Debt Maturity

$$V(b,d,\theta) = \max_{n,c,b',d',\tau} \; u(c,n) + \theta\, w(n-c) + \beta\, \mathbb{E}\left[V(b',d',\theta')\right] \quad (1)$$

s.t. $$1-\tau = -\frac{u_n(c,n)}{u_c(c,n)} \rightarrow \text{solve for } n \text{ as } f(c,\tau)$$

$$q_b = \beta\, \frac{\mathbb{E}\left[u_c(c',n')\right]}{u_c(c,n)}$$

$$q_d = \beta\, \frac{\mathbb{E}\left[u_c(c',n')\left(\kappa + (1-\ell)\, q_d'\right)\right]}{u_c(c,n)}$$

$$n - c + b + \kappa d = \tau n + q_b b' + q_d\left(d' - (1-\ell)d\right)$$

$$u(c,n) = \frac{c^{1-\gamma}}{1-\gamma} + \psi\, \frac{(1-n)^{1-\gamma}}{1-\gamma}$$

$$u_c = c^{-\gamma}$$

$$u_n = -\psi\,(1-n)^{-\gamma}$$

$$\Rightarrow \quad 1-\tau = \psi\left(\frac{1-n}{c}\right)^{-\gamma} \quad (1)$$

- $1-n = \left(\frac{\psi}{1-\tau}\right)^{\frac{1}{\gamma}} c$
- $n(1-\tau) = q_b b' + q_d\left(d' - (1-\ell)d\right) - b - \kappa d + c \quad (2)$

max (1) by choice of (c,b,d), get (n,τ) from (1, 2)

$$1-\tau = \psi\left(\frac{1-n}{c}\right)^{-\gamma} \quad (1)$$

- $$1-n = \left(\frac{\psi}{1-\tau}\right)^{\frac{1}{\gamma}} c$$

- $$n(1-\tau) = \underbrace{q_b b' + q_d(d' - (1-\rho)d) - b - \kappa d + c}_{(*)}$$

if $\gamma = 1$ $\qquad n = 1 - \frac{\psi}{1-\tau} c$

$$\Rightarrow \quad n(1-\tau) = 1-\tau - \psi c$$

given $(c, b, d) \rightarrow n(1-\tau) = (*) \rightarrow 1 - (*) + \psi c = \tau$

$$\rightarrow n = 1 - \frac{\psi}{1-\tau} c$$