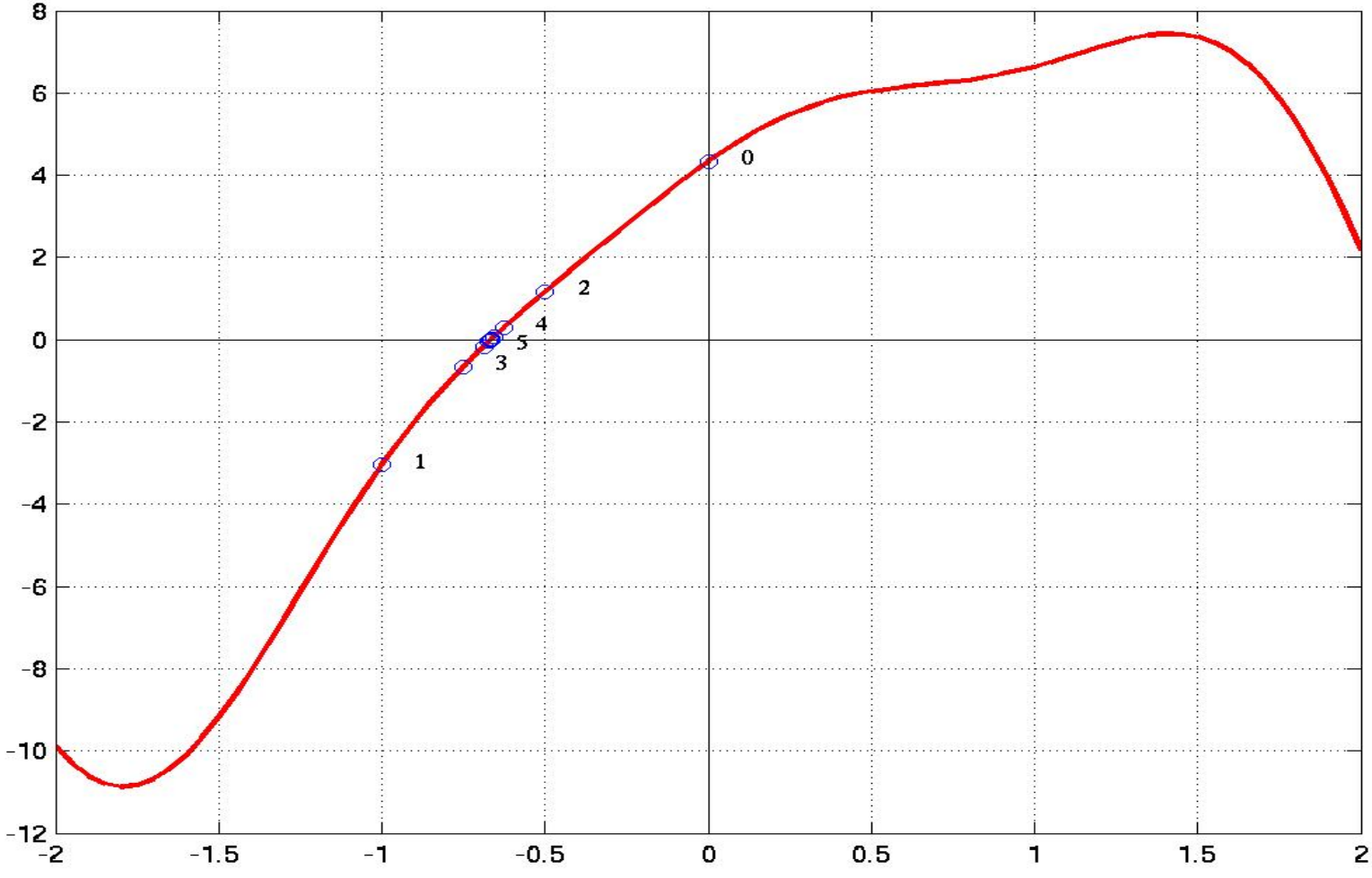
Análisis Numérico

3. Resolución numérica de ecuaciones no lineales

Abril, 2007

Método de bisección

$$f(x) = x^2 \sin(\pi x - 3) - (x - 7.5)\cos(x - \tfrac{3}{\pi}) = 0$$



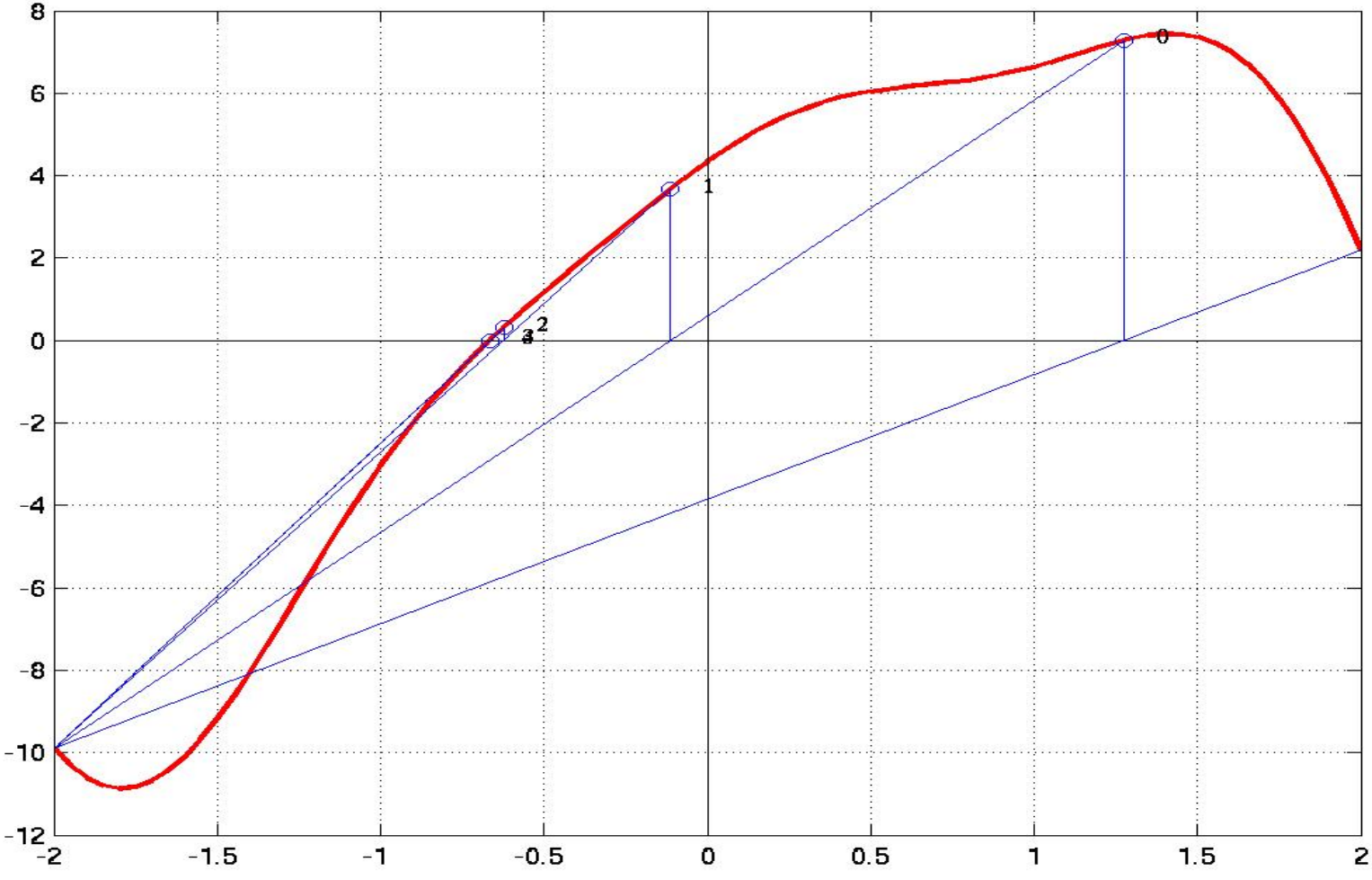
Método de bisección

$$f(x) = x^2 \sin(\pi x - 3) - (x - 7.5)\cos(x - \frac{3}{\pi}) = 0 \qquad \begin{cases} f(-2) < 0 \\ f(2) > 0 \end{cases}$$

i	x_i	$f(x_i)$
0	0.0	4.3325
1	-1.0	-3.0443
2	-0.5	1.1724
3	-0.75	-0.6534
4	-0.625	0.3042
5	-0.6875	-0.1599
6	-0.65625	0.0754
7	-0.671875	-0.0414
8	-0.6640625	0.0172
9	-0.66796875	-0.0120
10	-0.666015625	0.0026
11	-0.6669921875	-0.0047
12	-0.66650390625	-0.0011
13	-0.666259765625	0.0008

