I) WITH SOURCES ONLY      TS(I) WITH SOURCES AND RES.
        1                     77.5
```

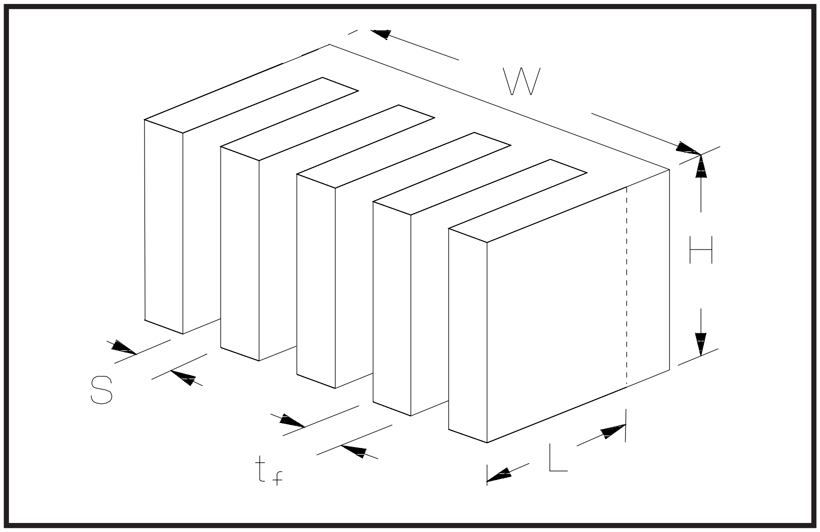
Example

Finned Heat Sink with Three Transistors

Solution Procedure

1. Set up plate (sink without fins) geometry with TPRO.
   a. Write file from TPRO.
2. Read TPRO/TAMS file into NSINK.
   a. Extablish fin details.
   b. Solve problem within NSINK, use "esecute" TAMS h1, h2e option.
   c. Write TAMS file from NSINK.
3. Start up TPRO again.
   a. Read TAMS file (the one written by NSINK).
   b. Write TAMS convergence file.
4. Solve convergence problem with TAMS.
5. Use TPRO to display convergence results.
   a. Adjust LMAX, MMAX in TPRO.
   b. Write TAMS input file from TPRO.
6. Solve problem with TAMS.

Heat Sink Geometry



Uniformly finned on one side. Compute thermal resistance for only finned side convecting.

$N = 20$ fins

$S = 0.34\ in.$

$t_f = 0.08\ in.,\ t_b = 0.25\ in.$

$H = 7.0\ in.$

$W = 8.25\ in.$

$L = 1.50\ in.$

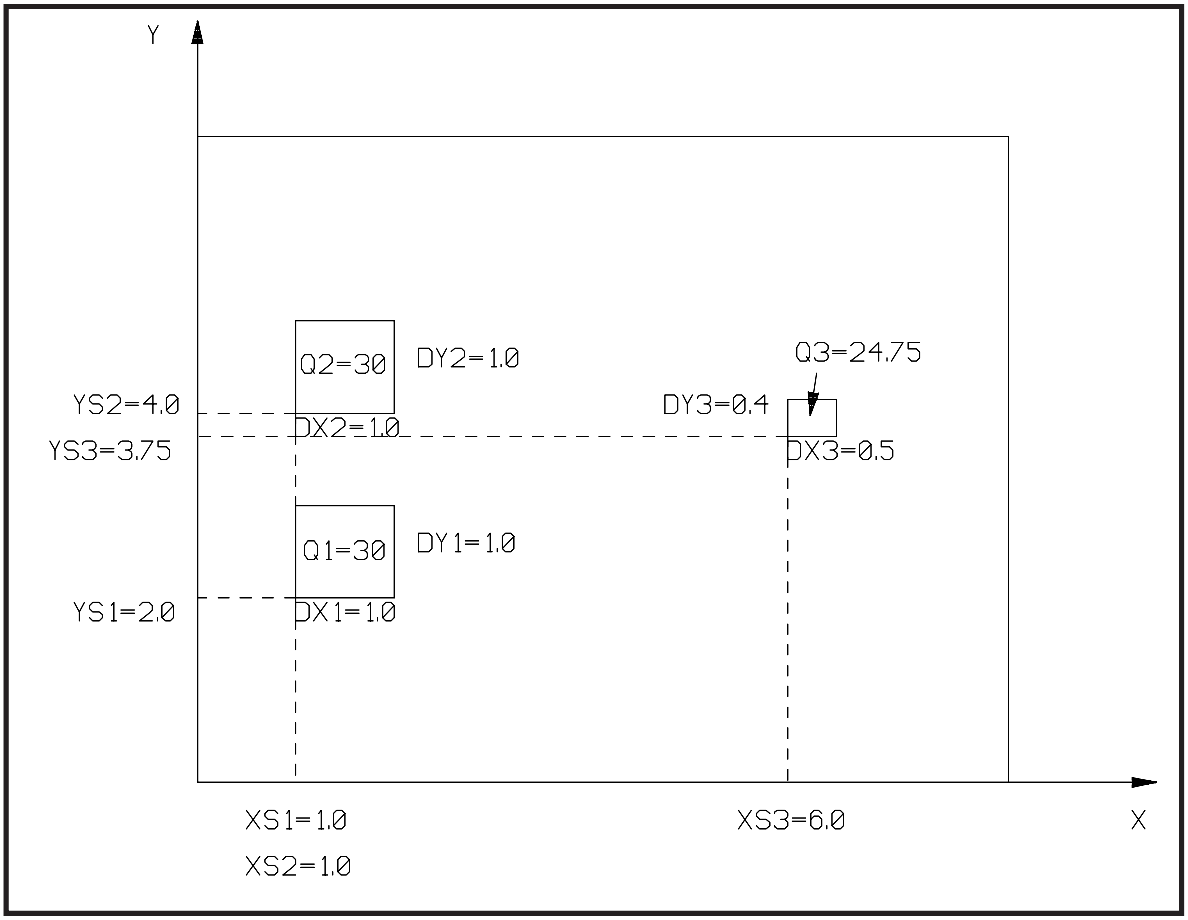
$\varepsilon = 0.8$

$T_A = 20^oC$

$k = 5.0\ W/in.\cdot^oC$

$Q = 2,\ 30\ W$ TO-3 transistors and 1, 24.75 W TO-220.

Transistor Layout for Heat Sink



TAMS Input File Prior To Loading Into NSINK

```
Finned Heat Sink
  1       0
100     100
 .8250E+01   .7000E+01   .6250E-01   .6250E-01   .6250E-01
.6250E-01
 .1000E-09   .1000E-09   .2000E+02
 .5000E+01   .5000E+01   .5000E+01   .5000E+01
  3       0       0       0
 .1000E+01   .1000E+01   .2000E+01   .1000E+01   .3000E+02
 .1000E+01   .1000E+01   .4000E+01   .1000E+01   .3000E+02
 .6000E+01   .5000E+00   .3750E+01   .4000E+00   .2475E+02
  1       2       3
```

Start Up NSINK (type NSINK)

```
                    WELCOME TO ETAP

You are now in the NSINK program which predicts
the thermal characteristics of vertically oriented,
finned heat sinks cooled by natural convection and
radiation.

This Electronics Thermal Analysis Package  (ETAP[1])
based on "Thermal Computations for Electronic
Equipment",  authored  by  Gordon N. Ellison, Robert
E. Krieger Co.,Inc., publishers, is  supplied as-is
and without warranty or representation of any kind.

The author  makes  no  representations  respecting
the programs and related  material and  expressly
disclaims  any  liability for damages from the use
of the programs  or  related material, or any part
thereof. The user must verify his/her own results.

[1] ETAP - Version 3.2 (C) Copyright 1986-1993 by
Thermal Computations, Inc., Hillsboro, Oregon.

             Press any key to continue.
```

```
                 Select from NSINK Menu ------------------->

INIT             Initialize NSINK dimensions, data ---------
E                Edit heat sink dimensions, data -----------
LIST             List heat sink dimensions, data -----------
LISTD            List current directory information --------
LISTF            List files for any directory --------------
READ             Read NSINK data file from disk ------------
READT            Read TAMS input file from disk ------------
WRITE            Write NSINK data file to disk -------------
WRITET           Write TAMS input file to disk -------------
EX               Execute - compute sink temp., resistance -
EXHT             Execute - compute TAMS H1,H2E -------------
EXCN             Execute - compute temp., conductance ARRAY
                 for TNETFA
HELP             Print this list again ---------------------
QUIT             Exit NSINK (return to DOS) ----------------
```

```
  NSINK
  Menu

A. INIT
B. E.....
C. LIST
D. LISTD
E. LISTF
F. READ
G. READT
H. WRITE
I. WRITET
J. EX
K. EXHT
L. EXCN

?  HELP
\  QUIT
```

Select READT (and provide proper TAMS file name).

```
          Select from EDIT Menu ---------------------->

H         Sink height ---------------------------------
W         Sink width ----------------------------------
L         Fin length (perp. to base) ------------------
TF        Fin thickness -------------------------------
S         Fin spacing ---------------------------------
NF        Number of fins ------------------------------
E1        Emissivity, non-finned side -----------------
E2        Emissivity, finned side ---------------------
QSINK     Total sink dissipation ----------------------
KS        Heat sink (fins) thermal conductivity -------
TA1       Ambient temperature, non-finned side --------
TA2       Ambient temperature, finned side ------------
C         Heat transfer configuration: ----------------
             1 for heat transfer from fins only to TA2;
             2 for heat transfer from fins to TA2 &
             flat side to TA1
LIST      List dimensions and thermal parameters ------
HELP      Display this list ---------------------------
EXIT      Return to main option control ---------------
```

```
   E
  Menu

A. H
B. W
C. L
D. TF
E. S
F. NF
G. E1
H. E2
I. QSINK
J. KS
K. TA1
L. TA2
M. C



N. LIST
?  HELP
\  EXIT
```

```
ENTER L:1.5
ENTER TF:0.08
ENTER NF:20
ENTER E2:0.8
ENTER KS:5.0
ENTER "1" OR "2" FOR C:1
```

```
    E
   Menu

A. H
B. W
C. L
D. TF
E. S
F. NF
G. E1
H. E2
I. QSINK
J. KS
K. TA1
L. TA2
M. C


N. LIST
?  HELP
\  EXIT
```

```
        VARIABLE LIST

   VARIABLE           VALUE

     H               7.0000
     W               8.2500
     L               1.5000
     TF               .0800
     S                .3500
     NF                  20
     E1                 .10
     E2                 .80
     QSINK            84.80
     KS                5.00
     TA2              20.00
     HEAT TRANSFER FROM FINS ONLY
```

```
    E
   Menu

A. H
B. W
C. L
D. TF
E. S
F. NF
G. E1
H. E2
I. QSINK
J. KS
K. TA1
L. TA2
M. C


N. LIST
?  HELP
\  EXIT
```

Use EXHT to solve heat sink problem,
answer Y to update "internal" TAMS file -

```
                    VARIABLE          VALUE

                      H                7.0000
                      W                8.2500
                      L                1.5000
                      TF                .0800
                      S                 .3500
                      NF                   20
                      E1                  .10
                      E2                  .80
                      QSINK             84.80
                      KS                 5.00
                      TA2               20.00
                      HEAT TRANSFER FROM FINS ONLY

QSINK= 84.8: TS= 66.6, RSINK=   .55, H2E= .3152E-01

DO YOU WANT THE TAMS H1,H2 UPDATED?y

   H1 SET AT  .1000E-09
   H2 SET AT  .3152E-01
   TA SET AT  .2000E+02
```

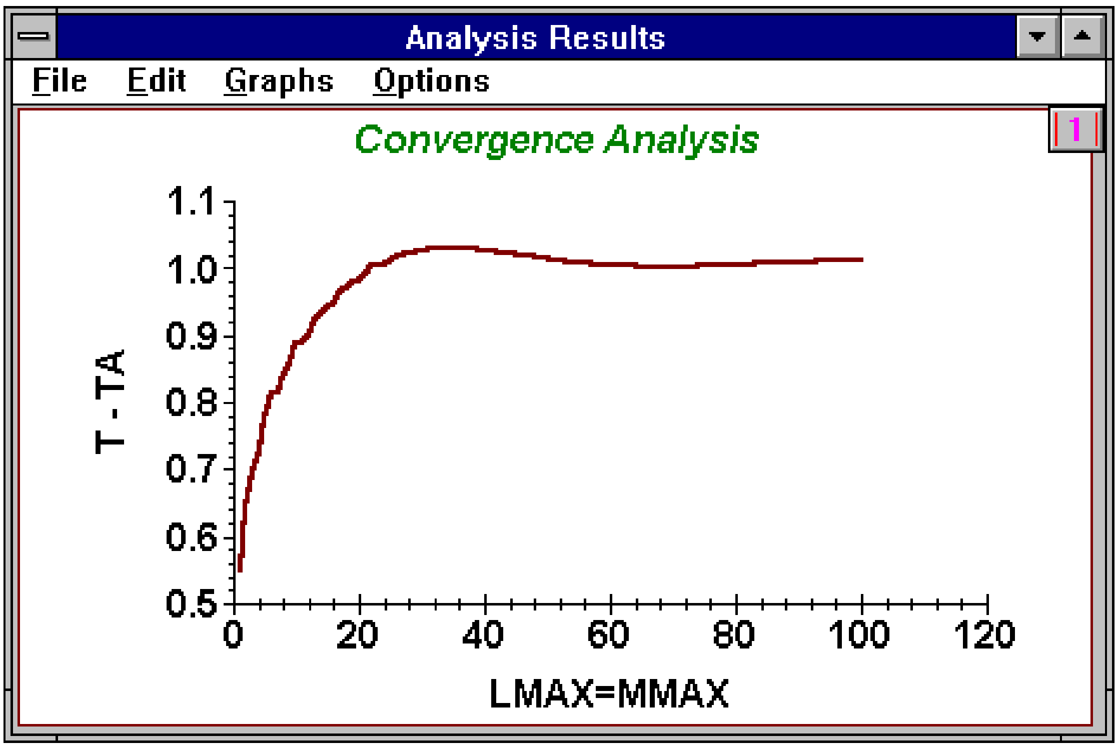
```
 NSINK
 Menu

A. INIT
B. E.....
C. LIST
D. LISTD
E. LISTF
F. READ
G. READT
H. WRITE
I. WRITET
J. EX
K. EXHT
L. EXCN

?  HELP
\  QUIT
```

Use WRITET to save as TAMS input file -

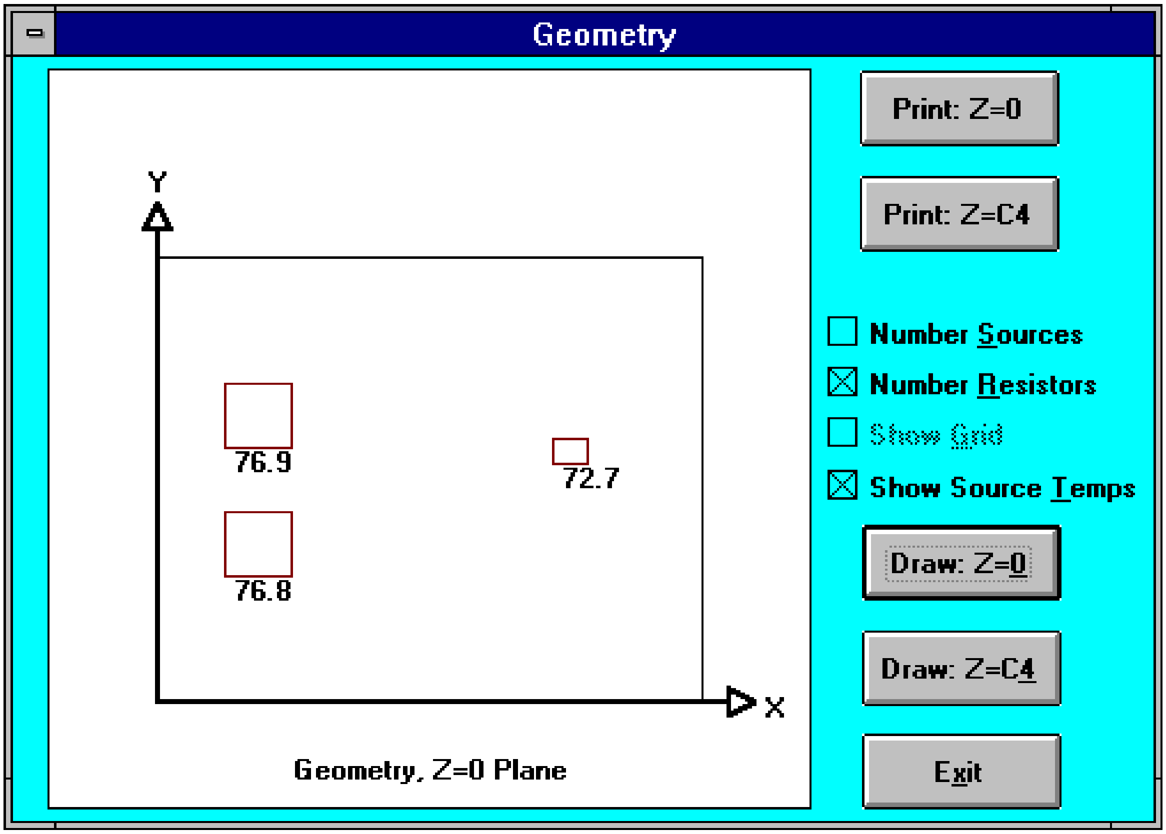
```
ENTER FILE NAME:din
```

```
 NSINK
 Menu

A. INIT
B. E.....
C. LIST
D. LISTD
E. LISTF
F. READ
G. READT
H. WRITE
I. WRITET
J. EX
K. EXHT
L. EXCN

?  HELP
\  QUIT
```

1. Read TAMS file (previously written to disk from NSINK) into TPRO.
2. Write TPRO convergence file to disk.
3. Run TAMS to solve convergence problem.
4. Use TPRO "Read Convergence Results" option to read convergence results and complete TAMS file.



5. Enter TPRO and change LMAX and MMAX to 30 and 30.
6. Save file as DIN file.
7. Run TAMS.
8. Read "Analysis Results" in TPRO and display geometry with source temperatures displayed.


Geometry
Print: Z=0
Print: Z=C4
Y
76.9
72.7
76.8
X
Number Sources
Number Resistors
Show Grid
Show Source Temps
Draw: Z=0
Draw: Z=C4
Exit
Geometry, Z=0 Plane

Actual TAMS Input File (DIN)

```
Finned Heat Sink
 1             0
 30            30
8.2500E+00    7.0000E+00    6.2500E-02    6.2500E-02    6.2500E-02    6.2500E-02
1.0000E-10    3.1520E-02    2.0000E+01
5.0000E+00    5.0000E+00    5.0000E+00    5.0000E+00
 3             0             0             0
1.0000E+00    1.0000E+00    2.0000E+00    1.0000E+00    3.0000E+01
1.0000E+00    1.0000E+00    4.0000E+00    1.0000E+00    3.0000E+01
6.0000E+00    5.0000E-01    3.7500E+00    4.0000E-01    2.4750E+01
  1  2  3
```

TAMS Output File (D1OUT)

```
******************************************************************************
****         Electronics Thermal Analysis Package - PC TAMS V4.0         ****
****           (C) Copyright 1996 by Thermal Computations, Inc.          ****
****                         Hillsboro, Oregon                           ****
******************************************************************************

Finned Heat Sink

  THERMAL ANALYSIS FOR NEWTON’S LAW COOLING AT Z=0 AND C4.
  SOURCES AND LEADS AT Z=0,C4.

