Sovereign Debt Tolerance with Potentially Permanent Costs of Default*

Marcos Chamon† Francisco Roldán‡

IMF IMF

May 2024

Abstract

We investigate the effect of uncertainty about the nature of output costs of sovereign default on debt tolerance. While the theoretical literature assumes output losses lasting until market access is restored, the empirical evidence points to persistent effects, and output may not return to its pre-default trend. We include such uncertainty in a model of sovereign default and find that it can significantly boost equilibrium debt levels. We also consider a government which is averse to this type of uncertainty and seeks robust decision rules. We calibrate the model to match evidence on the output trajectory around debt restructuring episodes and infer output costs of about the size found in the empirical literature, alongside significant uncertainty about their permanence and a strong desire for robustness. Uncertainty and robustness contribute about a quarter of observed debt tolerance.

JEL Classification E43, E44, F34, H63

Keywords Sovereign debt, default, debt tolerance, permanent costs, robustness

*For comments and suggestions we thank the editor Andrei Levchenko, our discussant Francesco Giavazzi, two anonymous referees, Tamon Asonuma, Stelios Fourakis, Javier García-Cicco, Leonardo Martinez, Ignacio Presno, Francisco Roch, as well as participants at the IMF's 24th Jacques Polak Annual Research Conference in honor of Ken Rogoff, UdeSA, and RedNIE. Zhuo Chen provided excellent research assistance. The views expressed in this paper are those of the authors and should not be attributed to the IMF, its Executive Board, or its Management.

†e-mail: *mchamon@imf.org*

‡e-mail: *froldan@imf.org*

Introduction

A fundamental question in the sovereign debt literature, and one to which Rogoff made substantial contributions, is what supports debt repayments. In the absence of international enforcement of sovereign debts, costs of default, actual and perceived, physical and reputational, determine how much debt can be sustained as well as the terms at which countries borrow.

In two influential papers, Bulow and Rogoff (1989a,b) show the limits of reputation to sustain otherwise unenforceable payments on sovereign debt and propose the inclusion of other costs affecting the economy, such as direct sanctions interfering with trade. The idea that defaults entail costs for the real economy still underpins most of the literature on sovereign debt.[1] Most of the theoretical literature assumes an output cost of default which lasts only until credit market access is restored. It is well documented that economic growth declines around the time of restructurings or defaults (Borensztein and Panizza, 2009) but the exact nature of such costs is not so clear. Subsequent studies with different methodologies (De Paoli et al., 2009; Asonuma et al., 2023; Farah-Yacoub et al., 2022; Cerra and Saxena, 2008) find growth slowdowns following a debt restructuring, some of which are quite persistent.[2]

This paper focuses on how the possibility of permanent scarring effects of a default heightens the ex-ante costs of restructuring. Motivated by the lack of conclusive empirical evidence one way or the other, we consider uncertainty about how permanent such costs are. We draw implications for the amount of sustainable debt in equilibrium and find that the possibility of permanent costs has a large impact on "debt tolerance" (Reinhart, Rogoff, and Savastano, 2003), especially when combined with realistic risk attitudes for the debtor country.

We evaluate the effect of potentially permanent costs within a sovereign default framework à la Eaton and Gersovitz (1981) with trend growth combined with uncertainty about the type of costs associated with default. In the model, a government chooses whether to repay its debt without knowing if defaulting would trigger a transitory or permanent decline in output. In addition, motivated by the difficulties faced by the empirical literature in estimating the magnitude and persistence of default costs, we consider a policy maker who is also unwilling to place a fully

[1]For surveys of the literature on sustainable public debt and sovereign default, see the handbook chapters by Aguiar and Amador (2014), Aguiar, Chatterjee, Cole, and Stangebye (2016), D'Erasmo, Mendoza, and Zhang (2016), and Martinez, Roch, Roldán, and Zettelmeyer (2023). Costs can also involve broader spillovers affecting the rest of the economy, as in Cole and Kehoe (1998).

[2]In practice, countries go to great lengths to remain current on their debts. If anything, they tend to wait until restructurings come "too late" (IMF, 2013), which suggests large perceived costs of restructurings, even though we do not observe much in the way of direct punishments, certainly not in the magnitude of the defaulted debt.

trusted prior distribution over the type of default costs. Instead, the government mistrusts its prior and seeks robust decision rules as in Hansen and Sargent (2001).[3]

These departures from the canonical model allow us to match reduced-form evidence on the trajectory of output following a debt restructuring. We compute observed trajetories of output relative to a pre-default trend for emerging-market economies which defaulted on their sovereign debts in 1990-2020. We then calibrate our model to generate these same endogenous dynamics as well as average levels of the external debt-to-GDP ratio and spreads on the debt. Our parametrization replicates these patterns, including both the decline on impact and 5 years out, which are targeted, as well as the general, untargeted dynamics of the output response.

In the calibrated model, we decompose the trajectory of output after a default according to the contributions of underlying shocks, 'causal' default costs, and the statistical effect of a trend estimated on a pre-restructuring history. The decomposition exemplifies the forces underlying the difficulties of the empirical literature: that the trend is endogenous and that underlying shocks drive both output and the decision to default. Ultimately, we find that the causal cost of default contributes about 35% to the total observed output decline.

Our calibration finds that about a quarter of the debt tolerance is due to robustness and uncertainty about default costs. We measure a probability of permanent costs of about two-thirds, and that concerns for robustness make the government act as it was in the 80-85% range. A calibration without robustness generates only a slightly worse fit but requires a probability of permanent costs of almost 90%, which seems high and at odds with the difficulties encountered by the empirical literature. In comparative statics, we find that making the costs permanent for sure leads to an increase in sustained debt levels of about a half relative to fully transitory costs, but that uncertainty and especially robustness have a much milder effect on equilibrium default probabilities.

The remainder of the paper is organized as follows. Section 2 discusses some of the possible costs associated with a restructuring and reviews the findings from the empirical literature on the costs of default. Section 3 presents the model and defines the equilibrium, while Section 4 describes its calibration. Section 5 discusses the quantitative results of the model. Section 6 concludes.

[3] Robustness is also featured in Pouzo and Presno (2016) and Roch and Roldán (2023) in the context of sovereign debt, but on the lender side.

2. Output losses and other costs around restructurings

The empirical literature estimates sizable impacts of defaults on economic activity, but deciding whether such effects are permanent is a statistically daunting task.[4] Borensztein and Panizza (2009) estimate a 2.6 percentage point decline in growth in the first year of the episode, but that effect (on growth) is short lived with no statistically significant effect on lagged variables. Levy Yeyati and Panizza (2011) also find in quarterly data that much of the contraction actually takes place before the default event in question. Asonuma and Trebesch (2016) show that restructurings that take place preemptively before payments are missed tend to be quicker and have better growth outcomes than those that take place after a default. Asonuma et al. (2023) estimate local projections on the level of log GDP relative to pre-crisis trend to investigate the persistence of a level effect. The adverse effects tend to be mild and temporary for preemptive restructurings, but large and persistent for restructurings involving a default (which historically has been the case in most restructurings). They estimate GDP to remain 4 percent below its pre-crisis trend five years after the default, which suggests these losses are not reversed.[5] Farah-Yacoub et al. (2022) use synthetic controls to estimate the economic and social costs of default on a long historical sample. They estimate a cumulative 8.4 percentage point gap within the first three years, with substantial scarring effects even after a decade.

Most of the literature has focused on external debt restructurings. One notable exception is Reinhart and Rogoff (2011), which show that the output decline after domestic restructurings is much larger than for external ones. Domestic debt restructurings were relatively rare during the 1980s and early 1990s, but have become more common as countries have started to rely more on domestic financing (IMF, 2021). Domestic restructurings can more directly lead to major financial distruption which can by itself entail significant output costs (Pérez, 2018). But following most of the literature, our paper focuses on external debt restructurings.

There is a wide range of outcomes following a debt restructuring. Some countries do experience mild and temporary declines, while for others the impact is deeper and more protracted. A few even experience a growth acceleration. Figure 1 plots the evolution of the log of GDP relative to the pre-restructuring trend for different countries experiencing an external debt restructuring.[6] Time is measured relative to the restructuring event which takes place at $t = 0$. Figure 10 in the

[4] Following most of the literature we use the term default loosely to refer to any distressed debt restructuring.

[5] Most sources of catch-up growth such as pent-up demand should have already materialized in that horizon.

[6] The events are based on the Asonuma and Trebesch dataset which includes both restructurings that take place after payments are missed (default) as well as distressed exchanges that take place preemptively prior to payments being missed. The latter is less frequent, but has become relatively more common over time.

Appendix presents the same information in calendar time.

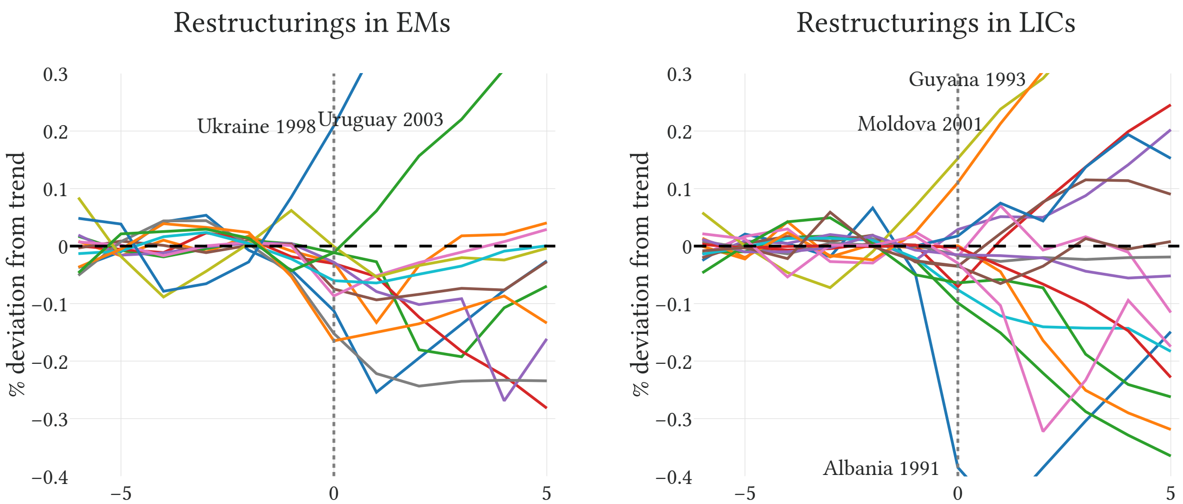

Figure 1: Growth outcomes around debt restructurings

Note: Outlier trajectories censored and marked

The left panel covers Emerging Market economies (EMs) and the right one, Low-Income Countries (LICs).[7] Each line traces the evolution of a country from 5 years prior to its restructuring to 10 years afterwards. When there are multiple restructurings within that time frame, we count each event separately. The log GDP series is detrended using a linear time trend based on the 5 years prior to the restructuring.[8] Hence by construction, the series hovers around zero prior to the restructuring, but can deviate significantly after that event.

There are fundamental differences between EM and LIC restructurings, especially in the historical sample. The official sector plays a much larger role in LICs, and many of those countries remained deeply insolvent until their debt crises were eventually resolved as part of the Heavily Indebted Poor Countries (HIPC) initiative. HIPC, combined with the Multilateral Debt Relief Initiative (MDRI) provided deep debt writedowns (enough to bring debt down to 150 percent of exports, sometimes even less). This helps explain the weak performance of LICs prior to the late 1990s/early 2000s, and their relatively strong performance afterwards. Given the particular features of LIC restructurings, we focus our attention on the sample of EMs, as shown in Figure

[7] We define these two groups based on whether they are covered by the IMF-World Bank LIC Debt Sustainability Framework or the one for market access countries. The sample is listed in Appendix A.