Método de *regula falsi*

$$f(x) = x^2 \sin(\pi x - 3) - (x - 7.5)\cos(x - \tfrac{3}{\pi}) = 0$$



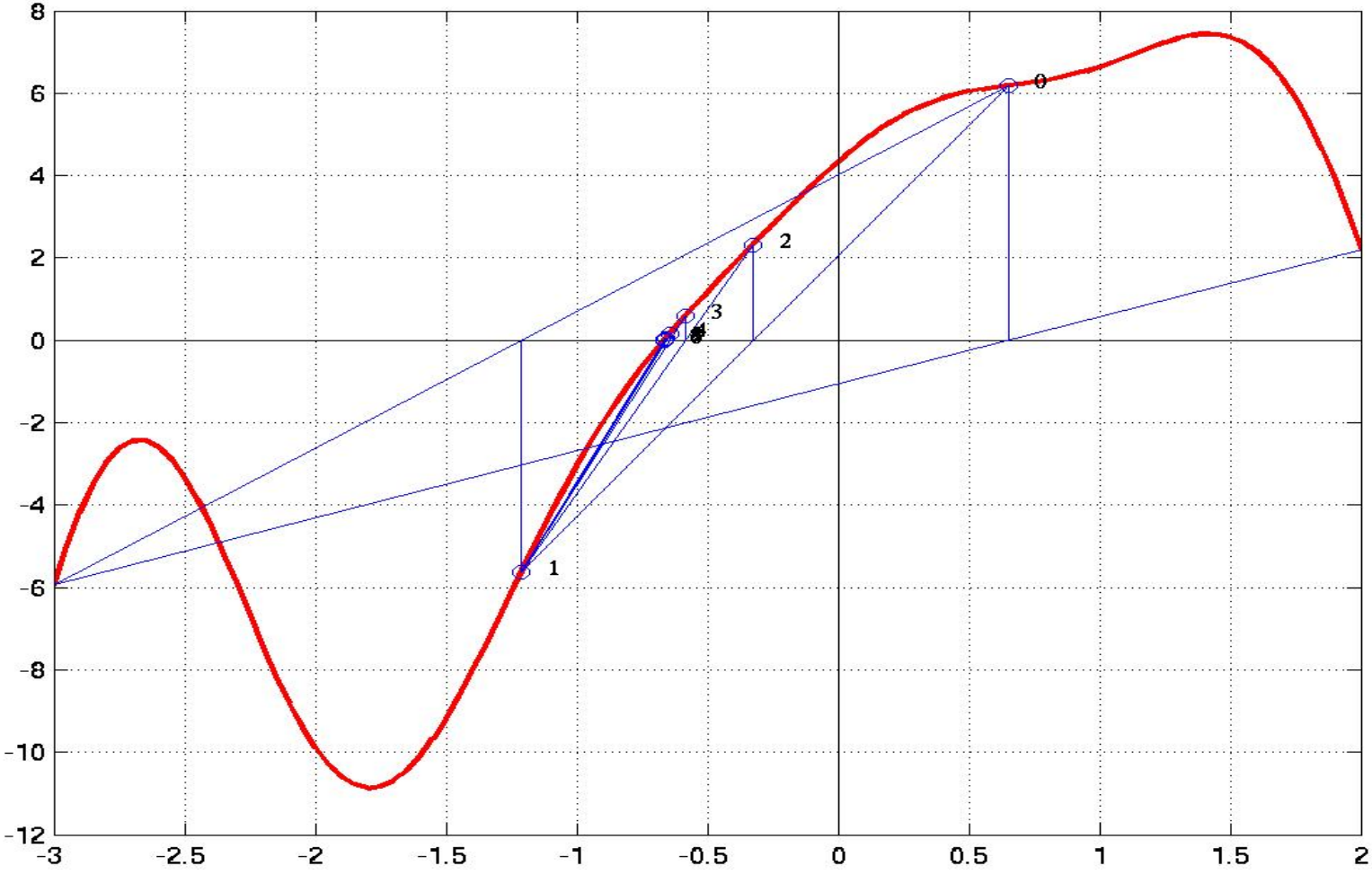
Método de *regula falsi*

$$f(x) = x^2 \sin(\pi x - 3) - (x - 7.5)\cos(x - \frac{3}{\pi}) = 0 \qquad \begin{cases} f(-2) < 0 \\ f(2) > 0 \end{cases}$$

i	x_i	$f(x_i)$
	-2.0	-9.89945543171094
	2.0	2.19564593546223
0	1.273872663384	7.27936700412511
1	-0.1134005755449	3.66926385936114
2	-0.6235762919663	0.31445168121281
3	-0.6659517261922	0.00307332292478
4	-0.6663657579253	-0.00002639878523

