MODIFIED SPECTRAL FATIGUE METHODS FOR S-N CURVES WITH MIL-HDBK-5J COEFFICIENTS
27-30 September 2016, Toulouse, France

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ABSTRACT

The rainflow method is used for counting fatigue cycles from a stress response time history, where the fatigue cycles are stress-reversals. The rainflow method allows the application of Palmgren-Miner's rule in order to assess the fatigue life of a structure subject to complex loading. The fatigue damage may also be calculated from a stress response power spectral density (PSD) using the semi-empirical Dirlik, Single Moment, Zhao-Baker and other spectral methods. These methods effectively assume that the PSD has a corresponding time history which is stationary with a normal distribution. This paper shows how the probability density function for rainflow stress cycles can be extracted from each of the spectral methods. This extraction allows for the application of the MIL-HDBK-5J fatigue coefficients in the cumulative damage summation. A numerical example is given in this paper for the stress response of a beam undergoing random base excitation, where the excitation is applied separately by a time history and by its corresponding PSD. The fatigue calculation is performed in the time domain, as well as in the frequency domain via the modified spectral methods. The result comparison shows that the modified spectral methods give comparable results to the time domain rainflow counting method.

1. INTRODUCTION

The material S-N curve is required for the fatigue calculation in either the time or frequency domain. The S-N curve for a given material expresses the threshold stress level for a given number of cycles. Fatigue failure occurs if the threshold is exceeded, but a safety margin may be applied for conservatism. This approach is straightforward for the idealized case of a steady sinusoidal response. But rainflow cycle counting [1] is needed for the case of a complex response. The Palmgren-Miners cumulative damage method can be used for time-varying stress cases, such as random oscillations. A simple way to represent S-N Curves is a straight line in log-log format, called a Basquin curve, which is defined by two coefficients, the fatigue strength coefficient and the fatigue exponent. The Dirlik and other spectral methods [2] assume a Basquin curve for their respective cumulative damage calculations. Note that this approach does not account for the stress ratio in equation (1) or an endurance

limit. But documents such as [3] give equivalent equations for S-N curves with up to four coefficients via the following formulas. The material-dependent coefficients are A, B, C, and P. Note that C is the endurance limit and is zero for some materials.

2. S-N CURVE EQUATION

The stress ratio R is

$$R = S_{min} / S_{max} \tag{1}$$

S_{min} and S_{max} are the respective minimum and maximum stresses in a particular stress cycle. The stress magnitude is equal to both S_{max} and the absolute value of S_{min} for R = -1, which represents fully reversed stress cycles. Note that R in equation (1) is calculated separately for each rainflow cycle for a time domain damage analysis. A constant value of R = -1 can be used for the spectral damage calculation, which effectively shifts the stress range of each cycle so that its stress midpoint is zero. Any discrepancy between the damage between the time and spectral methods is at least partially due to the R values. The equivalent stress S_{eq} is given in [3] by

$$S_{eq} = S_{max} (1 - R)^P \tag{2}$$

The equivalent stress is not a true peak-to-peak range. Neither is it an amplitude stress (range/2). Rather it is a stress metric which weighs the contributions of the tension and compression components for the purpose of the S-N curve equation. The critical number of cycles $N_{f,i}$ for a given stress level is

$$\log N_{f,i} = A - B \log (S_{eq,i} - C) \tag{3}$$

$$N_{f,i} = 10 \char`\^ \left[A - B \log (S_{eq,i} - C) \right] \tag{4}$$

The coefficients A, B, C and P vary by material and are given in [3].

