Geomorphometry

BIO401-01/598-02

2021-03-24 Wed

Git

Three copies

1. server copy : course materials enrichment/update
2. local src. copy (~/SE_data) : sync. w/ server
3. local working copy (/media/sf_LVM_Shared/my_SE_data) : for your work

Synchronisation

w/ server

```
$ cd  ~/SE_data ; git pull
```

in case of pull failure (likely due to you made changes)

```
$ rm -rf ~/SE_data
$ git clone https://github.com/selvaje/SE_data
```

local sync.

```
$ myCopy=/media/sf_LVM_Shared/my_SE_data
$ rsync -hvrPt --ignore-existing  ~/SE_data/* ${myCopy}
```

Probability Theory

- Proposition 3

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

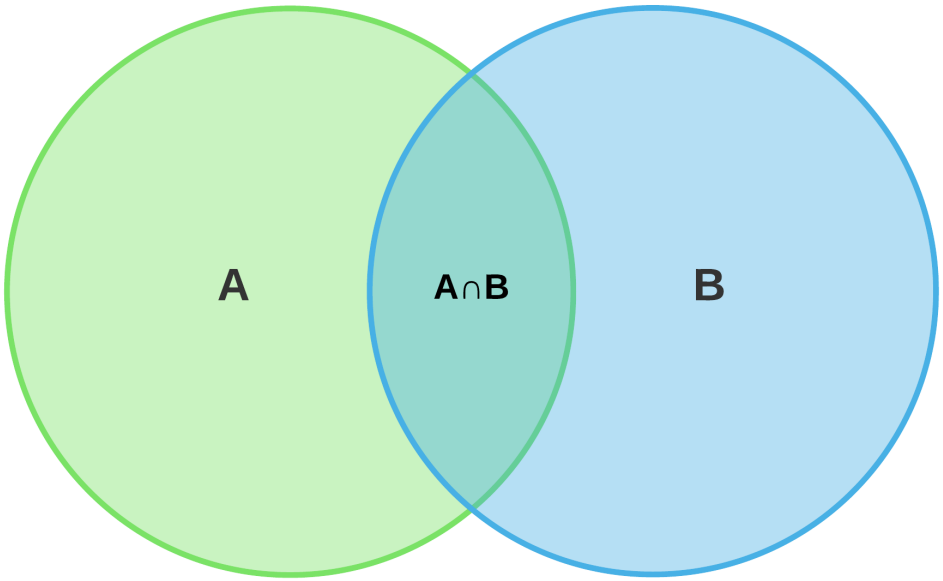
- Independent Events

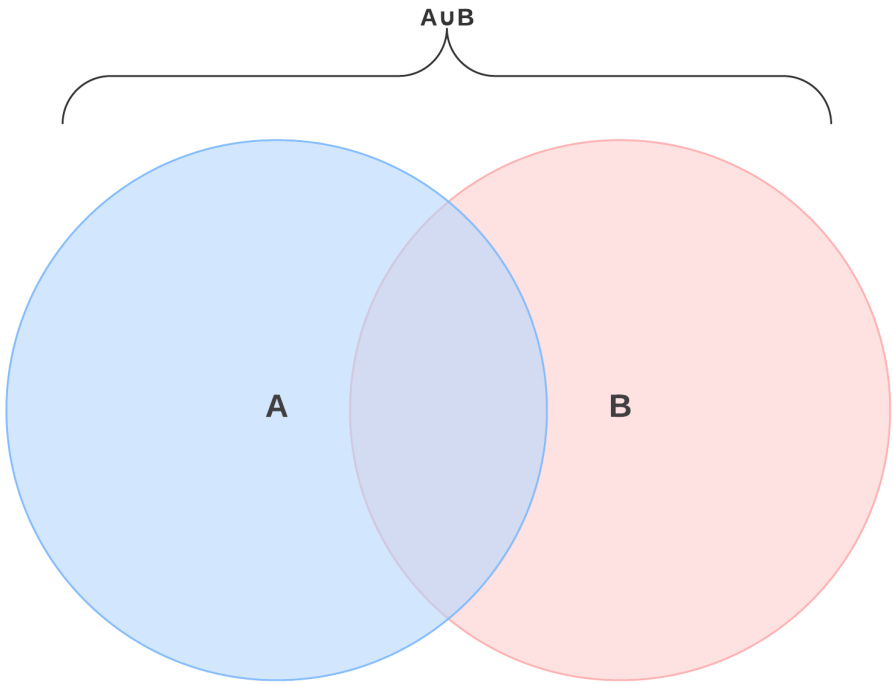
$$P(A \cap B) = P(A)P(B)$$

- Law of Total Probability

$$P(B) = \sum_{k=1}^{\infty} P(B|A_i)P(A_i)$$

Venn diagram





Exercise

Two neighbours (A and B) are chatting over their fence when a man passes by. A says to B, "This is the former senator." B responds, "Yes. He is the former senator who got into a scandal and had to resign." "Oh, Right", says A, "That sounds more probable." "No", says B, "I think you mean less probable."

