Risk Aversion in Sovereign Debt and Default

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Why risk aversion? Why in sovereign debt?

- In most RBC models, macro-financial separation holds
  - Elasticity of intertemporal substitution determines allocations
  - Risk aversion determines asset prices

- Sovereign debt literature typically inherits this line of thinking
  - CRRA preferences frequent, typically $\gamma = 2$

- If MFS holds in sovereign debt, macro outcomes robust to different preferences
  - In particular, calibration of output/utility costs of default
  - Less clear about welfare effects

    … losses from default, debt dilution

    … welfare effects of banning debt, introducing state-contingent bonds

Wanting risk prices in sovereign debt

This paper

- Show that macro-financial separation breaks in the sovereign debt model
- Understand the impact of preferences consistent with significant risk premia
- Find that risk aversion affects equilibria in unexpected ways

Model

Framework

- Sovereign default model without default [reduces to an income-fluctuations problem]

$$v(b,z) = \max_{b'} u(c) + \beta \mathbb{E}\left[v(b',z') \mid z\right]$$

$$\begin{aligned} \text{subject to} \quad & c + \kappa b = q(b',z)(b' - (1-\rho)b) + y(z) \\ & b' \leq \bar{b} \\ \text{with} \quad & q(b',z) = \frac{1}{1+r} \end{aligned}$$

- We consider parametrizations of the model to vary risk aversion
  ... with CRRA preferences $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$
  ... With robustness, $u(c) = \log c$; replace $\mathbb{E}$ with $\mathbb{T}\left[X \mid \mathcal{F}\right] = -\frac{1}{\theta}\log\left(\mathbb{E}\left[\exp\left(-\theta X\right) \mid \mathcal{F}\right]\right)$

Macro-financial separation without default

Autarky

- Start from log-log [$\theta = 0$]: RA moves asset prices and welfare, not the macro

	loglog	$\theta = 1$	$\theta = 2$	$\theta = 3$
Average spread (bps)	0.0276	0.031	0.0406	0.138
Corr. NX,Y (%)	0.00777	0.00916	0.0114	0.0147
Rel. vol. cons (%)	1.59	1.62	1.65	1.66
Risk premium (p.p.)	0.0769	2.03	3.84	5.44
Debt-to-GDP (%)	29.7	29.5	29.2	28.9
Corr. deficit, y (%)	−0.0119	−0.0141	−0.0177	−0.0231
Welfare	1.034	1.008	0.9867	0.971

… welfare in autarky at $\theta = 3$ is 6pp lower than loglog or CRRA

Macro-financial separation without default (cont'd)

- Start from log-log [$\gamma = 1$]: EIS+RA moves mostly macro, not asset prices and welfare

	loglog	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Average spread (bps)	0.0276	0.0273	0.0269	0.0271	0.0285
Corr. NX,Y (%)	0.00777	0.0154	0.0852	0.397	0.668
Rel. vol. cons (%)	1.59	1.56	1.35	0.965	0.727
Risk premium (p.p.)	0.0769	0.227	0.627	1.02	1.67
Debt-to-GDP (%)	29.7	28.8	25.9	19.3	8.75
Corr. deficit, y (%)	−0.0119	−0.0251	−0.162	−0.605	−0.774
Welfare	1.034	1.03	1.021	1.01	0.9918

... in fully Epstein-Zin, move only EIS for even less effect on asset prices and welfare

Models with default

- Option value of default (with small pref. shocks for numerical performance)

$$\mathcal{V}(b,z) = \max\{v_R(b,z) + \epsilon_R, v_D(b,z) + \epsilon_D\}$$

- Similar equation for value of repayment v_R, debt prices reflect default probabilities

$$q(b',z) = \frac{1}{1+r}\mathbb{E}\left[(1 - \mathbb{1}_{\mathcal{D}'})\left(\kappa + (1-\rho)q(b'',z')\right) \,\middle|\, z\right]$$

