

Geomorphometry

BIO401-01/598-02

2021-03-24 Wed

Three copies

1. server copy : course materials enrichment/update
2. local src. copy (~ /SE_data) : sync. w/ server
3. local working copy (/media/sf_LVM_Shared/my_SE_data) :
for your work

Synchronisation

w/ server

```
$ cd ~/SE_data ; git pull
```

in case of pull failure (likely due to you made changes)

```
$ rm -rf ~/SE_data
```

```
$ git clone https://github.com/selvaje/SE_data
```

local sync.

```
$ myCopy=/media/sf_LVM_Shared/my_SE_data
```

```
$ rsync -hvrPt --ignore-existing ~/SE_data/* ${myCopy}
```

- Proposition 3

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

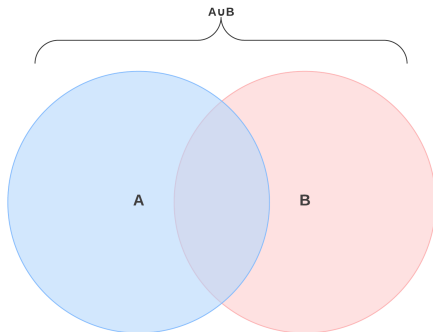
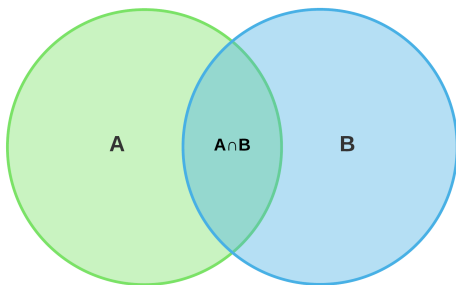
- Independent Events

$$P(A \cap B) = P(A)P(B)$$

- Law of Total Probability

$$P(B) = \sum_{k=1}^{\infty} P(B|A_i)P(A_i)$$

Venn diagram



Exercise

Two neighbours (A and B) are chatting over their fence when a man passes by. A says to B, "This is the former senator." B responds, "Yes. He is the former senator who got into a scandal and had to resign." "Oh, Right", says A, "That sounds more probable." "No", says B, "I think you mean less probable."

Please use the knowledge of probability to explain the rationale behind the last statement of B.

Exercise : Simpson's Paradox

Table of admission rates (A) for males and females on difficult and easy subjects

	Male (M)	Female (F)
Easy (E)	864/1385	106/133
Difficult (D)	334/1036	551/1702

$$P(A|M) \approx 0.49 > P(A|F) \approx 0.35$$

$$P(A|F \cap D) \approx 0.32 \text{ and } P(A|F \cap E) \approx 0.80$$

$$P(A|M \cap D) \approx 0.32 \text{ and } P(A|M \cap E) \approx 0.62$$

$$\text{So, } P(A|M \cap D) = P(A|F \cap D) \text{ and } P(A|M \cap E) < P(A|F \cap E)$$

but why $P(A|M) > P(A|F)$?

Simpson's Paradox (contd.)

Hint

$$P(A|F) = P(A|F \cap D)P(D|F) + P(A|F \cap E)P(E|F)$$

$$P(A|M) = P(A|M \cap D)P(D|M) + P(A|M \cap E)P(E|M)$$

$$P(D|F) = 1702/1832 \approx 0.93 \text{ and } P(E|F) \approx 0.07$$

$$P(D|M) = 1385/2421 \approx 0.57 \text{ and } P(E|M) \approx 0.43$$

- central problem in ecology
- spatiotemporal dimension of a pattern or process

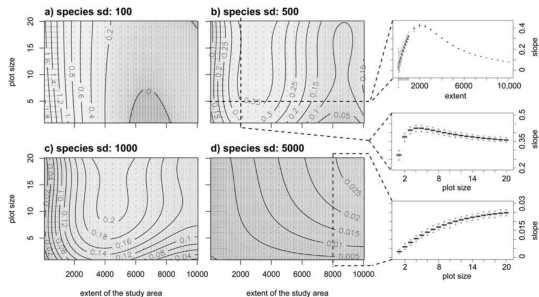
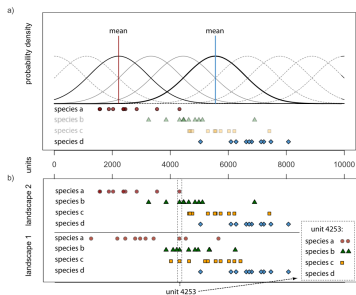
Tobler's First Law of Geography

everything is related to everything else,
but near things are more related than distant things

Species distribution modelling

relating environmental variables to observed species abundance

distance decay



- vector : a sequence of numbers
- matrix : a rectangular array of numbers

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Digital Elevation Model (DEM)

two-dimensional discrete function of elevation
of the topographic surface

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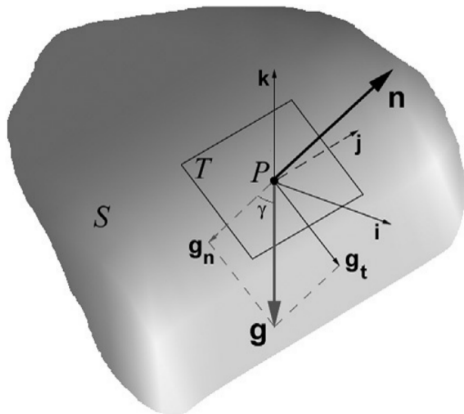
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two-dimensional discrete function of elevation
of the topographic surface

