Pattern Matching with Typed Holes

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1 INTRODUCTION

Today's programming environment either only provides meaningful feedback for complete programs or simply takes heuristic approach to achieve similar results. By modeling incomplete programs as typed expressions with *holes*, Omar et al. [2017] describes a static semantics for incomplete functional programs. Based on that, Omar et al. [2019] develops a dynamic semantics to evaluate such incomplete programs without exceptions.

Pattern matching is a cornerstone of functional programming languages including ML family. An expression of product or sum types often relies on pattern matching to be eliminated. However, Omar et al. [2019] only introduces a rudimentary form of pattern matching that only allows case analysis on binary sum type and doesn't support nested patterns.

$$\begin{array}{l}\texttt{match}((\texttt{inl}_{\texttt{num}}(\text{⦶}^{u}), 2))\{\\ \mid (\texttt{inr}(x), \_) \Rightarrow x\\ \mid (\_, x) \Rightarrow x\\ \}\end{array} \quad (1) \qquad \begin{array}{l}\texttt{match}((\texttt{inl}_{\texttt{num}}(1), 2))\{\\ \mid (\texttt{inl}(x), \_) \Rightarrow x\\ \mid (\_, \text{⦶}^{w}) \Rightarrow \text{⦶}^{u}\\ \}\end{array} \quad (2) \qquad \begin{array}{l}\texttt{match}((\texttt{inl}_{\texttt{num}}(1), 2))\{\\ \mid (\texttt{inl}(x), \_) \Rightarrow x\\ \mid (\texttt{inl}(\text{⦶}^{w_1}), \text{⦶}^{w_2}) \Rightarrow \text{⦶}^{u}\\ \}\end{array} \quad (3)$$

Holes in match expression don't have to stop the match expression from being evaluated. For a match expression shown in Eq. 1, the scrutinee contains a hole. By considering two rules in order, we observe that no matter how the hole u is filled, $(\texttt{inl}_{\texttt{num}}(\text{⦶}^{u}), 2)$ doesn't match $(\texttt{inr}(x), \_) \Rightarrow x$, and always matches $(\_, x)$. And thus the value of the match expression is 2.

Besides the computation result, the real time feedback as we enter a match expression can help programmers correct mistakes. However, it is subtle and complicated to check if the rules in Eq. 2 cover all the possibilities, i.e. *exhaustiveness*, or if the second rule in Eq. 3 can never be reached, i.e. *redundancy*.

This abstract focuses on the formalism of full-fledged pattern matching with typed holes so that the programming environment could give feedback on incomplete match expressions.

2 STATIC SEMANTICS

Part of the definition of the internal language is shown in Fig. 1. Different from the examples in Sec. 1, the constituent rules in a match expression has a pointer to indicate which rule is being considered. The pointer would move as we evaluate the match expression.

Note that the premise of Rule TMatchZPre and Rule TMatchNZPre involves judgement of the form $\top \models^{\dagger}_{?} \xi$. That means, the rules in match expression either must be or may be exhaustive. Take Eq. 2 as an example, by filling the pattern hole w, the rules may cover all the possibilities of scrutinee and we shouldn't generate warning about non-exhaustiveness.

Fig. 2 describes the inductive construction of rules. The premise $\xi_r \not\models \xi_{pre}$ in Rule TRules ensures that rule r is not redundant with respect to rules rs_{pre}. Take Eq. 3 as an example, all sub-expressions that match or may match $\texttt{inl}(\text{⦶}^{w_1})$ also matche $\texttt{inl}(x)$. Even if the second rule is not complete, we know it is redundant and thus we can suggest the programmer to either rewrite or eliminate it.

