Risk Aversion in Sovereign Debt and Default

Francisco Roch
UTDT

Francisco Roldán
IMF

Winter SED
UTDT, December 2024

Why risk aversion? Why in sovereign debt?

- In most RBC models, macro-financial separation holds
  - Elasticity of intertemporal substitution determines allocations
  - Risk aversion determines asset prices

- Sovereign debt literature typically inherits this line of thinking
  - CRRA preferences frequent, typically $\gamma = 2$

- If MFS holds in sovereign debt, macro outcomes robust to different preferences
  - In particular, calibration of output/utility costs of default
  - Less clear about welfare effects

    … losses from default, debt dilution

    … welfare effects of banning debt, introducing state-contingent bonds

Wanting risk prices in sovereign debt

This paper

- Show that macro-financial separation breaks in the sovereign debt model
- Understand the impact of preferences consistent with significant risk premia

Model

Framework

- Sovereign default model without default [reduces to an income-fluctuations problem]

$$v(b,z) = \max_{b'} u(c) + \beta \mathbb{E}\left[v(b',z') \mid z\right]$$

$$\text{subject to} \quad c + \kappa b = q(b',z)(b' - (1-\rho)b) + y(z)$$

$$\text{with} \quad q(b',z) = \frac{1}{1+r}$$

- We consider parametrizations of the model to vary risk aversion
  … with CRRA preferences $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$
  … With robustness, $u(c) = \log c$; replace $\mathbb{E}$ with $\mathbb{T}\left[X \mid \mathcal{F}\right] = -\frac{1}{\theta}\log\left(\mathbb{E}\left[\exp\left(-\theta X\right) \mid \mathcal{F}\right]\right)$

Macro-financial separation without default

Autarky

- Start from log-log $[\theta = 0]$: RA moves asset prices and welfare, not the macro

	loglog	$\theta = 1$	$\theta = 2$	$\theta = 3$
Average spread (bps)	0.0276	0.031	0.0406	0.138
Corr. NX,Y (%)	0.00777	0.00916	0.0114	0.0147
Rel. vol. cons (%)	1.59	1.62	1.65	1.66
Risk premium (p.p.)	0.0769	2.03	3.84	5.44
Debt-to-GDP (%)	29.7	29.5	29.2	28.9
Welfare	1.034	1.008	0.9867	0.971

… welfare in autarky at $\theta = 3$ is 6pp lower than loglog or CRRA

Macro-financial separation without default (cont'd)

- Start from log-log [$\gamma = 1$]: EIS+RA moves mostly macro, not asset prices and welfare

	loglog	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Average spread (bps)	0.0276	0.0273	0.0269	0.0271	0.0285
Corr. NX,Y (%)	0.00777	0.0154	0.0852	0.397	0.668
Rel. vol. cons (%)	1.59	1.56	1.35	0.965	0.727
Risk premium (p.p.)	0.0769	0.227	0.627	1.02	1.67
Debt-to-GDP (%)	29.7	28.8	25.9	19.3	8.75
Welfare	1.034	1.03	1.021	1.01	0.9918

… in fully Epstein-Zin, move only EIS for even less effect on asset prices and welfare

Models with default

- Option value of default (with small pref. shocks for numerical performance)

$$\mathcal{V}(b, z) = \max\{v_R(b, z) + \epsilon_R, v_D(b, z) + \epsilon_D\}$$

- Similar equation for value of repayment v_R, debt prices reflect default probabilities

$$q(b', z) = \frac{1}{1+r}\mathbb{E}\left[(1 - \mathbb{1}_{\mathcal{D}'})\left(\kappa + (1-\rho)q(b'', z')\right) \,\middle|\, z\right]$$

- Costs of default

$$v_D(b, z) = u(h(y(z))) + \beta\mathbb{E}\left[\mathbb{1}_R \mathcal{V}(B(b, z'), z') + (1 - \mathbb{1}_R) v_D(b, z') \,\middle|\, z\right]$$
$$h(y) = y(1 - d_0 - d_1 y)$$

- Risk aversion $\Longrightarrow$ no-smoothing in default costly $\Longrightarrow$ no macro-financial separation

