

Uncertainty Premia, Sovereign Default Risk, and State-Contingent Debt

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State-contingent debt instruments

- Decrease default risk
- Reduce cyclicalities of fiscal policy
- Improve risk-sharing

Why aren't they used?

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Unfavorable prices of state-contingent instruments

- These instruments are heavily **discounted** by markets
 - Costa, Chamon, and Ricci (2008) compute wide spreads for Argentine **GDP-warrants**
 - ~300-400bps from default risk of other securities
 - 600-1200bps residual: '**novelty**' premium

This paper proposes a framework that

- Rationalizes **pricing** of SCI + **welfare** analysis
 - With ingredients from resolutions of the equity premium puzzle
- Links unfavorable prices to common 'threshold' structure
 - Example: Argentina's GDP-warrants, also Ukraine, Greece. . .

► More

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A framework for pricing state-contingent debt

- Standard quantitative model of sovereign default with long-term debt
 - Aguiar and Gopinath (2006), Arellano (2008), Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012)
- International lenders with concerns about *model misspecification*
 - Preference for **robustness** Hansen and Sargent (2001), Pouzo and Presno (2016)
- Mechanism: lenders act *as if* the probability of states with low repayment was higher
 - With noncontingent debt, lenders overestimate the default probability
 - Pouzo and Presno (2016) uses robustness to reconcile **spreads** with default **frequencies**
 - In general, probability distortion depends on type and quantity of debt issued

Main findings

1. Robust lenders dislike repayment structures with **thresholds** in good times
 - Heavy discounts for these bonds \implies welfare **losses**
2. Explain most of the 'novelty premium' in Argentina's GDP warrants as **ambiguity** premia
 - Calibration of robustness from *noncontingent* debt only
3. Characterize the **optimal** design and how it changes with robustness
 - With high robustness, want to minimize ex-ante and ex-post contingency

Roadmap

- Stylized Model
- Probability Distortions
- Pricing and Welfare
- Quantitative Implementation
- Concluding Remarks

Stylized Model

The model

We consider a simple two-period model, small open economy

- Uncertain endowment $y(z)$ in the second period
- The government has access to **one** asset which promises a return $R(z)$.
- A few benchmarks

Noncontingent debt	$R(z)$	$=$	1
Linear indexing	$R^\alpha(z)$	$=$	$1 + \alpha(y(z) - 1)$
Threshold debt	$R^\tau(z)$	$=$	$\mathbb{1}(z > \tau)$
Optimal design	$R^*(z; \theta)$		chosen state-by-state

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The government's problem

- The government takes as given the **price schedule** $q(b)$

$$\begin{aligned} & \max_b u(c_1^b) + \beta_b \mathbb{E} [u(c_2^b)] \\ \text{subject to } & c_1^b = y_1 + q(b)b \\ & c_2^b = y_2(z) - h(z, \Delta)d(b, z) - (1 - d(b, z))R(z)b \end{aligned}$$

where

$$h(z, \Delta) = y_2(z)^2 \Delta$$

- In the second period, **default** if

$$\underbrace{u(y_2(z) - h(z, \Delta))}_{\text{v. default}} > \underbrace{u(y_2(z) - R(z)b)}_{\text{v. repayment}}$$

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The lenders' problem

Foreign lenders are less standard and have **multiplier preferences**

$$\begin{aligned} & \max c_1^L - \frac{\beta}{\theta} \log (\mathbb{E} [\exp(-\theta v_2^L)]) \\ & \text{subject to } v_2^L = c_2^L \\ & c_2^L = w_2 + (1 - d(b, z))R(z)b \\ & c_1^L = w_1 - q_1 b \end{aligned}$$

Lenders provide us with an **Euler equation** to price the debt

$$q(b; R) = \beta \mathbb{E} \left[\frac{\exp(-\theta c_2^L)}{\mathbb{E} [\exp(-\theta c_2^L)]} (1 - d(b, z))R(z) \right]$$