- Costs of default

$$v_D(b,z) = u(h(y(z))) + \beta\mathbb{E}\left[\mathbb{1}_R\mathcal{V}(B(b,z'),z') + (1 - \mathbb{1}_R)v_D(b,z') \,\middle|\, z\right]$$
$$h(y) = y(1 - d_0 - d_1 y)$$

- Risk aversion $\implies$ no-smoothing in default costly $\implies$ no macro-financial separation

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Quantitative properties

Calibration

- Keep the same discount rate, vary costs of default to match spreads and debt

	Parameter	$\gamma = 2$	loglog	$\theta = 3$
Sovereign's discount factor	β	0.9627	0.9627	0.9627
Sovereign's robustness parameter	θ	0	0	3
Sovereign's EIS	γ	2	1	1
Default output cost: linear	d_1	-0.2833	-0.2836	-0.247
Default output cost: quadratic	d_2	0.3253	0.3228	0.3029
Average spread (bps)	815	754	756	815
Debt-to-GDP ratio (%)	17.4	16.8	16.7	17.4

Comparative statics: CRRA

- Increasing EIS+RA: Less volatility, procyclical exports, more skewed debt outcomes

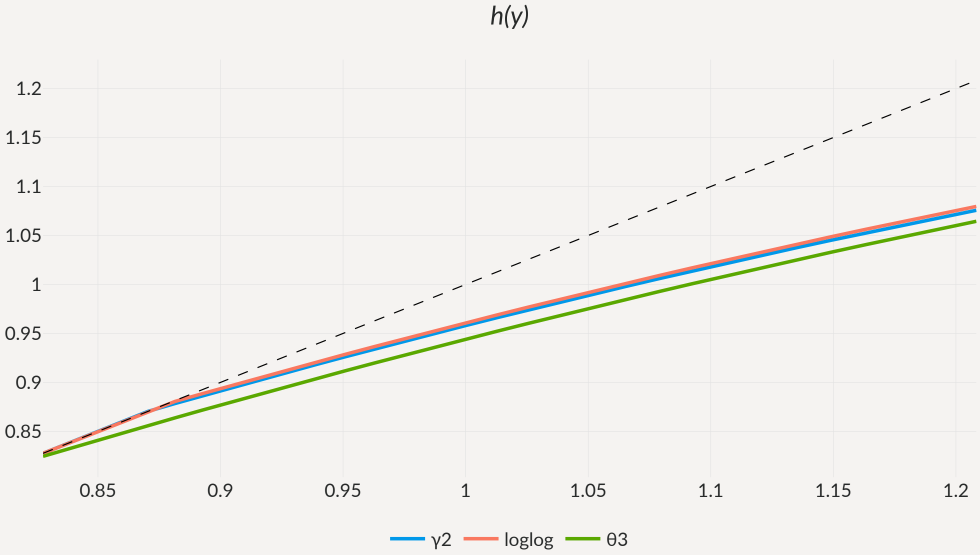
	loglog	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Avg. spread (bps)	756	800	912	974	1,057
Corr. NX,Y (%)	-0.285	-0.302	-0.21	0.0726	0.416
Rel. vol. cons (%)	1.5	1.37	1.18	1.04	0.921
Risk premium (p.p.)	0.652	0.789	1.02	1.28	2.38
Debt-to-GDP (%)	16.7	15.7	12.4	7.62	3.25
Corr. deficit, y (%)	0.391	0.391	0.217	-0.21	-0.627
Default freq. (%)	4.4	4.41	4.17	3.45	2.7
Std. dev. spreads (bps)	448	538	877	1,209	1,816
Welfare	1.013	1.01	1.002	0.9918	0.9728

