The Perils of Bilateral Sovereign Debt*

Francisco Roldán†
IMF

César Sosa-Padilla‡
University of Notre Dame
and NBER

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Abstract

We study the interaction between private and official sovereign debts. We develop a quantitative sovereign default model augmented by a large senior creditor with whom borrowing terms are negotiated. We use this model to evaluate concerns surrounding the emergence of new official lenders not bound by the Paris Club. The dynamics of the bilateral surplus lead the government to choose higher sovereign risk, ultimately creating welfare losses. This relational overborrowing effect arises due to an endogenous cross-elasticity of bilateral terms to market debt, which can be tested in practice to evaluate new forms of bilateral sovereign debt.

JEL Classification F34, F41, G15
Keywords Sovereign debt, debt dilution, bilateral bargaining, official debt

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†e-mail: *froldan@imf.org*
‡e-mail: *cesarspa@gmail.com*

Introduction

A large fraction of sovereign borrowing in emerging-market economies takes the form of official debt, including loans from other governments, regional development banks, or multilateral institutions such as the IMF and the World Bank. The past few decades have witnessed the rise of new sovereign creditors operating outside of the Paris Club institutional framework and with ambitions to redesign the financial architecture of sovereign debt (Horn et al., 2021b; Gelpern et al., 2021). The claims to senior creditor status by these new lenders as well as the opacity of their borrowing terms have raised concerns about welfare implications for borrowing countries (Horn et al., 2021a, 2023).

This paper evaluates such concerns in the context of a quantitative sovereign debt and default model, augmented by the presence of a large lender who offers an alternative source of funds. Relative to creditors in international capital markets, the large lender possesses a superior enforcement technology, which supports its claim to senior creditor status. In addition, in our baseline setup we assume that borrowing terms with the large creditor are determined through bargaining, which gives a crucial role to the dynamics of the bilateral relationship and its surplus.

We focus on the interaction between the market and the large lender as two possible financing sources. While the availability of bilateral loans from the large lender affects the government's behavior in debt markets, outcomes in debt markets also influence threat points in the bilateral negotiation. The interest rate charged by the large lender is therefore constrained by implicit competition from debt markets. But when default risk pushes up yields on marketable debt, the large lender is also able to charge a premium. Because there is lower or no default risk on this loan, such a premium only reflects the borrower's outside options.

The baseline model yields two main results: welfare losses and relational overborrowing. While bilateral loans can and are often used on the equilibrium path to avoid costly defaults, the borrowing country is worse off when the large lender is present. One reason for this is that the possibility of borrowing from the large lender while excluded from markets raises the value of default, which increases default incentives and lowers debt prices. But there is another, more fundamental reason. Even imposing the constraint that bilateral loans are unavailable during default, welfare for the government is still reduced by the presence of the large lender due to a "relational overborrowing" effect. Borrowing terms with the large lender are determined by splitting the bilateral surplus. This surplus is largest when sovereign risk is present and the government is paying high spreads on its debt, which is precisely when the large lender is most needed. Consequently, when it expects the government to face high spreads in the future, the large lender values the relationship more and is willing to invest in it by lending more cheaply. This endogenous elasticity of bilateral loan terms to indebtedness in markets acts as a countervailing force to the market discipline of spreads upon large issuances of debt. The presence of the large lender then leads the government to take on more debt in markets and to delever more slowly. The end result is an increase in the default frequency and therefore in spreads, which ultimately create welfare losses.

The dynamics and welfare effects we describe result from three critical assumptions regarding the large creditor and the funding it offers. Compared to competitive markets, we assume that it is more difficult (or impossible) to default on the large creditor, that its loans are of shorter maturity, and that the terms of borrowing result from bilateral bargaining rather than competition and a zero-profit condition. The prime example of these are the Central Bank swap lines that Horn et al. (2021b) identify as one of the main forms under which official lending, largely from China, has returned to the fore since the 2000s.

Other forms of official debt share these seniority and duration characteristics, including notably IMF programs and certain lines and repo facilities with large central banks such as the Fed or the ECB. In these cases, however, borrowing terms are fixed in advance by institutional frameworks such as the IMF surcharges policy (IMF, 2024). The Fed's liquidity lines typically involve a spread of 25bps over policy rates (Bahaj and Reis, 2023) and are reported in real-time on the FRB New York's website. We capture this feature in an extension of the model in which we replace the bargaining protocol by fixed rules for determining bilateral borrowing terms. This extension allows us to ask questions about the optimal design of these facilities and also sheds light on the dynamics underlying our main results.

When the interest rate of the bilateral loan does not respond aggressively to indebtedness in debt markets, the relational overborrowing effect is muted or disappears. In this case, the fact that the large lender can help avoid defaults on marketable debt becomes the dominating force, and the equilibrium features lower default frequencies, better market financing terms, and welfare gains for the government.

A key quantity in the model is the government's bargaining power relative to the large lender. When the government can make take-it-or-leave-it offers (i.e., when it has full bargaining power), the large lender simply provides nondefaultable loans at the risk-free rate, recovering the model of Hatchondo, Martinez, and Önder (2017). In this case, the government's welfare is also higher relative to the equilibrium without the large lender. In our preferred calibration, however, such gains quickly dissipate as the government's bargaining power declines.

In the baseline model, we assume that the large lender is purely motivated by profits. In fact, it shares the objectives, risk attitudes, and intertemporal preferences of the competitive creditors who lend in debt markets. We take this assumption to focus on the pure effect of market structure but discuss in the context of the extension what to expect if the large lender had other motivations. However, the feedback from debt levels to the bilateral surplus which gives rise to the relational overborrowing effect will be present unless the large lender has a strong motivation to avoid default on the government's marketable debts. In this sense, our main results appear robust to a much more realistic description of the large lender in a specific application.

Quantitatively, the model shows how bilateral loans that are small relative to the stock of marketable debt can still have an important impact on outcomes in sovereign debt markets. The difference in maturities between bilateral loans and marketable debt is critical in generating this result. The longer maturity of debt means that the debt service coming due in a particular period is of the same order of magnitude as the amount coming due on the bilateral loan. As a result, changes in the interest rate of the bilateral loan have an impact on the current-period budget constraint of similar magnitude to changes in the interest rate on marketable debt. In an extension, we show that with short-term market debt, the relational overborrowing effect is again largely muted. In reality, bilateral loans such as swap lines are typically of a much shorter duration than marketable debt.

The model implies a simple heuristic for determining whether a particular type of bilateral debt is likely to be beneficial to the government. The welfare effects of bilateral loans are determined by the elasticity of their interest rate to the level of *market* debt. If bilateral loans become cheaper when market debts are high, the relational overborrowing effect is likely to be present. Conversely, if bilateral loans are cheaper when market debts are low, they are more likely to support debt sustainability. These forces act in addition to better-known effects operating through the value of default, as emphasized for example in the Lending Into Arrears policies of the IMF (2022).

Discussion of the Literature We contribute to a nascent literature on the interaction of different types of sovereign debt. Excellent surveys of the broader literature on sustainable public debt and sovereign default can be found in handbook chapters by Aguiar and Amador (2014), Aguiar, Chatterjee, Cole, and Stangebye (2016), D'Erasmo, Mendoza, and Zhang (2016), and Martinez, Roch, Roldán, and Zettelmeyer (2023).

Hatchondo, Martinez, and Önder (2017) find that introducing a limited amount of undefaultable debt improves the government's welfare but only for a short period of time. Boz (2011) and Fink and Scholl (2016) study the interaction of market debt with a type of senior debt coming with conditionality. Kirsch and Rühmkorf (2017) and Roch and Uhlig (2018) investigate the role of official lending or bailout agencies in eliminating equilibrium multiplicity, as do Corsetti, Guimarães, and Roubini (2006). More generally, Kovrijnykh and Szentes (2007) and Faria-e-Castro et al. (2024) describe related situations featuring strategic behavior of creditors.

More recently, Liu, Liu, and Yue (2025) show how the best subgame-perfect equilibrium of a sovereign debt model with market, bilateral, and multilateral creditors decentralizes the constrained-efficient allocation with imperfect information and moral hazard. Arellano and Barreto (2025) combine market and official debt in a model of partial default (Arellano et al., 2023) and find that longer maturites, like those associated with bilateral lending from the Paris Club, endogenously make official debts less risky despite more favorable restructuring terms.

