

## Stress-Life Diagram (S-N Diagram)

The basis of the Stress-Life method is the Wohler S-N diagram, shown schematically for two materials in Figure 1. The S-N diagram plots nominal stress amplitude  $S$  versus cycles to failure  $N$ . There are numerous testing procedures to generate the required data for a proper S-N diagram. S-N test data are usually displayed on a log-log plot, with the actual S-N line representing the mean of the data from several tests.

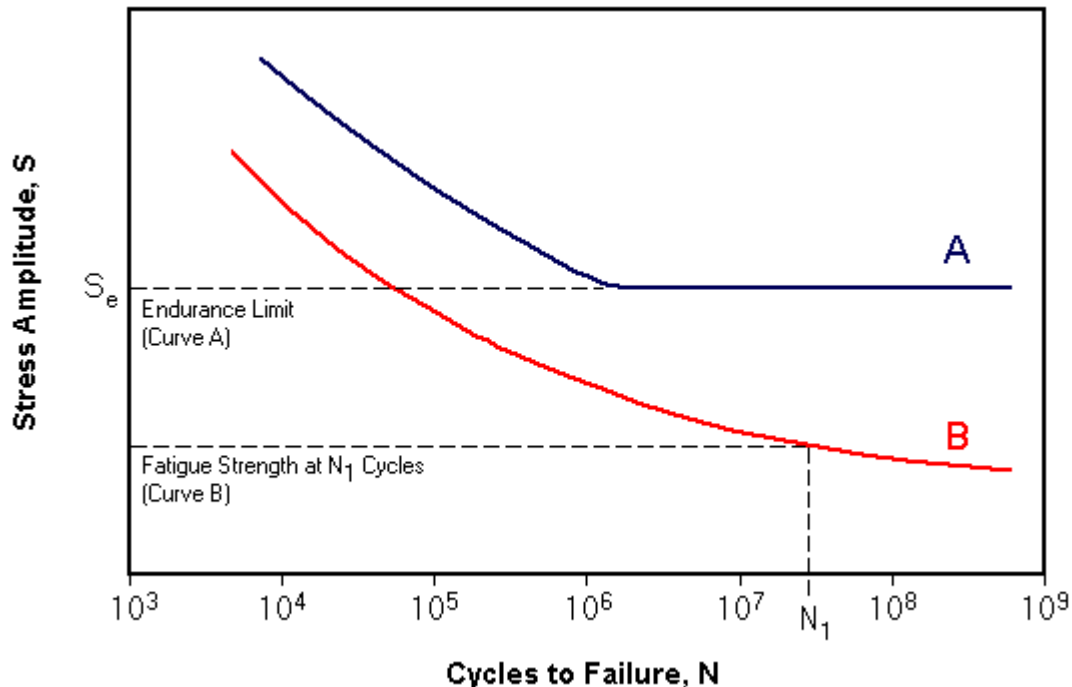


Figure 1: Typical S-N Curves

### Endurance Limit

Certain materials have a fatigue limit or *endurance limit* which represents a stress level below which the material does not fail and can be cycled infinitely. If the applied stress level is below the endurance limit of the material, the structure is said to have an *infinite life*. This is characteristic of steel and titanium in benign environmental conditions. A typical S-N curve corresponding to this type of material is shown Curve A in Figure 1.

Many non-ferrous metals and alloys, such as aluminum, magnesium, and copper alloys, do not exhibit well-defined endurance limits. These materials instead display a continuously decreasing S-N response, similar to Curve B in Figure 1. In such cases a *fatigue strength*  $S_f$  for a given number of cycles must be specified. An effective endurance limit for these materials is sometimes defined as the stress that causes failure at  $1 \times 10^8$  or  $5 \times 10^8$  loading cycles.

