Uncertainty Premia, Sovereign Default Risk, and State-Contingent Debt

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State-contingent debt instruments

- Decrease default risk
- Reduce cyclicality of fiscal policy
- Improve risk-sharing

Why aren't they used?

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Unfavorable prices of state-contingent instruments

- These instruments are heavily discounted by markets
  - Costa, Chamon, and Ricci (2008) compute wide spreads for Argentine GDP-warrants
    - $\sim$300-400bps from default risk of other securities
    - 600-1200bps residual: ‘novelty’ premium

This paper proposes a framework that

- Rationalizes pricing of SCI + welfare analysis
  - With ingredients from resolutions of the equity premium puzzle
- Links unfavorable prices to common ‘threshold’ structure
  - Example: Argentina’s GDP-warrants, also Ukraine, Greece. . . More

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A framework for pricing state-contingent debt

- Standard quantitative model of sovereign default with long-term debt
  - Aguiar and Gopinath (2006), Arellano (2008), Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012)
- International lenders with concerns about *model misspecification*
  - Preference for robustness   Hansen and Sargent (2001), Pouzo and Presno (2016)

- **Mechanism:** lenders act *as if* the probability of states with low repayment was higher
  - With noncontingent debt, lenders overestimate the default probability
  - Pouzo and Presno (2016) uses robustness to reconcile spreads with default frequencies
  - In general, probability distortion depends on type and quantity of debt issued

Main findings

1. Robust lenders dislike repayment structures with thresholds in good times
    - Heavy discounts for these bonds $\implies$ welfare losses
2. Explain most of the 'novelty premium' in Argentina's GDP warrants as ambiguity premia
    - Calibration of robustness from *noncontingent* debt only
3. Characterize the optimal design and how it changes with robustness
    - With high robustness, want to minimize ex-ante and ex-post contingency

Roadmap

- Stylized Model
- Probability Distortions
- Quantitative Implementation
- Concluding Remarks

Stylized Model

The model

We consider a simple two-period model, small open economy

- Uncertain endowment $y(z)$ in the second period
- The government has access to one asset which promises a return $R(z)$.
- A few benchmarks

Noncontingent debt	$R(z)$	$=$	1
Linear indexing	$R^{\alpha}(z)$	$=$	$1+\alpha(y(z)-1)$
Threshold debt	$R^{\tau}(z)$	$=$	$\mathbb{1}(z>\tau)$
Optimal design	$R^{\star}(z;\theta)$	chosen state-by-state	

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The government's problem

- The government takes as given the price schedule $q(b)$

$$\max_{b} u(c_1^b) + \beta_b \mathbb{E}\left[u(c_2^b)\right]$$

$$\text{subject to } \; c_1^b = y_1 + q(b)b$$

$$c_2^b = y_2(z) - h(z,\Delta)d(b,z) - (1-d(b,z))R(z)b$$

where

$$h(z,\Delta) = y_2(z)^2\Delta$$

- In the second period, default if

$$\underbrace{u\left(y_2(z) - h(z,\Delta)\right)}_{\text{v. default}} > \underbrace{u\left(y_2(z) - R(z)b\right)}_{\text{v. repayment}}$$

The lenders' problem

Foreign lenders are less standard and have multiplier preferences

$$\max c_1^L - \frac{\beta}{\theta} \log \left(\mathbb{E} \left[\exp(-\theta v_2^L) \right] \right)$$

$$\text{subject to } \quad v_2^L = c_2^L$$

$$c_2^L = w_2 + (1 - d(b, z)) R(z) b$$

$$c_1^L = w_1 - q_1 b$$

Lenders provide us with an Euler equation to price the debt

$$q(b; R) = \beta \mathbb{E} \left[\frac{\exp(-\theta v_2^L)}{\mathbb{E}\left[\exp(-\theta v_2^L)\right]} (1 - d(b, z)) R(z) \right]$$