  SUBSTRATE DIMENSIONS AND PHYSICAL CONSTANTS
     A = .8250E+01  B = .7000E+01
     T1= .6250E-01  T2= .6250E-01  T3= .6250E-01  T4= .6250E-01
     H1= .1000E-09  H2= .3152E-01
     K1= .5000E+01  K2= .5000E+01  K3= .5000E+01  K4= .5000E+01
     TA=  20.0

     NUMBER OF SOURCES=  3  NS1=  3  NS2=  0
     NUMBER OF RES.   =  0  NR1=  0  NR2=  0
     LMAX=  30  MMAX=  30

  SOURCE DATA
  SOURCE NO. I     XS(I)     DXS(I)      YS(I)     DYS(I)     Q(I)
        1         1.0000     1.0000     2.0000     1.0000   30.000
        2         1.0000     1.0000     4.0000     1.0000   30.000
        3         6.0000      .5000     3.7500      .4000   24.750
                                                   TOTAL Q= 84.750

  TEMPERATURES CALCULATED AT SOURCE CENTERS
  SOURCE NO. I       TS(I) WITH SOURCES ONLY      TS(I) WITH SOURCES AND RES.
        1                     76.8
        2                     76.9
        3                     72.7
```

Note results:

NSINK predicted an average
heat sink temperature of 66.6 ^{o}C.

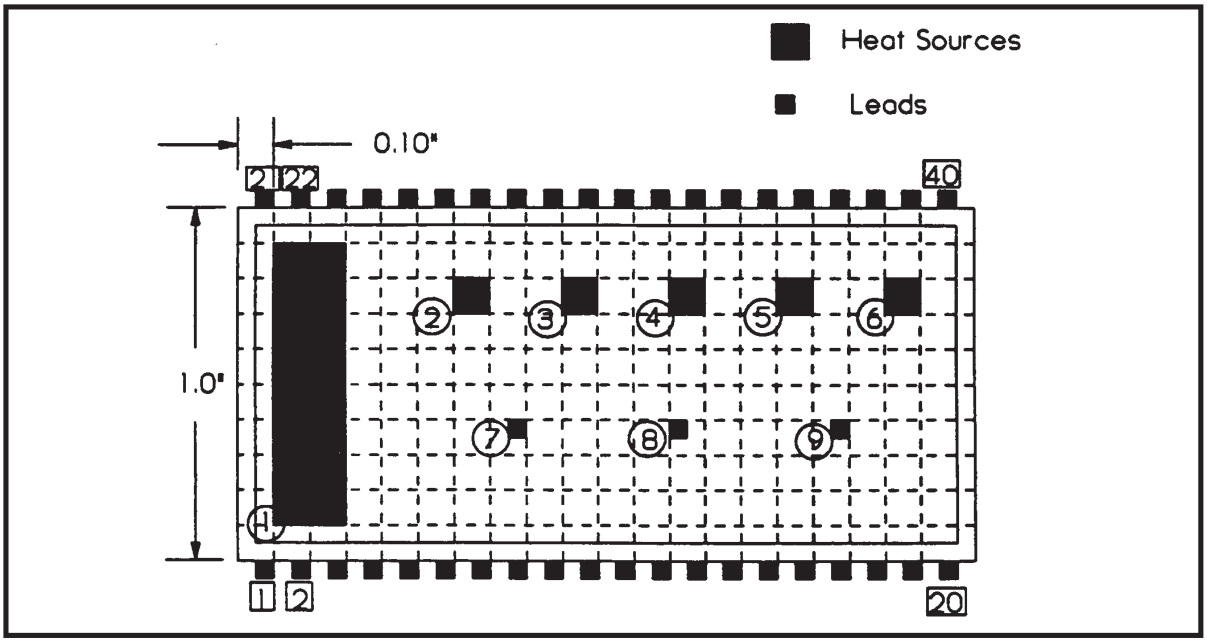
TAMS predicted transistor cases at

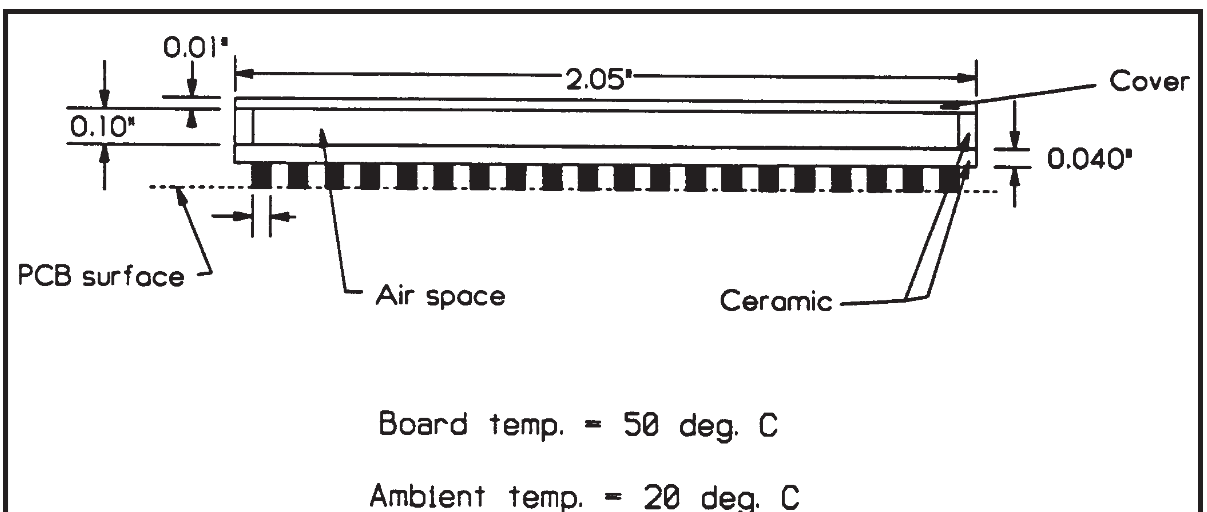
Q1: 76.8 ^{o}C
Q2: 76.9 ^{o}C
Q3: 72.6 ^{o}C

Example

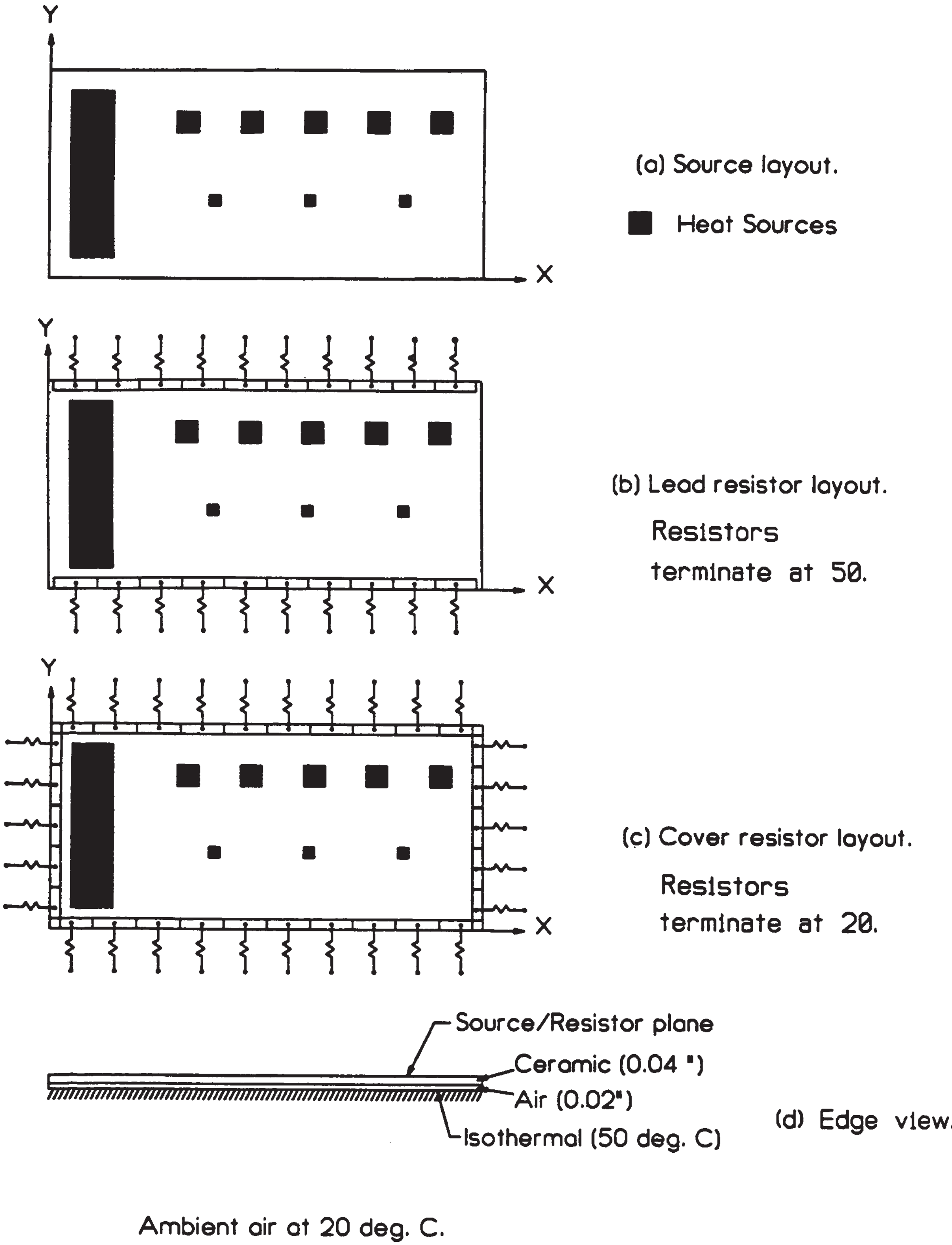
TAMS Model Result Compared with FEM
(FEM includes substrate-cover air gap,
which is ignored by TAMS)

Hybrid Geometry -





TAMS Model -



Y
X
(a) Source layout.
Heat Sources
Y
X
(b) Lead resistor layout.
Resistors
terminate at 50.
Y
X
(c) Cover resistor layout.
Resistors
terminate at 20.
Source/Resistor plane
Ceramic (0.04 ")
Air (0.02")
Isothermal (50 deg. C)
(d) Edge view.
Ambient air at 20 deg. C.

Thermal Parameters -

Chips -

Chip No.	Dissipation (W)
1 (resistor)	1.0
2	1.0
3	1.0
4	2.0
5	2.0
6	3.0
7	1.0
8	2.0
9	1.0

Conductivities -

Material	Conductivity ($W/in.\ ^oC$)
Ceramic substrate	1.0
Kovar cover	0.5
Copper leads	10.0
Substrate-board air	6.95×10^{-4}

Heat transfer coefficient -

$h = 0.01\ W/in.^2\ ^oC$, approximately appropriate for a low, forced air velocity.

Lead Resistances -

Forty leads modeled as twenty thermal resistances.

$$R_{Total\ Leads} = \frac{1}{40} r_l \text{ where } r_l = \text{Resistance of one lead}$$

$$R_L = 20\ R_{Total\ Leads} = \frac{20}{40} r_l = \frac{1}{2} r_l$$

$$= \left(\frac{1}{2}\right) \frac{l}{kA_k} = \left(\frac{1}{2}\right) \frac{0.02 in.}{\left(10.0\ W / in.\cdot {}^oC\right)(0.05\ in.)(0.01\ in.)} = 2\ {}^oC/W$$

Cover -

Modeled by attaching thirty resistors from substrate edge to ambient at 20 ^{o}C.

Model based on conversion from conduction-distributed convection in a disk (TCEE, section 4.6).

Model requires R_{SQ}, R_S:

$$R_{SQ} = \frac{1}{kt} = \frac{1}{\left(0.5\ W / in.\cdot {}^oC\right)(0.01\ in.)}$$

$$= 200\ {}^oC/W$$

$$R_S = \frac{1}{hA_s}$$

$$= \frac{1}{\left(0.01\ W/in.^2\cdot {}^oC\right)(2.05\ in.\ x\ 1.0\ in.)}$$

$$= 48.78\ {}^oC/W$$

From TCEE, Fig. 4-14

$$R_{\mathrm{cov}} = 1.15R_s = 1.15(48.78) = 56\ {}^oC/W$$

Each of 30 resistors is

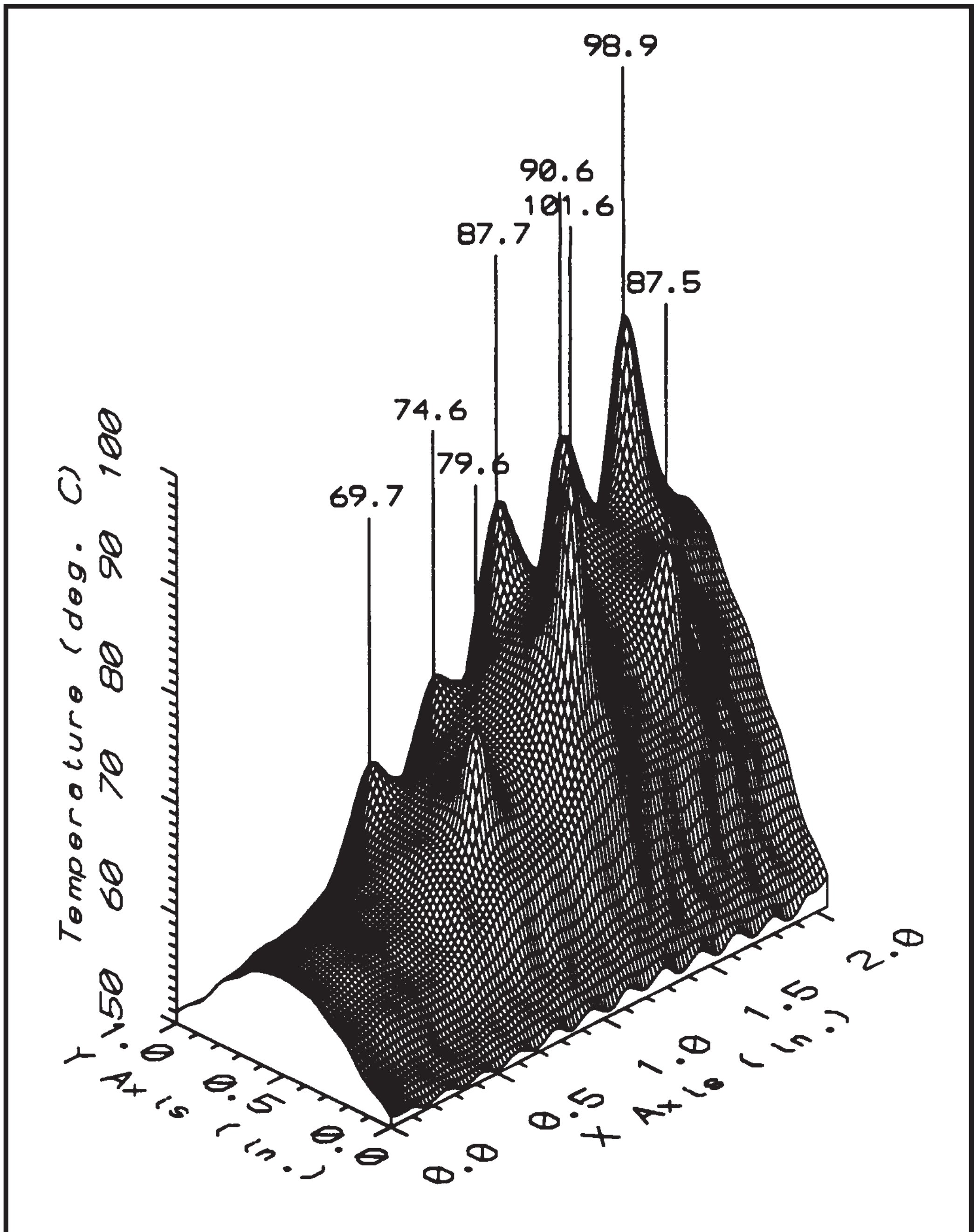
$$R_c = 30(56) = 1680\ {}^oC/W$$

Comments on TAMS Model -

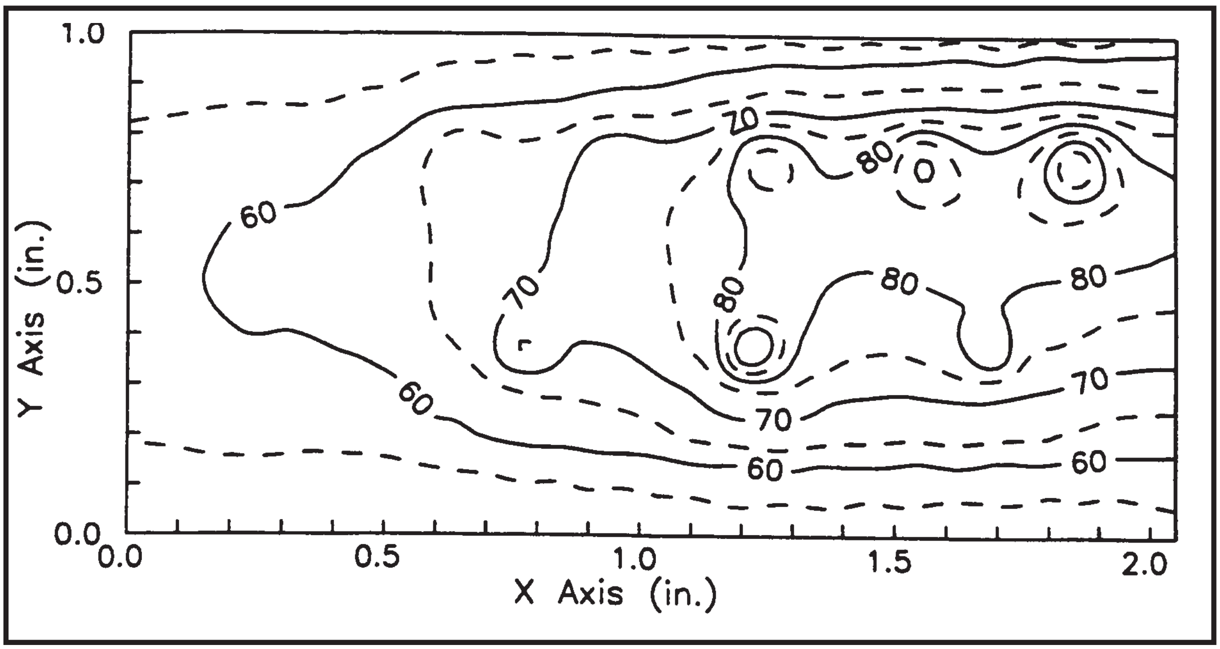
Two layer model

Layer 1: ceramic
Layer 2: air gap

TAMS Temperature Map of Source Plane -



TAMS Temperature Contours of Source Plane -



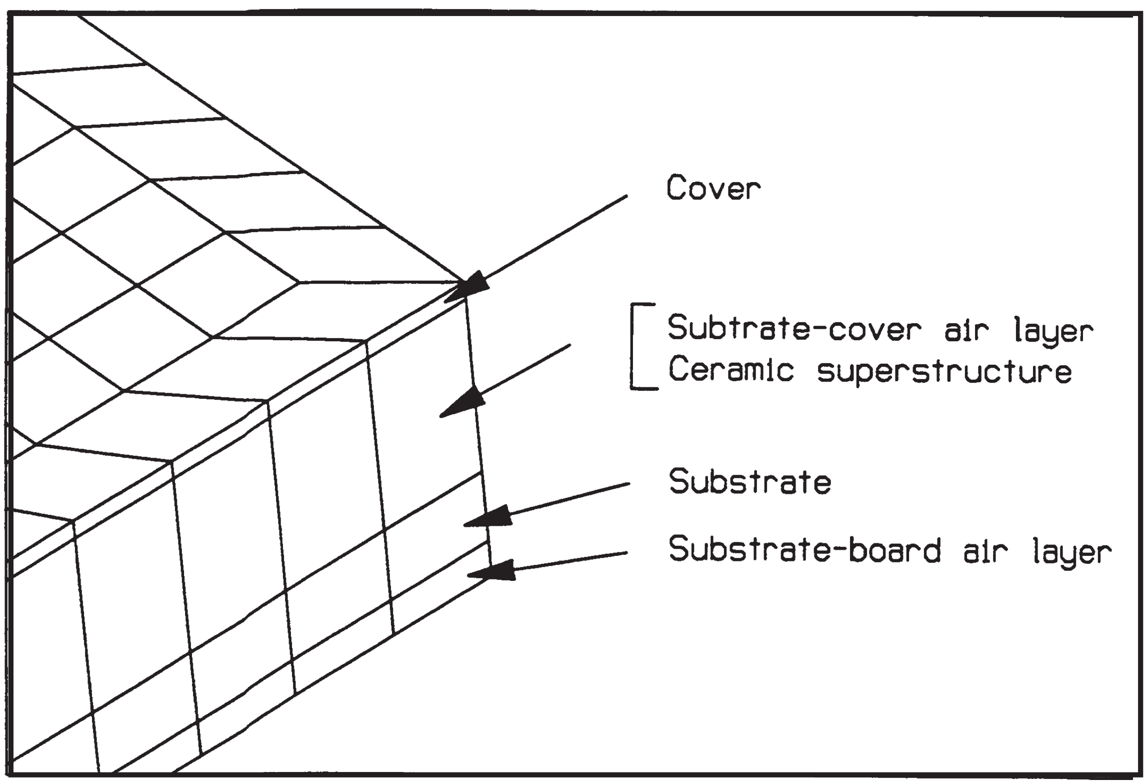
Finite Element Model -

Mesh resulted in 3264, 8-node brick elements, 4370 nodes.

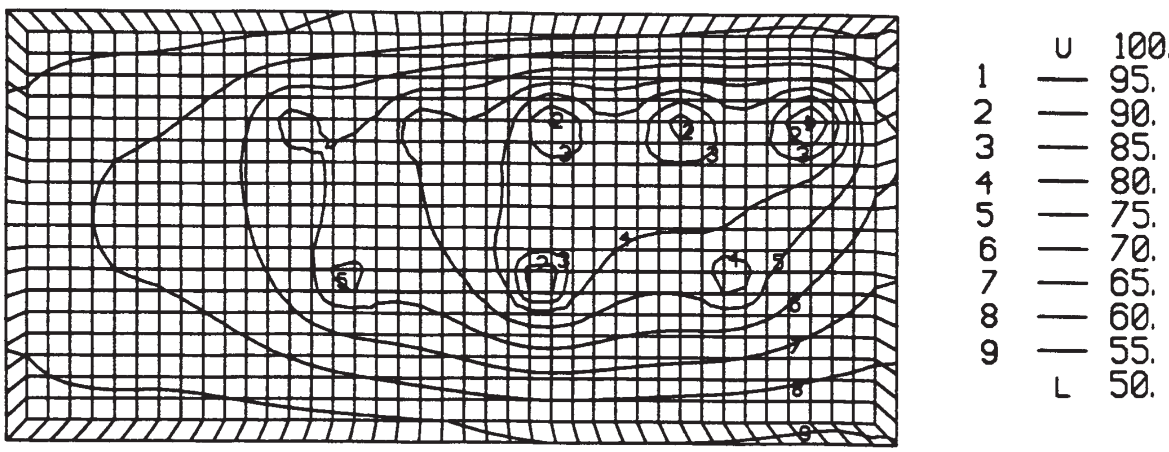
Kovar cover: 1 layer of elements.
Substrate to cover air gap: 1 layer of elements.
Substrate: 1 layer of elements.
Substrate to board air layer: 1 layer of elements.

Program used: ALGOR SuperSAP.

Portion of FEM model illustrating element thicknesses -



FEM Contours in Source Plane -



Peak Source Temperatures from TAMS and FEM Models -

Source	TAMS	FEM
1	60.5	60.0
2	69.7	70.0
3	74.6	77.5
4	87.7	90.7
5	90.6	92.5
6	98.9	96.8
7	79.6	76.5
8	101.6	92.4
9	87.5	82.2

TAMS Input File (DIN)

This file must be run with the MS Windows version of TAMS due to # of res.

