Reputation and the Credibility of Inflation Plans[*]

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Abstract

We study the optimal design of inflation targets by a planner who lacks commitment and exerts imperfect control over inflation. The government's reputation for being committed evolves as the public compares realized inflation to the targets. Reputation is valuable as it helps curb inflation expectations. However, plans that are more tempting to break lead to faster reputational losses in the ensuing equilibrium. The government announces a plan which balances low inflation promises with incentives to enhance credibility. We find that, despite the absence of private sources of inflation inertia, a gradual disinflation is preferred even in the zero-reputation limit.

JEL Classification E52, C73

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Introduction

Macroeconomic models give expectations about future policy a large role in the determination of current outcomes. Policy is then generally set under one of two assumptions: commitment to future actions or discretion. Attempts to model policy departing from these extreme cases have found limited success.

However, governments actively attempt to influence beliefs about future policy. Examples include forward guidance and inflation targets but also fiscal rules and the timing of introduction of policies. Such promises rarely constrain future choices, yet they can shift expectations substantially. Standard macroeconomic models cannot capture this idea directly, as expectations of the public are fully determined by the policy chosen with commitment, or with discretion as part of an equilibrium. In both cases the public understands that announcements do not bind the government in any way. In other words, announcements do not grant any additional credibility to the policy maker, as the public is convinced of her course of action.

In this paper we develop a rational-expectations theory of government credibility and apply it to the question of optimal policy announcements. Our notion of credibility is based on the concept of reputation in game theory (Kreps and Wilson, 1982; Milgrom and Roberts, 1982). In our model the government (or central bank, or planner) could be rational and strategic, or one of many possible behavioral types described by a policy that they stubbornly follow. The public is uninformed about the government's type and makes statistical inference about it after observing the government's announcements and actions. This inference is central to our analysis because it turns out to be in the best interest of the rational type to pretend to be one of the behavioral types.

We consider a stylized environment. In the initial period, the government makes an announcement of its policy targets and is then free to choose policy. However, the private sector knows that if the government is behavioral it announced exactly what it will implement. As a consequence, the rational type has an ex-post incentive to stay close to any announced targets, which might earn it a reputation for being committed to them. The incentive exists at any positive level of reputation, though its strength depends on the reputation level, as well on the announced sequence of targets. In anticipation of these interactions, the rational type chooses carefully which targets to announce. Our main question concerns the optimal policy announcement in the presence of these reputational concerns.

We set our model of reputation in a modern version of the classic environment of Barro (1986) and Backus and Driffill (1985), where a central bank sets inflation subject to an expectations-augmented Phillips curve. The monetary authority dislikes inflation but constantly faces an opportunity to engineer surprise inflation, which would deliver output closer to potential. We model these features through the standard, cashless-limit New Keynesian setup for the private economy. To focus on incentives and reputation dynamics, we abstract from an IS curve and let the government control inflation directly.

A natural definition of the government's reputation is the private sector's belief that the government is indeed the behavioral type whose plan was announced. We refer to the total, ex-ante probability of the behavioral types as the government's initial reputation. The credibility of a plan instead measures the proximity of expected inflation to the targets. While credibility generally increases with reputation, the insights of the reputation literature imply that our notion of credibility need not vanish as reputation approaches zero.

A key assumption we introduce is that the government exerts imperfect control over inflation, perhaps due to underlying shocks to money demand. Imperfect control masks the government's choice of policy: the private sector understands that realized inflation is only an imperfect signal of intended inflation. We consider additive and normally distributed noise which implies that the public can never be fully certain of the government's action. This assumption distinguishes us in technical terms from the early studies of reputation in monetary policy referenced above, where the public perfectly observes the inflation chosen by the government. But, crucially, imperfect control also creates a smooth tradeoff for the government: overshooting the target by more creates, in expectation, a larger boom accompanied by larger reputational losses.

When designing policy, the planner takes into account its own future behavior, which it can influence but not control. 'Future' governments have complete freedom and will only respect promises made at time 0 to the extent that it suits them. Preserving reputation turns out to be a powerful disciplining force for the planner's future self. Crucially, the value of reputation depends on the entire plan in place. Plans differ in the outcomes they intend to deliver and in how closely they are expected to be followed in the future, i.e. their credibility. Both features contribute to current outcomes through the private sector's expectations. These forces lead the planner to weigh a plan's intended outcomes against the reputation dynamics it generates.

Our main result is that the government announces a policy under which inflation starts high and diminishes gradually. Plans with gradual disinflation are more credible: having a higher target for today than tomorrow boosts the gains from sticking to the plan. This slows down the pace of reputational losses sufficiently to offset the negative effect of higher announcements on expected inflation. In an extension, we show that the planner also benefits from gradual feedback rules: promises to shift subsequent targets after one has been missed. This modification effectively increases the plan's gradualism and, consequently, its credibility. This second source of gradualism affects equilibrium plans beyond the early stages of the game and constitutes a clear lesson for policy from this model.

The gradualism of our optimal policy might lead an outside observer to conclude that there is substantial inflation inertia in the economy and that the government avoids a costly recession when bringing inflation down. However, in our model past inflation does not enter the Phillips curve. Instead, gradual disinflation is a result of the dynamic incentives of the government.

A second result concerns the limit as initial reputation becomes arbitrarily small. At zero initial reputation, the only Markov equilibrium is a repetition of the static Nash equilibrium with high inflation and output at the natural level. However, as is usual in the reputation literature, even a small amount of reputation creates a large departure in behavior from the Nash equilibrium. In particular, we show that the gradualist nature of optimal announcements and the corresponding credibility dynamics are preserved at arbitrarily low levels of initial reputation. The limiting announcement, which can be interpreted as the announcement in a fully rational model where the government has mild credibility concerns, also exhibits gradualism.

Discussion of the Literature. We contribute to a long literature dealing with issues of commitment, imperfect credibility, and reputation. The time-inconsistency of optimal policy (Kydland and Prescott, 1977) has long been recognized by researchers, who have set out to ask whether reputation can be a substitute for commitment.

Barro (1986) and Backus and Driffill (1985) were the first studies of monetary policy to introduce reputation via behavioral types committed to a certain policy. These and many subsequent studies (Cukierman and Liviatan, 1991; Sleet and Yeltekin, 2007; King et al., 2008; Dovis and Kirpalani, 2019) assume the government has perfect control of inflation. Thus, any deviations are detected by the private sector and fully destroy the government's reputation. In contrast, our assumption of imperfect control enables distinct tradeoffs that shape the gradualism of optimal plans. Moreover, the reputation literature typically considers the limit as the long-lived player becomes arbitrarily patient (Fudenberg and Levine, 1989), while we use a fixed discount factor for the planner.

Another line of research studies monetary policy with imperfect control by considering uncertainty about the preferences of the planner which is distinct from reputation (Cukierman and Meltzer, 1986; Faust and Svensson, 2001; Phelan, 2006). We view reputation as more directly suited to address optimal announcements, which was not the goal of the above papers.

Most closely related is the work of Lu, King, and Pastén (2016) and King and Lu (2020) who consider reputational models with imperfect control. However, their optimizing type has commitment power and the type that lacks commitment follows a fixed rule in Lu, King, and Pastén (2016), and behaves myopically in King and Lu (2020). This reversal of roles changes the underlying tradeoffs. In these papers the planner announces (and commits to) a plan that promotes separation from the alternative type. In our model, the planner directly chooses the behavioral type at the announcement stage and mimics its policy to convince the public that it is committed to it. This makes the model a natural setting for studying whether reputation-building incentives can substitute for commitment, as well as the credibility of different plans. In addition, Lu, King, and Pastén (2016) and King and Lu (2020) obtain the Ramsey plan in the limit as the planner becomes known to be the optimizing type, whereas the corresponding limiting plan in our model resembles neither commitment nor discretion.

An alternative view of reputation is given by the notion of sustainable plans (Chari and Kehoe, 1990; Phelan and Stacchetti, 2001). This literature considers subgame perfect equilibria in games between the government and the private sector applying the tools of Abreu, Pearce, and Stacchetti (1990). This typically generates a large set of equilibria. In fact, reputational models are often used to refine the equilibrium set. Faingold and Sannikov (2011) study a general model of reputation in continuous time which maps to our framework of monetary policy with imperfect control. They find conditions for a unique equilibrium which is Markovian in reputation, providing justification for our focus on Markov equilibria. Their model restricts considers behavioral types with static behavior, so it cannot address the dynamic announcements we are interested in.

Even though the announcements in our model do not constrain the actions of the rational government, they are not cheap talk, as they can be sent by only one of the behavioral types. This distinguishes us from cheap talk models of monetary policy such as Stein (1989) and Turdaliev (2010).

Finally, the gradualism featured by our equilibrium plans is reminiscent of the allocations arising from organizational equilibria described by Bassetto et al. (2018). Over time these allocations move further from the discretion outcome and closer to the commitment outcome without reaching the latter; similarly, our equilibrium plans transition away from the static Nash outcome and converge to a long-run rate of inflation above the first-best rate of 0. Organizational equilibria are based on equilibrium refinements from the renegotiation-proofness literature (Bernheim and Ray, 1989; Farrell and Maskin, 1989; Kocherlakota, 1996). Our work suggests that these dynamics can be generated endogenously by modelling reputational concerns directly.

Layout. The rest of the paper is structured as follows. Section 2 introduces our model of reputation. Notions of equilibrium are defined and discussed in Section ??. Section 6 lays out our main results and Section 7 discusses how optimal plans depend on parameters. Section 8 dissects our main result by studying an extension which highlights the role played by incentives and provides more concrete policy implications. Finally, Section 9 concludes.

2. Model

We set our model of credibility in the standard New Keynesian setup, where the presence of a Phillips curve creates time-inconsistency in the choice of inflation. We consider a two-stage setting in which inflation targets are announced initially but the government chooses policy sequentially. We describe outcomes with an announcement in place before moving on to evaluating different types of announcements in terms of the outcomes they induce, their credibility, and their welfare implications.

2.1 *Setting*

A policy *announcement* $\vec{a} = (a_t)_{t=0}^{\infty}$ is made in the first stage. Let $\mathcal{A}$ denote the set of possible announcements. Unless otherwise specified we consider $\mathcal{A} = \mathbb{R}^{\infty}$.

In the second (post-announcement) stage a policy maker and the private sector play an infinitely repeated game in discrete time $t = 0, 1, \ldots$. The policy maker can be one of two types. The rational type is free to choose any policy $g_t \in \mathbb{R}$ at time t. The behavioral type is committed to following the announcement $\vec{a}$, so it sets $g_t = a_t$. Let p_0 denote the initial probability of the behavioral type, whose possible origins we discuss below.

The policy maker's choice, or action, g_t influences inflation π_t as follows:

$$\pi_t = g_t + \sigma \epsilon_t, \tag{1}$$

where $\epsilon_t \overset{iid}{\sim} \mathcal{N}(0, 1)$ is a control shock and $\sigma \geq 0$. The policy maker has perfect control of inflation if $\sigma = 0$ and imperfect control if $\sigma > 0$. Let f_ϵ denote the density of ϵ_t.

The private sector forms expectations e_t of next-period inflation π_{t+1} conditional on observed inflation $\pi_0, .., \pi_t$ up to, and including, time t. Output y_t is determined implicitly by a Phillips curve[1]

$$\pi_t = \kappa y_t + \beta e_t, \tag{2}$$

where $\kappa \geq 0$ is the slope of the Phillips curve, and $\beta \in (0, 1)$ is the discount factor of the economy.