Comparative statics: robustness

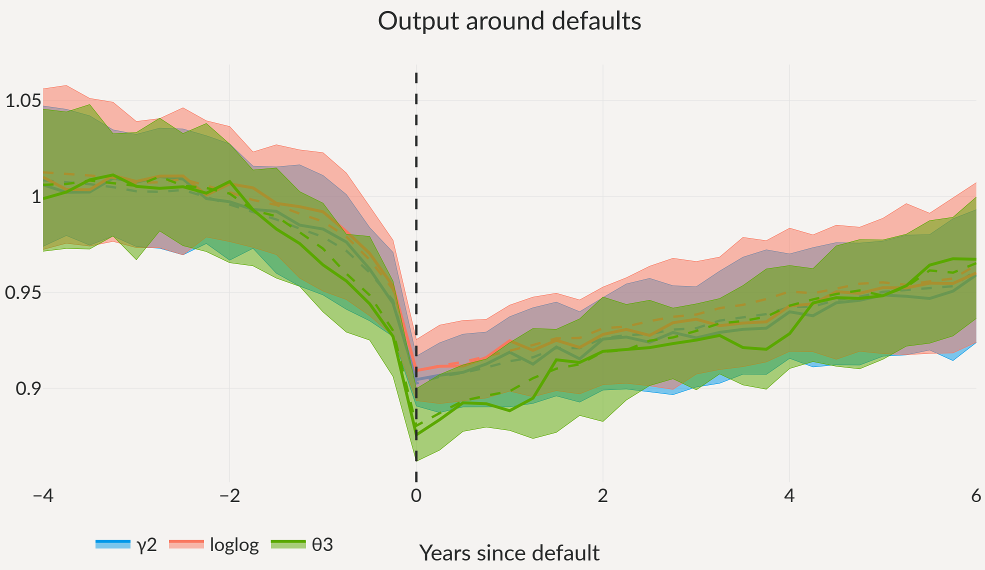
- Increasing RA: less debt tolerance, limited effect on volatilities

	loglog	$\theta = 1$	$\theta = 2$	$\theta = 3$
Avg. spread (bps)	756	1,683	20,240	36,331
Corr. NX, y (%)	-0.285	-0.227	-0.0901	-0.227
Rel. vol. cons (%)	1.5	1.38	1.26	1.46
Risk premium (p.p.)	0.652	2.92	4.43	6.99
Debt-to-GDP (%)	16.7	14.2	9.09	9.57
Corr. deficit, y (%)	0.391	0.292	0.118	0.266
Default freq. (%)	4.4	5.88	3.57	2.47
Std. dev. spreads (bps)	448	2,561	103,509	189,131
Welfare	1.013	0.9848	0.9629	0.9469

