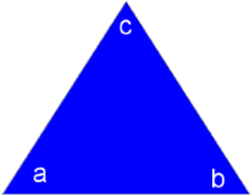
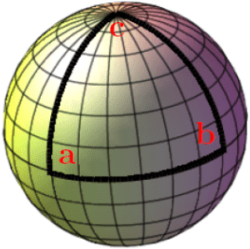
Dynamic Time Warping

GeoComput & ML

2021-05-18 Tue



$$a + b + c = 180 \text{ degrees}$$



$$a + b + c = 270 \text{ degrees!}$$

Configuration

Dynamic time warping (DTW) is a technique designed for optimally aligning two time sequences across their time points under certain restrictions.

Given two time sequences,

$$X = x_1, x_2, \ldots, x_N$$
$$Y = y_1, y_2, \ldots, y_M$$

we define a non-negative local dissimilarity function between element pairs x_i and y_j

$$d(i, j) \geq 0$$

Time normalised distance

We seek a warping function ϕ to remap the time indices of X and Y

$$\phi(k) = (\phi_x(k), \phi_y(k))$$

where $\phi_x(k) \in \{1 \ldots N\}$ and $\phi_y(k) \in \{1 \ldots M\}$

such that

$$D(X,Y) = \min_{\phi} \sum_{k=1}^{T} d(\phi_x(k), \phi_y(k)) m_{\phi}(k) \Big/ M_{\phi}$$

Constraints

- monotonicity

$$\begin{aligned}\phi_x(k+1) \geq& \phi_x(k)\\ \phi_y(k+1) \geq& \phi_y(k)\end{aligned}$$

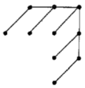
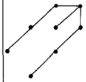
- continuity

$$\phi_x(k+1) - \phi_x(k) \leq 1$$
$$\phi_y(k+1) - \phi_y(k) \leq 1$$

- boundary conditions

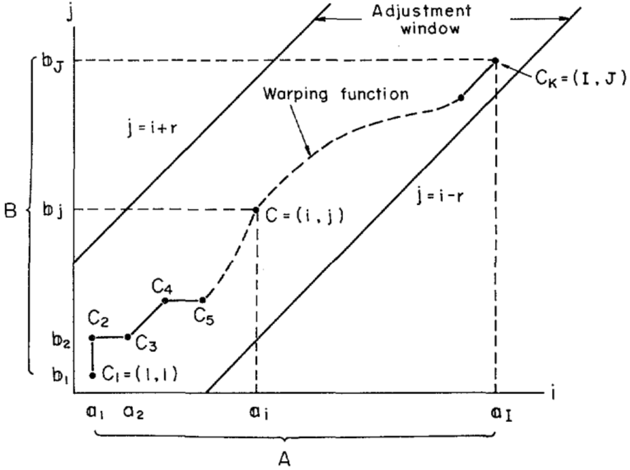
$$\phi_x(1) = 1;\ \phi_y(1) = 1$$
$$\phi_x(T) = N;\ \phi_y(T) = M$$

