Representation theories of some towers of algebras related to the symmetric groups and their Hecke algebras

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Sg, Hecke

Def Heckesg

Tout mettre ensemble dans la même algèbre.

Dimension of H$\mathfrak{S}_n$?

Natural basis of $H\mathfrak{S}_n$

Relation: $s_i\pi_i = \pi_i$

Thus: $\sigma\pi_\tau \rightsquigarrow \sigma'\pi_\tau$ if $\text{Des}(\sigma) \cap \text{Rec}(\tau) \neq \emptyset$

Theorem
Basis of $H\mathfrak{S}_n$: $\{s\pi_\tau, \quad \text{Des}(s) \cap \text{Rec}(\tau) = \emptyset\}$

Proof?

Linear independence: easy (triangularity)
Straightening relations:

$$\pi_1 s_1 \rightsquigarrow 1 + s_1 - \pi_1$$
$$\pi_2 s_1 \rightsquigarrow \pi_2 \pi_1 + s_1 s_2 \pi_1 \pi_2 - \pi_1 \pi_2 \pi_1$$
$$\pi_1 s_2 \rightsquigarrow \pi_1 \pi_2 + s_2 s_1 \pi_2 \pi_1 - \pi_1 \pi_2 \pi_1$$

Presentation of $H\mathfrak{S}_n$?
Démo: c'est le bordel

Structural definition of Heckesg

Équations de préservation anti-symmétries:
Demo: pile-poil le bon nombre
=> transposée: symmetry preserving

But: representation theory?

Démo desossage $=>$ n=3
Lancement desossange n=4
$=>$ suggère le treillis des sous-ensemble
Projectifs: P_I: sous-espace I-anti-symmétrique
Simples: $S_I := P_I/...$
Base? Histoire de montrer un élément typique par somme alternée
$=>$ n! idempotents orthogonaux
Décomposition de l'identité
Morita équivalent à l'algèbre d'incidence of the boolean lattice
(explicitement à coup d'idempotents)
Deux slides

Non decreasing functions

Non decreasing function $f : \{1, \ldots, n\} \mapsto \{1, \ldots, n\}$
Monoid algebra of non decreasing functions: $\mathbb{C}[NDF_n]$
Suggests to look for a faithful representation on subsets.
Just the smallest faithful representation

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Link Heckesg NDF

Dessiner pi, overline-pi

Relations similaires HeckeSym

Démo: pi->pi morphisme d'algèbre c'est un quotient de HeckeSym

Démo: Image de Sg par ce morphisme? Représentation équerre
(action sur les produits extérieurs).

Diagramme complet
Démo: triangularization matrice incidence parking -> ordre

Hopf algebras

Original goal: construct new Hopf algebras
Result: no Hopf algebras, but new interpretations of existing bases
of Hopf algebras.

Conclusion

Result of radical computation for HeckSym, Sloane, and thanks to Mike and organisers.