Método de *regula falsi*

$$f(x) = x^2 \sin(\pi x - 3) - (x - 7.5)\cos(x - \tfrac{3}{\pi}) = 0$$



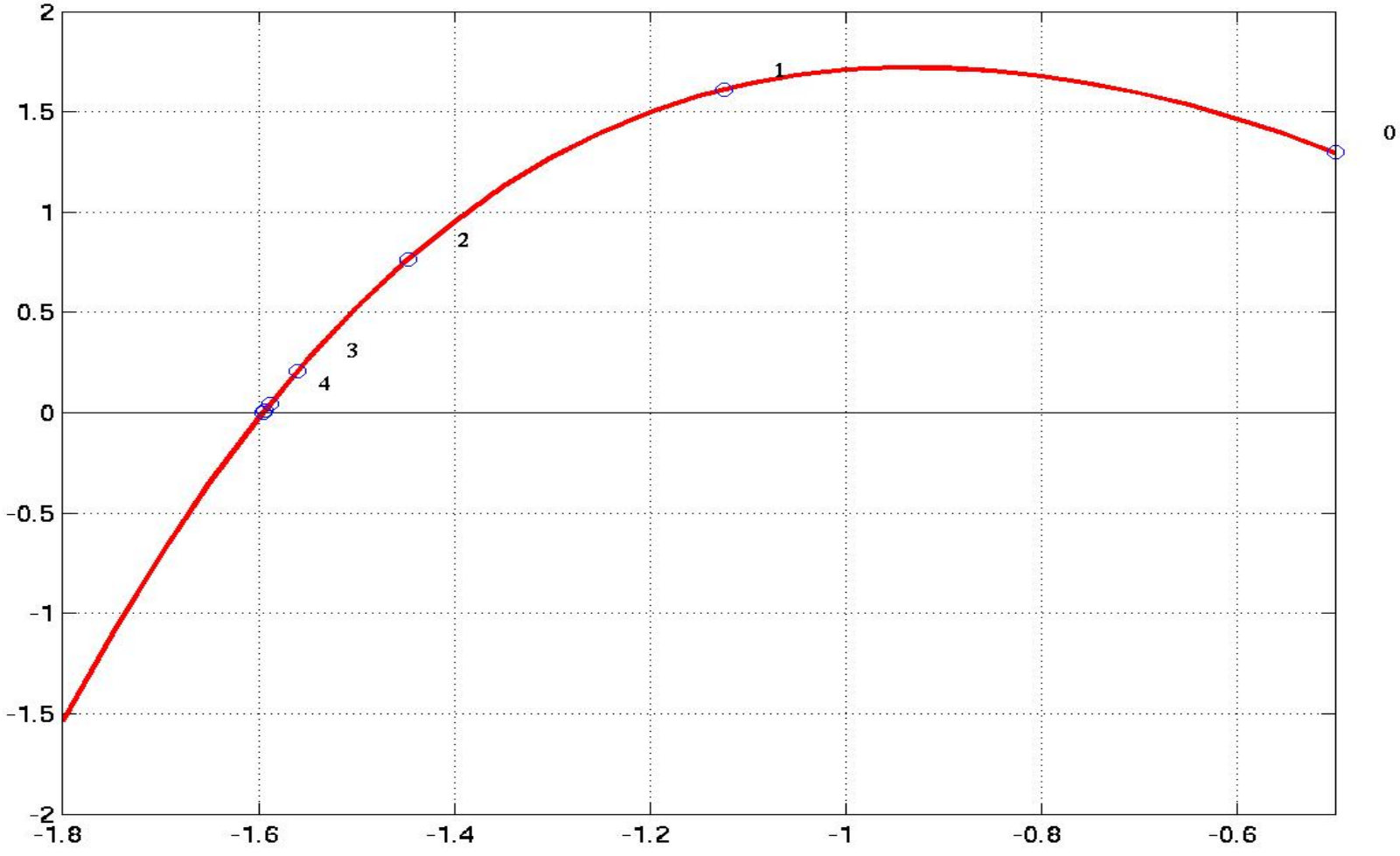
Método de punto fijo (o de iteración funcional)

$$f(x) = x^3 - x\sin^2 x + \ln(-x) + 2 = 0 \qquad g(x) = \left(x\sin^2 x - \ln(-x) - 2\right)^{1/3}$$