Empirically, Perks et al. (2021), and Bahaj and Reis (2021, 2023) document the network of Central Bank swap lines. Cesa-Bianchi et al. (2022) study swap lines among advanced economies, where they can serve a different purpose.

Layout The rest of the paper is structured as follows. Section 2 presents some evidence on official and marketable debts in emerging markets. Section 3 then introduces our model, starting with the case in which only bilateral loans are available. Section 4 describes the main model in which both types of debt coexist, while Section 5 analyzes its equilibrium, and Section 6 describes the extension with fixed rules for the terms of bilateral loans. Finally, Section 7 concludes.

2. Data

Figure 1 summarizes the evolution of all official sovereign debts captured in the World Bank's International Debt Statistics dataset. Debts include public and publicly-guaranteed debts (PPG) measured in constant dollars of 2023 and aggregated across all debtor countries.

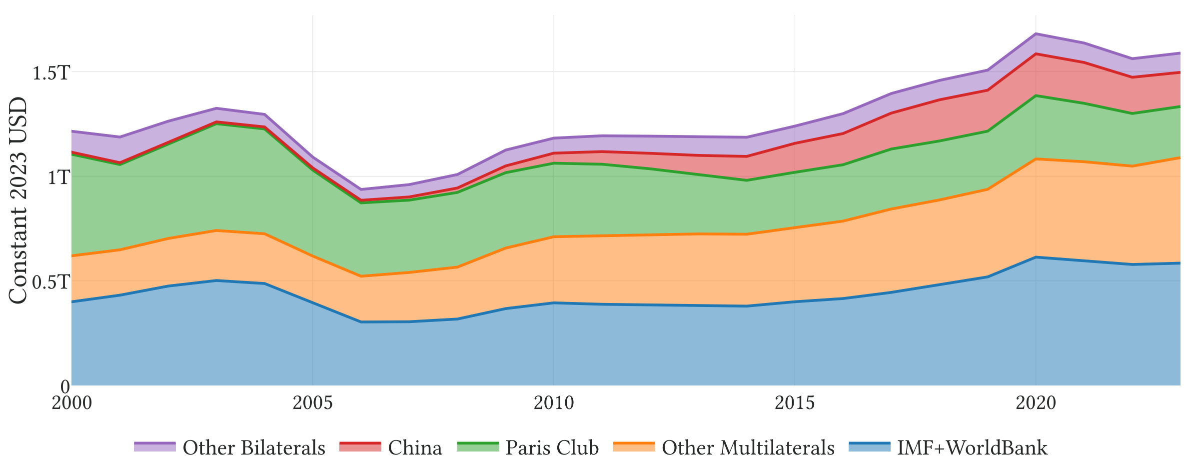

Figure 1: Total Official Debt

Source: International Debt Statistics, The World Bank

The rise of bilateral lenders outside of the Paris Club can be seen clearly in Figure 1. Starting in the mid to late 2000s, the importance of these lenders grows from a small share to roughly half of all bilateral debt owed by emerging market economies. As of end-2023, official debt is made up of three thirds: IMF and World Bank, other multilaterals, and bilaterals.

3. Model with bilateral loans only

We begin our analysis by studying a simple model in which only bilateral loans are possible. This first model serves to clarify the dynamics of bargaining in bilateral lending and the strategy through which the large lender extracts surplus: subsidized terms while debt accumulates, combined with high interest rates when the debt stock becomes large and the borrower reduces leverage.

We model a small open economy borrowing from a monopolist. The economy receives an endowment stream $y(z)$ where the state z follows an AR(1) process. Loans are short-term and therefore effectively continuously renegotiated. At the start of t, let $v(m, z)$ represent the value attained by the government (or sovereign, or borrower) at income state z and owing m to the monopolist. The lender similarly attains a value $h(m, z)$.

At the beginning of period t, borrower and lender negotiate over the terms of the loan. In the absence of default, payment of the full amount m extinguishes any debts and serves as a natural threat point. We use a simple Nash bargaining framework and set θ as the lender's bargaining power. The outcome of this negotiation is a transfer x and a new loan size m' determined by the solution to

$$\max_{x,m'} \mathscr{L}(x, m, m', z)^{\theta} \times \mathscr{B}(x, m, m', z)^{1-\theta} \tag{1}$$

where $\mathscr{L}$ and $\mathscr{B}$ represent the lender and borrower surplus functions. It will be useful to keep track of the implicit price ϕ and interest rate r of the loan which satisfy

$$x = \phi m' - m = \frac{1}{1+r} m' - m \tag{2}$$

After negotiations are concluded and transfers settled, consumption takes place. The lender finances the transfer x with a constant endowment a and thus consumes $c_L = a - x$. Conversely, the borrower receives the transfer so $c = y(z) + x$. Under risk neutral preferences for the lender,

$$\begin{aligned} \mathscr{L}(x, m, m', z) &= a - x + \beta_L \mathbb{E}\left[h(m', z') \mid z\right] - \left(a + m + \beta_L \mathbb{E}\left[h(0, z') \mid z\right]\right) \\ &= -x - m + \beta_L \mathbb{E}\left[h(m', z') - h(0, z') \mid z\right] \end{aligned}$$

and similarly

$$\mathscr{B}(x, m, m', z) = u(y(z) + x) - u(y(z) - m) + \beta \mathbb{E}\left[v(m', z') - v(0, z') \mid z\right]$$

where β_L and β are the discount factors of the lender and borrower, respectively. The borrower's utility function u is increasing and concave.

Notice that the choice of m' only involves continuation values v and h, while the choice of x only involves flow payoffs. Given a choice of m', the first-order condition for x is

$$\mathscr{B}(x, m, m', z)\theta = \mathscr{L}(x, m, m', z)u'(y(z) + x)(1 - \theta)$$

Given the solution $x(m, z), m'(m, z)$ to (1), the value functions satisfy

$$\begin{aligned} v(m, z) &= u(y(z) + x(m, z)) + \beta\mathbb{E}\left[v(m'(m, z), z') \mid z\right] \\ h(m, z) &= a - x(m, z) + \beta_L\mathbb{E}\left[h(m'(m, z), z') \mid z\right] \end{aligned} \tag{3}$$

Finally, we normalize $a = 0$, which allows us to interpret $h(m, z)$ as the expected present discounted value of transfers along the equilibrium path, or the lender's total expected profits.

3.1 *Equilibrium with bilateral loans only*

We solve the model with bilateral loans only at a parametrization that illustrates the forces at play. We choose $\theta = 0.5$ so the surplus is split equally between borrower and lender; we also set $\beta = \beta_L$ to isolate consumption smoothing and bargaining from the frontloading motive that would result if the borrower was relatively impatient, which in sovereign debt models tends to be the relevant case.

Figure 2 summarizes the terms of the new loan for each level of income z and initial loan size m. Unsurprisingly, the borrower economy delevers in high-income states and receives positive transfers in low-income ones. The monopolist makes intense use of the interest rate to extract surplus. When both debt and income are low, the monopolist offers subsidized and even negative rates. The benefit of incurring this cost is to induce high levels of debt, which make the borrower's threat point more costly to exercise. Once the loan size is large, repaying it in full becomes difficult and the monopolist is able to charge much higher interest rates.

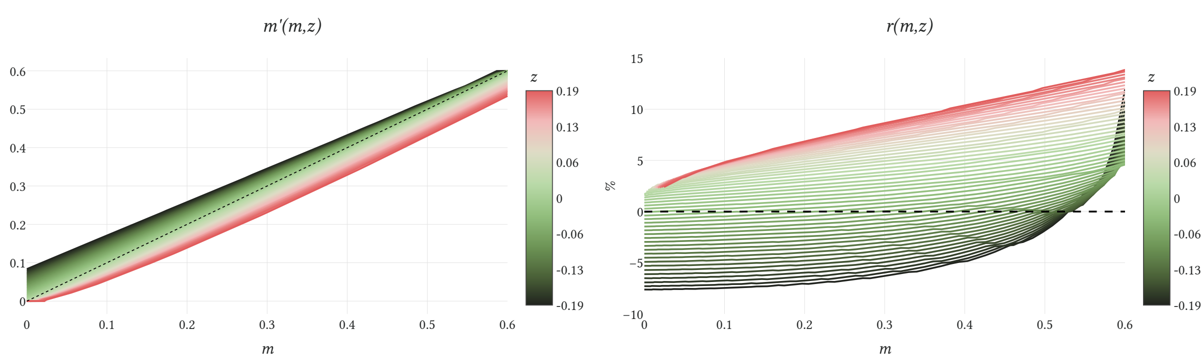

FIGURE 2: MONOPOLIST'S TERMS WITH $\theta = 0.5$

Figure 3 shows the value functions v and h for borrower and lender. As indebtedness m increases, the borrower's threat point becomes less credible. This increases the total surplus available (new transfers are more valuable) but also makes the lender 'stronger' in the negotiation. This allows the lender to charge higher interest rates. This effect creates convexity in the lender's profits and, hence, in the value function h.