The policy maker dislikes inflation as well as deviations of output from a target $y^\star > 0$. The resulting loss in period t is

$$(y_t - y^\star)^2 + \gamma \pi_t^2,$$

where $\gamma \geq 0$ is the relative weight on inflation. It will be convenient to express the loss as a function of π_t and e_t as follows using the Phillips curve (2):

$$l(\pi_t, e_t) = \left(\frac{\pi_t - \beta e_t}{\kappa} - y^\star \right)^2 + \gamma \pi_t^2.$$

2.2 *Post-announcement equilibrium*

A time-t history h^t is a sequence of realized inflation $(\pi_0, \ldots, \pi_{t-1})$ up to time t. Let $H^t = \mathbb{R}^t$ denote the set of time-t histories with $t > 0$, and $H^0 = \{\emptyset\}$ denote the singleton set containing the initial history. Let $H = \cup_{t=0}^{\infty} H^t$ be the set of all histories.

A strategy $e : H \setminus H^0 \to \mathbb{R}$ for the private sector assigns expectations of time-$t+1$ inflation to each history $(h^t, \pi_t) \in H^{t+1}$ consisting of a history $h^t \in H^t$ of inflation up to time t and inflation π_t observed at time t. A strategy $g : H \to \Delta\mathbb{R}$ for the rational type assigns a (potentially random) policy to each history, where $\Delta\mathbb{R}$ denotes the set of probability distributions over $\mathbb{R}$ with finite support. The strategy of the behavioral type is described by the announcement $\vec{a} = (a_t)_{t=0}^{\infty}$.

We now outline the the equilibrium conditions for the strategies of the policy maker and the private sector. A strategy g is optimal given expectations e if it minimizes expected discounted losses from any history $h^t \in H^t$ given the private sector's expectations, i.e.

$$g \in \underset{g'}{\operatorname{argmin}} \, \mathbb{E}_{h^t}^{g'} \sum_{s=t}^{\infty} \beta^t l\Big(\pi_s, e\big(h^t, \pi_t, .., \pi_s\big)\Big),$$

[1]Because π_t may contain information useful for the private sector's forecasts, we have to be explicit about our timing assumptions. Here we assume that π_t is used to form e_t. We believe this formulation to be closest to the narrative of the New Keynesian model, in which the policy maker moves first by setting the nominal interest rate. Then, firms and unions observe this and update prices, if they can. Output is determined by the demand of each good at these prices.

where $\mathbb{E}_{h^t}^{g'}$ denotes expectation over inflation $(\pi_{t+s})_{s=0}^{\infty}$ from history h^t given the strategy g' and the shocks $(\epsilon_{t+s})_{s=0}^{\infty}$.

At the beginning of each period t the private sector has beliefs $p(h^t)$ representing the probability of the behavioral type as a function of the history h^t of inflation observed so far, where $p_0(\emptyset) = p_0$ is the prior belief. After observing inflation π_t, the private sector updates beliefs via Bayes' rule if possible, forming a posterior $p(h^t, \pi_t)$ as follows. If the policy maker is the behavioral type, observed inflation π_t indicates that the shock was $\epsilon_t = \pi_t - a_t$. If, instead, observed inflation resulted from a policy g_t in the support of the rational type's strategy $g(h^t)$, the shock must have been $\epsilon_t = \pi_t - g_t$. Hence, the posterior belief is given by

$$p(h^t, \pi_t) = \frac{p(h^t) \cdot f_\epsilon(\pi_t - a_t)}{p(h^t) \cdot f_\epsilon(\pi_t - a_t) + (1 - p(h^t)) \cdot \int_{g_t} f_\epsilon(\pi_t - g_t) dg(h^t)},$$

where the expectation refers to the potentially stochastic nature of g_t if the rational type is following a mixed strategy. If $\sigma = 0$, $\pi_t \neq a_t$ and π_t is outside the support of $g(h^t)$, then Bayes' rule does not apply and we set $p(h^t, \pi_t) = 0$.

We say the expectations e are rational given g and a if

$$e(h^t) = p(h^t) a_t + (1 - p(h^t)) \int_{g_t} g_t dg(h^t)$$

for any time t and history $h^t \in H^t$.

We are now ready to state our equilibrium definition.[2]

Definition. A *post-announcement equilibrium* is a strategy g for the rational type, an announcement $\vec{a} \in \mathcal{A}$, and an expectations function e such that

- g is optimal given the expectations e
- expectations e are rational given g and $\vec{a}$

2.3 *Optimal announcements*

So far we have defined an equilibrium from the post-announcement stage of the game. We now turn to the initial stage where the announcement is determined. We assume that a benevolent

[2]Definition 2.2 describes a Perfect Public Equilibrium of the game between the policy maker and the private sector. The equilibrium is public because the strategy of the policy maker depends only on observed inflation. In principle, the policy maker could also condition on choices that are unobserved by the private sector (or, equivalently, on the control shocks). This would have no effect on the equilibrium payoffs (Mailath and Samuelson book p.XXX)

planner chooses an announcement in $\mathcal{A}$ to minimize the policy maker's loss. The announcement is chosen without knowledge of the policy maker's type, so it minimizes an average of the expected discounted losses of both types given the prior p_0, shared by the planner and the private sector. To formalize this, let $L^r(g, \vec{a}, e)$ and $L^b(g, \vec{a}, e)$ be the rational and behavioral type's (expected) discounted losses in an equilibrium $(g, \vec{a}, e)$. The average expected discounted loss is

$$\tilde{L}(g, \vec{a}, e) = p_0 L^b(g, \vec{a}, e) + (1 - p_0) L^r(g, \vec{a}, e).$$

The potential multiplicity of post-announcement equilibria may create (pre-announcement) equilibria with different average losses. We are interested in the optimal equilibrium and its associated optimal announcement. We can now define an equilibrium of the game from the initial stage prior to the announcement.

Definition. A (pre-announcement) *equilibrium* is a collection of strategies $\{g^a, e^a\}_{a\in\mathcal{A}}$ and an announcement a^* such that

- (g^a, a, e^a) is a post-announcement equilibrium for all $a \in \mathcal{A}$.
- $a^* \in \operatorname*{argmin}_a \tilde{L}(g^a, a, e^a)$.

In each equilibrium, the planner understands which expectations e the private sector will hold and which policy g the rational type will follow after every announcement. The equilibrium announcement minimizes the planner's expected loss. In an optimal equilibrium, the announcement on path is followed by a payoff that is maximal among all announcements and all equilibria.

Definition. An equilibrium $\{\{g^a, e^a\}_{a\in\mathcal{A}}, a^\star\}$ is *optimal* if it minimizes $\tilde{L}(g^{a^\star}, a^\star, e^{a^\star})$ among all equilibria. An announcement $a^\star$ is optimal if there exists an optimal equilibrium $\{\{g^a, e^a\}_{a\in\mathcal{A}}, a^\star\}$.

One interpretation is that the planner is a government that appoints a central banker – the policy maker. The government and the central banker share the same preferences about output and inflation. The government can choose a central banker who advocates for a specific inflation policy (the announcement), but it does not know whether the central banker is tough, i.e. committed to follow this policy. This interpretation also maps to a model like Kambe (1999), where the planner may become committed to its announcement after it is made.

3. Ramsey plan

Suppose the policy maker is committed to follow $\vec{a} = (a_t)_{t=0}^{\infty}$, i.e. $p_0 = 1$. Then expected time-$t+1$ inflation is simply a_{t+1}, so the average discounted loss is

$$\mathcal{L}(a) := \sum_{t=0}^{\infty} \beta^t l(a_t, a_{t+1}). \tag{3}$$

The optimal announcement a^R in this setting is called the Ramsey plan. Proposition 1 below characterizes it by using the Bellman equation

$$R(a) = \min_{a' \in \mathbb{R}} l(a, a') + \beta R(a') \tag{4}$$

and its associated policy function ϕ.

Proposition 1. *There exists a unique function $R : \mathbb{R} \to \mathbb{R}_+$ satisfying (4). Its associated policy function ϕ is unique.*

The unique Ramsey plan $(a_t^R)_{t=0}^{\infty}$ is given by $a_0^R = \operatorname{argmin}_x R(x)$ and $a_{t+1}^R = \phi(a_t^R)$. It satisfies $a_t^R > a_{t+1}^R > 0$ for all t and $\lim a_t^R = 0$.

The proofs of Proposition 1 and subsequent results are in the Appendix.

The Ramsey plan is shown in Figure XXX. It starts from a positive level of inflation and decreases over time, approaching zero inflation. This stimulates output in every period at the cost of creating inflation that vanishes in the long run. Inflation costs can be minimised if inflation is zero throughout, but such a policy is suboptimal because increasing inflation at time 0 would bring output closer to the desired level y^*, while having no effect on expected inflation due to the forward-looking Phillips curve.

4. Perfect control

Suppose that $\sigma = 0$, so that there is perfect control of inflation. The following result describes optimal equilibrium behavior.

Proposition 2. *Suppose $\sigma = 0$. Then the policy maker follows the Ramsey plan with probability 1 in any optimal equilibrium. The Ramsey plan is an optimal announcement, and it is unique if $p_0 > 0$.*

Proposition 2 states that reputation is not beneficial when control over inflation is perfect. Regardless of initial reputation, it is optimal to announce the Ramsey plan.[3] On the path of the

[3] If $p_0 = 0$ the announcement is immaterial, so it need not equal the Ramsey plan.

optimal equilibrium the private sector expects that the Ramsey plan will continue to be followed and reputation never changes. Hence, any initial level of reputation results in the same optimal equilibrium outcome.

Proposition 2 is enabled by the existence of an equilibrium where the rational type, who lacks commitment, is willing to follow the announced Ramsey plan. This is driven by the timing assumption inherent in the New Keynesian model. The private sector sets expectations after observing the chosen policy, so these expectations can be used to immediately punish deviations from the Ramsey plan. When, in contrast, expectations are set before observing the policy, punishments do not take effect until the subsequent period so deviations can temporarily boost output. In these models the Ramsey outcome is not attainable without commitment unless the policy maker is sufficiently patient. Dovis and Kirpalani (2019) consider such a model and show that the optimal announcement and equilibrium behavior depend on the intial level of reputation.

Even though the Ramsey outcome is attainable, it is not obvious that it minimizes average losses. In the presence of reputation there exists a post-announcement equilibrium that is preferred by the rational type over the Ramsey outcome. For instance, consider an announcement a with $a_0 = a_0^R$ and $a_t = 0$ for all $t > 0$. It is possible to construct a post-announcement equilibrium (g, a, e) where the rational type follows the Ramsey plan. In the initial period the rational type follows the announcement $a_0 = a_0^R$, so he preserves his initial reputation $p_0 > 0$ after the private sector observes a_0. Hence, he enjoys lower expectations relative to the Ramsey outcome because $a_1 < a_1^R$. In the second period the rational type deviates from the announcement by choosing a_1^R but there exists a favorable continuation equilibrium where the private sector expects him to continue following the Ramsey plan.[4] Hence, compared to the Ramsey outcome, the rational type is better off in the initial period and just as well off in the following periods. His welfare can be improved even further by raising the announcement a_0 for the initial period.

It follows that if the planner wishes to minimize the loss of the rational type, the optimal announcement can differ from the Ramsey plan. However, any improvement to the rational type's welfare comes at the cost of decreasing the welfare of the behavioral type. Proposition 2 states that this would be detrimental to average welfare.

Proposition 2 differs from standard results in the reputation literature.

consider po->0 and beta->1 Both are necessary, in general, because higher po->higher payoff and lower beta means incentives break down here things are simpler

NOT the standard reputational result... commitment as beta 0, not a limit here

[4]See the XXXX

5. Imperfect control

Suppose that $\sigma > 0$, so that control over inflation is imperfect. This amounts to a reputational model with imperfect monitoring. These models are rarely tractable analytically outside of the patient limit. Therefore, this section will focus on numerical results. In addition, we will make a few restrictions for the purposes of computational tractability.