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$$\begin{aligned} &\max c_1^L - \frac{\beta}{\theta} \log \left(\mathbb{E} \left[\exp(-\theta v_2^L) \right] \right) \\ \text{subject to } \; & v_2^L = c_2^L \\ & c_2^L = w_2 + (1 - d(b, z)) R(z) b \\ & c_1^L = w_1 - q_1 b \end{aligned}$$

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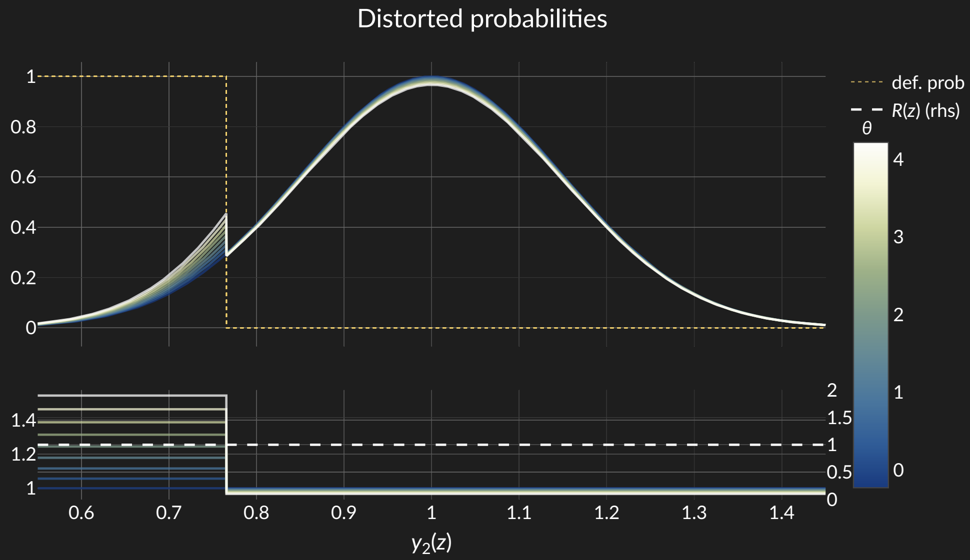
Probability Distortions

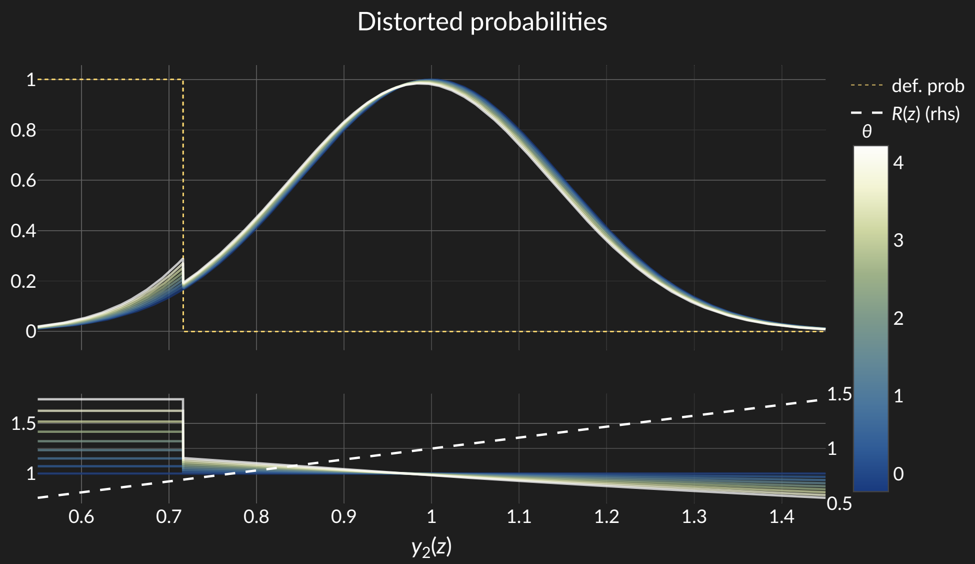
Distorted probabilities – noncontingent debt