3. DIRLIK METHOD

The Dirlik histogram formula n(S) for stress cycles ranges is

$$n(S) = \nu_p \cdot T \cdot p_{dk} (S) \tag{5}$$

where

ν_p is the expected peak rate

T   is the duration

S   is the stress cycle range (peak-to-peak)

Note that the expected peak rate ν_p is

$$\nu_p = \sqrt{m_4/m_2} \qquad (6)$$

The *j*th spectral moment m_j for a PSD is

$$m_j = \int_0^\infty f^j\, G(f)\, df \qquad (7)$$

where

        f is frequency

        $G(f)$ is the one-sided PSD

The Dirlik equation is based on the weighted sum of the exponential Gaussian, Rayleigh and exponential probability distributions. The continuous probability function $p_{dk}\big(Z(S)\big)$ prior to normalization is

$$p_{dk}\big(Z(S)\big) = \left\{ \frac{D_1}{Q}\exp\left(-\frac{Z}{Q}\right) + \frac{D_2\, Z}{R^2}\exp\left(\frac{-Z^2}{2R^2}\right) + D_3\, Z\exp\left(\frac{-Z^2}{2}\right) \right\} \frac{1}{2\sqrt{m_0}} \qquad (8)$$

The coefficients and variables are

$$Z = \frac{S}{2\sqrt{m_0}} \qquad\qquad D_1 = \frac{2\left(x_m - \gamma^2\right)}{1+\gamma^2}$$

$$D_2 = \frac{1-\gamma-D_1+D_1^2}{1-R} \qquad\qquad D_3 = 1 - D_1 - D_2$$

$$Q = \frac{1.25\left(\gamma - D_3 - D_2 R\right)}{D_1} \qquad\qquad R = \frac{\gamma - x_m - D_1^2}{1-\gamma-D_1+D_1^2}$$

$$\gamma = \frac{m_2}{\sqrt{m_0\, m_4}} \qquad\qquad x_m = \frac{m_1}{m_0}\sqrt{\frac{m_2}{m_4}}$$

Now transform $p_{dk}\big(Z(S)\big)$ into the corresponding digital probability function $p_{dk,i}$.

$$p_{dk,i} = \left\{ \frac{D_1}{Q}\exp\left(-\frac{Z_i}{Q}\right) + \frac{D_2\, Z_i}{R^2}\exp\left(\frac{-Z_i^2}{2R^2}\right) + D_3\, Z_i\exp\left(\frac{-Z_i^2}{2}\right) \right\} \frac{1}{2\sqrt{m_0}} \qquad (9)$$

where q is the total number of stress range bins

The Dirlik cumulative damage equation D_{DK} is

$$D_{DK} = v_p \ T \sum_{i=1}^{q} p_{dk,i} \left\{ 10^{\wedge}\left[A - B \log\left(S_{eq,i} - C\right) \right] \right\}^{-1} \Delta Z_i \tag{10}$$

Equation (9) effectively integrates the stress range histogram.

4. NARROWBAND BASED METHODS

4.1 Narrowband Method

The following methods are taken from [2]. The narrowband histogram formula n(S) for stress cycles ranges is

$$n(S) = v_0^+ \cdot T \cdot p_{nb}\left(Z(S)\right) \tag{11}$$

The rate of zero up-crossings v_0^+ can be estimated as

$$v_0^+ = \sqrt{m_2 / m_0} \tag{12}$$

The narrowband spectral method is derived using only the Rayleigh term from equation (6).

$$p_{nb,i} = Z_i \, \exp\left(\frac{-Z_i^{\,2}}{2} \right) \frac{1}{2\sqrt{m_0}} \tag{13}$$

The narrowband cumulative damage equation for the narrowband case D_{NB} is

$$D_{NB} = v_p \ T \sum_{i=1}^{q} p_{nb,i} \left\{ 10^{\wedge}\left[A - B \log\left(S_{eq,i} - C\right) \right] \right\}^{-1} \Delta Z_i \tag{14}$$

where q is the total number of stress range bins

4.2 Single-Moment Method

The Larsen-Lutes, single-moment damage D_{LL} is

$$D_{LL} = \lambda_L D_{NB} \tag{15}$$

The scale factor λ_L is

$$\lambda_L = \frac{\left(M_{2/m}\right)^{m/2}}{\sigma_S^m \, \nu_0^+} \tag{16}$$

Note that the $\left(M_{2/m}\right)^{m/2}$ term is evaluated using $j = 2/m$ in equation (7). σ_S^m is the RMS stress raised to the fatigue exponent.