i	x_i	$f(x_i)$
0	-0.50000000000000	1.29677724297302
1	-1.12445959044937	1.61046001879680
2	-1.44739719539093	0.76299846545740
3	-1.55983831975864	0.20899747260005
4	-1.58796083669615	0.04571053764627
5	-1.59398047252296	0.00941444636750
6	-1.59521463211523	0.00191377755248
7	-1.59546527981062	0.00038798858250
8	-1.59551608512537	0.00007861561288
9	-1.59552637908295	0.00001592760546
10	-1.59552846463334	0.00000322687701
11	-1.59552888715788	0.00000065375099

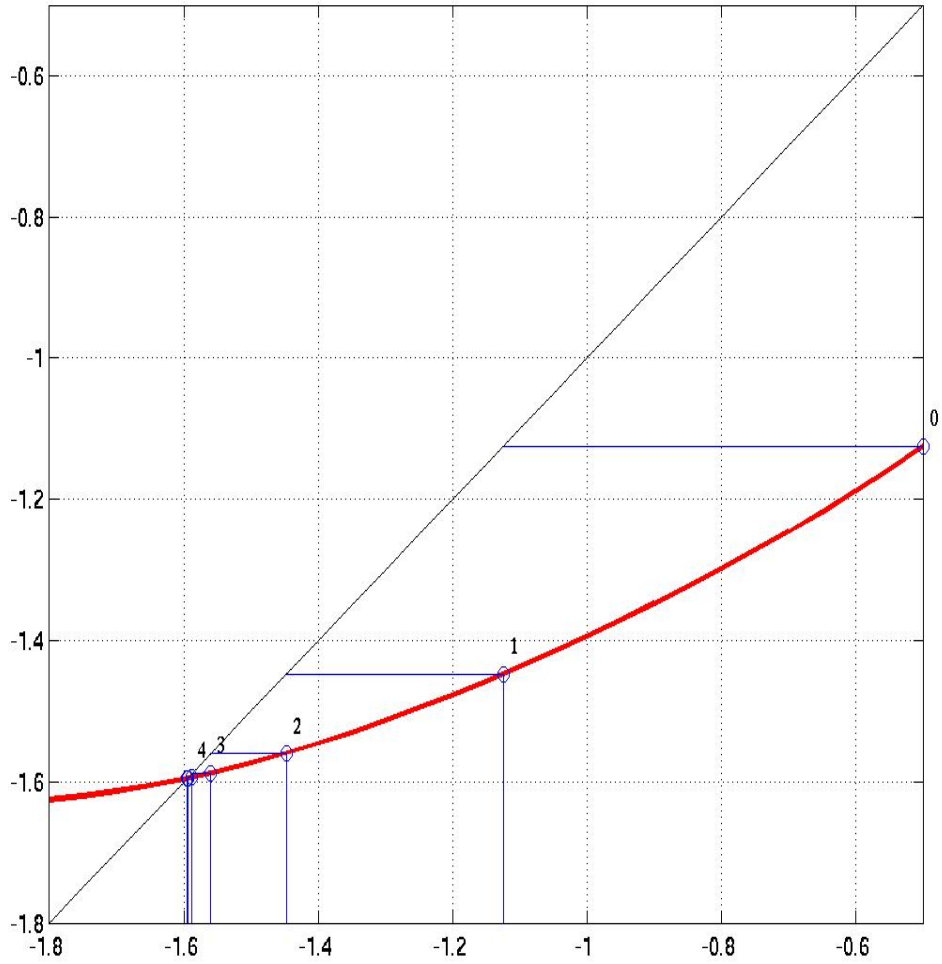
Método de punto fijo (o de iteración funcional)

$$f(x) = x^3 - x\sin^2 x + \ln(-x) + 2 = 0 \qquad x_0 = -0.5$$



Método de punto fijo (o de iteración funcional)

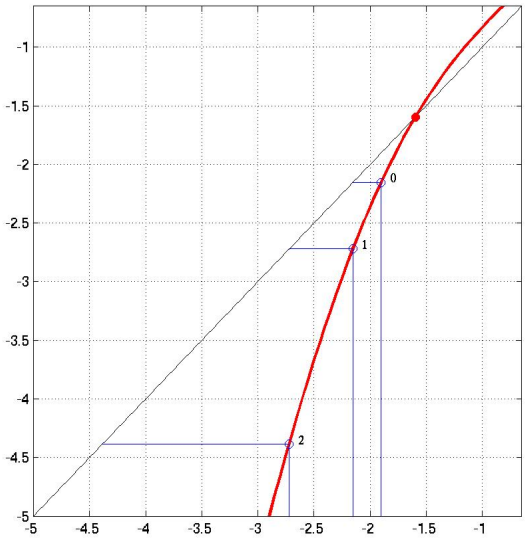
$$g(x) = \left(x \sin^2 x - \ln(-x) - 2\right)^{1/3} \qquad x_0 = -0.5$$



