Reserve Accumulation with a Private IIP*

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Abstract

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1. Model with government borrowing only

1.1 Model summary

Government choices Main problem in repayment

$$V^R(b,a,\mathbf{s}) = \max_{b',a',c_T,h} u(c_T, F(h)) + \beta \mathbb{E}\left[\mathcal{V}\left(b',a',\mathbf{s}'\right)\right] \tag{1}$$

$$\text{subject to } c_T + q_a a' + \kappa b = a + y_T(\mathbf{s}) + q_b(b',a',\mathbf{s})(b' - (1-\delta)b)$$

$$h \leq \mathcal{H}(c_T, \bar{w})$$

Main problem in default

$$V^D(b,a,\mathbf{s}) = \max_{a',c_T,h} u(c_T, F(h)) + \beta \mathbb{E}\left[\theta \mathcal{V}(b,a',\mathbf{s}') + (1-\theta) V^D(b,a',\mathbf{s})\right] \tag{2}$$

$$\text{subject to } c_T + q_a a' = a + \mathcal{D}(y_T(\mathbf{s}))$$

$$h \leq \mathcal{H}(c_T, \bar{w})$$

Extreme value preference-for-default shocks yield default probabilities

$$\mathcal{P}(b,a,\mathbf{s}) = \frac{\exp(V^D((1-\hbar)b,a,\mathbf{s})/\sigma^V)}{\exp(V^D((1-\hbar)b,a,\mathbf{s})/\sigma^V) + \exp(V^R(b,a,\mathbf{s})/\sigma^V)} \tag{3}$$

Finally, the value of entering a period with access to markets (and hence an option to default) is

$$\mathcal{V}(b,a,\mathbf{s}) = (1 - \mathcal{P}(b,a,\mathbf{s}))V^R(b,a,\mathbf{s}) + \mathcal{P}(b,a,\mathbf{s})V^D((1-\hbar)b,a,\mathbf{s}) \tag{4}$$

Foreigners, debt prices Stochastic discount factor

$$m(\mathbf{s},\mathbf{s}') = \exp(-r - \nu(\psi\epsilon' + 0.5\psi^2\sigma_\epsilon^2)) \qquad \epsilon' = \log(y') - \rho\log(y) - (1-\rho)\mu_y \tag{5}$$

$$q_a = \exp(-r) \tag{6}$$

$$q_b(b',a',\mathbf{s}) = \mathbb{E}\left[m(\mathbf{s},\mathbf{s}')\left(\mathbb{1}_{\mathcal{D}'}(1-\hbar)q_b(b',a',\mathbf{s}') + (1-\mathbb{1}_{\mathcal{D}'})(\kappa + (1-\delta)q_b(b',a',\mathbf{s}'))\right)\right] \tag{7}$$

Private economy Preferences

$$u(c_T, c_N) = \left[\varpi_N c_N^{-\eta} + \varpi_T c_T^{-\eta}\right]^{-\frac{1}{\eta}} \qquad p_N = \frac{\varpi_N}{\varpi_T}\left(\frac{c_T}{c_N}\right)^{1+\eta} \tag{8}$$

for $\varpi_N + \varpi_T = 1$.

Production and default costs

$$y_N(h, \mathbb{1}_\mathcal{D}) = (1 - \Delta\mathbb{1}_\mathcal{D})h^\alpha \qquad \mathcal{D}(y) = (1-\Delta)y \tag{9}$$

Wage rigidities

$$h \leq \mathcal{H}(c_T, \bar{w}, \mathbb{1}_\mathcal{D}) \qquad \mathcal{H}(c_T, \bar{w}, \mathbb{1}_\mathcal{D}) = \left(\frac{\varpi_N}{\varpi_T}(1-\Delta\mathbb{1}_\mathcal{D})\frac{\alpha}{\bar{w}}\right)^{\frac{1}{1+\alpha\eta}} c_T^{\frac{1+\eta}{1+\alpha\eta}} \tag{10}$$