The concept of an endurance limit is used in infinite-life or *safe stress* designs. It is due to interstitial elements (such as carbon or nitrogen in iron) that pin dislocations, thus preventing the slip mechanism that leads to the formation of microcracks. Care must be taken when using an endurance limit in design applications because it can disappear due to:

- Periodic overloads (unpin dislocations)
- Corrosive environments (due to fatigue corrosion interaction)
- High temperatures (mobilize dislocations)

The endurance limit is not a true property of a material, since other significant influences such as surface finish cannot be entirely eliminated. However, test values ( $S_e'$ ) obtained from polished specimens provide a baseline to which other factors can be applied. Influences that can affect the endurance limit include:

- Surface Finish
- Temperature
- Stress Concentration
- Notch Sensitivity
- Size
- Environment
- Reliability

Such influences are represented by reduction factors,  $k$ , which are used to establish a working endurance strength  $S_e$  for the material:

$$S_e = k S_e'$$

### Power Relationship

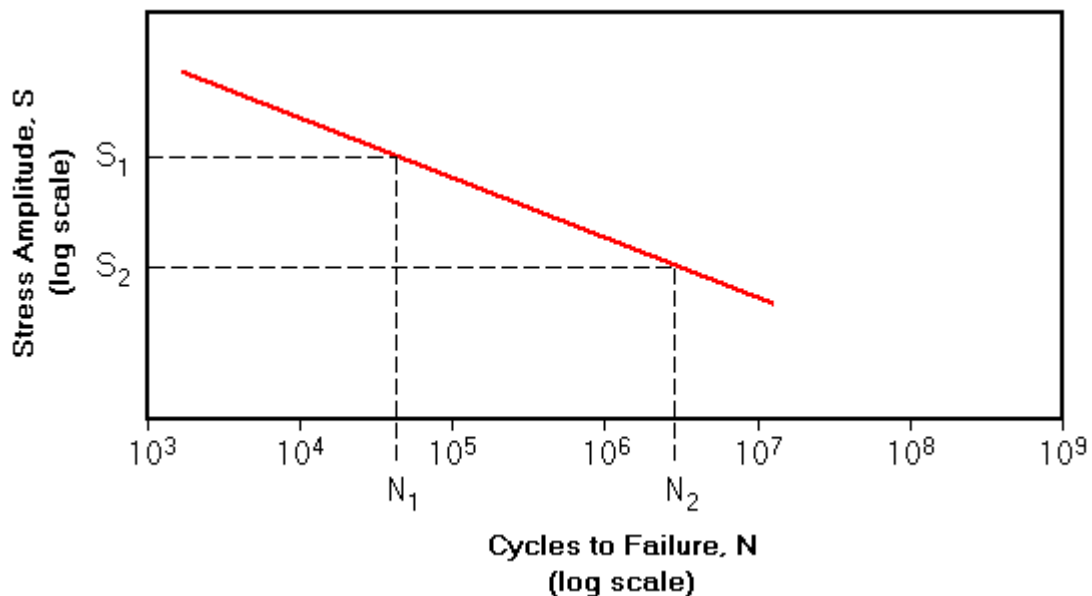
When plotted on a log-log scale, an S-N curve can be approximated by a straight line as shown in Figure 3. A power law equation can then be used to define the S-N relationship:

$$N_1 = N_2 \left( \frac{S_1}{S_2} \right)^{\frac{1}{b}}$$

where  $b$  is the slope of the line, sometimes referred to as the *Basquin slope*, which is given by:

$$b = \frac{-(\log S_1 - \log S_2)}{\log N_2 - \log N_1}$$

Given the Basquin slope and any coordinate pair  $(N, S)$  on the S-N curve, the power law equation calculates the cycles to failure for a known stress amplitude.



**Figure 3: Idealized S-N Curve**

The power relationship is only valid for fatigue lives that are on the design line. For ferrous metals this range is from  $1 \times 10^3$  to  $1 \times 10^6$  cycles. For non-ferrous metals, this range is from  $1 \times 10^3$  to  $5 \times 10^8$  cycles. Note the empirical relationships and equations described above are only estimates. Depending on the level of certainty required in the fatigue analysis, actual test data may be necessary.