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- The lenders' Euler equation explains the sources of the **spreads** they charge
- Call $M = \beta \frac{\exp(-\theta c_2^L)}{\mathbb{E}[\exp(-\theta c_2^L)]}$ the stochastic discount factor

$$\begin{aligned}
 q(b; R) &= \beta \mathbb{E} \left[\frac{\exp(-\theta c_2^L)}{\mathbb{E}[\exp(-\theta c_2^L)]} (1 - d(b, z)) R(z) \right] \\
 &= \underbrace{\beta \mathbb{E}[(1 - d)R]}_{= q_{RE}} + \underbrace{(1 - \mathbb{P}(d)) \text{cov}(M, R)}_{= q_{\theta}^{\text{cont}}} - \underbrace{\mathbb{E}[R] \text{cov}(M, d)}_{= -q_{\theta}^{\text{def}}}
 \end{aligned}$$

- The debt price is a rational-expectations price and two sources of **ambiguity** premia

Interpret lenders' stochastic discount factor as **probability distortions**

- For a random variable X

$$\tilde{\mathbb{E}}[X] = \mathbb{E} \left[\frac{\exp(-\theta v_2^L)}{\mathbb{E}[\exp(-\theta v_2^L)]} X \right]$$

- $\tilde{\mathbb{E}}$ **tilts** probabilities towards *less-favorable* states for lenders
- Obs The tilting is endogenous to the lenders' **outcomes**

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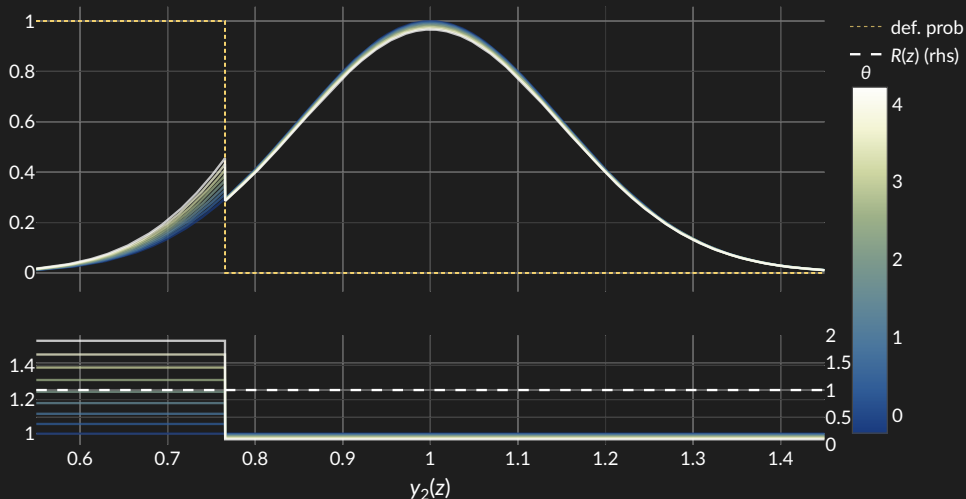
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Probability Distortions

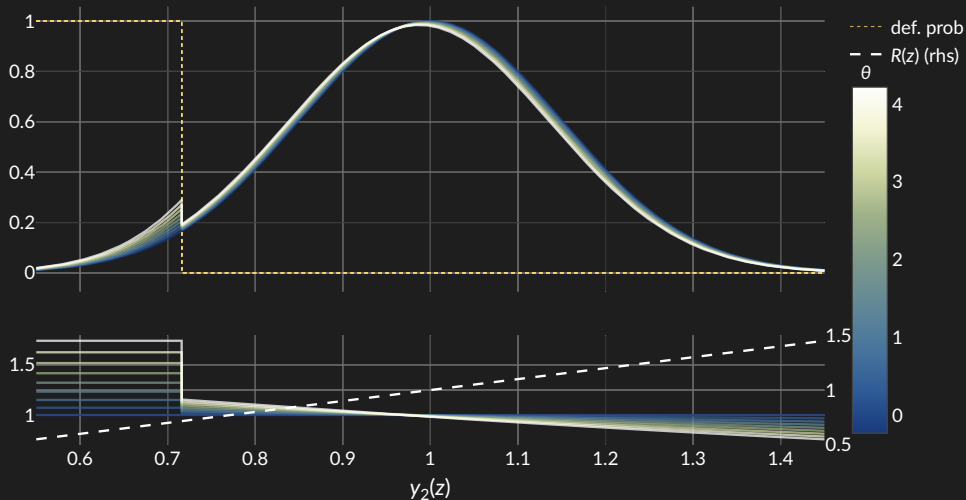
Costa, Chamon, and Ricci (2008) study the GDP-warrants issued by **Argentina**

