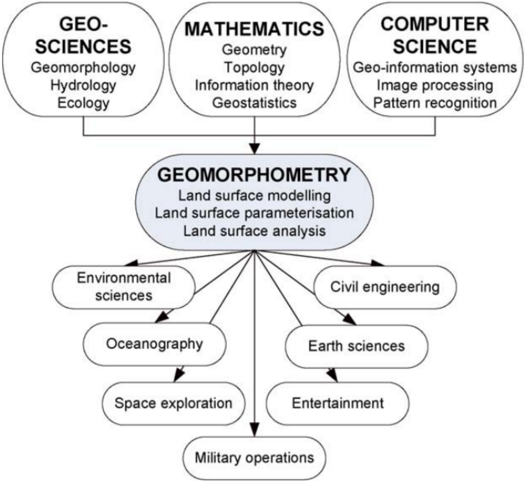
Terrain Description

BIO401-01/598-02

2021-03-29 Mon

REVIEW : Geomorphometry

- science of quantitative land-surface analysis



REVIEW : Vectors

- Projection

$$|\boldsymbol{OP}| = \sqrt{a_p^2 + b_p^2 + c_p^2}$$

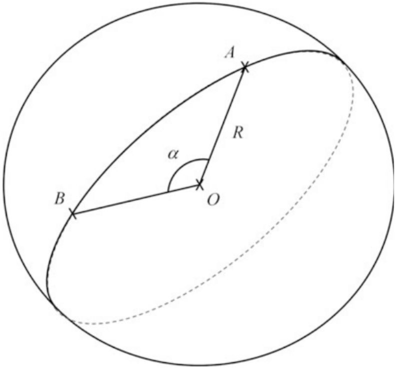
$$a_p = |\boldsymbol{OP}|cos(\alpha), \quad b_p = |\boldsymbol{OP}|cos(\beta), \quad c_p = |\boldsymbol{OP}|cos(\gamma)$$

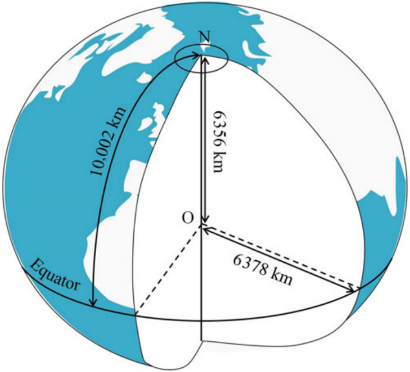
- Dot product

Given two vectors, $\boldsymbol{a} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}$ and $\boldsymbol{b} = b_x\boldsymbol{i} + b_y\boldsymbol{j} + b_z\boldsymbol{k}$.

$$\begin{aligned} \boldsymbol{a} \cdot \boldsymbol{b} &= |\boldsymbol{a}||\boldsymbol{b}|cos(\theta) \\ &= a_x b_x + a_y b_y + a_z b_z \end{aligned}$$

REVIEW : Arc Length

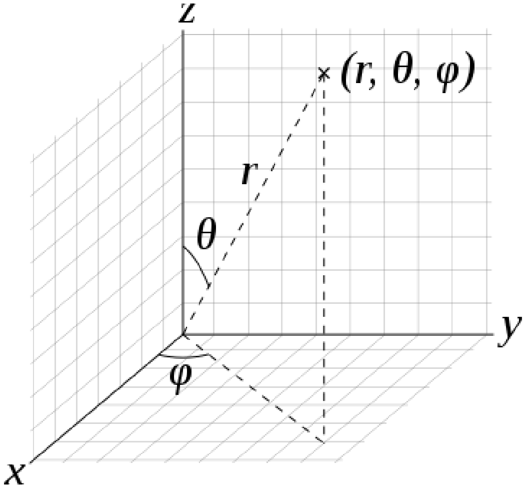




- radian : length $\widehat{AB} = \alpha R$
- degree : length $\widehat{AB} = \alpha R \pi/180$

- 1 degree of arc :
  $6357\,\pi/180 = 110.95$

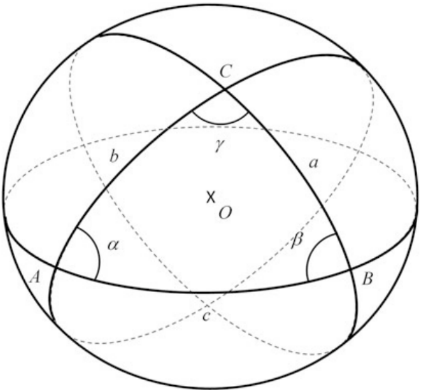
Spherical Coordinates



conversion :
Cartesian to Spherical

$$\begin{cases} x = r\,sin(\theta)\,cos(\phi) \\ y = r\,sin(\theta)\,sin(\phi) \\ z = r\,cos(\theta) \end{cases}$$