### Fatigue Ratio (Relating Fatigue to Tensile Properties)

Through many years of experience, empirical relations between fatigue and tensile properties have been developed. Although these relationships are very general, they remain useful for engineers in assessing preliminary fatigue performance.

The ratio of the endurance limit  $S_e$  to the ultimate strength  $S_u$  of a material is called the *fatigue ratio*. It has values that range from 0.25 to 0.60, depending on the material.

For steel, the endurance strength can be approximated by:

$$S'_{e,steel} \approx 0.5S_u \quad \text{for } S_u < 200 \text{ ksi} \\ (S_u < 1400 \text{ MPa})$$

and:

$$S'_{e,steel} \approx 100 \text{ ksi} \quad \text{for } S_u > 200 \text{ ksi} \\ (700 \text{ MPa}) \quad (S_u > 1400 \text{ MPa})$$

In addition to this relationship, for wrought steels the stress level corresponding to 1000 cycles,  $S_{1000}$ , can be approximated by:

$$S_{1000,steel} = 0.9S_u$$

Utilizing these approximations, a generalized S-N curve for wrought steels can be created by connecting the  $S_{1000}$  point with the endurance limit, as shown in Figure 4.

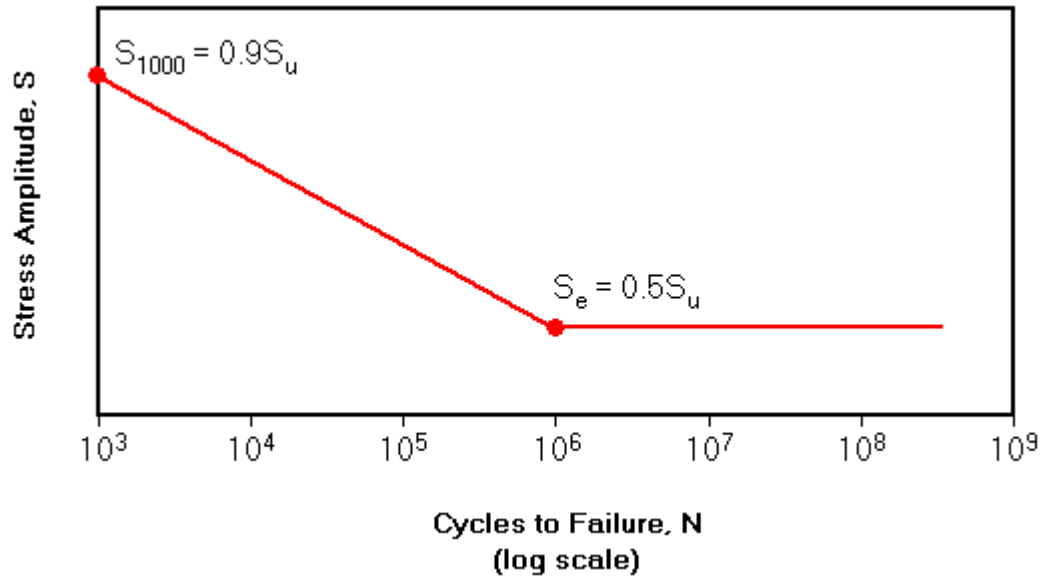


Figure 4: Generalized S-N Curve for Wrought Steels

### Mean Stress Effects

Most basic S-N fatigue data collected in the laboratory is generated using a fully-reversed stress cycle. However, actual loading applications usually involve a *mean stress* on which the oscillatory stress is superimposed, as shown in Figure 5. The following definitions are used to define a stress cycle with both alternating and mean stress.