Reoptimizing

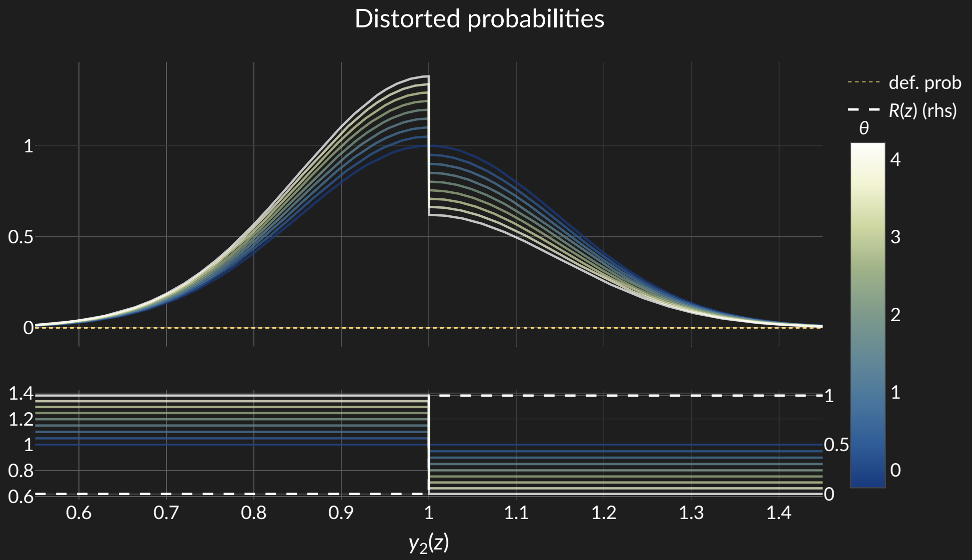


Distorted probabilities – linearly indexed debt

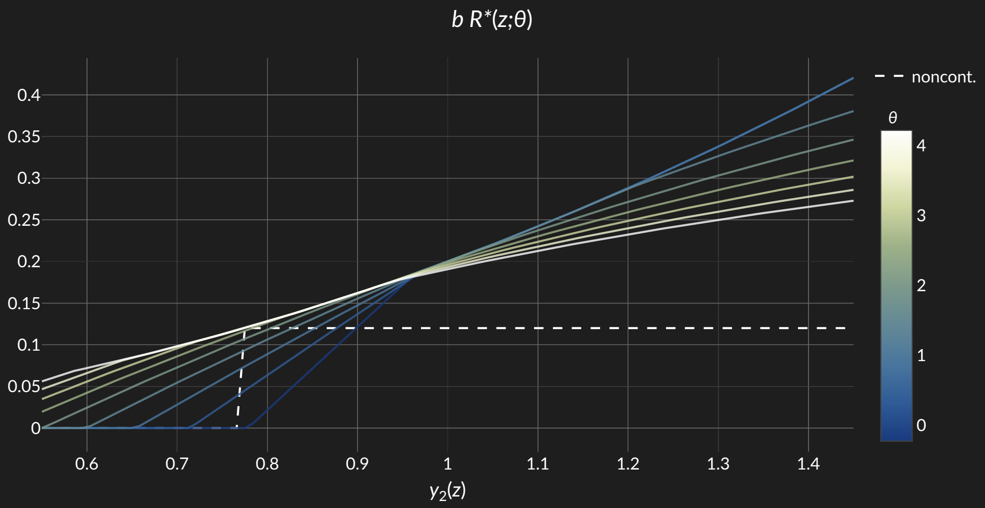
Reoptimizing



Distorted probabilities – threshold debt



Design of debt



Quantitative Implementation

Quantitative Model

- Infinite horizon, small-open economy
- Robust lenders as before
- Long-term debt, debt issued at t pays coupon at $t + s$

$$\max \left\{0, (1-\delta)^{s-1}(1+\alpha(y_s - 1))\mathbb{1}(y_s > \tau)\right\}$$

  - Noncontingent debt: $\alpha = 0, \tau = -\infty$
- Default triggers exclusion + output costs for a random amount of periods $\sim Geo(\psi)$

Robustness in the quantitative model

▸ CE calibration

	Rational Expectations			$\theta = 1.6155$ (benchmark)		
Statistic	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread	8.1	0.36	7.2	8.15	11.1	7.1
Std Spread	4.5	0.23	3.7	4.6	1.58	3.6
Debt	48.7	116.5	50.8	44.0	67.6	46.1
Std(c)/Std(y)	1.24	0.82	1.22	1.25	0.84	1.23
Default Prob	5.5	0.3	5.3	3.0	0.0	2.6
Welfare Gains	-	1.19	0.09	-	-0.37	0.07

Table 1: Statistics from calibrated model simulations

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with alpha = 1.