[8] A linear trend is preferable to using more sophisticated filters, such as HP, which would use observations after the crisis to inform its trend. One alternative could be to use one-sided filters using only backward-looking data, but for the purposes of this illustration a simple linear trend suffices.

1A.

The growth performance of EMs tends to be weaker following restructurings, which were much more common in the 1980s (see Figure 10 in the Appendix for the calendar time dimension of these events). There are several cases where GDP remains well below pre-crisis trends, reflecting the severity of that systemic event and the long time it took for its eventual resolution. It is also worth noting that external financial conditions were particularly unfavorable at that time (due to very high U.S. interest rates), and that many of the affected countries had deep structural problems (for example, very closed economies particularly in Latin America). From the 1990s onwards, we see relatively fewer restructuring episodes. We still see quite a few cases where GDP remains below trend, but there are also a number of cases where the initial decline is not that large or where it starts to revert sooner. There were a number of important innovations in the debt restructuring technology over time, including the adoption of Collective Action Clauses (CACs) on bonds and eventually enhanced CACs which allowed aggregation of votes across series (making it more difficult for any one creditor to hold out when there was widespread support from other creditors to a restructuring). IMF (2020) takes stock of developments in sovereign debt restructurings involving the private-sector creditors and notes that compared with previous periods, recent restructurings have generally proceeded smoothly and quicker, and were more likely to be preemptive.

On balance, the output costs are substantial in the historical sample. There are a number of steps a debtor can take to minimize those costs, such as collaboratively engaging with creditors towards restructuring before payments are missed.[9] But there are many things beyond the debtor's control, and restructurings are inherently complicated processes. Thus, even these costs have come down in expectation, there is still a tangible risk that things could go wrong and the costs turn out to be large. That cost can be further amplified by the risk and ambiguity aversion of the debtor, and help sustain more debt in equilibrium.

2.1 Other costs

Our focus has been on credit market exclusion combined with output costs, since these are easier to directly map into a model. As Bulow and Rogoff (1989a,b) and Rogoff (2022) point out, credit exclusion in itself cannot yield a large enough cost to sustain observed levels of debt, which is why researchers tend to combine it with direct costs on productivity or income. But there are

[9]For example, Trebesch and Zabel (2017) and Gordon and Guerrón-Quintana (2019) study the different implications of "hard" vs "soft" defaults

other potential sources of costs.

Direct sanctions have been proposed as another channel that could help sustain debt. Such punishments could translate into a decline in output, financial exclusion, and depending on the nature of the sanctions have a direct effect on welfare over and beyond the decline in output. In practice, we do not observe much in the way of such punishments on debtors. Even high profile cases of litigation involving asset seizures involve sums that are nowhere near the order of magnitude of the debt claims. The main example of a successful and disruptive litigation is Argentina vs NML Capital, where a 2014 ruling prevented the debtor from repaying the restructured bonds (from its 2001 default) without repaying the litigants in full. Such litigation can enforce credit market exclusion as a punishment (since it discourages new credit by interfering with repayments to new creditors). The debtor settled with that holdout creditor in 2016. Since then, New York courts have clarified that the decision to repay some creditors but not others in and of itself does not breach the contractual clause that underpinned NML's strategy. And that successful strategy will become increasingly difficult to pursue as enhanced CACs become more widespread. In almost all cases, litigants were not able to seize assets or disrupt ongoing restructurings (IMF, 2020).

It is possible that direct sanctions do not play a significant role because borrowers do follow procedures and norms that have come to be expected during a debt restructuring. Debtors typically ask only for the relief needed to restore sustainability with reasonable buffers, and most restructurings take place in the context of an IMF-supported program to facilitate the adjustment (and where a debt sustainability analysis informs the envelope of relief needed to restore sustainability based on the feasible adjustment). In practice, even debtors that take a fairly adversarial position with respect to their creditors follow these norms and are eager to bring the restructuring to a close. This is illustrated by the fact that if anything, restructurings tend to deliver "too little" relief (IMF, 2013; Asonuma and He, 2023). EM bond restructurings are rarely associated with significant face-value reductions, with much of the net present value relief coming from the rescheduling of debt at a moderate interest rate (significantly lower than the country would face if it were to issue new debt in the absence of a restructuring).

If countries were to deviate from these norms, e.g. ask for significant relief where it is not needed, they would likely face stronger pressures from the international community. But it is hard to test such hypothesis in the absence of counterfactuals. Even if such considerations play a role in practice, we still have the question of what costs can sustain debt relative to the choice of restructuring following established norms.

3. Model

To study the effect of uncertainty about costs of default on debt tolerance, we consider a model of a small open economy whose government borrows from competitive international lenders to frontload consumption and smooth shocks. The debt takes the form of long-term noncontingent, but defaultable, bonds. To this standard setup, we add the possibility that defaulting on the debt may carry permanent or transitory costs.

Resources The economy receives an endowment stream following a stochastic process with trend and cycle components

$$Y_t = \exp(z_t)\Gamma_t \tag{1}$$

with

$$z_t = \rho z_{t-1} + \epsilon_t^z$$
$$\log(\Gamma_t) = \log(\Gamma_{t-1}) + \log(g_t)$$

where $\epsilon_t^z \overset{iid}{\sim} \mathcal{N}(0, \sigma_z)$ and $\log(g_t) \overset{iid}{\sim} \mathcal{N}(0, \sigma_g)$. We keep the formulation for the trend Γ as parsimonious as possible since its only purpose is to enable possibly permanent costs of default, which would manifest as ex-post decreases in Γ.

We first present the model in non-stationary form. In Section 3.2, we normalize variables by Γ_t.

Assets The government borrows from international lenders in the form of a defaultable bond which promises to pay a noncontingent stream of geometrically-decaying coupons as in Leland (1998), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012). A bond issued in period t pays $(1-\rho)^{s-1}\kappa$ units of the good in period $t+s$, which effectively makes a one-period-old bond a perfect substitute of $(1-\rho)$ units of newly-issued debt. The coupon rate $\kappa = r + \rho$, where r is the international risk-free rate, is chosen so that the price of a bond that is expected to never default is $q^\star = 1$.

Upon default, the government loses access to international capital markets and faces a loss of output. There is uncertainty about whether this loss of output is permanent or transitory,[10] as

[10] When deciding to default, there are different sources of uncertainty about what the default spell might look like, including whether the default event will be viewed as “hard” or “soft” (Trebesch and Zabel, 2017; Gordon and Guerrón-Quintana, 2019), or what will be the stance of creditors in the ensuing negotiations (Asonuma and Joo, 2020). We capture these in the form of a shock to the type of default costs.

well as about the length of the exclusion period. Most of the literature assumes that the resumption of debt service payments, the recovery of market access, and the dissipation of any output costs all happen at the same time. We keep this assumption when the default costs turn out to be temporary. When costs are permanent, the resumption of debt payments and the return to markets still happen at the same time.

Government The government is benevolent and makes choices on a sequential basis to maximize the utility of a representative household with preferences given by

$$V_t = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(u(C_{t+s}) + \Gamma_{t+s}^{1-\gamma} \varepsilon_{t+s} \right) \right] \tag{2}$$

where $\mathbb{E}$ denotes the expectation operator, C_t represents the household's consumption, β is a discount factor, and ε is a preference shock for default or repayment,[11] scaled by $\Gamma^{1-\gamma}$ to ensure that its importance for the government's decisions remains commensurate to relevant quantities in the face of a non-stationary endowment process (see below).

While the government is not in default, it chooses whether to repay the debt and attains a value

$$V(B, z, \Gamma) = \mathbb{E} \left[\max \left\{ V_R(B, z, \Gamma) + \Gamma^{1-\gamma} \epsilon_R, V_D(z, \Gamma) + \Gamma^{1-\gamma} \epsilon_D \right\} \right] \tag{3}$$

where the ϵ's follow a Type 1 Extreme Value distribution as in Chatterjee et al. (2018), yielding closed forms for the value function in the normalized formulation below

Debt issuances If the government chooses the repay the debt, it can access capital markets and issue new debt B', so that

$$\begin{aligned} V_R(B, z, \Gamma) = &\max_{B'} u(C) + \beta \mathbb{E} \left[V(B', z', \Gamma g') \mid z \right] \\ \text{subject to} \quad &C + \kappa B = Y(z, \Gamma) + q(B', z, \Gamma)(B' - (1 - \rho)B) \end{aligned} \tag{4}$$

[11] We follow Chatterjee et al. (2018) and Dvorkin et al. (2021) and introduce preference shocks for repayment and default to improve the numerical convergence of the algorithm used to solve the model. While these papers also introduce preference shocks for the borrowing decision, this is not strictly needed for convergence in our case. Rather, the preference shocks for repayment are sufficient to replace the two-tiered income process used early in the literature of sovereign default with long-term debt (see for example Chatterjee and Eyigungor, 2012). As is also common, we keep the variance of the preference shocks small to ensure that they do not modify the quantitative properties of the model solution. Plausible stories to microfound a more meaningful inclusion of such shocks include shifting political preferences, developments in domestic debt markets or in the banking system, or changes in the external borrowing terms faced by private firms. Fourakis (2023) uses preference shocks with a larger variance to obscure the actions taken by two possible 'types' of the government and thus enable a signalling dimension to government borrowing, default, and renegotiation.

Default decision If the government chooses to default, it loses access to international capital markets, which it then recovers with a constant hazard ψ. While it is excluded, the borrowing economy suffers a loss of output Δ. Upon default, a shock $k \in \{T, P\}$ determines whether this loss of output is transitory or permanent, which happen with probability p and $1-p$, respectively. We therefore maintain the usual assumption that the resumption of payments and the renewed access to international markets happen simultaneously.[12] Our departure from the standard model only involves that the output costs of default do not necessarily dissipate when market access is restored.

Regardless of their persistence, the output costs of default are always of the same size and reduce output by a factor of Δ. This linearity assumption allows to cleanly separate transitory costs affecting z from permanent ones which impact Γ. In other words, if output in default is $Y^D = \exp(z)\Gamma(1-\Delta)$, we can express it as a reduction in z when the costs are transitory and a reduction in Γ when they are permanent. Below we show how this separation helps us save a state variable.

We assume that all debt is destroyed after a default for tractability and to more readily compare with other results in the literature.[13] The ex-ante and ex-post (with respect to the realization of the default cost type k) values of default is

$$V_D(z,\Gamma) = pV_D^T(z,\Gamma) + (1-p)V_D^P(z,\Gamma(1-\Delta)) \tag{5}$$

and

$$V_D^k(z,\Gamma) = u(Y(z,\Gamma)(1-1_{(k=T)}\Delta)) + \beta\mathbb{E}\left[\psi V(0,z',\Gamma g') + (1-\psi)V_D^k(z',\Gamma g') \mid z\right] \tag{6}$$

which illustrate how the permanent component of output Γ is reduced (in equation 5) when the costs are permanent, while output itself is reduced (in equation 6), for the duration of the exclusion spell, when costs are transitory.

Lenders Bonds issued by the government are purchased by deep-pocketed, risk-neutral foreign investors who equate the expected return of the debt to their cost of funds r, yielding a debt price

$$q(B',z,\Gamma) = \frac{1}{1+r}\mathbb{E}\left[(1-\mathcal{P}(B',z',\Gamma g'))(\kappa + (1-\rho)q(B'',z',\Gamma g')) \mid z\right] \tag{7}$$

[12]See Mihalache (2020) for a case in which market access is restored after debt service payments have resumed.