The *stress range* is the algebraic difference between the maximum and minimum stress in a cycle:

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$$

The *stress amplitude* is one-half the stress range:

$$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

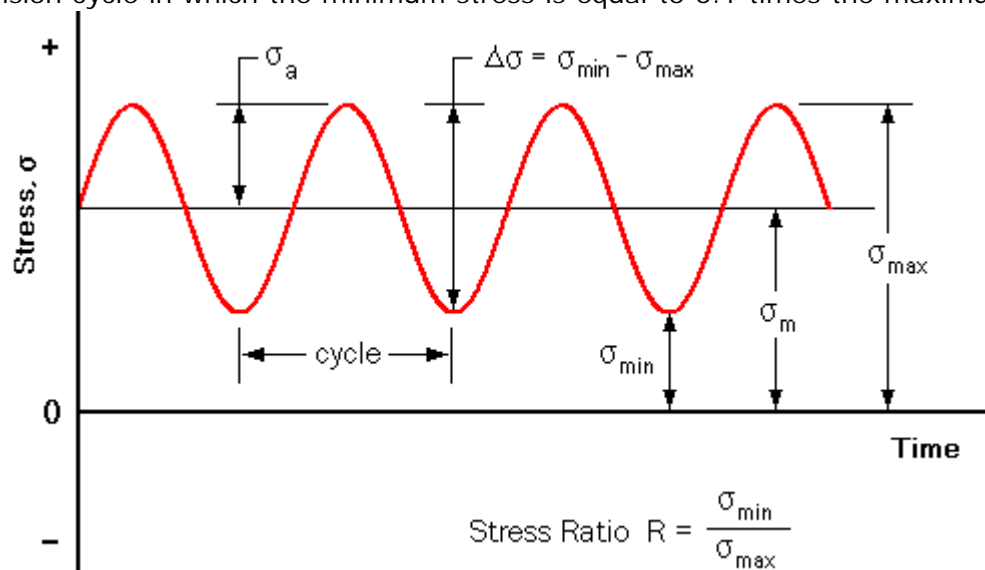
The *mean stress* is the algebraic mean of the the maximum and minimum stress in the cycle:

$$\sigma_m = \frac{\sigma_{\min} + \sigma_{\max}}{2}$$

Two ratios that are often defined for the representation of mean stress are the *stress ratio*  $R$  and the *amplitude ratio*  $A$ :

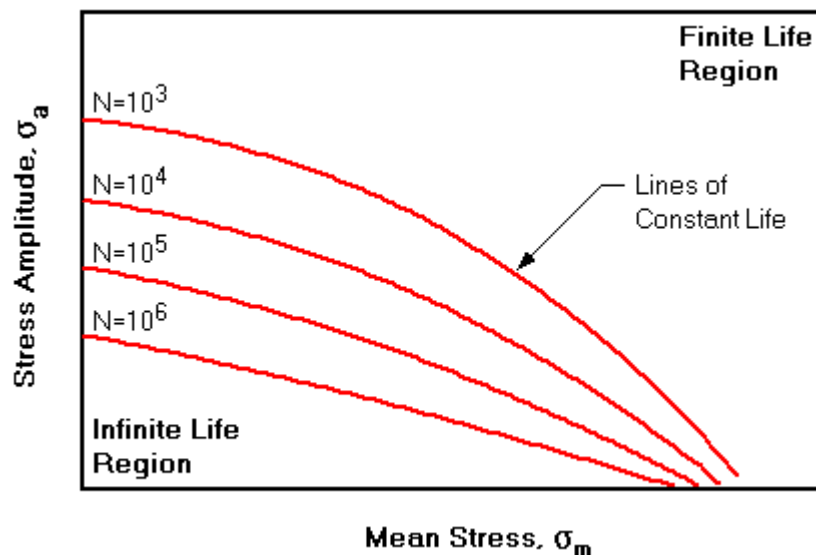
$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \\ A = \frac{\sigma_a}{\sigma_m} = \frac{1-R}{1+R}$$

For fully-reversed loading conditions,  $R$  is equal to  $-1$ . For static loading,  $R$  is equal to  $1$ . For a case where the mean stress is tensile and equal to the stress amplitude,  $R$  is equal to  $0$ . A stress cycle of  $R = 0.1$  is often used in aircraft component testing, and corresponds to a tension-tension cycle in which the minimum stress is equal to  $0.1$  times the maximum stress.