- The warrant paid if
 - Output **growth** above pre-set level (4.3% initially, later 3%)
 - Output *level* above the compounded cutoff growth
 - There is also a cap on total payments

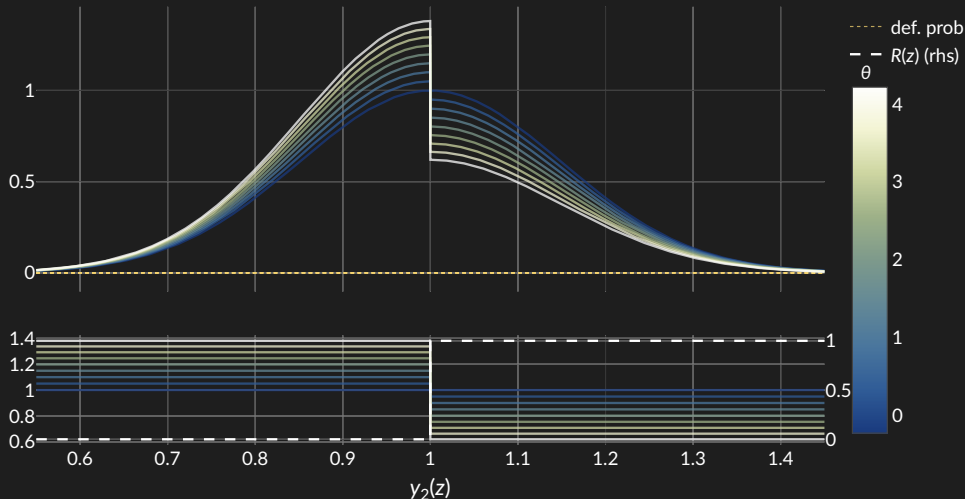
Distorted probabilities



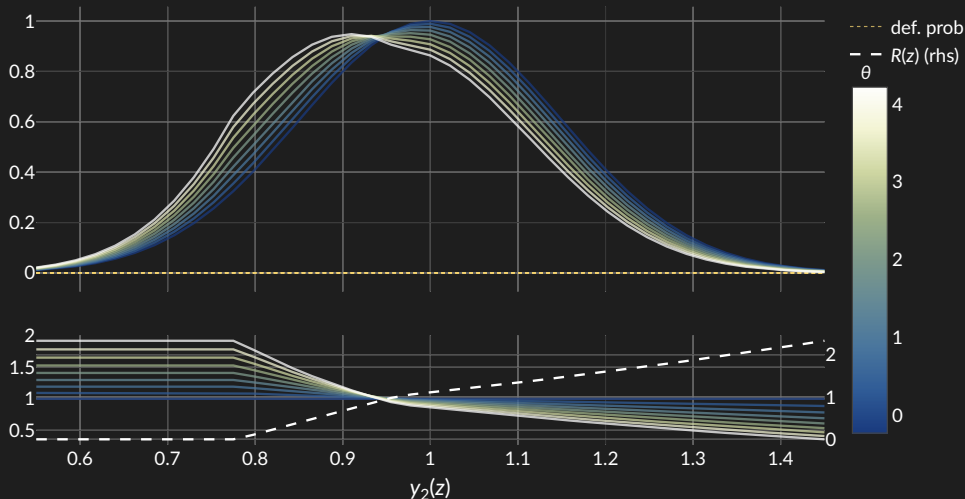
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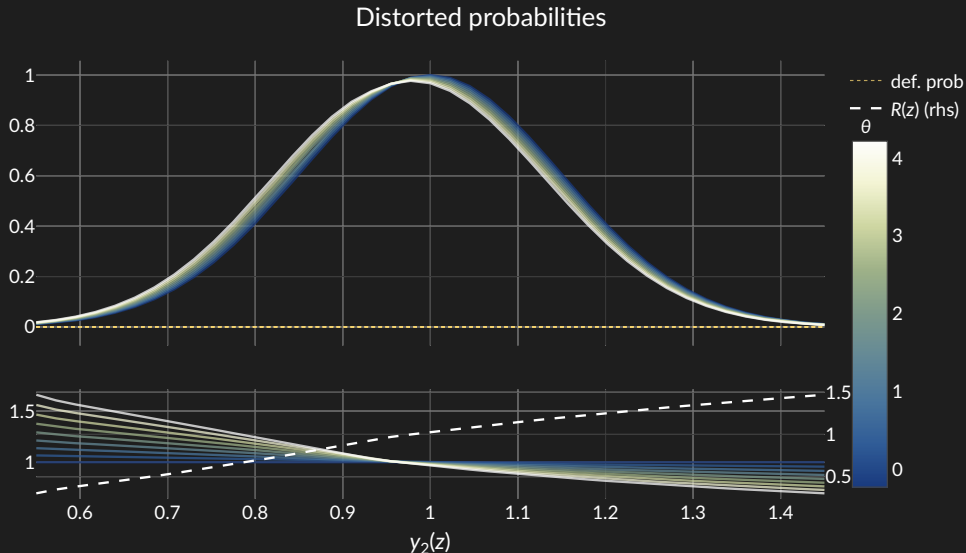