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Optimal design of state-contingent debt

Statistic	Rational Expectations $\tau = 0.875, \alpha = 7$	Robustness $\tau = 0.875, \alpha = 5$
Spread	0.1	2.8
Std Spread	0.04	0.13
Debt	79.3	65.9
Std(c)/Std(y)	0.76	0.96
Default Prob	0.1	0.23
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Table 2: Statistics under the optimal state-contingent bond for different types of lenders

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Concluding Remarks

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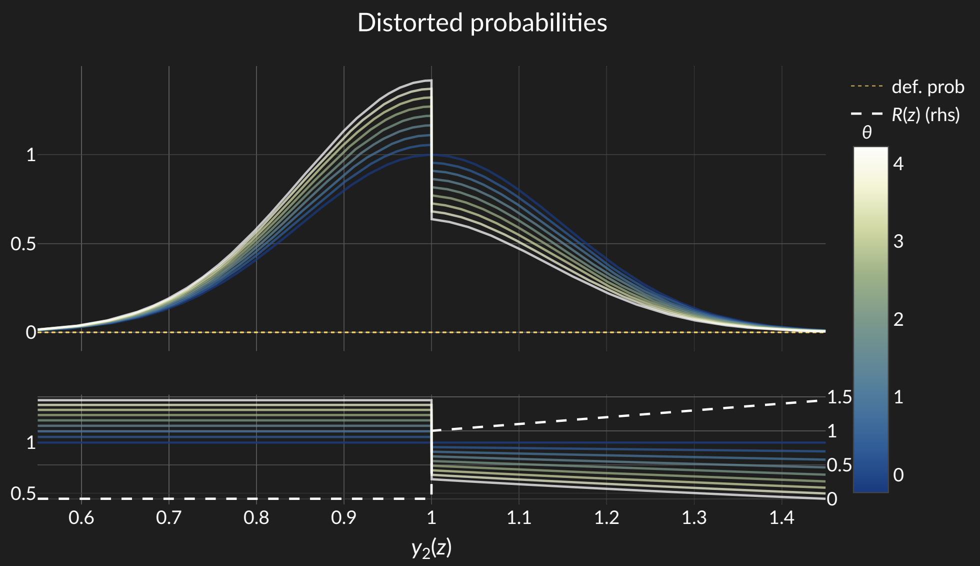
- Standard sovereign debt model augmented with robust lenders
  1. Accounts for spreads on typical threshold SCDIs
  2. Rationalizes part of the ‘novelty’ premium as a premium for ambiguity
  3. Links unfavorable prices to common *threshold* structure
  4. Welfare gains of SCDI decreasing in robustness
     - Both for given instrument and for optimally-designed debt

- Optimal design
  - With realistic robustness, lower thresholds and flatter indexation than RE
  - With extreme robustness, eliminate contingency ex-ante (*stipulated*) and ex-post (*default*)
  - In general, tradeoff between contingency and risk-sharing

Distorted probabilities – threshold+linear debt

Back



Quantitative model

Back

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Spread	8.5	0.6	6.8	8.4	15.5	7.1
Std Spread	4.3	0.4	3.0	4.4	2.3	3.1
Debt	69.9	159.6	74.4	62.6	87.7	67.2
Std(c)/Std(y)	1.24	0.83	1.21	1.25	0.82	1.22
Corr(y,c)	0.98	0.53	0.98	0.98	0.94	0.98
Corr(y,tb/y)	-0.7	0.52	-0.62	-0.67	0.58	-0.6
Corr(y,spread)	-0.77	-0.87	-0.78	-0.75	-0.61	-0.77
Default Prob	5.8	0.56	5.3	2.3	0.12	1.8
Welfare Gains	-	1.86	0.27	-	-0.87	0.2

Table 3: Statistics based on Chatterjee and Eyigungor (2012)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with alpha = 1.

