Uncertainty Premia, Sovereign Default Risk, and State-Contingent Debt

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Why do governments borrow noncontingent?

State-contingent debt instruments

- Decrease default risk
- Reduce cyclicality of fiscal policy
- Improve risk-sharing

Why aren't they used?

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Unfavorable prices of state-contingent instruments

- It seems that these instruments are heavily discounted by markets
  - Costa, Chamon, and Ricci (2008) compute wide spreads for Argentine GDP-warrants
    - $\sim$300-400bps from default risk of other securities
    - 600-1200bps residual: 'novelty' premium

This paper proposes a framework that

- Rationalizes pricing of SCI + welfare analysis
  - With ingredients from resolutions of the equity premium puzzle
- Links unfavorable prices to common 'threshold' structure
  - Example: Argentina's GDP-warrants
- Informs optimal design of state-contingent bonds

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A framework for pricing state-contingent debt

- Standard quantitative model of sovereign default with long-term debt
  - Aguiar and Gopinath (2006), Arellano (2008), Hatchondo and Martinez (2012)
- International lenders with concerns about *model misspecification*
  - Preference for robustness   Hansen and Sargent (2001), Costa (2009), Pouzo and Presno (2016)

- **Mechanism:** lenders act *as if* the probability of states with low repayment was higher
  - With noncontingent debt, lenders overestimate the default probability
  - Pouzo and Presno (2016) uses robustness to reconcile spreads with default frequencies
  - In general, probability distortion depends on type and quantity of debt issued

Main findings

1. Robust lenders dislike repayment structures with thresholds in good times
   - Heavy discounts for these bonds $\implies$ welfare losses

2. Explain most of the 'novelty premium' in Argentina's GDP warrants as ambiguity premia
   - Calibration of robustness from *noncontingent* debt only

3. Characterize the optimal design and how it changes with robustness
   - With high robustness, want to minimize ex-ante and ex-post contingency

Roadmap

- The Model
- Probability Distortions
- Pricing and Welfare
- Quantitative Results
- Concluding Remarks

The Model

The model

We consider a simple two-period model

- The government of a small open economy faces
  - Uncertain endowment z in the second period
  - A stochastic preference ξ for defaulting on debt
- The government has access to one asset which promises a return $R(z)$.
- A few benchmarks

Noncontingent debt	$R(z)$	$=$	1
Linear indexing	$R^{\alpha}(z)$	$=$	$1 + \alpha(y(z) - 1)$
Threshold debt	$R^{\tau}(z)$	$=$	$\mathbb{1}(z > \tau)$
Optimal design	$R^{\star}(z; \theta)$		chosen state-by-state

The government's problem

- The government takes as given the price schedule $q(b)$

$$
\begin{aligned}
& \max_b u(c_1^b) + \beta_b \mathbb{E}\left[u(c_2^b) - \xi d(b,z,\xi)\right] \\
\text{subject to } \quad & c_1^b = y_1 + q(b)b \\
& c_2^b = y_2(z) - h(z,\Delta)d(b,z,\xi) - (1 - d(b,z,\xi))R(z)b
\end{aligned}
$$

where

$$h(z,\Delta) = \phi y_2(z)\Delta + (1-\phi)y_2(z)^2\Delta$$

- In the second period, default if

$$\underbrace{u\left(y_2(z) - h(z,\Delta)\right)}_{\text{v. default}} - \xi > \underbrace{u\left(y_2(z) - R(z)b\right)}_{\text{v. repayment}}$$

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$$

The lenders' problem

Foreign lenders are less standard and have multiplier preferences

$$
\begin{aligned}
& \max c_1^L - \frac{\beta}{\theta} \log\left(\mathbb{E}\left[\exp(-\theta v_2^L)\right]\right) \\
\text{subject to } \; & v_2^L = c_2^L \\
& c_2^L = w_2 + (1 - d(b, z, \xi))R(z)b \\
& c_1^L = w_1 - q_1 b
\end{aligned}
$$

Lenders provide us with an Euler equation to price the debt

$$
q(b; R) = \beta \mathbb{E}\left[\frac{\exp(-\theta c_2^L)}{\mathbb{E}\left[\exp(-\theta c_2^L)\right]}(1 - d(b, z, \xi))R(z)\right]
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$$