Please use the knowledge of probability to explain the rationale behind the last statement of B.

Exercise : Simpson's Paradox

Table of admission rates (A) for males and females on difficult and easy subjects

	Male (M)	Female (F)
Easy (E)	864/1385	106/133
Difficult (D)	334/1036	551/1702

$P(A|M) \approx 0.49 > P(A|F) \approx 0.35$
$P(A|F \cap D) \approx 0.32$ and $P(A|F \cap E) \approx 0.80$
$P(A|M \cap D) \approx 0.32$ and $P(A|M \cap E) \approx 0.62$

So, $P(A|M \cap D) = P(A|F \cap D)$ and $P(A|M \cap E) < P(A|F \cap E)$
but why $P(A|M) > P(A|F)$?

Simpson's Paradox (contd.)

Hint

$$P(A|F) = P(A|F \cap D)P(D|F) + P(A|F \cap E)P(E|F)$$
$$P(A|M) = P(A|M \cap D)P(D|M) + P(A|M \cap E)P(E|M)$$

$P(D|F) = 1702/1832 \approx 0.93$ and $P(E|F) \approx 0.07$
$P(D|M) = 1385/2421 \approx 0.57$ and $P(E|M) \approx 0.43$

Scaling

- central problem in ecology
- spatiotemporal dimension of a pattern or process

Tobler's First Law of Geography

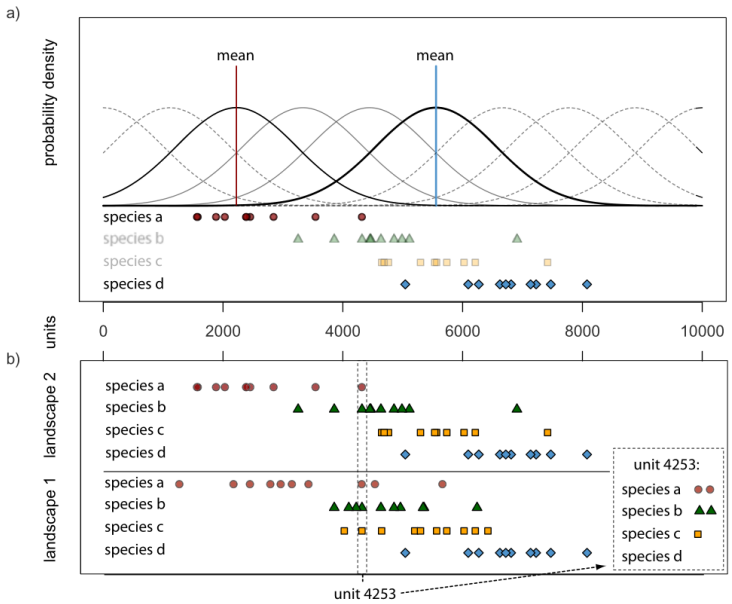
`everything is related to everything else,`
`but near things are more related than distant things`