1.2 Model results

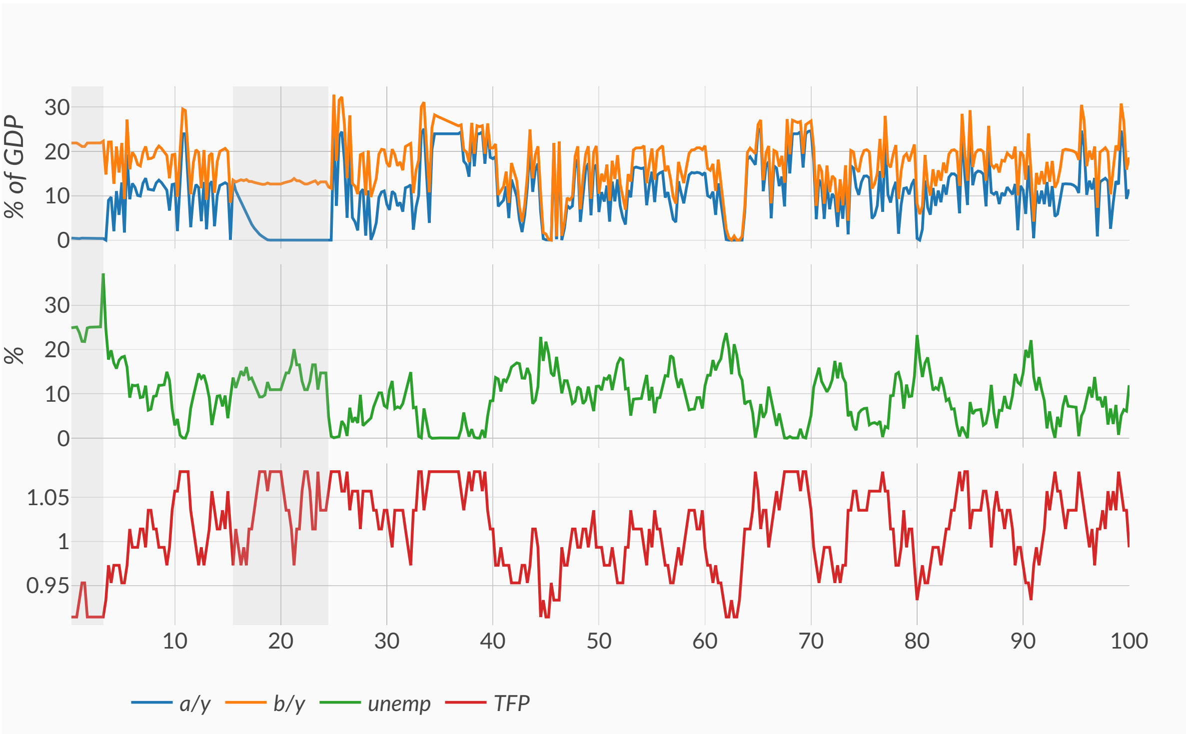

FIGURE 1: A SIMULATED PATH

2. MODEL WITH PRIVATE SAVINGS

We maintain the assumption that the government collects lump-sum taxes from the representative agent. But now the representative agent is also able to buy risk-free securities in international markets.

Because of the possibility of lump-sum taxation, the state variable for the economy is the sum of government-held and private-held 'reserves.' When choosing the savings position of the economy, the government is bound by the private sector's Euler equation, except if the constraint on non-negative savings binds for the representative agent. Therefore,

$$\begin{aligned} V^R(b, a, \mathbf{s}) = &\max_{b', a', c_T, h} u(c_T, F(h)) + \beta \mathbb{E}\left[\mathcal{V}(b', a', \mathbf{s}')\right] \qquad (11)\\ &\text{subject to } c_T + q_a a' + \kappa b = a + y_T(\mathbf{s}) + q_b(b', a', \mathbf{s})(b' - (1-\delta)b) \\ &h \leq \mathcal{H}(c_T, \bar{w}) \\ &u_T(c_T, F(h))q_a \geq \beta \mathbb{E}\left[u_T(c'_T, F(h'))\right] \end{aligned}$$

