BOLETIN 9

[SISTEMAS]

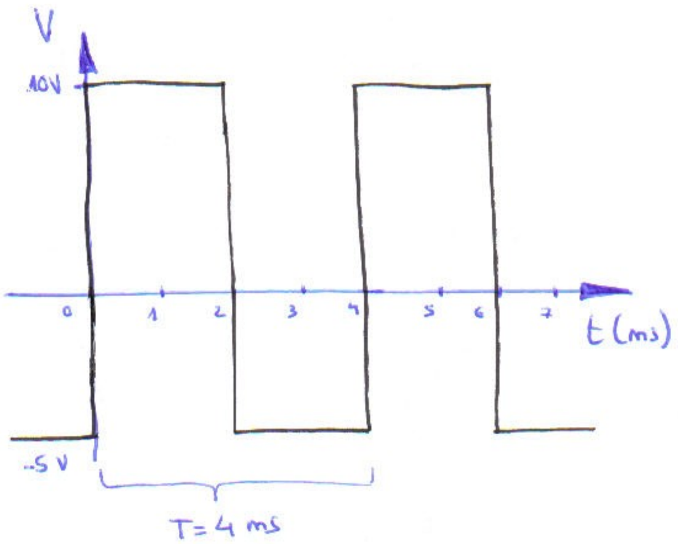
① Calcular para cada una de las señales de la figura:

a) Valor medio (V_m)

b) Valor eficaz (V_{ef})

c) Valor pico a pico (V_{pp})

1



a) $$V_m = \frac{1}{T}\int_0^T x(t)\cdot dt = \frac{1}{4\,ms}\cdot\int_0^{4ms} x(t)\cdot dt =$$

$$= \frac{1}{4\,ms.}\left[\int_0^{2ms} 10\,V\cdot dt + \int_{2\,ms}^{4ms} -5\,V\cdot dt\right] = \frac{1}{4\cancel{ms}}\left[10\,V\cdot 2\cancel{ms} - 5\,V\cdot 4\cancel{ms} + 5\,V\cdot 2\cancel{ms}\right] =$$

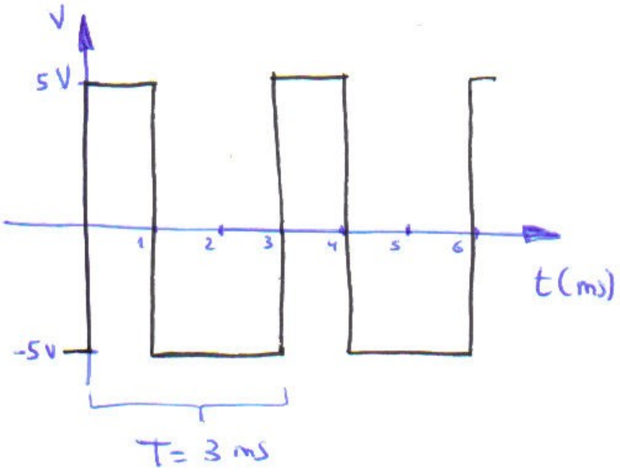
$$= \frac{10}{4}\,V = 2{,}5\,V$$

b) $$V_{ef} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\cdot dt} = \sqrt{\frac{1}{4ms}\left[\int_0^{2ms} 10^2 V^2\cdot dt + \int_{2ms}^{4ms} (-5)^2 V^2\cdot dt\right]} =$$

$$= \sqrt{\frac{1}{4\cancel{ms}}\left[100\,V^2\cdot 2\cancel{ms} + 25\,V^2\cdot 2\cancel{ms}\right]} = \sqrt{\frac{250}{4}V^2} = \frac{5\sqrt{10}}{2}\,V$$

c) $$V_{pp} = V_{max} - V_{min} = 10 - (-5) = 15\,V$$

$\boxed{2}$



a) $V_m = \frac{1}{T}\int_0^T x(t)\cdot dt = \frac{1}{3ms}\left[\int_0^{1ms} 5V\cdot dt + \int_{1ms}^{3ms} (-5V)\, dt\right] =$

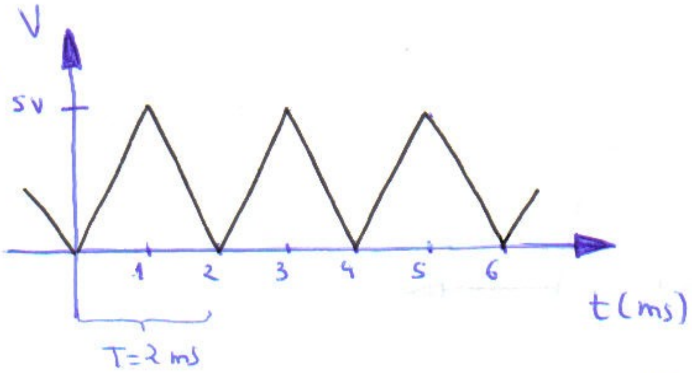
$= \frac{1}{3\cancel{ms}}\left[5V\cdot 1\cancel{ms} + (-5)V\cdot 3\cancel{ms} + 5V\cdot 1\cancel{ms}\right] = -\frac{5}{3}\ V$

b) $V_{ef} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\cdot dt} = \sqrt{\frac{1}{3ms}\int_0^{1ms} 25V^2\cdot dt + \int_1^{3ms} 25V^2\cdot dt} =$

$= \sqrt{\frac{1}{3}\int_0^3 25V^2\cdot dt} = \sqrt{\frac{25}{\cancel{3}}\cancel{3}\cdot V^2} = 5\ V$

c) $V_{pp} = V_{max} - V_{min} = 5 - (-5) = 10V$

$\boxed{3}$



a) $V_m = \frac{1}{T}\int_0^T x(t)\cdot dt = \frac{1}{2}\left[\int_0^1 5t\cdot dt + \int_1^2 (-5t+10)\,dt\right] =$

$= \frac{1}{2}\left[\left[5\frac{t^2}{2}\right]_0^1 - \left[\frac{5t^2}{2}\right]_1^2 + \left[10t\right]_1^2\right] = \frac{5}{2}\ V$

b) $V_{ef} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\cdot dt} = \sqrt{\frac{1}{2}\left[\int_0^1 25t^2\cdot dt + \int_1^2 (-5t+10)^2 dt\right]} =$

$= \sqrt{\frac{1}{2}\left[\left[\frac{25t^3}{3}\right]_0^1 + \int_1^2 (25t^2+100-100t)\,dt\right]} = \sqrt{\frac{1}{2}\left(\frac{25}{3}+\frac{175}{3}+100-50\right)} =$

$= \sqrt{\frac{1}{2}\left(\frac{350}{3}\right)} = \sqrt{\frac{175}{3}} = 5\cdot\sqrt{\frac{7}{3}}\ V$

c) $V_{pp} = V_{max} - V_{min} = 5 - 0 = 5V$

4

$5 \cdot \text{sen}(2\pi f t) =$
$= 5 \cdot \text{sen}\left(\frac{2\pi}{T} t\right) = 5 \cdot \text{sen}\, \pi t$

a) $V_{med} = \frac{1}{T} \int_0^T x(t) \cdot dt = \frac{1}{2} \left[\int_0^1 5 \cdot \text{sen}(\pi t) \cdot dt + \int_1^2 0 \cdot dt \right] =$

$= \frac{5}{2\pi} \left[-\cos \pi t \right]_0^1 = \frac{5}{\cancel{2}\pi} \cdot \cancel{2} = \frac{5}{\pi} \text{ V}$

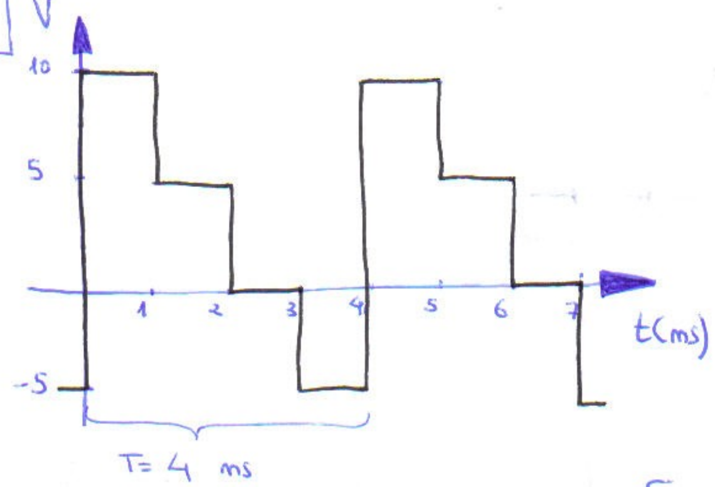
b) $V_{ef} = \sqrt{\frac{1}{T} \int_0^T x^2(t) \cdot dt} = \sqrt{\frac{1}{2} \left[\int_0^1 25 \cdot \text{sen}^2 \pi t \cdot dt + \int_1^2 0^2 \cdot dt \right]} =$

$= \sqrt{\frac{25}{2} \int_0^1 \text{sen}^2 \pi t \cdot dt} = \sqrt{\frac{25}{2} \int_0^1 \frac{1 - \cos 2\pi t}{2}} = \sqrt{\frac{25}{4} \left[\int_0^1 dt - \int_0^1 \cos 2\pi t \right]} =$

$= \sqrt{\frac{25}{4} + 0} = \sqrt{\frac{25}{4}} = \frac{5}{2} = 2{,}5 \text{ V}$

c) $V_{pp} = V_{max} - V_{min} = 5 - 0 = 5 \text{ V}$

5



a) $V_{med} = \frac{1}{T}\int_0^T x(t)\cdot dt = \frac{1}{4}\left[\int_0^1 10V\cdot dt + \int_1^2 5V\cdot dt + \int_2^3 0V\cdot dt + \int_3^4 -5V\cdot dt\right] =$