4.3 Alpha 0.75 Method

The Benasciutti & Tovo $\alpha_{0.75}$ damage D_{AL} is

$$D_{AL} = \lambda_\alpha D_{NB} \tag{17}$$

The scale factor λ_α is

$$\lambda_\alpha = \alpha_{0.75}^2 \tag{18}$$

where

$$\alpha_{0.75} = \frac{m_{0.75}}{\sqrt{m_0 \, m_{1.5}}} \tag{19}$$

5. BEAM BENDING EXAMPLE

Again, Dirlik and other spectral methods assume a Basquin curve for their respective cumulative damage calculations. The Palmgren-Miners calculation is effectively embedded in the final form of their respective equation. The key is to extract the rainflow stress-cycles histogram from the spectral method of interest. The histogram can then be divided into a series of bars with a center stress for the base and the number of cycles for the height. A large number of bins is recommended. The number in the following example is 400.
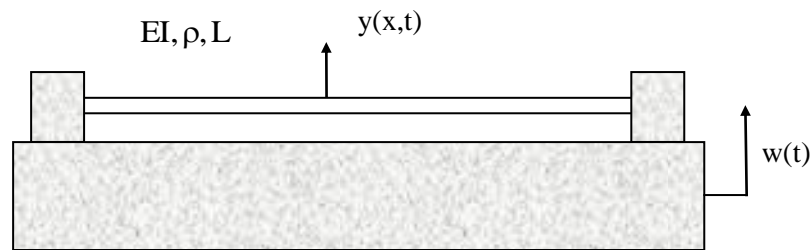

Fig. 1. Fixed-Fixed Beam Subjected to Base Excitation

The beam properties are shown in Table 1. The natural frequencies for the first five modes are given in Table 2, as calculated using the method in [4]. The fundamental bending mode is shown in Fig. 2.

Table 1. Beam Properties	
Material	Aluminum 7075-T6
Length	28 inches
Cross Section	Solid Cylinder
Diameter	0.5 inches
Amplification Factor	Q=10 for all modes

Table 2. Natural Frequencies		
Mode	Natural Frequency (Hz)	Effective Modal Mass Percent
1	111.5	69.0
2	307.5	0
3	602.8	13.2
4	996.4	0
5	1488	5.4

NORMALIZED BENDING MODE
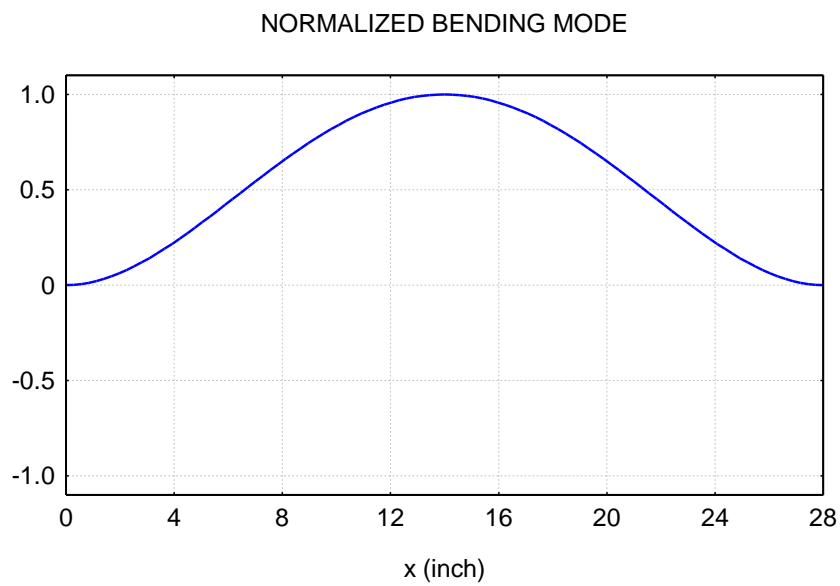
Fig. 2. Fundamental Bending Mode, 111.5 Hz