Digital Terrain Model (DTM)

two-dimensional discrete function of morphometric variables,
to describ the topographic surface
produced based on DEM

Slope



\mathbf{n} : norm direction

\mathbf{g} : gravitational acceleration vector

\mathbf{g}_t : tangential component

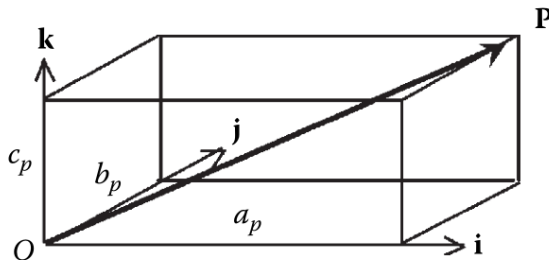
\mathbf{g}_n : normal component

$\mathbf{i}, \mathbf{j}, \mathbf{k}$: unit vectors in directions of x,y,z coordinates

T : tangent plane

Vectors

- vector : dimensional quantity associated with directions, like velocity
- scalar : no directions, like weight



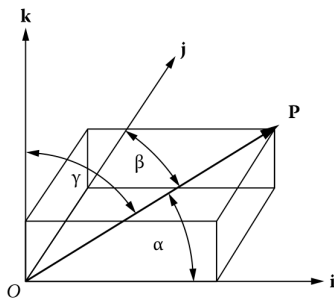
$$\mathbf{OP} = a_p \mathbf{i} + b_p \mathbf{j} + c_p \mathbf{k}$$

where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors.

Projection

- Pythagoras

$$|\mathbf{OP}| = \sqrt{a_p^2 + b_p^2 + c_p^2}$$



$$a_p = |\mathbf{OP}| \cos \alpha, \quad b_p = |\mathbf{OP}| \cos \beta, \quad c_p = |\mathbf{OP}| \cos \gamma$$

Dot Product

Given two vectors, $\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$ and $\mathbf{b} = b_x\mathbf{i} + b_y\mathbf{j} + b_z\mathbf{k}$. Their dot product is thus defined as.

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= |\mathbf{a}||\mathbf{b}|\cos(\theta) \\ &= a_x b_x + a_y b_y + a_z b_z\end{aligned}$$

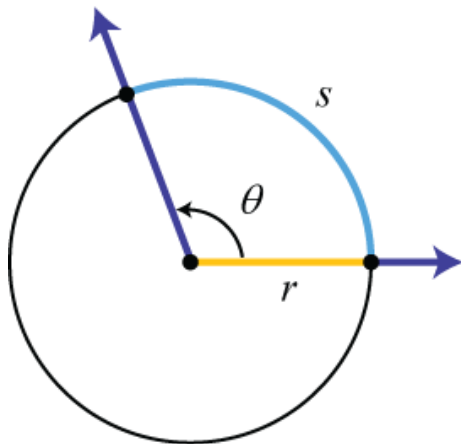
Example

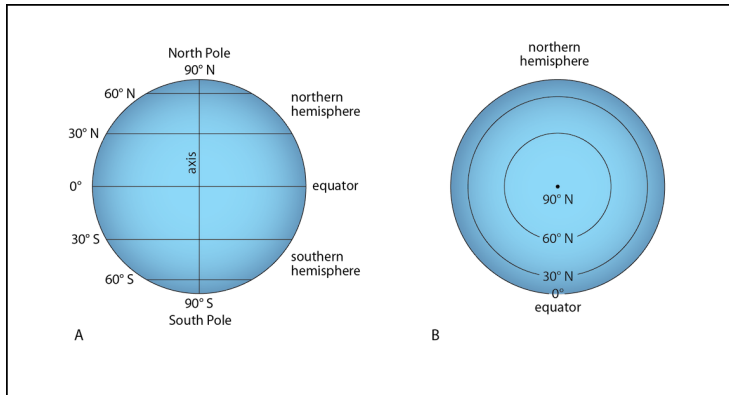
Two vectors P(5,9,2) and Q(3,7,4) in a 3d-space. Find out the angle between them.

$$\begin{aligned}OP \cdot OQ &= 15 + 63 + 8 = 86 \\ &= \sqrt{110} \sqrt{74} \cos\theta \\ \cos\theta &= 0.953 \Rightarrow \theta = \cos^{-1}0.953 = 17.6^\circ\end{aligned}$$

Radian

$$\theta \text{ (radians)} = \frac{s}{r}$$





$$^{\circ}arc = 2\pi r\theta/360$$

$$r_{eq} = 6357$$

$$^{\circ}arc = 110.9506$$

References



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