Step Patterns

P	Schematic explanation	Symmetric / Asymmetric	DP-equation $g(i, j) =$
0		Symmetric	$\min\left[\begin{array}{l} g(i,j-1)+d(i,j) \\ g(i-1,j-1)+2d(i,j) \\ g(i-1,j)+d(i,j) \end{array}\right]$
		Asymmetric	$\min\left[\begin{array}{l} g(i,j-1) \\ g(i-1,j-1)+d(i,j) \\ g(i-1,j)+d(i,j) \end{array}\right]$
1/2		Symmetric	$\min\left[\begin{array}{l} g(i-1,j-3)+2d(i,j-2)+d(i,j-1)+d(i,j) \\ g(i-1,j-2)+2d(i,j-1)+d(i,j) \\ g(i-1,j-1)+2d(i,j) \\ g(i-2,j-1)+2d(i-1,j)+d(i,j) \\ g(i-3,j-1)+2d(i-2,j)+d(i-1,j)+d(i,j) \end{array}\right]$
		Asymmetric	$\min\left[\begin{array}{l} g(i-1,j-3)+(d(i,j-2)+d(i,j-1)+d(i,j))/3 \\ g(i-1,j-2)+(d(i,j-1)+d(i,j))/2 \\ g(i-1,j-1)+d(i,j) \\ g(i-2,j-1)+d(i-1,j)+d(i,j) \\ g(i-3,j-1)+d(i-2,j)+d(i-1,j)+d(i,j) \end{array}\right]$
1		Symmetric	$\min\left[\begin{array}{l} g(i-1,j-2)+2d(i,j-1)+d(i,j) \\ g(i-1,j-1)+2d(i,j) \\ g(i-2,j-1)+2d(i-1,j)+d(i,j) \end{array}\right]$
		Asymmetric	$\min\left[\begin{array}{l} g(i-1,j-2)+(d(i,j-1)+d(i,j))/2 \\ g(i-1,j-1)+d(i,j) \\ g(i-2,j-1)+d(i-1,j)+d(i,j) \end{array}\right]$
2		Symmetric	$\min\left[\begin{array}{l} g(i-2,j-3)+2d(i-1,j-2)+2d(i,j-1)+d(i,j) \\ g(i-1,j-1)+2d(i,j) \\ g(i-3,j-2)+2d(i-2,j-1)+2d(i-1,j)+d(i,j) \end{array}\right]$
		Asymmetric	$\min\left[\begin{array}{l} g(i-2,j-3)+2(d(i-1,j-2)+d(i,j-1)+d(i,j))/3 \\ g(i-1,j-1)+d(i,j) \\ g(i-3,j-2)+d(i-2,j-1)+d(i-1,j)+d(i,j) \end{array}\right]$

Warping Window

$$\phi_x(k) - \phi_y(k) \leq T_0$$



Computational Performance

- quite fast
- 4G RAM + 4G swap : 6000x6000 in 10s
- larger problems : downsampling, breaking-up and interative matching, etc.

Wavelet Transformation

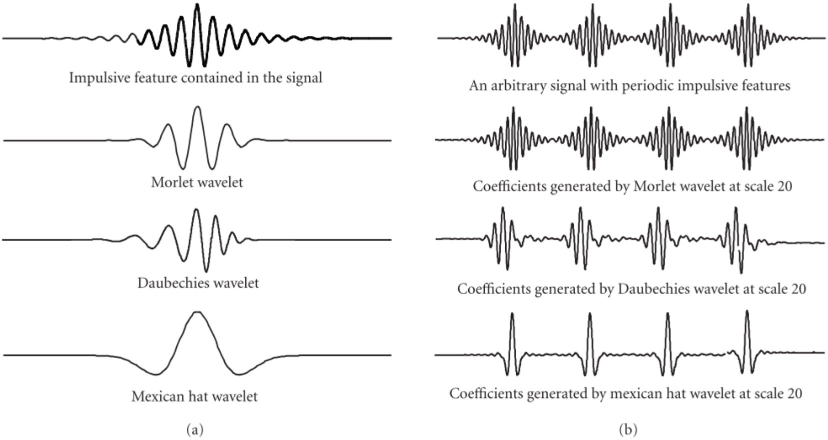
Let ϕ be the mother wavelet and her daughter wavelets can be constructed via dilation and translation.

$$\psi_{a,b} = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$$

Wavelet transformation of a signal $x(t)$ is defined as the inner product as follows.

$$WT(a,b) = \langle \psi_{a,b}, x(t) \rangle$$

Maximum Matching



cw-DTW

Algorithm

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Input s1, s2, r
Repeat DTW(CW(s1,r), C(s2,r))
Until max matching
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References

H. Sakoe, S. Chiba. IEEE. Trans. Acous. Speech. SigProc. (1978) 26, 43

B. Berndt, J. Clifford. AAA1-94 Workshop on Knowledge discovery in databases (1994) 229

T. Giorgino, J. Stat. Soft. (2009) 31, 1

R. Han, *et. al.*, Bioinformatics, (2018), 34, i722

W. Yang, X. Ren, EURASIP J. Appl. Sig. Proc. (2014) 8, 1156

L. Rabiner, B-H. Juang. Fundamentals of speech recognition (1993)

E. Maharaj. Time series clustering and classification (2019)

https://www.hawking.org.uk/in-words/lectures/space-and-time-warps

https://www.thephysicsmill.com/2015/12/27/measuring-the-curvature-of-spacetime-with-the-geodetic-effect/

https://en.wikipedia.org/wiki/Ricker_wavelet

https://en.wikipedia.org/wiki/Wavelet_transform