Uncertainty Premia, Sovereign Default Risk, and State-Contingent Debt

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Why do governments borrow noncontingent?

State-contingent debt instruments

- Decrease default risk
- Reduce cyclicality of fiscal policy
- Improve risk-sharing

Why aren't they used?

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Why aren't they used?

Recent experiences with state-contingent instruments

- Issuances of these instruments in 21st century featured a threshold structure
  - … Bonds only pay if high output growth
  - … Argentina 2005, Greece 2012, Ukraine 2015
- Heavily discounted by markets:
  - Costa, Chamon, and Ricci (2008) compute wide spreads for Argentine GDP-warrants
    - … $\sim$300-400bps from default risk of other securities
    - … 600-1200bps residual: 'novelty' premium

A framework for evaluating state-contingent debt

This paper proposes a framework that

- Rationalizes pricing of observed SCI + welfare analysis
  - Standard quantitative model of sovereign default with long-term debt
    - Aguiar and Gopinath (2006), Arellano (2008), Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012)
  - International lenders with concerns about *model misspecification*
    - Preference for robustness   Hansen and Sargent (2001), Pouzo and Presno (2016)

- **Mechanism**: lenders act *as if* the probability of states with low repayment was higher
  - With noncontingent debt, lenders overestimate the default probability
  - Pouzo and Presno (2016) uses robustness to reconcile spreads with default frequencies
  - In general, probability distortion depends on type and quantity of debt issued

Main findings

1. Robust lenders dislike repayment structures with thresholds in good times
   - Heavy discounts for these bonds $\implies$ welfare losses
2. Explain most of the 'novelty premium' in Argentina's GDP warrants as ambiguity premia
   - Calibration of robustness from *noncontingent* debt only
3. Characterize the optimal design and how it changes with robustness
   - With high robustness, want to minimize ex-ante and ex-post contingency

Roadmap

- Stylized Model
- Probability Distortions
- Quantitative Implementation
- Concluding Remarks

Stylized Model

The model

We consider a simple two-period model, small open economy

- Uncertain endowment $y(z)$ in the second period
- The government has access to one asset which promises a return $R(z)$.
- A few benchmarks

Noncontingent debt	$R(z)$	$=$	1
Linear indexing	$R^{\alpha}(z)$	$=$	$1+\alpha(y(z)-1)$
Threshold debt	$R^{\tau}(z)$	$=$	$\mathbb{1}(z>\tau)$
Optimal design	$R^{\star}(z;\theta)$		chosen state-by-state

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The government's problem

- The government takes as given the price schedule $q(b)$

$$
\begin{aligned}
& \max_b u(c_1^b) + \beta_b \mathbb{E}\left[u(c_2^b)\right] \\
\text{subject to } \; & c_1^b = y_1 + q(b)b \\
& c_2^b = y_2(z) - h(z,\Delta)d(b,z) - (1-d(b,z))R(z)b
\end{aligned}
$$

  where

$$
h(z,\Delta) = y_2(z)^2\Delta
$$

- In the second period, default if

$$
h(z,\Delta) < R(z)b
$$

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The lenders' problem

Foreign lenders are less standard and have multiplier preferences

$$\max c_1^L - \frac{\beta}{\theta} \log \left(\mathbb{E} \left[\exp(-\theta v_2^L) \right] \right)$$

$$\text{subject to} \quad v_2^L = c_2^L$$

$$c_2^L = w_2 + (1 - d(b,z))R(z)b$$

$$c_1^L = w_1 - q_1 b$$

Lenders provide us with an Euler equation to price the debt

$$q(b;R) = \beta \mathbb{E} \left[\frac{\exp(-\theta v_2^L)}{\mathbb{E}\left[\exp(-\theta v_2^L)\right]} (1 - d(b,z))R(z) \right]$$

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Lenders provide us with an Euler equation to price the debt

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$$

Debt prices

CARA

- The lenders' Euler equation explains the sources of the spreads they charge
- Call $M = \beta \frac{\exp(-\theta v_2^L)}{\mathbb{E}\left[\exp(-\theta v_2^L)\right]}$ the stochastic discount factor

$$
\begin{aligned}
q(b;R) &= \beta \mathbb{E}\left[\frac{\exp(-\theta v_2^L)}{\mathbb{E}\left[\exp(-\theta v_2^L)\right]}(1-d(b,z))R(z)\right] \\
&= \underbrace{\beta \mathbb{E}\left[(1-d)R\right]}_{=q_{RE}} + \underbrace{(1-\mathbb{P}(d))\operatorname{cov}(M,R)}_{=q_\theta^{\text{cont}}} - \underbrace{\mathbb{E}\left[R\right]\operatorname{cov}(M,d)}_{=-q_\theta^{\text{def}}}
\end{aligned}
$$