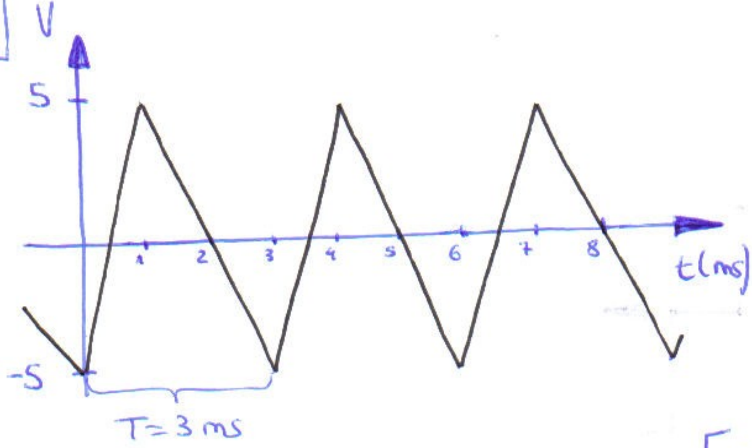
$= \frac{10}{4} + \cancel{\frac{5}{4}} + 0 - \cancel{\frac{5}{4}} = \frac{10}{4} = 2,5\ V$

b) $V_{ef} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\cdot dt} = \sqrt{\frac{1}{4}\left[\int_0^1 100\,dt + \int_1^2 25\,dt + \int_2^3 0\cdot dt + \int_3^4 25\cdot dt\right.} =$

$= \sqrt{25\cdot 1 + \frac{25}{4} + \frac{25}{4}} = \sqrt{\frac{150}{4}} = \frac{5\sqrt{6}}{2}\ V$

c) $V_{pp} = V_{max} - V_{min} = 10 - (-5) = 15\,V$

6



a) $V_{med} = \frac{1}{T}\int_0^T x(t)\cdot dt = \frac{1}{3}\left[\int_0^1 (10t-5)\,dt + \int_1^2 (-5t+10)\,dt\right] =$

$= \frac{1}{3}\left[10\frac{t^2}{2} - 5t\right]_0^1 + \frac{1}{3}\left[-5\frac{t^2}{2}+10t\right]_1^2 = \frac{1}{3}\underbrace{\left[\frac{10\cdot1^2}{2} - 5\cdot 1\right]}_{0} +$

$+ \frac{1}{3}\left[-5\frac{2^2}{2}+10\cdot2 - \left(-5\frac{1^2}{2}+10\cdot1\right)\right] = \frac{1}{3}\left(\cancel{10} + \frac{5}{2} - \cancel{10}\right) = \frac{5}{6}\ V$

b) $V_{ef} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\cdot dt} = \sqrt{\frac{1}{3}\int_0^1 (10t-5)^2 dt + \frac{1}{3}\int_1^2 (10-5t)^2 dt} =$

$= \sqrt{\frac{1}{3}\int_0^1(100t^2+25-100t)\,dt + \frac{1}{3}\int_1^2 (100+25t^2-100t)\,dt} =$

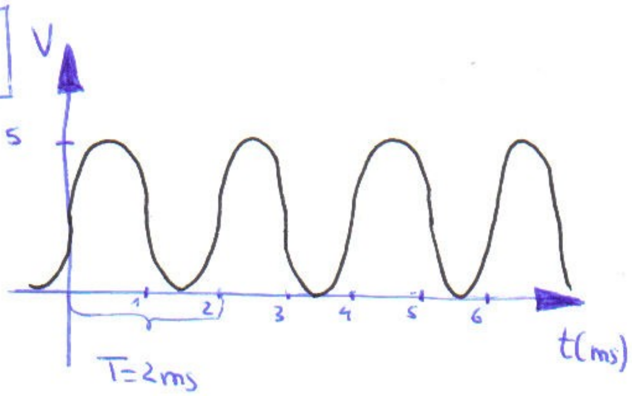
$=\sqrt{\frac{1}{3}\left[100\frac{t^3}{3}+25t-100\frac{t^2}{2}\right]_0^1 + \frac{1}{3}\left[100t+25\frac{t^3}{3}-100\frac{t^2}{2}\right]_1^2} =$

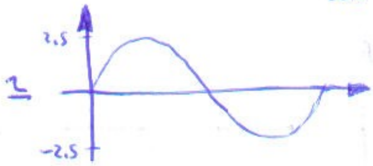
$=\sqrt{\frac{1}{3}\left[\frac{100}{3}+25-50\right]+\frac{1}{3}\left[\cancel{200}+\frac{200}{3}-\cancel{200}-\left(100+\frac{25}{3}-50\right)\right]} =$

$=\sqrt{\frac{1}{3}\left(\frac{25}{3}\right)+\frac{1}{3}\left(\frac{25}{3}\right)} = \sqrt{2\cdot\frac{25}{9}} = \frac{5\sqrt{2}}{3}\ V$

c) $V_{pp} = V_{max} - V_{min} = 5-(-5) = 10\ V$

7





$2{,}5\ \text{sen}\left(\frac{2\pi}{T}t\right) =$

$= 2{,}5 \cdot \text{sen}(\pi t)$

$2{,}5 \cdot \text{sen}\,\pi t + 2{,}5$

a) $V_{med} = \frac{1}{T}\int_0^T x(t)\cdot dt = \frac{1}{2}\int_0^2 2{,}5\ \text{sen}\,\pi t \cdot dt + \frac{1}{2}\int_0^2 2{,}5\, dt =$

$= \frac{2{,}5}{2\pi}\int_0^2 \text{sen}\,\pi\cdot t \cdot dt + \frac{2{,}5}{2}\int_0^2 dt = \underbrace{\frac{2{,}5}{2\pi}\Big[-\cos \pi\cdot t\Big]_0^2}_{0} + \frac{2{,}5}{\cancel{2}}\cdot\cancel{2} =$

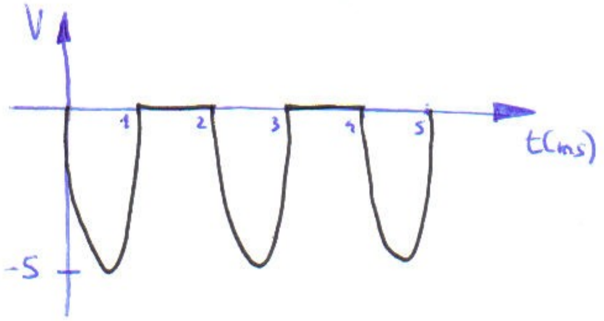
$= 2{,}5\ V$

b) $V_{ef} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\cdot dt} = \sqrt{\frac{1}{2}\int_0^2 (2{,}5\cdot\text{sen}\,\pi t + 2{,}5)^2\, dt} =$

$= \sqrt{\frac{1}{2}\int_0^2 \left(6{,}25\cdot \text{sen}^2\pi t + 6{,}25 + 13{,}5\ \text{sen}\,\pi t\right) dt}$

c) $V_{pp} = V_{max} - V_{min} = 5 - 0 = 5V$

$\boxed{8}$ Vemos que es el $\boxed{4}$ pero negativo:

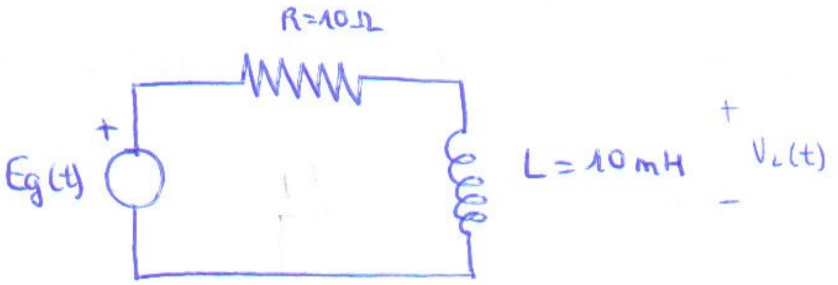


a) $V_{med} = -V_{med\boxed{4}} = -\frac{5}{\pi}$ V

b) $V_{ef} = -V_{ef\boxed{4}} = -2{,}5$ V

c) $V_{pp} = V_{max} - V_{min} = 0 - (-5) = 5$ V

- EJERCICIO DE EXAMEN: Circuito RL



a) Escalón de amplitud $V=10\,V$, V_L?

b) Exponencial $E_g = 10\cdot e^{-2t}$

(Suponer que $i_L(t=0)=0$, t=ms)

$$H(s) = \frac{V_L}{E_g}$$

$$V_L = I_L \cdot R_L = \frac{E_g}{R+S\cdot L}\cdot S\cdot L$$

$R = 10\,\Omega$

$Z_L = S\cdot L = 10\cdot 10^{-3}\cdot S$

$$H(s) = \frac{\frac{E_g}{R+S\cdot L}\cdot S\cdot L}{E_g} = \frac{S\cdot L}{R+S\cdot L} = \frac{S}{S+\frac{R}{L}}$$

---

a) $V = 10\,V$

$E_g(t) = E_g\cdot e^{st}$, $s=0$, $E_g = 10$

$$\boxed{V_L(t) = V_{L,p}(t) + V_{L,n}(t)}$$

$V_{L,p}(t) = H(s=0)\cdot E_g = 0\cdot E_g = 0 \Rightarrow V_L(t) = V_{L,n}(t)$

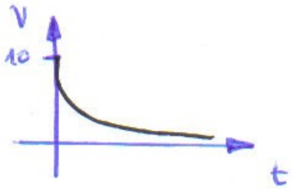
$V_{L,n}(t) = A\cdot e^{P_1\cdot t}$ $\quad P_1$ = Polo del denominador de $H(s) = -\frac{R}{L}$