[13]In practice, restructurings tend to involve substantial recovery rates. Meyer, Reinhart, and Trebesch (2022) estimate an average recovery above 50 in a long historical sample. Adding a recovery rate of that magnitude would reduce the benefits of restructuring, and hence help support more debt in equilibrium. But a positive recovery rate should not interact with the forces we highlight in this model.

where B'' and $\mathcal{P}$ denote the government's issuance and default policy expected by lenders. We implicitly assume these are private investors.[14]

3.1 Equilibrium

We define an equilibrium of the economy in its original non-stationary form before proceeding to normalizing the model to compute a solution.

Definition. *A* Markov-Perfect equilibrium *consists of a set of value functions V, V_R, V_D, V_D^T and V_D^P, a default probability $\mathcal{P}$, a borrowing rule B', and a bond price function q, such that*

i. Given the price of debt, the value functions satisfy functional equations (3)-(6).

ii. Given the default probability and borrowing rules, the price of debt satisfies the creditors' Euler equation (7).

iii. Given the price of debt, the default probability and borrowing rules correspond to the policy functions solving (3) and (4).[15]

3.2 Normalization

We detrend variables by Γ_t and denote normalized values with lowercase. For example, $y_t = Y_t/\Gamma_t = \exp(z_t)$. We also specialize the utility function to a CRRA form, and obtain

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma}\frac{(C/\Gamma)^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma}u(c)$$

which implies a normalization constant $\Gamma_t^{1-\gamma}$ for the value functions, as laid out in more detail in Appendix B.

[14] In principle, the model could apply to any risk-neutral investor. The processes for restructuring private and official bilateral debt are typically different, but to the extent that our model assumes no recovery after a default that would not make a difference. There likely are other factors influencing the decision to lend in the case of official creditors that are not captured by our model. For recent studies exploring the interaction of private and different types of official sovereign borrowing, see Boz (2011), Hatchondo, Martinez, and Onder (2017), Arellano and Barreto (2023) and Roldán and Sosa-Padilla (2023).

[15] Aguiar and Amador (2020) show that in an Eaton-Gersovitz model with long-term debt, there may be multiple Markov Perfect equilibria. We rule out this possibility by focusing on the one that arises as the limit of the equilibrium of the finite-horizon economy.

Normalizing variables by Γ leads to a formulation with state variables (b, z, g). At the beginning of a period, the government faces the choice of default and repayment (3), where we guess-and-verify (along the lines of Appendix B) that $V(B, z, \Gamma) = \Gamma^{1-\gamma} v(b, z, g)$ and multiply both sides by $\Gamma^{\gamma-1}$ to obtain

$$v(b, z, g) = \mathbb{E}\left[\max\left\{v_R(b, z, g) + \epsilon_R, v_D(z, g) + \epsilon_D\right\}\right] \tag{8}$$

As in Chatterjee et al. (2018), the distributional assumption for the ϵ's implies closed forms for the value function $v(b, z, g)$ and the (ex-post) default probability $\mathcal{P}(b, z, g)$

$$\begin{aligned} v(b, z, g) &= \chi \log\left(\exp(v_D(z, g)/\chi) + \exp(v_R(b, z, g)/\chi)\right) \\ \mathcal{P}(b, z, g) &= \frac{\exp(v_D(z, g)/\chi)}{\exp(v_D(z, g)/\chi) + \exp(v_R(b, z, g)/\chi)} \end{aligned} \tag{9}$$

where the ϵ's are distributed Extreme Value Type 1 with scale parameter χ.

Casting the equations describing the values for repayment and default in stationary form turns out to be less trivial. For (4), notice that normalizing by the current Γ creates a difference between the control variable B'/Γ which appears in the budget constraint and the following period's state B'/Γ' which appears in the continuation value function. To reserve the symbol b' for the second meaning, we denote the current period's issuance B'/Γ as x. Multiplying both sides of (4) by $\Gamma^{\gamma-1}$ yields

$$\begin{aligned} \Gamma^{\gamma-1} V_R(B, z, \Gamma) &= \max_{B'} \Gamma^{\gamma-1} u(C) + \beta \mathbb{E}\left[\Gamma^{\gamma-1}\left(\frac{g'}{g'}\right)^{\gamma-1} V(B', z', \Gamma g') \mid z\right] \\ v_R(b, z, g) &= \max_{B'/\Gamma} u(c) + \beta \mathbb{E}\left[(g')^{1-\gamma} v(B'/\Gamma', z', g') \mid z\right] \\ v_R(b, z, g) &= \max_{x} u(c) + \beta \mathbb{E}\left[(g')^{1-\gamma} v(x/g', z', g') \mid z\right] \end{aligned} \tag{10}$$

where the maximization is subject to the budget constraint

$$\begin{aligned} \frac{C + \kappa B}{\Gamma} &= \frac{Y(z, \Gamma) + q(B', z, \Gamma)(B' - (1-\rho)B)}{\Gamma} \\ c + \kappa b &= y(z) + q(x, z, g)(x - (1-\rho)b) \end{aligned}$$

where we also used that $Y(z, \Gamma)/\Gamma = \exp(z)$ which does not depend on g. Clearly, it is equivalent to choose b' or x as long as the state-contingent relationship between them, $b' = x/g'$, is imposed where appropriate.[16]

[16] When the government chooses the level of debt B' for the following period, it does so without knowing the shock g' which would be used to normalize this level and construct b'. This drives the need to distinguish x from b' in the normalized formulation.

Notice that, as usual in non-stationary models, the stochastic discount factor $\beta(g')^{1-\gamma}$ incorporates the growth rate of the economy. Trend growth and impatience impact borrowing decisions in the same way. We choose for simplicity a formulation without drift in the stochastic process for g.[17]

Finally, to cast the value of default in stationary form, notice that the value of Γ, by which we are normalizing, varies when the costs turn out to be permanent. Starting from (5) and multiplying by $\Gamma^{\gamma-1}$, we have

$$\begin{aligned}\Gamma^{\gamma-1}V_D(z,\Gamma) &= p\Gamma^{\gamma-1}V_D^T(z,\Gamma) + (1-p)\Gamma^{\gamma-1}\left(\frac{\Gamma(1-\Delta)}{\Gamma(1-\Delta)}\right)^{\gamma-1} V_D^P(z,\Gamma(1-\Delta)) \\ v_D(z,g) &= pv_D^T(z,g) + (1-p)(1-\Delta)^{1-\gamma}v_D^P(z,g(1-\Delta))\end{aligned} \tag{11}$$

and the conditional value of default follows similarly

$$v_D^k(z,g) = u(y(z)(1-1_{(k=T)}\Delta)) + \beta\mathbb{E}\left[(g')^{1-\gamma}\left(\psi v(0,z',g') + (1-\psi)v_D^k(z',g')\right) \mid z\right] \tag{12}$$

The price of debt (7) is relatively unaffected by the normalization but features the distinction between $x = B'/\Gamma$, the debt chosen in the current period, and $b' = B'/\Gamma'$, the debt outstanding at the beginning of the following period

$$q(x,z,g) = \frac{1}{1+r}\mathbb{E}\left[(1-\mathcal{P}(x/g',z',g'))\left(\kappa + (1-\rho)q(x',z',g')\right) \mid z\right] \tag{13}$$

Notice that because g is *iid* the costs of default can be applied to g or z depending on their permanence. In addition, since the factor Δ is independent of both z and g, there is no need to carry g as a state variable in any of the value or price functions. Conversely, autocorrelated g shocks would require conditioning all expectations on the current g, and more general costs (for example, strictly convex costs as a function of Y), would require a renormalization factor in (11) with an explicit dependence on g (and possibly z). With these assumptions, we drop redundant dependencies on g in what follows.

Although advantageous, our formulation with linear costs is less flexible than the strictly convex costs sometimes used in the literature. Convex costs make it 'cheaper' to default when income is low and therefore induce more countercyclical default probabilities and spreads. For the same reason, convex costs make ex-post default more dependent on the following period's income shocks; in other words they induce more uncertainty in default probabilities. This feature can be seen as similar to the uncertainty introduced by the preference shocks and manifested by

[17] Such a formulation effectively loads mean growth onto the discount factor. In the data, we remove mean growth as part of our estimation.

the ex-post default probability $\mathcal{P}$. However, the variance of our preference shocks is chosen to be small, to ensure that $\mathcal{P}$ mostly takes values close to 0 and 1.

3.3 *Model with robust preferences*

There are five shocks in the model, ϵ^z and g for the endowment process, the preference shock $\epsilon_R - \epsilon_D$ for repayment, the reentry to international markets after a default, and whether the costs of defaulting are permanent or transitory. We assume that, while the true stochastic processes governing these shocks are unknown, all agents in the economy take the descriptions above to be approximating models. While foreign creditors trust the approximating model and use it to compute the expectation in (7), the government does not. The government is instead concerned about potential misspecification of (some of) these processes and seeks robust decision rules.

We represent such doubts and objectives with *multiplier preferences* (Hansen and Sargent, 2001) which capture ambiguity aversion with a single parameter. In this framework, robust decision rules emerge from confronting the decision maker with an 'evil agent' alter-ego. This fictitious, auxiliary agent chooses a worst-case model to minimize the decision maker's utility for each possible choice of action. Critically, the evil agent faces an entropy cost $\frac{1}{\theta}$ which only allows it to pick distortions from the approximating model (worst-case models) which are difficult to statistically distinguish from it. When the marginal cost of this relative entropy is infinite (when $\theta \to 0$), we recover the standard expected-utility decision maker. In what follows, we refer to the inverse of the marginal cost of relative entropy, θ, as the robustness parameter.

Moreover, the government potentially places different degrees of trust in its model for the endowment process, which is observed continuously, and in its model for the default costs, which are observed infrequently.[18] We therefore denote as θ_s and θ_c the two robustness parameters for evaluating expectations involving (z', g') and k, respectively. Finally, we assume that the government does not doubt the stochastic process for reentry to markets after a default. While it is difficult to empirically determine whether output eventually catches up to its pre-default trend, exclusion periods are generally better understood (see Cruces and Trebesch, 2013).

[18] We assume in all cases that the remaining shocks are not subject to potential misspecification. The preference shocks (ϵ_R, ϵ_D) are only introduced for numerical performance and with a low variance.

With multiplier preferences, the government's problems and value functions are[19]

$$v_R(b,z) = \max_x u(c) + \beta \frac{1}{-\theta_s} \log \left(\mathbb{E} \left[\exp \left(-\theta_s (g')^{1-\gamma} v(x/g', z') \right) \mid z \right] \right)$$
$$\text{subject to} \quad c + \kappa b = y(z) + q(x,z)(x - (1-\rho)b) \tag{14}$$

$$v_D(z) = \frac{1}{-\theta_c} \log \left(p \exp \left(-\theta_c v_D^T(z) \right) + (1-p) \exp \left(-\theta_c (1-\Delta)^{1-\gamma} v_D^P(z) \right) \right) \tag{15}$$

and

$$v_D^k(z) = u(y(z)(1 - 1_{(k=T)}\Delta)) + \beta \frac{1}{-\theta_s} \log \left(\mathbb{E} \left[\exp \left(-\theta_s (g')^{1-\gamma} \left(\psi v(0,z') + (1-\psi) v_D^k(z') \right) \right) \mid z \right] \right) \tag{16}$$

It is easy to show that this formulation with robustness converges back to the standard expected utility case as $(\theta_s, \theta_c) \to 0$. Appendix D discusses the numerical solution of the model in this general case.