- The debt price is a rational-expectations price and two sources of ambiguity premia

Distorted probabilities

Interpret lenders' stochastic discount factor as probability distortions

- For a random variable X

$$\tilde{\mathbb{E}}[X] = \mathbb{E}\left[\frac{\exp(-\theta v_2^L)}{\mathbb{E}\left[\exp(-\theta v_2^L)\right]} X\right]$$

- $\tilde{\mathbb{E}}$ tilts probabilities towards *less-favorable* states for lenders

- Obs The tilting is endogenous to the lenders' outcomes *and to the debt design*

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- **Obs** The tilting is endogenous to the lenders' outcomes *and to the debt design*

Probability Distortions

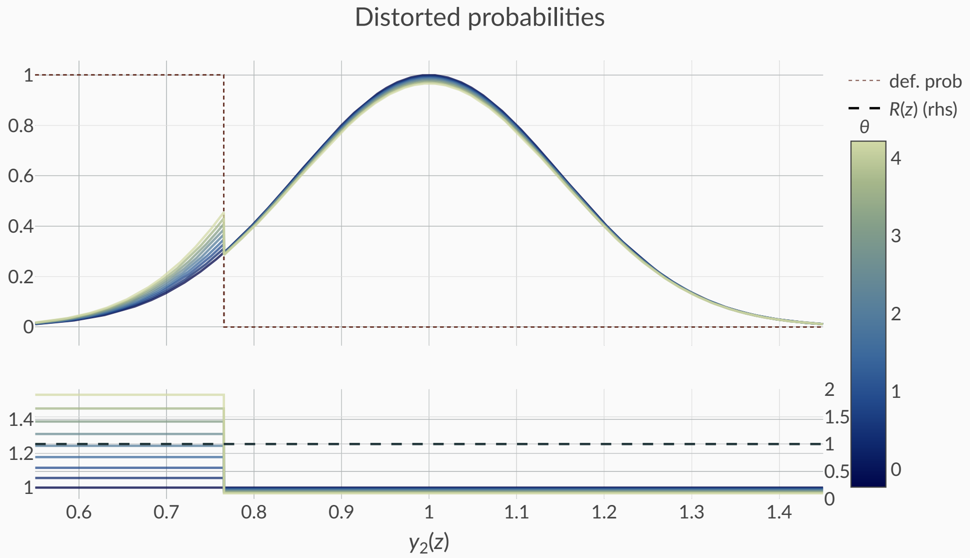
Parametrization

▸ Calibration

Costa, Chamon, and Ricci (2008) study the GDP-warrants issued by Argentina

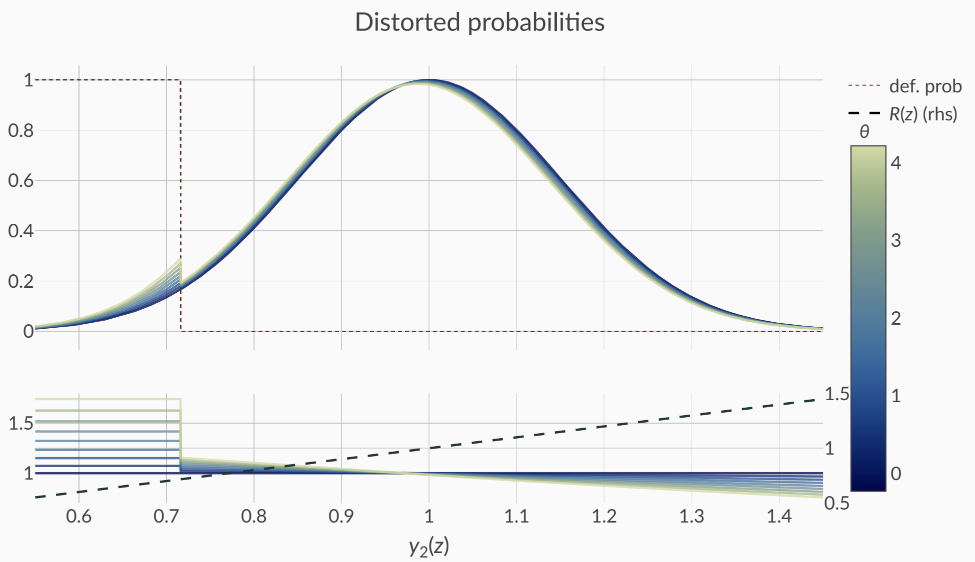
- The warrant paid if
    - Output growth above pre-set level (4.3% initially, later 3%)
    - Output *level* above the compounded cutoff growth
    - There is also a cap on total payments