ACCELERATION POWER SPECTRAL DENSITY 48.1 GRMS Overall
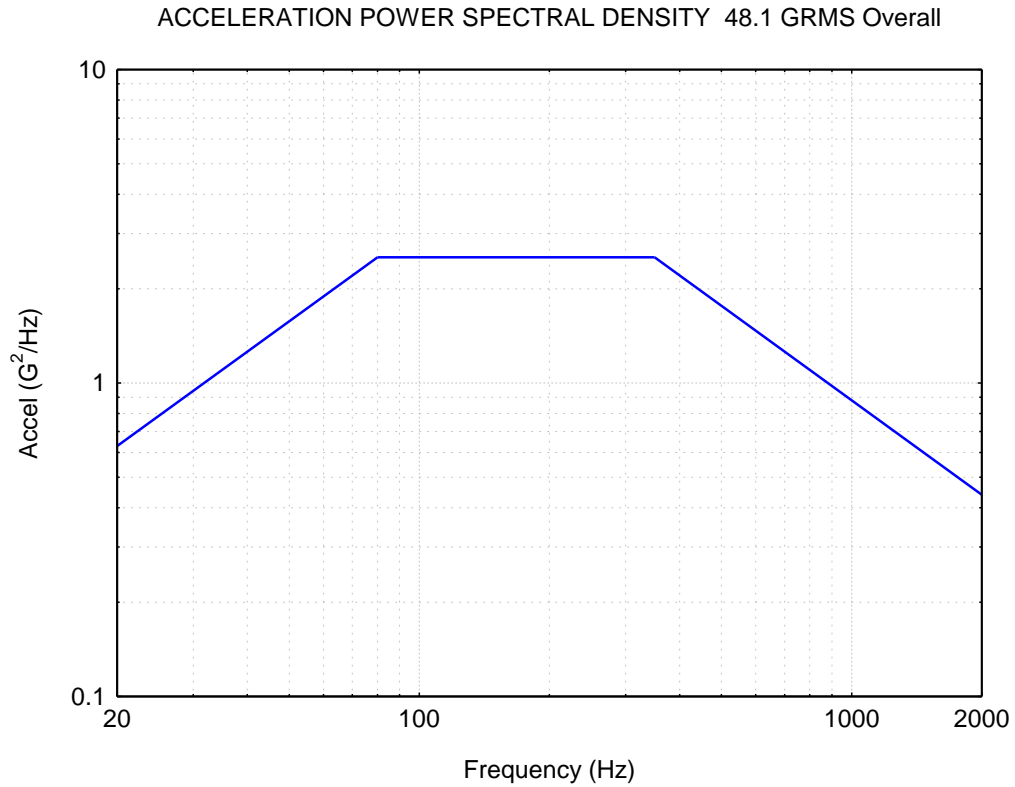
Power Spectral Density	
Frequency (Hz)	Accel (G^2/Hz)
20	0.63
80	2.52
350	2.52
2000	0.44