In general, under certain conditions it can be shown that multiplier preferences are mathematically equivalent to risk-sensitive Epstein-Zin preferences (Strzalecki, 2011). While risk-sensitive preferences describe an agent with certain attitudes when confronted to risk, robustness links such attitudes to the agent's reluctance to fully trust the model underpinning expectations. As such, robustness becomes a natural assumption in the context of uncertainty about the nature of default costs, a topic on which research is still ongoing. Finally, robustness allows a more flexible approach relative to general risk-sensitive preferences, focusing on specific sources of ambiguity, by distinguishing the risk-attitudes towards uncertainty in the continuously-observed and better-understood endowment process, represented by θ_s, and towards uncertainty in the persistence of the output costs of default which are observed much more infrequently, represented by θ_c.[20] Below, we will starkly assume that the government trusts the model for the endowment shocks (effectively setting $\theta_s = 0$) but doubts the model for default costs, represented by $\theta_c > 0$.

As a benchmark, we maintain the assumption of risk-neutral expected-utility preferences for the creditors. In other words, while everyone in the model shares the same approximating or baseline model, only the government mistrusts (particular aspects of) it. The government understands that creditors fully trust the baseline model and may sometimes note with relief that borrowing costs would be higher if the creditors shared its concerns about misspecification.[21] In

[19]For more details about how multiplier preferences yield a Bellman equation of the form of (14), see Appendix C.

[20]Distinguishing two robustness parameters in this way can also be viewed as an extreme form of robustness to "structured uncertainty" (Hansen and Sargent, 2015) when the decision maker's misspecification concerns focus on particular features of its beliefs.

[21]The fact that worst-case models are endogenous makes it difficult to exploit gains from trade arising from differ-

general, the fact that debt prices do not incorporate an ambiguity premium from creditors' robustness does not sway the government's concerns as the differences in trust are common knowledge to all agents. In this particular case, moreover, potential concerns about misspecification of the type of default costs would not materially affect the lenders' problem, as they care about the probability of default in different states but not about the reasons why the default occurred.

3.3.1 *Worst-case models*

An appealing feature of robustness relative to risk-sensitive preferences is that it is possible to recover the belief distortions underpinning particular actions, the 'worst-case model.' Let the *distorted expectation* of random variable X at information set $\mathcal{F}$ be the objective expectation of the product of X with a likelihood ratio

$$\tilde{\mathbb{E}}[X \mid \mathcal{F}] = \mathbb{E}\left[\frac{\exp(-\theta v)}{\mathbb{E}\left[\exp(-\theta v) \mid \mathcal{F}\right]} X \,\middle|\, \mathcal{F}\right] \tag{17}$$

where v is the government's value function at that information set.

As compared to an expectation taken with the objective probability measure (the approximating or baseline model), the distorted expectation magnifies the likelihood of states for which the government's utility, measured by the value function v, is low. This procedure results in endogenously pessimistic beliefs which sustain robust decision-making. The role of the robustness parameter θ can also be clearly understood from (17). A larger value for θ means stronger misspecification concerns on the part of the government, and consequently a more distorted expectation. Conversely, as $\theta \to 0$, misspecification concerns vanish and the distorted expectation converges back to the objective one.

3.3.2 *Model detection-error probabilities*

As is common in the robustness literature, we compute model detection-error probabilities (DEP) to help inform and discipline our choice of θ_s and θ_c.[22] When the government follows robust decision rules, it does so by computing an auxiliary worst-case model. The DEP captures the probability that an agent, with a limited amount of data, mistakes data generated by one of the

ent beliefs within a certain equilibrium. Manipulating the counterpart's worst-case model is a central theme in Roch and Roldán (2023), where a government designs GDP-linked bonds to borrow from robust creditors, or in Karantounias (2023), where a policymaker manages the private sector's expectations in order to relax forward-looking constraints.

[22]We leave the model description general even though we only consider positive values for θ_c.

models as coming from the other one. A DEP of 50% means that the baseline and worst-case models are observationally equivalent while a DEP of 0 means that an agent can perfectly distinguish both models. A high DEP suggests that the amount of misspecification implicit in the distorted model is plausible and validates the decision maker's desire for robustness.

We compute model detection-error probabilities as the probability of misclassifying a sample from the approximating model A as coming from the (endogenous) worst-case model W and viceversa. In each case we compute the likelihood $L(x \mid M)$ of data x being generated by model M to classify the sample as coming from the model which produces the maximum likelihood. Finally, the detection-error probability is the average of the probability of misclassification when the true model is W and when it is A.[23]

$$DEP = \frac{\mathbb{P}\big(L(x \mid A) > L(x \mid W) \mid W\big) + \mathbb{P}\big(L(x \mid W) > L(x \mid A) \mid A\big)}{2} \tag{18}$$

We note that it seems natural that distortions to the stochastic process governing the type of default costs, which as we show below can have a large impact on the government's decisions, can be difficult to detect as this shock is only observed infrequently. On the other hand, distortions of similar magnitude applied to the continuously-observed shocks governing the endowment would be much more salient. This asymmetry motivates our approach of assuming different robustness parameters for these two forms of uncertainty, while recognizing that a fuller approach in which the government's evil agent has, for example, an entropy budget to allocate to both types of shocks, so as to analyze which shock is distorted more, is left outside the scope of this paper.

4. Calibration

4.1 *Output trajectories around restructuring episodes*

We summarize the evidence on output costs of restructuring by computing deviations of output around debt restructuring episodes from a pre-restructuring trend in the Asonuma and Trebesch (2016) dataset.[24] We purposefully abstract from causality. Instead, our approach is to parametrize the model to ensure that it generates the same correlations we find in the data. This allows us to

[23] We implement these calculations below on 60-year samples (which is more than the length of quarterly GDP data for most EM countries), after a 500-period burn-in to ensure draws from the ergodic distribution of each model, over 2000 repetitions.

[24] We consider all restructuring episodes in their dataset, including those that take place preemptively to avoid a default.

indirectly infer the underlying causal parameters determining the size and persistence of default costs.

We consider a panel of emerging market economies (EMs) which have had a sovereign restructuring event in 1990-2020 for which data on external debt, standing in international capital markets, GDP, and spreads are all available for at least 10 years previous to the default event. For each restructuring episode occuring at time t in country i, we construct a pre-restructuring trend for output by regressing log output in years $t-j$, for $1 \leq j \leq 6$, against a linear function of time

$$\log Y_{i,t-j} = \alpha_{it} + \beta_{it}(t-j) + \epsilon_{itj} \tag{19}$$

We then use this pre-restructuring trend to obtain fitted values $\hat{Y}_{i,t+k}$ and compare realized output $Y_{i,t+k}$ against this trend at horizon k. We focus on horizons $k=1$ and 5 and take the median across all episodes. We complement these calculations with values for external debt (as share of GDP) and spreads, taken as averages between $t-10$ and $t-5$.

Table 1 summarizes our results, which we later use as calibration targets for the model presented in Section 3. The output deviation is sensitive to how the trend is defined, as the loss may be spread over many years and some of the decline relative to trend may take place in the run-up to the restructuring. We are therefore careful to estimate the same statistics in the model-generated data. We recover the structural causal cost-of-default parameters by matching the output deviations, while the overall output trajectory can be used for validation of the model. Finally, as all debt in the model is external, we compute a target for the external-debt-to-GDP ratio.[25]

Table 1: Calibration targets

Description	**Calculation**	**Value**
Output deviation, 1-year horizon, %	$Y_{t+1}/\hat{Y}_{t+1} - 1$	8.27
Output deviation, 5-year horizon, %	$Y_{t+5}/\hat{Y}_{t+5} - 1$	7.6
Average external debt-to-GDP ratio, %	$\overline{B_{t-j}/Y_{t-j}}$	23.4
Average spread, bps	$\overline{Spread_{t-j}}$	793

[25]This number is of course lower than the headline debt-to-GDP ratio at which most countries tend to experience sovereign debt defaults.

4.2 *Baseline calibration with robustness*

We parametrize our model at a quarterly frequency. We externally set most parameters (the sovereign's risk aversion γ, the risk-free interest rate r, the duration of debt ρ and the reentry probability ψ) to standard values in models of sovereign default. We choose a small value for the preference shock scale parameter χ to minimize the impact of this numerical device on the equilibrium.

We estimate the endowment process parameters $(\rho_z, \sigma_z, \sigma_g)$ for each country in our sample in the period before the first default, starting in 1960, and take averages across the episodes.In each case, we run the log of output through an HP filter and estimate an AR(1) process on the cycle component to obtain (ρ_z, σ_z) and retrieve σ_g as the standard deviation of the residuals of the trend component against a linear time trend. The results for each country are reported in the Appendix.[26]

Finally, we assume that the government places full trust in its approximating model for the income process, so $\theta_s = 0$. This leaves us the sovereign's discount factor β, the level of default costs Δ, the probability of a transitory cost p (under the approximating model), and the robustness parameter θ_c. We set these parameters to match the moments from Table 1: the local projections for output at 1 and 5 years after a default event, relative to a pre-default trend, the average level of spreads and the average debt-to-GDP ratio while in repayment. Table 2 summarizes our parametrization.

To assess the match to the moments from Table 1, we generate simulated data from the model[27] and filter it through the same procedure used for the sample of EM default episodes. The calibrated model matches those patterns closely, as shown in Table 3. We note that our structural estimate for the costs of default ($\Delta = 4.11\%$) falls well within the range of empirical estimates of the causal effect, and quite close to the value found by Asonuma et al. (2023) which controls for a number of variables as well as the endogeneity of the choice of restructuring.

Figure 2 plots the median trajectory for output around default events in the data, with a shaded area indicating the interquantile range, as well as in the model. In the model we take the average

[26] Alternatively, the endowment process can be estimated with state space methods. However, the results are noisy, with large variations across countries, and also quite sensitive to the time sample used within each country, given the limited time series (see García-Cicco et al., 2010, for details on how this can be expected). Imposing a little structure, as described above, provides more stable results and a more meaningful estimation given the data constraints.

[27] We take 2000 simulations of 250 years in each case.

Table 2: Baseline parameter values

Externally chosen		
	Parameter	**Value**
Sovereign's risk aversion	γ	2
Preference shock scale parameter	χ	0.01
Risk-free interest rate	r	0.01
Robustness parameter: income shocks	θ_s	0
Duration of debt	ρ	0.05
Reentry probability	ψ	0.0385
Income autocorrelation coefficient	ρ_z	0.9256
Standard deviation of z_t	σ_z	0.0231
Standard deviation of g_t	σ_g	0.0211
Internally calibrated		
	Parameter	**Value**
Sovereign's discount factor	β	0.902
Default cost	Δ	0.0411
Probability of transitory shock	p	0.339
Robustness parameter: default costs	θ_c	7.6

Table 3: Model fit

	Data	**Model**
Output deviation, 1-year horizon, %	8.27	9.75
Output deviation, 5-year horizon, %	7.6	7.99
Average external debt-to-GDP ratio, %	23.4	21.3
Average spread, bps	793	813

simulation path conditional on default at year 0 and no defaults in the previous 5 years to replicate the selection done in the data. The model-generated dynamics match the data counterparts well, especially taking into account that the (annual) data reflects some time-averaging relative to our quarterly model. Both data and model exhibit a slight acceleration of output in the run-up to the default (between years -5 and -2) followed by a series of negative shocks starting around 1.5 years before the default. Both model and data then show a slow recovery in the years following the episode.