Calibrated output costs of default with robustness



Event-study of defaults

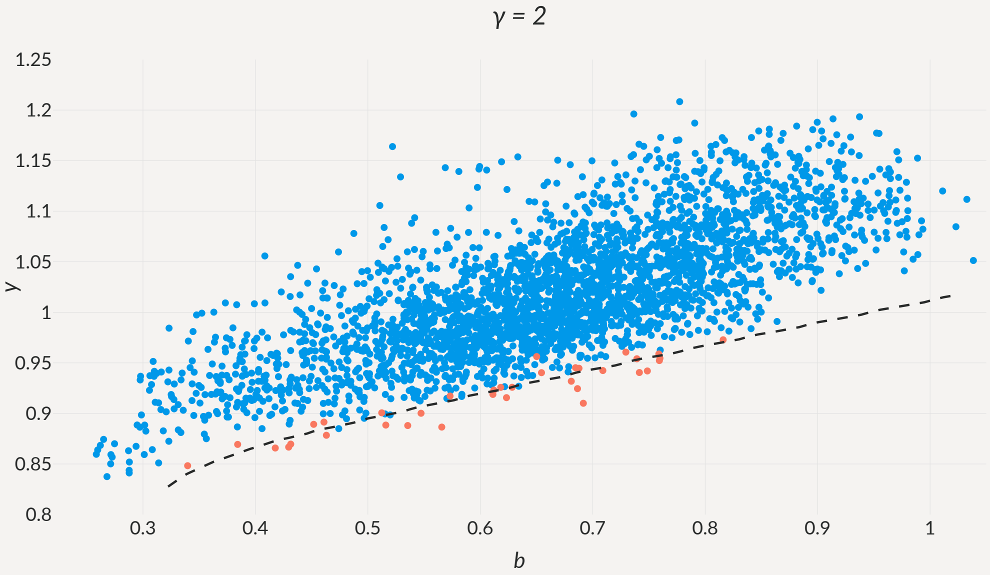


Calibrations with risk aversion

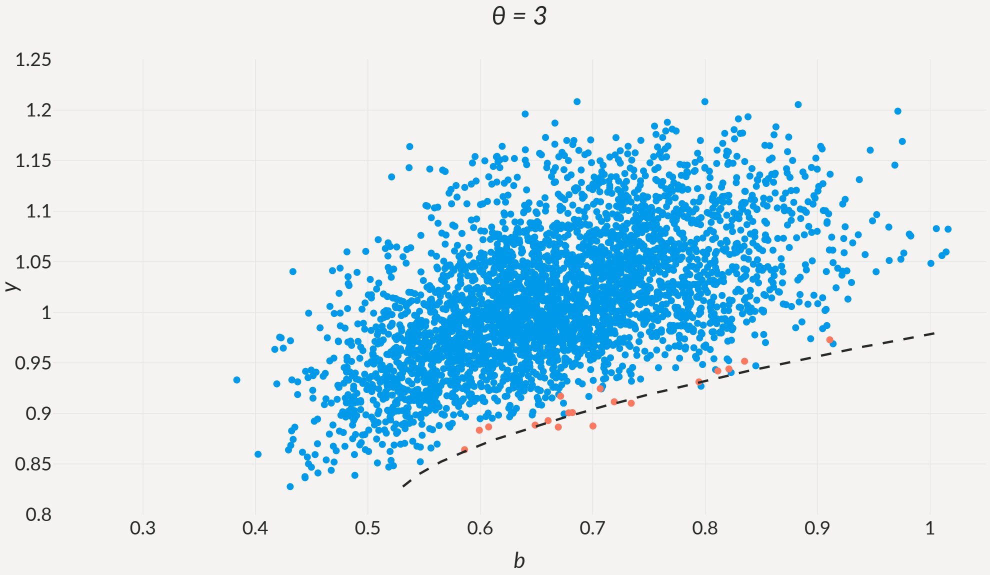
- Calibration with robustness: skewed debt outcomes, small decrease in macro volatility

	Data	$\gamma = 2$	loglog	$\theta = 3$
Avg. spread (bps)	815	754	756	815
Corr. NX,Y (%)	-	-0.314	-0.285	-0.194
Rel. vol. cons (%)	0.94	1.38	1.5	1.35
Risk premium (p.p.)	-	0.778	0.652	5.9
Debt-to-GDP (%)	17.4	16.8	16.7	17.4
Corr. deficit, y (%)	-	0.405	0.391	0.207
Default freq. (%)	-	4.21	4.4	1.51
Std. dev. spreads (bps)	443	496	447	2,026