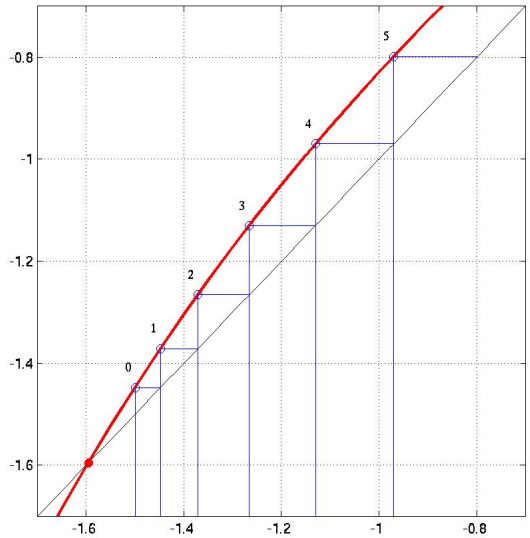
Método de punto fijo (o de iteración funcional)

$$g(x) = \frac{1}{10}\left[x^3 + \ln(-x) + 2 + x\left(10 - \sin^2 x\right)\right]$$

$x_0 = -1.90$

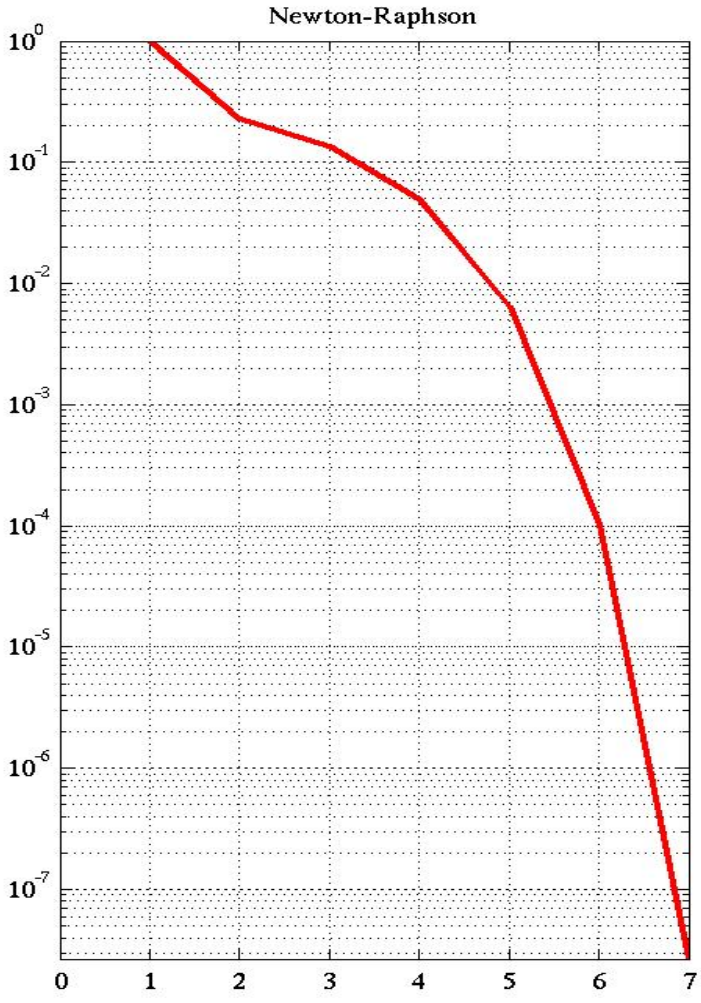