**Figure 5: Typical Cyclic Loading Parameters**

The results of a fatigue test using a nonzero mean stress are often presented in a *Haigh diagram*, shown in Figure 6. A Haigh diagram plots the mean stress, usually tensile, along the x-axis and the oscillatory stress amplitude along the y-axis. Lines of constant life are drawn through the data points. The infinite life region is the region under the curve and the finite life region is the region above the curve. For finite life calculations the endurance limit in any of the models can be replaced with a fully reversed ( $R = -1$ ) alternating stress level corresponding to the finite life value.



**Figure 6: Example of a Haigh Diagram**

A very substantial amount of testing is required to generate a Haigh diagram, and it is usually impractical to develop curves for all combinations of mean and alternating stresses. Several empirical relationships that relate alternating stress to mean stress have been developed to address this difficulty. These methods define various curves to connect the endurance limit on the alternating stress axis to either the yield strength,  $S_y$ , ultimate strength  $S_u$ , or true fracture stress  $\sigma_f$  on the mean stress axis. The following relations are available in the Stress-Life module:

**Goodman (England, 1899):** 
$$\frac{S_a}{S_e} + \frac{S_m}{S_u} = 1$$

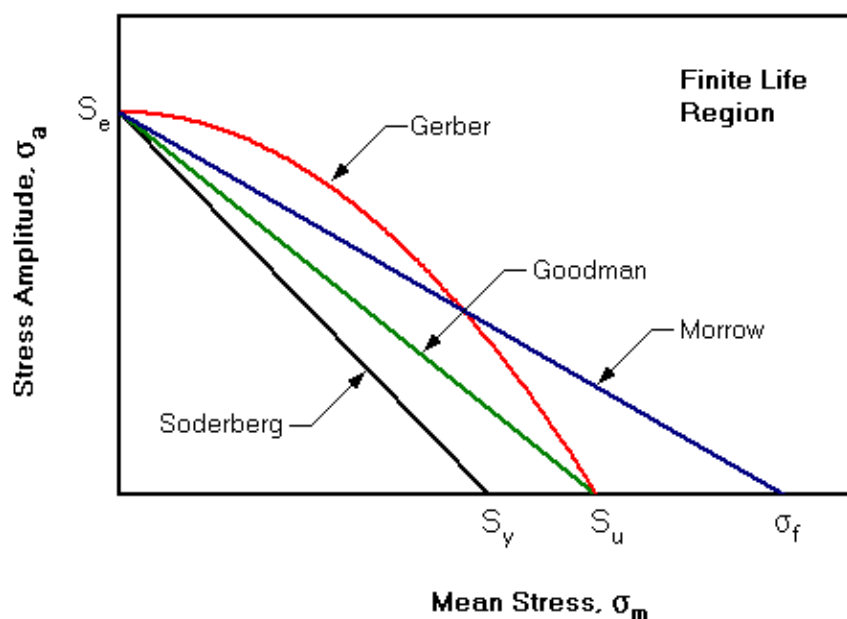
**Gerber (Germany, 1874):** 
$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_u}\right)^2 = 1$$

**Soderberg (USA, 1930):** 
$$\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1$$

**Morrow (USA, 1960s):** 
$$\frac{S_a}{S_e} + \frac{S_m}{\sigma_f} = 1$$

A graphical comparison of these equations is shown in Figure 7. The two most widely accepted methods are those of Goodman and Gerber. Experience has shown that test data tends to fall between the Goodman and Gerber curves. Goodman is often used due to mathematical simplicity and slightly conservative values. Other observations related to the mean stress equations include:

- All methods should only be used for tensile mean stress values.
- For cases where the mean stress is small relative to the alternating stress ( $R \ll 1$ ), there is little difference in the methods.
- The Soderberg method is very conservative. It is used in applications where neither fatigue failure nor yielding should occur.
- For hard steels (brittle), where the ultimate strength approaches the true fracture stress, the Morrow and Goodman curves are essentially equivalent. For ductile steels ( $\sigma_f > S_u$ ), the Morrow model predicts less sensitivity to mean stress.
- As the  $R$  approaches 1, the models show large differences. There is a lack of experimental data available for this condition, and the yield criterion may set design limits.