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$\boxed{\Gamma\,;\Delta \vdash e : \tau}$ e is of type τ

TMatchZPre

$$\frac{\Gamma\,;\Delta \vdash e : \tau_1 \qquad \Gamma\,;\Delta \vdash [\bot]r \mid rs : \tau_1[\xi] \Rightarrow \tau_2 \qquad \boxed{\top \models^{\dagger}_{?} \xi}}{\Gamma\,;\Delta \vdash \texttt{match}(e)\{\cdot \mid r \mid rs\} : \tau_2}$$

TMatchNZPre

$$\frac{\Gamma\,;\Delta \vdash e : \tau_1 \qquad e\ \texttt{final} \qquad \Gamma\,;\Delta \vdash [\bot]rs_{pre} : \tau_1[\xi_{pre}] \Rightarrow \tau_2 \qquad \Gamma\,;\Delta \vdash [\xi_{pre}]r \mid rs_{post} : \tau_1[\xi_{rest}] \Rightarrow \tau_2 \qquad \boxed{e \not\models^{\dagger}_{?} \xi_{pre}} \qquad \boxed{\top \models^{\dagger}_{?} \xi_{pre} \vee \xi_{rest}}}{\Gamma\,;\Delta \vdash \texttt{match}(e)\{rs_{pre} \mid r \mid rs_{post}\} : \tau_2}$$

Fig. 1. Typing of Internal Expressions

$\boxed{\Gamma\,;\Delta \vdash [\xi_{pre}]rs : \tau_1[\xi_{rs}] \Rightarrow \tau_2}$ rs is of type $\tau_1 \Rightarrow \tau_2$, following rules constrained by ξ_{pre} and constrained by ξ_{rs}

TRules

$$\frac{\Gamma\,;\Delta \vdash r : \tau_1[\xi_r] \Rightarrow \tau_2 \qquad \Gamma\,;\Delta \vdash [\xi_{pre} \vee \xi_r]rs : \tau_1[\xi_{rs}] \Rightarrow \tau_2 \qquad \boxed{\xi_r \not\models \xi_{pre}}}{\Gamma\,;\Delta \vdash [\xi_{pre}]r \mid rs : \tau_1[\xi_r \vee \xi_{rs}] \Rightarrow \tau_2}$$

Fig. 2. Typing of rules

$$\begin{array}{rcl} p & ::= & x \mid \underline{n} \mid \_ \mid (p_1, p_2) \mid \texttt{inl}(p) \mid \texttt{inr}(p) \mid (\!|\,|\!)^w \mid (\!|p|\!)^w \\ \xi & ::= & \top \mid \bot \mid \underline{n} \mid \not{\underline{n}} \mid \xi_1 \wedge \xi_2 \mid \xi_1 \vee \xi_2 \mid (\xi_1, \xi_2) \mid \texttt{inl}(\xi) \mid \texttt{inr}(\xi) \mid ? \end{array}$$

Fig. 3. Syntax of patterns and constraints

Constraints ξ is emitted from patterns. The syntax of them is given in Fig. 3. A constraint predict the set of expressions that match the corresponding pattern. For example, variable patterns or wild card patterns emit *Truth* constraint $\top$; pattern holes emit *Unknown* constraint ?; patterns in multiple rules altogether emit disconjunction of constraints $\xi_1 \vee \xi_2 \vee \cdots \vee \xi_n$.

The dual of a constraint, $\overline{\xi}$, predicts the complement of the set of expressions that match the corresponding pattern of ξ. e.g. $\overline{\underline{n}} = \not{\underline{n}}$ means all numbers other than n; $\overline{\xi_1 \vee \xi_2} = \overline{\xi_1} \wedge \overline{\xi_2}$ means the complement of the union of two constraints.

To give an idea of how rules is related to constraints: in Eq. 2, the corresponding constraint of two rules is $(\texttt{inl}(\top), \top) \vee (\top, ?)$; in Eq. 3, the constraint of the second rule is $(\texttt{inl}(?), ?)$ while the constraint of the first rule is $(\texttt{inl}(\top), \top)$. We will see in Sec. 3 how these constraints are used to check exhaustiveness and redundancy.

3 EXHAUSTIVENESS AND REDUNDANCY

Since constraints correspond to patterns, the relationship between an expression and a constraint should be defined in a way to represent that the expression match the associated pattern of the constraint.

We write $e \models \xi$ to denote that the expression e satisfies the constraint ξ, which is inductively defined by a set of rules. For example, any expression satisfies Truth constraint, $e \models \top$.