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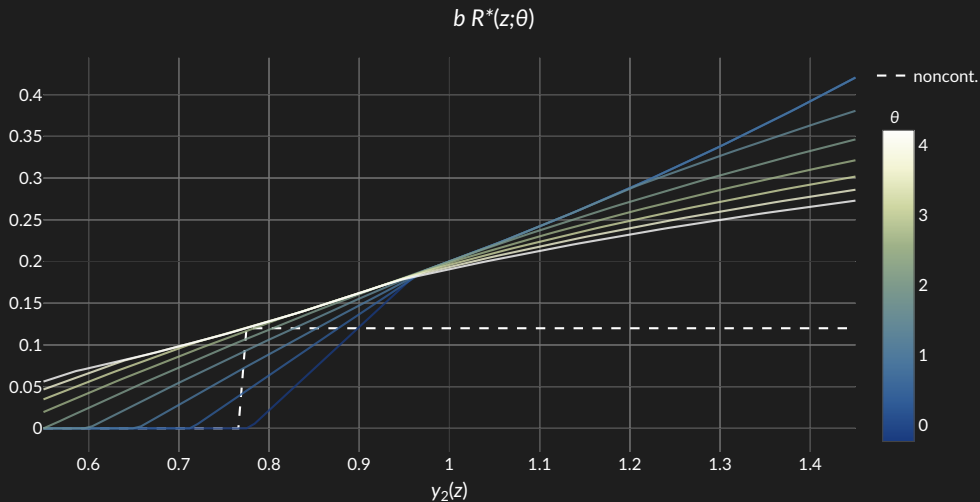


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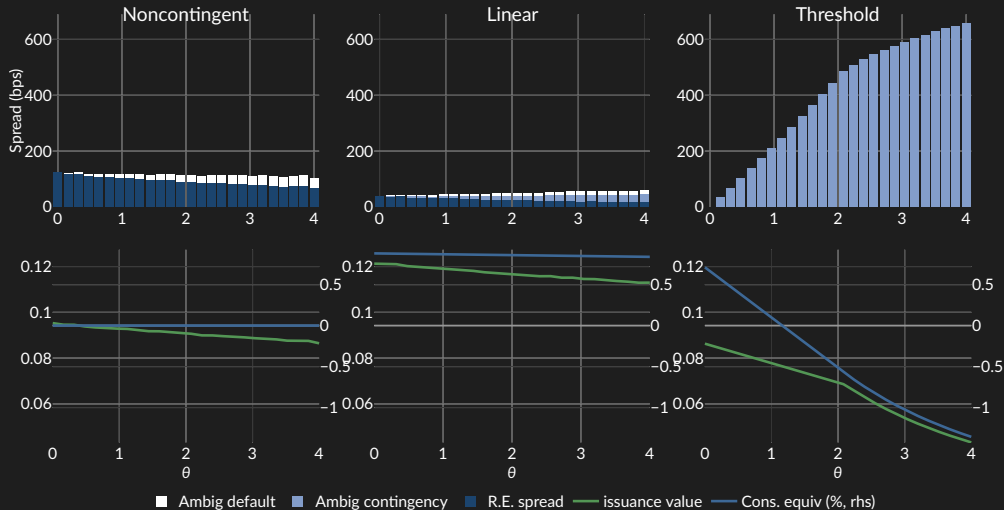