CARA

Back

Euler equations of a rational-expectations agent with CARA preferences and access to a risk-free bond

$$q = \beta \mathbb{E}\left[\frac{u'(c_2)}{u'(c_1)} R\right] = \beta \mathbb{E}\left[\frac{\exp(-\gamma c_2)}{\exp(-\gamma c_1)} R\right]$$

$$\frac{1}{1+r} = \beta \mathbb{E}\left[\frac{u'(c_2)}{u'(c_1)}\right]$$

hence

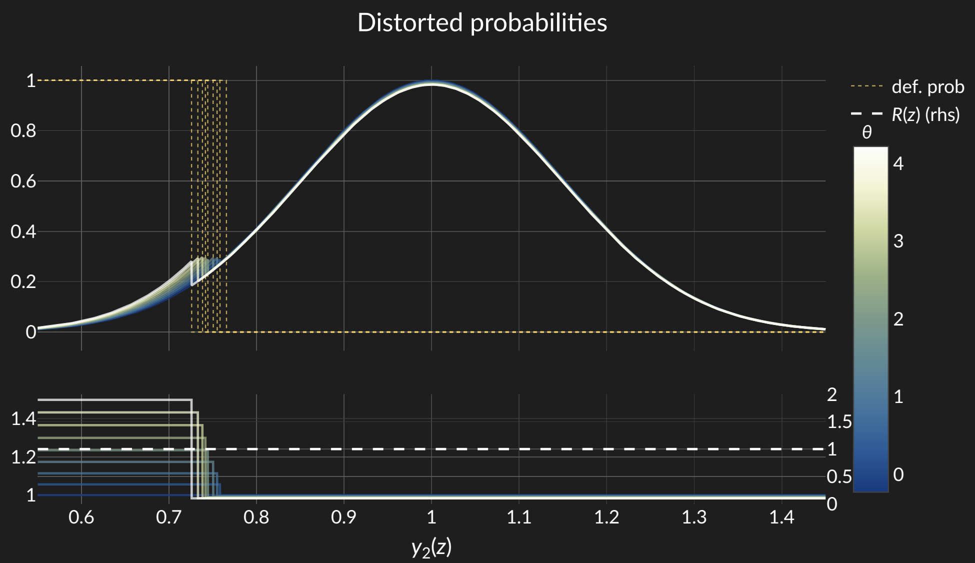
$$q = \beta \mathbb{E}\left[\frac{\exp(-\gamma c_2)}{\beta(1+r)\mathbb{E}\left[\exp(-\gamma c_2)\right]} R\right]$$

Same as robustness in two periods, in general the robust sdf is

$$q = \beta \mathbb{E}\left[\frac{\exp(-\theta v')}{\mathbb{E}\left[\exp(-\theta v')\right]} R\right]$$

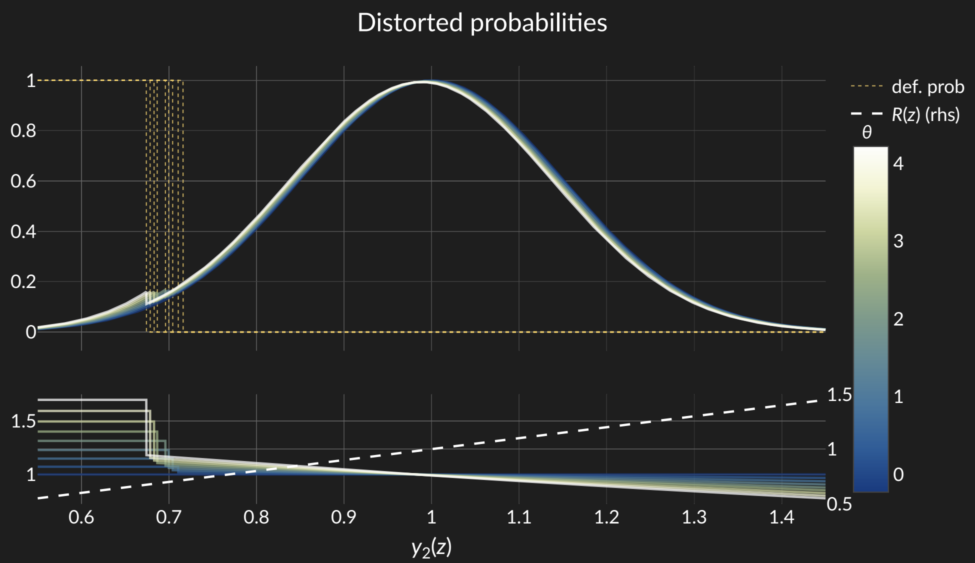
Distorted probabilities – noncontingent debt