$V_{L,n}(t) = A\cdot e^{-\frac{Rt}{L}} = V_L(t)$

Si $i_L(t=0)=0 \implies V_L(t=0^+) = E_g = 10\,V$

$$V_L(0) = A = 10 \quad , \quad \frac{R}{L} = \frac{10}{10\cdot 10^{-3}} = 10^3\ ms = 1\,s$$

$$V_L(t) = 10 \cdot e^{-t}$$



b) $E_g(t) = 10 \cdot e^{-2t} \quad , \quad E_g = 10 \,, \; s=-2$

$$V_L(t) = V_{L,p}(t) + V_{L,n}(t)$$

$$V_{L,p}(t) = H(s=-2) \cdot E_g = \frac{-2}{-2+1} \cdot 10 \cdot e^{-2t} = 2 \cdot 10 \cdot e^{-2t}$$

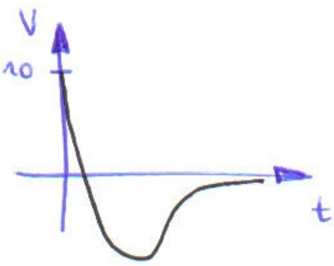
$$V_{L,n}(t) = A \cdot e^{p_1 t} = A \cdot e^{-t}$$

$$V_L(t) = 2 \cdot 10\, e^{-2t} + A \cdot e^{-t}$$

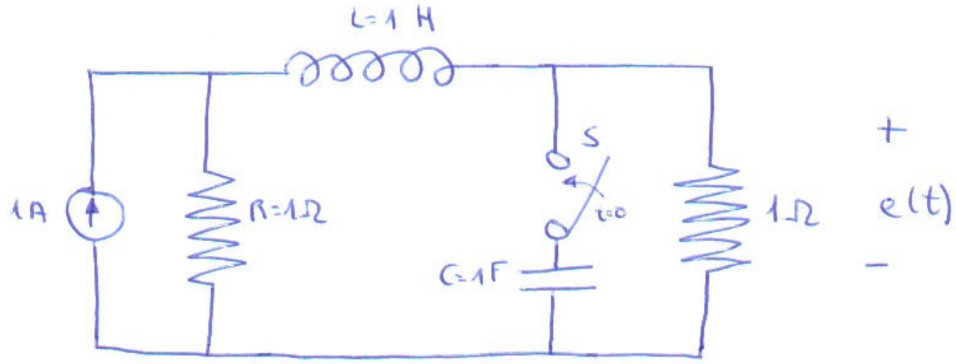
Si $i_L(t=0)=0 \implies V_L(t=0^+) = E_g = 10\ V$

$$V_L(0) = 20 + A = 10 \implies A = -10$$

$$V_L(t) = 20 \cdot e^{-2t} - 10 \cdot e^{-t}$$

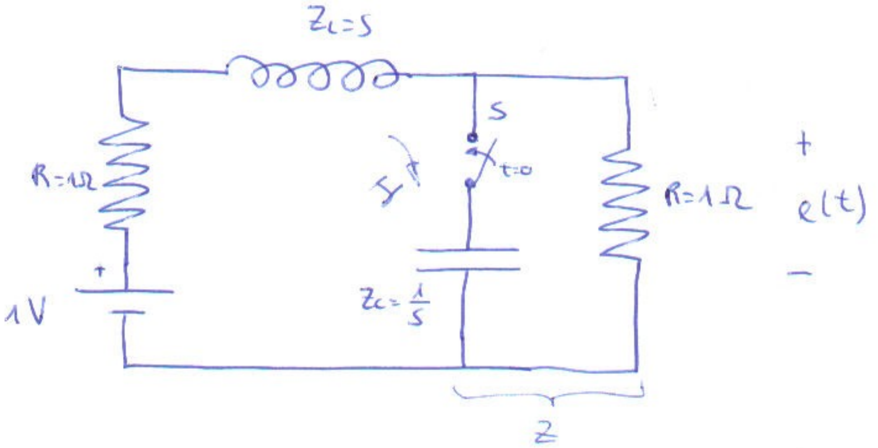


(4) En el circuito de la figura, calcular la evolución de e(t) a partir del instante en que se cierra el interruptor S (t=0). Si el condensador C inicialmente descargado:



$$i_g(t) = I_g \cdot e^{st} = 1 \cdot e^{0t} = 1 \implies \begin{matrix} I_g = 1 \\ s = 0 \end{matrix}$$

$$R = 1\,\Omega \qquad Z_L = s \cdot L = s \qquad Z_C = \frac{1}{Cs} = \frac{1}{s}$$



$(Z_R \parallel Z_C)$

$$Z = \frac{1 \cdot Z_C}{1+Z_C}$$

$$e_g(t) = E_g \cdot e^{st} \Big|_{s=0} = 1\,V$$

$$H(s) = \frac{E}{E_g} = \frac{I \cdot \frac{1 \cdot Z_C}{1+Z_C}}{E_g} = \frac{\frac{\cancel{E_g}}{R+Z_L+\frac{1 \cdot Z_C}{1+Z_C}} \cdot \frac{1 \cdot Z_C}{1+Z_C}}{\cancel{E_g}} = \frac{\frac{Z_C}{1+Z_C}}{R+Z_L+\frac{Z_C}{1+Z_C}} =$$

$$= \frac{Z_C}{(R+Z_L)(1+Z_C)+Z_C} = \frac{1/s}{(1+s)(1+\frac{1}{s})+\frac{1}{s}} = \frac{1}{(s+1)(s+1)+1} = \frac{1}{(s+1)^2+1} =$$

$$= \frac{1}{s^2+2 \cdot s+2} \qquad \frac{1}{0} = \infty$$

Polos: $s^2+2s+2=0 \implies s = \frac{-2 \pm 2j}{2} = -1 \pm j$

$$H(s) = \frac{1}{(s-p_1)(s-p_2)} \qquad \begin{array}{l} p_1 = -1 + j \\ p_2 = -1 - j \end{array}$$

$$e(t) = e_p(t) + e_n(t)$$

$$e_n(t) = A \cdot e^{p_1 \cdot t} + B \cdot e^{p_2 \cdot t} = A \cdot e^{(-1+j)t} + B \cdot e^{(-1-j)t}$$

$$e_p(t) = H(s=0) \cdot E_g = \frac{1}{2} \cdot E_g = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

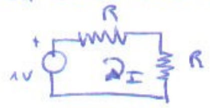
$$e(t) = \frac{1}{2} + A\, e^{(-1+j)t} + B \cdot e^{(-1-j)t}$$

$$V_c(t=0^-) = 0 \implies V_c(t=0^+) = 0$$ (No puede variar bruscamente)

$$V_c(t=0^-) = 0 = V_c(t=0^+) = e(t=0) = \frac{1}{2} + A + B$$

$$\boxed{① \ \frac{1}{2} + A + B = 0}$$

L en cortocircuito si $t<0$



$$i_L(t=0^-) = \frac{1}{2} =$$

$$= i_L(t=0^+) = i_C(t=0^+) = C \cdot \left.\frac{dV_c(t)}{dt}\right|_{t=0^+} =$$

$$= C \cdot \left.\frac{de(t)}{dt}\right|_{t=0^+} = \left. C \cdot A(-i+j) \cdot e^{(-i+j)t} + C \cdot B \cdot (-i-j) \cdot e^{(-i-j)t} \right|_{t=0^+} =$$

$$= \left. C \cdot A(-1+j) + C \cdot B(-i-1) \right|_{C=1F} = A(-1+j) + B(-1-j) = \frac{1}{2}$$

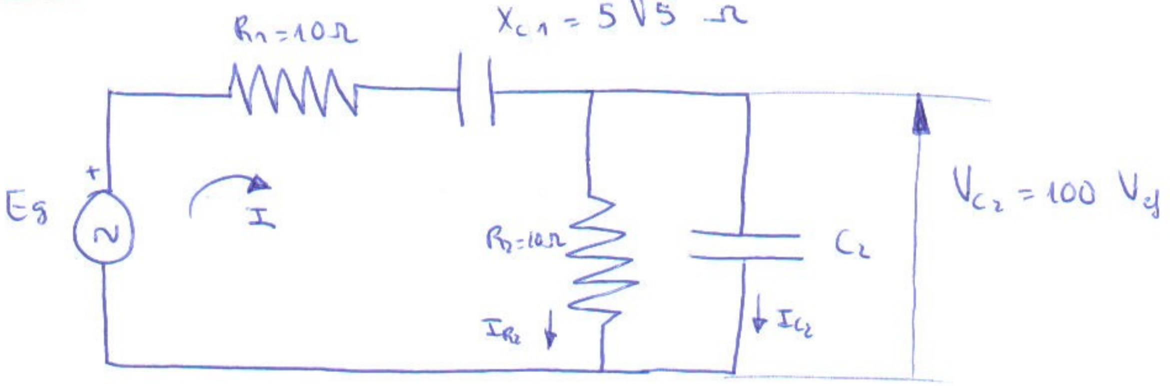
$$② \ \boxed{A(-1+j) + B(-1-j) = \tfrac{1}{2}}$$

$$\left\{ \begin{array}{l} ① \\ ② \end{array} \right. \implies \begin{array}{l} A = -\frac{1}{4} \\ B = -\frac{1}{4} \end{array}$$

$$e(t) = \frac{1}{2} - \frac{1}{4} \cdot e^{(-1+j)t} - \frac{1}{4} e^{(-1-j)t} = \frac{1}{2} - \frac{1}{4} e^{-t} \left[e^{jt} + e^{-jt} \right] =$$

$$= \frac{1}{2} - \frac{1}{2} e^{-t} \left[\frac{e^{jt} + e^{-jt}}{2} \right] = \frac{1}{2} \left(1 - e^{t} \cdot \cos t \right)$$