Debt prices

- The lenders' Euler equation explains the sources of the spreads they charge
- Call $M = \beta \frac{\exp(-\theta c_2^L)}{\mathbb{E}\left[\exp(-\theta c_2^L)\right]}$ the stochastic discount factor

$$
\begin{aligned}
q(b;R) &= \beta \mathbb{E}\left[\frac{\exp(-\theta c_2^L)}{\mathbb{E}\left[\exp(-\theta c_2^L)\right]} (1 - d(b,z,\xi)) R(z) \right] \\
&= \underbrace{\beta \mathbb{E}\left[(1-d)R\right]}_{= q_{RE}} + \underbrace{\mathbb{E}\left[1-d\right] \operatorname{cov}(M,R)}_{= q_\theta^{\text{cont}}} + \underbrace{\mathbb{E}\left[R\right] \operatorname{cov}(1-d, M)}_{= q_\theta^{\text{def}}}
\end{aligned}
$$

- The debt price is a rational-expectations price and two sources of ambiguity premia

Distorted probabilities

Interpret lenders' stochastic discount factor as probability distortions

- For a random variable X

$$\tilde{\mathbb{E}}\left[X\right] = \mathbb{E}\left[\frac{\exp(-\theta c_2^L)}{\mathbb{E}\left[\exp(-\theta c_2^L)\right]} X\right]$$

- $\tilde{\mathbb{E}}$ tilts probabilities towards *less-favorable* states for lenders

- Obs The tilting is endogenous to the lenders' outcomes

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- **Obs** The tilting is endogenous to the lenders' outcomes

Probability Distortions

Parametrization

Costa, Chamon, and Ricci (2008) study the GDP-warrants issued by Argentina

- The warrant paid if
  - Output growth above pre-set level (4.3% initially, later 3%)
  - Output *level* above the compounded cutoff growth
  - There is also a cap on total payments
- Pricing
  - Spreads of about 1000bps since end-2006 (higher before)
  - About 300 bps explained by default risk (of other securities)

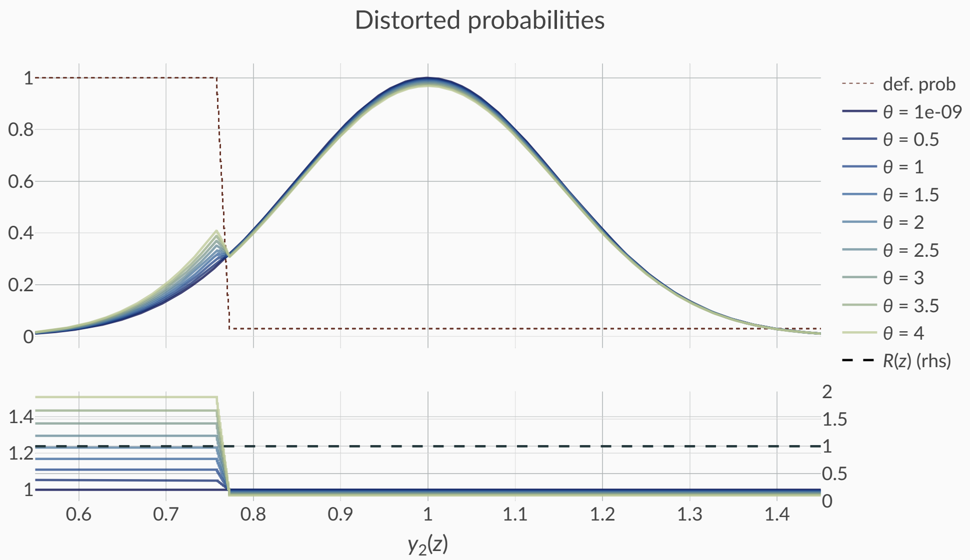
Parametrization

We represent this bond with threshold debt, one period = five years, and

Parameter	Target	Value
β_b	Borrower's discount rate	6% ann.
β	Risk-free rate	3% ann.
γ	Borrower's risk aversion	2
Δ	Output cost of default	20%
g	Expected growth rate	8% ann.
k	Threshold for repayment	50%