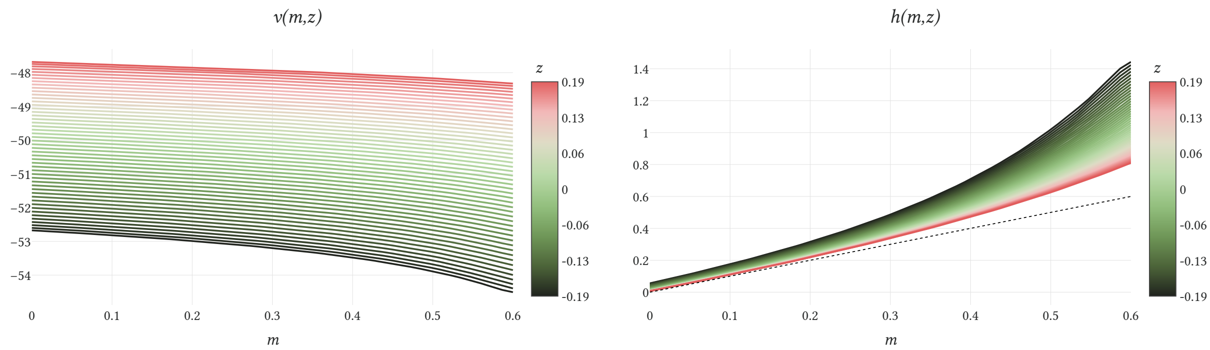

FIGURE 3: VALUE FUNCTIONS, $\theta = 0.5$

Convexity in the lender's value function implies endogenously risk-loving behavior. In equilibrium, the lender gambles for debt overhang. Subsidizing new loans in order to induce high indebtedness only pays off if the borrower's income takes a long time to recover. If the borrower receives a favorable income shock quickly, the loan is repaid before the monopolist has had an opportunity to raise rates and collect profits.

Figure 4 shows a simulation path, which further clarifies the lender's strategy. The loan is subsidized on the way up and, once debt has accumulated, the interest rate can increase to extract profits from the borrower. The borrower government anticipates these dynamics: the relationship between the initial subsidy and the expected high rates later on is disciplined by the requirement to deliver part of the surplus to the borrower, depending on the value of θ.

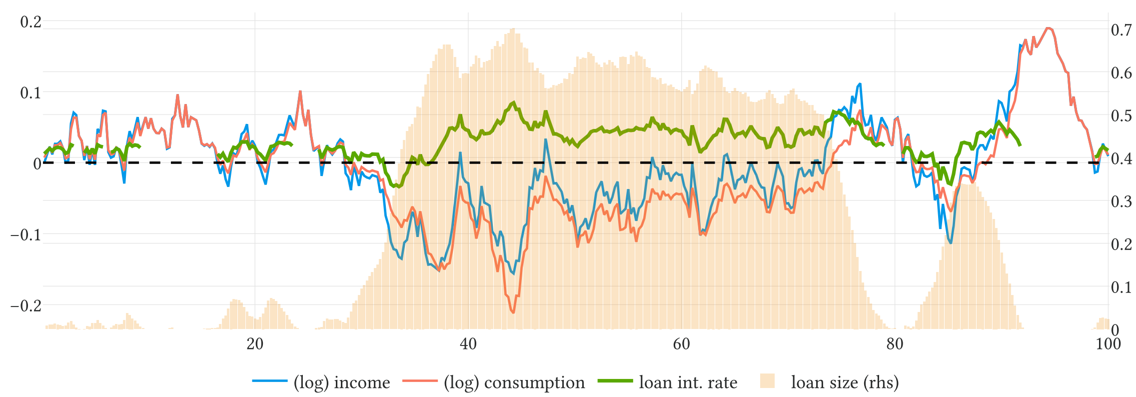

FIGURE 4: SIMULATED PATH, $\theta = 0.5$

Figure 14 in the Appendix, which simulates a model with $\theta = 0$, shows that when the borrower holds all the bargaining power, it is able to borrow at rate β_L^{-1} at all times. Because rates do not go up once the loan is large, they cannot be negative when it is still small. This effectively recovers an income fluctuations problem at the risk-free rate without default.

4. Model with bilateral loans and defaultable debt

In this section we present the full version of the model, in which the borrowing government has access to the monopolist as well as a competitive fringe of lenders. Default on the debt b held by competitive lenders is possible, subject to standard output costs of default. However, for the same reasons as before, bilateral loans m cannot be defaulted.

A period takes place as follows.

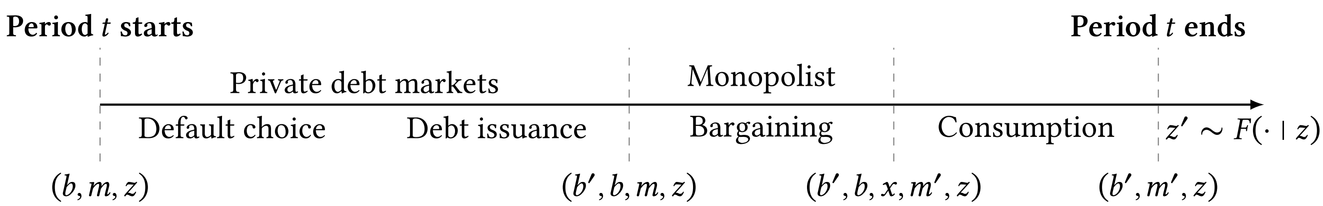

Figure 5: Timeline of events while not in default

At the start of t, the government owes m to the monopolist, b to the fringe, and observes the exogenous state z. Additionally, the economy can be in default ($\zeta = D$) or in repayment ($\zeta = R$). Let $v(b, m, z)$ and $h(b, m, z)$ represent the government's and the monopolist's value functions at the beginning of a period, when there is no default.

Private markets In the morning of t, first, the government decides default for the current period if it is in good standing with the market.

$$v(b, m, z) = \max \{v_R(b, m, z) + \epsilon_R, v_D(m, z) + \epsilon_D\} \tag{4}$$

where the ϵ's follow a Type 1 Extreme Value distribution, yielding closed forms for $v(b, m, z)$ and the ex-post default probability $\mathscr{P}(b, m, z)$

$$v(b, m, z) = \chi \log \big(\exp(v_D(m, z)/\chi) + \exp(v_R(b, m, z)/\chi) \big)$$
$$\mathscr{P}(b, m, z) = \frac{\exp(v_D(m, z)/\chi)}{\exp(v_D(m, z)/\chi) + \exp(v_R(b, m, z)/\chi)}$$

If it is not in default, the government issues new debt b' to the fringe of lenders understanding the value of entering negotiations with the monopolist having issued debt b'

$$v_R(b, m, z) = \max_{b'} w_R(b', b, m, z) \tag{5}$$

where debt is a perpetuity of geometrically-decaying coupons (Leland, 1998; Hatchondo and Martinez, 2009; Arellano and Ramanarayanan, 2012). A unit of debt issued at t promises to repay $\kappa(1-\delta)^{s-1}$ is period $t+s$, effectively making a unit issued at t a perfect substitute for $(1-\delta)$ units issued at $t-1$. We normalize $\kappa = r + \delta$ so that the price of debt equals 1 when the government repays with certainty.