· EJERCICIO DE EXAMEN:



$E_g = E_0 \cdot sen(50t + \varphi)$

a) Sabiendo que $P_{E_g} = 3250\ W$, determinar (C_2, E_0 y φ)

NOTA: Tómese V_{C_2} en el origen de fases ($\varphi = 0$)

$$|I| = \sqrt{|I_{R_2}|^2 + |I_{C_2}|^2}$$

$$P_{R_2} = |I_{R_2}|^2 \cdot R_2 = \frac{|V_{C_2}|^2}{|R_2|^2} \cdot R_2 = \frac{100^2}{10} = 1000 = 10^3\ W$$

$$P_{E_g} = P_{R_2} + P_{R_1} \Rightarrow P_{R_1} = 3250 - 1000 = 2250\ W = |I|^2 \cdot R_1 = |I|^2 \cdot 10$$

$$|I|^2 = 225 \Rightarrow |I| = 15\ A$$

$$|I_{C_2}| = \sqrt{|I|^2 - |I_{R_2}|^2} = \sqrt{15^2 - 10^2} = \sqrt{125} = 5\sqrt{5}\ A$$

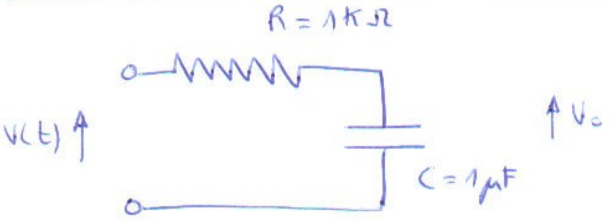
$$I_{R_2} = \frac{V_{C_2}}{R_2} = 10\ A$$

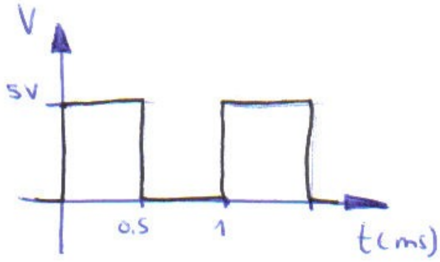
$$I = 10 + 5\sqrt{5}\,j$$

$$E_g = I \cdot R_1 + I \cdot \underbrace{Z_{C_1}}_{X \cdot j} + I_{R_2} \cdot R_2$$

...

• EJERCICIO DE EXAMEN:

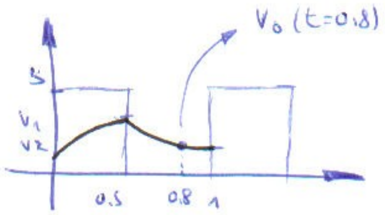




$f = 1\ kHz$

c) $t = 0{,}8$ ms , ¿V_0?

INICIO



$$\left[V_a = V_f + (V_i - V_f) \cdot e^{-t/\tau} \right]$$ FÓRMULA CARGA Y DESCARGA CONDENSADOR

$$\tau = R \cdot C = 10^{3} \cdot 10^{-6} = 10^{-3} = 1\ ms$$

$$V_f = 5\ V \ , \ V_i = 0$$

$t = \frac{1}{2}$, $V_a = 5 - 5 \cdot e^{-0,5} = 1{,}967\ V$

$$V_0 = V_0(t = 0{,}8\ ms) \underset{V_f = 0}{=} V_a \cdot e^{0,5-0,8} = 1{,}967 \cdot e^{-0,3} = 1{,}457\ V$$

e) V_1 y V_2 ?

$$\begin{cases} V_2 = 5 + (V_1 - 5) \cdot e^{-t/2\tau} \\ V_1 = 0 + (V_2 - 0) \cdot e^{-t/2\tau} \end{cases} \Longrightarrow \begin{matrix} V_1 = 3{,}11\ V \\ V_2 = 1{,}89\ V \end{matrix}$$

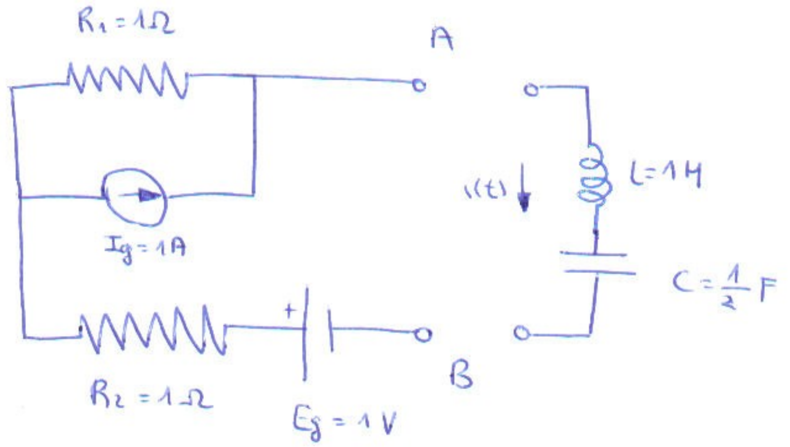
f) $$V_m = \frac{1}{T} \int_0^T x(t)\, dt = \frac{1}{1} \int_0^{T/2} 5 + (V_1 - 5) \cdot e^{-t/2}\, dt + \frac{1}{1} \int_{T/2}^{T} V_2 \cdot e^{-t/2}\, dt =$$

$$= \left[\left(5 \cdot t + (V_1 - 5) \cdot e^{-t/2} \Big/ \left(-\frac{1}{2}\right) \right]_0^{T} + \left[\frac{V_2 \cdot e^{-t/2}}{1/2} \right]_{T/2}^{T} =$$

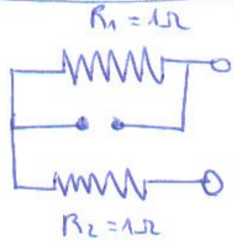
$V_1 + V_2 = 5$
$V_1 - 5 = V_2$

$$= \left[\frac{(5t + V_1 - 5) \cdot e^{t/2}}{-1/2} + \frac{V_1 \cdot e^{-t/2}}{-1/2} \right]_0^{T/2} = \frac{5}{2}\ V$$

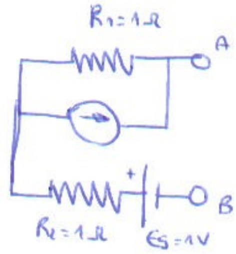
EJERCICIO DE EXAMEN:



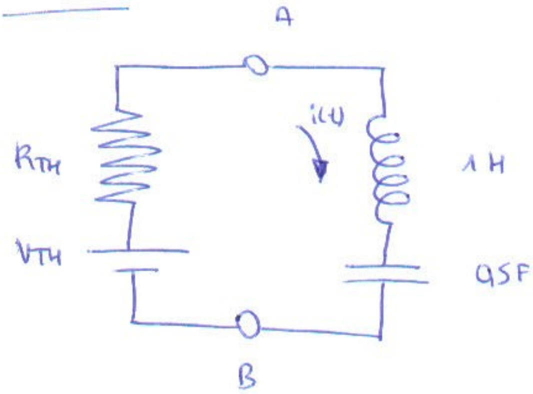
• Equivalente Thévenin:



$R_{TH} = R_1 + R_2 = 2\Omega$



$V_{TH} = V_{AB} = I_g \cdot R_2 + E_g = 1 + 1 = 2\ V$



$Z_L = S \cdot L = S$

$Z_C = \frac{1}{C \cdot S} = \frac{2}{S}$

$$H(S) = \frac{I}{E} = \frac{\frac{\cancel{E}}{2+S+\frac{2}{S}}}{\cancel{E}} = \frac{1}{2+S+\frac{2}{S}} =$$

$$= \frac{S}{S^2 + 2 \cdot S + 2}$$

Raíces: $s^2 + 2s + 2 = 0 \Rightarrow p_1 = -1 + j \qquad p_2 = -1 - j$

$i(t) = i_n(t) + i_p(t)$

$i_n(t) = A \cdot e^{p_1 t} + B \cdot e^{p_2 t} = A e^{(-1+j)t} + B \cdot e^{(-1-j)t}$

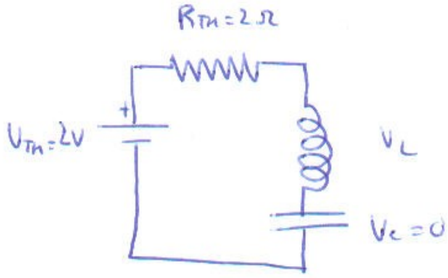
$i_p(t) = H(S=0) \cdot E \Big|_{E = V_{TH}} = \frac{0}{2} = 0$