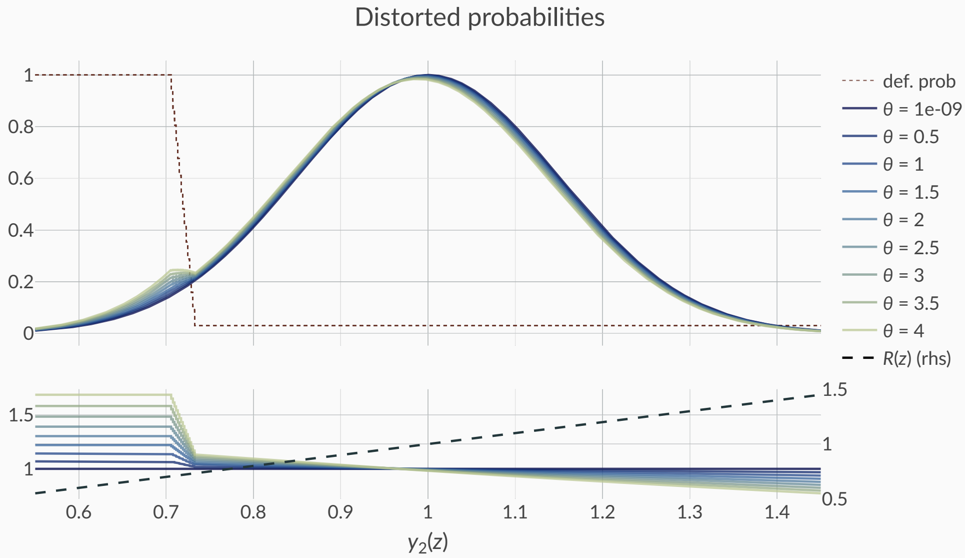
Distorted probabilities – noncontingent debt

Reoptimizing



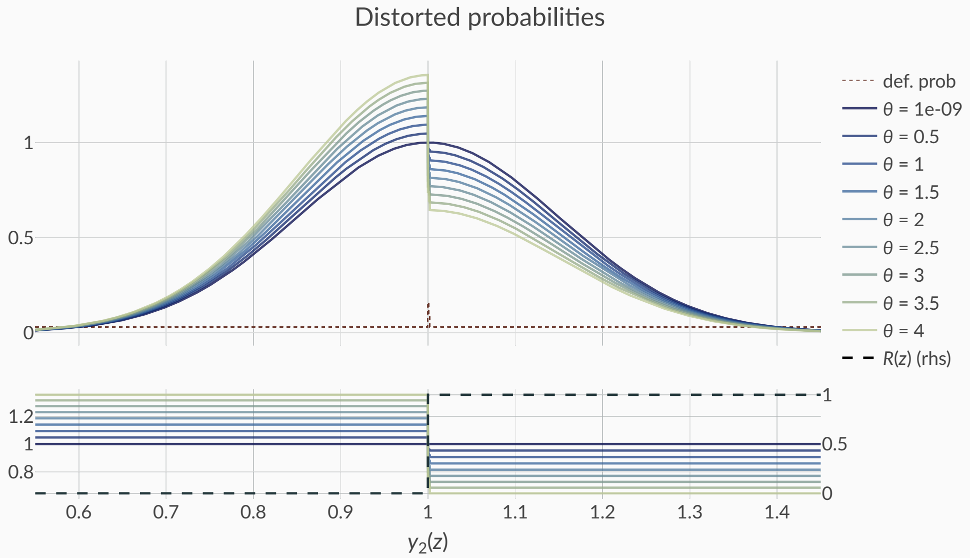
Distorted probabilities – linearly indexed debt

Reoptimizing



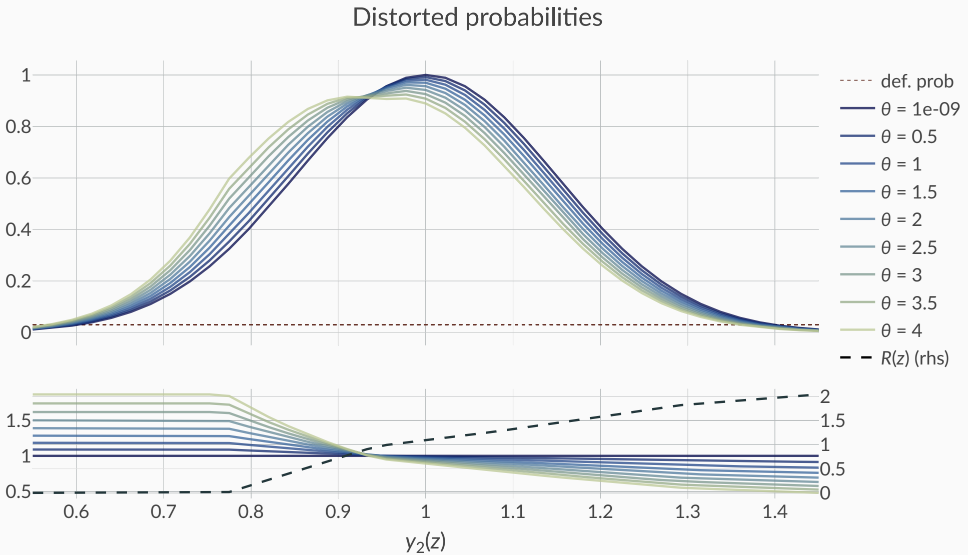
Distorted probabilities – threshold debt