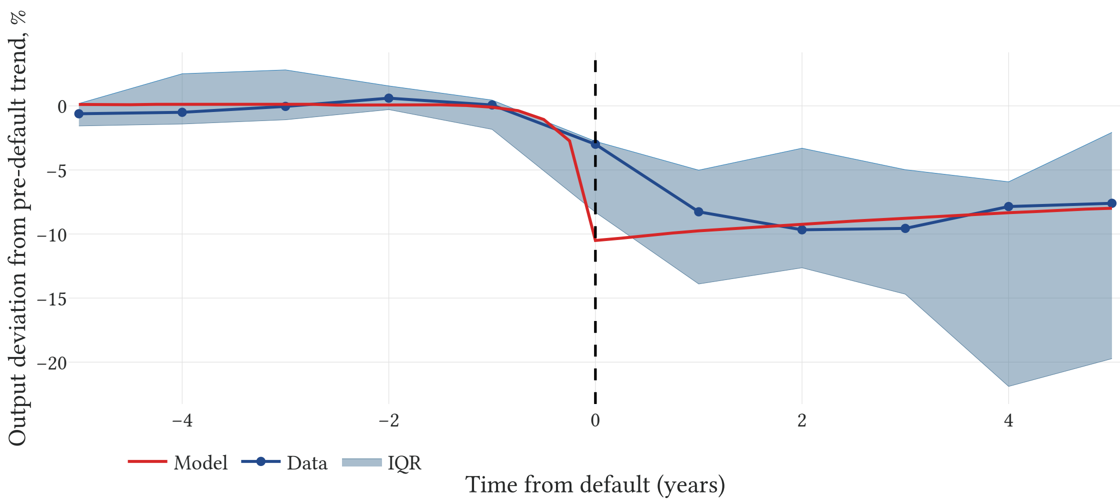

FIGURE 2: OUTPUT TRAJECTORIES AROUND DEFAULTS

5. QUANTITATIVE RESULTS

We begin by presenting the moments targeted in the calibration for our benchmark model, along with those same moments for a version without robustness (in which we simply set $\theta_c = 0$) and for a version without robustness, but where we recalibrate the remaining three parameters, and similarly for uncertainty (by setting $p = 1$). Table 4 summarizes the results.[28]

TABLE 4: TARGETED MOMENTS

	Data	**Benchmark**	$\theta_c = 0$	$\theta_c = 0$, **recal.**	$p = 1$	$p = 1$, **recal.**
Output dev., 1-year, %	8.27	9.75	9.72	9.64	8.02	10.2
Output dev., 5-year, %	7.6	7.99	8.12	8.13	5.46	7.27
Avg. ext. debt/GDP, %	23.4	21.3	20.6	20.8	15.7	21.6
Avg. spread, bps	793	813	835	815	1021	814

[28]Notice that, because robustness has no effect if there is no underlying uncertainty, we do not solve the model introducing only robustness.

Table 4 shows how uncertainty and robustness help match the four calibration targets. Our benchmark model reproduces the observed patterns reasonably well, although the calibration slightly underpredicts debt levels while overpredicting costs. Perhaps convex costs (which as discussed above could complicate the model and its solution) could allow a better fit, in that case complementing the set of targeted moments with the volatility of spreads, as is standard with convex costs.[29]

In the benchmark model with robustness turned off, marked $\theta_c = 0$, perceived costs of default diminish and so does debt tolerance: on average the government sustains a lower amount of debt and pays higher spreads. This also affects the states in which the government defaults, which induces a deeper measured contraction at the 5-year horizon. When we recalibrate the remaining parameters (Table 7 in the Appendix), the discount factor β and the size of default costs Δ only change marginally, but the probability of a transitory cost p is reduced to about 11%.[30] This raises (actual) costs of default and allows for slightly larger debt levels at lower spreads, but still short of both the data and the benchmark model with robustness, as can be seen on the fourth column of Table 4. Moreover, with more persistent default costs, the output contraction continues to be too large at the 5-year horizon. Without robustness, the calibration is less able to replicate the observed dynamics of output after default while also requiring a very low, perhaps implausibly so, value for the probability p of a transitory cost.[31]

The last two columns of Table 4 describe the behavior of the model when the cost of default is transitory with certainty,[32] once more keeping the baseline parameters and recalibrating (see Table 8 in the Appendix). With the baseline parameters, debt tolerance is much lower: the economy sustains less debt on average at much wider spreads. Recalibrating in this case continues to require costs of default that generate too large deviations of output, although in this case the effect can be seen only on impact at the 1-year horizon, as the medium-term impact is fully determined by the fact that costs are always transitory in this case.

All in all, the model with certainly transitory costs is unable to replicate the dynamics of output at the observed levels of debt and spreads. The version with uncertainty is much closer to the data, although it predicts lower debt levels with a cost of default that implies too deep an output contraction in the medium-run, while requiring a probability of transitory costs which

[29]The spread volatility generated by this calibration is indeed on the low side, as shown in Table 5.

[30]Figure 6 reveals that in the benchmark model with robustness, the government acts as if p was about 17-20% when $p \approx 2/3$.

[31]From a robustness point of view, a probability of transitory costs as low as 11% seems inconsistent with the difficulties encountered by the empirical literature in its estimation.

[32]In this case, once uncertainty is taken away, robustness has no effect.

seems implausibly low. The model with uncertainty and robustness improves the fit in all these dimensions, although as discussed above it does not generate the four targets exactly either.

Table 5 provides a comparison of the general performance of these models along typical moments in the sovereign debt literature: the standard deviation of spreads, the cyclicality of spreads and net exports, and the volatility of consumption relative to output. Given that information on all variables for all countries in the sample is not available for the same period as the calibration target moments, to avoid changing the sample we do not report data counterparts, but rather comapre these moments to typical findings in this literature.

TABLE 5: OTHER ERGODIC MOMENTS

	Benchmark	$\theta_c = 0$	$\theta_c = 0$, **recal.**	$p = 1$	$p = 1$, **recal.**
Spread st. dev., bps	115	118	115	138	113
Corr. spreads, growth, %	-51.6	-51.2	-50.6	-51.3	-54.9
Corr. NX, growth, %	-23.5	-22.9	-21.5	-26.2	-31.2
Rel. cons. volatility, %	139	137	137	114	138

The ergodic moments seem mostly in line with typical calibrations in this literature, with countercyclical spreads and net exports and a volatility of consumption (here measured as the relative volatility of the consumption share to output growth) that is larger than that of output. The volatility of the spread is lower than usual calibrations based on Argentina, but seems plausible given that in the data we are averaging with countries with more stable spread levels. Comparing the models, the benchmark calibration with robustness delivers slightly more countercyclical spreads and net exports, and slightly more volatile consumption, relative to the models without robustness or the model without uncertainty but with the same remaining parameters. Recalibrating the parameters in a model without uncertainty yields even more countercyclical spreads and net exports.

5.1 *Actual and perceived persistence of default costs*

Figure 3 shows policy functions for default: for each value of the income shock z, the line shows the level of debt at which the ex-post probability of default crosses 50%. The solid blue line shows our benchmark calibration, while the remaining lines show the cases of a higher value for the robustness parameter $\theta_c = 15$, no robustness ($\theta_c = 0$), as well as the cases of $p = 0$ (fully permanent cost) and $p = 1$ (fully transitory costs).

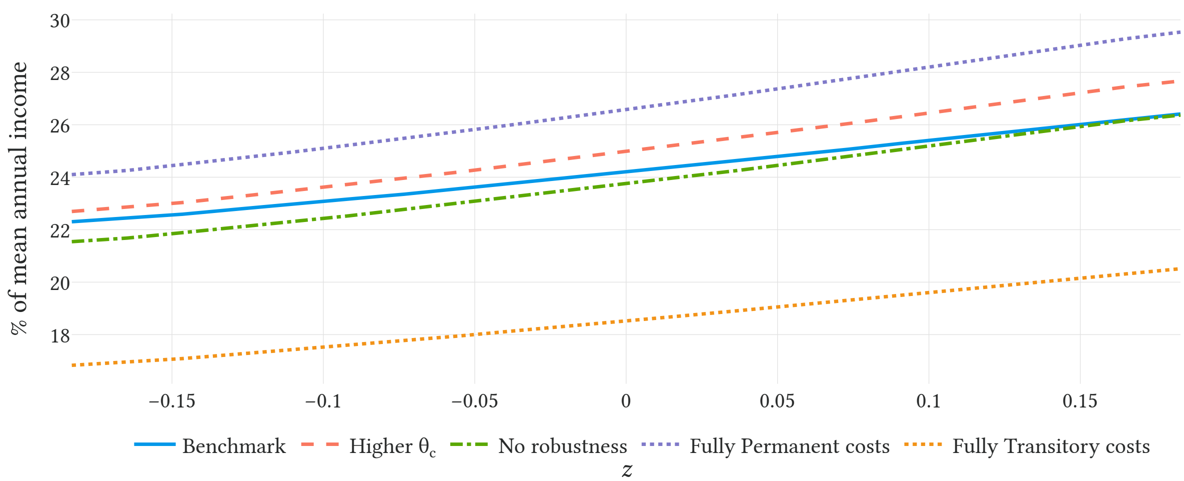

FIGURE 3: DEFAULT BARRIERS VARYING p AND θ_c

Figure 3 illustrates the effect of robustness and uncertainty on the government's default choice. Making the default costs actually more permanent (by changing p) or perceived to be more permanent (by changing θ_c) increases debt tolerance in the sense that the government defaults at higher levels of debt for a given income level z.

Figure 4 shows simulation results at the baseline parametrization, varying the probability p that default costs are transitory and the robustness parameter θ_c affecting this particular source of uncertainty. We solve and simulate the economy for each combination of (p, θ_c) and report the average debt-to-GDP ratio B/Y and the default frequency, both in yearly terms.

When the shock is more likely to be permanent, or the government has greater misspecification concerns, the left panel of Figure 4 shows that the economy sustains larger levels of debt. Robustness has a mechanically larger impact when the underlying uncertainty about the permanence of costs is greater (i.e. when p is closer to 50%). Our calibrated value for p is about 35% which is squarely in the range where robustness has more quantitative bite. Compared to a parametrization in which the government is convinced that the cost is transitory ($p = 1$, the standard case in the literature), the average level of debt increases by about a quarter. Making the cost permanent for sure would result in a further increase in average debt of about 15% from the baseline calibration level.

The right panel reveals that the default frequency is less sensitive to either of these param-

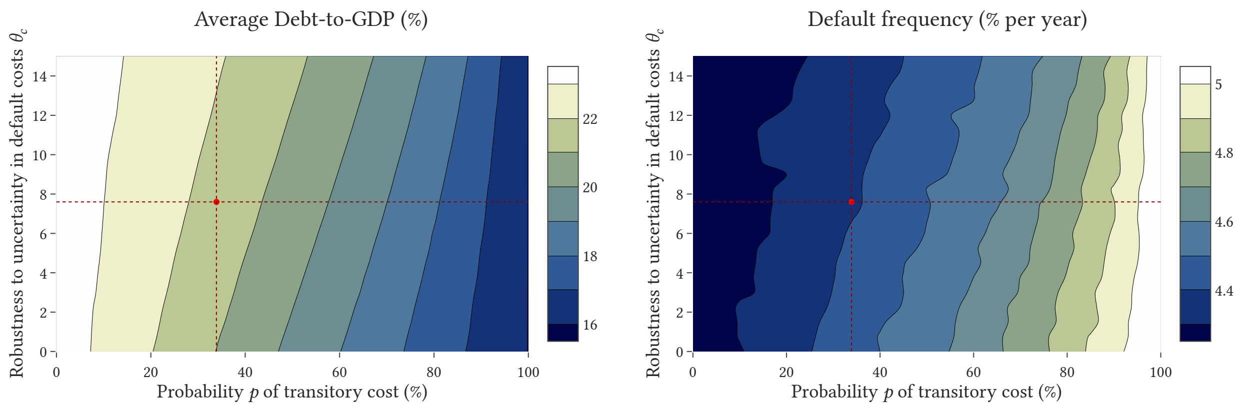

FIGURE 4: DEBT TOLERANCE VARYING p AND θ_c

Note: Red dots denote the baseline calibration.

eters, and relatively more insensitive to the degree of robustness.[33] Shifting the cost from certainly permanent to certainly transitory only increases the default frequency by about 17% (or about 0.3p.p.). Thus, most of the increase in perceived costs of default (both through actual persistence and robustness) translates into additional borrowing. The debtor government uses the extra 'credibility' afforded by the higher default costs to sustain higher levels of indebtedness, rather than keeping debt the same and facing the costs of default less frequently.