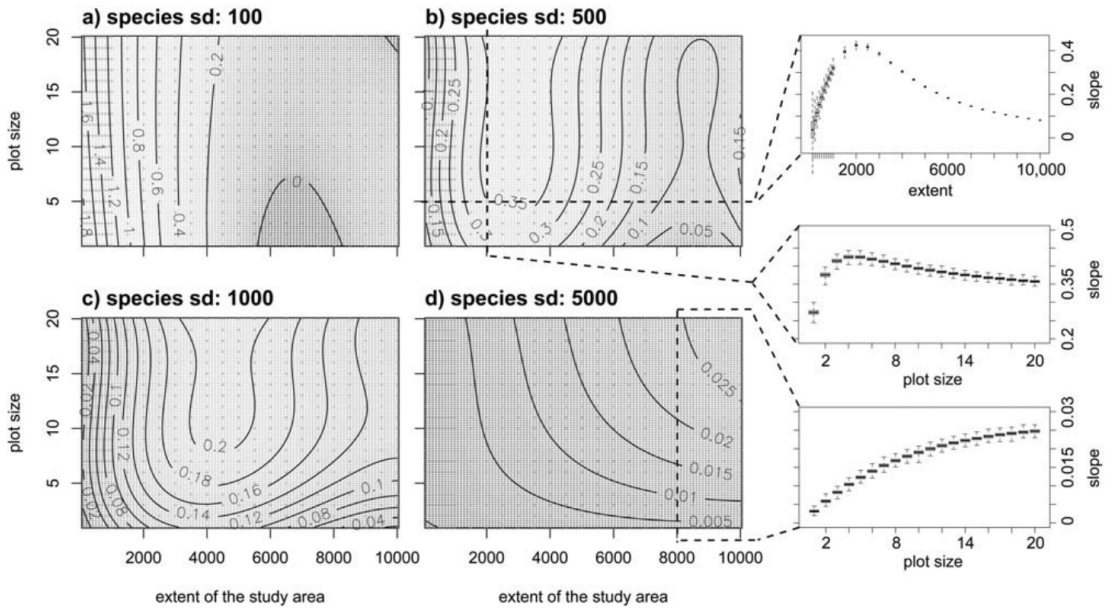
Species distribution modelling

relating environmental variables to observed species abundance

Scaling

- distance decay





Geomorphometry

- vector : a sequence of numbers
- matrix : a rectangular array of numbers

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Digital Elevation Model (DEM)

two-dimensional discrete function of elevation of the topographic surface

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Digital Elevation Model (DEM)