First, we restrict the space of announcements $\mathcal{A}$. We assume inflation targets for each period are in the interval $A = [0, \pi^N]$, where $\pi^N = y^\star \frac{\kappa}{1-\beta+\kappa^2\gamma}$ is inflation in the unique *stationary* post-announcement equilibrium of the game where the policy maker is known to be rational.[5] In addition, we assume that $\mathcal{A}$ only contains announcements $(a_t)_{t=0}^{\infty}$ parameterized by (a_0, ω, χ), where

$$a_t = \chi + e^{-\omega t}\left(a_0 - \chi\right).$$

These announcements include constant, decreasing, and increasing paths for inflation, demonstrated in Figure 1. Inflation starts from a_0 and converges towards χ with a exponential decay rate of ω. When it does not lead to confusion we identify a plan $(a_t)_{t=0}^{\infty}$ with the triple (a_0, ω, χ). This parametrization makes $\mathcal{A}$ finite-dimensional and allows us to write each plan recursively as

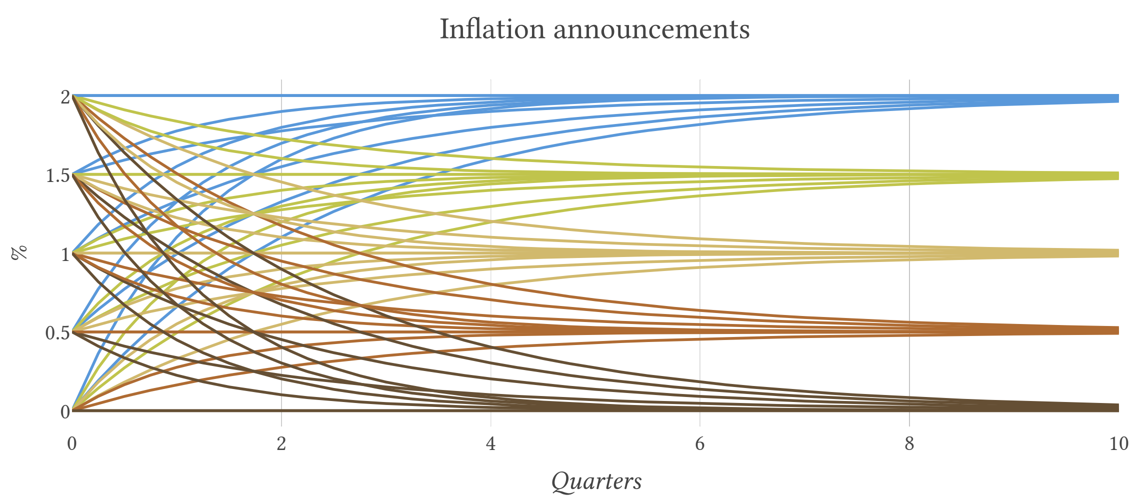


Figure 1: Possible behavioral types' announcements

$a_{t+1} = \phi(a_t)$, where

$$\phi(a) = \chi + e^{-\omega}\left(a - \chi\right).$$

[5]A stationary equilibrium requires that the same actions are played by the policy maker and the private sector in each period. When $p_0 = 0$, this equilibrium is Markovian in the state variable p representing the policy maker's reputation.

In Section XXX we consider an alternative set of announcements where targets can vary freely in a number of intial periods and become constant thereafter.

We also restrict attention to pure-strategy Markov equilibria, where the state variables are time and reputation. In the context of Definition 2.2 this means the distribution $g(h^t)$ is degenerate and depends only on time t and beliefs $p(h^t)$. Dependence on time is desirable due to the nonstationary plans we consider. The restriction to pure strategies is standard in the literature (XXX ????). While mixed strategies are necessary for equilibrium existence with perfect monitoring......

Computation. To find a continuation equilibrium, we take the infinite-horizon limit of the finite-horizon game. Given a continuation value $\mathcal{L}^c_{t+1}$ and policy $g^\star_{t+1}$, we plug them into the objective function and the Phillips curve constraint in (**??**). This creates an operator mapping values for g, representing the private sector's expectations in the Bayes' rule constraint, into optimal actions g' for the government. For each state (p, a), we find the new $g^\star_t(p, a)$ as the fixed point of this operator, and the new $\mathcal{L}^c_t(p, a)$ as the resulting minimum. Then we iterate (backward in time) until convergence.

A useful property of continuation equilibria follows from close observation of problem (**??**): given the decay and long-run target parameters, it is equivalent to start the plan at a different initial announcement a or to just have arrived at a current announcement a as the continuation equilibrium unfolded.

Observation. Suppose $(\mathcal{L}, g^\star)$ is a continuation equilibrium for announcement $c = (a_0, \omega, \chi) \in \mathcal{C}$. Then for any b_0, the same pair $(\mathcal{L}, g^\star)$ is a continuation equilibrium for plan $c' = (b_0, \omega, \chi)$.

Another immediate property of equilibrium is that the government cannot design a sequence of inflation targets that will increase its reputation over time. If the rational government follows the announcement exactly, i.e. if $g^\star(p, a) = a$, its reputation will stay unchanged because inflation is not a signal of its type. But if the equilibrium strategy calls for deviations from the plan, reputation will decline on average. In other words, rational expectations prevent the planner from accumulating reputation by consistently delivering on its promises, as such compliance would be anticipated by the private sector. What the planner can do is to design its plan in a way that provides incentives to deliver on it.

Observation. In any continuation equilibrium, the rational type's reputation is a supermartingale:

$$\mathbb{E}\left[p_{t+1} \mid \text{rational}, \mathcal{F}_t\right] \leq p_t,$$

where $\mathcal{F}_t$ denotes information up to time t. Thus, the planner cannot design an announcement that generates expected reputational gains in the ensuing continuation equilibrium.

6. Analysis and Numerical Results

We solve the model numerically for different announcements $c \in \mathcal{C}$. We parametrize our model following Lu, King, and Pastén (2016). Our preference and technology parameters $\gamma, \kappa, y^\star$ are consistent with the planner's objective function and Phillips curve in a standard New Keynesian economy calibrated to US data (Galí, 2015; Galí and Gertler, 1999). Table 1 summarizes our parameter choices. These parameters imply a level for Nash inflation of about 2% annualized.

Table 1: Benchmark calibration

Parameter	**Value**	**Definition**	**Source / Target**
β	0.995	Discount factor	2% real interest rate
γ	60	Inflation weight	Lu, King, and Pastén (2016)
σ	1%	Std of control shock	Lu, King, and Pastén (2016)
κ	0.17	Slope of Phillips curve	Lu, King, and Pastén (2016)
$y^\star$	5%	Output target	Lu, King, and Pastén (2016)

6.1 *Continuation equilibrium after announcement* c

Figure 2 shows a typical value function $\mathcal{L}^c(p, a)$ for an arbitrary plan c. All plots have current reputation p on the x-axis. Darker lines correspond to a lower current target a, which is measured relative to Nash inflation π^N. We draw three main lessons from this figure.

Firstly, $\mathcal{L}^c$ is decreasing in p. An increase in reputation generally decreases expected inflation leading to higher current output and, therefore, smaller losses.

Secondly, the loss function has a convex-concave shape reflecting the dynamics of reputation. When reputation is close to 0 or 1 the public is confident in its assessment of the government's type, and significant evidence is required to move beliefs. Conversely, near $\frac{1}{2}$, movements in reputation are fickle and can easily be reversed. Thus, the same change in reputation is more valuable the closer p is to the extremes, leading to steepness of the loss function.

Thirdly, at high levels of reputation, a lower target a is preferred. The reason for this is that, as reputation increases, beliefs place a higher weight on the behavioral type who sticks to the

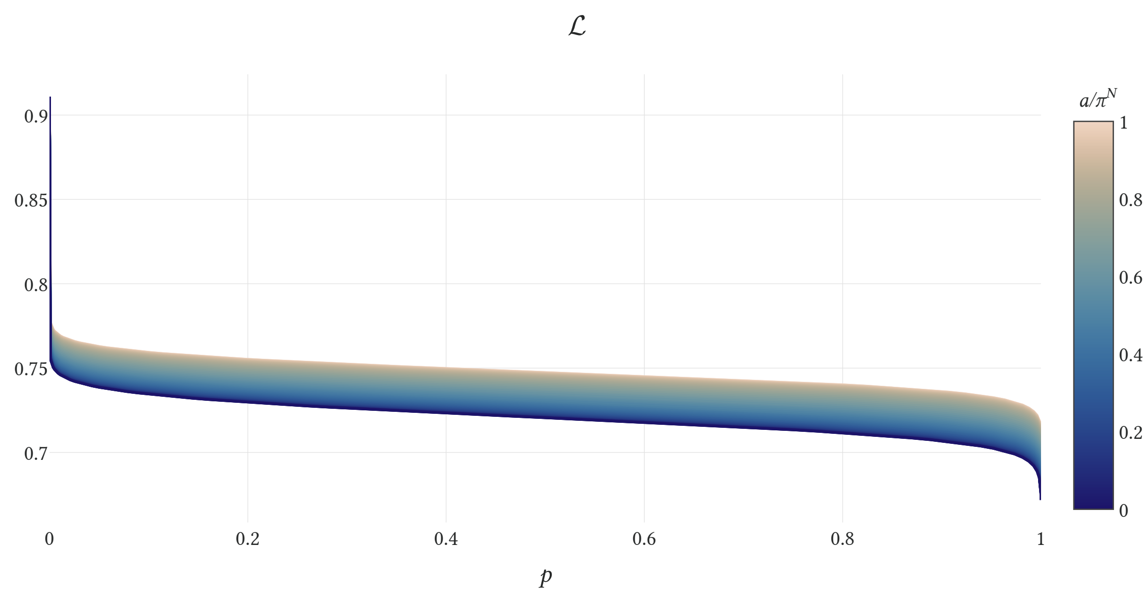


Figure 2: Loss function after announcement c

announcement. As the rational type's strategy becomes less important, the credibility tradeoff dissipates.

Finally, the range of values $\mathcal{L}$ across different targets is generally smaller at lower levels of reputation. One reason is that with lower reputation the current target becomes less relevant, as its weight in expected inflation decreases. Another, more nuanced reason is that the tradeoffs between stimulating output and sustaining reputation are more pronounced when the government is seen as less likely to be committed to the target. While lower targets are directly beneficial to inflation expectations, the government finds it more costly to deliver on low targets when reputation is low (see discussion below). Thus, the benefits of ambitious announcements with low targets can be offset more heavily at low levels of reputation.

Figure 3 shows how the current target a and reputation p affect $g^{\star}(p, a) - a$, the rational type's deviation from the current target. The deviation is generally decreasing in reputation, except when reputation becomes large and the government is able to produce surprise inflation largely undetected. There is a discontinuity at zero reputation, where the unique Markov equilibrium exhibits inflation π^N regardless of the announcement, since there are no reputational concerns. In contrast, at small but positive levels of reputation, the government benefits from staying somewhat close to the announcement.