**Figure 7: Comparison of Mean Stress Equations**

## Notches

Thus far, the discussion of Stress-Life fatigue analysis assumed a smooth, unnotched specimen. However, in practice most fatigue failures occur at notches or stress concentrations.

Almost all machine components and structural members contain some form of geometrical or microstructural discontinuities. These discontinuities, or *stress concentration factors*, often result in maximum local stresses  $S_{\max}$  that are many times greater than the nominal stress  $S$  of the member. In ideally elastic members, the ratio of these stresses is designated the theoretical stress concentration factor  $K_T$ :

$$K_T = \frac{S_{\max}}{S}$$

The theoretical stress concentration factor is solely dependent on the geometry and the mode of loading (axial, in-plane bending, etc.)

In the Stress-Life approach, the effect of notches is accounted for by the *fatigue notch factor*  $K_f$  (also known as the fatigue stress concentration factor). The fatigue notch factor relates the unnotched fatigue strength (the endurance limit for ferrous metals) of a member to its notched fatigue strength:

$$K_f = \frac{S_e(\text{unnotched})}{S_e(\text{notched})}$$

In almost all cases, the fatigue notch factor is less than the stress concentration factor, and is less than 1. That is:

$$1 \leq K_f \leq K_T$$

The stress concentration factor  $K_T$  can be related to the fatigue notch factor  $K_f$ .

Unlike the stress concentration factor  $K_T$ , the fatigue notch factor  $K_f$  is dependent on the type of material and notch size. To account for these additional effects, a *notch sensitivity factor*  $q$  was developed:

$$q = \frac{K_f - 1}{K_T - 1}$$

The values of  $q$  range from 0 (no notch effect,  $K_f = 1$ ) to 1 (full theoretical effect,  $K_f = K_T$ ). A number of researchers have proposed analytical relationships for the determination of  $q$ , based on correlation to experimental data. The most common relationships are those proposed by Peterson and Neuber. Both the Peterson and Neuber relations are empirical curve fits to data. When used for analysis there is little difference to the approaches. Both methods show that  $q$  is related to material, notch geometry, and notch size. Thus, two notches can have the same  $K_T$  value, but different  $K_f$  values because of the different  $q$  values. Both approaches also appear to support a limiting value for  $K_f$ . The limiting value is dependent on material, but is generally between 5 and 6.

The Peterson Equation for notch factor sensitivity is:

$$q = \frac{1}{1 + \frac{a}{r}}$$

where  $r$  is the notch root radius and  $a$  is a material constant.

The constant  $a$  depends on the material strength and ductility and is obtained experimentally from long life fatigue tests on both notched and unnotched specimens. Sharp notches in hard metals tend to be notch sensitive. For ferrous-based wrought metals, the constant  $a$  is given by:

$$a \approx 0.001 \left( \frac{300}{S_u(\text{ksi})} \right)^{1.8} \text{ inch} \approx 0.001 \left( \frac{300}{0.5 \text{ BHN}} \right)^{1.8} \text{ inch}$$

The Neuber Equation for notch factor sensitivity is:

$$q = \frac{1}{1 + \sqrt{\frac{\rho}{r}}}$$

where  $r$  is the notch root radius and  $\rho$  is a material constant that is related to the grain size of the material.

The notch factor  $K_f$  is usually used to correct the fatigue strength for the notched member.

$$S_e = \frac{S'_e}{K_f}$$

Although  $K_f$  can be used to correct the entire S-N curve, results tend to be conservative. Note that this module will calculate  $K_f$ , but will not incorporate it into any solutions. The user must divide the unnotched fatigue strength (and  $S_{1000}$  if desired) by the calculated  $K_f$  value to account for the effect of notches.