Similarly, we inductively define a maybe satisfaction judgement $e \models_? \xi$ to denote that the expression e may satisfy the constraint ξ, i.e. e may match the associated pattern. For example, an expression hole, no matter it is empty or not, may satisfy any constraint, i.e. $(\!|\,|\!)^u \models_? \xi$ and $(\!|e|\!)^u \models_? \xi$, since we can always fill the hole with an expression that matches a given pattern.

Since pattern matching only works with final expressions, the judgements mentioned above only makes sense when e `final`, i.e. e `val` or e `indet`. e `indet` means that e can't be further evaluated due to the existence of holes [Omar et al. 2019].

For Eq. 2, we notice that for any final expression e, either $e \models (\texttt{inl}(\top), \top)$ or $e \models_? (\top, ?)$. Therefore, we can say Eq. 2 may be exhaustive. Definition 3.1 gives the formal definition of exhaustiveness or maybe exhaustiveness.

Definition 3.1 (Exhaustiveness or Maybe Exhaustiveness)**.** $\top \models^{\dagger}_{?} \xi$ *iff* $\xi : \tau$ *and for all* e *such that* $\cdot ; \Gamma \vdash e : \tau$ *and* e `final` *we have* $e \models \xi$ *or* $e \models_? \xi$.

By comparing two constraints $(\texttt{inl}(\top), \top)$ and $(\texttt{inl}(?), ?)$ that correspond to two rules in Eq. 3, we notice that $e \models_? (\texttt{inl}(?), ?)$ implies $e \models (\texttt{inl}(\top), \top)$, which means that no matter how we fill the holes in $(\texttt{inl}((\!|\,|\!)^{w_1}), (\!|\,|\!)^{w_2})$, all expressions that matches the completed pattern also matches $(\texttt{inl}(x), \_)$. Hence, the second rule must be redundant. Definition 3.2 gives the formal definition of redundancy.

However, to determine if the redundancy judgement is true, we have to apply *material entailment of constraints* and $\xi_r \models \xi_{pre}$ is equivalent to $\top \models \overline{\xi_r} \vee \xi_{pre}$. Then, we only need to determine if $\overline{\xi_r} \vee \xi_{pre}$ can be satisfied by all expressions.

Definition 3.2 (Redundancy)**.** $\xi_r \models \xi_{pre}$ *iff* $\xi_r : \tau$ *and* $\xi_{pre} : \tau$ *and for all* e *such that* $\cdot ; \Gamma \vdash e : \tau$ *and* e `val` *we have* $e \models \xi_r$ *or* $e \models_? \xi_r$ *implies* $e \models \xi_{pre}$.

While Definition 3.1 ensures that the scrutinee at least matches or may match one of the constinuent rules, redundancy doesn't have anything to do with the scrutinee, it is simply the property of a single rule with respect to previous rules. Therefore, we only consider the expression that is already a value and recognize the redundant rule with confidence.

4 DYNAMIC SEMANTICS

As for pattern matching, either the scrutinee matches or may match or doesn't match the pattern. When there are only one remaining rule, the exhaustiveness or maybe exhaustiveness checking ensures that the pattern of the rule either must be matched or may be matched by the scrutinee.

Theorem 4.1 (Preservation)**.** *If* $\cdot ; \Delta \vdash e : \tau$ *and* $e \mapsto e'$ *then* $\cdot ; \Delta \vdash e' : \tau$

Theorem 4.2 (Progress)**.** *If* $\cdot ; \Delta \vdash e : \tau$ *then either* e `final` *or* $e \mapsto e'$ *for some* e'.

5 DISCUSSION

The abstract explores how constraint helps reasoning pattern matching with typed holes. One main contribution of the work is extending the match constraint language [Harper 2012] with Unknown constraint and introduce a three-way logic, including the concept of "maybe". The other contribution is the formalism of pattern matching with typed holes based on the development of the extended match constraint language.

Next, type holes and dynamic type casting would be added and our type system would be turned into a gradual type system. Recently, pattern matching statements have also been proposed to be added to Python, whose *Type Hints* is taken from the idea of gradual typing [Bucher et al. 2020; Siek and Taha 2006; van Rossum et al. 2014].

As a result, the work lays a foundation for integrating full-fledged pattern matching into the Hazel programming environment described in Omar et al. [2019].

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