The effect of the current target a on deviations is unambiguous: a lower target a causes the rational type to deviate farther from it. In comparison to an equilibrium with a higher target,

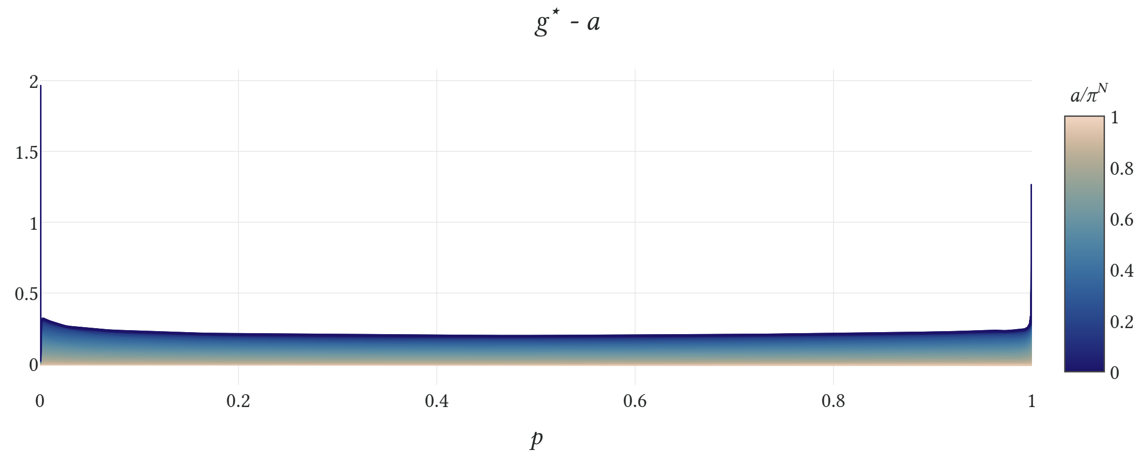


Figure 3: Inflation deviations

the government needs to inflate less to maintain the same evolution of reputation (which would yield broadly the same level of output), holding expectations of future policy fixed. Thus, there are lower costs to creating surprise inflation when the target is low.

Current reputation p affects optimal deviations through several different channels. On the one hand, a larger stock of reputation makes the planner more inclined to spend it by creating substantial surprise inflation. On the other hand, higher reputation anchors expectations more tightly and makes it less costly for the government to preserve its reputation by staying close to the target, especially when the target is high.

Figure 4 shows the average change in reputation $\mathbb{E}[p' - p]$, again as a function of the current reputation p and target a. As previously noted, the government's reputation declines on average: $\mathbb{E}\left[p'\right]$ is never above p. In particular, reputation is preserved when $p' = p$ in all contingencies, which only happens when $p \in \{0, 1\}$ (because no update is possible) or when the current target a equals Nash inflation π^N, so that incentives to create surprise inflation are eliminated. Also, as a consequence of Bayes' rule, changes in reputation are smaller when initial reputation is closer to 0 or 1. Finally, lower announcements are associated with a larger expected reputation loss. Lower, more ambitious targets generate weaker incentives: as the temptation to inflate grows larger, the government prefers to spend more of its reputation to achieve higher output.

6.2 K-*equilibrium announcements*

Figure 5 shows K-equilibrium announcements. Each announcement minimizes the government's loss function conditional on starting with reputation p_0. The top panel shows the decay rate

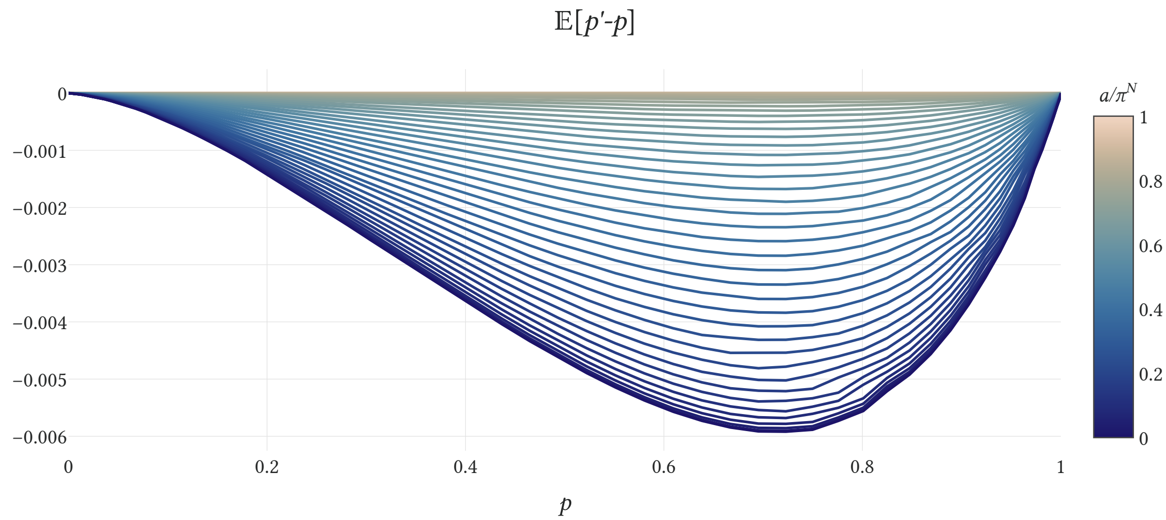


Figure 4: Expected reputation losses

$1-e^{-\omega}$ (in percent terms), while the bottom panel shows the choice of initial and long-run inflation targets a_0 and χ.

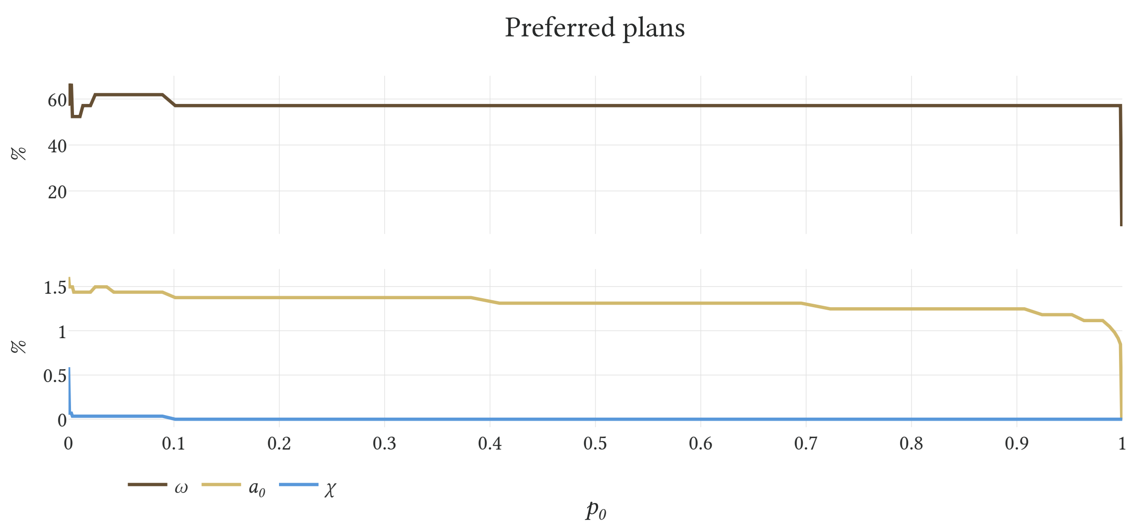


Figure 5: K-equilibrium announcements

At $p_0 = 1$, any announcement is fully believed by the private sector, regardless of expectations about the behavior of the rational type. The government sets expectations at their most advantageous level by promising zero inflation throughout. Since the private sector is convinced the government is committed to the announcement, the rational type has a clear incentive to create surprise inflation. When $p_0 < 1$, such a deviation incurs reputational losses. Thus, the government announces plans with inflation above zero, evidenced by the initial target $a_0 > 0$.

Positive inflation targets help the incentives of the government's future selves to stay closer to the announcement, conserving reputation. Since $a_0 > \chi$, the optimal inflation targets decrease over time. This structure of the optimal announcements is preserved even as initial reputation p_0 approaches zero. As p_0 becomes small, the planner raises the level of long-run inflation χ as the lack of credibility of an asymptote of 0 overwhelms its benefits.

Figure 6 shows the determination of the K-equilibrium when p_0 is small. For each initial and long-run level of inflation a_0 and χ we plot the minimized loss function $\min_\omega \mathcal{L}(p_0, (a_0, \omega, \chi))$. The minimum is achieved at a point with $a_0 > \chi > 0$: in the K-equilibrium, the initial planner promises a gradual disinflation which converges to a level above the first-best level of zero inflation.

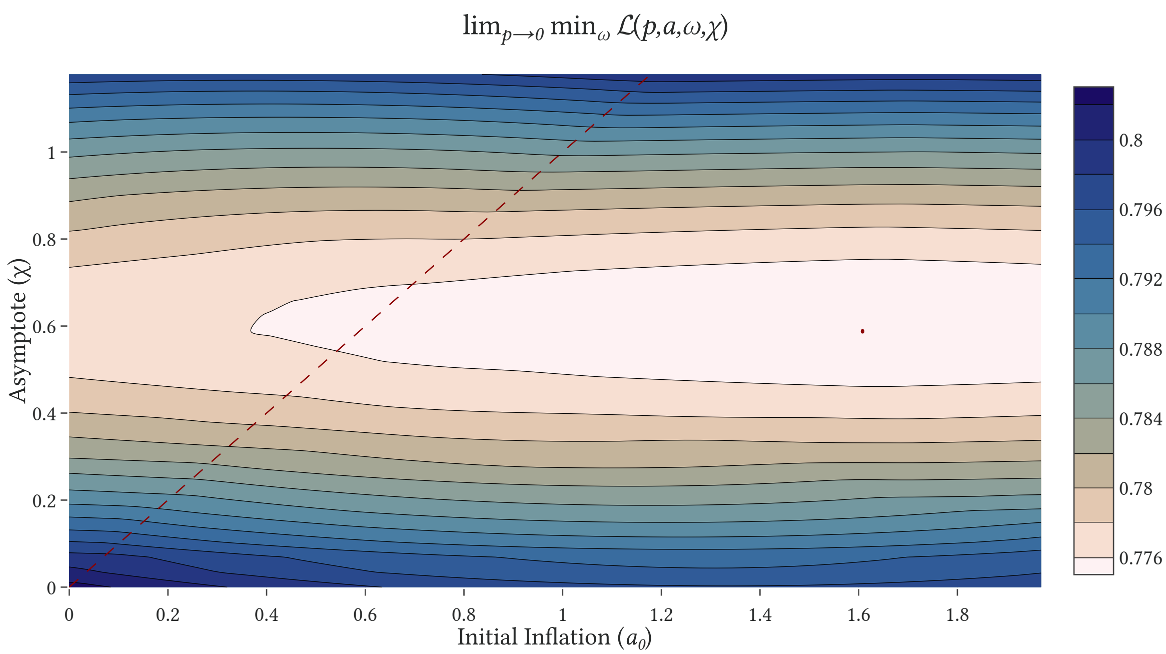


Figure 6: Loss function across announcements

When χ is small, plans eventually call for ambitiously low levels of inflation which are difficult to sustain. Consequently, reputation is quickly lost giving rise to unfavorable continuation values. High levels of long-run inflation, on the other hand, are easier to sustain but provide less benefit. For intermediate levels of long-run inflation χ, the benefit of a higher initial level $a_0 > \chi$ is visible. Such levels of initial inflation are valuable as they enhance the planner's ability to stick to a plan which eventually delivers the right level of inflation. When the long-run level is either too high or too low, the planner is less affected by the starting point.

The optimal plan involves an intermediate announcement of long-run inflation, balancing the desire for low expectations with incentives to preserve reputation. The choice of decay rate, shown in Figure 15 in the Appendix, also matters. The planner chooses a slope in its targets that is steep enough to boost incentives in the initial periods when inflation targets are above their long-run levels.

6.3 Credibility

DEFINE CREDIBILITY HERE While the government's reputation describes the likelihood it is committed to the plan, it does not reflect how closely the plan is followed on average across both types. To obtain a measure of this, we define the credibility of a plan as the ratio of announced and (expected) realized inflation, normalized by their distance from Nash inflation.

Definition. Given a plan c, its *remaining credibility* in state (p, a) is defined recursively as follows:

$$\begin{aligned} C(p, a; c) &= \mathbb{E}\left[(1-\beta)\frac{\pi^N - \pi}{\pi^N - a} + \beta C(p_c'(p, a), \phi_c(a); c)\right] \\ &= (1-\beta)\frac{\pi^N - [pa + (1-p)g_c^\star(p, a)]}{\pi^N - a} + \beta\mathbb{E}\left[C(p_c'(p, a), \phi_c(a); c)\right] \end{aligned} \tag{5}$$

where π^N is Nash inflation. The *credibility* of a plan in a K-equilibrium is given by

$$C^K(c) = \lim_{p\to 0} C(p, a_0(c); c)$$

and the credibility of a reputational equilibrium μ_z is

$$C^\star = \lim_{z\to 0}\int C(p_0(c), a_0(c); c)\, d\mu_z(c).$$

In our simulations $g_c^\star(p, a) \in [a, \pi^N]$ for all (p, a) and all plans c, so credibility lies in $[0, 1]$.