Fig. 3. NAVMAT P-9492 PSD + 18 dB

The base input is the NAVMAT P-9492 PSD + 18 dB for 610 seconds, as shown in Fig. 3. A time history is synthesized to meet this specification via [5] as shown in Fig. 4. The PSD and time history are applied separately as base excitation to the beam. The stress response for the PSD is shown in Fig. 5. The time domain response is given in Fig. 6, as analyzed using [6]. The highest stress occurs at each of the beam's boundaries. Calculate the bending stress at the left boundary. Then perform a cumulative damage analysis separately for the PSD and time history inputs. A classical modal solution is used for a continuous beam using Matlab scripts using the formulas in [6] and [7].
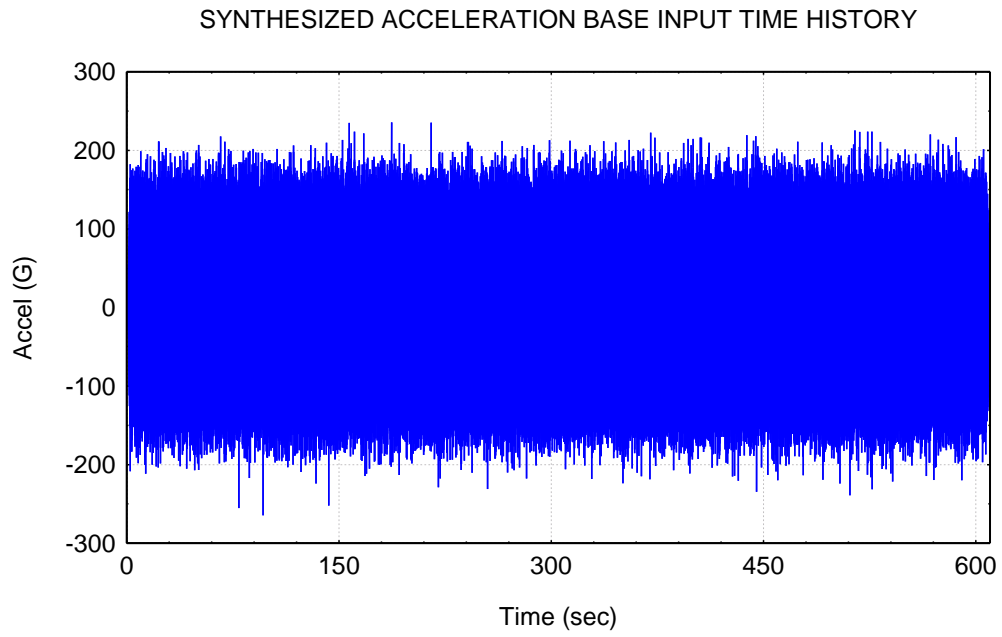
SYNTHESIZED ACCELERATION BASE INPUT TIME HISTORY
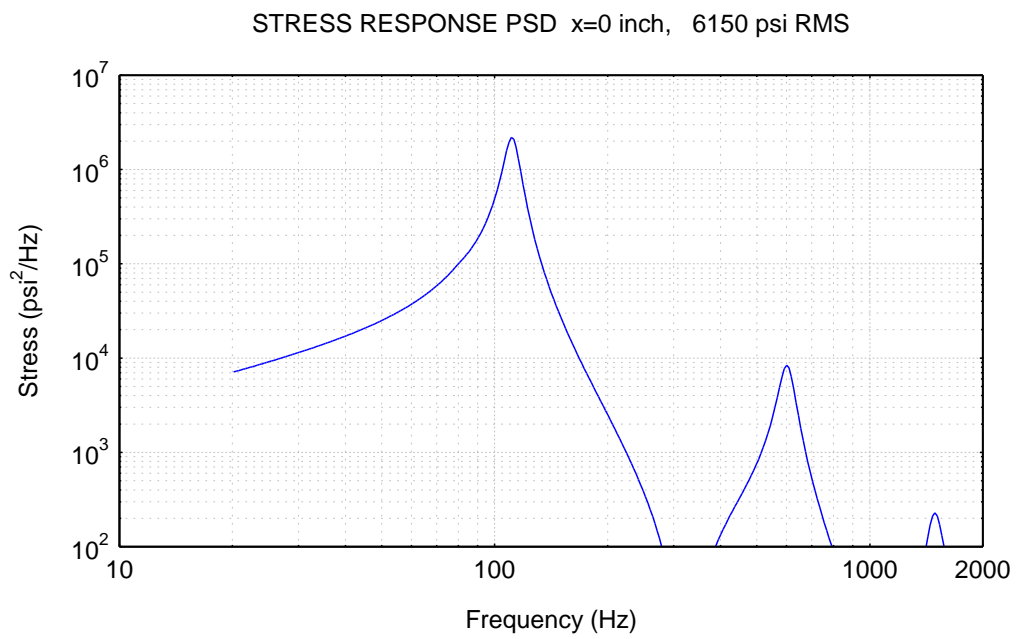