$i(t) = i_n(t) = A \cdot e^{(-1+j)t} + B \cdot e^{(-1-j)t}$

$i_L(t=0^-) = 0 = i_L(t=0^+) \Rightarrow i(0) = A + B = 0$

$V_c\,(t=0^-) = 0 = V_c\,(t=0^+)$

$$V_L(t=0^+) = V_{TH} = 2V \Big|_{V_L=0}$$



$$V_L\,(t=0^+) = 2 = L \cdot \frac{di(t)}{dt}\Big|_{t=0^+} = A(-1+j) + B(-1-j)$$

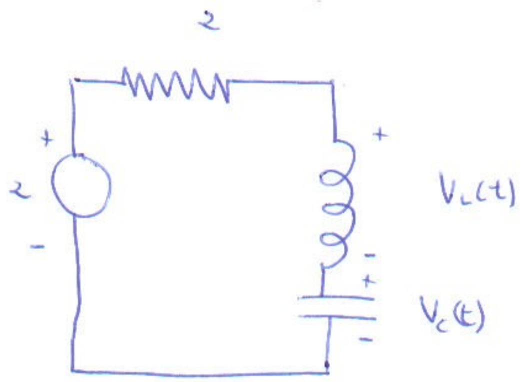
$$\begin{cases} A+B=0 \\ A(-1+j)+B(-1-j)=2 \end{cases} \implies A=\frac{1}{j} \qquad B=-\frac{1}{j}$$

$$i(t) = \frac{1}{j}e^{(-1+j)t} - \frac{1}{j}e^{(-1-j)t} = e^{-t}\left[\frac{e^{jt}-e^{-jt}}{j}\right] =$$

$$= 2\cdot e^{-t}\left[\frac{e^{jt}-e^{-jt}}{2j}\right] = 2\cdot e^{-t}\cdot \text{sen}\,t$$

- $V_L(t)$, $V_c(t)$:

$$V_L(t) = L\cdot\frac{di(t)}{dt} = 2\cdot\left[e^{-t}\cos t - \text{sen}\,t\cdot e^{-t}\right] = 2\cdot e^{-t}(\cos t - \text{sen}\,t)$$



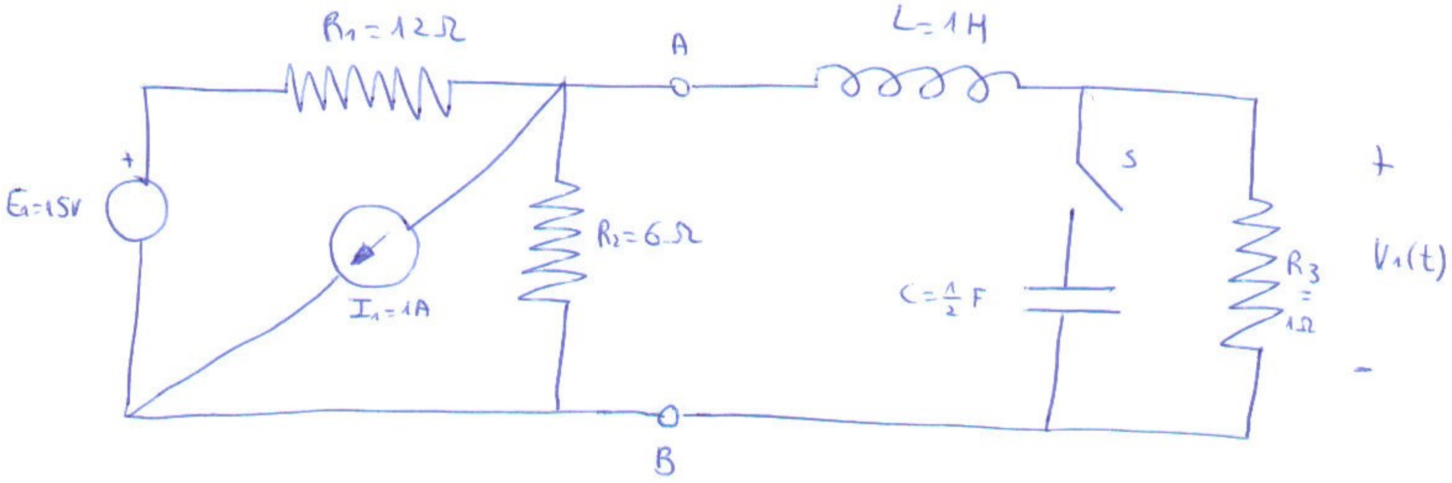
$$2 = i(t)\cdot 2 + V_L(t) + V_c(t)$$

$$V_c(t) = 2 - i(t) - V_L(t) =$$

$$= 2 - 4\cdot e^{-t}\,\text{sen}\,t - 2\cdot e^{-t}(\cos t - \text{sen}\,t) =$$

$$= 2 - 2\cdot e^{-t}(\text{sen}\,t + \cos t)$$

• EJERCICIO DE EXAMEN:



a) (1 pto) Función de transferencia $H(s)$

b) (0,5 ptos) Eq. Th. a la izquierda de A y B

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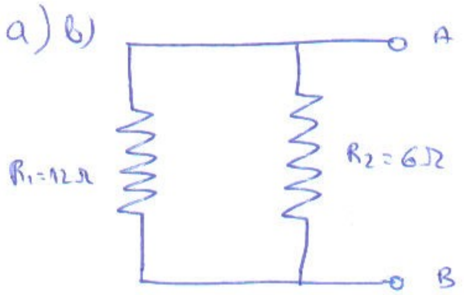
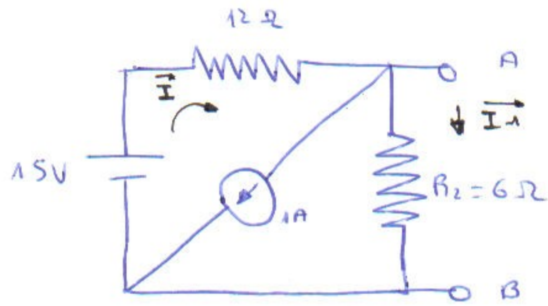
a) b)



$$R_{TH} = 6 \parallel 12 = \frac{6 \cdot 12}{6+12} = 4\,\Omega$$

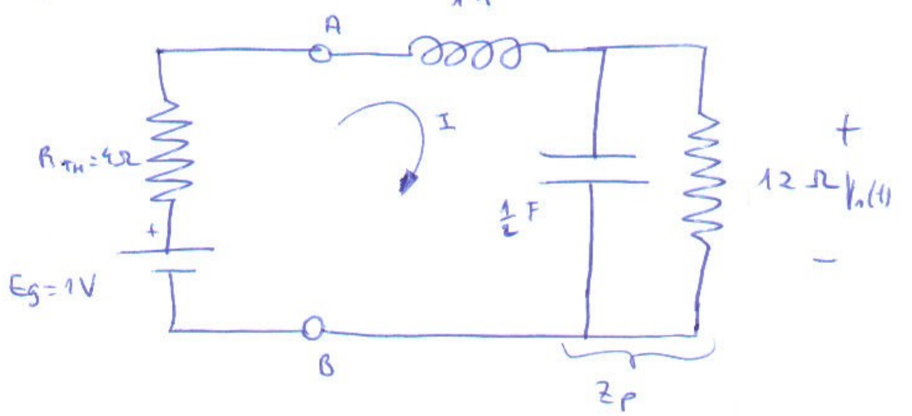


$$15 = 12 \cdot I + (I-1) \cdot 6 \Rightarrow$$

$$\Rightarrow I = \frac{21}{18}\,A$$

$$V_{TH} = V_{AB} = (I-1) \cdot 6 = 1\,V$$

b) (1 pto) ¿$V_1(t)$?



$$Z_L = L \cdot S = S$$

$$Z_C = \frac{1}{CS} = \frac{2}{S}$$

$$H(s) = \frac{V_1}{E_g}$$

$$Z_p = \frac{2}{s} \parallel 1 = \frac{2}{2+s}$$

$$I = \frac{E_g}{4+s+\frac{2}{2+s}} \qquad H(s) = \frac{\overbrace{I_{AB} R_{AB}}^{V_1}}{E_g} = \frac{\frac{\cancel{E_g}}{4+s+\frac{2}{2+s}} \cdot \frac{2}{2+s}}{\cancel{E_g}} =$$

$$= \frac{2}{(4+s)(2+s)+2} = \frac{2}{s^2+6s+10}$$

$$V_1(t) = V_{1n}(t) + V_{1p}(t)$$

Polos: $s^2+6s+10=0$

$s = -3 \pm j$

$$V_{1,n}(t) = A \cdot e^{p_1 t} + B e^{p_2 t} = A \cdot e^{(-3+j)t} + B \cdot e^{(-3-j)t}$$

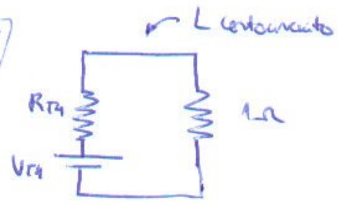
$$V_{1,p}(t) = H(s=0) \cdot E_g = \frac{1}{5} \cdot 1 = \frac{1}{5} = 0{,}2\ V$$

$$V_1(t) = 0{,}2 + A \cdot e^{(-3+j)t} + B e^{(-3-j)t}$$

$$V_c(t=0^-) = 0 = V_c(t=0^+) = V_1(t=0^+) = 0{,}2 + A + B$$

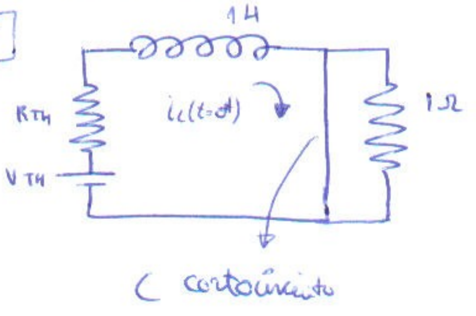
① $A + B + 0{,}2 = 0$

$\boxed{t=0^-}$



$$i_L(t=0^-) = \frac{1}{5}$$

$\boxed{t=0^+}$



$$i_L(t=0^-) = \frac{1}{5} = i_L(t=0^+) = I_c(t=0^+) =$$

$$= C \cdot \left.\frac{dV_c(t)}{dt}\right|_{t=0^+} = C \cdot \left.\frac{dV_1(t)}{dt}\right|_{t=0^+} = \left.\frac{1}{2}\left((-3+j)Ae^{(-3+j)t} + (-3-j)Be^{(-3-j)t}\right)\right|_{t=0^+} =$$