Our setup distinguishes reputation p, the posterior belief that the government is the behavioral type announced at the start of the game, from credibility $C(p, a; c)$, the expected discounted deviations from plan c at reputation p and current target a, as defined in (5). Figure 7 plots the credibility of different plans at vanishingly small reputation, as a function of initial and long-run inflation a_0 and χ, for the correspondig loss-minimizing decay rate ω.

Plans with a lower asymptote are less credible as they eventually imply too low levels of inflation and a quick loss in reputation. Moreover, especially when long-run inflation χ is low, plans with decreasing targets ($a_0 > \chi$) are more credible.

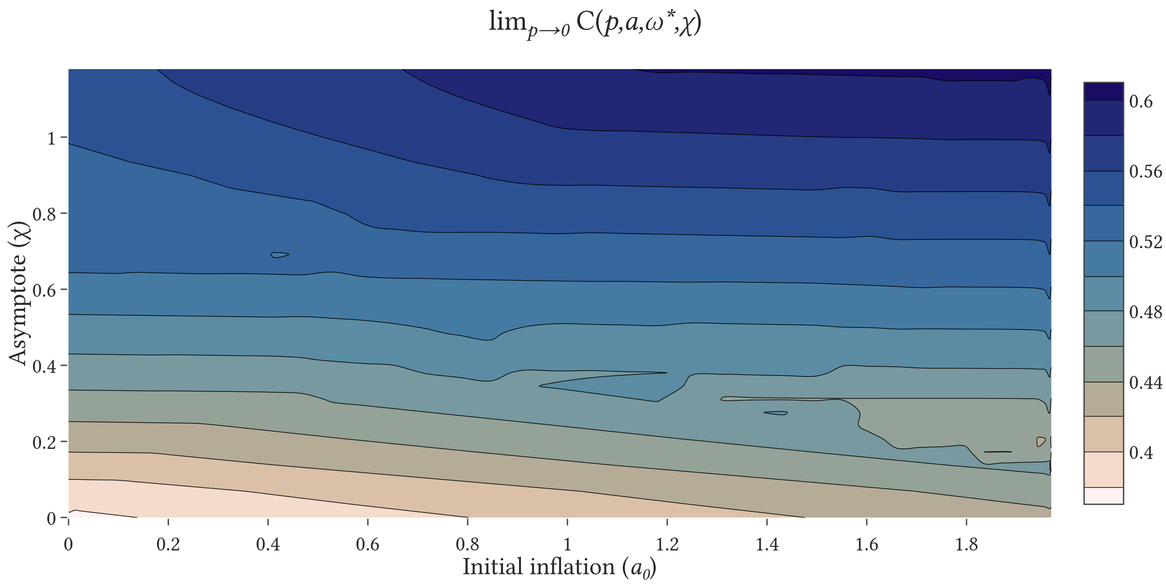


FIGURE 7: CREDIBILITY

6.4 *Distribution of announcements in the reputational equilibrium*

We now turn to a description of the reputational equilibrium and the associated distributions μ_z of announcements by the rational type. Figure 8 plots the average plan as a function of initial reputation z prior to the announcement.

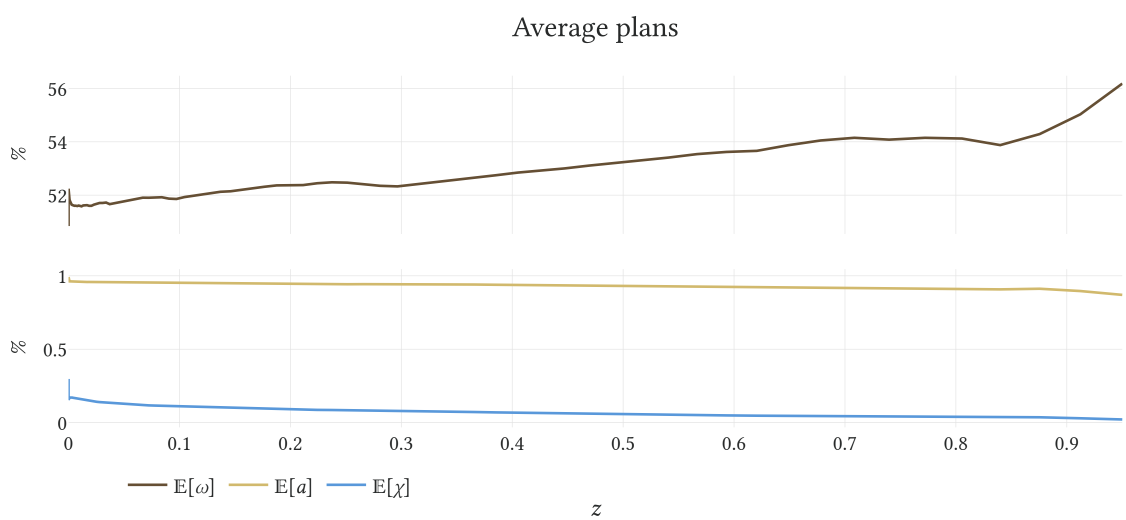
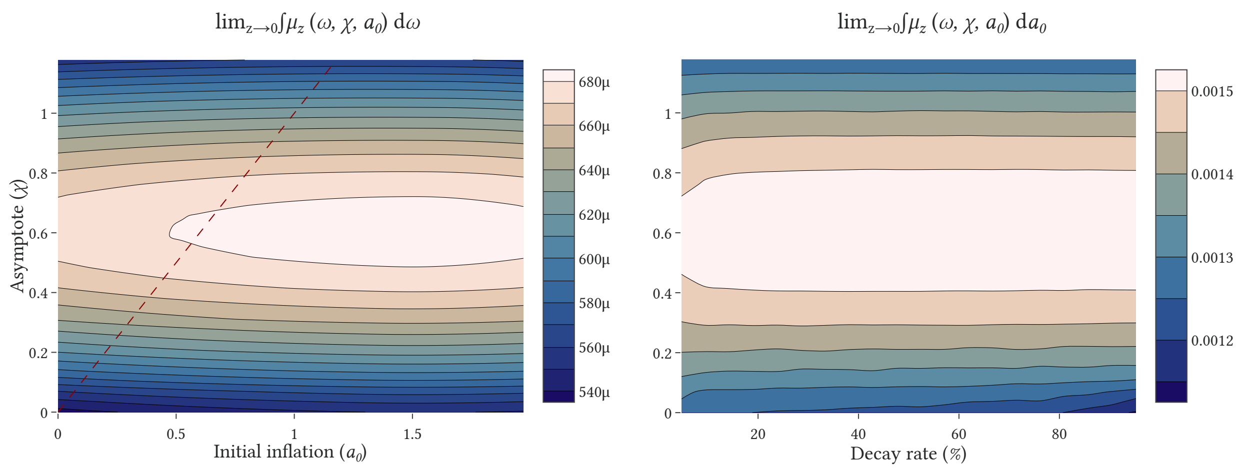


FIGURE 8: REPUTATIONAL EQUILIBRIUM ANNOUNCEMENTS

For intermediate values of initial reputation, the planner chooses (on average) a disinflation path that starts from about half of Nash inflation π^N and converges towards a tenth of it by about half the distance each period. As initial reputation becomes small, the planner starts to put more weight on plans that converge toward a higher long-run target χ, similarly to the optimal K-equilibrium announcement in Figure 5.

Figure 9 on the left shows the limiting distribution $\mu^{\star} = \lim_{z \to 0} \mu_z$ of announcements as initial reputation vanishes. The left panel shows the distribution of types as a function of the asymptote χ and initial inflation a_0, integrating over the decay rate ω, while the right panel integrates over initial inflation.



(A) (B)

FIGURE 9: DISTRIBUTION OF TYPES

Figure 9a reveals that the government tends to choose gradual plans with higher initial inflation a_0 than long-run inflation χ. This probability is $\mathbb{P}\left(a_0 > \chi\right) = 64.9\%$. Plans with initial inflation of even five times the asymptote are still announced quite often: $\mathbb{P}\left(a_0 > 5\chi\right) = 16.6\%$. While the level of initial inflation has a fairly wide distribution, the asymptote is more precisely set: the density of a_0 and χ in the reputational equilibrium announcement falls sharply for χ away from the optimum, while it stays flat over many more values of initial inflation a_0.

Figure 9a bears a close resemblance to Figure 6 which plotted the K-equilibrium loss function at low p_0. Announcements with a lower loss in the K-equilibrium are good for the planner, so they are chosen more often in the reputational equilibrium. Hence, the government starts with lower reputation in those plans, which lowers their value in the equilibrium. This initial update of reputation (from z to p_0) makes the planner indifferent across all equilibrium announcements, which ultimately justifies the mixed strategy. Similarly to the K-equilibrium, there is not much variation in the value of announcements along the decay rate dimension, evidenced by the nearly flat density $\mu^{\star}$. One exception is that very small decay rates are significantly less desirable, in contrast to the K-equilibrium.

On the right, Figure 9b shows that, given the 'right' long-run target χ, the planner chooses decay rates almost uniformly (in the background, different distributions of initial inflation a_0 adjust the choice for each decay rate).

7. Comparative Statics

Figure 10 shows the average plan announced in the reputational equilibrium as a function of the variance of the control shock σ around its baseline value of 1%. More noise in the control makes deviations from targets harder to detect. Therefore, the level of adherence to plans is decreasing in σ. This makes the planner choose less ambitious plans when the control over inflation is less tight. These plans have higher inflation throughout, as they feature a higher asymptote χ, a marginally higher initial inflation a_0, and slower decay ω.

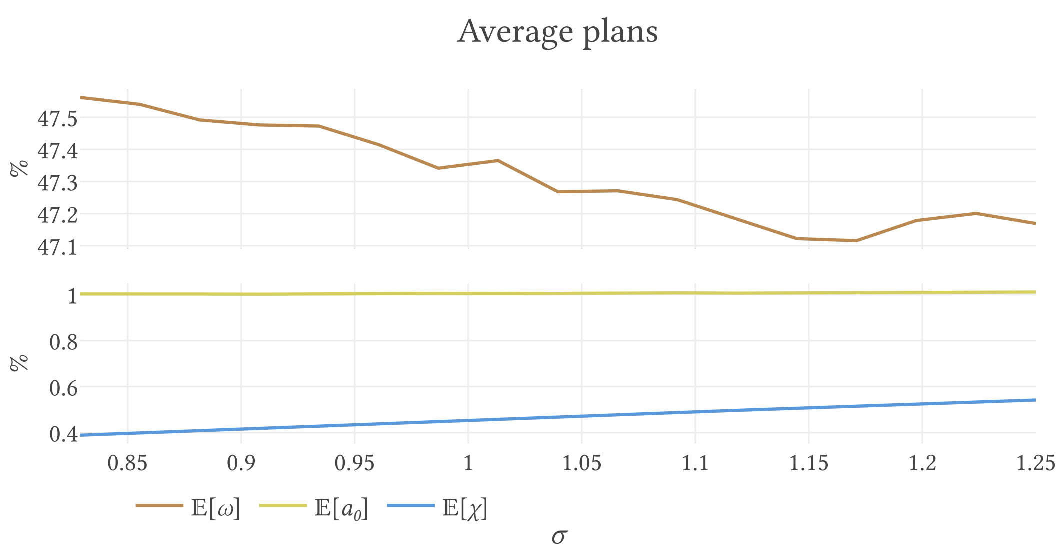
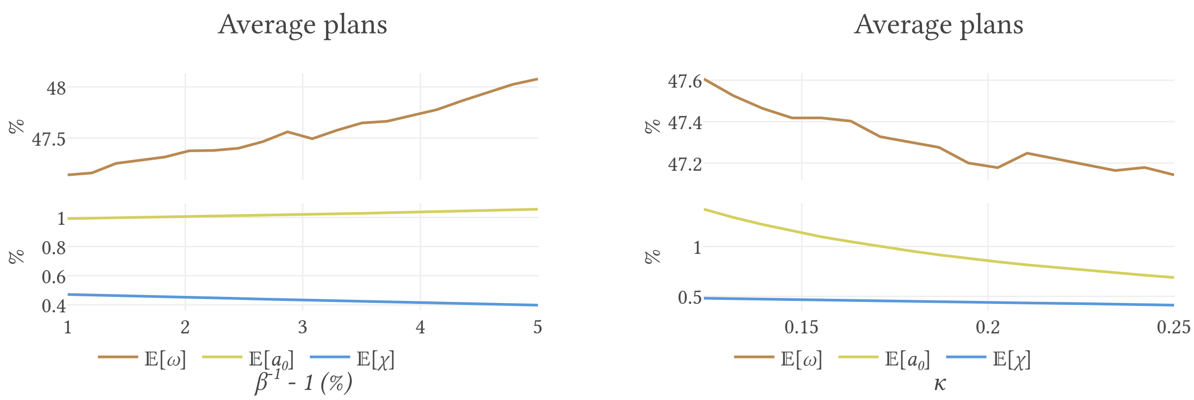