Distorted probabilities – noncontingent debt



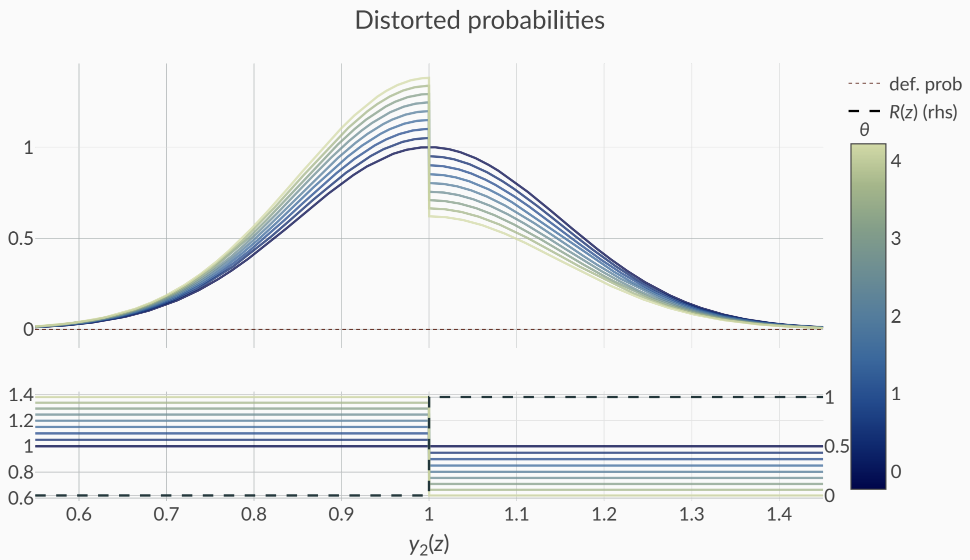
Distorted probabilities – linearly indexed debt: $\alpha = 1$