Design of debt

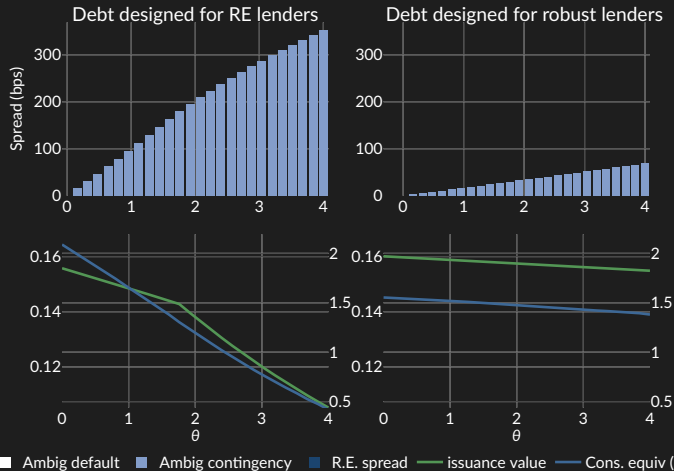


Pricing and Welfare

Parametric debt types



Optimal debt designs



Quantitative Implementation

- Infinite horizon, small-open economy
- **Robust** lenders as before
- Long-term debt, debt issued at t pays coupon at $t + s$

$$\max \{0, (1 - \delta)^{s-1} (1 + \alpha(y_s - 1)) \mathbb{1}(y_s > \tau)\}$$

- Noncontingent debt: $\alpha = 0, \tau = -\infty$
- Default triggers exclusion + output costs for a random amount of periods $\sim \text{Geo}(\psi)$

	Data	Benchmark	Rational Expectations
Spread	8.15	8.15	8.1
Std Spread	4.58	4.6	4.5
Debt	46	44	48.7
Std(c)/Std(y)	0.87	1.25	1.24
Corr(y,c)	0.97	0.98	0.98
Corr(y,tb/y)	-0.77	-0.68	-0.71
Corr(y,spread)	-0.72	-0.76	-0.77
Default Prob	3.0	3.0	5.5
DEP	-	31%	-

Note: Statistics computed in the model with noncontingent debt

Statistic	Rational Expectations			$\theta = 1.6155$ (benchmark)		
	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread	8.1	0.36	7.2	8.15	11.1	7.1
Std Spread	4.5	0.23	3.7	4.6	1.58	3.6
Debt	48.7	116.5	50.8	44.0	67.6	46.1
Std(c)/Std(y)	1.24	0.82	1.22	1.25	0.84	1.23
Default Prob	5.5	0.3	5.3	3.0	0.0	2.6
Welfare Gains	-	1.19	0.09	-	-0.37	0.07
DEP	-	-	-	31%	20%	30%

Table 1: Statistics from calibrated model simulations

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with $\alpha = 1$.

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Optimal design of state-contingent debt

Statistic	Rational Expectations	Robustness
	$\tau = 0.875, \alpha = 7$	$\tau = 0.875, \alpha = 5$
Spread	0.1	2.8
Std Spread	0.04	0.13
Debt	79.3	65.9
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Table 2: Statistics under the optimal state-contingent bond for different types of lenders