Figure 10: Average plans and the control shock variance

Figure 11 repeats the exercise varying the discount factor β and the slope of the Phillips curve κ. It reveals some subtleties in the manipulation of the three parameters that describe our plans. Figure 11a shows the average plan as a function of the discount rate $1/\beta - 1$ (whose benchmark value is 2% in annual terms). As the planner becomes more impatient, average plans start higher but converge to lower inflation, with a faster decay rate. With more impatience, the public expects a larger inflation bias. For this reason, the planner tends to choose plans that are more resilient. Increasing initial inflation makes the plan easier to keep, while decreasing asymptotic inflation makes it more costly to deviate early on. A steeper descent of inflation targets contributes to both objectives.

Figure 11b shows that when the slope of the Phillips curve is increased from its baseline value of 0.17, the planner announces lower inflation throughout. When the Phillips curve is steeper, there are weaker incentives to create surprise inflation, as it results in a smaller output boom. Thus, the planner lowers expectations through lower targets without increasing reputational losses.



(A) Varying the discount factor

(B) Varying the slope of the Phillips curve

Figure 11: Average plans

8. Inspecting the Mechanism

Our benchmark model of reputation with imperfect control yields gradual disinflation plans. We dissect this result to by comparing our model to the Ramsey plan, which yields a gradual disinflation for very different reasons.

8.1 The Ramsey plan

Figure 12 plots the Ramsey plan against the average announcement in the reputational equilibrium, as well as the announcement in the K-equilibrium.

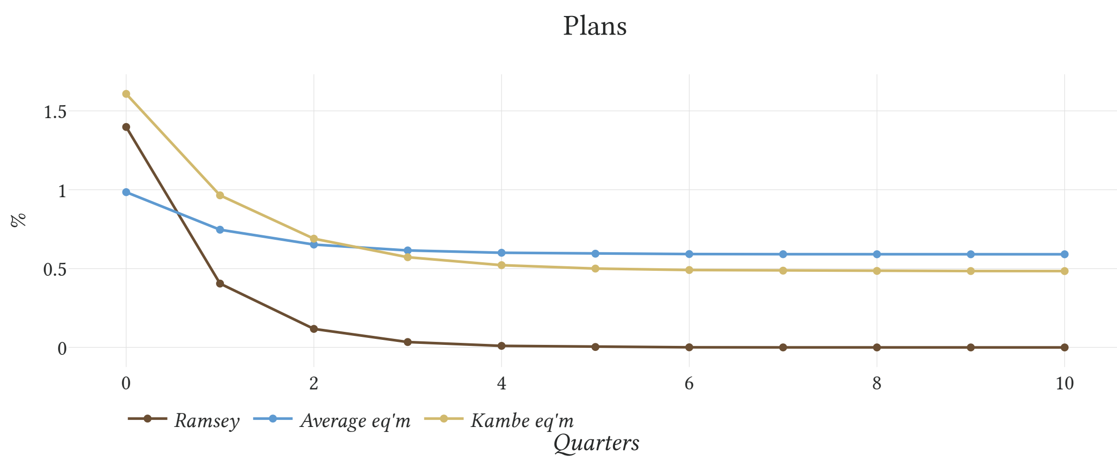


FIGURE 12: THE RAMSEY PLAN AND EQUILIBRIUM ANNOUNCEMENTS

8.2 *Reacting to shocks*

Our notions of equilibrium with reputation yield optimal plans that start from a high initial level of inflation and gradually decrease towards positive long-run values. The Ramsey plan has a similar shape. However, in this section we demonstrate that gradualism is driven by equilibrium incentives that the Ramsey planner does not face.

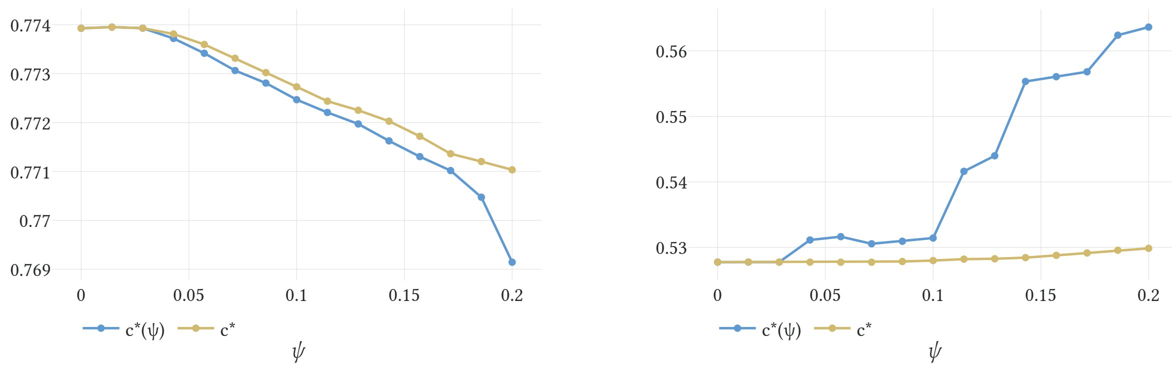
To investigate the importance of incentives, we augment the space of behavioral types to include gradual feedback rules. These types follow plans where future targets respond to deviations of inflation from the current target

$$a' = \chi + e^{-\omega}(a - \chi) + \psi(\pi - a) \tag{6}$$

so that whenever realized inflation π differs from its current target a, a share ψ of the difference is embedded in the target for the following period.

The Ramsey planner does not benefit from conditioning on deviations from the targets. Even beyond quadratic utility, the reason why the Ramsey plan starts high and converges to zero is because the benefits of high initial inflation do not come at a cost in terms of expected inflation in the past (this is also why the Ramsey planner sets $\pi_{t+1} \leq \pi_t$). With gradual feedback rules, the public's expectations incorporate the possibility that shocks will shift future targets. Gradual feedback rules which create benefits at time $t > 0$ also involve costs on the equilibrium path for all times $0 \leq j < t$, which is conceptually why the Ramsey planner does not use them.

In a continuation equilibrium, however, introducing gradual feedback is helpful. Figure 13 shows the value function $\mathcal{L}$ on the left panel along with initial credibility $\mathcal{C}$ on the right. Both are plotted as a function of the target updating parameter ψ, either reoptimizing the plan for each ψ (labeled $c^\star(\psi)$) or fixing the parameters (ω, χ, a_0) of the original K-equilibrium plan at $\psi = 0$ (labeled $c^\star$). Gradual feedback rules induce paths where inflation targets come down gradually after a high control shock ϵ. This increases the credibility of the announcement and is hence preferred by the planner.



(A) Loss function $\mathcal{L}$

(B) Credibility $\mathcal{C}$

Figure 13: Gains from gradual feedback rules

Figure 14 provides more detail into how the introduction of gradual feedback rules affects the optimal plan. As before, we show the decay rate ω on the top panel and the initial and long-run targets a_0 and χ on the bottom panel, as a function of ψ. The introduction of gradual feedback rules is a substitute for other forms of gradualism: as ψ increases, the planner chooses plans which converge more slowly from the initial to the long-run targets. There is a also a marginal shift up of the entire plan, since both a_0 and χ increase slightly with ψ.

9. Concluding Remarks

This paper addresses an old question: can reputation be a substitute for commitment? We find that a simple model of reputation combined with imperfect control on the part of the government creates incentives for staying close to announced targets. The optimal policy after a plan has been announced trades off the benefits of surprise inflation against the possibility that a deviation becomes known to the public. In this way, the government's reputation becomes an important state variable in the problem of optimal policy under discretion.

Various characteristics of announced plans come to bear when determining the value of reputation. We find that a pervasive feature of optimal plans is gradualism. In anticipation of the

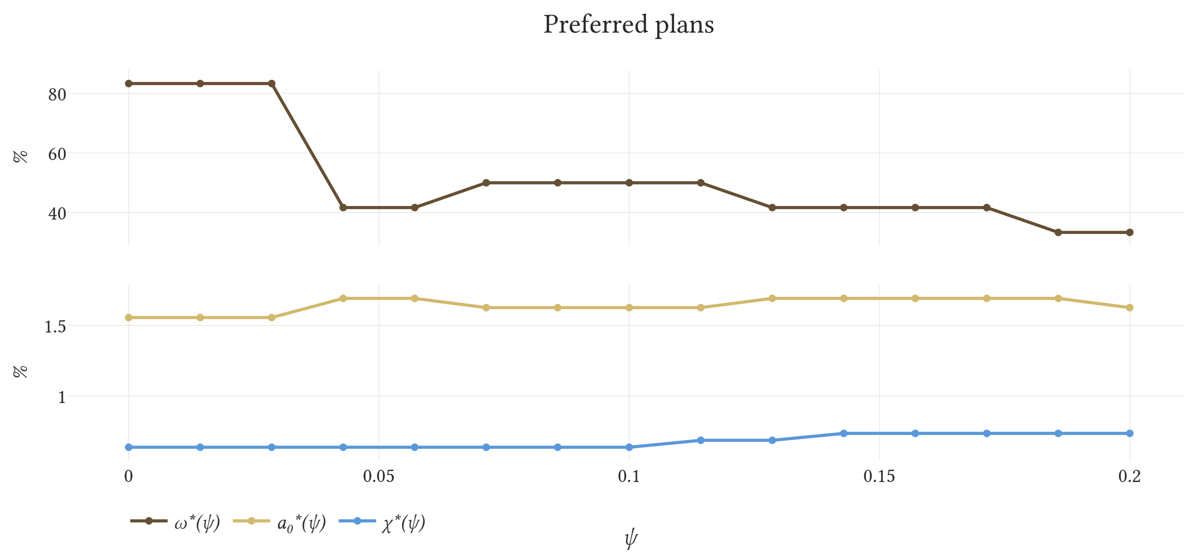


Figure 14: Implied plans with gradual feedback rules

continuation equilibrium, the planner finds it desirable to set itself up in situations where keeping its reputation is both easy and valuable. These are situations where announced inflation for the current period is higher than in the future. The resulting gradualism is therefore an artifact of incentives and not a reflection of inflation inertia. Understanding how the presence of sources of true inertia might interact with our results is one of our goals going forward.