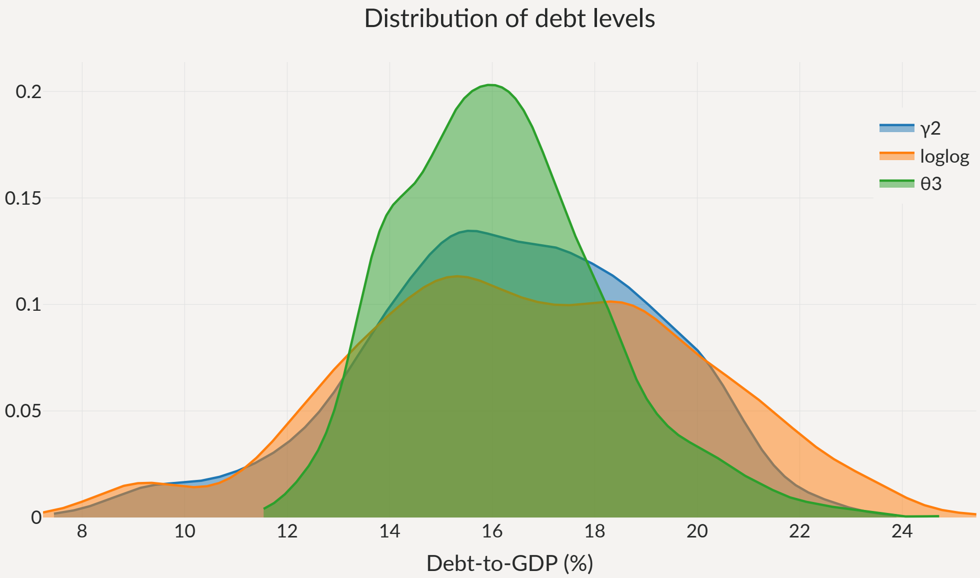
Ergodic distribution for debt in CRRA model



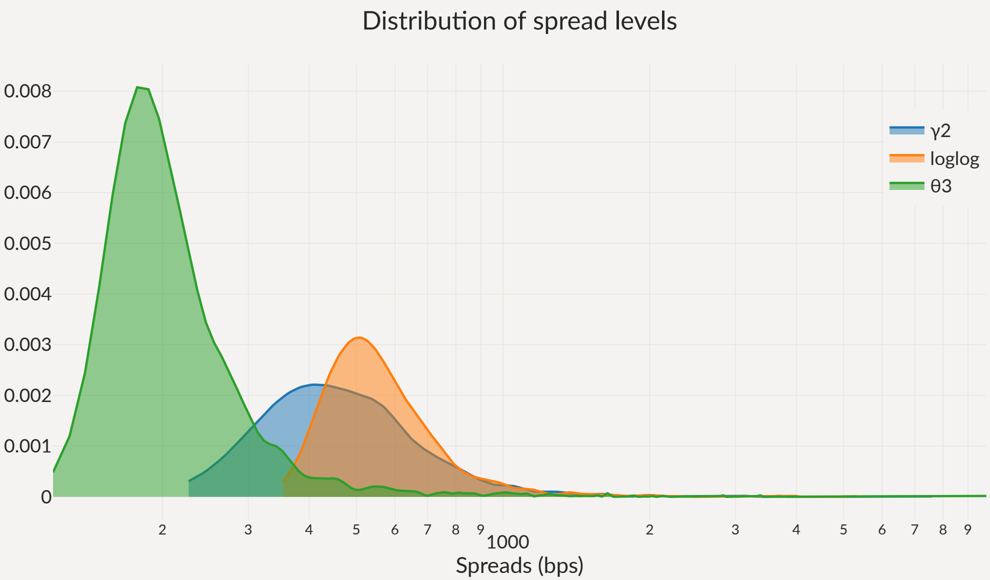
Ergodic distribution for debt with robustness