$$= \frac{1}{2}(-3+j)A + \frac{1}{2}(-3-j)B = \frac{1}{5}$$

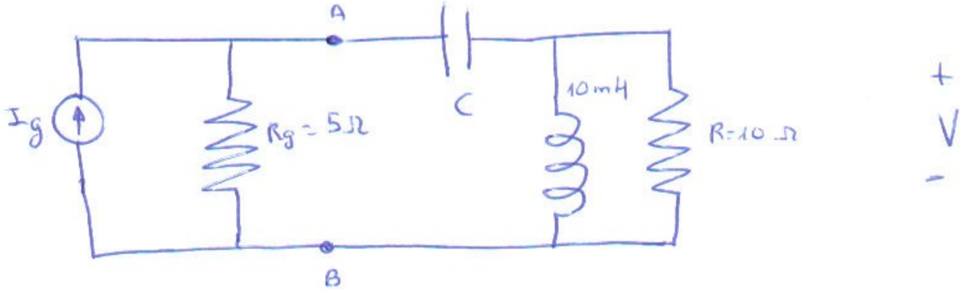
$$\begin{cases} ① \\ ② \ \frac{1}{2}(-3+j)A + \frac{1}{2}(-3-j)\cdot B = \frac{1}{5} \end{cases} \Rightarrow \begin{array}{l} B = \frac{1}{10}(-1-j) \\ A = \frac{1}{10}(-1+j) \end{array}$$

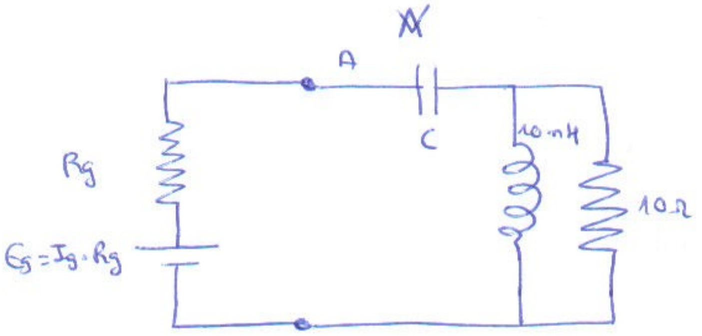
$$V_1(t) = \frac{1}{5} + \frac{1}{10}(-1+j) \cdot e^{(-3+j)t} + \frac{1}{2}(-1-j) \cdot e^{(-3-j)t} = \frac{1}{5} + \frac{\sqrt{2}}{10} \cdot e^{-3t}\left[\frac{e^{j(t+\frac{3\pi}{4})} + e^{-j(t+\frac{3\pi}{4})}}{2}\right] \cdot 2 =$$

$$= \frac{1}{5} + \frac{\sqrt{2}}{5} \cdot e^{-3t} \cdot \cos\left(t + \frac{3\pi}{4}\right)$$

• EJERCICIO DE EXAMEN:

b) (0,5 ptos) ¿C si existe adaptación?





Si existe adaptación

$Z_{AB} = Z_g^*$

$\omega = 10^3$ rad/s

$$Z_{AB} = -j \cdot X_C + R \parallel \underbrace{j \cdot X_L}_{\text{Imped bobina}} = -j \frac{1}{\omega C} + \frac{10j \cdot 10}{10 + j \cdot 10} = 5$$

Adaptación

$$\frac{1}{j\omega C} = -j \left(\frac{1}{\omega C}\right) = -j \cdot X_C$$

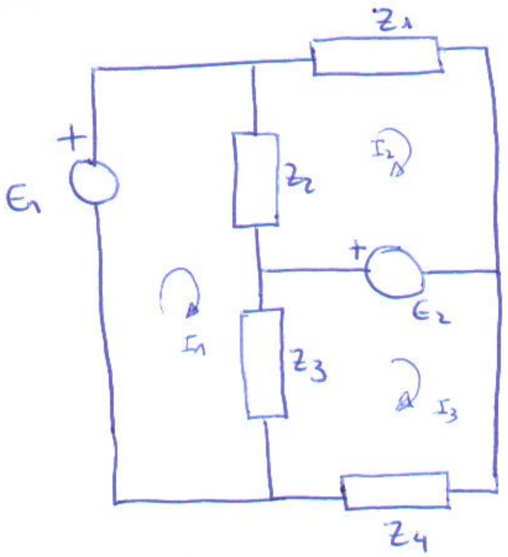
$$j\omega L = j X_L = j \cdot 10^3 \cdot 10^{-3} \cdot 10 = 10j$$

$$-j \cdot \frac{1}{10^3 C} + \frac{100j\,(10 - 10j)}{200} = 5$$

$$-j \frac{1}{10^3 C} + \frac{-1000j + 1000}{200} = -j \cdot \frac{1}{10^3 C} - 5j + \cancel{5} = \cancel{5}$$

$$\cancel{j} \frac{1}{10^3 C} = 5\cancel{j} \qquad C = \frac{1}{5 \cdot 10^3} = 200\,\mu F$$

- EJERCICIO DE EXAMEN:



$e_1 = \sqrt{2} \cdot \text{sen}\left(wt + \frac{\pi}{4}\right)$

$e_2 = \cos(wt)$

$Z_1 = 1 - j$

$Z_2 = j$

$Z_3 = -j$

$Z_4 = 1 + j$

b) Valores de las potencias del generador E_1:

$E_1 = \sqrt{2} \cdot e^{j\frac{\pi}{4}} = \sqrt{2} \cdot \left(\cos\frac{\pi}{4} + j \cdot \text{sen}\frac{\pi}{4}\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) = 1 + j$

$E_2 = e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j \cdot \text{sen}\frac{\pi}{2} = j$

① $1 + j = I_1 (j - j) - I_2 \cdot j + \underset{-(Z_3)}{j} \cdot I_3$

② $j = I_2 (j + 1 - j) - I_1 \cdot j$

③ $-j = I_3 (1 + j - j) - I_1(-j)$

$$\begin{cases} 1 + j = -I_2 \cdot j + j \cdot I_3 \\ j = I_2 - I_1 \cdot j \\ -j = I_3 + I_1 \cdot j \end{cases}$$

$I_1 = -\frac{1}{2} + \frac{1}{2}j = \frac{\sqrt{2}}{2} \cdot e^{j \cdot \frac{3\pi}{4}}$

$\boxed{1}$ Pot. media:

$P_1 = \frac{|E_1||I_1|}{2} \cdot \cos\varphi = \frac{\sqrt{2} \cdot \frac{\sqrt{2}}{2}}{2} \cdot \cos\left(\frac{\pi}{4} - \frac{3\pi}{4}\right) = \frac{1}{2}\cos\left(-\frac{\pi}{2}\right) = 0$

$\boxed{2}$ Pot. reactiva:

$Q_1 = \frac{|E_1| \cdot |I_1|}{2} \cdot \text{sen}\,\varphi = \frac{1}{2} \cdot \text{sen}\left(-\frac{\pi}{2}\right) = \frac{1}{2} \cdot (-1) = -\frac{1}{2}$ VAr (Voltiamperios reactivos)

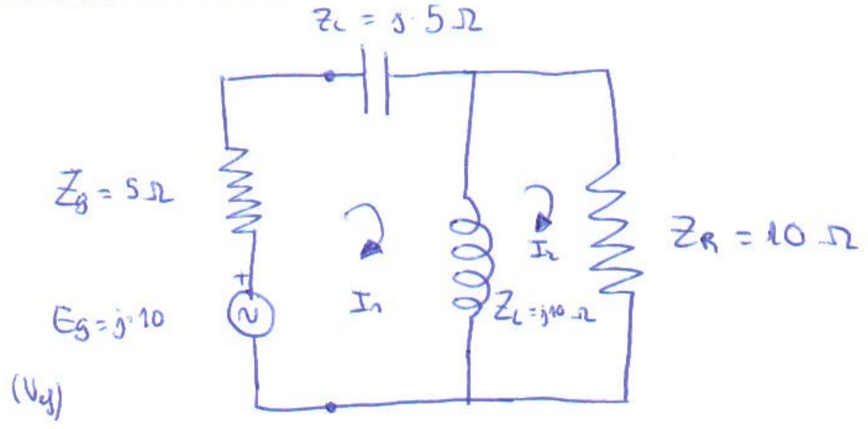
$\boxed{3}$ Pot. aparente:

$|S_1| = \frac{|E_1| \cdot |I_1|}{2} = \frac{1}{2}$ VA

$S_1 = \frac{1}{2} \cdot E_1 \cdot I_1^* = \frac{1}{2}\sqrt{2} \cdot e^{j\frac{\pi}{4}} \cdot \frac{\sqrt{2}}{2} \cdot e^{-j\frac{3\pi}{4}} = P_1 + j \cdot Q_1$

Pot. compleja

- EJERCICIO DE EXAMEN:



a) (1 pto) Kirchoff

b) (0,5 ptos) $(E) = (Z) \cdot (I)$

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b) $\begin{cases} E_g = I_1 \cdot Z_g + I_1 \cdot Z_c + Z_L \cdot I_1 - Z_L \cdot I_2 \\ 0 = I_2 \cdot Z_L - I_1 \cdot Z_L + I_2 \cdot Z_R \end{cases}$

$$\begin{cases} 10j = I_1(5 - 5j + 10j) - I_2 \cdot 10 \cdot j \\ 0 = I_2(10 + 10j) - I_1 \cdot 10j \end{cases} \iff \begin{cases} 10j = I_1(5 + 5j) - I_2 \cdot 10j \\ 0 = -j \cdot 10\, I_1 + (10 + 10j) I_2 \end{cases}$$

$$\begin{pmatrix} j10 \\ 0 \end{pmatrix} = \begin{pmatrix} 5+5j & -j10 \\ -j10 & 10+10j \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

