# Numerical recipes 11.1 : Jacobi transformations of a symmetric matrix

$$
\begin{array}{c}
\begin{array}{cc} \text{p column} & \text{q column} \end{array} \\
M_{pq}(\theta) = \left[\begin{array}{ccccccc}
1 & & & & & & \\
& 1 & & & & & \\
& & \cos\theta & \cdots & \sin\theta & & \\
& & \vdots & 1 & \vdots & & \\
& & -\sin\theta & \cdots & \cos\theta & & \\
& & & & & 1 & \\
& & & & & & 1
\end{array}\right]
\begin{array}{l} \\ \\ \leftarrow \text{p row} \\ \\ \leftarrow \text{q row} \\ \\ \\ \end{array}
\end{array}
$$

Let $a_{pq}$ be the largest magnitude off diagonal element (and thus $a_{pq} \neq 0$)

$$A' = M_{pq}^T A M_{pq}$$

annulate the off-diagonal element

$$a'_{pq} = \underbrace{(\cos^2\theta - \sin^2\theta)}_{\cos 2\theta} a_{pq} + \underbrace{\sin\theta\cos\theta}_{\frac{\sin 2\theta}{2}}(a_{pp} - a_{qq}) = 0$$

$$\Rightarrow \quad \cos 2\theta \quad a_{pq} = (a_{qq} - a_{pp}) \frac{\sin 2\theta}{2}$$

$$\Rightarrow \quad \cot 2\theta = \frac{\cos 2\theta}{\sin 2\theta} = \frac{a_{qq} - a_{pp}}{2 a_{pq}} := \phi$$

$$\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta} = \phi$$

$$\cot^2\theta - 2\phi\cot\theta - 1 = 0 \quad \Rightarrow \quad \cot\theta = \phi \pm \sqrt{\phi^2 + 1}$$

We want small angles, $\tan\theta$ is safer to compute

$$\Rightarrow \tan\theta = \frac{1}{\phi \pm \sqrt{\phi^2+1}}$$ ; let us take the smallest magnitude solution:

$$\tan\theta = \frac{\operatorname{sign}(\phi)}{|\phi| + \sqrt{\phi^2+1}} \quad \Rightarrow \quad \begin{cases} \cos\theta = \dfrac{1}{\sqrt{\tan^2\theta + 1}} \\ \sin\theta = \cos\theta \cdot \tan\theta \end{cases}$$