5.2 *Decomposition of output costs of default*

The calibrated model matches the output trajectory estimated from the data (Figure 2). In the model, we can decompose the output deviations according to[34]

$$\log Y_t - \log \hat{Y}_t = z_t + \log \Gamma_t + \log(1 - \Delta) 1_{(D_t=1)} - \log \hat{Y}_t \tag{20}$$

where $1_{(D_t=1)}$ refers to whether costs of default are active in period t, which could result from the economy being excluded in the current period or from a permanent cost of default in the current episode. We further normalize Y_t, $\hat{Y}_t$, and Γ_t to the trend level at the start of each episode such that $\Gamma_{t-21} = 1$.

Figure 5 plots the decomposition and reveals that actual, 'causal' default costs account for about 35% of the output deviation from trend. Another 40% comes from underlying shocks which

[33] At the calibrated level of p, shifting θ_c moves debt levels but leaves the default frequency relatively unchanged.

[34] To obtain an exact decomposition in this case, we measure the output deviation from trend as log differences, which leads to slightly different headline numbers than in the calibration and Figure 2, where we measured such deviations as ratios.

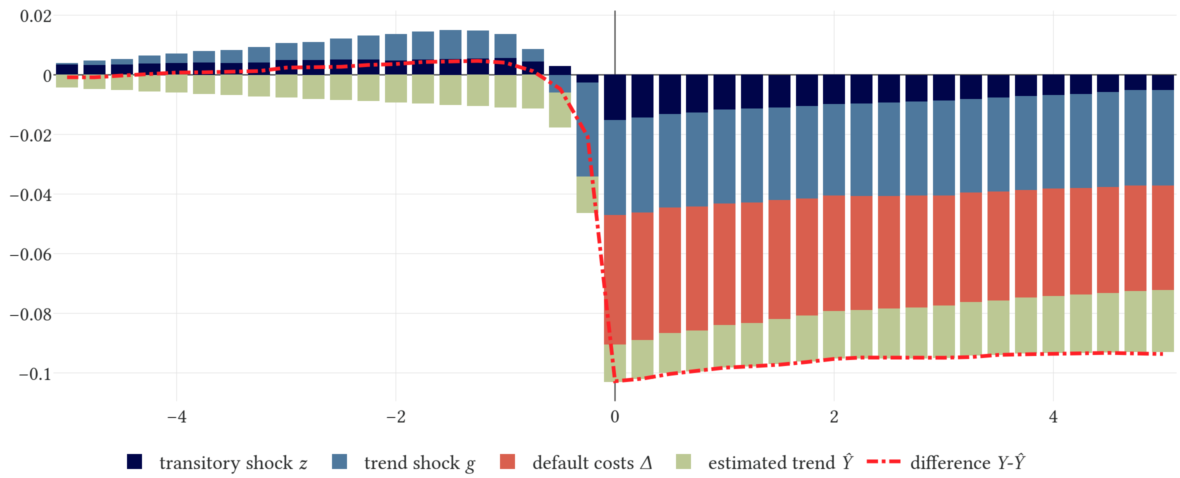

Figure 5: Decomposition of output deviations from trend

tend to be negative at the time of default. Finally, about 25% of the deviation comes from the fact that output is growing (more than usual) in the pre-default period, leading to an upward trend later on. By construction, our sampling includes a restructuring at $t = 0$ but not before, which selects a relatively strong pre-crisis trend. The level shocks g_t then turn negative as we approach the time of the default event $t = 0$. These considerations explain why it would be hard for output to recover to the pre-crisis trend even if the default costs were to completely disappear, and how simple before-and-after growth comparisons can be misleading.

5.3 *The effect of robustness*

We focus on how robustness amplifies the perceived probability of a permanent cost of default. Because this variable $k \in \{P, T\}$ is only realized after a default has taken place, the appropriate value function to use in (17) is v_D^k,with the renormalization factor $(1-\Delta)^{1-\gamma}$ in case the cost is permanent. Letting $\tilde{p}(z)$ denote the distorted probability that the cost is transitory (i.e. $k = T$) at state z,

$$\tilde{p}(z) = \tilde{\mathbb{E}}[1_{(k=T)} \mid z] = \mathbb{E}\left[\frac{\exp(-\theta v_D^k(z)(1 - 1_{(k=P)}\Delta)^{1-\gamma})}{\mathbb{E}\left[\exp(-\theta v_D^k(z)(1 - 1_{(k=P)}\Delta)^{1-\gamma})\right]} 1_{(k=T)} \,\middle|\, z\right]$$

Figure 6 shows that the government's misspecification concerns make it act as if the probability of a permanent cost of default was larger by a factor of about a quarter across the state space. The distortion is larger when current income is lower, as a rebound in income (which is aided by a transitory cost) is more valuable then. This effect is the key to the differential effect of robustness θ_c and the actual probability of a permanent cost $1-p$: the worst-case probability of a permanent cost increases more in low states, which is where default is more likely. As (perceived) costs become relatively larger in low states, the level of debt at which default happens becomes less dependent on the current income state z. This makes it easier to forecast whether a certain level of debt will be repaid in the future. As a consequence, when default costs increase via robustness, the government is more able to 'extract' the increase in debt tolerance while keeping the default frequency relatively unchanged. When they increase via the actual probability p, the government chooses a lower default frequency as well as a higher debt level (Figure 4).

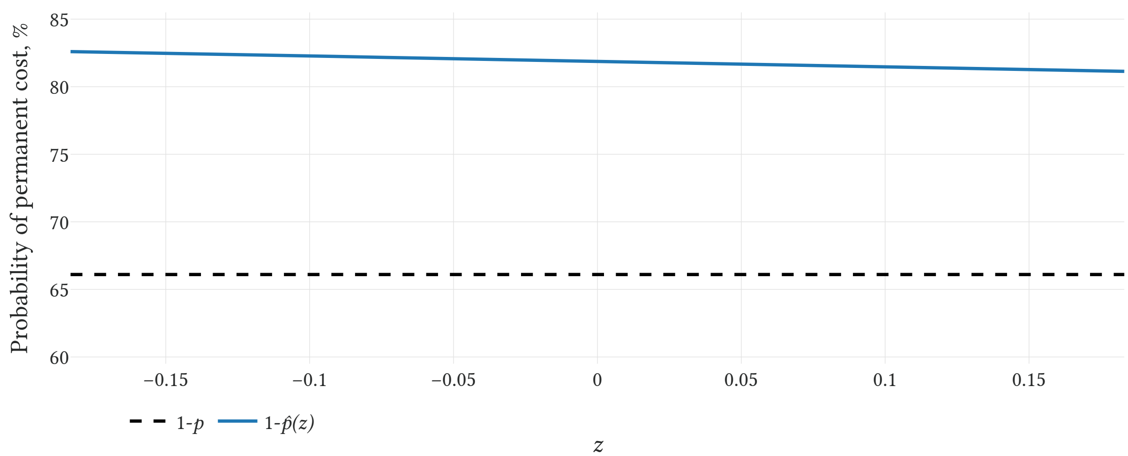

FIGURE 6: WORST-CASE PROBABILITY OF A TRANSITORY COST, $\tilde{p}$

Figure 7 illustrates the relationship between borrowing terms faced by the government and the costs of default, actual and perceived, it faces. For a moderate level of debt (but in the region where spreads start to increase), spreads decrease when the shock is more likely to be permanent. They also decrease when the government has stronger misspecification concerns. As before, the power of robustness is dampened when the government is more certain about the nature of costs (i.e. when p is closer to 0 or 1). The response of spreads becomes very non-linear as we approach high values of p (especially at low values for θ_c). At the chosen debt level (19% of mean income), the default probability would be almost 1 at such parameter values. Figure 8 recasts this same information in terms of the debt issuance level at which spreads become very large, which is perhaps easier to read in terms of debt tolerance.

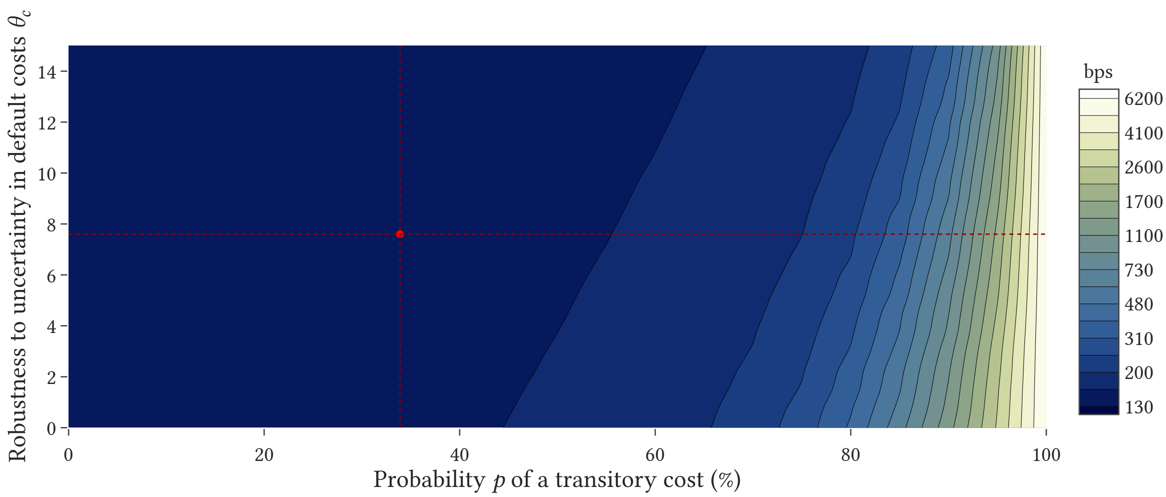

Figure 7: Spreads on government debt

Note: spreads computed at mean income and debt at 19% of income in each case, red dots denote the baseline calibration

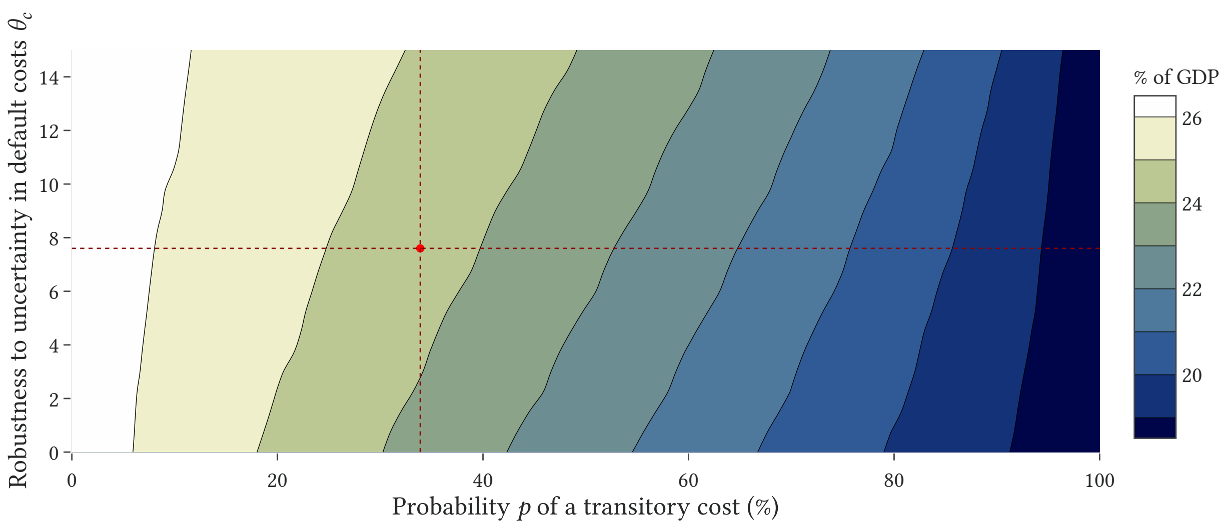

Figure 8: Debt at which spreads cross 1000bps

Note: Red dots denote the baseline calibration.