The price faced by the borrower government reflects its lenders' expectations of repayment, discounted with a risk-neutral kernel

$$q(b', b, m, z) = \frac{1}{1+r}\mathbb{E}\left[(1 - \mathbb{1}_{\mathscr{D}}(b', g_m(b', b, m, z), z'))\left(\kappa + (1-\delta)q(b'', b', g_m(b', b, m, z), z')\right) \mid z\right] \tag{6}$$

where $\mathbb{1}_{\mathscr{D}}(b, m, z)$ denotes the government's default policy as perceived by lenders, $g_m(b', b, m, z)$ is the expected result of negotiations with the monopolist, to happen in the afternoon, and $b'' = g_b(b', g_m(b', b, m, z), z')$ is the expected debt issuance in the following period.

Bilateral loan In the afternoon of t, the government meets with the monopolist to negotiate the loan m. As before, the outcome of their negotiation is a transfer x and new loan size m' which solve the following Nash bargaining problems, depending on default status vis-à-vis the market

$$\begin{gathered} \max_{m',x} \mathscr{L}_R(b', x, m, m', z)^{\theta}\ \mathscr{B}_R(b', b, x, m, m', z)^{1-\theta} \\ \text{or} \\ \max_{m',x} \mathscr{L}_D(x, m, m', z)^{\theta}\ \mathscr{B}_D(x, m, m', z)^{1-\theta} \end{gathered} \tag{7}$$

As before the monopolist's surplus is

$$\begin{aligned} \mathscr{L}_R(b', x, m, m', z) &= -x - m + \beta_L \mathbb{E}\left[h(b', m', z') - h(b', 0, z') \mid z\right] \\ \mathscr{L}_D(x, m, m', z) &= -x - m + \beta_L \mathbb{E}\left[\psi\left(h(0, m', z') - h(0, 0, z')\right) + (1-\psi)\left(h_D(m', z') - h_D(0, z')\right) \mid z\right] \end{aligned}$$

while the borrower's surplus now also reflects outcomes in debt markets

$$\begin{aligned} \mathscr{B}_R(b', b, x, m, m', z) &= u(y(z) + P(b', b, m, z) + x) - u(y(z) + P(b', b, m, z) - m) + \\ &\quad + \beta \mathbb{E}\left[v(b', m', z') - v(b', 0, z') \mid z\right] \\ \mathscr{B}_D(x, m, m', z) &= u(y_D(z) + x) - u(y_D(z) - m) + \\ &\quad + \beta \mathbb{E}\left[\psi\left(v(0, m', z') - v(0, 0, z')\right) + (1-\psi)\left(v_D(m', z') - v_D(0, z')\right) \mid z\right] \end{aligned}$$

where the function $y_D(z) = y(z) - \xi(z)$ is output in default and P summarizes net transfers from the competitive lenders received in the morning. Our assumptions on debt maturity yield $P(b', b, m, z) = q(b', b, m, z)(b' - (1 - \delta)b) - \kappa b$. In default, opportunities to reaccess markets arrive with probability ψ. The bargaining problems yield new terms for the bilateral loan $x_R(b', b, m, z)$, $m'_R(b', b, m, z)$ and $x_D(m, z)$, $m'_D(m, z)$ respectively in repayment and in default.

The most important way in which the presence of debt markets affects the bargaining stage is through the revenues from debt issuance $P(b', b, m, z)$. These revenues modulate the government's threat point: after a successful issuance which raises large revenues, the government is in a strong position to negotiate as repaying m is less costly.

After the negotiation is done and transfers settled, consumption takes place. The borrower's value functions for entering negotiations are given by

$$\begin{aligned} c_\zeta(b', b, m, z) &= y(z) - \mathbb{1}_{\mathscr{D}}\xi(z) + P(b', b, m, z) + x_\zeta(b', b, m, z) \quad \text{for } \zeta \in \{R, D\} \\ w_R(b', b, m, z) &= u(c_R(b', b, m, z)) + \beta\mathbb{E}\left[v\left(b', m'_R(b', b, m, z), z'\right) \mid z\right] \\ w_D(m, z) = v_D(m, z) &= u(c_D(m, z)) + \beta\mathbb{E}\left[\psi v\left(0, m'_D(m, z), z'\right) + (1 - \psi)v_D\left(m'_D(m, z), z'\right) \mid z\right] \end{aligned} \tag{8}$$

while for the monopolist we have

$$\begin{aligned} h(b, m, z) &= \mathscr{P}(b, m, z)h_D(m, z) + (1 - \mathscr{P}(b, m, z))h_R(b'(b, m, z), b, m, z) \\ h_R(b', b, m, z) &= a - x_R(b', b, m, z) + \beta_L\mathbb{E}\left[h(b', m'_R(b', b, m, z), z') \mid z\right] \\ h_D(m, z) &= a - x_D(m, z) + \beta_L\mathbb{E}\left[\psi h(0, m'_D(m, z), z') + (1 - \psi)h_D(m'_D(m, z), z') \mid z\right] \end{aligned} \tag{9}$$

5. Quantitative results

We parametrize our model at a quarterly frequency following standard strategies in the sovereign default literature (most parameters taken from the calibration to the 2001 Argentina default in Roch and Roldán, 2023). Table 1 summarizes our parametrization.

Table 2 presents some statistics from simulating the model with and without bilateral loans, for different values of the lender's bargaining weight θ. Statistics correspond to pre-default samples of 35 quarters and the term 'loans' refers to bilateral loans m. Welfare gains are reported in equivalent consumption terms and calculated averaging the ergodic distribution of the only-market equilibrium conditional on repayment, comparing with the economies with both types of debt at $m = 0$. The leftmost column corresponds to the equilibrium with only market debt and as such generates the business-cycle properties of Argentina in the years preceding its 2001 default.

The remaining columns show that even with a relatively large bargaining weight for the government, the availability of bilateral loans dramatically boosts the frequency of default. As a consequence, the government pays higher and more volatile spreads, despite slightly lower debt

Table 1: Baseline parameter values

	Parameter	Value
Sovereign's discount factor	β	0.9504
Sovereign's risk aversion	γ	2
Preference shock scale parameter	χ	0.025
Lender's bargaining power	θ	0.5
Risk-free interest rate	r	0.01
Duration of debt	ρ	0.05
Income autocorrelation coefficient	ρ_z	0.9484
Standard deviation of y_t	σ_z	0.02
Reentry probability	ψ	0.0385
Default cost: linear	d_0	-0.24
Default cost: quadratic	d_1	0.3

Table 2: Business cycle statistics with and without bilateral loans

	No swap	Unrestricted, $\theta = 0.25$	Unrestricted, $\theta = 0.5$
Avg spread (bps)	802	1,841	2,375
Std spread (bps)	454	1,075	1,516
$\sigma(c)/\sigma(y)$ (%)	112	110	110
Debt to GDP (%)	21.5	21	20.3
Swap to GDP (%)	0	3.75	3.29
Swap spread (bps)	–	-37.2	-460
Corr. swap & spreads (%)	–	54.1	62.7
Default frequency (%)	6.27	12.5	14.6
Welfare gains (rep)	–	-0.081%	-0.41%

levels and relatively modest amounts borrowed from the large lender. As a result, even with $\theta = 0.25$ the government prefers the equilibrium in which the large lender is not present.

Part of the increase in default frequencies is due to an improvement in the value of default when the large lender is present. Figure 6 compares an economy with access to market debt and bilateral loans but with current level $m = 0$, on the left, to an economy in which only the market is available, on the right. It shows that the presence of the large lender raises the option value of default: the economy with access to bilateral loans defaults at lower levels of debt (or higher levels of income) than the one without it.