Spherical Geometry



Spherical law of cosines

$$cos(\gamma) = cos(\alpha)\, cos(\beta) + sin(\alpha)\, sin(\beta)\, cos(c)$$

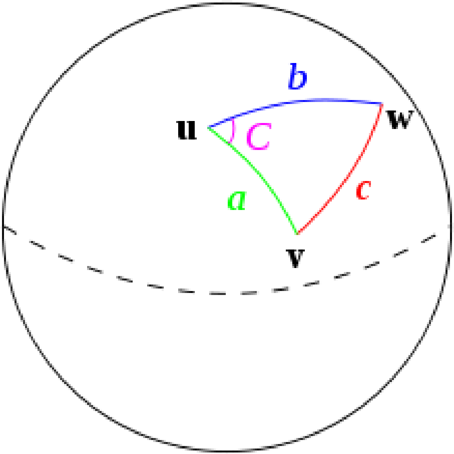
$$cos(c) = cos(a)\, cos(b) + sin(a)\, sin(b)\, cos(\gamma)$$

Geodesic : d(X,Y) being the distance between two points on Earth

$$d(X, Y) = R \cos^{-1}\{sin(\phi_X)\, sin(\phi_Y) + cos(\phi_X)\, cos(\phi_Y)\, cos(\Delta\lambda)\}$$

where ϕ_X and ϕ_Y are latitudes for X and Y, and $\Delta\lambda$ is the longitude difference.

Proof



Let $\boldsymbol{u}, \boldsymbol{v}$ and $\boldsymbol{w}$ be unit vectors.
Rotate to set $\boldsymbol{u}$ at north pole and $\boldsymbol{v}$ aligned with the prime meridian

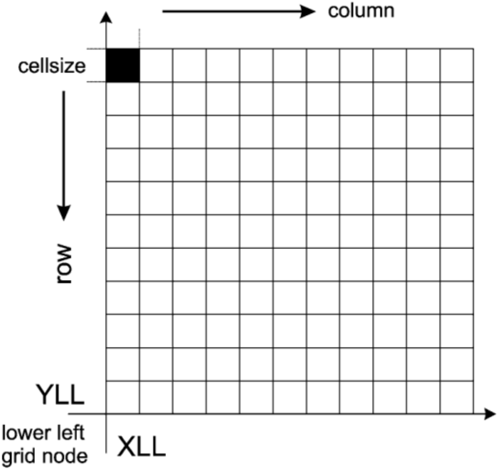
$$\boldsymbol{v}(r, \theta, \phi) = (1, a, 0)$$
$$\boldsymbol{w}(r, \theta, \phi) = (1, b, C)$$

$$\boldsymbol{v}(x, y, z) = (sin(a), 0, cos(a))$$
$$\boldsymbol{w}(x, y, z) = (sin(b)cos(C), sin(b)sin(C), cos(b))$$

$$\begin{aligned} cos(c) &= \boldsymbol{v} \cdot \boldsymbol{w} \\ &= sin(a)\, sin(b)\, cos(C) + cos(a)\, cos(b) \end{aligned}$$