5.4 *Model detection-error probabilities*

Figure 9 plots the model detection-error probability for different values of the objective probability p of a transitory cost, as a function of the choice of the robustness parameter θ_c. Typically, in the robust control literature, acceptable values for DEP are of 20% or above (Barillas et al., 2009). Our choices of θ_c stay well within this constraint suggesting we rely on reasonable amounts of probability distortions.

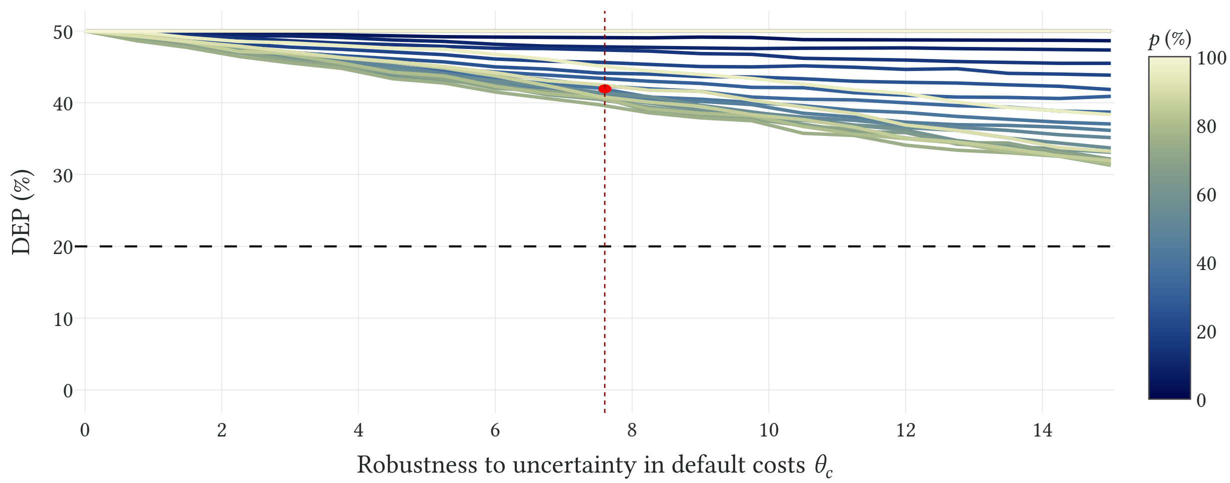

Figure 9: Debt and DEPs for different prior probabilities of a transitory cost

For a given baseline probability p of a permanent shock, increasing robustness leads to higher levels of debt sustained in equilibrium. The government sustains these higher levels of debt because the worst-case model arising from the government's robustness concerns features a larger probability of permanent costs (Figure 4). This feature also makes it more easily distinguishable from the baseline model. Indeed, Figure 9 shows that the DEP decreases with the robustness parameter θ_c. Moreover, as p grows towards 1, there is more scope to distort beliefs in a pessimistic way. The more the cost looks transitory, the more there is to fear that it is permanent. This leads to lower values for the DEP when p is large, holding robustness constant.

6. Concluding remarks

This paper shows how the potential permanence of output costs following a debt restructuring can significantly amplify a country's willingness to repay its debt and, hence, its debt capacity. Debt tolerance with fully permanent output costs can be almost one half larger than with purely transitory costs. In practice, decision makers are unlikely to have a clear picture of how permanent these costs are, similarly to researchers in sovereign debt. Aversion to this ambiguity boosts debt tolerance, by making the government treat costs as perhaps being more persistent. While these channels have received little attention in the literature, our results suggest that they may be key drivers of debt tolerance.

We find that the persistence of output costs, and the uncertainty surrounding it, has a sizable effect on debt tolerance, but perhaps not as much as one would have expected. We parametrize the model to an exclusion period of 6.5 years on average, following widely used estimates from Cruces and Trebesch (2013). The additional impact of making the cost permanent after the exclusion period is limited by the discounting of a relatively impatient borrower, which is also a common feature of calibrated sovereign debt models.[35] With a more patient borrower (or a counterfactually shorter exclusion period), permanent default costs would boost debt tolerance even more. This interaction could be a key driver of what explains differences in debt tolerance across countries (e.g. "graduation" effects).

The focus of this paper, like much of the literature, has been on external debt. Domestic debt restructurings can be much more disruptive, given the implications for domestic financial stability, which can potentially lead to much larger and protracted output costs relative to an external debt restructuring. Modeling domestic debt introduces complications to the model, but would be an interesting direction for future work, and where both channels proposed could have an even larger amplifying impact on debt tolerance.

References

Aguiar, M. and M. Amador (2014): "Sovereign Debt," *Handbook of International Economics*, 4, 647–87.

——— (2020): "Self-Fulfilling Debt Dilution: Maturity and Multiplicity in Debt Models," *American Economic Review*.

Aguiar, M., S. Chatterjee, H. Cole, and Z. Stangebye (2016): "Quantitative Models of Sovereign Debt Crises," *Handbook of Macroeconomics*, 2B, 1697–1755, chapter 21.

Anderson, E. W., L. P. Hansen, and T. J. Sargent (2003): "A Quartet of Semigroups for Model Specification, Robustness, Prices of Risk, and Model Detection," *Journal of the European Economic Association*, 1, 68–123.

Arellano, C. and L. Barreto (2023): "Official Sovereign Debt," Mimeo.

Asonuma, T., M. Chamon, A. Erce, and A. Sasahara (2023): "Costs of Sovereign Defaults: Restructuring Strategies and the Credit-Investment Channel," Mimeo.

[35]Such low discount factors can be interpreted as representing political-economy considerations.

Asonuma, T. and C. He (2023): ""Too Little" Sovereign Debt Restructurings," Mimeo.

Asonuma, T. and H. Joo (2020): "Sovereign Debt Restructurings: Delays in Renegotiations and Risk Averse Creditors [Sovereign Debt]," *Journal of the European Economic Association*, 18, 2394–2440.

Asonuma, T. and C. Trebesch (2016): "Sovereign Debt Restructurings: Preemptive or Post-Default," *Journal of the European Economic Association*, 14, 175–214.

Barillas, F., L. Hansen, and T. Sargent (2009): "Doubts or Variability?" *Journal of Economic Theory*, 144, 2388–2418.

Borensztein, E. and U. Panizza (2009): "The Costs of Sovereign Default," *IMF Staff Papers*, 56, 683–741.

Boz, E. (2011): "Sovereign default, private sector creditors, and the IFIs," *Journal of International Economics*, 83, 70–82.

Bulow, J. and K. Rogoff (1989a): "Sovereign Debt: Is to Forgive to Forget?" *American Economic Review*, 79, 43–50.

——— (1989b): "A Constant Recontracting Model of Sovereign Debt," *Journal of Political Economy*, 97, 155–178.

Cerra, V. and S. C. Saxena (2008): "Growth Dynamics: The Myth of Economic Recovery," *American Economic Review*, 98, 439–57.

Chatterjee, S., D. Corbae, J.-V. Rios-Rull, and K. Dempsey (2018): "A Theory of Credit Scoring and the Competitive Pricing of Default Risk," 2018 Meeting Papers 550, Society for Economic Dynamics.

Chatterjee, S. and B. Eyigungor (2012): "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 102, 2674–99.

Cole, H. and P. Kehoe (1998): "Models of Sovereign Debt: Partial versus General Reputations," *International Economic Review*, 39, 55–70.

Cruces, J. J. and C. Trebesch (2013): "Sovereign Defaults: The Price of Haircuts," *American Economic Journal: Macroeconomics*, 5, 85–117.

De Paoli, B., G. Hoggarth, and V. Saporta (2009): "Output costs of sovereign crises: some empirical estimates," Bank of England working papers 362, Bank of England.

D'Erasmo, P., E. Mendoza, and J. Zhang (2016): "Quantitative Models of Sovereign Debt Crises," *Handbook of Macroeconomics*, 2B, 2493–2597, chapter 32.

Dvorkin, M., J. M. Sánchez, H. Sapriza, and E. Yurdagul (2021): "Sovereign Debt Restructurings," *American Economic Journal: Macroeconomics*, 13, 26–77.

Eaton, J. and M. Gersovitz (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *The Review of Economic Studies*, 48, 289–309.

Farah-Yacoub, J. P., C. M. H. Graf Von Luckner, R. Ramalho, and C. M. Reinhart (2022): "The Social Costs of Sovereign Default," Policy Research Working Paper Series 10157, The World Bank.

Fourakis, S. (2023): "Sovereign Default and Government Reputation," Mimeo.

García-Cicco, J., R. Pancrazi, and M. Uribe (2010): "Real Business Cycles in Emerging Countries?" *American Economic Review*, 100, 2510–31.

Gordon, G. and P. Guerrón-Quintana (2019): "A Quantitative Theory of Hard and Soft Sovereign Defaults," 2019 Meeting Papers 412, Society for Economic Dynamics.

Hansen, L. P. and T. J. Sargent (2001): "Robust Control and Model Uncertainty," *American Economic Review*, 91, 60–66.

——— (2015): "Four types of ignorance," *Journal of Monetary Economics*, 69, 97–113.

Hatchondo, J. C. and L. Martinez (2009): "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 79, 117–125.

Hatchondo, J. C., L. Martinez, and Y. K. Onder (2017): "Non-defaultable debt and sovereign risk," *Journal of International Economics*, 105, 217–229.

IMF (2013): "Sovereign Debt Restructuring - Recent Developments and Implications for the Fund's Legal and Policy Framework," available at: https://www.imf.org/-/media/Websites/IMF/imported-full-text-pdf/external/np/pp/eng/2013/_042613.ashx, IMF Policy Papers.

——— (2020): "The International Architecture for Resolving Sovereign Debt Involving Private-Sector Creditors—Recent Developments, Challenges, And Reform Options," available at: https://www.imf.org/-/media/Files/Publications/PP/2020/English/PPEA2020043.ashx, IMF Policy Paper No. 2020/043.

——— (2021): "Issues in Restructuring of Sovereign Domestic Debt," available at: https://www.imf.org/-/media/Files/Publications/PP/2021/English/PPEA2021071.ashx, IMF Policy Paper No. 2021/071.

Karantounias, A. G. (2023): "Doubts about the model and optimal policy," *Journal of Economic Theory*, 210, 105643.

Leland, H. E. (1998): "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance*, 53, 1213–1243.

Levy Yeyati, E. and U. Panizza (2011): "The elusive costs of sovereign defaults," *Journal of Development Economics*, 94, 95–105.