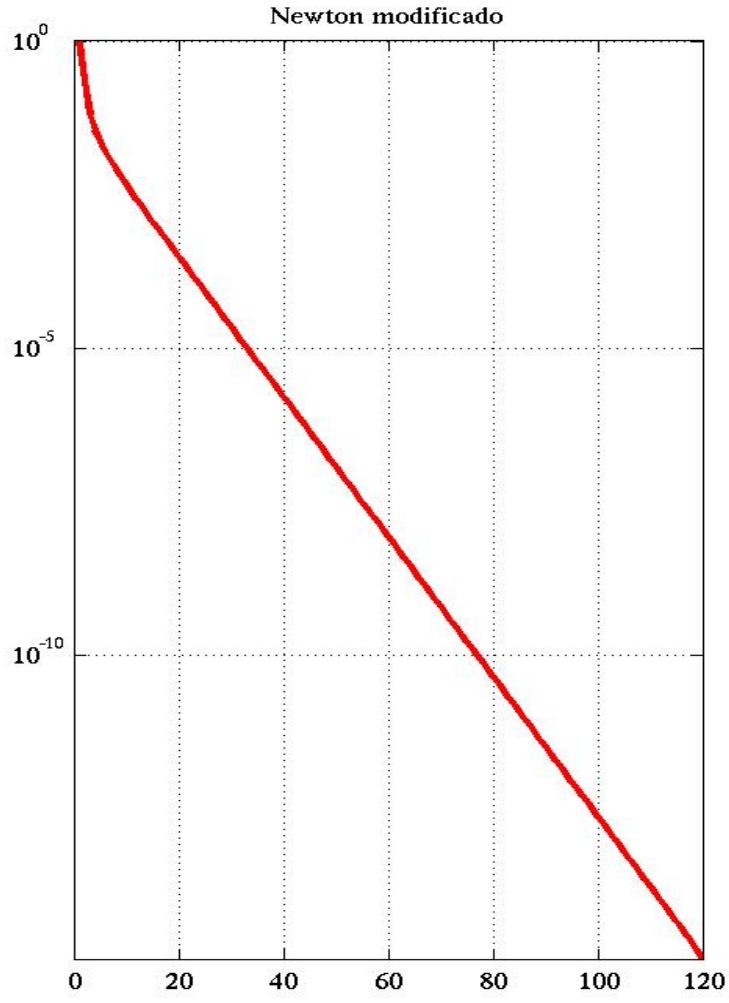
$x_0 = -1.50$



Métodos de Newton y de Newton modificado

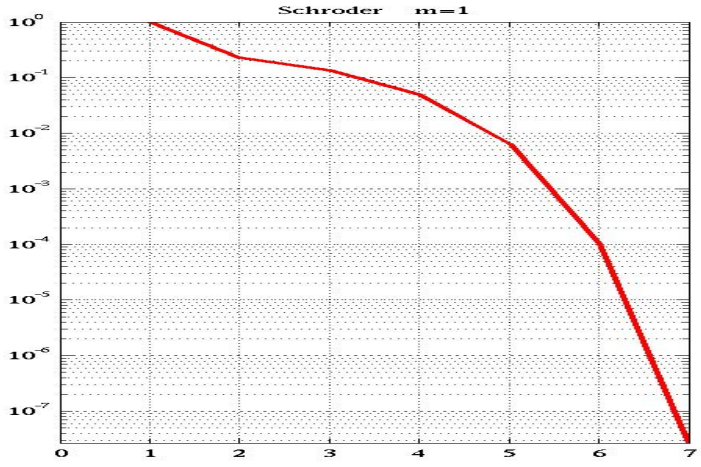
$$f(x) = -x^3 + 9x^2 - 15x = 0 \qquad x_0 = 10$$