```
 2    0
100   100
 .2050E+01  .1000E+01  .4000E-01  .6667E-03  .6667E-03  .6667E-03
 .1000E-09  .1000E-09  .5000E+02
 .1000E+01  .6950E-03  .6950E-03  .6950E-03
 9    0   50    0
 .1000E+00  .2000E+00  .1000E+00  .8000E+00  .1000E+01
 .6000E+00  .1000E+00  .7000E+00  .1000E+00  .1000E+01
 .9000E+00  .1000E+00  .7000E+00  .1000E+00  .1000E+01
 .1200E+01  .1000E+00  .7000E+00  .1000E+00  .2000E+01
 .1500E+01  .1000E+00  .7000E+00  .1000E+00  .2000E+01
 .1800E+01  .1000E+00  .7000E+00  .1000E+00  .3000E+01
 .7500E+00  .5000E-01  .3500E+00  .5000E-01  .1000E+01
 .1200E+01  .5000E-01  .3500E+00  .5000E-01  .2000E+01
 .1650E+01  .5000E-01  .3500E+00  .5000E-01  .1000E+01
 1    2    3    4    5    6    7    8
 9
 .2500E-01  .2000E+00  .0000E+00  .5000E-01  .2000E+01  .5000E+02
 .2250E+00  .2000E+00  .0000E+00  .5000E-01  .2000E+01  .5000E+02
 .4250E+00  .2000E+00  .0000E+00  .5000E-01  .2000E+01  .5000E+02
 .6250E+00  .2000E+00  .0000E+00  .5000E-01  .2000E+01  .5000E+02
 .8250E+00  .2000E+00  .0000E+00  .5000E-01  .2000E+01  .5000E+02
 .1025E+01  .2000E+00  .0000E+00  .5000E-01  .2000E+01  .5000E+02
 .1225E+01  .2000E+00  .0000E+00  .5000E-01  .2000E+01  .5000E+02
 .1425E+01  .2000E+00  .0000E+00  .5000E-01  .2000E+01  .5000E+02
 .1625E+01  .2000E+00  .0000E+00  .5000E-01  .2000E+01  .5000E+02
 .1825E+01  .2000E+00  .0000E+00  .5000E-01  .2000E+01  .5000E+02
 .2500E-01  .2000E+00  .9500E+00  .5000E-01  .2000E+01  .5000E+02
 .2250E+00  .2000E+00  .9500E+00  .5000E-01  .2000E+01  .5000E+02
 .4250E+00  .2000E+00  .9500E+00  .5000E-01  .2000E+01  .5000E+02
 .6250E+00  .2000E+00  .9500E+00  .5000E-01  .2000E+01  .5000E+02
 .8250E+00  .2000E+00  .9500E+00  .5000E-01  .2000E+01  .5000E+02
 .1025E+01  .2000E+00  .9500E+00  .5000E-01  .2000E+01  .5000E+02
 .1225E+01  .2000E+00  .9500E+00  .5000E-01  .2000E+01  .5000E+02
 .1425E+01  .2000E+00  .9500E+00  .5000E-01  .2000E+01  .5000E+02
 .1625E+01  .2000E+00  .9500E+00  .5000E-01  .2000E+01  .5000E+02
 .1825E+01  .2000E+00  .9500E+00  .5000E-01  .2000E+01  .5000E+02
 .0000E+00  .2050E+00  .0000E+00  .5000E-01  .1680E+04  .2000E+02
 .2050E+00  .2050E+00  .0000E+00  .5000E-01  .1680E+04  .2000E+02
 .4100E+00  .2050E+00  .0000E+00  .5000E-01  .1680E+04  .2000E+02
 .6150E+00  .2050E+00  .0000E+00  .5000E-01  .1680E+04  .2000E+02
 .8200E+00  .2050E+00  .0000E+00  .5000E-01  .1680E+04  .2000E+02
 .1025E+01  .2050E+00  .0000E+00  .5000E-01  .1680E+04  .2000E+02
 .1230E+01  .2050E+00  .0000E+00  .5000E-01  .1680E+04  .2000E+02
 .1435E+01  .2050E+00  .0000E+00  .5000E-01  .1680E+04  .2000E+02
```

.1640E+01	.2050E+00	.0000E+00	.5000E-01	.1680E+04	.2000E+02
.1845E+01	.2050E+00	.0000E+00	.5000E-01	.1680E+04	.2000E+02
.0000E+00	.2050E+00	.9500E+00	.5000E-01	.1680E+04	.2000E+02
.2050E+00	.2050E+00	.9500E+00	.5000E-01	.1680E+04	.2000E+02
.4100E+00	.2050E+00	.9500E+00	.5000E-01	.1680E+04	.2000E+02
.6150E+00	.2050E+00	.9500E+00	.5000E-01	.1680E+04	.2000E+02
.8200E+00	.2050E+00	.9500E+00	.5000E-01	.1680E+04	.2000E+02
.1025E+01	.2050E+00	.9500E+00	.5000E-01	.1680E+04	.2000E+02
.1230E+01	.2050E+00	.9500E+00	.5000E-01	.1680E+04	.2000E+02
.1435E+01	.2050E+00	.9500E+00	.5000E-01	.1680E+04	.2000E+02
.1640E+01	.2050E+00	.9500E+00	.5000E-01	.1680E+04	.2000E+02
.1845E+01	.2050E+00	.9500E+00	.5000E-01	.1680E+04	.2000E+02
.0000E+00	.5000E-01	.0000E+00	.2000E+00	.1680E+04	.2000E+02
.0000E+00	.5000E-01	.2000E+00	.2000E+00	.1680E+04	.2000E+02
.0000E+00	.5000E-01	.4000E+00	.2000E+00	.1680E+04	.2000E+02
.0000E+00	.5000E-01	.6000E+00	.2000E+00	.1680E+04	.2000E+02