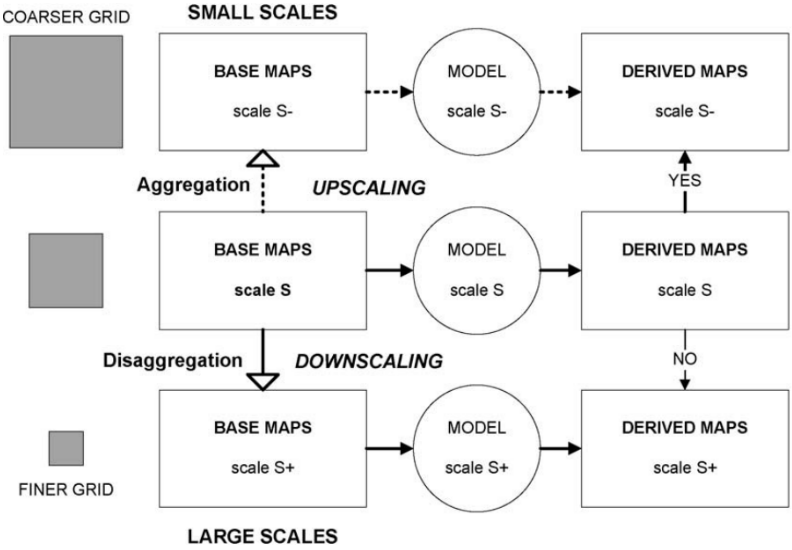
Cell size

- cell size : distance between two grid nodes



for example :
cell size = 0.5 mm
25 m resolution
So : 1:50,000 scale

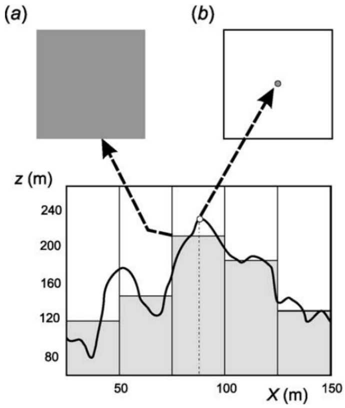
Rescaling



COARSER GRID
SMALL SCALES
BASE MAPS
scale S-
MODEL
scale S-
DERIVED MAPS
scale S-
Aggregation
UPSCALING
YES
BASE MAPS
scale S
MODEL
scale S
DERIVED MAPS
scale S
Disaggregation
DOWNSCALING
NO
BASE MAPS
scale S+
MODEL
scale S+
DERIVED MAPS
scale S+
FINER GRID
LARGE SCALES

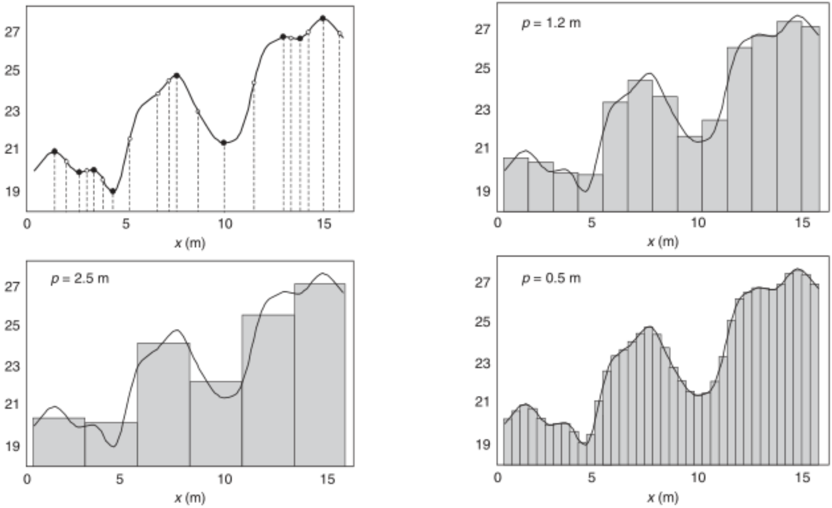
Support size

- land-cover surface : infinite
- finite sampling
- support size : a area/volume of the land being sampled



(a) : topo. image ; (b) : LiDAR

Sampling effects



Sampling frequency

Nyquist-Shannon sampling theorem

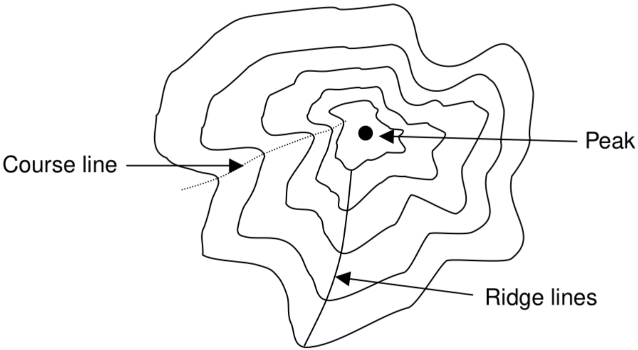
Grid resolution should be at least half the average spacing between the inflection points.

$$\Delta s \leq \frac{l}{2\, n(\delta z)}$$

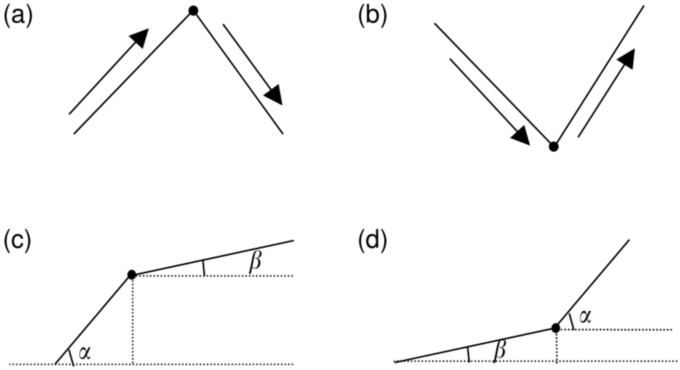
where Δs is the grid cell size, l is the length of the transect and $n(\delta z)$ is the number of observed inflection points.