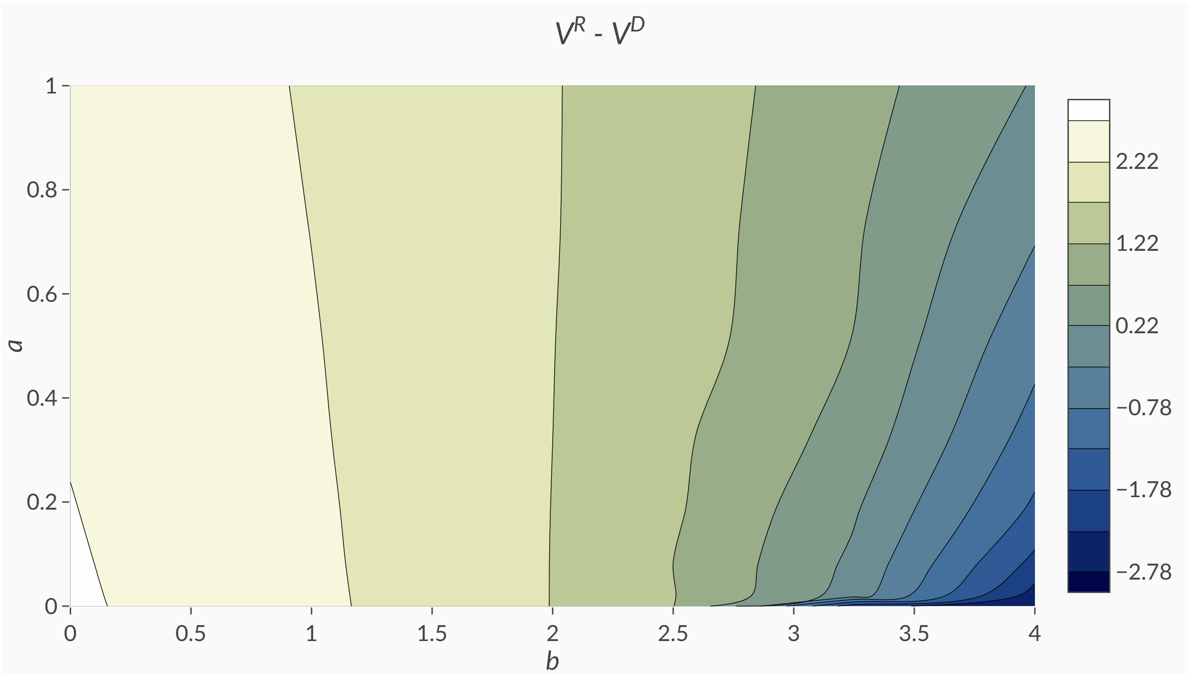

FIGURE 2: DEFAULT INCENTIVES

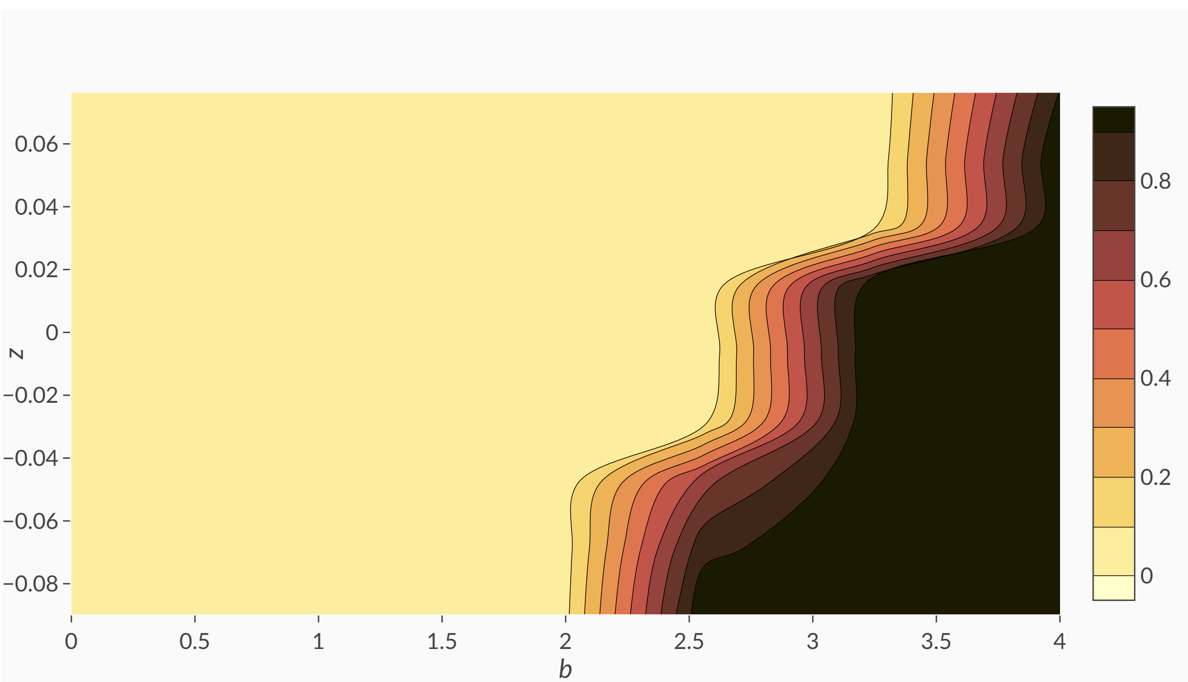

FIGURE 3: DEFAULT PROBABILITY

Similarly, the value in default is now

$$
\begin{aligned}
V^D(b,a,\mathbf{s}) = \max_{a',c_T,h} \; & u(c_T, F(h)) + \beta \mathbb{E}\left[\theta \mathcal{V}(b,a',\mathbf{s}') + (1-\theta) V^D(b,a',\mathbf{s})\right] \\
\text{subject to } & c_T + q_a a' = a + \mathcal{D}(y_T(\mathbf{s})) \\
& h \leq \mathcal{H}(c_T, \bar{w}) \\
& u_T(c_T, F(h)) q_a \geq \beta \mathbb{E}\left[u_T(c_T', F(h'))\right]
\end{aligned}
\tag{12}
$$

Choice variables As before, in equilibrium $h = \min\{1, \mathcal{H}(c_T, \bar{w})\}$. So a two-step procedure could be (i) find the set $A(b) = [a_{\min}(b), a_{\max}(b)]$ where a can be chosen respecting the private Euler equation (using the previous formula and the budget constraint for h and c_T) and (ii) choose b and $\theta \in [0,1]$ (with $a = a_{\min}(b) + \theta(a_{\max}(b) - a_{\min}(b))$.

Decentralized formulation The government has reserves a, the representative agent has savings k. The government collects lump-sum taxes T. Their choices are summarized by