▸ Linear+threshold ▸ Reoptimizing



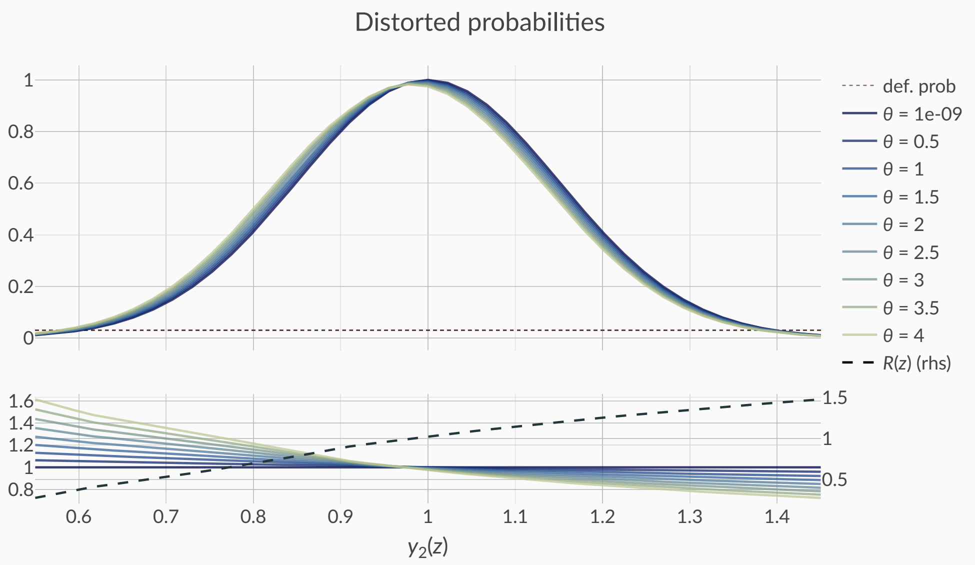
Distorted probabilities – debt for RE lenders