.0000E+00	.5000E-01	.8000E+00	.2000E+00	.1680E+04	.2000E+02
.1000E+01	.5000E-01	.0000E+00	.2000E+00	.1680E+04	.2000E+02
.1000E+01	.5000E-01	.2000E+00	.2000E+00	.1680E+04	.2000E+02
.1000E+01	.5000E-01	.4000E+00	.2000E+00	.1680E+04	.2000E+02
.1000E+01	.5000E-01	.6000E+00	.2000E+00	.1680E+04	.2000E+02
.1000E+01	.5000E-01	.8000E+00	.2000E+00	.1680E+04	.2000E+02

PTAMS - A <u>P</u>cb <u>T</u>hermal <u>A</u>nalyzer for <u>M</u>ultilayer <u>S</u>tructures

Geometry is Very Similar to TAMS

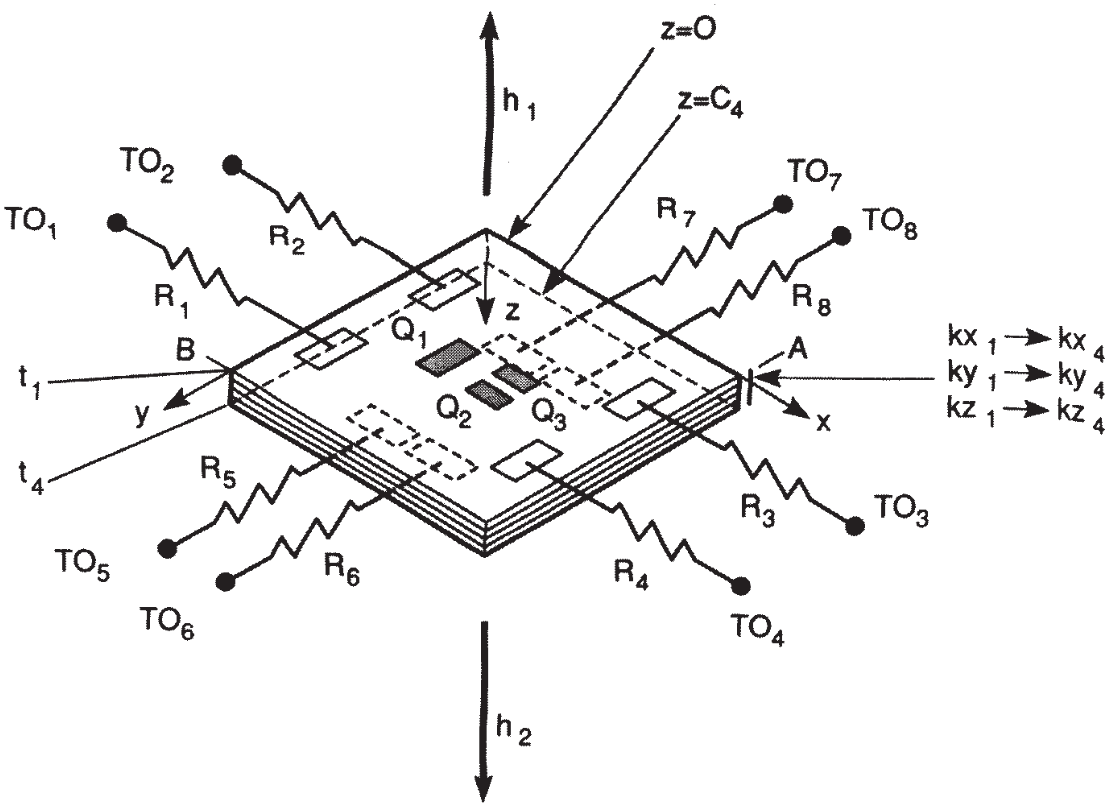
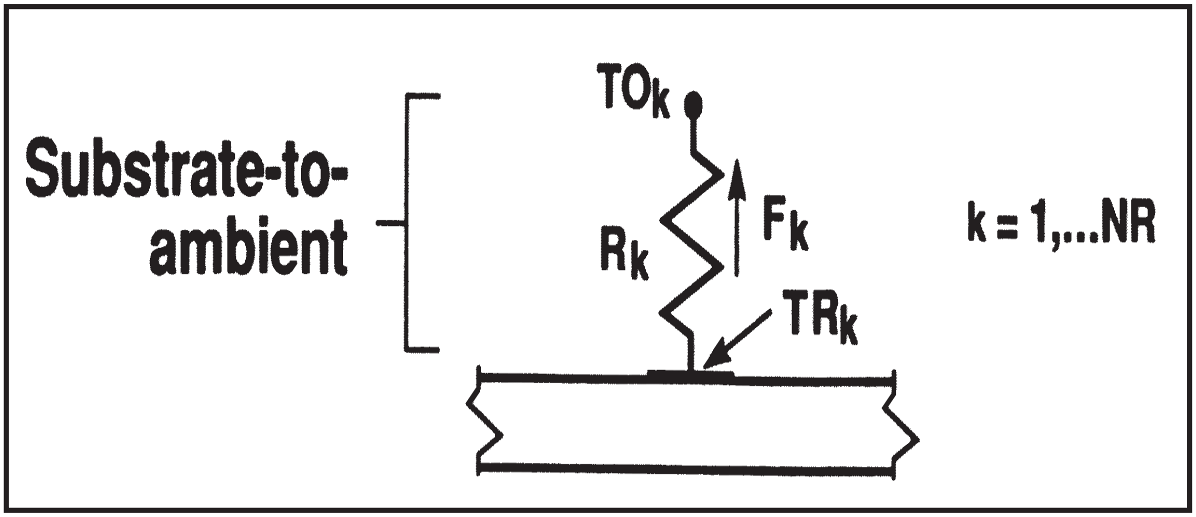
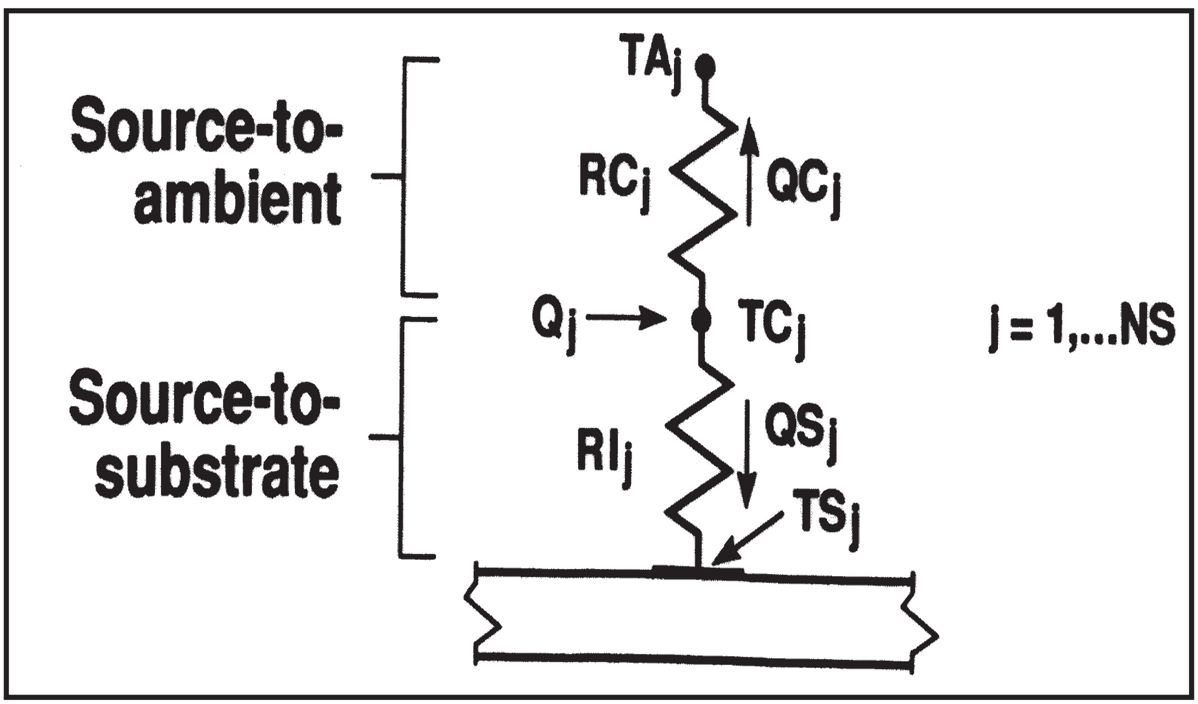


Figure 3.13 Geometry and revelant heat transfer quantities for heat sources and lumped parameter thermal resistances on a multilayer substrate [Ellison (1984)] [Reprinted with premission of the International Society for Hybrid Microelectronics, Reston, VA].

....Except for Source Input (ITEST is also restricted to a value of 1 or 11)





PPRO and PTAMS are nearly identical to TPRO and TAMS except:

TPRO/TAMS (VWin5.0)	PPRO/PTAMS (VWin5.0)
1. ITEST=1,2,...6 (for convergence analysis, add 10 to ITEST.	1. ITEST=1 or 11 only, i.e. always surface flux with finite h_1, h_2 .
2. LMAX, MMAX virtually unlimited.	2. LMAX, MMAX <,= 200.
3. h_1, h_2 used internally in TAMS as-is.	3. h_1, h_2 dimenished internally in PTAMS according to source size coverage, i.e. $\Delta x * \Delta y$.
4. Source input requires $x, \Delta x, y, \Delta y, Q$.	4. Source input requires $x, \Delta x, y, \Delta y, RI, RC, Q, TAL$.
5. Virtually unlimited number of sources.	5. Maximum of 100 sources on each side of board.
6. Virtually unlimited number of resistors	6. Maximum of 100 resistors on each side of board.
7. Gridding/profiling option.	7. No gridding/profiling option, but TPRO/TAMS can be used.

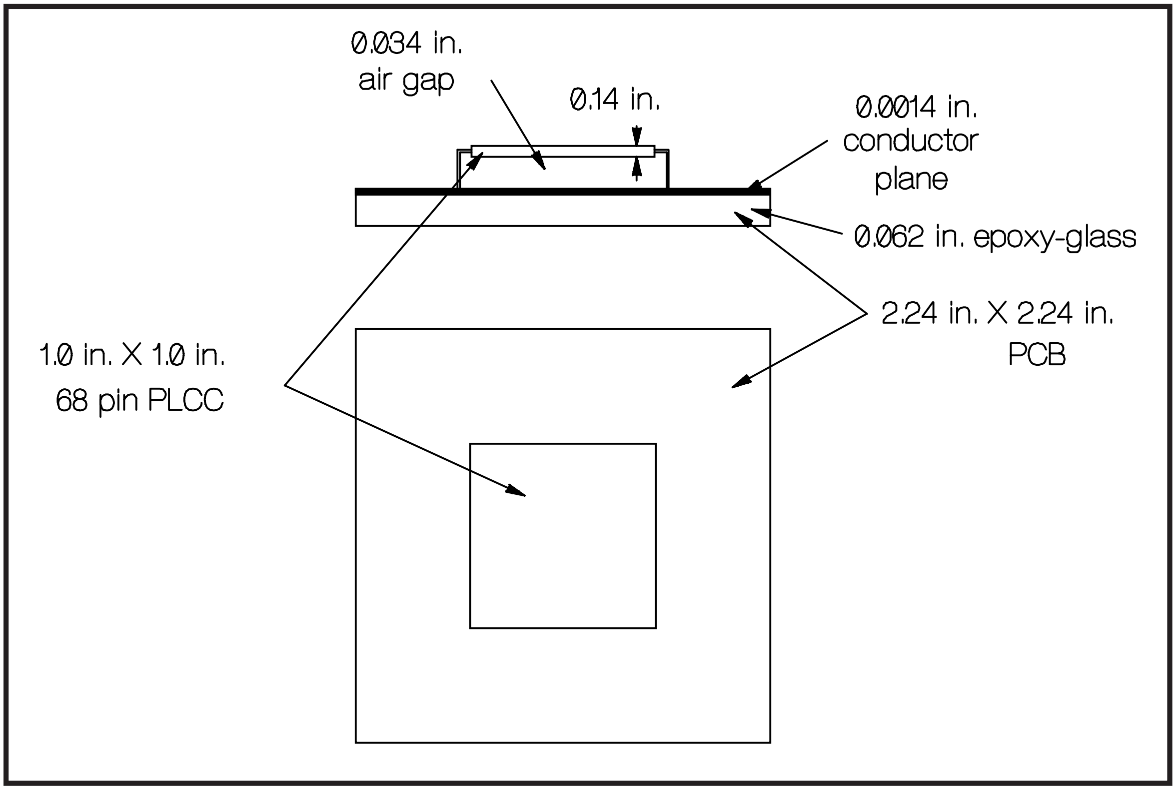
Example

PTAMS Model of a Single

68 Pin PLCC on a PCB.

Calculations Compared with Vendor Spec.

Geometry of Package Externals and Board -



Conductor plane covers about 50% of PCB and is solder coated.

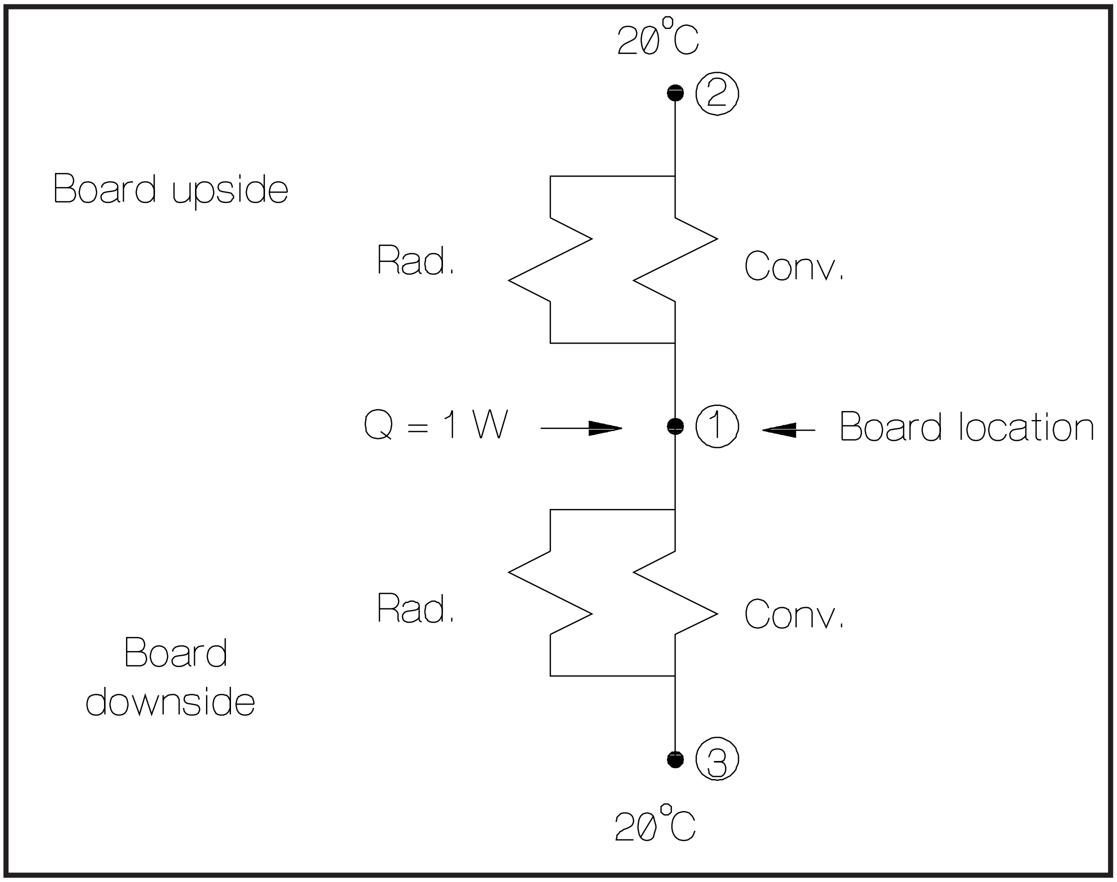
Opposite board plane is bare epoxy-glass.

Heat transfer from PCB is natural convection/radiation.

Thermal Network Model to Calculate h_{Total}

Horizontal plate parameter:

$$P = \frac{WL}{2(WL)} = \frac{(2.4\ in.)^2}{2(4.8\ in.)} = 0.56\ in.$$



Up: $\varepsilon A_s = \varepsilon_{E-G} A_s = 0.5\left[(0.5)(2.24\ in.\, x 2.24\ in.)\right] = 1.25\ in.^2$

Down: $\varepsilon A_s = (0.5)(2.24\ in.\, x 2.24\ in.) = 2.51\ in.^2$

Thermal network model solution -

"Small device" h_c required.

The TNETFA program was used to solve the problem:

Results from output file:

$$T_1 = 37.45\ {}^oC$$

Board upside:

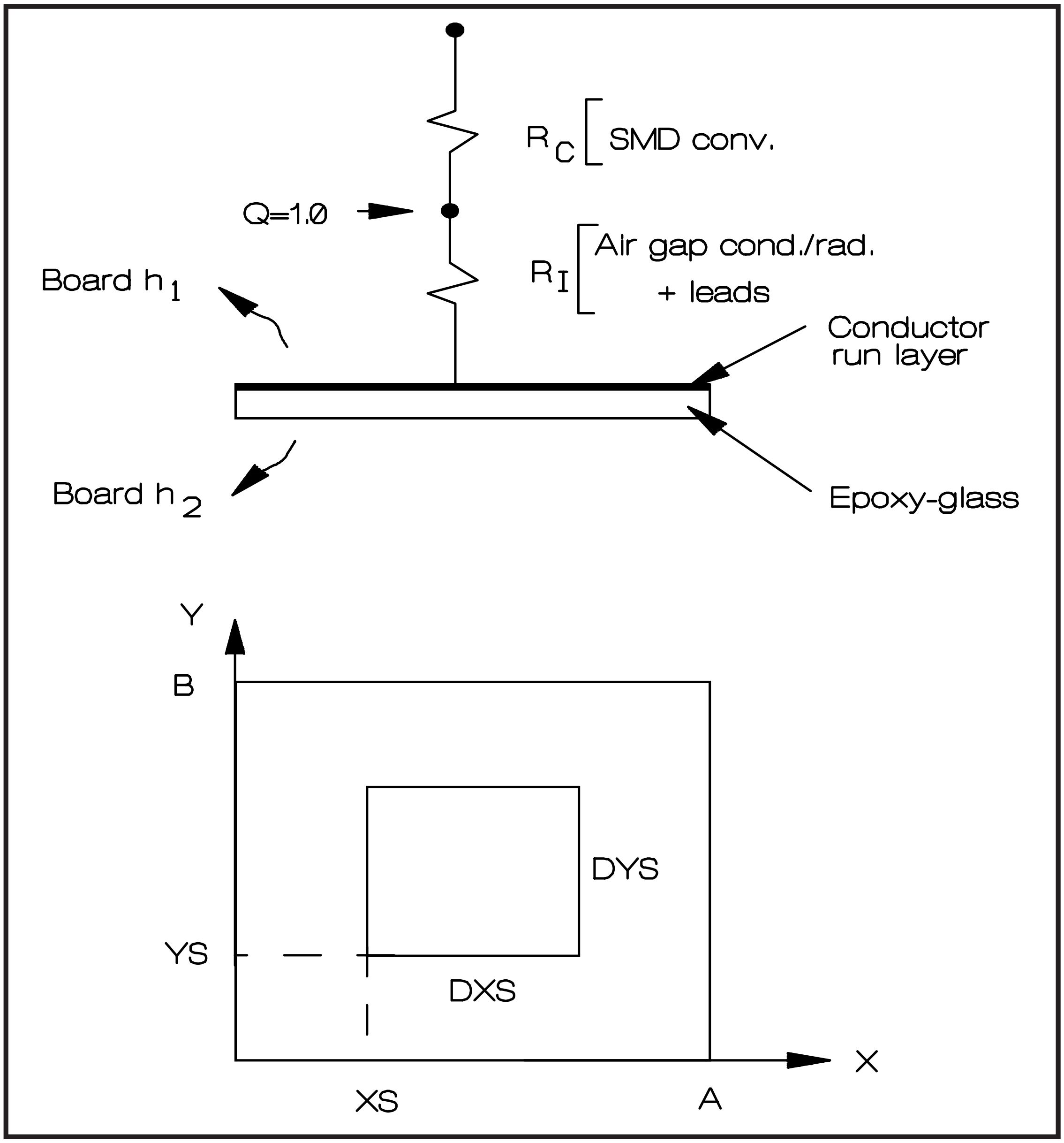
$$\begin{aligned} h_T &= 0.00560 + (0.25)(0.00402) \\ &= 0.0066\ W/in.^2 \cdot {}^oC \end{aligned}$$

Board downside:

$$\begin{aligned} h_T &= 0.00280 + (0.5)(0.00402) \\ &= 0.00481\ W/in.^2 \cdot {}^oC \end{aligned}$$

Fourier Series Solution of PCB Problem
Using the PTAMS Program

PTAMS Model -



PTAMS Input -

$$\text{Board } h_1 = 0.0066\ W/in.^2 \cdot {}^oC$$

$$h_2 = 0.0048\ W/in.^2 \cdot {}^oC$$

R_c (SMD) -

$$R_c = \frac{1}{h_T A_s} = \frac{1}{(0.0066)\left(1.0\ in.^2\right)} = 152\ {}^oC/W$$

R_I −

$$\frac{1}{R_I} = \frac{1}{R_{cond.}} + \frac{1}{R_{rad.}} + \frac{1}{R_{leads}}$$

$$R_{cond.} = \frac{t}{kA} = \frac{0.034\ in.}{\left(6x10^{-4}\ W/in.\cdot{}^oC\right)\left(1.0\ in.^2\right)} = 56.7\ {}^oC/W$$