Reoptimizing



Distorted probabilities – threshold debt

▸ Linear+threshold ▸ Reoptimizing



Distorted probabilities – debt for RE lenders

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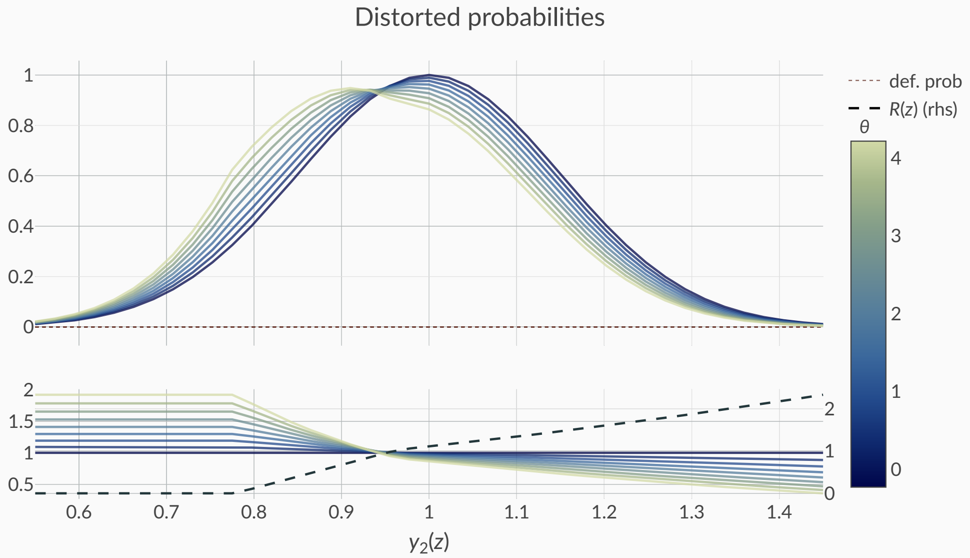
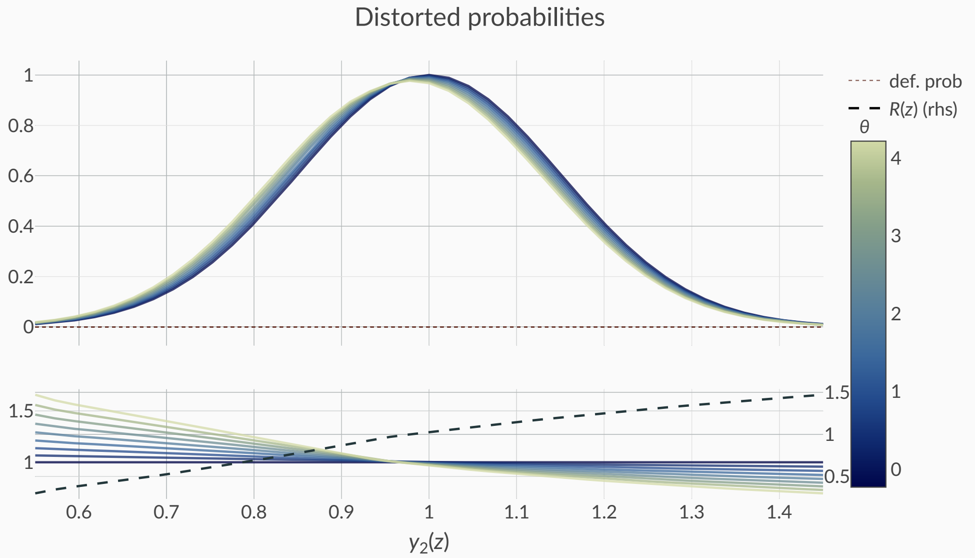
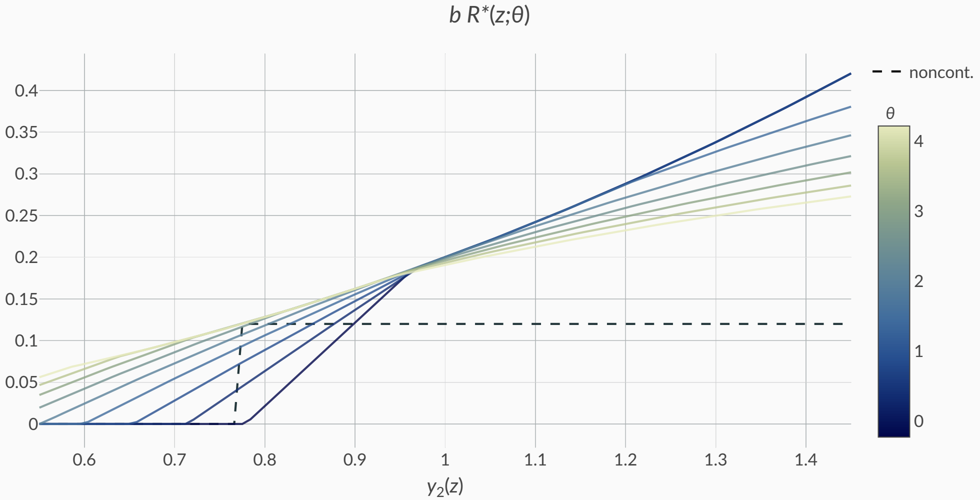


Distorted probabilities – debt for robust lenders

Reoptimizing



Design of debt



Quantitative Implementation

Quantitative Model

- Infinite horizon, small-open economy
- Robust lenders as before
- Long-term debt, debt issued at t pays coupon at $t+s$

$$\max\left\{0, (1-\delta)^{s-1}(1+\alpha(y_s-1))\mathbb{1}(y_s>\tau)\right\}$$

    - Noncontingent debt: $\alpha = 0, \tau = -\infty$
- Default triggers exclusion + output costs for a random amount of periods $\sim Geo(\psi)$

Calibration

	Data	Benchmark	Rational Expectations
Spread	8.15	8.15	8.1
Std Spread	4.58	4.6	4.5
Debt	46	44	48.7
Std(c)/Std(y)	0.87	1.25	1.24
Corr(y,c)	0.97	0.98	0.98
Corr(y,tb/y)	-0.77	-0.68	-0.71
Corr(y,spread)	-0.72	-0.76	-0.77
Default Prob	3.0	3.0	5.5
DEP	-	31%	-

Note: Statistics computed in the model with noncontingent debt

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Robustness in the quantitative model