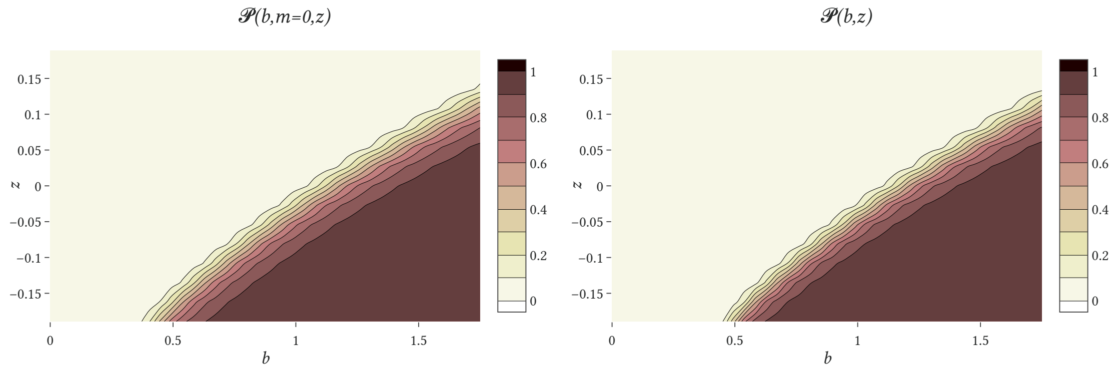

Figure 6: Default probability at $m = 0$ with (left) and without (right) bilateral loans

Moreover, since the large lender keeps a share of the surplus generated by the loan, the borrower economy is somewhat reluctant to use it. In a typical simulation path, conditional on no default the amount borrowed bilaterally is 3.3% of annual income with a standard deviation of 1.6%. Figure 7 shows that this changes significantly around default events: the loan size m shoots up around the moment of default. The large lender heavily subsidizes this accumulation of bilateral debt. As before, the large lender provides negative interest rates at first, while bilateral

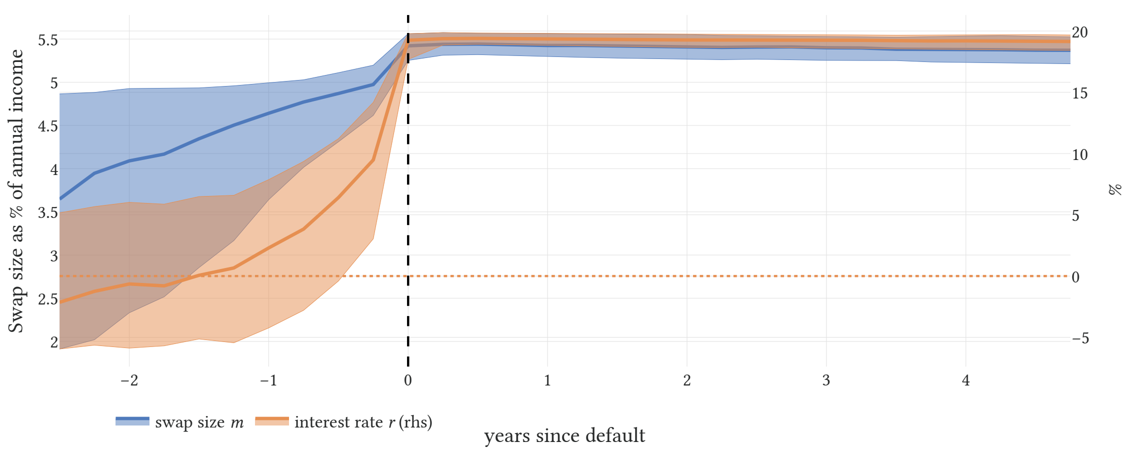

Figure 7: Loans around default events

debt is increasing, to then raise them as default grows closer. In this case, the gamble for debt overhang has a twist: when the economy recovers market access, it immediately issues market debt to pay off the loan (see Figure 15 in the Appendix). The large lender is then gambling that income will not revert and that the exclusion period will be long.

While most of the use of the bilateral loan m occurs during default, Figure 7 shows that default episodes are preceded by bilateral issuance in an effort to avoid or postpone default (Figure 7 does not show the defaults that were avoided as a consequence of the large lender's presence).

Bilateral loans affect the economy in two ways: on the one hand, they provide extra financing when default risk makes borrowing in private markets costly. But they also provide funds in default, which raises the value of being excluded from private markets and consequently spreads. To disentangle these effects, we consider a variant of the model in which the large creditor is only willing to lend while the economy is in good standing with private debt markets, in other words, a variant with the extra constraint that $m'_D(m, z) = 0$. This implies in particular that whenever the government default, it must at the same time repay all of its bilateral loans.

Table 3 presents some statistics from simulating the model with and without bilateral loans (now marked 'Unrestricted'), as well as the variant in which bilateral loans are 'Limited.'

Table 3: Business cycle statistics with bilateral loans

	No swap	**Unrestricted,** $\theta = 0.5$	**Limited,** $\theta = 0.5$
Avg spread (bps)	802	2,375	1,211
Std spread (bps)	454	1,516	753
$\sigma(c)/\sigma(y)$ (%)	112	110	113
Debt to GDP (%)	21.5	20.3	21.8
Swap to GDP (%)	0	3.29	1.05
Swap spread (bps)	–	-460	408
Corr. swap & spreads (%)	–	62.7	70.1
Default frequency (%)	6.27	14.6	9.17
Welfare gains (rep)	–	-0.41%	-0.084%

In the version with Limited loans in default, the usage of bilateral loans declines by more than two-thirds on average, as repaying the large creditor after defaulting becomes even more costly (and can even act as an extra cost of default). However, bilateral loans are still mostly used when spreads are high, as shown by the high correlation between spreads and loans. The Limited version still creates welfare losses for the economy, but these are much lower than in the Unrestricted case.

5.1 Default probabilities and debt prices

Figure 8 shows ex-post default regions for private debt, when the bilateral loan $m = 0$. Solid lines represent the case without bilateral loans, while dotted and dashed lines correspond to the versions with Unrestricted and Limited bilateral loans. When bilateral loans are Unrestricted, their presence leads to an increase in default as the government is able to sustain lower levels of debt. The Limited variety leaves the government's default policy virtually unchanged.

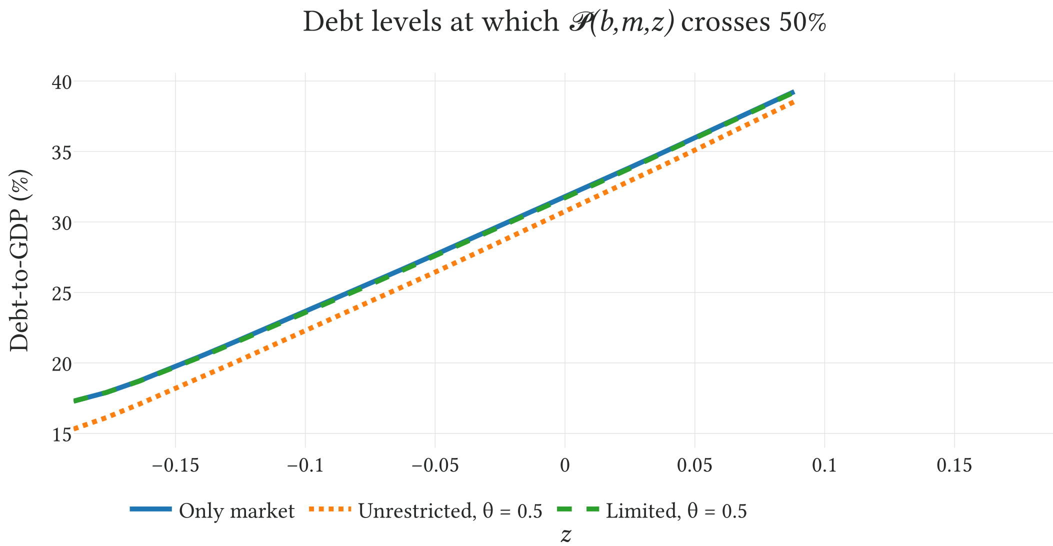

Figure 8: Default regions

Figure 8 highlights the negative impact of the bilateral loans. In the Unrestricted version, loans are available during default, which makes market debt repayment less attractive. The higher default probability translates into lower prices for debt, as shown in Figure 9. This effect is muted in the Limited variant, when bilateral loans are not available in default. However, as shown in Figure 9, prices remain lower in the Limited variant relative to the model without the large creditor, especially when debt is low. This means that even though the one-period-ahead default probability may not increase when bilateral loans are introduced in the Limited variant, policies (notably, future borrowing) are altered in a way that still creates more default risk later on. In other words, the option of bilateral loans, even in the Limited variant, induces the government to take on more default risk.

Figure 10 presents the ergodic distribution of the debt/GDP ratio in simulations of the three models, conditional on repayment. When bilateral loans are available, in both variants, the economy spends longer in the region of the state space where default risk is large.