▸ Reoptimizing

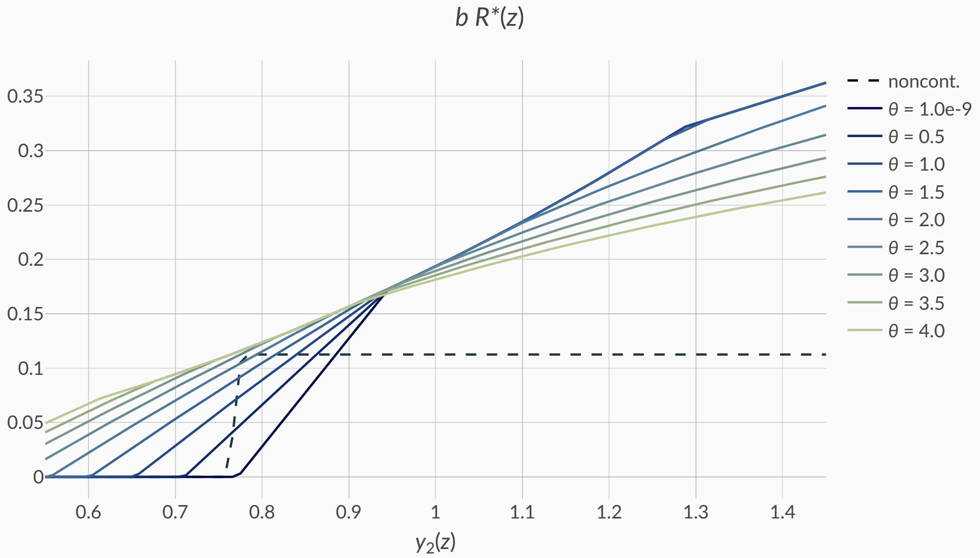


Distorted probabilities – debt for robust lenders

Reoptimizing

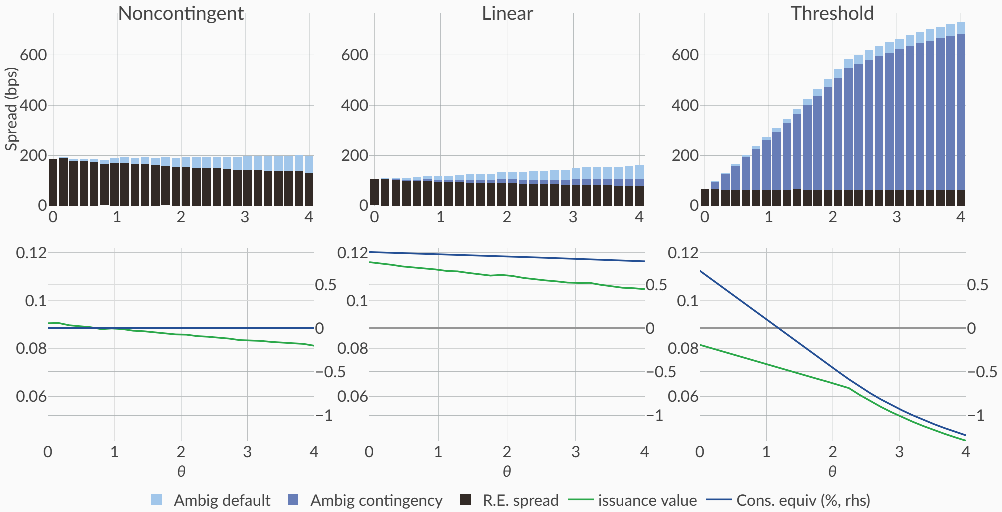


Design of debt

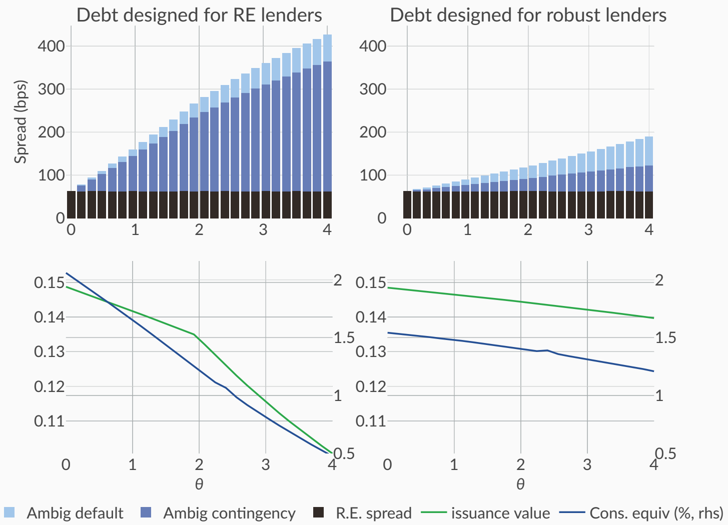


Pricing and Welfare

Parametric debt types



Optimal debt designs



Quantitative Results

Quantitative Model

- Infinite horizon, small-open economy
- Long-term debt, debt issued at t pays coupon at $t+s$

$$\max\left\{0, (1-\delta)^{s-1}(1+\alpha(y_s-1))\mathbb{1}(y_s>\tau)\right\}$$

- Default triggers exclusion + TFP costs for an amount of periods $\sim Geo(\psi)$

Calibration

	Parameter	Chatterjee and Eyigungor (2012)	Pouzo and Presno (2016)
Sovereign's risk aversion	γ	2	2
Interest rate	r	0.01	0.01
Income autocorrelation coefficient	ρ	0.9485	0.9484
Standard deviation of innovations	σ_ϵ	0.027	0.02
Reentry probability	ψ	0.0385	0.0385
Duration of debt	δ	0.05	0.05
Discount factor	β	0.95402	0.9627
Default cost: linear	d_0	-0.18819	-0.255
Default cost: quadratic	d_1	0.24558	0.296
Degree of robustness	θ	0	1.62
Linear coupon indexation	α	0	0
Coupon repayment threshold	τ	$-\infty$	$-\infty$

Table 1: Parameter values for the baseline parametrizations.

Robustness in the quantitative model

CE calibration

	Rational Expectations			$\theta = 1.6155$ (benchmark)		
Statistic	**Noncontingent**	**Threshold**	$\alpha = 1$	**Noncontingent**	**Threshold**	$\alpha = 1$
Spread	8.1	0.36	7.2	8.15	11.1	7.1
Std Spread	4.5	0.23	3.7	4.6	1.58	3.6
Debt	48.7	116.5	50.8	44.0	67.6	46.1
Std(c)/Std(y)	1.24	0.82	1.22	1.25	0.84	1.23
Corr(y,c)	0.98	0.55	0.98	0.98	0.93	0.98
Corr(y,tb/y)	-0.71	0.54	-0.67	-0.68	0.52	-0.64
Corr(y,spread)	-0.77	-0.87	-0.79	-0.76	-0.63	-0.77
Default Prob	5.5	0.3	5.3	3.0	0.0	2.6
Welfare Gains	-	1.19	0.09	-	-0.37	0.07

Table 2: Statistics based on Pouzo and Presno (2016)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with alpha = 1.