The gradualist property of optimal plans holds at positive levels of reputation and also in the limit as initial reputation vanishes. We interpret this limit case as a sensible refinement of the game between a rational government and the private sector.

Finally, we show that a target-updating rule can improve the performance of the optimal plan in a reputational equilibrium. By letting future targets respond to deviations of inflation, the plan reallocates more challenging tasks to the states in which reputation has increased. This property increases the overall credibility of the plan, improves expectations and, through them, outcomes. This new source of gradualism, which does not vanish after the first few periods and continues to affect equilibrium plans even in the long run, constitutes a potential lesson for policy.

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A. Omitted Proofs

Proof of Proposition 1

For each $n \geq 0$ let $\mathcal{F}_n$ denote the set of bounded functions $f \colon [-n, n] \to \mathbb{R}_+$. Let $B_n : \mathcal{F}_n \to \mathcal{F}_n$ be the operator associated with the Bellman equation (4):

$$B_n f(a) = \min_{a' \in [-n,n]} l(a, a') + \beta f(a'). \tag{7}$$

Standard dynamic programming arguments establish that B_n is a contraction with modulus β and it has a unique fixed point R_n that is continuously differentiable and strictly convex. Let ϕ_n be the associated policy function and let ϕ_n^t be its t-fold composition with $\phi^0(a) := a$.

Lemma 1. *For each $n \geq 0$ and $a \in [-n, n]$*

(a) $\phi_n(a)$ is the unique minimizer of $l(a, a') + \beta R_n(a')$.

(b) If $a > 0$, then $0 < \phi_n(a) < a$.

(c) If $a < 0$, then $a < \phi_n(a) < 0$.

(d) If $a = 0$, then $\phi_n(a) = 0$. Moreover, $R_n'(0) = -\frac{2}{\kappa} y^*$.

(e) $\phi_n^t(a) \to 0$ as $t \to \infty$.

Proof. For simplicity, we assume that $\operatorname{argmin}_{a'} l(a, a') + \beta R_n(a')$ is interior; similar arguments apply for corner solutions. The FOC is

$$2\left(y^* - \frac{1}{\kappa}(a - \beta a')\right)\frac{\beta}{\kappa} + \beta R_n'(a') = 0. \tag{8}$$

Notice that the LHS is strictly increasing in a' due to the strict convexity of R_n'. Hence, part (a) holds. The envelope condition is

$$R_n'(a) = -2\left(y^* - \frac{1}{\kappa}(a - \beta a')\right)\frac{1}{\kappa} + 2\gamma a \tag{9}$$

Combining (8) and (9) yields

$$2\gamma a = R_n'(a) - R_n'(a') \tag{10}$$

It follows from the strict convexity of R_n that $a' < a$ when $a > 0$, $a' > a$ when $a < 0$, and $a' = 0$ when $a = 0$. It follows from (8) that $R_n'(0) = -\frac{2}{\kappa}y^*$, completing the proof of part (d). If $a > 0$, then

$$2\left(y^* - \frac{1}{\kappa}a\right)\frac{1}{\kappa} + R_n'(0) < 0,$$

so (8) implies that $a' > 0$, completing the proof of part (b). Similarly, $a' < 0$ when $a < 0$, completing the proof of part (c).

If $a = 0$, then part (d) implies that $\phi_n^t(a) = 0$ for all t. If $a > 0$, then part (b) implies that $(\phi_n^t(a))_{t=0}^{\infty}$ is positive and decreasing, so it converges to some $\underline{a} \geq 0$ as $t \to \infty$. The continuity of R_n' implies that $|R_n'(\phi_n^{t+1}(a)) - R_n'(\phi_n^t(a))| \to 0$. It follows from (10) that $\underline{a} = 0$, as required. A symmetric argument shows that $\phi_n^t(a) \to 0$ when $a < 0$, thereby completing the proof of part (e). $\square$

Consider the functions $R : \mathbb{R} \to \mathbb{R}_+$ and $\phi : \mathbb{R} \to \mathbb{R}$ given by $R(a) = R_n(a)$ and $\phi(a) = \phi_n(a)$ for $n \in \mathbb{N}$ such that $n - 1 \leq |a| < n$.

Lemma 2. *The restrictions of R and ϕ to $[-n, n]$ equal R_n and ϕ_n, respectively, for any $n \geq 0$.*

Proof. Let $m > n \geq 0$. For any $a \in [-n, n]$ Lemma 1 implies that $|\phi_m(a)| \leq |a| \leq n$, so

$$R_m(a) = \min_{a' \in [-m,m]} l(a, a') + \beta R_m(a') = \min_{a' \in [-n,n]} l(a, a') + \beta R_m(a').$$

Since R_n is the unique fixed point of B_n and its associated policy function ϕ_n is unique by Lemma 1(a), the restrictions of R_m and ϕ_m to $[-n, n]$ equal R_n and ϕ_n, respectively. The desired result now follows. $\square$

Lemma 2 implies that

$$l(a, a') + \beta R(a') = l(a, a') + \beta R_n(a') > R_n(a)$$

for all $a' \neq \phi_n(a) = \phi(a)$ and $n = \max\{|a|, |a'|\}$. Hence, $\phi(a)$ is the unique minimizer $a' \in \mathbb{R}$ of $l(a, a') + \beta R(a')$. Lemma 2 also implies that

$$R(a) = R_n(a) = l(a, \phi_n(a)) + \beta R_n(\phi_n(a)) = l(a, \phi(a)) + \beta R_n(\phi(a)) = \min_{a' \in \mathbb{R}} l(a, a') + \beta R(a')$$

for all $a \in \mathbb{R}$ and $n \geq |a|$. Hence, R satisfies the Bellman equation (4) and has a unique policy function ϕ. R is strictly convex and continuously differentiable because it inherits these properties from R_n on each interval $[-n, n]$ by Lemma 2.

Now we define the Ramsey plan $a^R = (a^R_t)_{t=0}^\infty \in \mathcal{A}$ as follows. Since R is strictly convex and bounded below by 0, there exists a unique minimizer a^R_0. Lemma 2 and Lemma 1(d) imply that $R'(0) < 0$, so $a^R_0 > 0$. The rest of the sequence is defined inductively via $a^R_{t+1} = \phi(a^R_t)$. It follows from Lemma 2 and Lemma 1(b) that $a^R_t > a^R t + 1 > 0$ for all t, and it follows from Lemma 1(e) that $a^R_t \to 0$ as $t \to \infty$.

It remains to show that a^R minimizes $\mathcal{L}$ and that R is the unique solution to (4). The former is implied by Lemma 4 below, and the latter is shown in Lemma 5.

Lemma 3. *Let $a^*_0 \in \mathbb{R}$ and $a^*_{t+1} = \phi(a^*_t)$ for all t. Then $\mathcal{L}(a) > \mathcal{L}(a^*)$ for all $a \in \mathcal{A}$ distinct from $a^* = (a^*_t)_{t=0}^\infty$ with $a_0 = a^*_0$ and $\sup_t |a_t| < \infty$.*

Proof. Let $n = \sup_t |a_t|$. Standard dynamic programming arguments imply that the minimum of $\mathcal{L}(a')$ over sequences $(a'_t)_{t=0}^\infty$ with $a'_0 = a^*_0$ and $a'_t \in [-n, n]$ for all t equals $R_n(a^*_0)$. It is uniquely attained by the sequence with $a'_t = a^*_t$ for all t by Lemma 1(a). It follows from Lemma 2 that $\mathcal{L}(a) > R(a^*_0)$. On the other hand,

$$R(a^*_0) = R_m(a^*_0) = \sum_{t=0}^{\infty} \beta^t l\left(\phi^t_m(a^*_0), \phi^{t+1}_m(a^*_0)\right) = \mathcal{L}(a^*)$$

where $m = |a^*_0|$. The first and last equality follow from Lemma 2, while the second equality follows from the boundedness of R_m. This completes the proof. □

Lemma 4. *Let $a^*_0 \in \mathbb{R}$ and $a^*_{t+1} = \phi(a^*_t)$ for all t. Then $\mathcal{L}(a) > \mathcal{L}(a^*)$ for all $a \in \mathcal{A}$ distinct from $a^* = (a^*_t)_{t=0}^\infty$ with $a_0 = a^*_0$.*

Proof. Let $a \neq a^*$ with $a_0 = a^*_0$. Then there exists τ with $a_\tau \neq a^*_\tau$. Suppose, without loss of generality, that $\mathcal{L}(a) < \infty$. Let

$$\hat{\mathcal{A}} = \left\{ (\hat{a}_t)_{t=0}^\infty \in \mathcal{A} \,\middle|\, \hat{a}_t = a_t \text{ for all } t \leq \tau \text{ and } \sup_t |\hat{a}_t| < \infty \right\}$$

be the space of bounded announcements that agree with a up to time τ. The proof proceeds in two steps. First, we show that the loss from a can be approximated with arbitrary precision by the loss from an announcement in $\hat{\mathcal{A}}$. Secondly, we show that the loss from each announcement in $\hat{\mathcal{A}}$ is bounded away from $\mathcal{L}(a^*)$.

Towards the first step, note that $\sum_{t=0}^\infty \beta^t \gamma a_t^2 < \infty$ because $\mathcal{L}(a) < \infty$. It follows that

$$\beta^T \gamma a_T^2 < \epsilon \tag{11}$$

for sufficiently large T and sufficiently small $\epsilon > 0$. Consider the announcement $\hat{a} \in \hat{\mathcal{A}}$ given by

$$\hat{a}_t = \begin{cases} a_t & \text{if } t \leq T \\ 0 & \text{if } t > T \end{cases}.$$

Since $l(0,0) = (y^*)^2$, it follows that

$$\begin{aligned} \mathcal{L}(\hat{a}) - \mathcal{L}(a) &\leq \beta^T \left(y^* - \frac{1}{\kappa} a_T \right)^2 + \frac{\beta^{T+1}}{1-\beta} (y^*)^2 \\ &\leq \frac{\beta^T}{1-\beta} (y^*)^2 + \beta^T \frac{2}{\kappa} y^* |a_T| + \beta^T \frac{1}{\kappa^2} a_T^2 \\ &< \frac{\beta^T}{1-\beta} (y^*)^2 + \frac{2\beta^{T/2} y^*}{\kappa} \sqrt{\frac{\epsilon}{\gamma}} + \frac{\epsilon}{\kappa^2 \gamma} \end{aligned}$$

where the last inequality follows from (11) when T is large and ϵ is small. It follows that for every $\eta > 0$, there exists T sufficiently large and $\epsilon > 0$ sufficiently small, so that the above estimate implies $\mathcal{L}(\hat{a}) \leq \mathcal{L}(a) + \eta$.