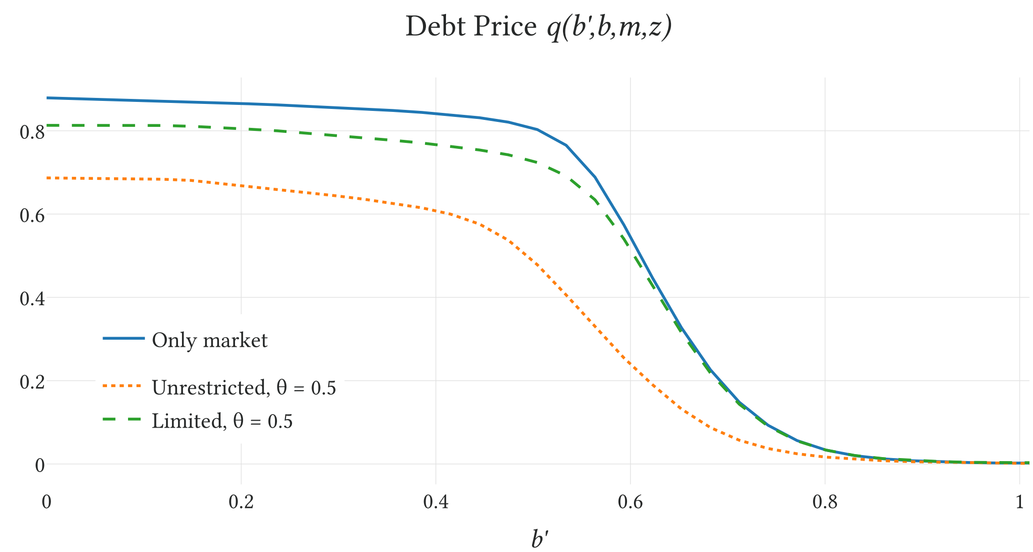

FIGURE 9: DEBT PRICES

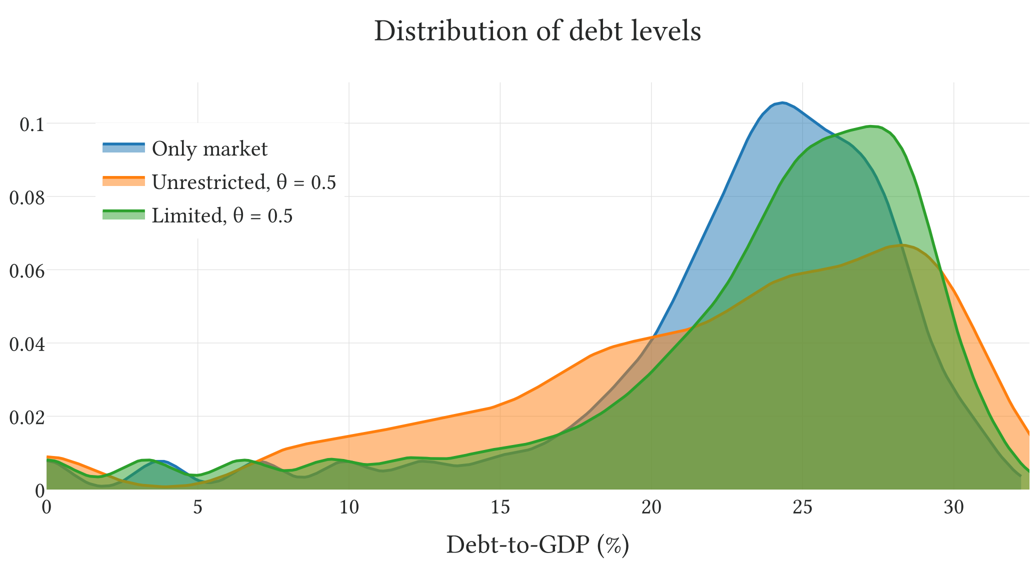

FIGURE 10: DISTRIBUTION OF DEBT LEVELS

5.2 Dynamics with bilateral loans

In the model with bilateral loans, the government issues debt in a riskier manner and spends more time in the regions where default risk is likely. Figure 11 plots the monopolist's value (or profit) function (9) as a function of debt, as always at $m = 0$.

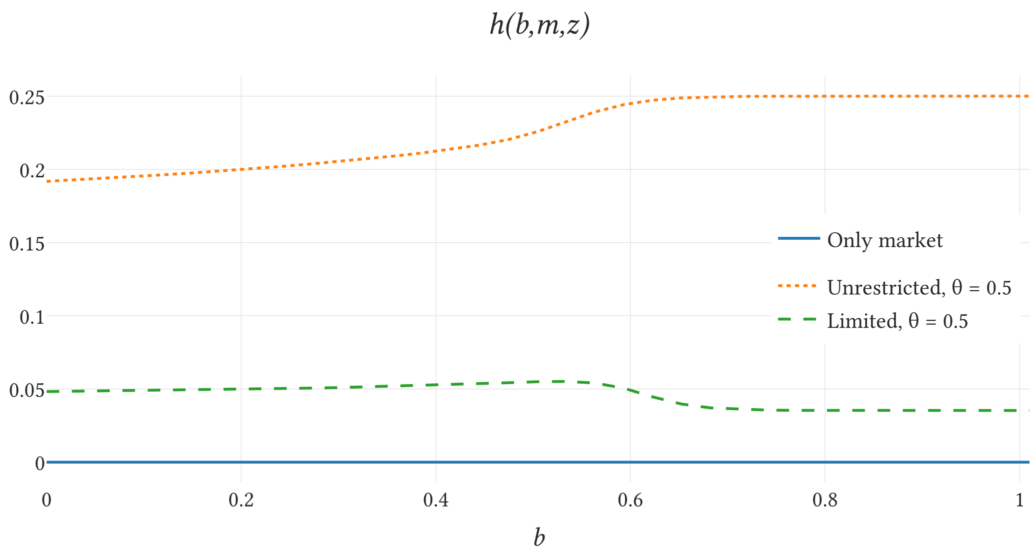

FIGURE 11: MONOPOLIST'S PROFITS

In both versions (both when loans are Unrestricted or Limited in default), the large lender's profits are increasing in market debt b, at least when debt is low enough that the one-period-ahead default probability remains contained. In this region, as spreads open up, the presence of the large lender is more valuable to the government, which increases total surplus in the negotiations. As debt continues to increase and default becomes likely, profits for the large lender decrease in the Limited version, as any bilateral lending m must be repaid at face value upon default, but increase sharply in the Unrestricted case, in which large payments can be extracted from the borrower during the default spell.

When the government has a safe level of debt and pays low spreads, there is little for the monopolist to offer that private markets do not already provide at a competitive price. But when default risk drives up spreads, the monopolist can provide financing through the bilateral loan, which is undefaultable, and charge an interest rate between the risk-free rate and the rate that the government is paying on its market debt.

Relational overborrowing At the bargaining stage, the choice of market debt b' has already been made by the government. As a result, if negotiations break down the borrower is left with a consumption level of $y(z) + q(b', b, m, z)(b' - (1 - \delta)b) - \kappa b - m$. In periods when the government is trying to reduce market debt b', it enters negotations with the large lender in a 'weak' position as consumption at the government's threat point is low. This generates large gains from any transfer x provided by the large lender. Thus, in order to split the surplus such transfers must come at a high interest rate. Conversely, when the government has issued a large amount of bonds b', it expects a high level of consumption even if negotiations break down, which results in a strong position and good borrowing terms with the large lender.

Figure 12 shows the interest rate that results from bilateral negotiations as a function of the choice of b'. It confirms the discussion above: borrowing terms with the large lender respond aggressively to the government's choice of indebtedness in debt markets.