Martinez, L., F. Roch, F. Roldán, and J. Zettelmeyer (2023): "Sovereign Debt," in *The Research Handbook of Financial Markets*, ed. by R. S. Gürkaynak and J. H. Wright, Edward Elgar Publishing, chap. 17, 378–405.

Meyer, J., C. M. Reinhart, and C. Trebesch (2022): "Sovereign Bonds since Waterloo," *The Quarterly Journal of Economics*, 137.

Mihalache, G. (2020): "Sovereign default resolution through maturity extension," *Journal of International Economics*, 125C, 103326.

Pérez, D. (2018): "Sovereign Debt, Domestic Banks and the Provision of Public Liquidity," mimeo, NYU.

Pouzo, D. and I. Presno (2016): "Sovereign Default Risk and Uncertainty Premia," *American Economic Journal: Macroeconomics*, 8, 230–66.

Reinhart, C., K. Rogoff, and M. A. Savastano (2003): "Debt Intolerance," *Brookings Papers on Economic Activity*, 34, 1–74.

Reinhart, C. M. and K. S. Rogoff (2011): "The Forgotten History of Domestic Debt," *Economic Journal*, 121, 319–350.

Roch, F. and F. Roldán (2023): "Uncertainty Premia, Sovereign Default Risk, and State-Contingent Debt," *Journal of Political Economy Macroeconomics*, 1, 334–370.

Rogoff, K. (2022): "Issues in the theory of sovereign debt and post-covid workouts," *Journal of Policy Modeling*, 44, 804–811.

Roldán, F. and C. Sosa-Padilla (2023): "Central Bank Swap Lines as Bilateral Sovereign Debt," Mimeo.

Strzalecki, T. (2011): "Axiomatic Foundations of Multiplier Preferences," *Econometrica*, 79, 47–73.

Trebesch, C. and M. Zabel (2017): "The output costs of hard and soft sovereign default," *European Economic Review*, 92, 416–432.

A. Output around restructuring events

Figure 10 presents the same output deviations from Figure 1 but in calendar time, and also including those restructuring events ocurring before 1990, which are associated with higher volatility in outcomes.

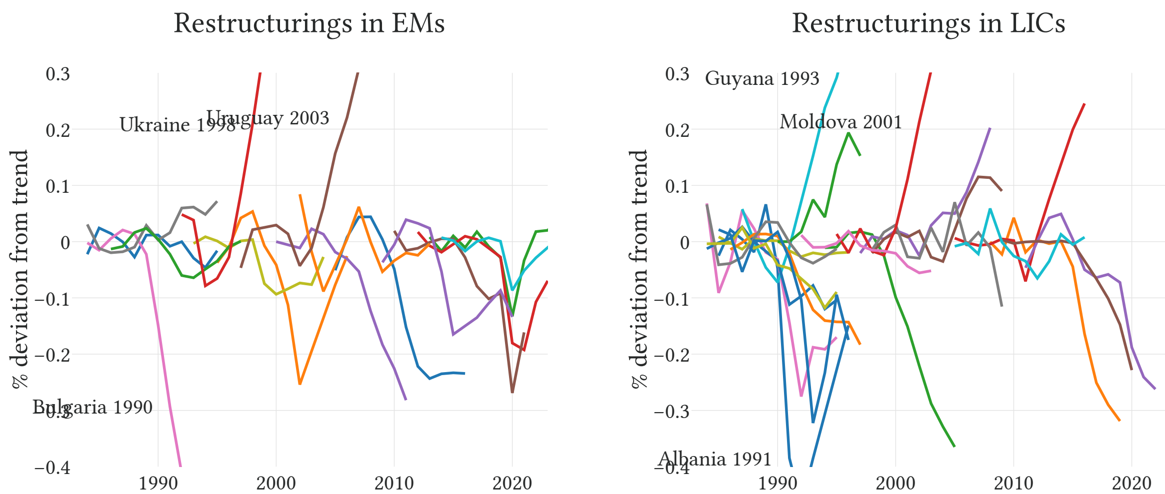

Figure 10: Growth outcomes around debt restructurings

Note: Outlier trajectories censored and marked

The events in our sample for EMs (or Market-Access Countries) are Argentina 2001, Argentina 2019, Barbados 2018, Belize 2006, Belize 2016, Ecuador 1999, Ecuador 2020, Greece 2011, Seychelles 2008, South Africa 1992, Ukraine 1998, Ukraine 2015, and Uruguay 2003. Those classified as LICs are Albania 1991, Chad 2014, Côte d'Ivoire 2000, Côte d'Ivoire 2011, Dominica 2003, Dominican Republic 2004, Grenada 2004, Guinea 1991, Guyana 1993, Kenya 1992, Mauritania 1992, Moldova 2001, Mongolia 2017, Mozambique 2015, Pakistan 1998, St. Kitts and Nevis 2011, and Togo 1991.

B. Normalization details

We normalize all variables by Γ_t, denote normalized values with lowercase, and notice that $y_t = \exp(z_t)$ and

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma}\frac{(C/\Gamma)^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma}u(c)$$

so in a typical Bellman equation we can guess and verify (denoting $x = B'/\Gamma$) forms like $V\Gamma^{\gamma-1} = v$

$$\begin{aligned}
V(B, z, \Gamma) &= \max_{B'} u(C) + \beta \mathbb{E}\left[V(B', z', \Gamma')\right] \\
V(B, z, \Gamma) &= \max_{B'} \Gamma^{1-\gamma} u(c) + \beta \mathbb{E}\left[V(B', z', \Gamma')\right] \\
\Gamma^{\gamma-1} V(B, z, \Gamma) &= \max_{B'} u(c) + \beta \mathbb{E}\left[(\Gamma')^{\gamma-1} \left(\Gamma/\Gamma'\right)^{\gamma-1} V(B', z', \Gamma')\right] \\
v(b, z, g) &= \max_{x} u(c) + \beta \mathbb{E}\left[(g')^{1-\gamma} v(b'(x, g'), z', g')\right] \\
v(b, z, g) &= \max_{x} u(c) + \beta \mathbb{E}\left[(g')^{1-\gamma} v(b'(x, g'), z', g')\right]
\end{aligned}$$

while for the budget constraint we have

$$\begin{aligned}
C + \kappa B &= Y + q(B' - (1-\rho)B) \\
c + \kappa b &= y + q(B'/\Gamma - (1-\rho)b) \\
c + \kappa b &= y + q(b'(\Gamma'/\Gamma) - (1-\rho)b) \\
c + \kappa b &= y + q(b'g' - (1-\rho)b)
\end{aligned}$$

This budget constraint makes it clear that $x = b'g'$ (simply substituting $x = B'/\Gamma$ in line 2) or $b'(x, g') = x/g'$.

C. Robustness

For a technical exposition of robustness, see Hansen and Sargent (2001). Less formally, consider a decision maker facing a state (vector) x with a (trusted) law of motion $\mu(x' \mid x, c)$ and a Bellman equation of the form

$$v(x) = \max_{c} u(c) + \beta \int v(x')\mu(x' \mid x, c) dx'$$

To include doubts about the law of motion μ and guard against misspecification, the decision maker seeks an action that would perform reasonably well if the true law of motion was instead *any* μ'. The alternative is left unidentified but could be any probability distribution that is difficult to distinguish statistically from μ. To obtain an action that performs well under all possible μ', the decision maker enlists the help of an auxiliary evil agent who chooses a probability distortion m that minimizes the utility given the action c chosen. By behaving in this pessimistic way (each action is evaluated according to the distortion that yields the lowest utility), the decision maker obtains lower bounds for each action.

Finally, in order to only allow distortions that are difficult to identify from data, the evil agent faces a cost for the relative entropy of the original model μ and the distorted one, the so-called worst-case model $m\mu$. The inverse marginal cost of relative entropy, θ, determines how robust the resulting decision will be. The decision maker takes into account the reaction of the evil agent and solves

$$v(x) = \max_c \min_{m\geq 0} u(c) + \beta \int v(x')m(x')\mu(x' \mid x, c)dx' + \frac{1}{\theta}\text{ent}(m\mu, \mu)$$
$$\text{subject to} \quad \int m(x')\mu(x' \mid x, c)dx' = 1$$

The solution to the minimization problem yields a worst-case model $\hat{m}(x') \propto \exp(-\theta v(x'))$. As Hansen and Sargent (2001) show, plugging in the worst-case model yields the robust Bellman equation

$$v(x) = \max_c u(c) + \frac{1}{-\theta}\beta \log\left(\int \exp\left(-\theta v(x')\right)\mu(x' \mid x, c)dx'\right)$$

D. Solution algorithm

We focus on Markov Perfect equilibria which we find as the limit of a finite-horizon version of the model to handle possible multiplicity.

We start at time T from a policy of always repaying the debt before the world ends, meaning that $v(b, z) = v_R(b, z)$ is given by the utility of consuming the endowment net of coupon payments $y(z) - \kappa b$. Because the world ends, at T the price of debt is $q(x, z) = 0$ in all states.

We iterate backward in time. At each period, we first use the default probabilities $\mathcal{P}$ and issuance x' policies, along with the current guess of the price q, to compute the new debt price from (13). Then with new debt prices in hand and the current guess of the value function v we move to updating v_R and x' from (14). Using the guesses for v, v_D^T and v_D^P, we can also update the values of default v_D^T and v_D^P from (16) and subsequently v_D from (15). Finally, with updated values for default and repayment, we update the beginning-of-period value function v as well as the default probability $\mathcal{P}$ from (9).

The preference shocks, as represented by the closed forms (9), produce enough smoothing in the value function to allow this algorithm to converge, while avoiding the problems highlighted by Chatterjee and Eyigungor (2012). Our formulation with preference shocks produces a similar smoothing as the two-tiered income process introduced by Chatterjee and Eyigungor (2012) and commonly used in discrete state space solutions of Eaton-Gersovitz models, but the closed-form solutions for $\mathcal{P}$ and v avoid the computational complication introduced by the inclusion of the extra state required by a two-tiered income process.

E. Estimation Details

We estimate the endowment process parameters $(\rho_z, \sigma_z, \sigma_g)$ as described in Section 4.2 for each country in our sample (sample selection described in Section 4.1). The parameter μ refers to a drift in the random walk process which we clean out from the data. Table 6 below reports all coefficients, cast in quarterly terms.

Table 6: Country-by-country estimation

Country	ρ_z	σ_z	μ	σ_g
Greece	0.96	0.017	0.0077	0.055
South Africa	0.94	0.012	0.0079	0.017
Argentina	0.89	0.035	0.0048	0.014
Chile	0.91	0.031	0.0061	0.0036
Ecuador	0.94	0.022	0.01	0.025
Uruguay	0.95	0.021	0.005	0.0026
Barbados	0.96	0.015	0.0054	0.064
Belize	0.96	0.024	0.014	0.032
Algeria	0.91	0.026	0.0099	0.0067
Seychelles	0.92	0.025	0.01	0.014
Bulgaria	1.0	0.0058	0.016	0.019
Ukraine	0.72	0.043	-0.036	5.4e-5
Average	0.93	0.023	0.0051	0.021

F. More model results

Table 7: Calibration without robustness

	Parameter	**Value**
Sovereign's discount factor	β	0.903
Default cost	Δ	0.039
Probability of transitory shock	p	0.115
Robustness parameter: default costs	θ_c	0

Table 8: Calibration without uncertainty

	Parameter	Value
Sovereign's discount factor	β	0.901
Default cost	Δ	0.0515
Probability of transitory shock	p	1
Robustness parameter: default costs	θ_c	0