---

c) (0,5 ptos) I_1 e I_2: (Por Cramer)

$$I_1 = \frac{\begin{vmatrix} j10 & -j10 \\ 0 & 10+10j \end{vmatrix}}{\begin{vmatrix} 5+5j & -j10 \\ -j10 & 10+10j \end{vmatrix}} = j = 1 \underline{|90^\circ}$$

$$I_2 = \frac{\begin{vmatrix} 5+5j & j10 \\ -j10 & 0 \end{vmatrix}}{\begin{vmatrix} 5+5j & -j10 \\ -j10 & 10+10j \end{vmatrix}} = -\frac{1}{2} + \frac{1}{2}j = \frac{\sqrt{2}}{2} \cdot e^{j\,\operatorname{arctg}\left(\frac{1/2}{-1/2}\right)} = \frac{\sqrt{2}}{2} e^{j \cdot \frac{-3\pi}{4}} = \frac{\sqrt{2}}{2} \underline{|135^\circ}$$

d) (0,5 ptos). Determinar: el factor de potencia, pot gen. por el generador y la disipada por el circuito.

$$V = E_g - I \cdot Z_g = j \cdot 10 - j \cdot 5 = 5j = 5 \angle 90^\circ$$

$$I_2 = j = 1 \angle 90^\circ$$

$$\cos\varphi = \cos(\varphi_v - \varphi_i) = \cos(90^\circ - 90^\circ) = 1$$

$$\text{Pot. entregada} = \frac{|V|}{\sqrt{2}} \cdot \frac{|I|}{\sqrt{2}} \cdot \cos\varphi = \frac{|V| \cdot |I|}{2} \cdot \cos\varphi = V_{ef} \cdot I_{ef} \cos\varphi = 5 \cdot 1 \cdot 1 = 5\,W$$

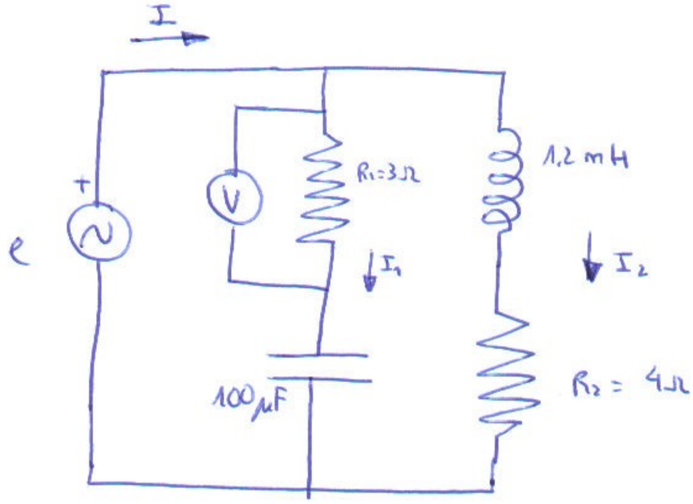
$$\text{Pot. disipada} = |I_2|^2 \cdot R = \frac{1}{2} \cdot 10 = 5\,W$$

$$I_2 = \frac{\sqrt{2}}{2} \angle 135^\circ$$

(11) En el circuito de la figura, la lectura del voltímetro es de 30 V. Si la frecuencia del generador de tensión es de $f = \frac{1{,}25}{\pi}$ KHz, determinar:

a) Corriente entregada por el generador.

b) Pot. entregada por el generador y absorbida por el circuito.



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a) $R_1 = 3\Omega = Z_1$

$$C = 100\,\mu F \Rightarrow Z = \frac{1}{j \cdot W \cdot C} = \frac{1}{j \cdot 2\pi f \cdot C} = \frac{1}{j \cdot 2\cancel{\pi} \frac{1{,}25}{\cancel{\pi}} 10^{3} \cdot 100 \cdot 10^{-6}} = \frac{10}{j \cdot 2{,}5} = -j \cdot 4$$

$$L = 1{,}2\,mH \Rightarrow Z_3 = j \cdot W \cdot L = j \cdot 2\pi f \cdot L = j \cdot 2\cancel{\pi} \cdot \frac{1{,}25}{\cancel{\pi}} \cdot \cancel{10^{3}} \cdot 1{,}2 \cdot \cancel{10^{-3}} = j \cdot 3$$

$R_2 = 4\Omega = Z_4$

$$\frac{V_{volt}}{R_1} = \frac{30}{3} = 10\,A \Rightarrow I_1 = 10\,A_{ef} \qquad I_1 = 10\sqrt{2}\,A$$

$$V_{ef} = \frac{V_{volt}}{\sqrt{2}}$$

$$Z_1 \cdot I_1 + Z_2 \cdot I_1 = Z_3 \cdot I_2 + Z_4 \cdot I_2$$

$$(Z_1 + Z_2) \cdot I_1 = (Z_3 + Z_4) \cdot I_2$$

$$I_2 = \frac{(Z_1 + Z_2)}{(Z_3 + Z_4)} \cdot I_1 = \frac{3 - 4j}{4 + 3j} \cdot I_1$$

$$I = I_1 + I_2 = I_1 + \frac{3-4j}{4+3j} I_1 = I_1 \left(1 + \frac{3-4j}{4+3j}\right) = I_1 \left(\frac{7-j}{4+3j}\right) =$$

$$= \frac{\sqrt{50} \cdot e^{j\,\text{arctg}(-1/7)}}{5 \cdot e^{j\,\text{arctg}(3/4)}} \cdot I_1 = \frac{\sqrt{50} \cdot e^{j(-8{,}13^\circ)}}{5 \cdot e^{j(36{,}87^\circ)}} \cdot I_1 = \frac{\sqrt{50}}{\sqrt{25}} e^{j(-8{,}13^\circ - 36{,}87^\circ)} \cdot I_1 =$$

$$= \sqrt{2} \cdot e^{j(-45^\circ)} \cdot 10 \cdot \sqrt{2} = 20 \cdot e^{-j(45^\circ)}$$

b) $$E = Z_1 \cdot I_1 + Z_2 \cdot I_1 = (Z_1 + Z_2) \cdot I_1 = (3 - j4)\, 10\sqrt{2} =$$

$$= \sqrt{25} \cdot e^{j\,\text{arctg}\left(\frac{-4}{3}\right)} \cdot 10\sqrt{2} = 50\sqrt{2} \cdot e^{-j(53{,}13^\circ)}$$

$$I = 20 \cdot e^{-j45^\circ}$$

$$\cos\varphi = \cos(\varphi_V - \varphi_i) = \cos(-53{,}13^\circ + 45^\circ) = \cos(-8{,}13^\circ) = \cos(8{,}13^\circ) = 0{,}9899$$

$$\text{Pot. entreg. gen} = \frac{|V| \cdot |I|}{2} \cdot \cos\varphi = \frac{50\sqrt{2} \cdot 20}{2} \cdot 0{,}9899 = 700\ W$$

$$\text{Pot. absorb} = \frac{1}{2} I_1^2 \cdot 3 + \frac{1}{2} I_2^2 \cdot 4 \overset{*}{=} \frac{1}{2} \cdot 200 \cdot 3 + \frac{1}{2} \cdot 4 \cdot 200 = 300 + 400 = 700\ W$$

(Pot entreg = Pot absorb)

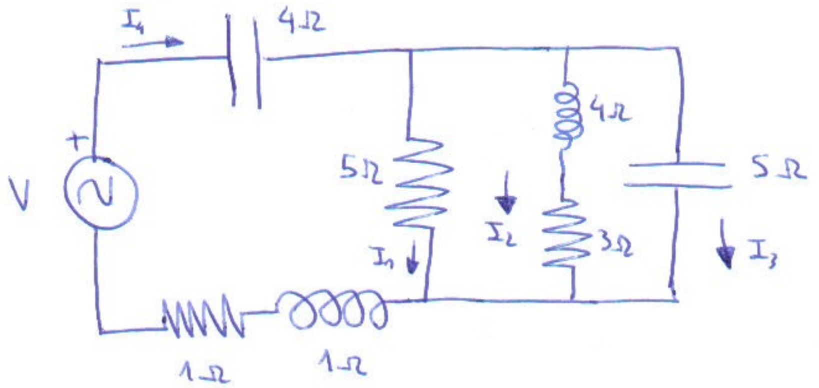
$$* \left[I_2 = \frac{3-4j}{4+3j} I_1 \qquad |I_2| = \frac{\cancel{\sqrt{25}}}{\cancel{\sqrt{25}}} |I_1| \quad \Rightarrow \quad I_1 = 2\sqrt{10} \quad |I_1|^2 = 200 \right]$$

(12) En el circuito de la figura, si $I_1 = 5$ A, determinar:

a) Valor de I_4.

b) Pot. absorbida por el circuito.

c) Factor de potencia.