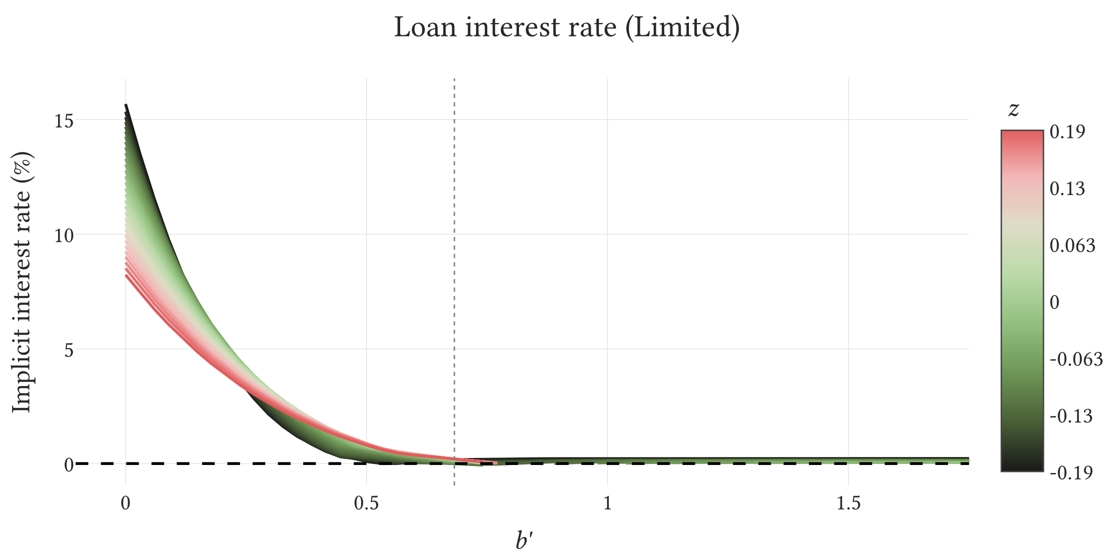

Figure 12: Interest rate on the bilateral loan

As a result of this elasticity of loan terms to debt, new terms appear in the government's Euler equation for market debt b':

$$u'(c)\left(q + \frac{\partial q}{\partial b'}i + \frac{\partial x}{\partial b'}\right) = \beta \mathbb{E}\left[u'(c)(1 - \mathbb{1}_{\mathscr{D}})\left(\kappa + (1-\delta)q' - i'\left(\frac{\partial q'}{\partial b} - \frac{\partial m'}{\partial b'}\frac{\partial q'}{\partial m}\right) - \frac{\partial x'}{\partial b} + \frac{\partial m'}{\partial b'}\frac{\partial x'}{\partial m}\right)\right] \tag{10}$$

where $i = b' - (1-\delta)b$ represents new debt issuances. Relative to a world with only market debt, there are three new effects arising from the presence of the large lender.

First, issuing debt affects transfers x received from the large lender in the afternoon of the current period. Second, issuing debt also affects the desired amount of loans m' taken from the large creditor. This flows into debt prices q' and transfers x' in the following period through the chain-rule terms. Finally, because debt prices and borrowing terms also depend on the initial level of indebtedness b, issuing new debt affects both the price at which market debt will be placed q' as well as transfers x' in the following period.

The left-hand side of (10) describes a countervailing force to the market discipline of spreads. In a model with only market debt, the benefit of issuing an extra unit of debt is $u'(c)\left(q + \frac{\partial q}{\partial b'}\right)$: the government understands that issuing more debt may increase spreads (deccrease the price q) and this mitigates issuances. With the large lender, the benefit of issuing an extra bond includes additionally the term $\frac{\partial x}{\partial b'}$ as the extra issuance also affects the threat point at the bargaining stage. Notice that, by writing the transfer x as a price ϕ times the new loan m' (as in (2)), the marginal benefit of debt term becomes $u'(c)\left(q + \frac{\partial q}{\partial b'} i + \phi \frac{\partial m'}{\partial b'} + \frac{\partial \phi}{\partial b'} m'\right)$, which emphasizes how debt issuances affect revenues from the market and from the large lender in a strictly symmetrical way (notice also that $\frac{\partial i}{\partial b'} = 1$).

5.3 *Welfare effects of bilateral loans*

The forces discussed above combine to produce welfare effects of bilateral loans. Figure 13 shows the government's value function as a function of debt b when it owes $m = 0$ to the large lender. The government prefers bilateral loans to be Unavailable during default, except of course when a default in the current period is very likely.

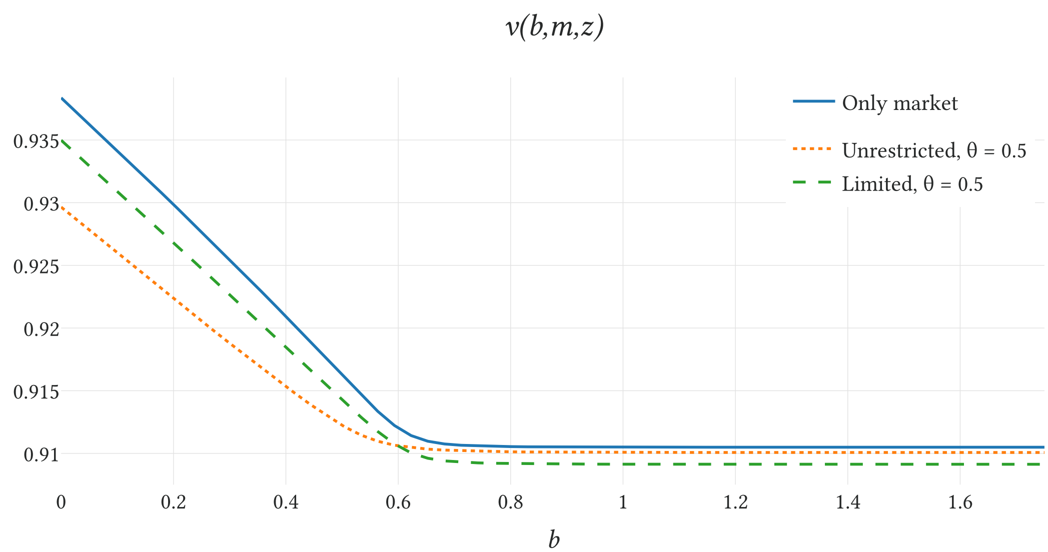

Figure 13: Value functions

Figure 16 in the Appendix confirms this intuition about debt dilution. In particular, it shows that with short-term debt there are welfare gains for the variant where loans are Unavailable during default. The Unrestricted variant produces very small welfare losses in this case.

6. Programming the large lender

We have shown how the combination of market debt and bilateral loans can create welfare losses for the government through the relational overborrowing effect which incentivizes risk-taking in debt issuances.

In this section we replace the bargaining protocol with fixed rules for the terms of bilateral loans, with two objectives. First, to better understand the dynamics of the relational overborrowing effect in a simpler case. Second, to explore design questions surrounding bilateral loans. Do there exist rules that contain spreads on market debt by curbing sovereign default risk? Do there exist designs that can create welfare gains for the government? What is the optimal design of bilateral loans? Are there possible Pareto improvements, under which the government attains a higher value than without bilateral loans and also the large lender obtains profits?

To investigate these questions, we remove the bargaining protocol for the large lender and, instead, consider rules of the form

$$r(b', m') = \max\left\{r,\ \alpha_0 + \alpha_b b' + \alpha_m m'\right\}$$

With these rules, the government now faces the exogenous interest rate $r(b', m')$ when it chooses market debt b' and bilateral loan m'. Notice that because the bilateral loan always costs more than the risk-free rate r, the large lender makes non-negative profits.

At this moment, we consider a 'risk-inducing' rule with $\alpha_0 > 0, \alpha_b < 0, \alpha_m = 0$, which replicates the main properties of the equilibrium with bargaining, and a size-dependent rule which does not load on indebtedness in markets, with $\alpha_0 > 0, \alpha_b = 0, \alpha_m > 0$. Table 4 summarizes our findings.

The risk-inducing rule creates similar dynamics to the equilibrium with bargaining and, thus, welfare losses for the government. However, the size-dependent rule exemplifies the possibility of rules which induce lower default and spreads, improving the government's welfare, while also generating profits for the large lender.

7. Concluding remarks

To investigate the interaction between marketable and bilateral debts, we develop a model of sovereign debt in which the government can borrow from a large lender (or monopolist) as well as from a competitive fringe of private creditors. We find that both sources of funds are linked by strong interactions, even when quantitatively the amounts borrowed from the large lender remain an order of magnitude smaller than market debts.