$$R_{rad.} = \frac{1}{F h_r A_s} = \frac{1}{\left\{\dfrac{h_r A_s}{\left[\dfrac{1-\varepsilon_1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{A_1}{A_2}\right) + \dfrac{1}{F_{1-2}}\right]}\right\}}$$

$$\cong \frac{1}{(0.33)(0.005)(1.0)}$$

$$= 606\ {}^oC/W$$

R_{leads}

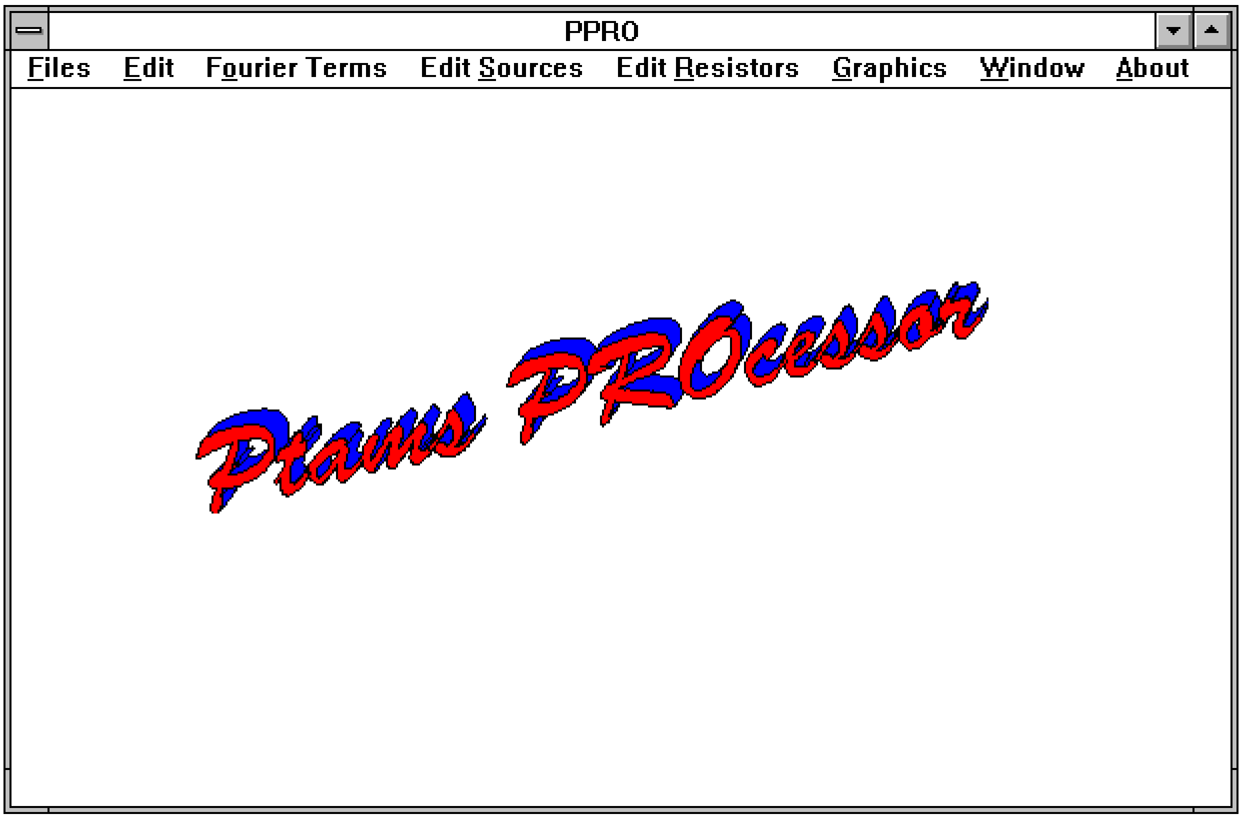
$$R_l = \frac{l}{68kA_k} = \frac{(0.034 + 0.0071)}{68(10)(0.03)(0.0078)}$$

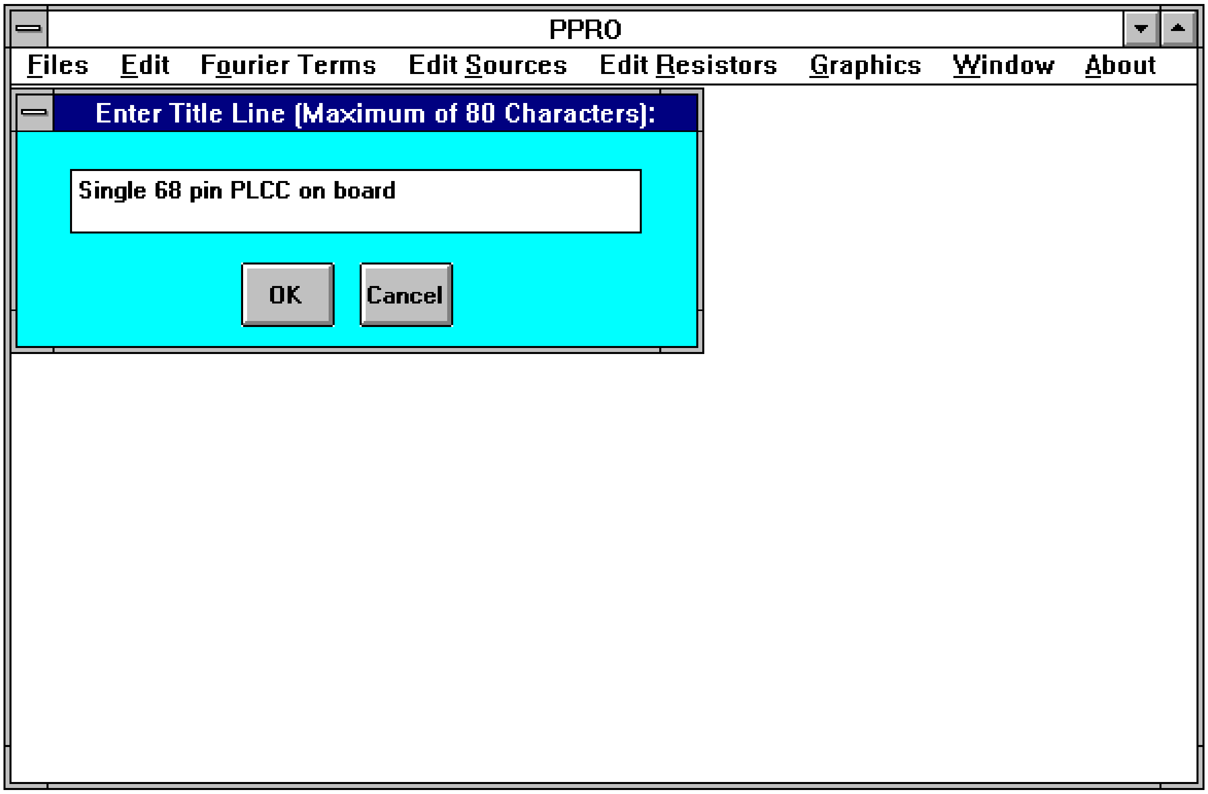
$$= 0.66\ {}^oC/W$$

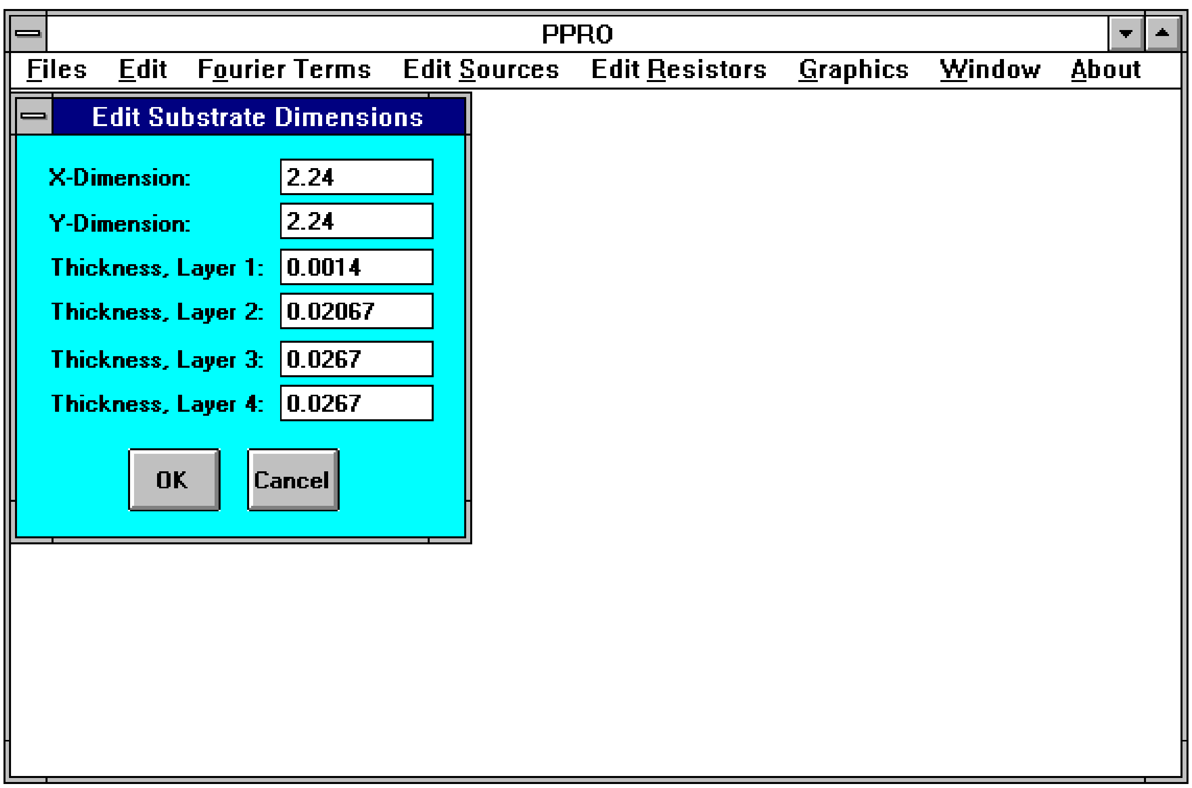
$$\frac{1}{R_I} = \frac{1}{56.7} + \frac{1}{606} + \frac{1}{0.66}$$

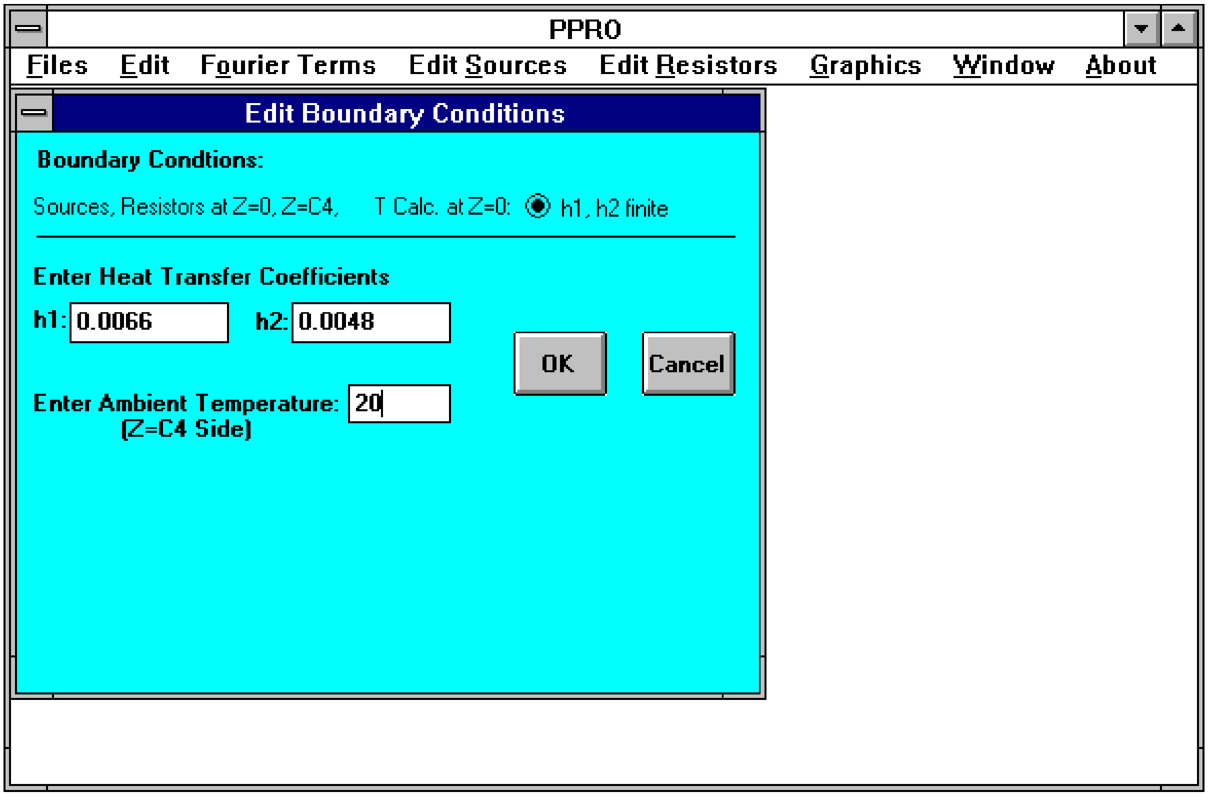
$R_I = 0.65\ {}^oC/W$, i.e. all lead conduction!

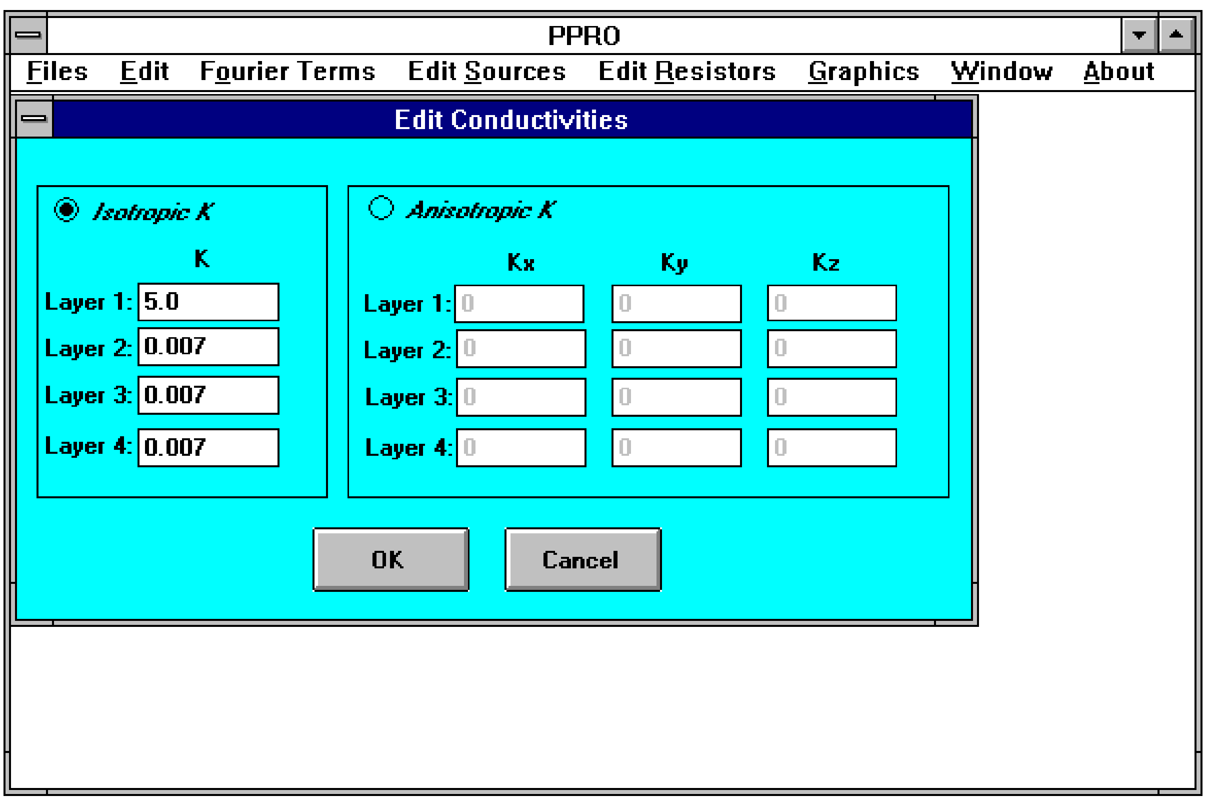
Double Click on *PPRO* Icon to Start PTAMS Processor

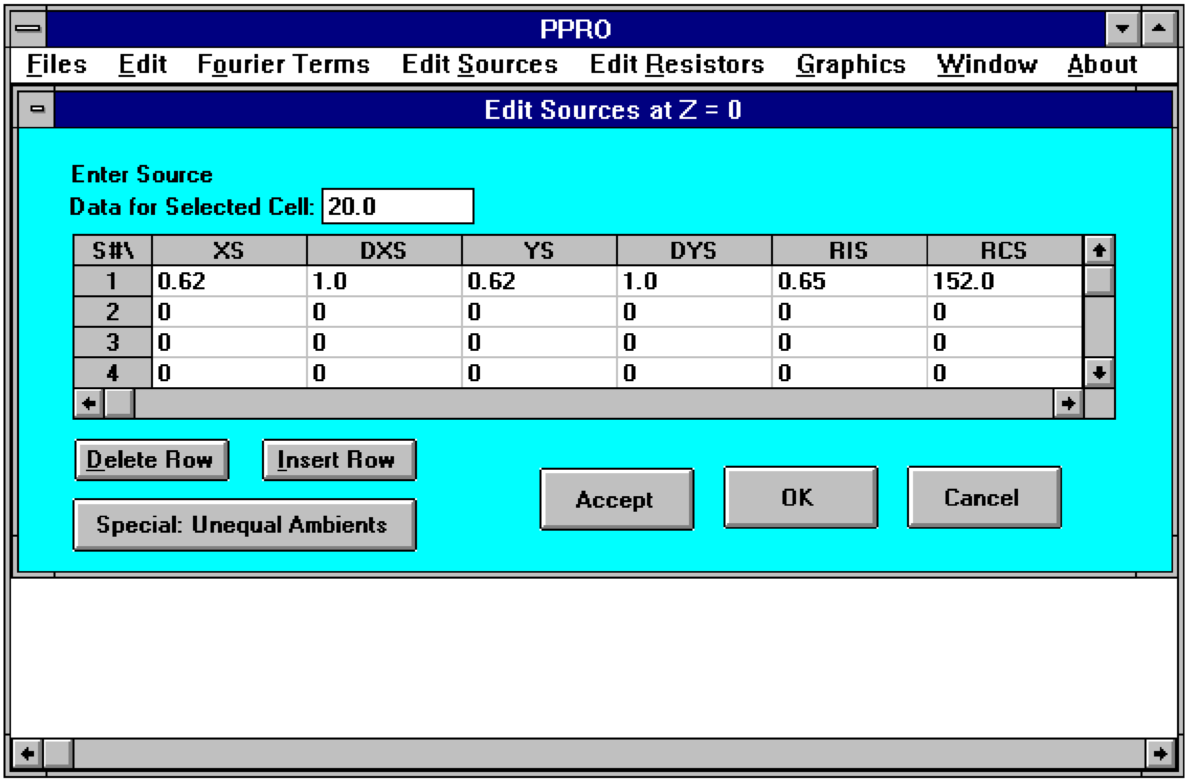


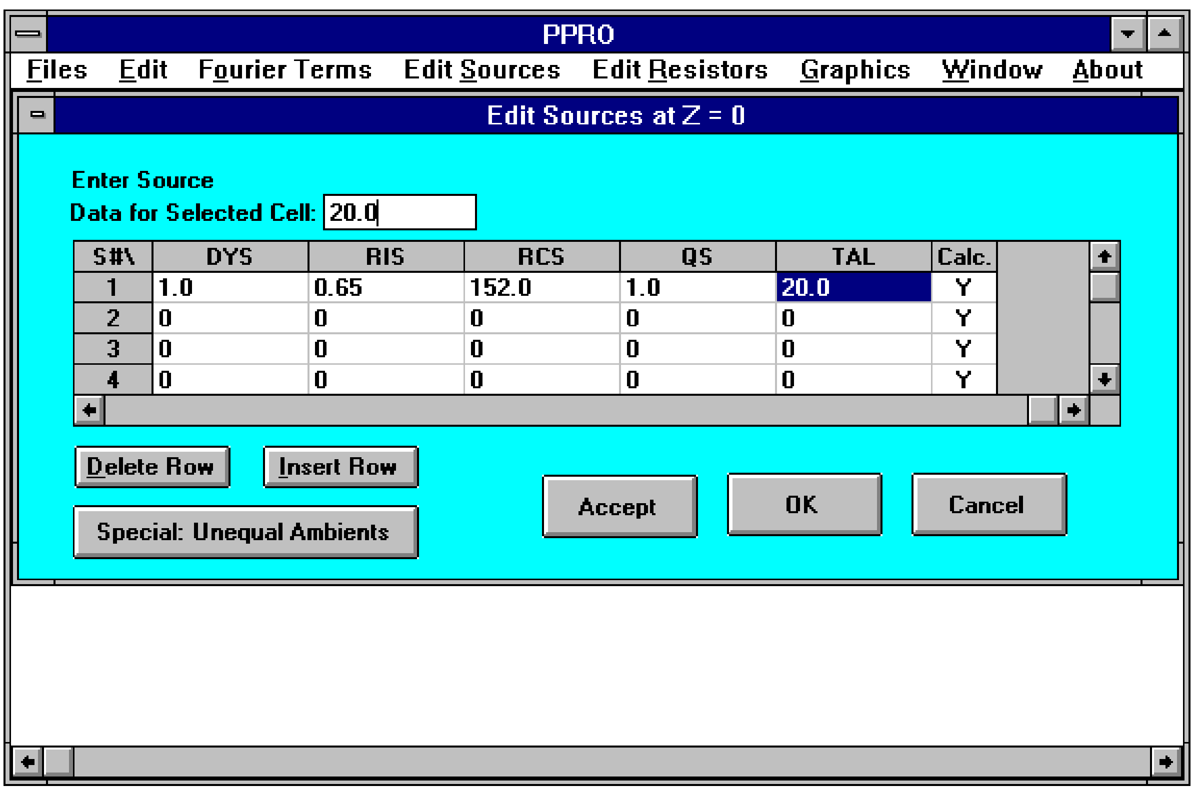

PPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Enter Title Line (Maximum of 80 Characters):
Single 68 pin PLCC on board
OK
Cancel

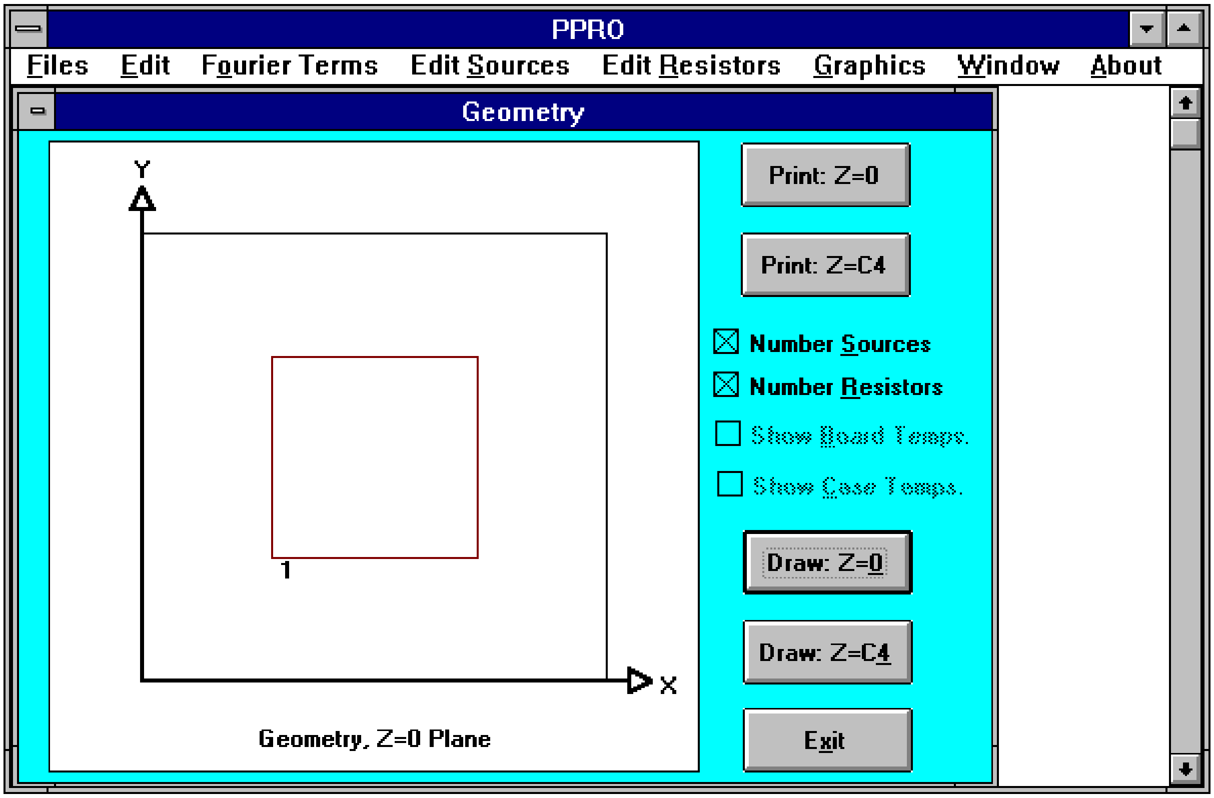

PPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Edit Substrate Dimensions
X-Dimension: 2.24
Y-Dimension: 2.24
Thickness, Layer 1: 0.0014
Thickness, Layer 2: 0.02067
Thickness, Layer 3: 0.0267
Thickness, Layer 4: 0.0267
OK
Cancel

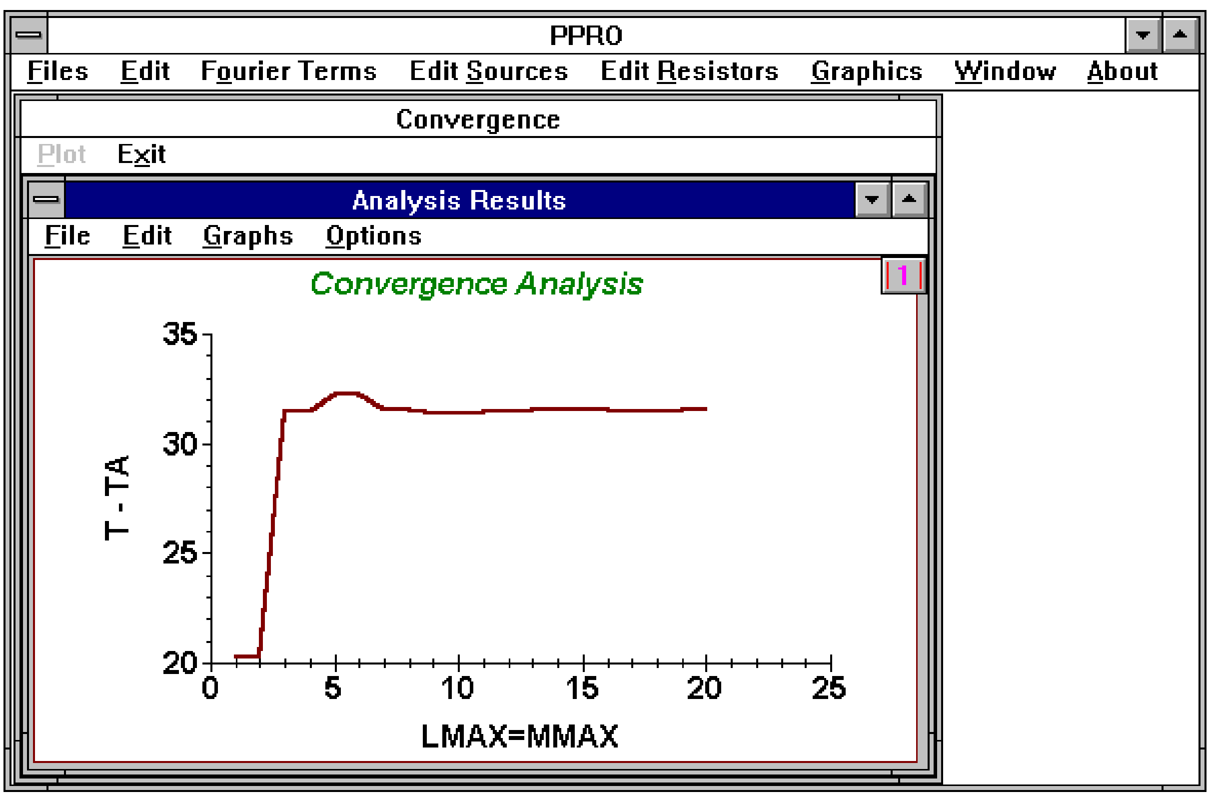

PPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Edit Boundary Conditions
Boundary Condtions:
Sources, Resistors at Z=0, Z=C4, T Calc. at Z=0: h1, h2 finite
Enter Heat Transfer Coefficients
h1: 0.0066 h2: 0.0048
OK
Cancel
Enter Ambient Temperature: 20
(Z=C4 Side)

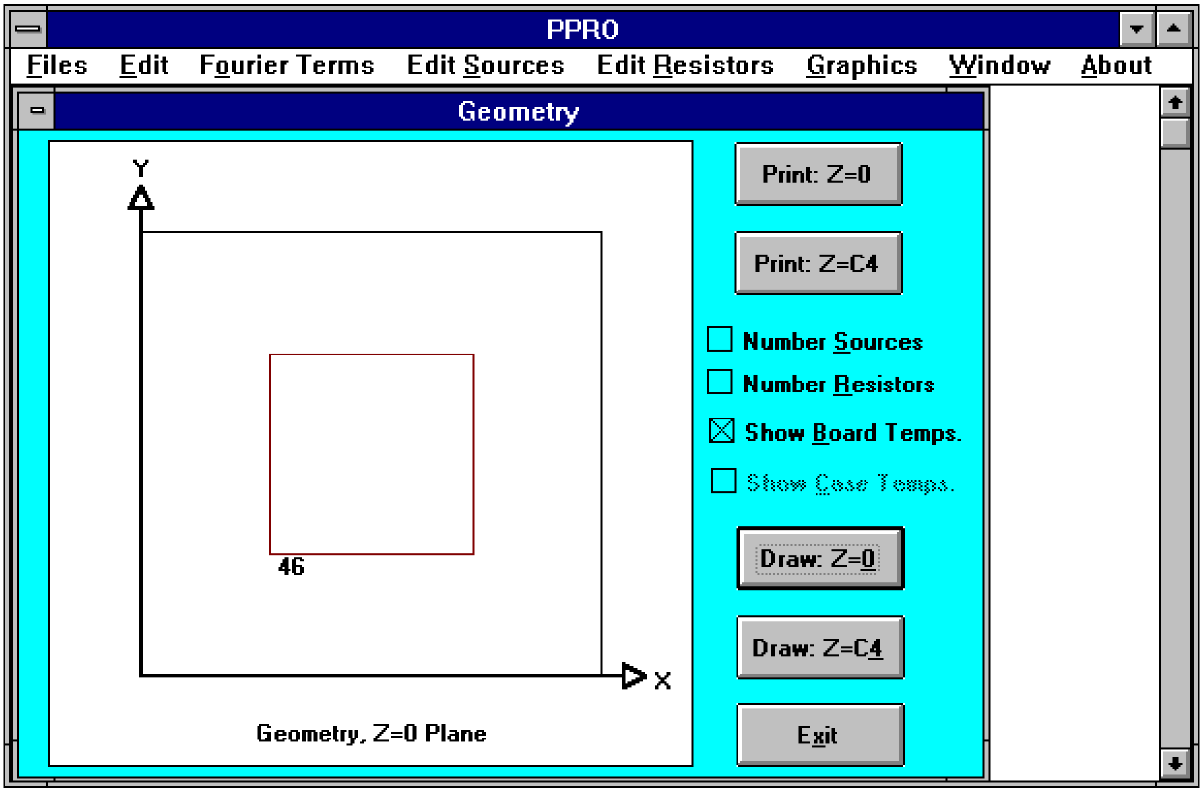

PPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Edit Conductivities
Isotropic K
K
Layer 1: 5.0
Layer 2: 0.007
Layer 3: 0.007
Layer 4: 0.007
Anisotropic K
Kx Ky Kz
Layer 1: 0 0 0
Layer 2: 0 0 0
Layer 3: 0 0 0
Layer 4: 0 0 0
OK
Cancel

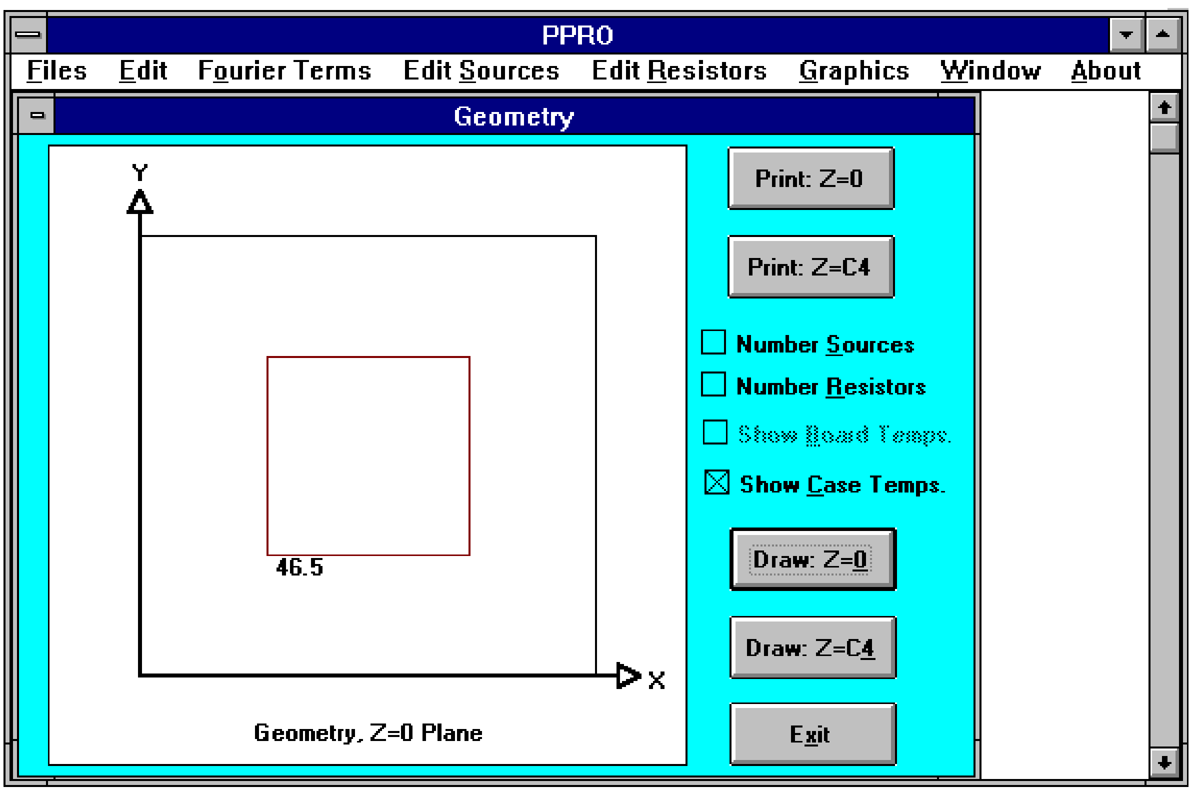

PPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Edit Sources at Z = 0
Enter Source
Data for Selected Cell: 20.0
S#\ XS DXS YS DYS RIS RCS
1 0.62 1.0 0.62 1.0 0.65 152.0
2 0 0 0 0 0 0
3 0 0 0 0 0 0
4 0 0 0 0 0 0
Delete Row
Insert Row
Special: Unequal Ambients
Accept
OK
Cancel


PPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Edit Sources at Z = 0
Enter Source
Data for Selected Cell: 20.0
S#\ DYS RIS RCS QS TAL Calc.
1 1.0 0.65 152.0 1.0 20.0 Y
2 0 0 0 0 0 Y
3 0 0 0 0 0 Y
4 0 0 0 0 0 Y
Delete Row
Insert Row
Special: Unequal Ambients
Accept
OK
Cancel


PPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Geometry
Y
X
1
Geometry, Z=0 Plane
Print: Z=0
Print: Z=C4
Number Sources
Number Resistors
Show Board Temps.
Show Case Temps.
Draw: Z=0
Draw: Z=C4
Exit


PPRO
Files Edit Fourier Terms Edit Sources Edit Resistors Graphics Window About
Convergence
Plot Exit
Analysis Results
File Edit Graphs Options
Convergence Analysis
1
T - TA
35
30
25
20
0
5
10
15
20
25
LMAX=MMAX


PPRO
Files
Edit
Fourier Terms
Edit Sources
Edit Resistors
Graphics
Window
About
Geometry
Y
46
X
Geometry, Z=0 Plane
Print: Z=0
Print: Z=C4
Number Sources
Number Resistors
Show Board Temps.
Show Case Temps.
Draw: Z=0
Draw: Z=C4
Exit

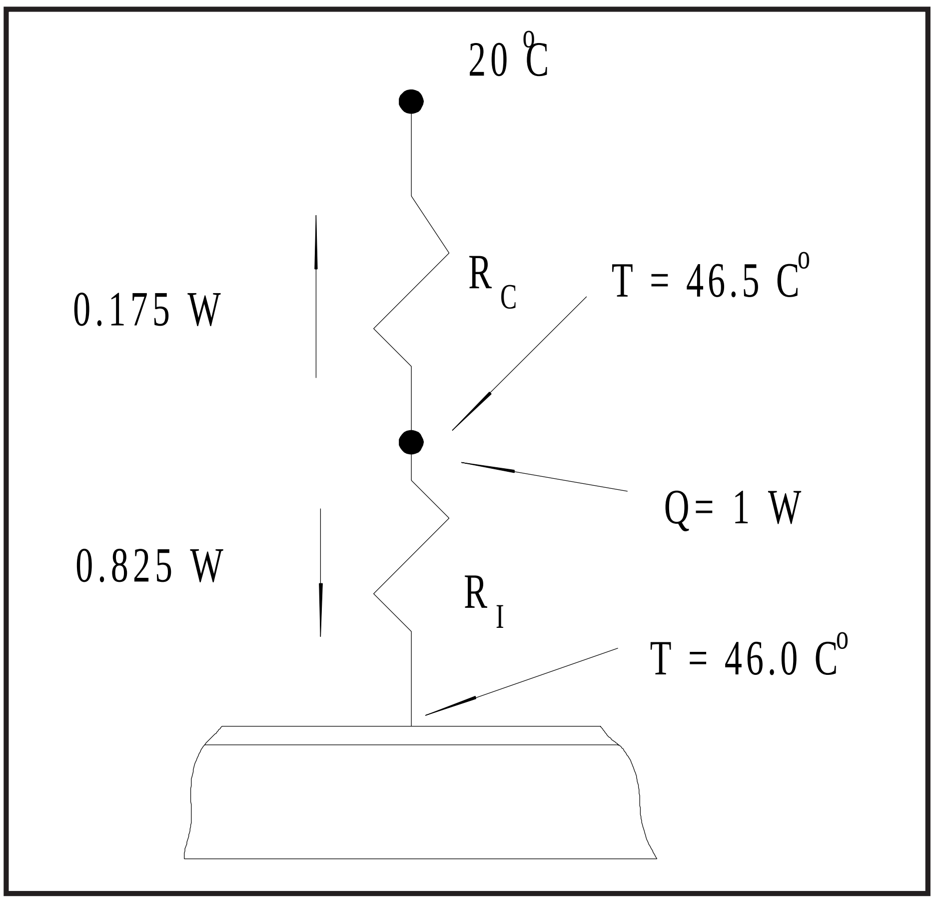

PPRO
Files
Edit
Fourier Terms
Edit Sources
Edit Resistors
Graphics
Window
About
Geometry
Y
46.5
X
Geometry, Z=0 Plane
Print: Z=0
Print: Z=C4
Number Sources
Number Resistors
Show Board Temps.
Show Case Temps.
Draw: Z=0
Draw: Z=C4
Exit

Before leaving PPRO, write file to disk (DIN.PCB).

Execute PTAMS by typing PTAMS.

The results are in a file named D1OUT.PCB.

PTAMS Results:



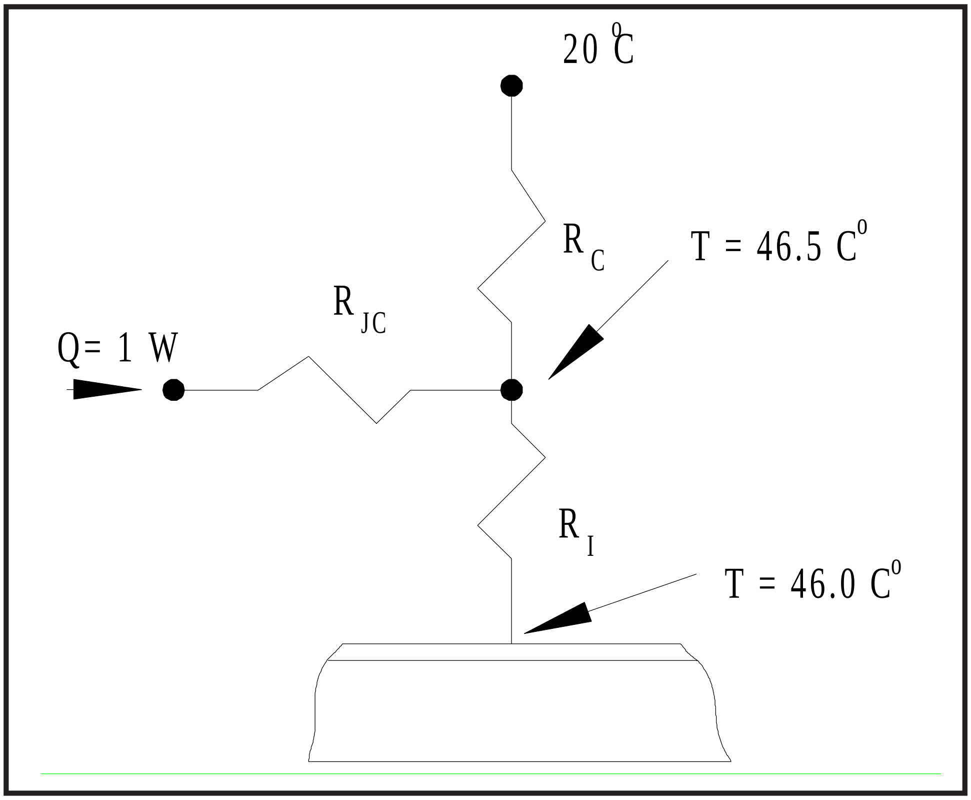
PTAMS Output File (D1OUT.PCB)