two-dimensional discrete function of elevation
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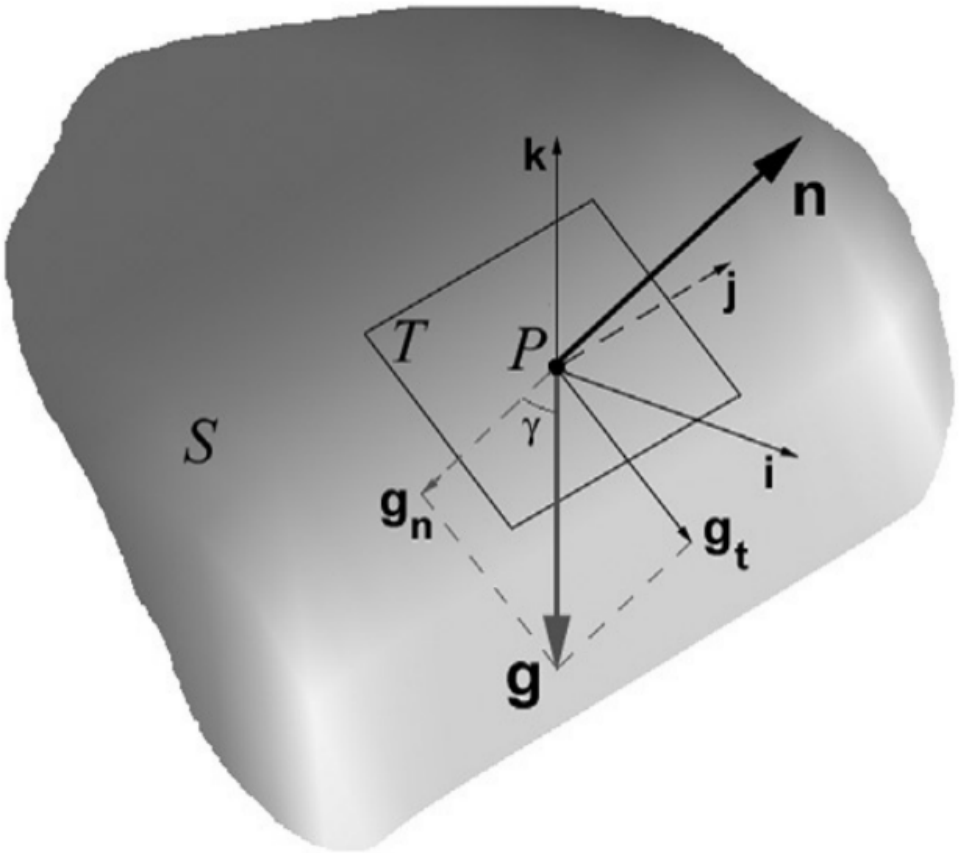
Digital Terrain Model (DTM)

two-dimensional discrete function of morphometric variables,
to describ the topographic surface
produced based on DEM

Slope



$\boldsymbol{n}$: norm direction

$\boldsymbol{g}$: gravitational acceleration vector

$\boldsymbol{g_t}$: tangential component

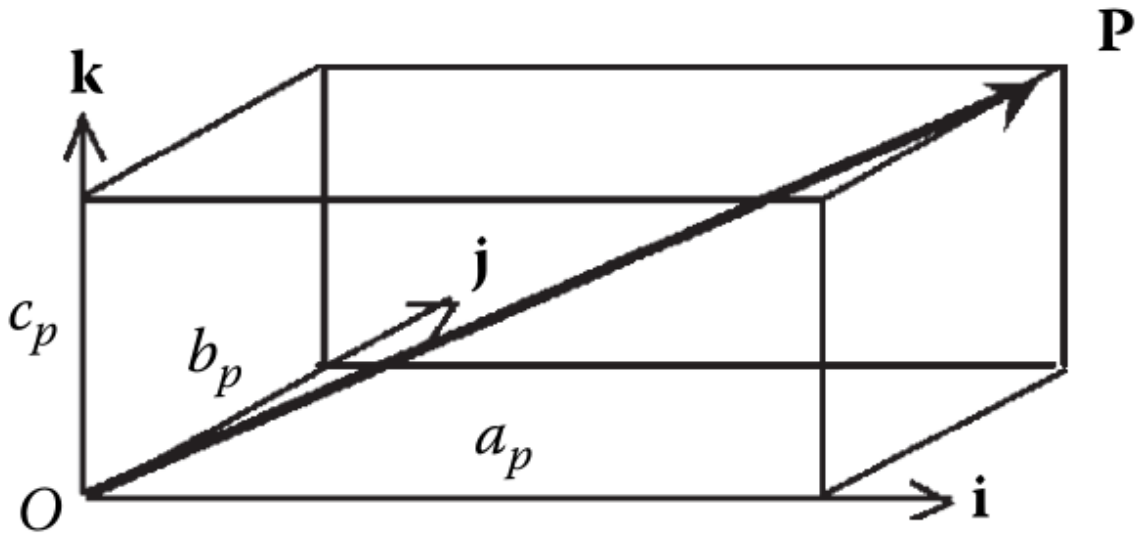
$\boldsymbol{g_n}$: normal component

$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$: unit vectors in directions of x,y,z coordinates