▸ CE calibration

	Rational Expectations			$\theta = 1.6155$ (benchmark)		
Statistic	**Noncontingent**	**Threshold**	$\alpha = 1$	**Noncontingent**	**Threshold**	$\alpha = 1$
Spread	8.1	0.36	7.2	8.15	11.1	7.1
Std Spread	4.5	0.23	3.7	4.6	1.58	3.6
Debt	48.7	116.5	50.8	44.0	67.6	46.1
Std(c)/Std(y)	1.24	0.82	1.22	1.25	0.84	1.23
Default Prob	5.5	0.3	5.3	3.0	0.0	2.6
Welfare Gains	-	1.19	0.09	-	-0.37	0.07
DEP	-	-	-	31%	20%	30%

Table 1: Statistics from calibrated model simulations

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with alpha = 1.

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Optimal design of state-contingent debt

Statistic	Rational Expectations $\tau = 0.875, \alpha = 7$	Robustness $\tau = 0.875, \alpha = 5$
Spread	0.1	2.8
Std Spread	0.04	0.13
Debt	79.3	65.9
Std(c)/Std(y)	0.76	0.96
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Table 2: Statistics under the optimal state-contingent bond for different types of lenders

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Concluding Remarks

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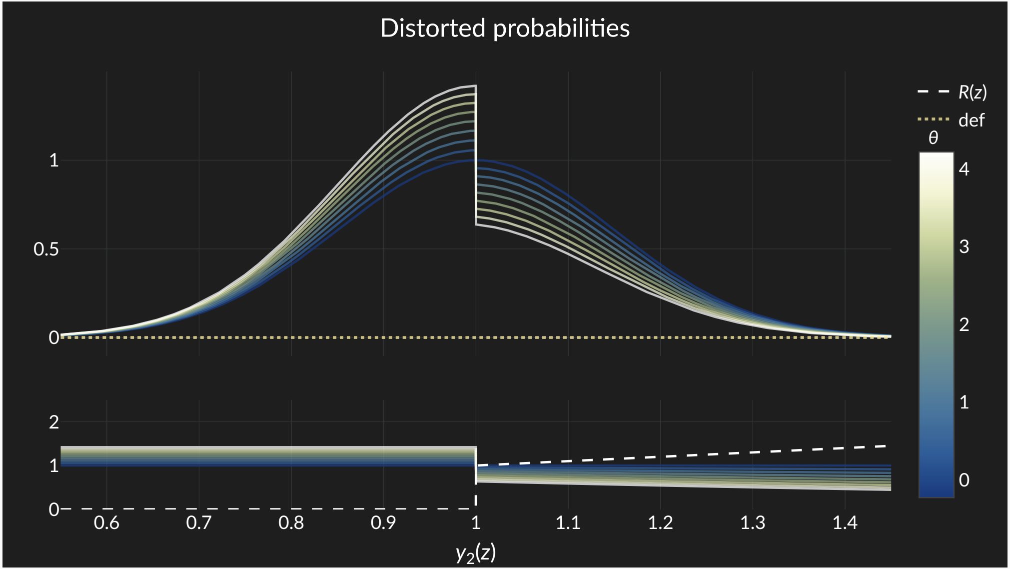
- Standard sovereign debt model augmented with robust lenders
  1. rationalizes lack of popularity of recent SCDI issuances
  2. links unfavorable prices to common *threshold* structure
  3. rationalizes part of the 'novelty' premium as a premium for ambiguity
  4. accounts for spreads on typical threshold SCDIs
  5. Welfare gains of SCDI decreasing in robustness
     - Both for given instrument and for optimally-designed debt

- Optimal design
  - With extreme robustness, eliminate contingency ex-ante (*stipulated*) and ex-post (*default*)
  - With general robustness, minimize variance imposed on lenders for given level of insurance.
  - At calibrated robustness, thresholds on far left tail, flatter indexation than RE

Distorted probabilities – threshold+linear debt

◂ Back



Quantitative model

◂ Back

Statistic	Rational Expectations (benchmark)			$\theta = 1.6155$		
	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread	8.5	0.6	6.8	8.4	15.5	7.1
Std Spread	4.3	0.4	3.0	4.4	2.3	3.1
Debt	69.9	159.6	74.4	62.6	87.7	67.2
Std(c)/Std(y)	1.24	0.83	1.21	1.25	0.82	1.22
Corr(y,c)	0.98	0.53	0.98	0.98	0.94	0.98
Corr(y,tb/y)	-0.7	0.52	-0.62	-0.67	0.58	-0.6
Corr(y,spread)	-0.77	-0.87	-0.78	-0.75	-0.61	-0.77
Default Prob	5.8	0.56	5.3	2.3	0.12	1.8
Welfare Gains	-	1.86	0.27	-	-0.87	0.2

Table 3: Statistics based on Chatterjee and Eyigungor (2012)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with alpha = 1.