Ergodic distribution for debt



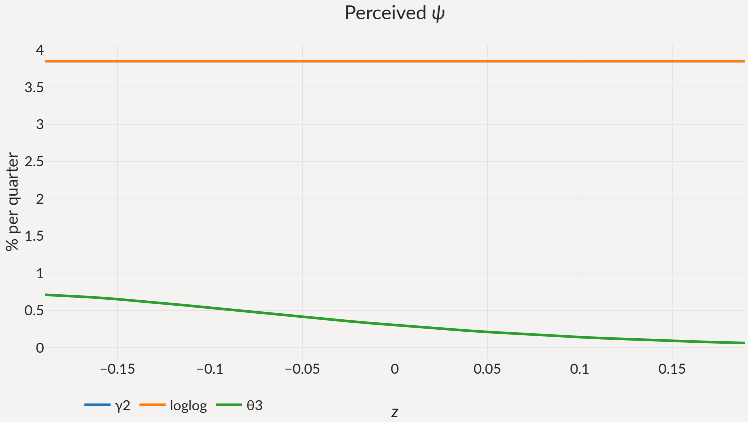
Ergodic distribution for spreads



Worst-case models

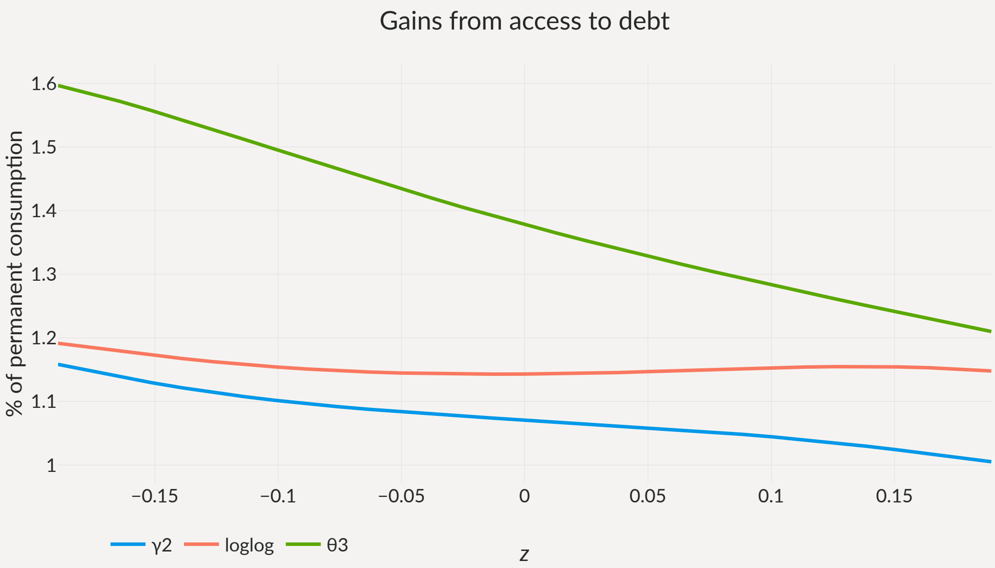
- Distorted expectation of X

$$\tilde{\mathbb{E}}\left[X \mid \mathcal{F}\right] = \mathbb{E}\left[\frac{\exp(-\theta v(s'))}{\mathbb{E}\left[\exp(-\theta v(s')) \mid \mathcal{F}\right]} X \;\middle|\; \mathcal{F}\right]$$