Optimal design of state-contingent debt

	Chatterjee and Eyigungor (2012)		Pouzo and Presno (2016)	
Statistic	Rational Expectations $\tau = 0.75, \alpha = 4$	Robustness $\tau = 0.8, \alpha = 3$	Rational Expectations $\tau = 0.875, \alpha = 7$	Robustness $\tau = 0.875, \alpha = 5$
Spread	0.02	2.83	0.1	2.8
Std Spread	0.02	0.11	0.04	0.13
Debt	119.8	95.7	79.3	65.9
Std(c)/Std(y)	0.8	0.99	0.76	0.96
Corr(y,c)	0.99	0.98	0.99	0.98
Corr(y,tb/y)	0.98	0.13	0.98	0.25
Corr(y,spread)	-0.42	-0.17	-0.91	-0.67
Default Prob	0.04	0.17	0.1	0.23
Welfare Gains	3.2	1.44	1.79	0.79

Table 3: Statistics based on Chatterjee and Eyigungor (2012) and Pouzo and Presno (2016) under the optimal state-contingent bond with and without robust lenders.

Concluding Remarks

Concluding Remarks

- Robustness is a viable explanation for high spreads on state-contingent debt
  - We explain about 60% of the spreads on Argentine GDP-warrants

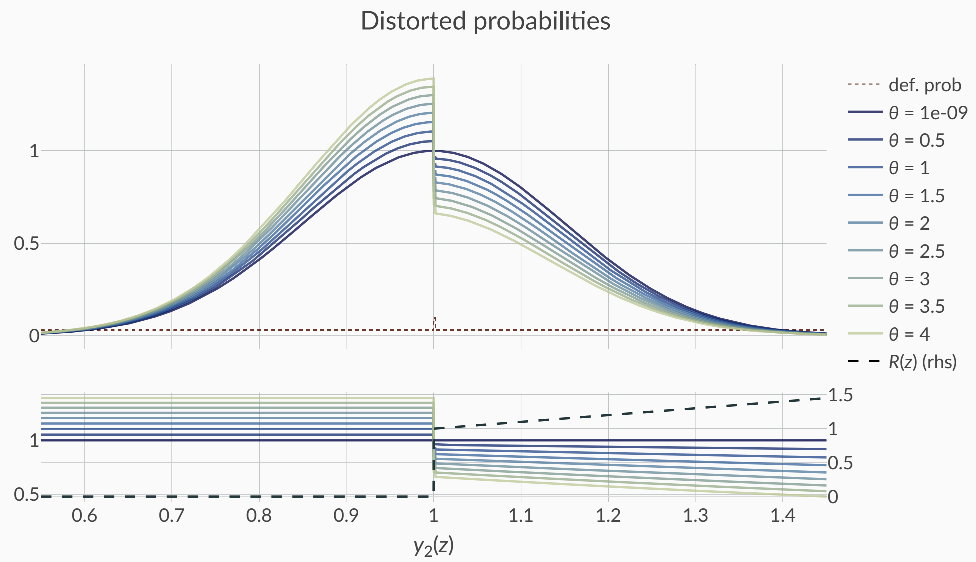
    $\ldots$ 90% with $\theta = 4$
  - Realistic parametrization but stylized model

- Key takeaway: robustness heavily discounts thresholds in likely states

- Other findings
  - 'Threshold' debt can worsen welfare relative to noncontingent
    - But good idea without robustness
  - 'Linear-indexed' debt can potentially do better
  - Characterized the optimal state-contingent instrument with robust lenders
    - Different than for rational-expectations lenders!