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a) $I_4 = I_1 + I_2 + I_3$

$5 \cdot I_1 = I_2 \cdot (\overset{X_L}{4j} + 3) \implies I_2 = \frac{5}{3+4j} I_1$

$5 \cdot I_1 = I_3 \; (\overset{X_C}{-5 \cdot j}) \implies I_3 = \frac{1}{-j} I_1 = j \cdot I_1$

$$I_4 = I_1 + \frac{5}{3+4j} I_1 + j \cdot I_1 = \left(1 + \frac{5}{3+4j} + j\right) \cdot I_1 = \left(\frac{3+4j+5+3j-4}{3+4j}\right) I_1 =$$

$$= \frac{4+7j}{3+4j} I_1 = \frac{\sqrt{16+49} \cdot e^{j \cdot \operatorname{arctg}\left(\frac{7}{4}\right)}}{\sqrt{9+16} \cdot e^{j \cdot \operatorname{arctg}\left(\frac{4}{3}\right)}} \cdot I_1 = \frac{\sqrt{65}}{\cancel{5}} \cdot \frac{e^{j \cdot 60'25°}}{e^{j \, 53'130}} \cdot \cancel{5} = \sqrt{65} \cdot e^{j(60,25° - 53,13°)} =$$

$$= \sqrt{65} \cdot e^{j \cdot 7'130} = \sqrt{65}\,\angle 7'13° = \sqrt{65} \cdot (\cos 7'13° + j \cdot \operatorname{sen} 7'13°) \text{ A}$$

b) $P = \frac{1}{2}(I_1)^2 \cdot 5 + \frac{1}{2}(I_2)^2 \cdot 3 + \frac{1}{2}(I_4)^2 \cdot 1 =$

$= \frac{1}{2} \cdot 5 \cdot 25 + \frac{1}{2} \left| \frac{5}{3+j4} \right| \cdot |I_1| + \frac{1}{2}(\sqrt{65})^2 \cdot 1 = \frac{1}{2} \cdot 5 \cdot 25 + \frac{1}{2} 5 + \frac{1}{2} 65 =$

$= 132.5$ W

c) $V = I_4 \cdot (-j \cdot X_C) + I_1 \cdot 5 + I_4 \cdot (1) + I_4 \cdot (j \cdot X_L) =$

$= I_4 \cdot (-j) \cdot 4 + I_1 \cdot 5 + I_4 \cdot j + I_4 = I_4 (1-3j) + I_1 \cdot 5 = 5 \cdot I_1 +$

$+ \dfrac{4+7j}{3+4j} I_1 (1-3j) = I_1 \cdot \left(5 + \dfrac{(4+7j)(1-3j)}{3+4j} \right) = I_1 \left(5 + \dfrac{4-12j+7j+21}{3+4j} \right) =$

$= I_1 \left(5 + \dfrac{25-5j}{3+4j} \right) = I_1 \cdot \left(\dfrac{15+j \cdot 20 + 25 - 5j}{3+4j} \right) = I_1 \cdot \left(\dfrac{40+15j}{3+4j} \right) =$

$= I_1 \cdot \dfrac{\sqrt{1600+225} \cdot e^{j \operatorname{arctg}\left(\frac{15}{40}\right)}}{\sqrt{9+16} \cdot e^{j \operatorname{arctg}\left(\frac{4}{3}\right)}} = \cancel{I_1} \cdot \dfrac{\sqrt{1825}}{\cancel{5}} \cdot \dfrac{e^{j \cdot 20{,}56^\circ}}{e^{j\, 53'13^\circ}} =$

$= \sqrt{1825} \cdot e^{j\,(20{,}56^\circ - 53{,}13^\circ)} = \sqrt{1825} \cdot e^{-j\,(32{,}57^\circ)}$

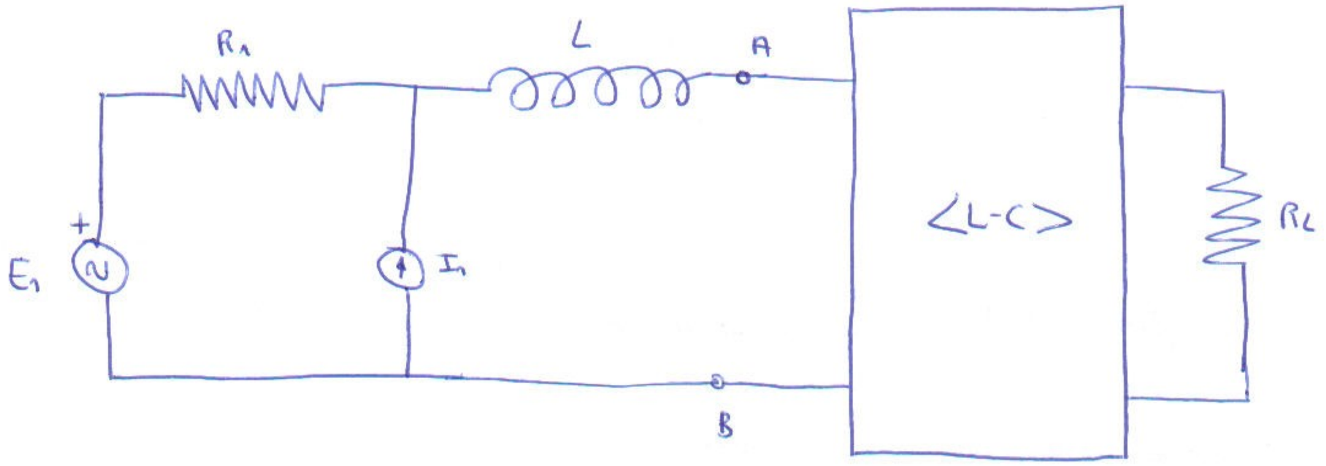
$\cos \varphi = \cos (\varphi_V - \varphi_I) = \cos (-32{,}57^\circ - 7{,}13^\circ) = \cos(-39{,}7^\circ) = \cos (39{,}7^\circ) =$

$= 0{,}7694$ (Factor de potencia)

(16) En el circuito de la figura, la red adaptadora L-C es tal que en R_L se disipa la máxima potencia que se puede extraer de la red que existe a la izquierda de la sección A-B. Se pide determinar:

a) Equiv. Th. entre A y B hacia la izquierda y el equiv Norton hacia la derecha

b) Pot. disipada en R_L.



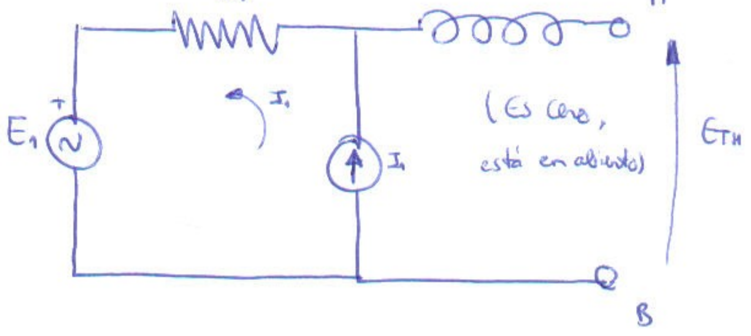
$e_1 = 30 \cos wt \Rightarrow e_1 = 30 \cdot \text{sen}(wt + \pi/2) \Rightarrow E_1 = 30_{\angle \pi/2} = 30 \cdot j$

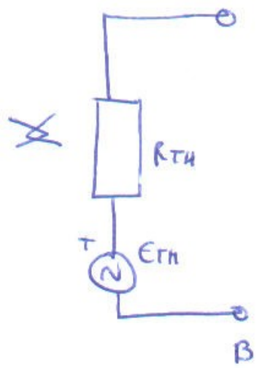
$i_1 = \text{sen}\, wt \Rightarrow I_1 = 1$

$R_1 = 30\,\Omega$

$X_L = 15\,\Omega \Rightarrow Z_L = j \cdot X_L = 15 \cdot j$

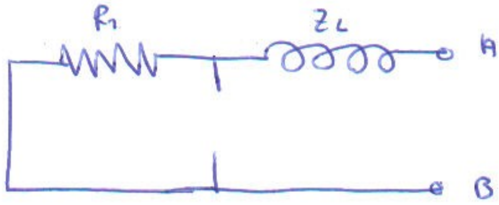
$R_L = 60\,\Omega$



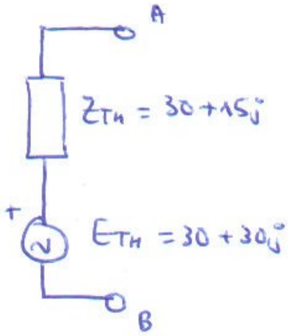


$J_1 \cdot R_1 + E_1 = E_{TH}$ $\qquad 1 \cdot 30 + 30j = 30 + 30j = E_{TH} = 30\sqrt{2}\,\angle \pi/4$

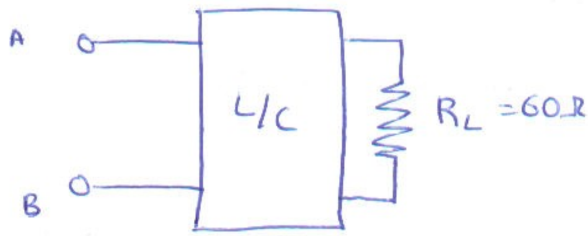
Para Z_{TH} eliminamos los I_i y cortocircuitamos los E_i:



$Z_{TH} = R_1 + Z_L = 30 + 15j$



Ahora tenemos un circuito pasivo (equivalente = impedancia)



Z_{AB}

$\Downarrow$

No se disipa energía ni en L ni en C, por tanto, es una impedancia sin parte real.

- La potencia de R_L recibida es la misma que genera E_{TH} (alcanzarán los max. a la vez)
- Cuando está adaptado el generador es cuando alcanza el máximo (conjugado):

$$Z_{AB} = Z_{TH}^{*} = 30 - 15j$$

b) $$P_{RL} = P_{TH} = P_{AB}\Big|_{máx} = \frac{(E_{TH})^{2\,*}}{8 \cdot R_{TH}}\Bigg|_{E_{TH} = 30\sqrt{2}} = \frac{30^2 \cdot \cancel{2}}{\cancel{8} \cdot 3\cancel{0}} = \frac{30}{4} = 7{,}5 \text{ W}$$

* Estamos trabajando con valores de pico.