The model describes a 'relational overborrowing' effect which arises in the presence of the large lender. This effect is due to an endogenous cross-elasticity of bilateral borrowing terms to

Table 4: Outcomes with exogenous rules for bilateral loans

	Only market	**Size dependent** r	**Risk inducing** r	**Limited,** $\theta = 0.5$
Avg spread (bps)	714	623	921	1,038
Std spread (bps)	399	315	552	612
$\sigma(c)/\sigma(y)$ (%)	113	115	115	113
Debt to GDP (%)	22.5	23.5	22.8	22.5
Loan to GDP (%)	0	0.71	0.972	1.06
Loan spread (bps)	–	682	1,264	536
Corr. loan & spreads (%)	–	62.5	48.1	71.1
Default frequency (%)	5.72	5.13	6.92	7.72
Welfare gains (rep)	–	0.21%	-0.079%	-0.2%

outcomes in debt markets which erodes the market discipline of spreads. Our model of bargaining with the large lender generates this cross-elasticity with three key ingredients: loans from the large lender are more costly (or impossible) to default, their terms result from bargaining, and they are of shorter maturity than marketable debts. The simpler version developed in Section 6 shows that the cross-elasticity is in itself sufficient to incentivize the government to overborrow, face more sovereign risk, and ultimately attain lower levels of welfare.

The simpler model of Section 6 also illustrates a broader point. Policy discussions surrounding bilateral debts often focus on the price of the loans. Our results emphasize that welfare losses cannot come from the level of the interest rate charged by the large lender, as long as the government retains the option to borrow from the market. Rather, the perverse incentives created by the endogenous cross-elasticity, which have been more overlooked, are crucial for the welfare losses we find.

Our results suggest that introducing more sources of indebtedness can be detrimental for the borrowing government. While bilateral loans can in some cases help a government fend off default, they can also make it more likely: either through reducing the effective costs of default (when it is possible to borrow from the large lender while excluded from markets) or through the relational overborrowing effect.

The welfare impact of the large lender's presence raises important policy questions and challenges. Limiting the use of bilateral debt during defaults is a clear welfare-enhancing policy in this model. Extending the maturity of bilateral loans would mute the incentivizing effect of their borrowing terms and could also disarm relational overborrowing. Because the relational overborrowing effect operates through increased default risk, the gains of fiscal rules that constrain market borrowing should be larger for countries with access to the type of bilateral debts we describe.

The model implies a simple test to gauge the likely effects of a new bilateral creditor or instrument. Bilateral loans whose interest rate is expected to be strongly decreasing in the amount (or spreads) of marketable debt will induce relational overborrowing and are thus likely to hurt welfare. More generally, it highlights the benefits of transparent rules for the terms of bilateral debts.

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A. More results

Figure 14 shows that when the borrower holds all the bargaining power, the loan interest rate is constant at β_L^{-1}. In this example with $\beta = \beta_L$, the upper bound on how large the loan m can be does not bind, but in the quantitative version with $\beta < \beta_L$, which at $\theta = 0$ recovers the model of Hatchondo, Martinez, and Önder (2017), the borrower prioritizes bilateral loans as a source of funding and thus quickly reaches the upper bound.

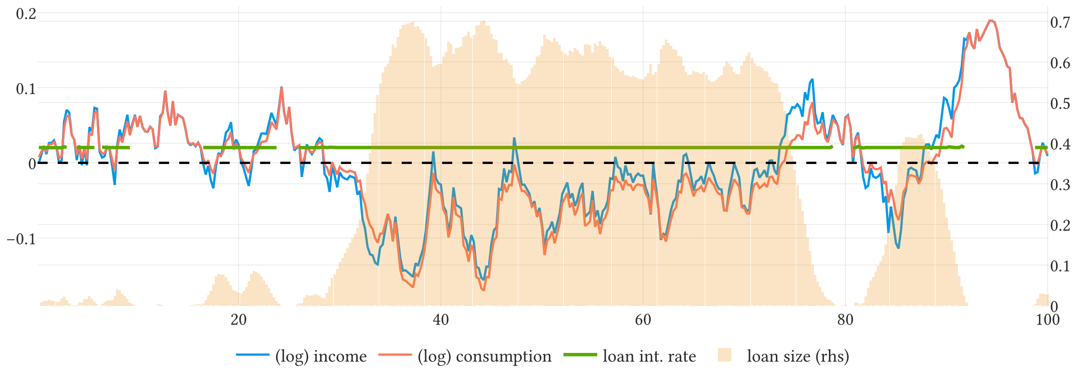

Figure 14: Simulated path, $\theta = 0$

Figure 15 shows that, further conditioning on an exclusion period of 2 years, the economy issues debt in the market in order to pay off the loan as soon as it recovers market access.

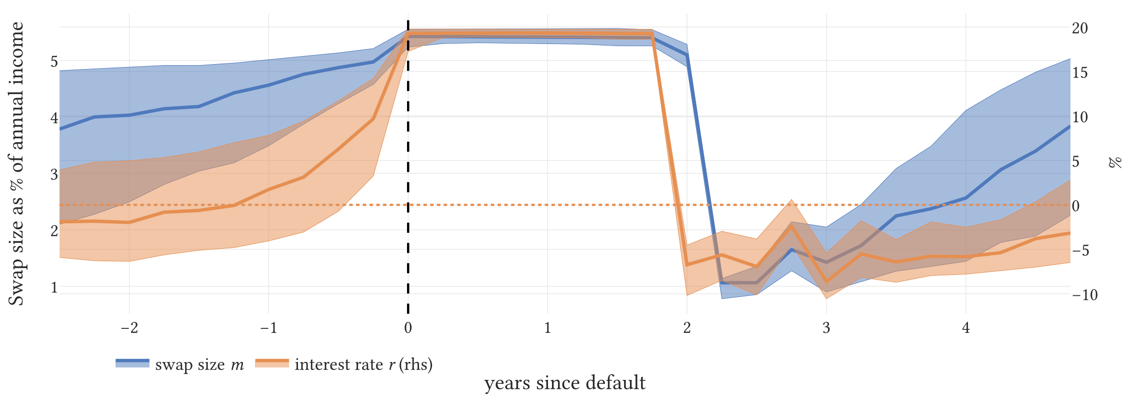

Figure 15: Loans around default events

Figure 16 shows that, (*i*) with short-term debt, allowing the loan to be drawn while in default provides welfare gains and (*ii*) there are welfare gains from the loan for a larger range of values of the bargaining weight, in particular for $\theta \leq 0.3$.

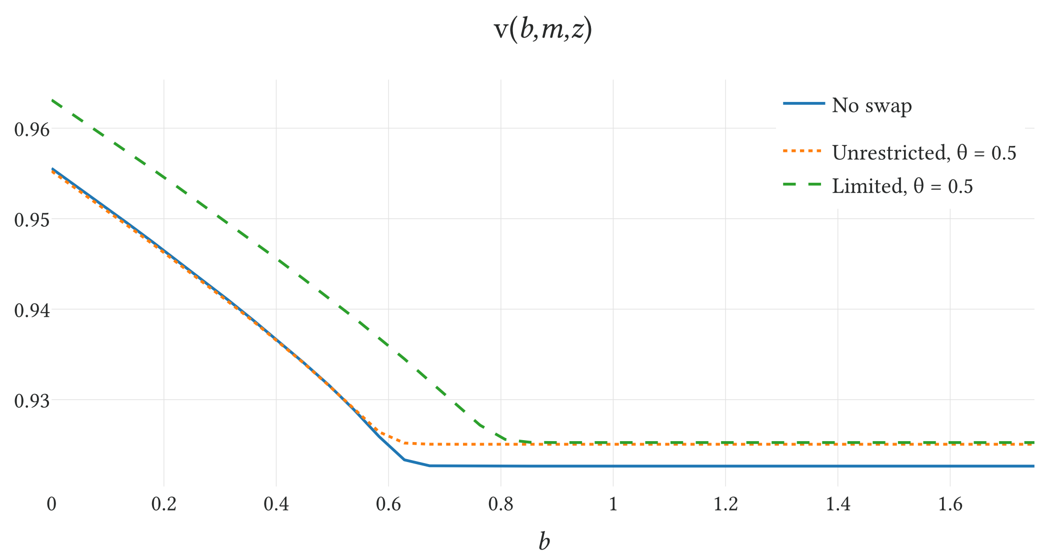

FIGURE 16: VALUE FUNCTIONS, SHORT-TERM DEBT