

# Modular representations, crystal bases, and the combinatorics of partitions

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**Abstract:** It has been realized in the past few years that the same combinatorics was underlying two apparently unrelated subjects: the representation theory of symmetric groups over a field  $F$  of finite characteristic  $n$ , and the affine Lie algebra  $\widehat{\mathfrak{sl}}_n$  [LLT96]. A precise connection between these theories can be formulated as follows: the direct sum of the complexified Grothendieck groups  $\bigoplus_{m \geq 0} G_0(F[S_m])$ , endowed with some refined restriction and induction operators originally defined by Robinson [Rob1], build up the basic representation of  $\widehat{\mathfrak{sl}}_n$ .

The representation theory of the type  $A$  Hecke algebra  $H_m(\zeta)$ , for  $\zeta$  a primitive  $n$ th root of unity also involves the same combinatorics, and there,  $n$  need not be a prime. Consideration of the global crystal basis (canonical basis) of the basic representation of  $U_q(\widehat{\mathfrak{sl}}_n)$  led to a conjectural description of the decomposition matrices of Hecke algebras at roots of unity [LLT96]. This conjecture was subsequently proven in a more general form by Ariki [Ar]. A proof was also announced by Grojnowski, relying on the results of [Groj]. Another extension [LT96] provided a conjectural description of the decomposition matrices of  $q$ -Schur algebras at roots of unity. It was given in terms of a new canonical basis of the Fock space representation of  $U_q(\widehat{\mathfrak{gl}}_n)$ , constructed by means of ribbon tableaux (see [LLT97]). This conjecture has been settled recently by Varagnolo and Vasserot [VV98].

Other generalizations and applications can be found in [FLOTW1, FLOTW2, LT97].

## REFERENCE

- [Ar] S. ARIKI, *On the decomposition numbers of the Hecke algebra of  $G(n, 1, m)$* , J. Math. Kyoto Univ. **36** (1996), 789–808.
- [FLOTW1] O. FODA, B. LECLERC, M. OKADO, J.-Y. THIBON and T. WELSH, *RSOS models and Jantzen-Seitz representations of Hecke algebras at roots of unity*, Lett. Math. Phys. (1998).
- [FLOTW2] O. FODA, B. LECLERC, M. OKADO, J.-Y. THIBON and T. WELSH, *Branching functions of  $A_{n-1}^{(1)}$  and Jantzen-Seitz problem for Ariki-Koike algebras*, preprint, 1997.
- [Groj] I. GROJNOWSKI, *Representations of affine Hecke algebras (and affine quantum  $GL_n$ ) at roots of unity*, Internat. Math. Res. Notices **5** (1994), 215–217.
- [LLT96] A. LASCoux, B. LECLERC and J.-Y. THIBON, *Hecke algebras at roots of unity and crystal bases of quantum affine algebras*, Commun. Math. Phys. **181** (1996), 205–263.