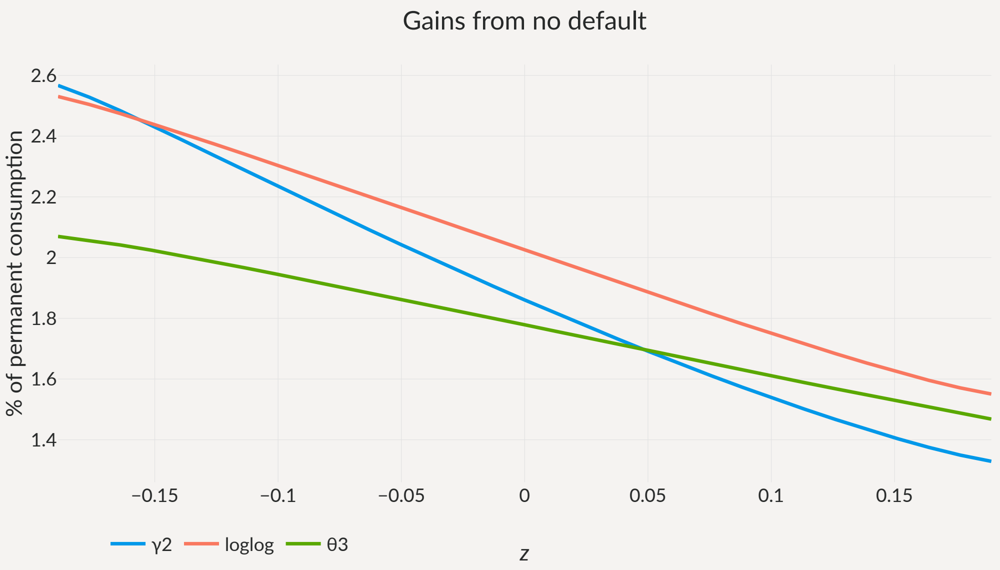


Welfare effects

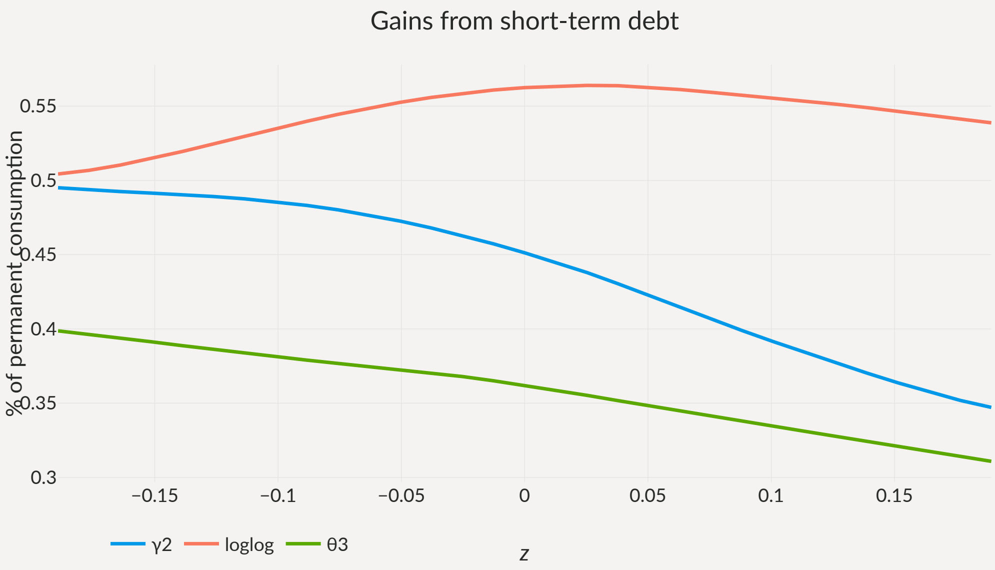
Welfare effects of debt



Welfare effects of banning defaults



Welfare effects of shortening maturity



Takeaways

With preferences consistent with positive risk premia

- *Lower* debt tolerance
  - … Larger default costs required

- Less staying at the edge of default
  - … More skewness in the distribution of debt and spreads

- More use of the debt for insurance
  - … Large gains from debt access, not so much for making debt safe

Welfare gains decomposition

Consumption without default costs c_t^R

$$c^R(b,z) = \mathbb{1}_{\mathcal{D}}(b,z)y(b,z) + (1 - \mathbb{1}_{\mathcal{D}}(b,z))c(b,z)$$

Evaluate value of consuming c^R [instead of c] and removing uncertainty

$$\begin{aligned} V_{NC}(b,z) &= u(c^R(b,z)) + \beta\mathbb{E}\left[V_{NC}(b',z') \mid z\right] \\ V_{NV}(b,z) &= u(c^R(b,z)) + \beta V_{NV}(b', \mathbb{E}\left[z' \mid z\right]) \end{aligned}$$