T : tangent plane

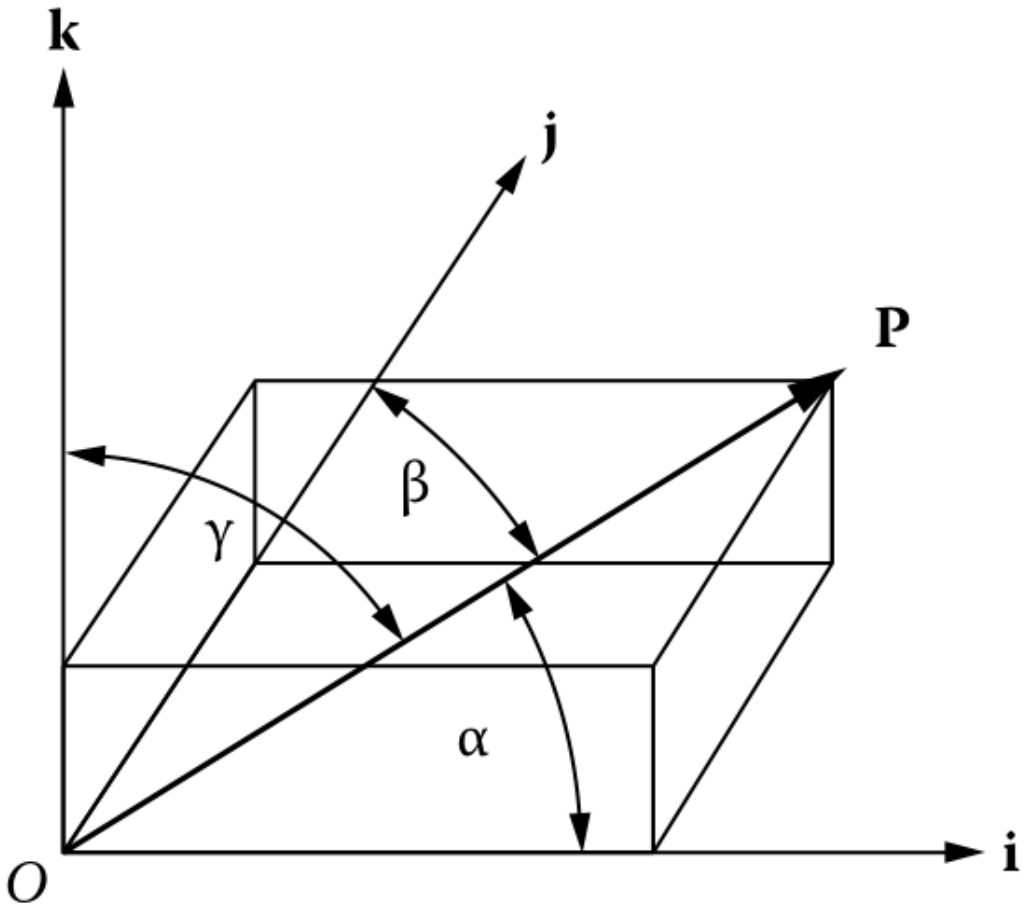
Vectors

- vector : dimensional quantity associated with directions, like velocity
- scalar : no directions, like weight



$\boldsymbol{OP} = a_p\boldsymbol{i} + b_p\boldsymbol{j} + c_p\boldsymbol{k}$
where $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are unit vectors.