Back



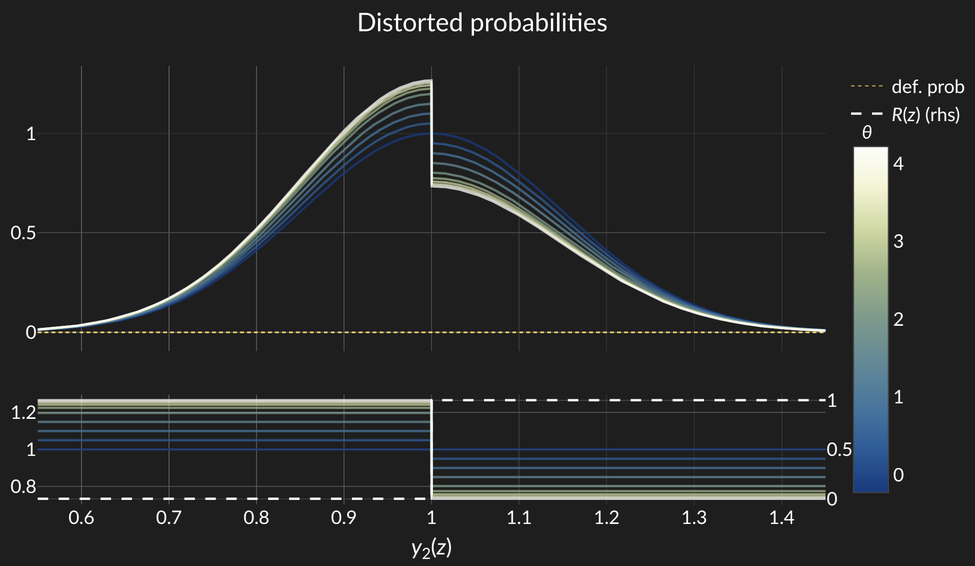
Distorted probabilities – linearly indexed debt

Back



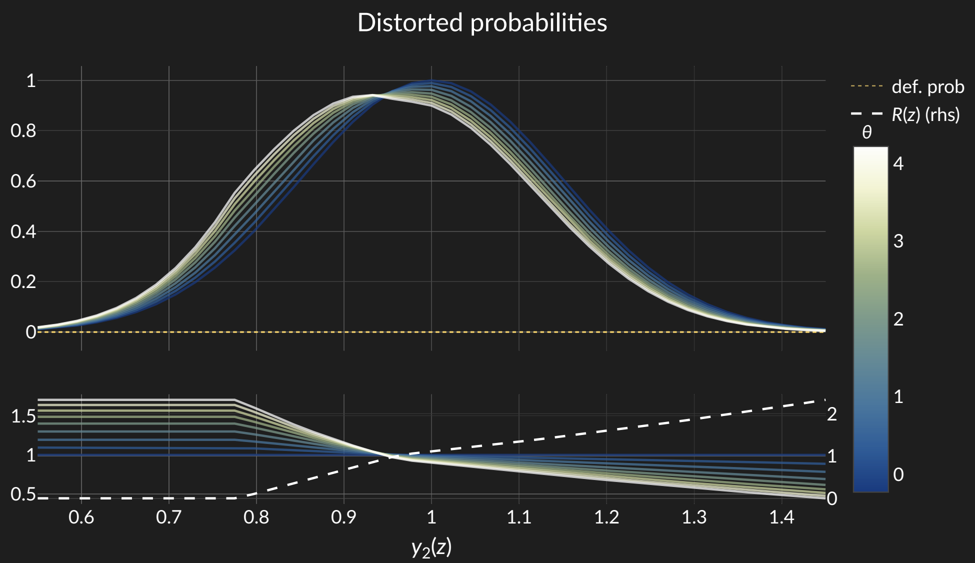
Distorted probabilities – threshold debt