$$
\begin{aligned}
w^R(b,a,k,T,\mathbf{s}) = \max_{c_T,c_N,k'} \; & u(c_T, c_N) + \beta \mathbb{E}\left[w(b',a',k',T',\mathbf{s}')\right] \\
\text{subject to } & c_T + q_a k' + p_N c_N = k + y_T(\mathbf{s}) + p_N y_N - T
\end{aligned}
\tag{13}
$$

We obtain the following first-order conditions, where also applying market clearing conditions $c_N = y_N$

$$
\begin{aligned}
p_N &= \frac{\varpi_N}{\varpi_T}\left(\frac{c_T}{F(h)}\right)^{1+\eta} \\
u_T(c_T, F(h)) q_a &\geq \beta \mathbb{E}\left[u_T(c_T', F(h'))\right]
\end{aligned}
$$

The intratemporal condition is the same as the planner's as this choice between consumption of both goods is undistorted in the equilibrium (the planner does not need a second tax instrument to match the wedge in the consumption allocation decision).

The planner's problem is

$$
\begin{aligned}
v^R(b,a,k,\mathbf{s}) = \max_{b',a',T} \; & u(c_T, F(h)) + \beta \mathbb{E}\left[v(b',a',k',\mathbf{s}')\right] \\
\text{subject to } & q_a a' + \kappa b = q_b b' + a + T
\end{aligned}
\tag{14}
$$