Fig. 4. Time History Synthesis

STRESS RESPONSE PSD x=0 inch,   6150 psi RMS

Fig. 5. Stress Response PSD
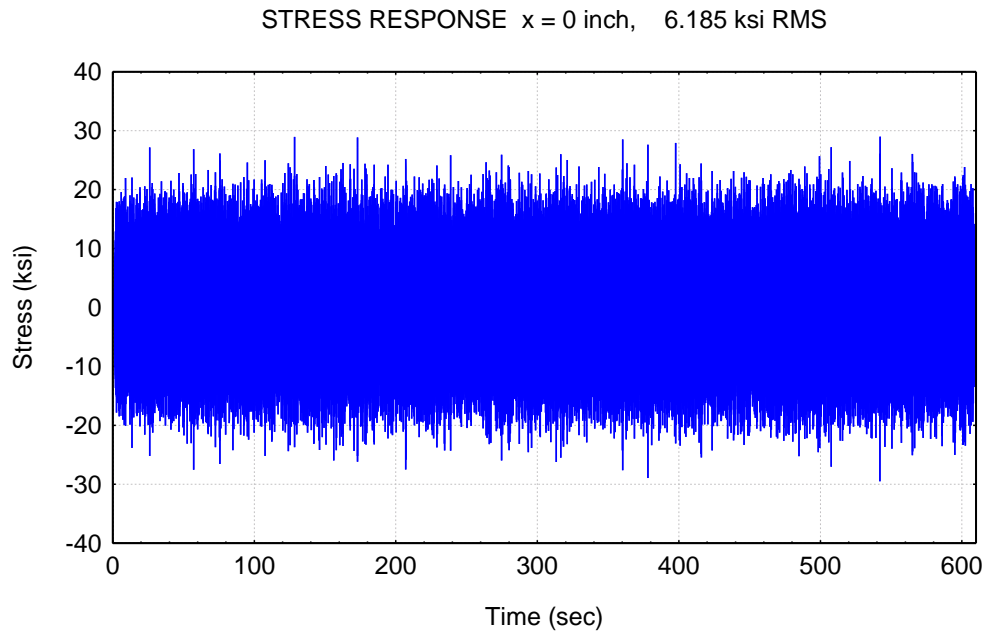
STRESS RESPONSE x = 0 inch,   6.185 ksi RMS

Fig. 6. Stress Response Time History, without Stress Concentration Factor

Assume a stress concentration factor of 2.5 at the beam's left boundary for conservatism. The stress concentration factor is not included in the response plots, but is applied to the fatigue calculations.   The damage results are given in Table 3.

Table 3. Damage Results, Al 7075-T6, SCF=2.5	
Analysis	Damage Rate (1/sec)
Dirlik Spectral	0.00030
Single Moment Spectral	0.00028
Alpha 0.75 Spectral	0.00032
Rainflow Time Domain	0.00033

Now repeat the example for Al 6061-T6 with a stress concentration factor of 1.5. The results are shown in Table 4.

Table 4. Damage Results, 6061-T6, SCF=1.5	
Analysis	Damage Rate (1/sec)
Dirlik Spectral	0.00018
Single Moment Spectral	0.00017
Alpha 0.75 Spectral	0.00019
Rainflow Time Domain	0.00020

For simplicity the two alloys were assumed to have the same elastic modulus, Possion's ratio and mass density. Thus the same bending stress time history and PSD could be used for each case. But the fatigue stress equations differ. Also note that Al 7075-T6 has an endurance limit, but Al 6061-T6 does not. Note that the total number of rainflow cycles in the time domain was 250,284 for each case.

6. CONCLUSIONS

The spectral and time domain methods gave similar results for each material in the examples. Further cases should run by varying the amplification factor, boundary conditions, base input, etc. In addition, the full array of spectral methods in [8] should be converted for use with the MIL-HDBK-5J S-N curves coefficients for a given material. Again, this requires extraction of their histogram functions. Further attention is needed to determine the sensitivity of each spectral method to the stress ratio R.

7. REFERENCES

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