```
 *****************************************************************************
 ****          Electronics Thermal Analysis Package - PCB PTAMS V4.0
****
 ****            (C) Copyright 1996 by Thermal Computations, Inc.
****
 ****                        Hillsboro, Oregon
****
 *****************************************************************************

 Single 68 pin PLCC on board

   SUBSTRATE DIMENSIONS AND PHYSICAL CONSTANTS
      A = .2240E+01  B = .2240E+01
      T1= .1400E-02  T2= .2067E-01  T3= .2670E-01  T4= .2670E-01
      H1= .6600E-02  H2= .4800E-02
      K1= .5000E+01  K2= .7000E-02  K3= .7000E-02  K4= .7000E-02
      TA=  20.0

      NUMBER OF SOURCES=  1  NS1=  1  NS2=  0
      NUMBER OF RES.   =  0  NR1=  0  NR2=  0
      LMAX=  20  MMAX=  20

   SOURCE DATA
   I    XS(I)    DXS(I)   YS(I)    DYS(I)   RI(I)    RC(I)    Q(I)    TAI(I)
   1  .620E+00 .100E+01 .620E+00 .100E+01 .650E+00 .152E+03 .100E+01 .200E+02
                                                    TOTAL Q= .100E+01

   TEMPERATURES CALCULATED AT SUBSTRATE AND CASE CENTERS
   I  TS(I): SOURCES  TS(I): SOURCES/RES.  TC(I): SOURCES  TC(I): SOURCES/RES
   1     .460E+02                            .465E+02

             SOURCE DISSIPATION
   I   TO BOARD  TO AMBIENT   TOTAL
   1   .825E+00   .175E+00   .100E+01
```

PTAMS Input File (DIN.PCB)

```
Single 68 pin PLCC on board
 1             0
 20            20
2.2400E+00    2.2400E+00    1.4000E-03    2.0670E-02    2.6700E-02    2.6700E-
02
6.6000E-03    4.8000E-03    2.0000E+01
5.0000E+00    7.0000E-03    7.0000E-03    7.0000E-03
 1             0             0             0
6.2000E-01    1.0000E+00    6.2000E-01    1.0000E+00    6.5000E-01
1.5200E+02    1.0000E+00    2.0000E+01
  1
```

Use PTAMS result and vendor R_{JC} to calculated R_{JA}



Vendor measured $R_{JC} = 16\ {}^oC/W$

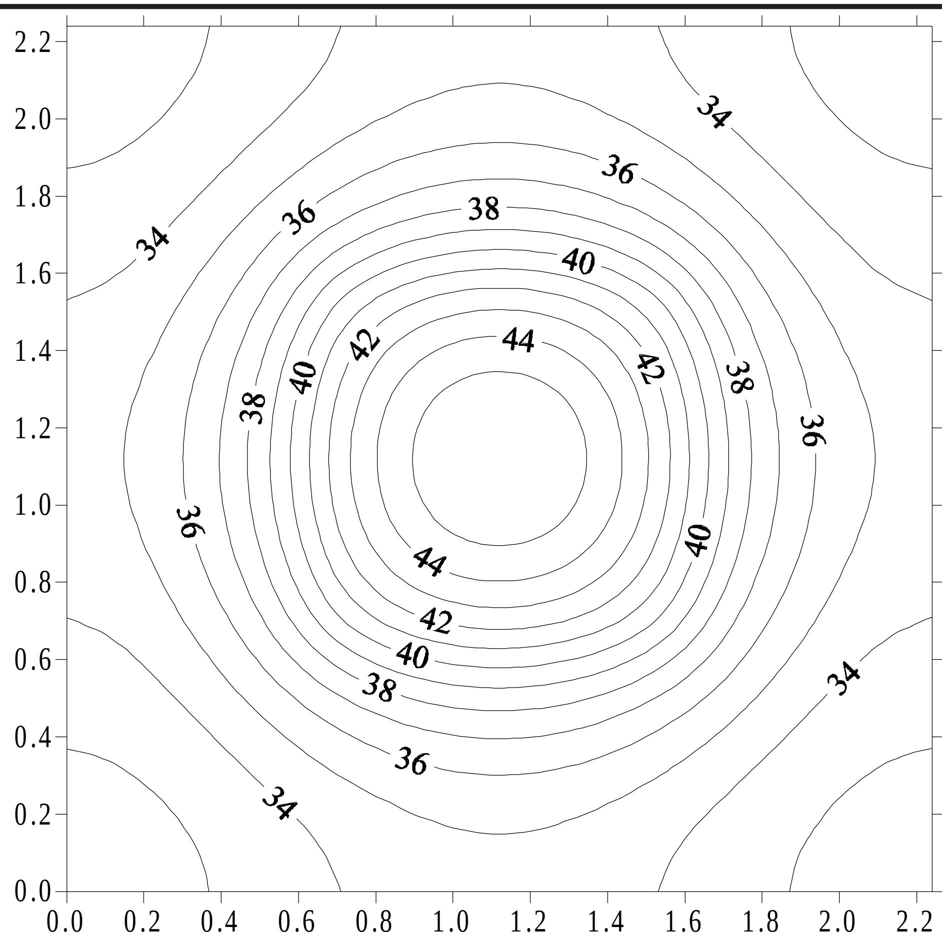
$$T_J = QR_{JC} + T_c$$
$$= (1\,W)\left(16\ {}^oC/W\right) + 46.5\ {}^oC$$
$$= 62.5\ {}^oC$$

$$R_{JA} = \frac{T_J - T_A}{Q} = \frac{62.5 - 20}{1.0} \cong 43\ {}^oC/W$$

Vendor specific test result is R_{JA}=46 $^oC/W$.

Contours constructed by:

1. Start up TPRO.
2. Read in PTAMS input file (DIN.PCB).
3. Use Graphics-Create Grid in TPRO to define 20x20 grid.
4. Write TAMS input file (DIN).
5. Read TAMS gridded output file (D4OUT) into SURFER*, grid and contour the file.

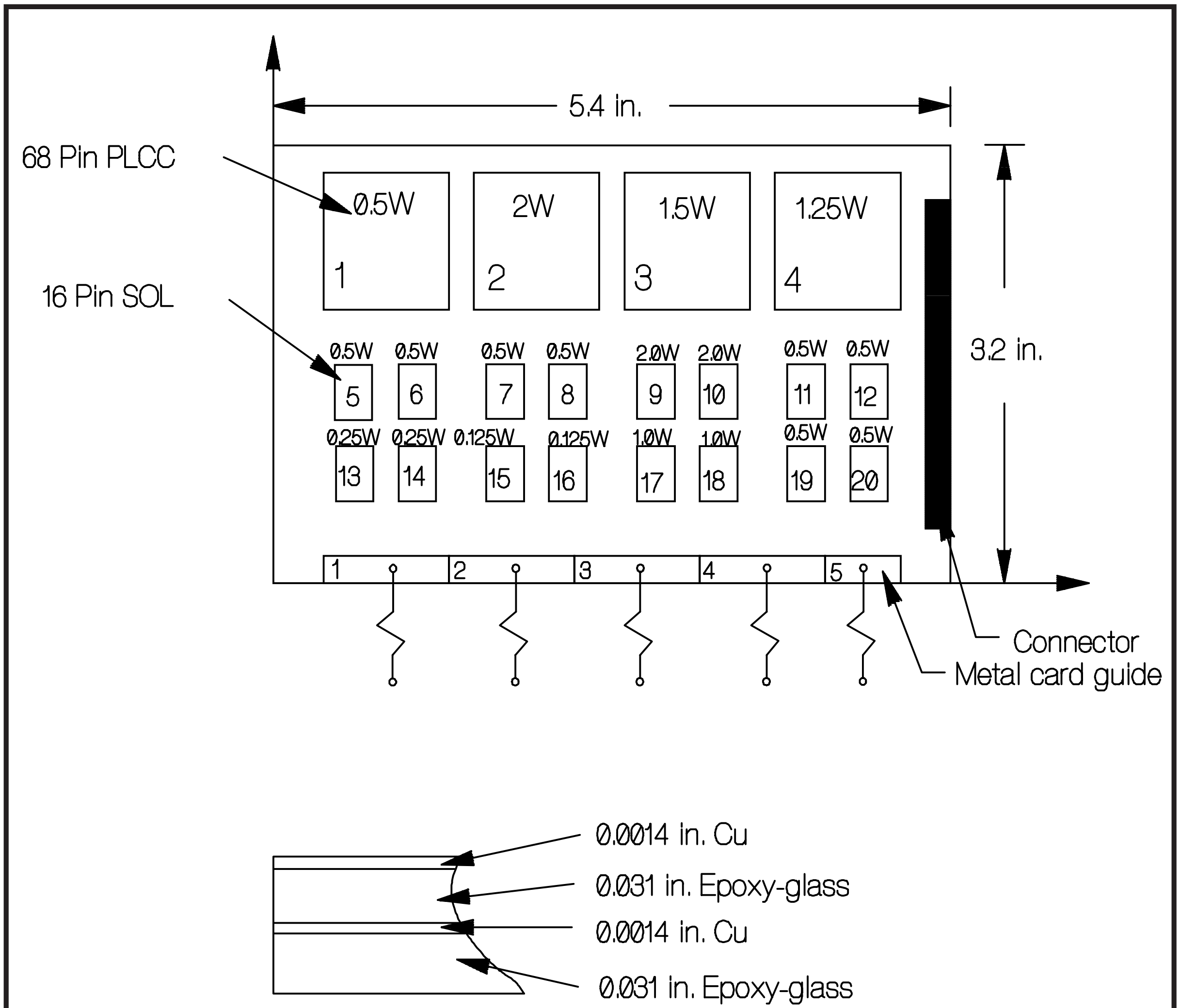


* Golden Software, Golden Colo.

Example

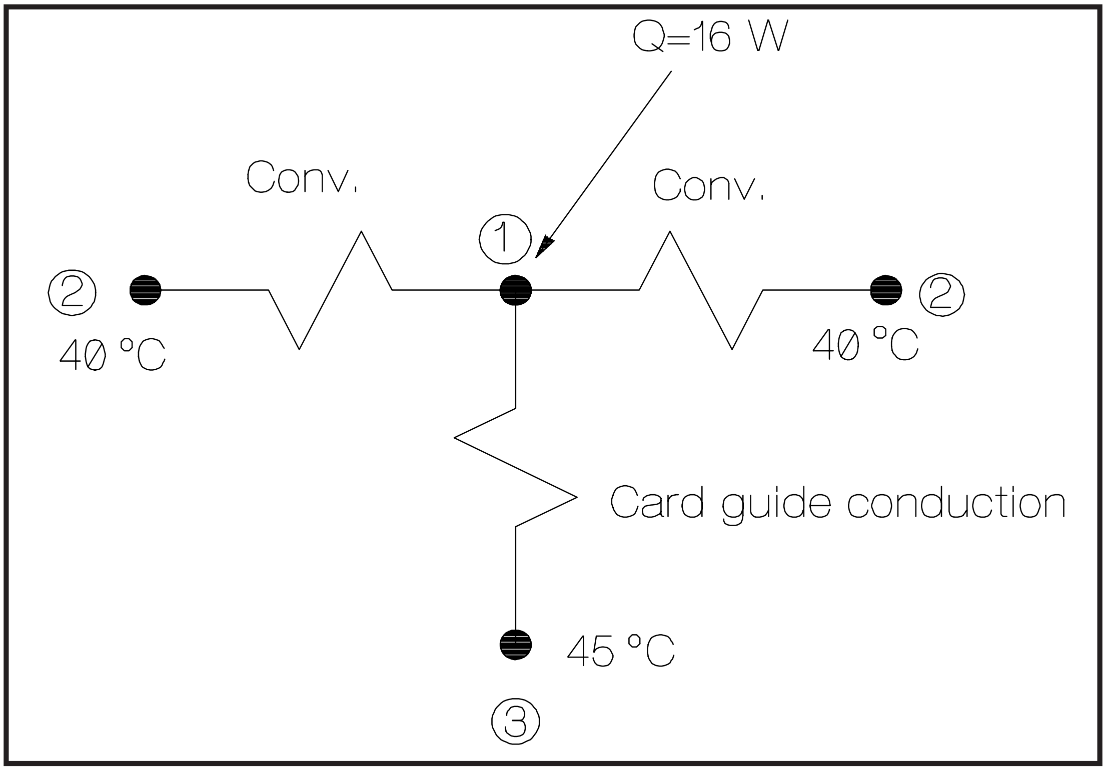
PTAMS Model of a 3x5 PCB
with
4, 68-Pin PLCCs and 16, 16-Pin SOLs

Board, source, and thermal resistance geometry -



Cooled by natural convection and conduction through card guide.

Simple Thermal Network Model to Determine
Natural Convection Heat Transfer Coefficients



Surrounding boards cause radiation shielding, therefore radiation is ignored.

Convection: "Small device" model element in TNETFA program used.
A_s = single-side board area = 17.28 in.^{2}

Card guide: R = 12 ^oC-*in./W* which results in 2.61 $^oC/W$ for the entire card guide.

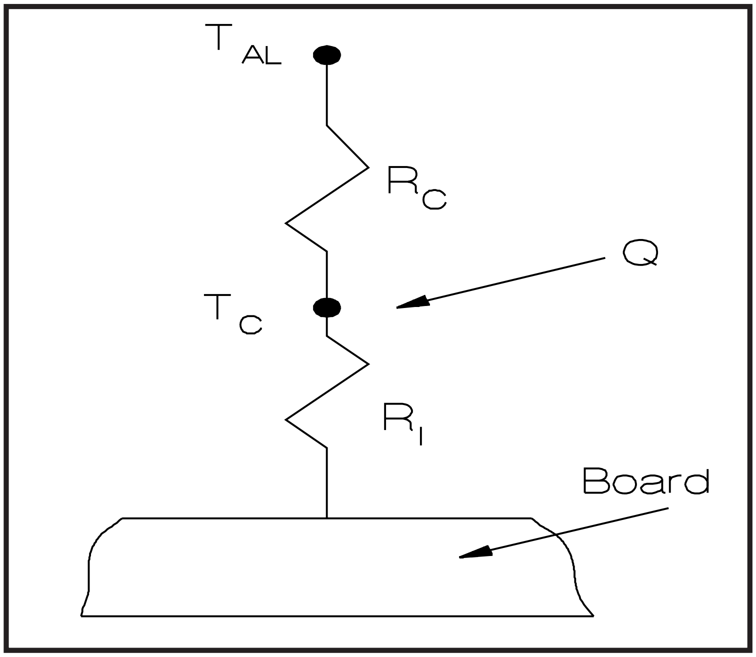
TNETFA results are $T_1 = 72.4\,^oC$, $h_{1-2} = 0.0049\ W/in.^2\,^oC$ per card side.

TNETFA Input File

4, 68 pin PLCCs and 12, 16 pin SOLs on board