Projection



- Pythagoras

$$|\boldsymbol{OP}| = \sqrt{a_p^2 + b_p^2 + c_p^2}$$

$$a_p = |\boldsymbol{OP}| cos\alpha$$
$$b_p = |\boldsymbol{OP}| cos\beta$$
$$c_p = |\boldsymbol{OP}| cos\gamma$$

Dot Product

Given two vectors, $\boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}$ and $\boldsymbol{b} = b_x\boldsymbol{i} + b_y\boldsymbol{j} + b_z\boldsymbol{k}$. Their dot product is thus defined as.

$$\begin{aligned}\boldsymbol{a}\cdot\boldsymbol{b} &= |\boldsymbol{a}||\boldsymbol{b}|cos(\theta)\\ &= a_xb_x + a_yb_y + a_zb_z\end{aligned}$$

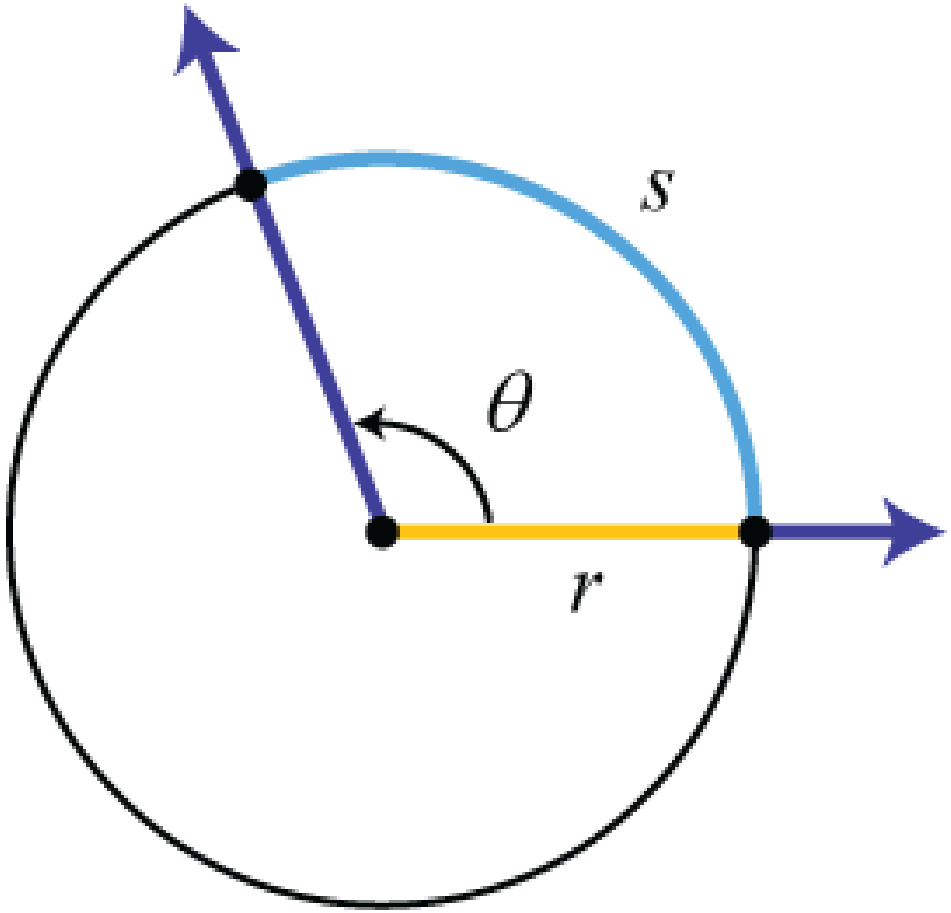
Example

Two vectors P(5,9,2) and Q(3,7,4) in a 3d-space. Find out the angle between them.