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Concluding Remarks

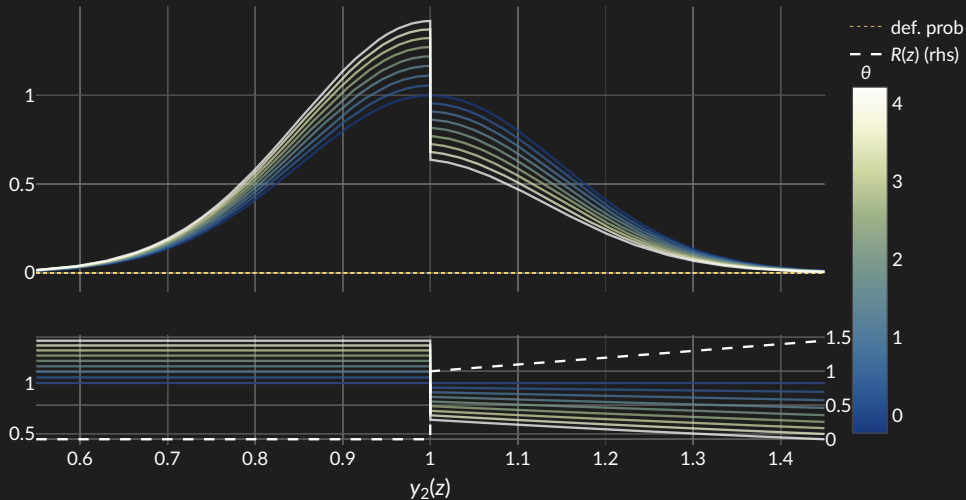
Concluding Remarks

- Standard sovereign debt model augmented with robust lenders
 1. accounts for **spreads** on typical threshold SCDIs
 2. rationalizes part of the 'novelty' premium as a premium for **ambiguity**
 3. links unfavorable prices to common *threshold* structure
 4. **Welfare** gains of SCDI decreasing in robustness
 - Both for given instrument and for optimally-designed debt
- Optimal design
 - With realistic robustness, lower thresholds and **flatter** indexation than RE
 - With extreme robustness, eliminate contingency ex-ante (*stipulated*) and ex-post (*default*)
 - In general, tradeoff between **contingency** and **risk-sharing**

Distorted probabilities – threshold+linear debt

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Distorted probabilities



Statistic	Rational Expectations (benchmark)			$\theta = 1.6155$		
	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread	8.5	0.6	6.8	8.4	15.5	7.1
Std Spread	4.3	0.4	3.0	4.4	2.3	3.1
Debt	69.9	159.6	74.4	62.6	87.7	67.2
Std(c)/Std(y)	1.24	0.83	1.21	1.25	0.82	1.22
Corr(y,c)	0.98	0.53	0.98	0.98	0.94	0.98
Corr(y,tb/y)	-0.7	0.52	-0.62	-0.67	0.58	-0.6
Corr(y,spread)	-0.77	-0.87	-0.78	-0.75	-0.61	-0.77
Default Prob	5.8	0.56	5.3	2.3	0.12	1.8
Welfare Gains	-	1.86	0.27	-	-0.87	0.2

Table 3: Statistics based on Chatterjee and Eyigungor (2012)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with $\alpha = 1$.

Euler equations of a rational-expectations agent with CARA preferences and access to a risk-free bond

$$q = \beta \mathbb{E} \left[\frac{u'(c_2)}{u'(c_1)} R \right] = \beta \mathbb{E} \left[\frac{\exp(-\gamma c_2)}{\exp(-\gamma c_1)} R \right]$$
$$\frac{1}{1+r} = \beta \mathbb{E} \left[\frac{u'(c_2)}{u'(c_1)} \right]$$

hence

$$q = \beta \mathbb{E} \left[\frac{\exp(-\gamma c_2)}{\beta(1+r) \mathbb{E} [\exp(-\gamma c_2)]} R \right]$$

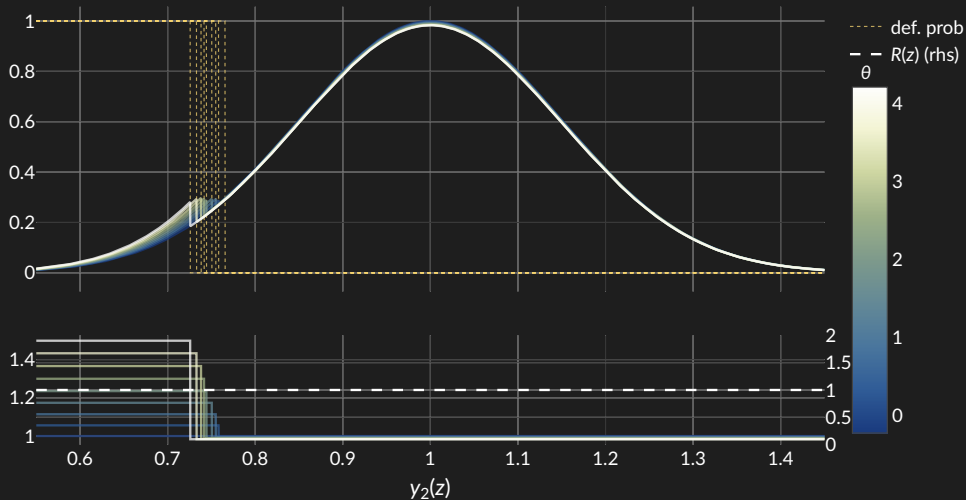
Same as robustness in two periods, in general the robust sdf is

$$q = \beta \mathbb{E} \left[\frac{\exp(-\theta \mathbf{v}')}{\mathbb{E} [\exp(-\theta \mathbf{v}')] } R \right]$$