```
11   2    0
3    2    1    0    1    1    0    1    0
.4000E+02    .0000E+00
2    .4000E+02
3    .4500E+02
1    .4000E+02    .1600E+02
0    0
2    1    0    2    0    .1728E+02    101
1    3    .3800E+00    0
6    .3200E+01
10   .1000E+01    .1000E-02    10
.0000E+00    .000E+00
10   1
```

Miscellaneous PTAMS Input



Board conductivities -

Run layer: k1 = (45 %)(10 *W/in.* ^oC)
First epoxy-glass layer: k2 = 0.007 *W/in.* ^oC
Second Cu layer: k3 = 10 *W/in.* ^oC
Second epoxy-glass layer: k4 = 0.007 *W/in.* ^oC

68 Pin PLCC -

R_I = 0.65 $^oC/W$ from before.
R_{JC} = 16 $^oC/W$ from vendor.
$R_C = 1/hA_s$ = 1/(0.0049 $W/in.^2\ ^oC$)(1.0 $in.^2$) = 204 $^oC/W$
T_{AL} = 40 ^oC

16 Pin SOL -

$$\frac{1}{R_I} = \frac{1}{R_{cond.}} + \frac{1}{R_{rad.}} + \frac{1}{R_l}$$

$R_{cond.} \equiv$ air gap conduction

$$= \frac{t}{kA} = \frac{8x10^{-3}\ in.}{\left(6x10^{-4}\ W/in.\cdot{}^{o}C\right)(0.4\ in.)(0.3\ in.)}$$

$$= 111\ {}^{o}C/W$$

$R_{rad.} \equiv$ radiation resistance

$$= \frac{1}{\mathcal{F} h_r A_s} \cong \frac{1}{(0.33)\left(0.005\ W/in.^2\cdot{}^{o}C\right)} = 5051\ {}^{o}C/W$$

$R_l \equiv$ lead conduction resistance

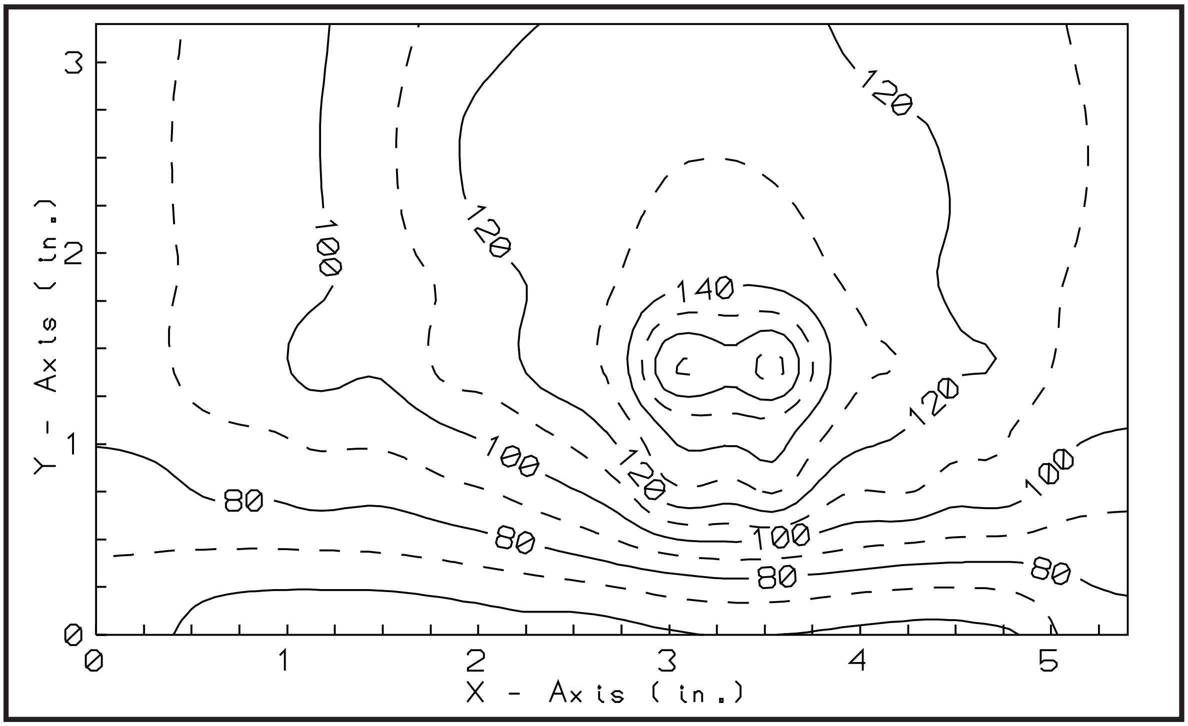
$$= \left(\frac{1}{16}\right)\frac{(0.008\ in.+0.09\ in.)}{\left(10\ W/in.\cdot{}^{o}C\right)(0.01\ in.)(0.017\ in.)}$$

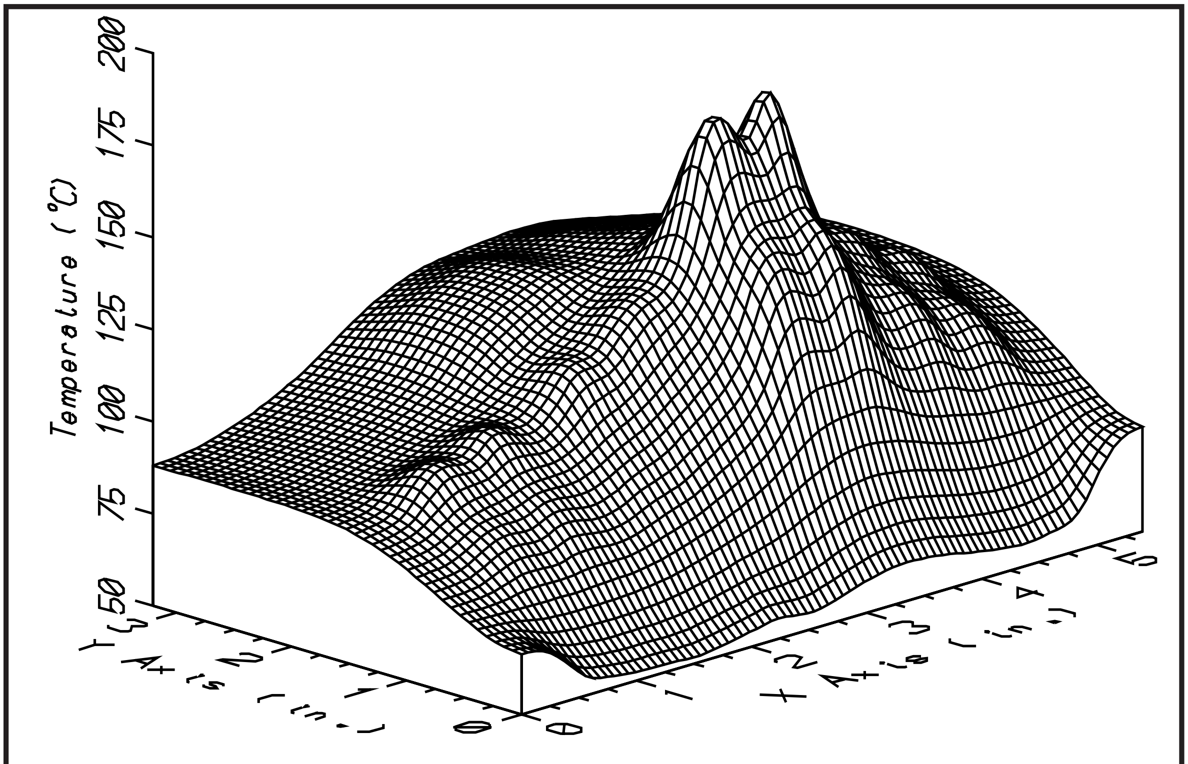
$$= 3.6\ {}^{o}C/W$$

$$\therefore R_I = 3.5\ {}^{o}C/W$$

$$R_c = \frac{1}{hA_s} = \frac{1}{\left(0.0049\ W/in.^2\cdot{}^{o}C\right)(0.4\ in.\ x 0.3\ in.)} = 1701\ {}^{o}C/W$$

Board Temperature Mapping
Using Golden Software's SURFER





Tabulation of Significant Results

PLCC Style	No.	Case Temp.	Board Temp.	Device Dis. (W)	Board Dis. (W)
68 Pin	1	95.8	95.6	0.5	0.228
	2	125	124	2.0	1.59
	3	131	130	1.5	1.06
	4	120	119	1.25	0.859
16 Pin	5	99.8	98.2	0.5	0.465
	6	105	103	0.5	0.462
	7	115	113	0.5	0.456
	8	126	124	0.5	0.449
	9	179	172	2.0	1.92
	10	180	173	2.0	1.92
	11	131	129	0.5	0.447
	12	121	119	0.5	0.452
	13	84.0	83.2	0.25	0.224
	14	86.4	85.6	0.25	0.223
	15	89.7	89.3	0.125	0.096
	16	98.6	98.3	0.125	0.091
	17	135	132	1.0	0.944
	18	137	134	1.0	0.943
	19	114	113	0.5	0.456
	20	108	106	0.5	0.460

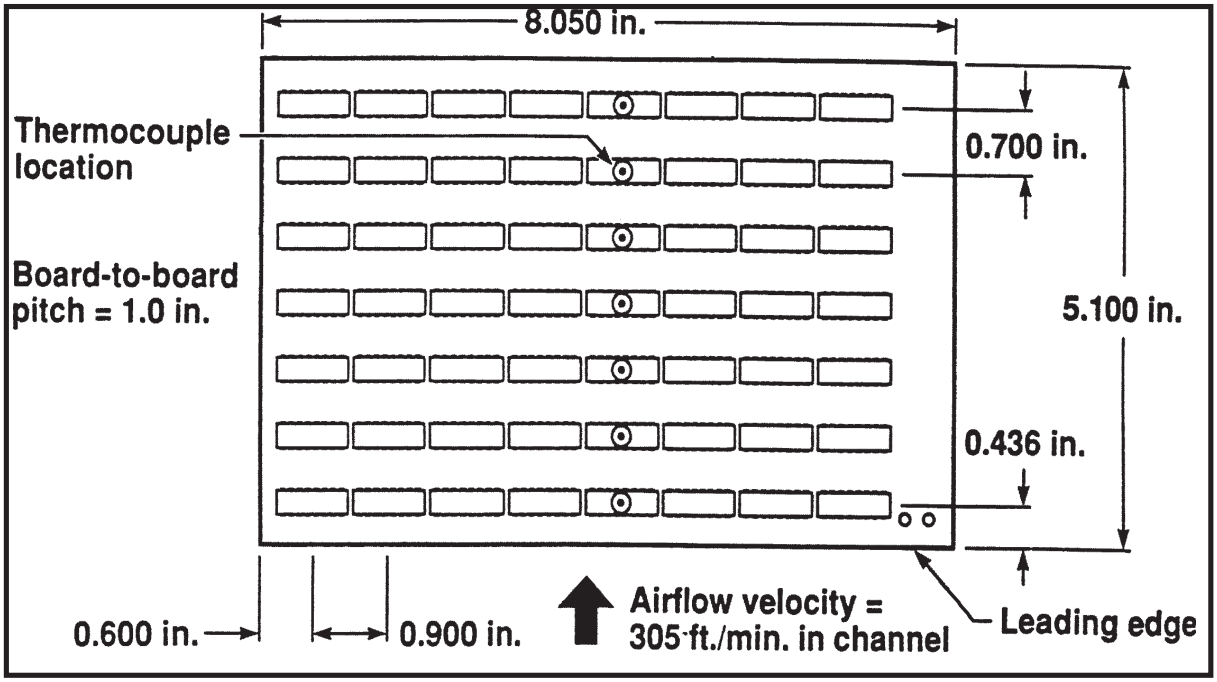
PTAMS Input File