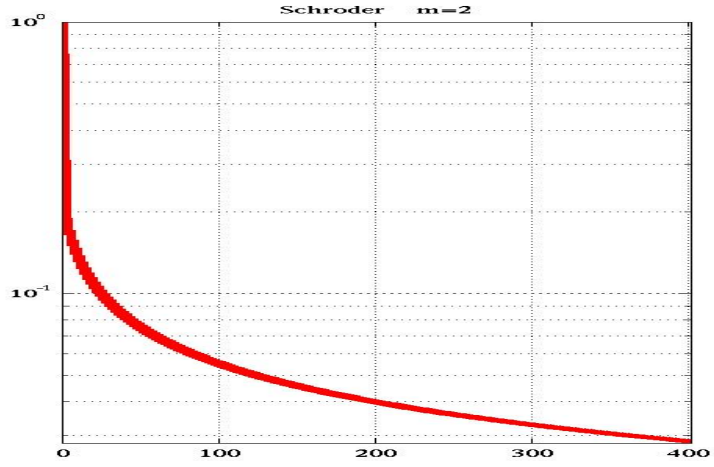




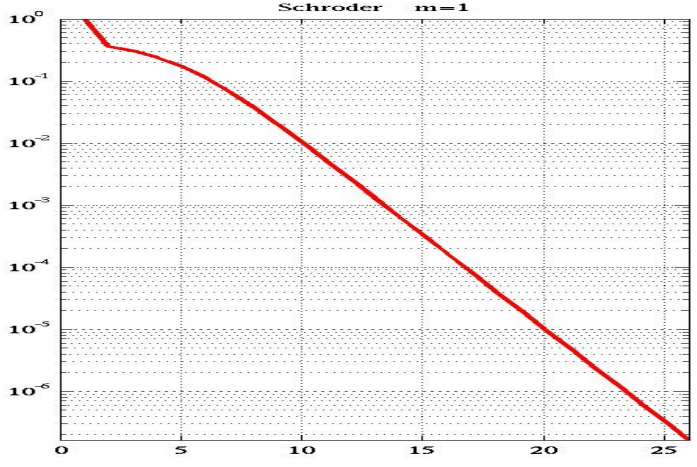
Método de Schröder

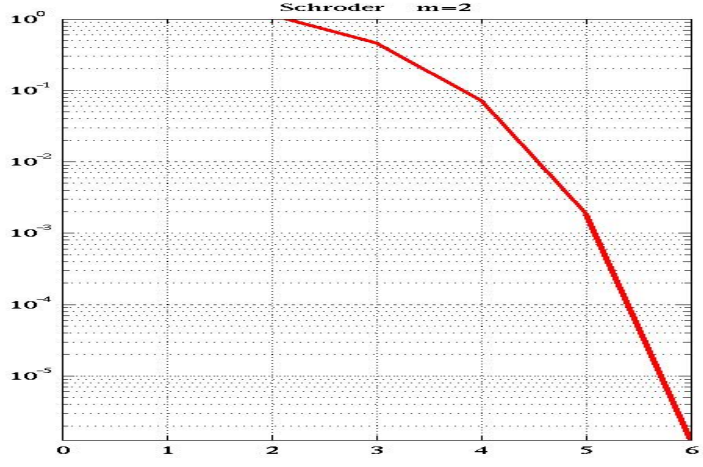
$$f(x) = -x^3 + 9x^2 - 15x = 0 \qquad x_0 = 10$$





$$f(x) = (x+1)(x-3)^2 = 0 \qquad x_0 = 10$$





Orden de convergencia

k	x_k	y_k
0	7.5	7.5
1	7.25	7.25
2	7.125	7.0625
3	7.0625	7.003906
4	7.03125	
5	7.015625	
6	7.0078125	
	$\alpha = 7$	$\alpha = 7$

$$\frac{|x_1 - \alpha|}{|x_0 - \alpha|^p} = \frac{0.25}{0.5^p} = 0.5 \qquad \text{si } p = 1$$

$$\frac{|x_2 - \alpha|}{|x_1 - \alpha|^p} = \frac{0.125}{0.25^p} = 0.5 \qquad \text{si } p = 1$$

$$\frac{|x_3 - \alpha|}{|x_2 - \alpha|^p} = \frac{|x_4 - \alpha|}{|x_3 - \alpha|^p} = \ldots = 0.5 \qquad \text{si } p = 1$$

Orden de convergencia

k	x_k	y_k
0	7.5	7.5
1	7.25	7.25
2	7.125	7.0625
3	7.0625	7.003906
4	7.03125	
5	7.015625	
6	7.0078125	
	$\alpha = 7$	$\alpha = 7$

$$\frac{|y_1 - \alpha|}{|y_0 - \alpha|^1} = \frac{0.25}{0.5^1} = 0.5$$

$$\frac{|y_2 - \alpha|}{|y_1 - \alpha|^1} = \frac{0.625}{0.25^1} \neq 0.5$$

$$\frac{|y_1 - \alpha|}{|y_0 - \alpha|^2} = \frac{0.25}{0.5^2} = 1$$

$$\frac{|y_2 - \alpha|}{|y_1 - \alpha|^2} = \frac{0.0625}{0.25^2} = 1$$

$$\frac{|y_3 - \alpha|}{|y_2 - \alpha|^2} = \frac{0.003906}{0.06255^2} \approx 1$$

Métodos de aceleración de la convergencia

$$f(x) = -x^3 + 9x^2 - 15x = 0 \qquad g(x) = (9x^2 - 15x)^{1/3} \qquad x_0 = 10$$