CARA

Back

Euler equations of a rational-expectations agent with CARA preferences and access to a risk-free bond

$$
\begin{aligned}
q &= \beta \mathbb{E}\left[\frac{u'(c_2)}{u'(c_1)} R\right] = \beta \mathbb{E}\left[\frac{\exp(-\gamma c_2)}{\exp(-\gamma c_1)} R\right] \\
\frac{1}{1+r} &= \beta \mathbb{E}\left[\frac{u'(c_2)}{u'(c_1)}\right]
\end{aligned}
$$

hence

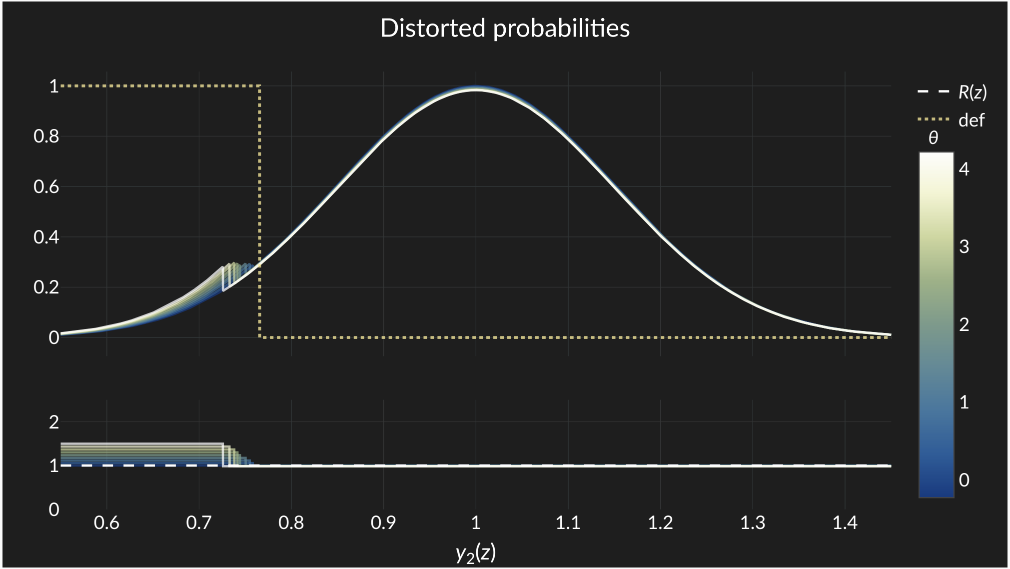
$$
q = \beta \mathbb{E}\left[\frac{\exp(-\gamma c_2)}{\beta(1+r)\mathbb{E}\left[\exp(-\gamma c_2)\right]} R\right]
$$

Same as robustness in two periods, in general the robust sdf is

$$
q = \beta \mathbb{E}\left[\frac{\exp(-\theta v')}{\mathbb{E}\left[\exp(-\theta v')\right]} R\right]
$$

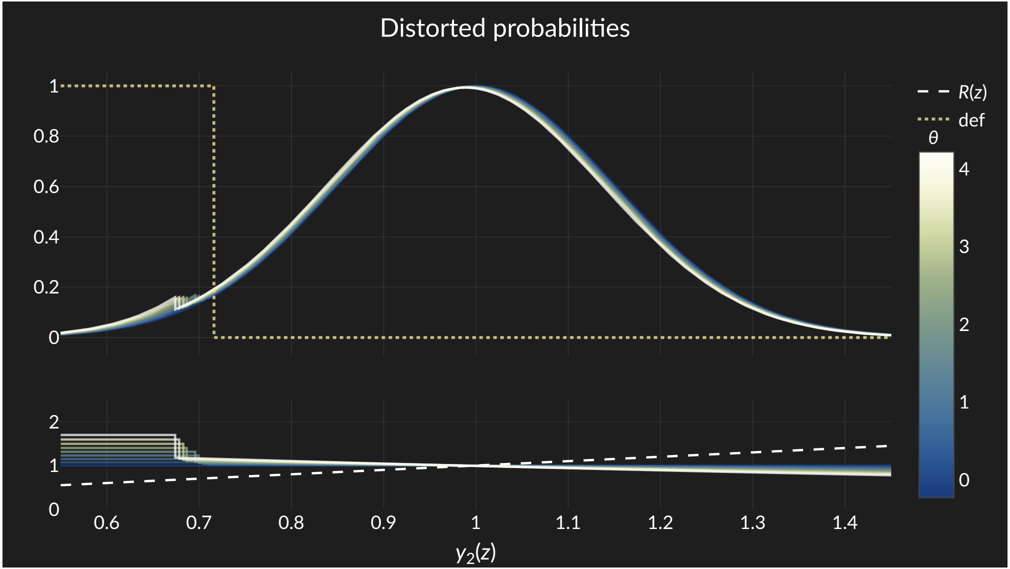
Distorted probabilities – noncontingent debt