```
Board Model for 68 pin PLCCs and 16 pin SOLs.
  1      0
 35     35
 .5400E+01  .3200E+01  .1400E-02  .3100E-01  .1400E-02  .3100E-01
 .4900E-02  .4900E-02  .4000E+02
 .4500E+01  .7000E-02  .1000E+02  .7000E-02
 20      0      5      0
 .4000E+00  .1000E+01  .2000E+01  .1000E+01  .6500E+00  .2040E+03  .5000E+00  .4000E+02
 .1600E+01  .1000E+01  .2000E+01  .1000E+01  .6500E+00  .2040E+03  .2000E+01  .4000E+02
 .2800E+01  .1000E+01  .2000E+01  .1000E+01  .6500E+00  .2040E+03  .1500E+01  .4000E+02
 .4000E+01  .1000E+01  .2000E+01  .1000E+01  .6500E+00  .2040E+03  .1250E+01  .4000E+02
 .5000E+00  .3000E+00  .1200E+01  .4000E+00  .3500E+01  .1704E+04  .5000E+00  .4000E+02
 .1000E+01  .3000E+00  .1200E+01  .4000E+00  .3500E+01  .1701E+04  .5000E+00  .4000E+02
 .1700E+01  .3000E+00  .1200E+01  .4000E+00  .3500E+01  .1701E+04  .5000E+00  .4000E+02
 .2200E+01  .3000E+00  .1200E+01  .4000E+00  .3500E+01  .1701E+04  .5000E+00  .4000E+02
 .2900E+01  .3000E+00  .1200E+01  .4000E+00  .3500E+01  .1701E+04  .2000E+01  .4000E+02
 .3400E+01  .3000E+00  .1200E+01  .4000E+00  .3500E+01  .1701E+04  .2000E+01  .4000E+02
 .4100E+01  .3000E+00  .1200E+01  .4000E+00  .3500E+01  .1701E+04  .5000E+00  .4000E+02
 .4600E+01  .3000E+00  .1200E+01  .4000E+00  .3500E+01  .1701E+04  .5000E+00  .4000E+02
 .5000E+00  .3000E+00  .6000E+00  .4000E+00  .3500E+01  .1701E+04  .2500E+00  .4000E+02
 .1000E+01  .3000E+00  .6000E+00  .4000E+00  .3500E+01  .1701E+04  .2500E+00  .4000E+02
 .1700E+01  .3000E+00  .6000E+00  .4000E+00  .3500E+01  .1701E+04  .1250E+00  .4000E+02
 .2200E+01  .3000E+00  .6000E+00  .4000E+00  .3500E+01  .1701E+04  .1250E+00  .4000E+02
 .2900E+01  .3000E+00  .6000E+00  .4000E+00  .3500E+01  .1701E+04  .1000E+01  .4000E+02
 .3400E+01  .3000E+00  .6000E+00  .4000E+00  .3500E+01  .1701E+04  .1000E+01  .4000E+02
 .4100E+01  .3000E+00  .6000E+00  .4000E+00  .3500E+01  .1701E+04  .5000E+00  .4000E+02
 .4600E+01  .3000E+00  .6000E+00  .4000E+00  .3500E+01  .1701E+04  .5000E+00  .4000E+02
  1      2      3      4      5      6      7      8
  9     10     11     12     13     14     15     16
 17     18     19     20
 .4000E+00  .1000E+01  .0000E+00  .2000E+00  .1200E+02  .4500E+02
 .1400E+01  .1000E+01  .0000E+00  .2000E+00  .1200E+02  .4500E+02
 .2400E+01  .1000E+01  .0000E+00  .2000E+00  .1200E+02  .4500E+02
 .3400E+01  .1000E+01  .0000E+00  .2000E+00  .1200E+02  .4500E+02
 .4400E+01  .6000E+00  .0000E+00  .2000E+00  .2000E+02  .4500E+02
```

Example

56 DIPs on a PCB

Calculations Compared with Experiment



Board-to-board spacing = 1.0 in.

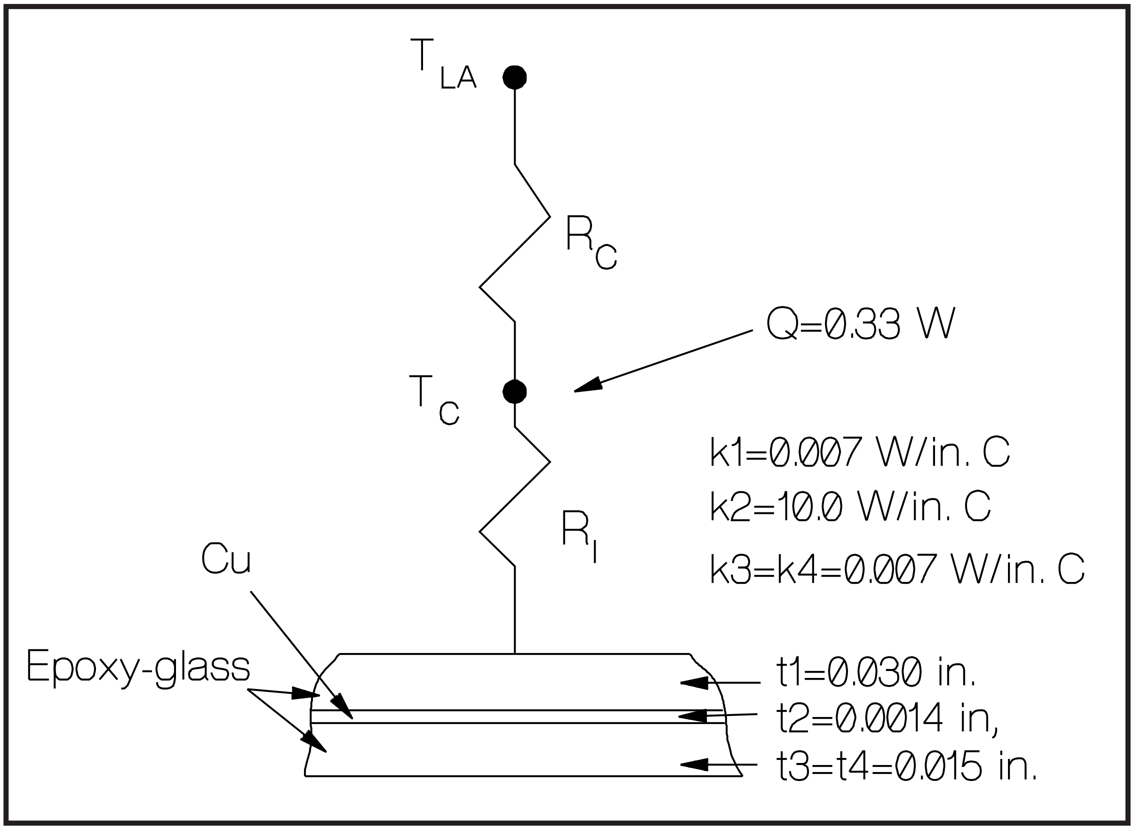
Board is 0.062 in. thick epoxy-glass with a nearly solid 0.0014 in. thick copper layer in the center.

The sources are 16 lead epoxy resistor DIPs with a 0.04 in. air gap.

DIP dimensions: L (direction of air flow)=0.24 in., W=0.82 in., H=0.123 in., Q (each)=0.33 W. Case area A=0.82*x*0.24+2*x*(0.82*x*0.123+0.24*x*0.123)=0.46 in.

Calculation of PTAMS Input

A piece of the problem -



$$\frac{1}{R_I} = \frac{1}{R_l} + \frac{1}{R_{air-gap}} + \frac{1}{R_{rad.}}$$

$$R_l = \frac{l}{16kA_k} = \frac{0.12}{16(10)(0.06)(0.01)} = 1.25\ {}^oC/W$$

$$R_{air-gap} = \frac{t}{kA_k} = \frac{0.04}{\left(6x10^{-4}\right)(0.82\ in.)(0.24\ in.)}$$

$$= 331\ {}^oC/W$$

$$R_{rad.} = \frac{1}{Fh_rA_s} \cong \frac{1}{(0.33)\left(0.005\ W/in.^2\cdot{}^oC\right)(0.82\ in.)(0.24\ in.)}$$

$$= 3006$$

for $\varepsilon_1 = \varepsilon_2 = 0.5$

Then $R_I = 1.25\ {}^oC/W$, i.e. mostly conduction!

Calculations Required for Wills and Flat Plate Representations of Components :

Board h computed as flat plate average

$$h_c = 0.001092\sqrt{V/L_{PCB}}\,f = 0.001092\sqrt{305/5.1}(1.6)$$
$$= 0.014$$

Component local ambients T_{AL} computed as well mixed, i.e.

$$T_{AL} - T_{Inlet} = \frac{1.76 Q_{upstream\,col}}{G_{col}}$$

$Q_{upstream\,col} \equiv$ heat dissipated for one column into channel upstream of device considered.

$G_{Col} \equiv$ channel flow rate (ft^3 / $min.$) for one column

$$= 305\,ft./\min.\left[\frac{(1.0in.-0.06in.-0.15in.)(8.05in./8)}{144in.^2/ft.^2}\right]$$
$$= 1.68\,ft.^3/\min.$$

Component PCB ambient T_A computed as well mixed air, i.e.

$$T_A = \Delta T_{Air} = \frac{1}{2}\left(\frac{1.76 Q_{Total}}{G_{Total}}\right) = 0.5\left(\frac{1.76\cdot 56\cdot 0.33}{8\cdot 1.68}\right) = 1.21^oC$$

Compute component h_c and R_c:

Case to ambient thermal resistance -

$$R_c = \frac{1}{h_c A_s}$$

$$A_s = 0.84in.\cdot 0.24in.+2\cdot 0.84in.\cdot 0.123in.+2\cdot 0.24in.\cdot 0.123in.$$
$$= 0.47in.^2$$

Wills correlation -

$$h_c = 3.42x10^{-3}\frac{L}{p}+1.60x10^{-4}\frac{V^{0.8}}{(Np)^{0.2}}\left[\left(\frac{p}{L}\right)-1\right]^{0.13}$$

where $L = 0.24in.$ (component length), $p = 0.68in.$ (component pitch), N = row number, $V = 305 ft./\min.$

Ellison's flat plate correlation with correction factor -

$$h_c = 0.000546\sqrt{\frac{V}{x}}f = 0.000546\sqrt{\frac{305}{x}}1.6$$

Summary of h_c and R_c calculations -

	Wills		Ellison		"Revised Duct"*	
Row	h_c	R_c	h_c	R_c	h_c	R_c
1	0.0195	110	0.022	97.4	0.047	45.5
2	0.0171	125	0.0136	157.4	0.018	118.9
3	0.0159	135	0.0107	200	0.011	200.0
4	0.0150	142	0.0091	235	0.0073	293.2
5	0.0144	148	0.0081	265	0.0054	396.3
6	0.0140	153	0.0073	294	0.0041	522.0
7	0.0136	158	0.0067	318	0.0033	649.0

* See Section I discussion of h for PCBs and ducts.

It can also be shown that $\bar{h}_{D_H} \approx 0.0039\, watts / in.^2 \cdot {}^oC$ for the Revised Duct.

Components analyzed using Richard Wirtz's adiabatic heat transfer coefficient and thermal wake function -

Bare board treated as a flat plate:

$$h_c = 0.001092\sqrt{V / L_{PCB}}\, f = 0.001092\sqrt{305/5.1}(1.6)$$
$$= 0.014$$

Use Q_{Total} from both channel surfaces so that board area near row seven "sees" total rise (gives most realistic T for last row):

$$T_A = \Delta T_{Air} = \left(\frac{1.76 Q_{Total}}{G_{Total}}\right) = \left(\frac{1.76 \cdot 56 \cdot 0.33}{8 \cdot 1.68}\right) = 2.4^o C$$

The reader should review the earlier section on forced convection cooling of PCBs and in particular the material referring to Wirtz's work.

In that earlier section, Wirtz's correlation was applied to the circuit board analyzed here. However, board conduction effects were not considered, i.e. each of the 0.33 W dissipated by each component was necessarily assumed to be convected directed to the local ambient air. In the current section we shall include board conduction effects and see that only a fraction of the heat dissipation by each component convects from the component case.

An iterative approach is required wherein a convective heat load is assumed for each component prior to the calculation of the adiabatic h and thermal wake functions. The procedure used for PPRO/PTAMS is outline below and the results listed.

1. The problem is started by assuming a convective heat transfer value for each IC package. In this problem it is 0.33 *W*/ package.

2. The adiabatic *h*, thermal convection resistance, and local ambient temperatures are then computed.

3. The resulting component convection resistances and local ambient temperatures are edited into the PTAMS input file using either a text editor or PPRO.

4. The program is executed. The output file "d1out.pcb" is examined for the calculated value for convective heat transfer. It will certainly be less than the initial 0.33 *W*.

5. The new convective heat transfer value is used as the source value in the adiabatic *h* and thermal wake function calculation to calculate new component convection resistances and local ambient for each component.

6. The new resistances and local ambients are edited into the PTAMS input file.

7. PTAMS is re-run and the d1out.pcb output file is examined again for the convective component of heat transfer.

8. This procedure (steps 5-7) is repeated until the convective heat transfer from the components changes very little from iteration to iteration.

An additional issue is that the bare board, component side, and opposite board surface also convect heat to some ambient air temperature. The university and industrial researchers using the adiabatic *h* and thermal wake function approach have not yet divulged a method of accurately calculating the board heat transfer coefficient and air ambient. In this problem the procedure selected was to used a flat plate *h* and an average of the local air temperatures calculated using the wake functions. This procedure is questionable, but in the absence of something better, it has been used here.

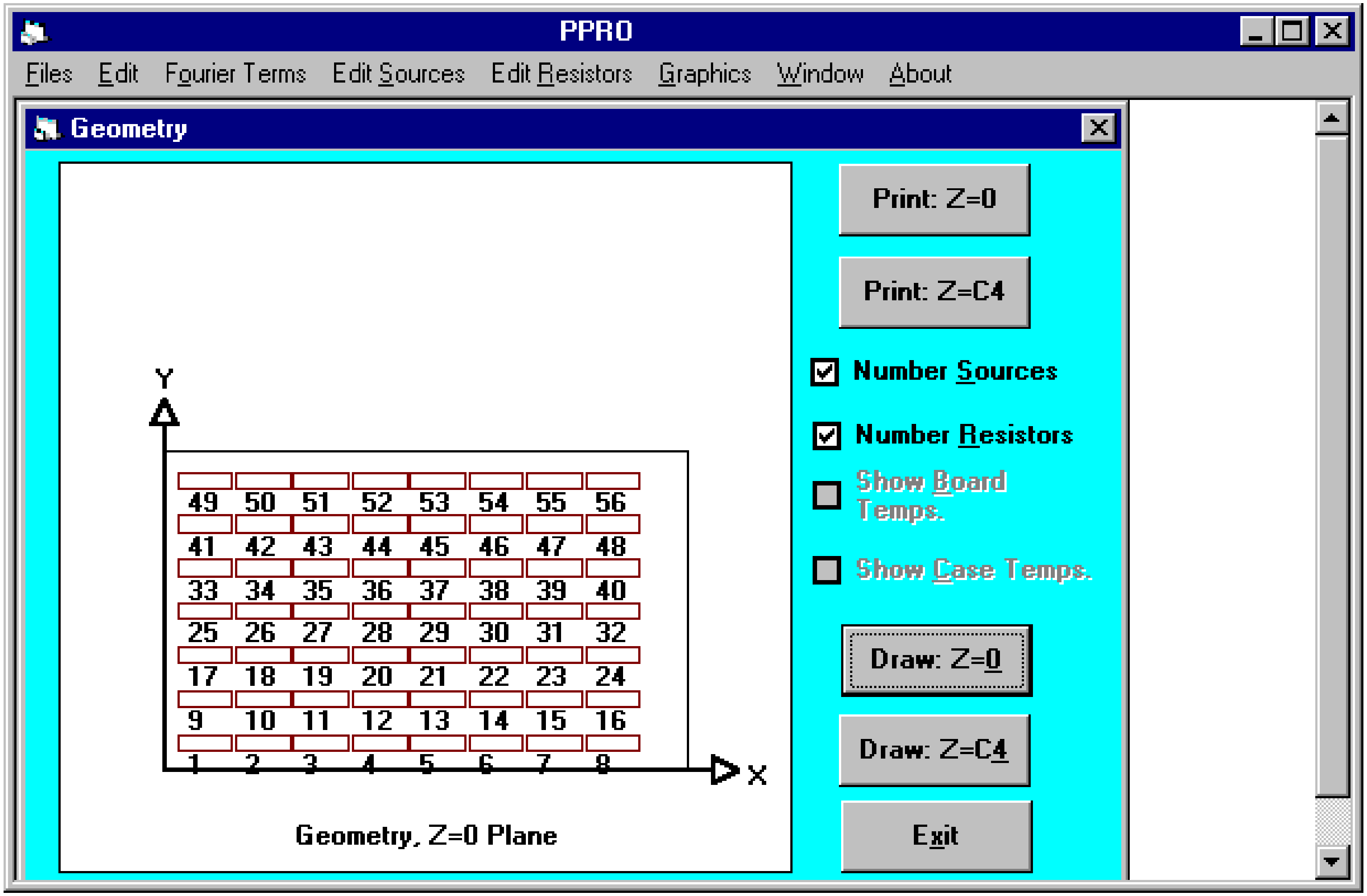
Mathcad has been used to calculate *h* , R_c, and the wake functions. The reader is advised to read the notes section concerning the PCB application of adiabatic *h*.

The result of applying the adiabatic h method in an iterative fashion with PPRO/PTAMS is listed in the following table:

Row	*Pkg.*	Q_{Conv}	R_C	$T_{Air} - T_0$	$T_{Case} - T_0$	$T'_{Case} - T_0$
1	5	0.238	49	0	11.5	16.2
2	13	0.199	55.7	2.87	13.9	22.6
3	21	0.172	61.2	5.41	15.9	28.8
4	29	0.157	61.2	7.89	17.5	33.6
5	37	0.140	61.2	10.29	18.8	38.6
6	45	0.121	61.2	12.64	20.0	43.9
7	53	0.095	61.2	14.95	20.8	49.6

where T'_{Case} is the computed result when board conduction/convection is not considered.

Board layout as seen from PPRO



Final PTAMS Input File (din.pcb) Generated from PPRO -

```
Sparse board: V = 185 ft/min. Vchannel=305. Wirtz h Tloc.
 1             0
 200           200
8.0500E+00    5.1000E+00    3.0000E-02    1.4000E-03    1.5000E-02    1.5000E-02
1.4000E-02    1.4000E-02    2.5000E+00
7.0000E-03    1.0000E+01    7.0000E-03    7.0000E-03
 56            0             0             0
1.8000E-01  8.4000E-01  3.1100E-01  2.5000E-01  1.2500E+00  4.9000E+01  3.3000E-01  0.0000E+00
1.0800E+00  8.4000E-01  3.1100E-01  2.5000E-01  1.2500E+00  4.9000E+01  3.3000E-01  0.0000E+00
1.9800E+00  8.4000E-01  3.1100E-01  2.5000E-01  1.2500E+00  4.9000E+01  3.3000E-01  0.0000E+00
2.8800E+00  8.4000E-01  3.1100E-01  2.5000E-01  1.2500E+00  4.9000E+01  3.3000E-01  0.0000E+00
3.7800E+00  8.4000E-01  3.1100E-01  2.5000E-01  1.2500E+00  4.9000E+01  3.3000E-01  0.0000E+00
4.6800E+00  8.4000E-01  3.1100E-01  2.5000E-01  1.2500E+00  4.9000E+01  3.3000E-01  0.0000E+00
5.5800E+00  8.4000E-01  3.1100E-01  2.5000E-01  1.2500E+00  4.9000E+01  3.3000E-01  0.0000E+00
6.4800E+00  8.4000E-01  3.1100E-01  2.5000E-01  1.2500E+00  4.9000E+01  3.3000E-01  0.0000E+00
1.8000E-01  8.4000E-01  1.0110E+00  2.5000E-01  1.2500E+00  5.5700E+01  3.3000E-01  2.8700E+00
1.0800E+00  8.4000E-01  1.0110E+00  2.5000E-01  1.2500E+00  5.5700E+01  3.3000E-01  2.8700E+00
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5.5800E+00  8.4000E-01  1.0110E+00  2.5000E-01  1.2500E+00  5.5700E+01  3.3000E-01  2.8700E+00
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```

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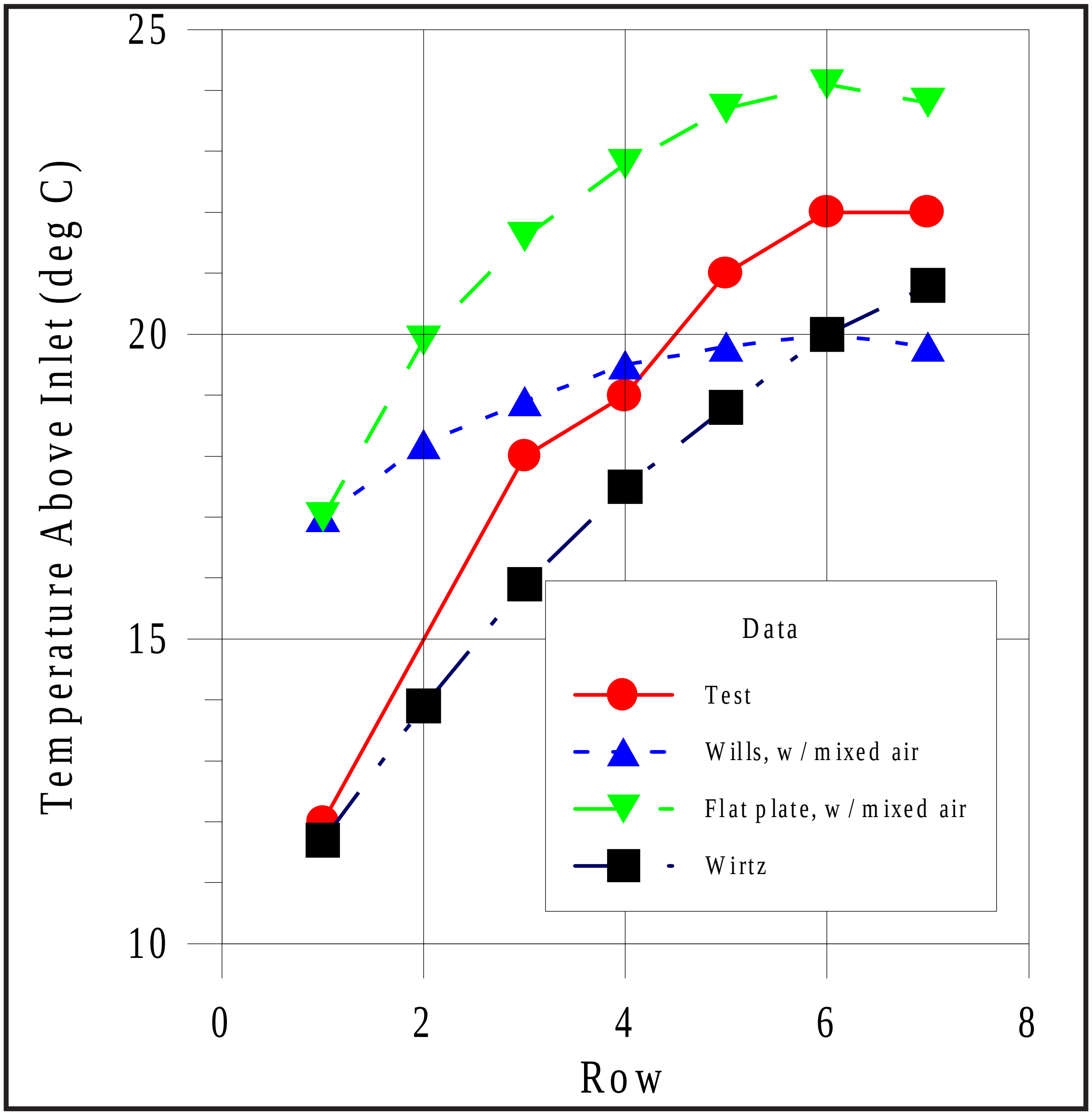
Computed and Measured Results Compared



Note: a much improved correlation may be obtained by using position-dependent convection resistances from board surface to local ambient.

Results using duct correlation omitted because of excessive disagreement with measurement.

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