Distorted probabilities – threshold+linear debt

◂ Back



Quantitative model

◂ Back

	Rational Expectations (benchmark)			$\theta = 1.6155$		
Statistic	**Noncontingent**	**Threshold**	$\alpha = 1$	**Noncontingent**	**Threshold**	$\alpha = 1$
Spread	8.5	0.6	6.8	8.4	15.5	7.1
Std Spread	4.3	0.4	3.0	4.4	2.3	3.1
Debt	69.9	159.6	74.4	62.6	87.7	67.2
Std(c)/Std(y)	1.24	0.83	1.21	1.25	0.82	1.22
Corr(y,c)	0.98	0.53	0.98	0.98	0.94	0.98
Corr(y,tb/y)	-0.7	0.52	-0.62	-0.67	0.58	-0.6
Corr(y,spread)	-0.77	-0.87	-0.78	-0.75	-0.61	-0.77
Default Prob	5.8	0.56	5.3	2.3	0.12	1.8
Welfare Gains	-	1.86	0.27	-	-0.87	0.2

Table 4: Statistics based on Chatterjee and Eyigungor (2012)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with alpha = 1.

CARA

Euler equations of a rational-expectations agent with CARA preferences and access to a risk-free bond

$$q = \beta \mathbb{E}\left[\frac{u'(c_2)}{u'(c_1)} R\right] = \beta \mathbb{E}\left[\frac{\exp(-\gamma c_2)}{\exp(-\gamma c_1)} R\right]$$
$$\frac{1}{1+r} = \beta \mathbb{E}\left[\frac{u'(c_2)}{u'(c_1)}\right]$$

hence

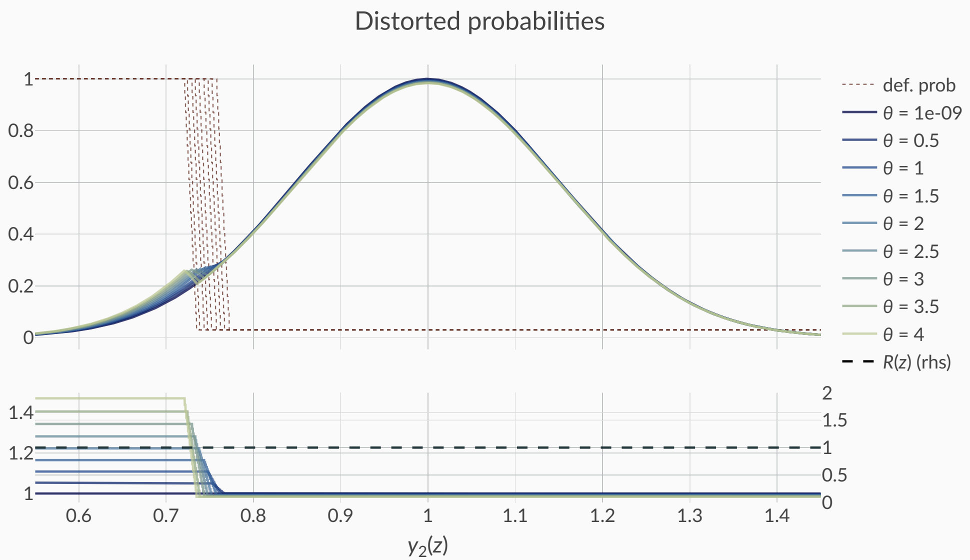
$$q = \beta \mathbb{E}\left[\frac{\exp(-\gamma c_2)}{\beta(1+r)\mathbb{E}\left[\exp(-\gamma c_2)\right]} R\right]$$

Same as robustness in two periods, in general the robust sdf is

$$q = \beta \mathbb{E}\left[\frac{\exp(-\theta v')}{\mathbb{E}\left[\exp(-\theta v')\right]} R\right]$$

Distorted probabilities – noncontingent debt

◂ Back



Distorted probabilities – linearly indexed debt

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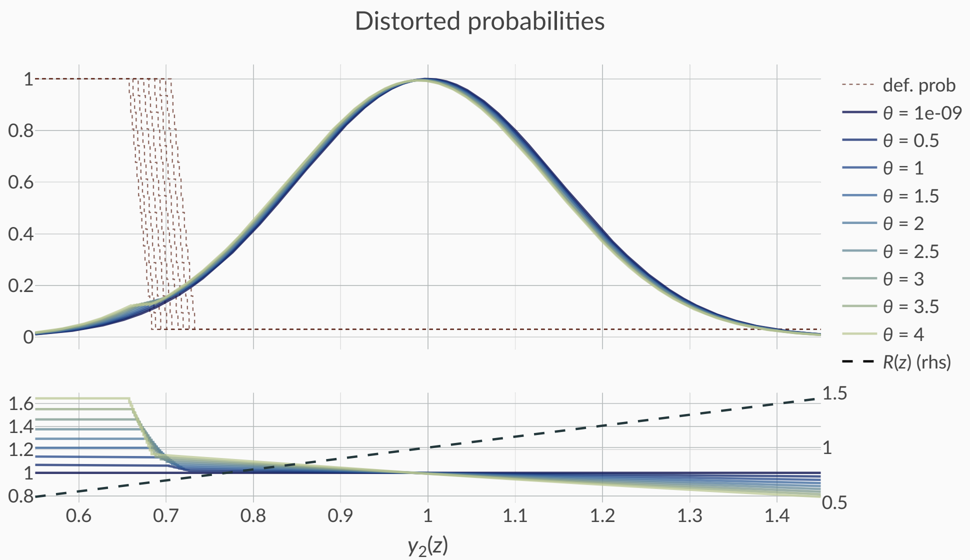