Back



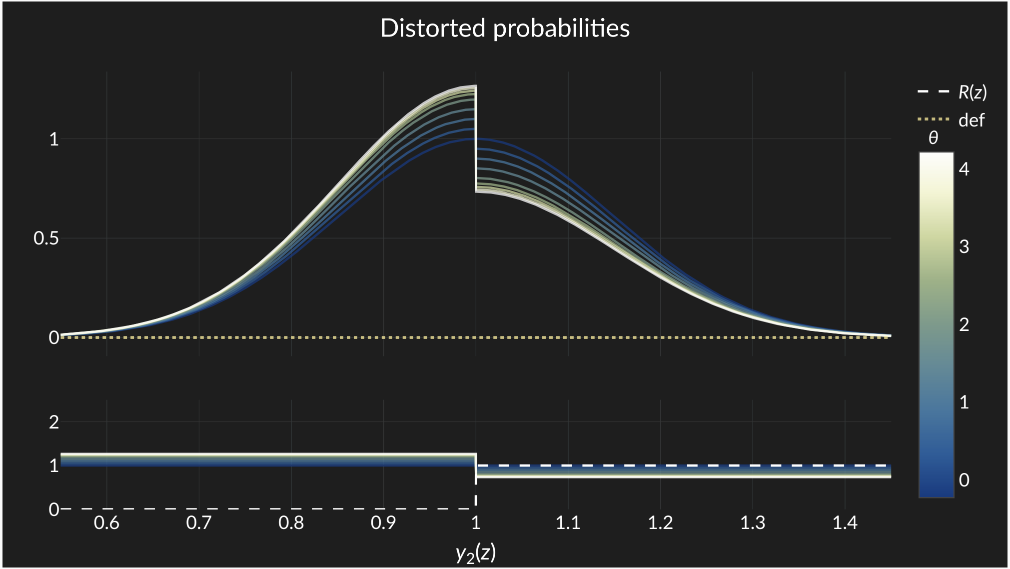
Distorted probabilities – linearly indexed debt

Back



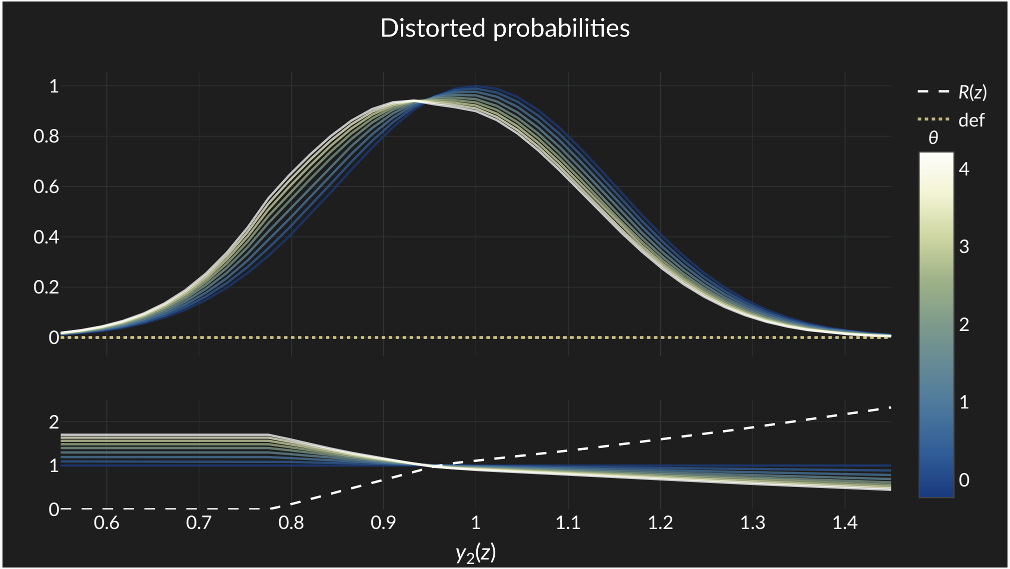
Distorted probabilities – threshold debt