Welfare gains between models/equilibria with value functions v and $v^\star$

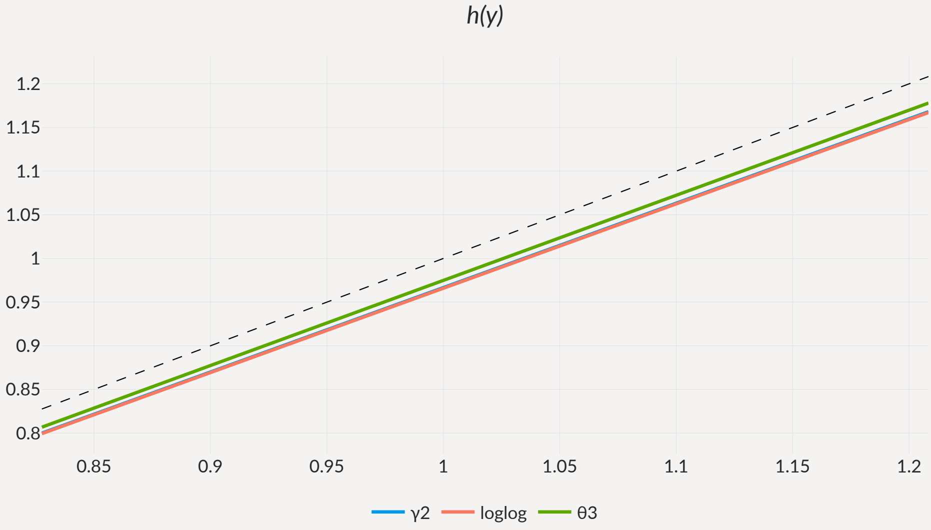
$$\frac{v^\star(b_0,z_0)}{v(b_0,z_0)} = \frac{v^\star(b_0,z_0)/v^\star_{NC}(b_0,z_0)}{v(b_0,z_0)/v_{NC}(b_0,z_0)} \times \frac{v^\star_{NC}(b_0,z_0)/v^\star_{NV}(b_0,z_0)}{v_{NC}(b_0,z_0)/v_{NV}(b_0,z_0)} \times \frac{v^\star_{NV}(b_0,z_0)}{v_{NV}(b_0,z_0)}$$

Welfare gains

	Total gains	From default costs	From volatility	From level
		$\gamma = 2$		
Access to markets	0.622	-0.273	0.218	0.679
No default	1.87	0.274	-0.292	1.89
Short-term debt	0.411	0.255	-0.448	0.606
		loglog		
Access to markets	0.663	-0.294	0.284	0.674
No default	2.04	0.295	-0.345	2.09
Short-term debt	0.519	0.272	-0.439	0.688
		$\theta = 3$		
Access to markets	0.961	-0.25	0.0354	1.18
No default	1.72	0.251	-0.0744	1.54
Short-term debt	0.262	0.233	-0.45	0.481

Model with linear costs

- Convex costs lower income volatility during defaults



Concluding remarks

Risk aversion in the sovereign debt model

- We evaluate preferences consistent with risk premia in the sovereign default model

  … mostly possible to match standard calibration targets with robustness

- Effect of robustness concentrated at higher-order moments

  … makes crises look like more abrupt events

- Innocent-looking features of the standard model weigh against large risks/distortions

  … convex costs of default mute post-default uncertainty

Macro-finanical separation with autarky

Back

	loglog	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Corr. NX,Y (%)	-0.00131	-0.00131	-0.00131	-0.00131	-0.00131
Rel. vol. cons (%)	1	1	1	1	1
Risk premium (p.p.)	0.0833	0.251	0.751	1.57	3.05
Welfare	1.002	1	0.9951	0.9868	0.9699

	loglog	$\theta = 1$	$\theta = 2$	$\theta = 3$
Corr. NX,Y (%)	-0.00131	-0.00131	-0.00122	-0.00073
Rel. vol. cons (%)	1	1	1	1
Risk premium (p.p.)	0.0833	2.02	3.81	5.32
Welfare	1.002	0.9769	0.9564	0.9411