Distorted probabilities – noncontingent debt

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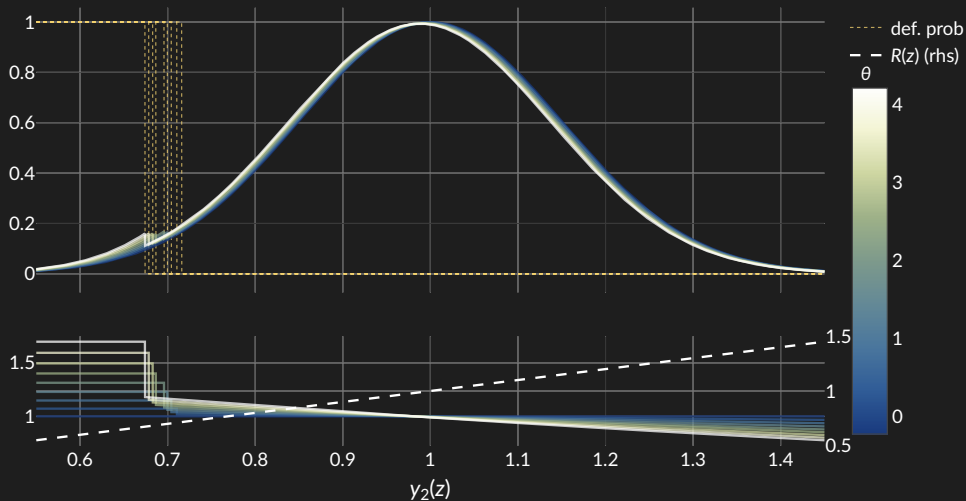
Distorted probabilities



Distorted probabilities – linearly indexed debt

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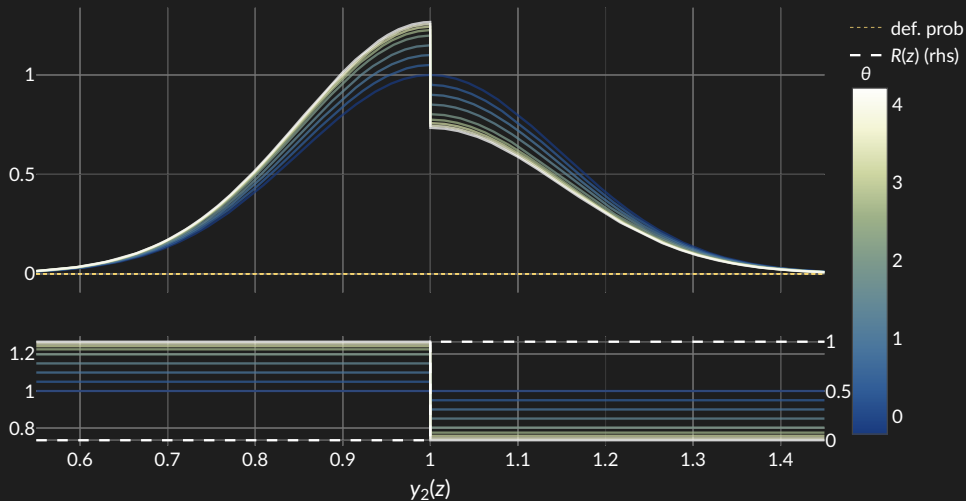
Distorted probabilities