Back



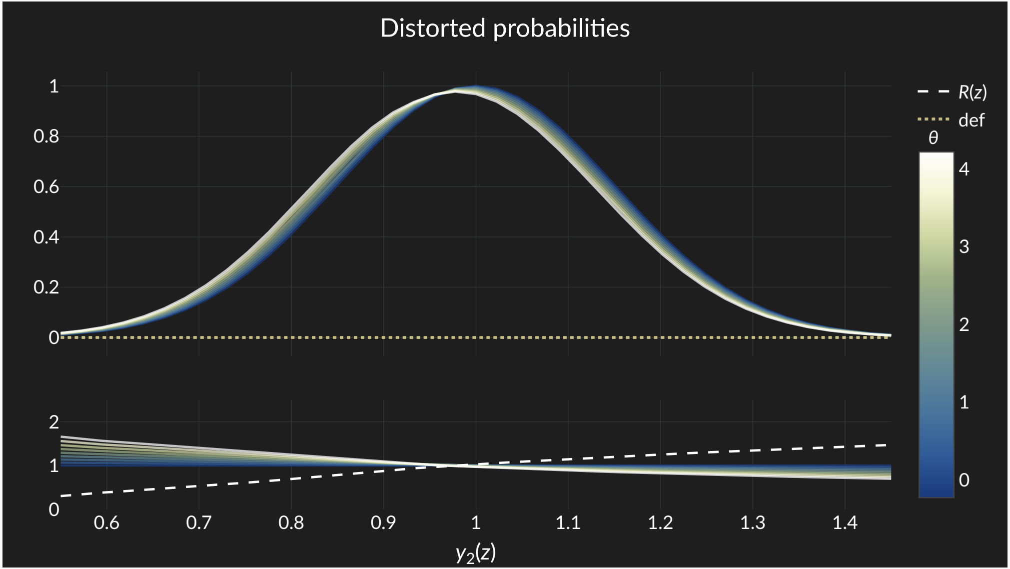
Distorted probabilities – debt for RE lenders

Back



Distorted probabilities – debt for robust lenders

Back



Parametrization

◂ Back

We represent this bond with threshold debt, one period = five years, and

Parameter	Target	Value
β_b	Borrower's discount rate	6% ann.
β	Risk-free rate	3% ann.
γ	Borrower's risk aversion	2
Δ	Output cost of default	20%
g	Expected growth rate	8% ann.
k	Threshold for repayment	50%

Decomposition of spreads

Back

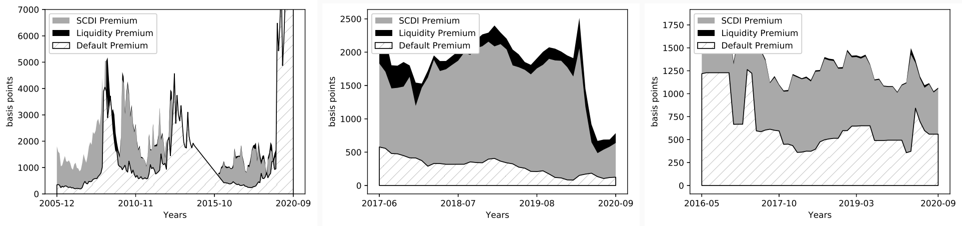


Figure 1: GDP-linked security premia.

The figure shows the estimated spread decomposition in Igan and Kim (2021) for the GDP-warrants issued by Argentina (left), Greece (middle) and Ukraine (right).

Lenders' problem

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Given a stochastic process for consumption $\{c_t\}_t$, lenders value is

$$v^L(c) = \min_m u(c_1) + \beta \mathbb{E}\left[m u(c_2) + \frac{1}{\theta} m \log m \right]$$

subject to $\mathbb{E}[m] = 1$

Lender chooses c, 'evil agent' chooses m with entropy penalty

Solution is $\hat{m} \propto \exp(-\theta u(c_2))$ Statistical Murphy's law

Lenders' problem

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