Distorted probabilities – threshold debt

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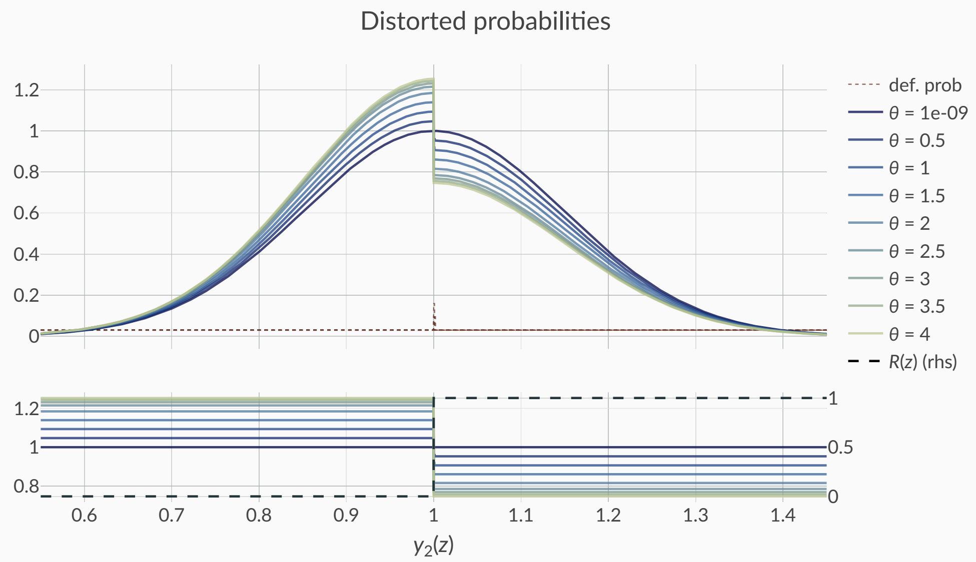
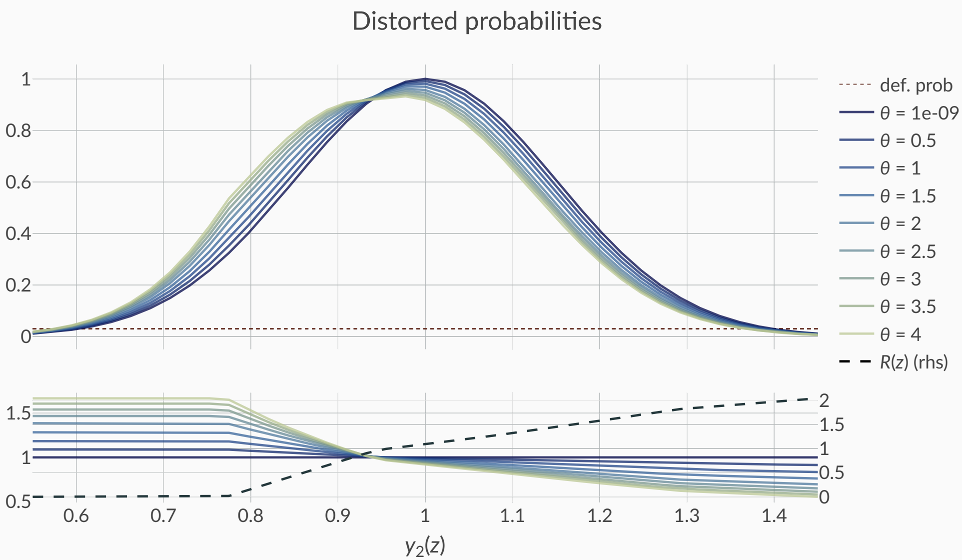


Distorted probabilities – debt for RE lenders

Back



Distorted probabilities – debt for robust lenders

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