Different points of view

- statistics : sample space
- geometry : contouring; square/triangle grids
- feature : feature specific (F-S) points/lines



F-S points

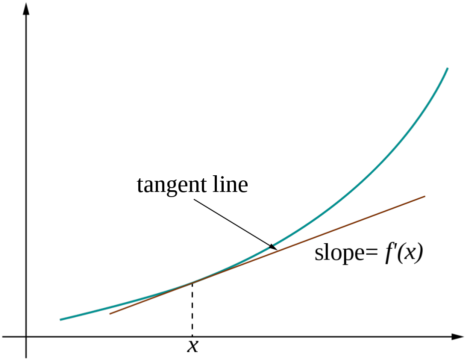


(a) : peak; (b) : pit; (c) concave point; (d) convex point

Derivative

- derivative measures the sensitivity to change of the function value w.r.t. its argument

$$f'(x) = lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$



Common derivatives

Function name	Expression	Derivative
Constant	$f(x) = a$	0
Linear	$f(x) = ax + b$	a
	$f(x) = 1.8x + 32$	$f'(x) = 1.8$
Quadratic	$f(x) = ax^2 + bx + c$	$f'(x) = 2ax + b$
	$z(t) = -\frac{1}{2}gt^2 + \frac{\sqrt{2}}{2}v_0 t + z_0$	$z'(t) = -gt + \frac{\sqrt{2}}{2}v_0$
Square root	$f(x) = \sqrt{x}$	$f'(x) = \frac{1}{2\sqrt{x}}$
Exponential	$f(x) = e^x$	$f'(x) = e^x$
	$f(x) = e^{ax}$	$f'(x) = ae^{ax}$
	$a(t) = a_0 \times \exp\left(-\frac{t}{\tau}\right)$	$a'(t) = -\frac{a_0}{\tau} \times \exp\left(-\frac{t}{\tau}\right)$
Logarithm	$f(x) = \ln(x)$	$f'(x) = \frac{1}{x}$
Power	$f(x) = x^a$	$f'(x) = ax^{a-1}$
Cosine	$f(x) = \cos x$	$f'(x) = -\sin x$
Sine	$f(x) = \sin x$	$f'(x) = \cos x$
Tangent	$f(x) = \tan x = \frac{\sin x}{\cos x}$	$f'(x) = \frac{1}{\cos^2 x} = 1 + \tan^2 x$

Rules

- multiplication

$$\frac{d\,(au)}{d\,x} = a\frac{d\,u}{d\,x}$$

- summation

$$\frac{d\,(u+v)}{d\,x} = \frac{d\,u}{d\,x} + \frac{d\,v}{d\,x}$$

- product

$$\frac{d\,(u\,v)}{d\,x} = v\frac{d\,u}{d\,x} + u\frac{d\,v}{d\,x}$$

- quotient

$$\frac{d\,(u/v)}{d\,x} = \left(v\frac{d\,u}{d\,x} - u\frac{d\,v}{d\,x}\right)/v^2$$

- chain

$$\frac{d\,(f(u))}{d\,x} = \frac{d\,f}{d\,u}\frac{d\,u}{d\,x}$$

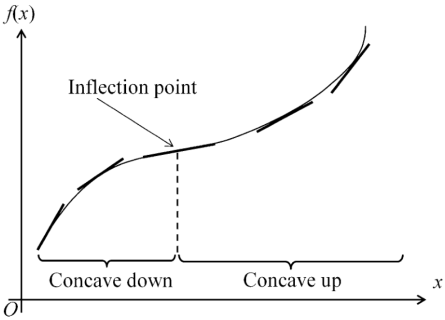
Derivative properties

Extrema

If $f(a)$ is local minimum or maximum for f, the $f'(a) = 0$

- Concavity

When the first derivative of a function is decreasing, then the function is concave ($f''(x) < 0$). When the first derivative of a function is increasing, then the function is convex ($f''(x) > 0$). Then change of concavity occurs at the inflection point, where $f''(x) = 0$



Partial Derivative

A partial derivative of a multivariate function is its derivative w.r.t. to one of it variables while holding others constant.

Example

A surface can be described by function $z = f(x, y) = x^2 - 2xy - 3y^2$. Compute it partial derivatives.

$$\frac{\partial z}{\partial x} = 2x - 2y$$
$$\frac{\partial z}{\partial y} = -2x - 6y$$

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