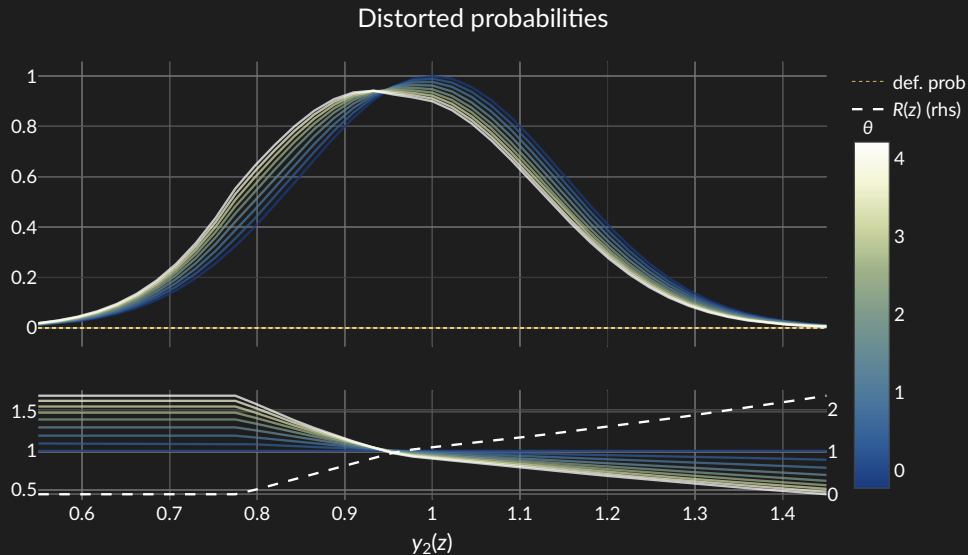
Distorted probabilities – threshold debt

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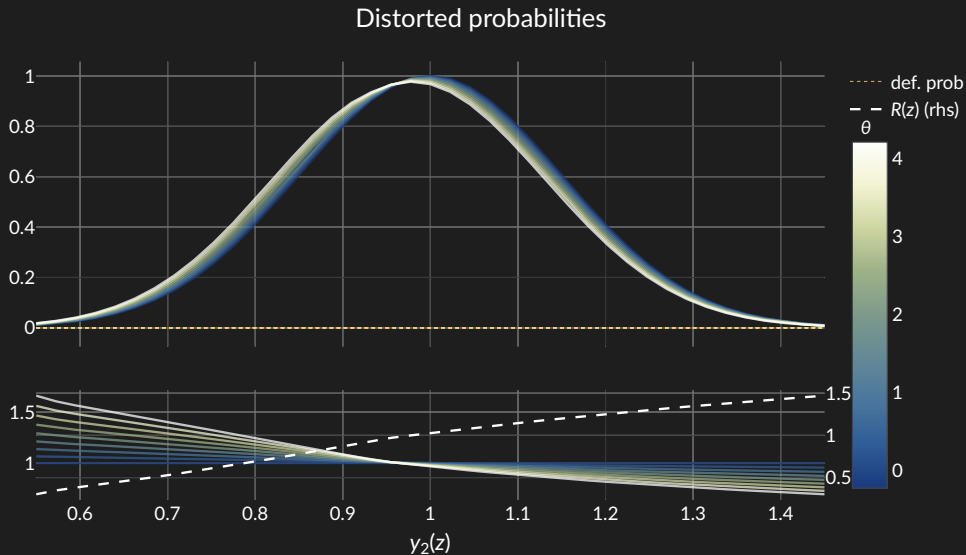
Distorted probabilities



Distorted probabilities – debt for RE lenders

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Distorted probabilities – debt for robust lenders

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We represent this bond with threshold debt, one period = five years, and

Parameter	Target	Value
β_b	Borrower's discount rate	6% ann.
β	Risk-free rate	3% ann.
γ	Borrower's risk aversion	2
Δ	Output cost of default	20%
g	Expected growth rate	8% ann.
k	Threshold for repayment	50%

Decomposition of spreads

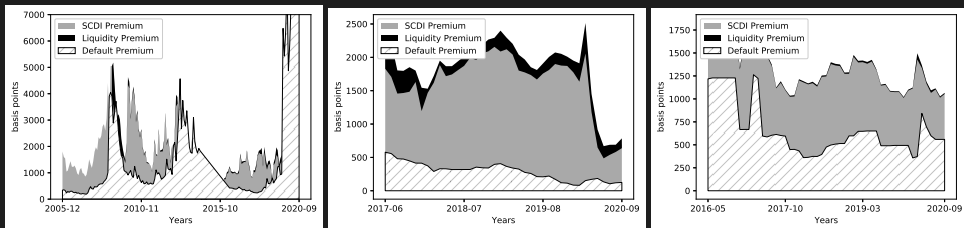


Figure 1: GDP-linked security premia.

The figure shows the estimated spread decomposition in Igan and Kim (2021) for the GDP-warrants issued by Argentina (left), Greece (middle) and Ukraine (right).