Towards the second step, consider the minimum loss $\mathcal{L}(\hat{a})$ over announcements $\hat{a} \in \hat{\mathcal{A}}$. Since $\hat{a}_t = a_t$ for all $t \leq \tau$, the problem amounts to minimizing the continuation loss from time τ onwards subject to setting $\hat{a}_\tau = a_\tau$. Lemma 3 implies that this continuation loss is uniquely minimised when $\hat{a}_{t+1} = \phi(\hat{a}_t)$ for all $t \geq \tau$. The resulting announcement $\hat{a}$ has $\hat{a}_0 = a_0^*$ and $\hat{a}_\tau \neq a_\tau^*$. It follows from Lemma 3 that $\mathcal{L}(\hat{a}) > \mathcal{L}(a^*)$. Hence, the loss from any announcement in $\hat{\mathcal{A}}$ is bounded away from $\mathcal{L}(a^*)$, thereby completing the proof. □

Lemma 5. *If $\hat{R} : \mathbb{R} \to \mathbb{R}_+$ satisfies $\hat{R}(a) = \min_{a' \in \mathbb{R}} l(a, a') + \beta \hat{R}(a')$ for all $a \in \mathbb{R}$, then $\hat{R} = R$.*

Proof. For every $a \in \mathbb{R}$

$$\begin{aligned} \hat{R}(a) &\leq l(a, \phi(a)) + \beta \hat{R}(\phi(a)) \\ &\leq \sum_{t=0}^{T} \beta^t l(\phi^t(a), \phi^{t+1}(a)) - \beta^T [l(\phi^T(a), \phi^{T+1}(a)) - l(\phi^T(a), 0)] + \beta^{T+1} \hat{R}(0). \end{aligned}$$

Since $\phi^{T+1}(a) \to 0$ as $T \to \infty$ by Lemma 1(e) and Lemma 2, it follows that

$$\hat{R}(a) \leq \sum_{t=0}^{\infty} \beta^t l(\phi^t(a), \phi^{t+1}(a)) = R(a).$$

Suppose, towards a contradiction, that $\hat{R}(a) < R(a)$. Since $\hat{R} \geq 0$, there exists $(a'_t)_{t=0}^{\infty}$ with $a'_0 = a$ such that

$$\hat{R}(a) \geq \sum_{t=0}^{T} \beta^t l(a'_t, a'_{t+1})$$

for all T. It follows that $\sum_{t=0}^{\infty} \beta^t l(a'_t, a'_{t+1}) > R(a'_0)$, which contradicts Lemma 4. □

Proof of Proposition 2

Consider any equilibrium (g, a, e). Let $a^t = (a_0, .., a_{t-1})$ denote the time-t history such that the announcement has been followed in each preceding period. Let $p_t = p(a^t)$ be the posterior beliefs at these histories. Let L_t^r and L_t^b be the (expected) continuation losses of both types from history a^t, and let $\tilde{L}_t = p_t L_t^b + (1 - p_t) L_t^r$ be the average loss. If $g(a^t)$ puts probability 1 on a_t, let $g_t = a_t$. Otherwise, let g_t be some policy different from a_t in the support of $g(a^t)$.

Lemma 6. *$L_t^b \geq L_t^r$ for all t.*

Proof. This follows from the optimality of the rational type's strategy, because following the announcement will result in the loss obtained by the behavioral type. □

Lemma 7. *$L_t^r \geq R(g_t)$ for all t such that $g_t \neq a_t$.*

Proof. After choosing $g_t \neq a_t$ at history a^t the private sector believes that the policy maker is rational for the rest of the game. Let $\underline{g}_t = g_t$ and let $\underline{g}_{t+1}$ be the lowest policy in the support of $g(a^t, \underline{g}_t)$. Proceeding inductively, let $\underline{g}_{s+1}$ be the lowest policy in the support of $g(a^t, \underline{g}_t, .., \underline{g}_s)$. Hence, $e(a^t, \underline{g}_t, .., \underline{g}_s) \geq \underline{g}_{s+1}$. Since the rational type randomizes only when indifferent, it follows that

$$L_t^r \geq \sum_{s=t}^{\infty} \beta^{s-t} l\left(\underline{g}_s, \underline{g}_{s+1}\right). \tag{12}$$

It follows from (3) that the RHS of (12) is at least $R(g_t)$, as required. □

Let $\psi(p', a', g') = \beta[p'R(a') + (1 - p')R(g') - R(p'a' + (1 - p')g')]$. It follows from the strict convexity of R (Lemma XXX) that $\psi(p', a', g') > 0$ whenever $a' \neq g'$ and $p' \in (0, 1)$ and $\psi(p', a', g') = 0$ otherwise.

Lemma 8.

$$l(\hat{a}, p'a' + (1 - p')g') + \beta[p'R(a') + (1 - p')R(g')] \geq R(\hat{a}) + \psi(p', a', g')$$

for any $a, a', g' \in \mathbb{R}$ and $p' \in (0, 1)$

Proof. The result follows from

$$\begin{aligned} &l(\hat{a}, p'a' + (1 - p')g') + \beta[p'R(a') + (1 - p')R(g')] \\ =&l(\hat{a}, \pi') + \beta R(\pi') + \psi(p', a', g') \geq R(\hat{a}) + \psi(p', a', g'), \end{aligned}$$

where $\pi' = p'a' + (1 - p')g'$ and the last inequality follows from the recursive formulation of the Ramsey problem (4). □

Lemma 9. *Suppose that*

$$\tilde{L}_{t+1} \geq p_{t+1}R(a_{t+1}) + (1-p_{t+1})R(g_{t+1}) + \eta$$

for some $\eta \in \mathbb{R}$ and time t such that $p_{t+1} < 1$. Then

$$\tilde{L}_t \geq p_t R(a_t) + (1-p_t)R(g_t) + p_t\psi(p_{t+1}, a_{t+1}, g_{t+1}) + \beta \min\{\eta, p_0\eta\}.$$

Proof. The behavioral type's payoff from history a^t is given by

$$L_t^b = l(a_t, p_{t+1}a_{t+1} + (1-p_{t+1})g_{t+1}) + \beta L_{t+1}^b. \tag{13}$$

Since, $p_{t+1} < 1$, the rational type chooses a_t with positive probability following history a^t, so

$$L_t^r = l(a_t, p_{t+1}a_{t+1} + (1-p_{t+1})g_{t+1}) + \beta L_{t+1}^r. \tag{14}$$

It follows from (13) and (14) that

$$\begin{aligned} p_{t+1}L_t^b + (1-p_{t+1})L_t^r &= l(a_t, p_{t+1}a_{t+1} + (1-p_{t+1})g_{t+1}) + \beta\tilde{L}_{t+1} \\ &\geq R(a_t) + \psi(a_{t+1}, g_{t+1}) + \beta\eta \end{aligned} \tag{15}$$

where the last inequality is implied by the hypothesis of this lemma and the result of Lemma 8.

If $p_t = p_{t+1}$, then the rational type plays a_t at history a^t with probability 1, so $g_t = a_t$. It follows from (15) that

$$\tilde{L}_t = p_{t+1}L_t^b + (1-p_{t+1})L_t^r \geq p_t R(a_t) + (1-p_t)R(g_t) + \psi(a_{t+1}, g_{t+1}) + \eta,$$

as required.

If $p_t < p_{t+1}$, then

$$\begin{aligned} \tilde{L}_t &= p_t L_t^b + (1-p_t)L_t^r \\ &\geq p_t[p_{t+1}L_t^b + (1-p_{t+1})L_t^r] + (1-p_t)L_t^r \\ &\geq p_t[R(a_t) + \psi(a_{t+1}, g_{t+1}) + \beta\eta] + (1-p_t)R(g_t), \end{aligned}$$

where the first inequality follows from Lemma 6 and the second inequality follows from (15), Lemma 8, and Lemma 7. □

Now we are ready to prove Proposition 2. Let $T = \inf\{t | p_{t+1} = \sup_s p_s\}$ be the last time beliefs change conditional on observing inflation matching the announcement. We adopt the convention $\inf \emptyset = \infty$. We consider two cases.

Case 1: Suppose $0 < T < \infty$. Then $p_T < p_{T+1}$, so $g_T \neq a_T$. Since $p_{T+1} = p_{T+2} = ..$, the private sector expects inflation a_s following history a^s with $s > T$. Hence, the behavioral type's continuation loss from a^T is

$$L^b_T \geq \sum_{s=T}^{\infty} \beta^{s-T} l(a_s, a_{s+1}) \geq R(a_T),$$

where the inequality follows from (3). Moreover, $L^r_T \geq R(g_T)$ by Lemma 7. Hence,

$$\tilde{L}_T \geq p_T R(a_T) + (1 - p_T) R(g_T)$$

An inductive application of Lemma 9 obtains

$$\tilde{L}_0 > p_0 R(a_0) + (1 - p_0) R(g_0)$$

since $g_T \neq a_T$ implies $\psi(a_T, g_T) > 0$. It follows that $\tilde{L}_0 > R(a^R_0)$.

Case 2: Suppose $T = \infty$. Then $p_t < 1$ for all t. It follows that for any $\epsilon > 0$, there exists t sufficiently large that the rational type follows the announcement from history a^t with probability at least $1 - \epsilon$. Hence, for every $\epsilon > 0$ there exists t with $g_t \neq a_t$ such that $L^b_t \geq R(a_t) - \epsilon$ and $L^b_t \geq R(g_t) - \epsilon$, so that

$$\tilde{L}_t \geq p_t R(a_t) + (1 - p_t) R(g_t) - \epsilon. \tag{16}$$

Consider a time τ with $g_\tau \neq a_\tau$ and let t satisfy (16) for $\epsilon = \beta^t p^\tau_0 \psi(p_\tau, g_\tau, a_\tau)/2$. It follows from Lemma 9 that $\tilde{L}_0 > R(a^R_0)$ by an argument similar to the previous case.

It follows that $\tilde{L}_0 > R(a^R_0)$ when $T > 0$. When $T = 0$ the rational type chooses a_t at history a^t for any t, i.e. he behaves identically to the behavioral type. In this case both types obtain loss $\sum_{t=0}^{\infty} \beta^t l(a_t, a_{t+1}) = \tilde{L}_0$. It follows from Proposition 1 and XXX that $\tilde{L}_0 \geq R(a^R_0)$ and $\tilde{L}_0 = R(a^R_0)$ iff $a = a^R$. Thus, the unique announcement is a^R_0 and any equilibrium that attains the lowest average loss $R(a^R_0)$ has the same on-path behavior where both types follow the announcement.

B. More results

Figure 15 shows that, at $a_0 = \chi^\star$ (about 0.6 in this example), the planner is indifferent between decay rates, as the decay rate matters more when a_0 is farther away from χ. Starting from the optimal a_0 (about 1.6), it prefers an intermediate decay rate: too slow would negate the incentives from rapidly decaying targets and make the plan target high inflation for too long, but too fast would not create incentives for long as the plan would rapidly become flat.

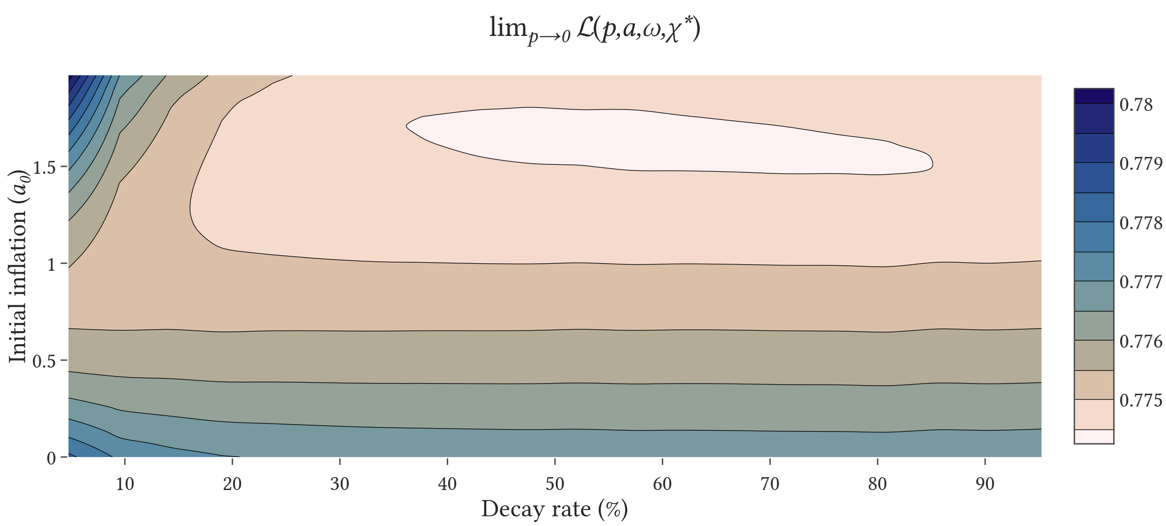

Figure 15: Loss function across announcements