Back



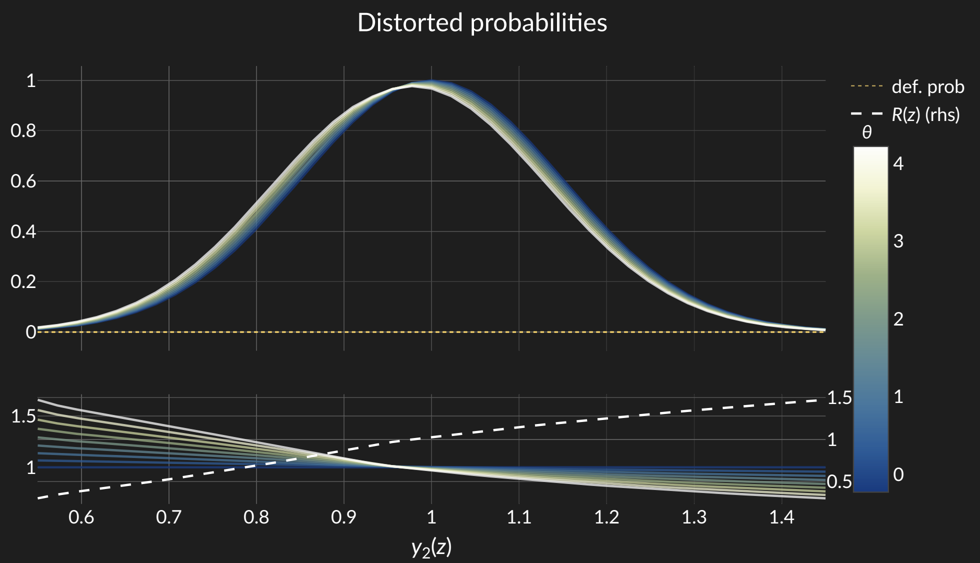
Distorted probabilities – debt for RE lenders

Back



Distorted probabilities – debt for robust lenders

◂ Back



Parametrization

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We represent this bond with threshold debt, one period = five years, and

Parameter	Target	Value
β_b	Borrower's discount rate	6% ann.
β	Risk-free rate	3% ann.
γ	Borrower's risk aversion	2
Δ	Output cost of default	20%
g	Expected growth rate	8% ann.
k	Threshold for repayment	50%

Decomposition of spreads

◂ Back

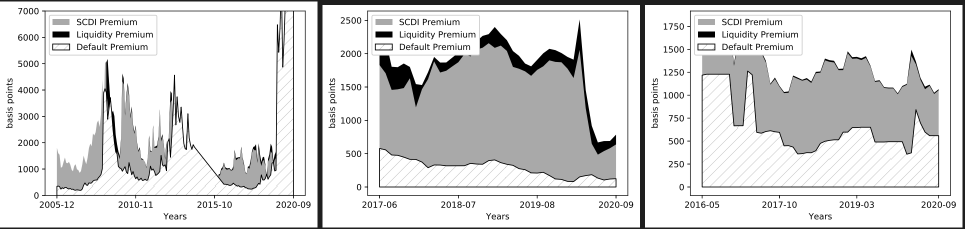

Figure 1: GDP-linked security premia.

The figure shows the estimated spread decomposition in Igan and Kim (2021) for the GDP-warrants issued by Argentina (left), Greece (middle) and Ukraine (right).