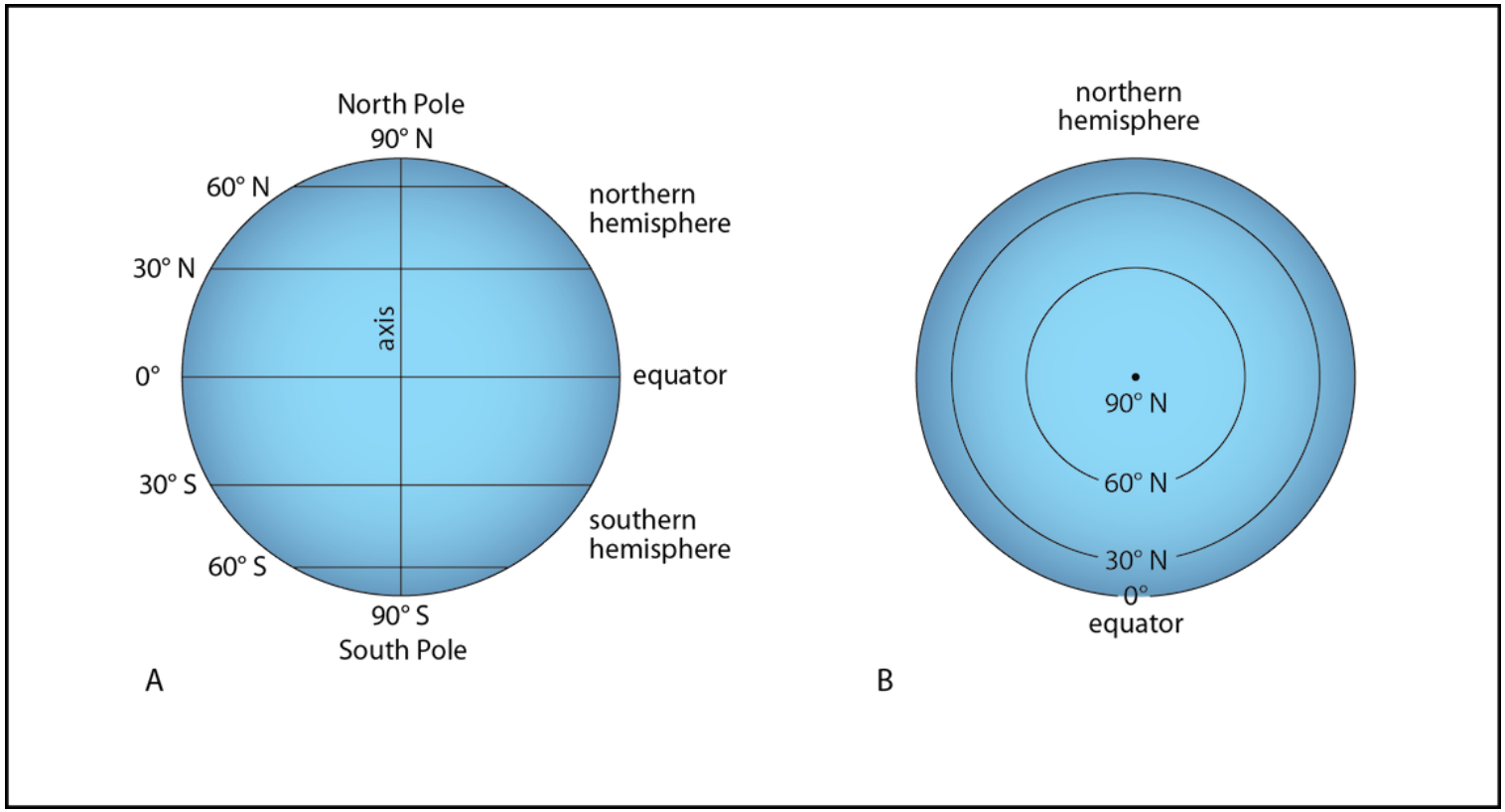
$$\begin{aligned}\boldsymbol{OP}\cdot\boldsymbol{OQ} &= 15 + 63 + 8 = 86\\ &= \sqrt{110}\sqrt{74}\,cos\theta\\ cos\theta = 0.953 \Rightarrow \theta &= cos^{-1}0.953 = 17.6^\circ\end{aligned}$$

Radian

$$\theta \text{ (radians)} = \frac{s}{r}$$



Earth



$$\left.\begin{aligned} arc^{\circ} &= 2\pi r\theta/360 \\ r_{eq} &= 6357 \end{aligned}\right\} \Rightarrow 1arc^{\circ} = 110.9506$$

References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