Models with default

- Option value of default (with small pref. shocks for numerical performance)

$$\mathcal{V}(b,z) = \max\{v_R(b,z) + \epsilon_R, v_D(b,z) + \epsilon_D\}$$

- Similar equation for value of repayment v_R, debt prices reflect default probabilities

$$q(b',z) = \frac{1}{1+r}\mathbb{E}\left[(1 - \mathbb{1}_{\mathcal{D}'})\left(\kappa + (1-\rho)q(b'',z')\right) \,\middle|\, z\right]$$

- Costs of default

$$v_D(b,z) = u(h(y(z))) + \beta\mathbb{E}\left[\mathbb{1}_R \mathcal{V}(B(b,z'),z') + (1 - \mathbb{1}_R)v_D(b,z') \,\middle|\, z\right]$$
$$h(y) = y(1 - d_0 - d_1 y)$$

- Risk aversion $\implies$ no-smoothing in default costly $\implies$ no macro-financial separation

Quantitative properties

Calibration

- Keep the same discount rate, vary costs of default to match spreads and debt

	Parameter	$\gamma = 2$	loglog	$\theta = 3$
Sovereign's discount factor	β	0.9627	0.9627	0.9627
Sovereign's robustness parameter	θ	0	0	3
Sovereign's EIS	γ	2	1	1
Default output cost: linear	d_1	–0.2833	–0.2836	–0.247
Default output cost: quadratic	d_2	0.3253	0.3228	0.3029
Average spread (bps)	815	754	756	815
Debt-to-GDP ratio (%)	17.4	16.8	16.7	17.4

Comparative statics: CRRA

- Increasing EIS+RA: Less volatility, procyclical exports, more skewed debt outcomes

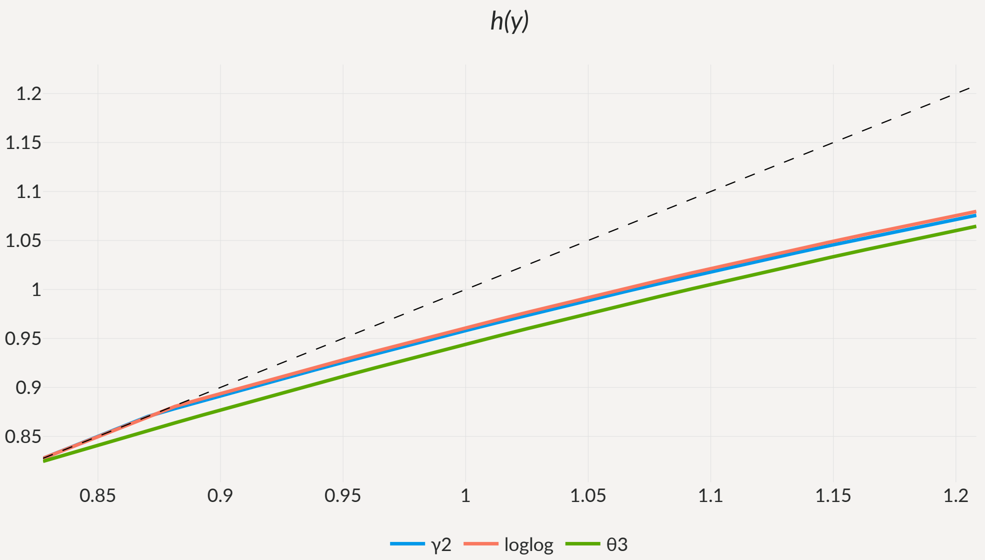
	loglog	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Avg. spread (bps)	756	800	912	974	1,057
Corr. NX,Y (%)	-0.285	-0.302	-0.21	0.0726	0.416
Rel. vol. cons (%)	1.5	1.37	1.18	1.04	0.921
Risk premium (p.p.)	0.652	0.789	1.02	1.28	2.38
Debt-to-GDP (%)	16.7	15.7	12.4	7.62	3.25
Default freq. (%)	4.4	4.41	4.17	3.45	2.7
Std. dev. spreads (bps)	448	538	877	1,209	1,816
Welfare	1.013	1.01	1.002	0.9918	0.9728

Comparative statics: robustness

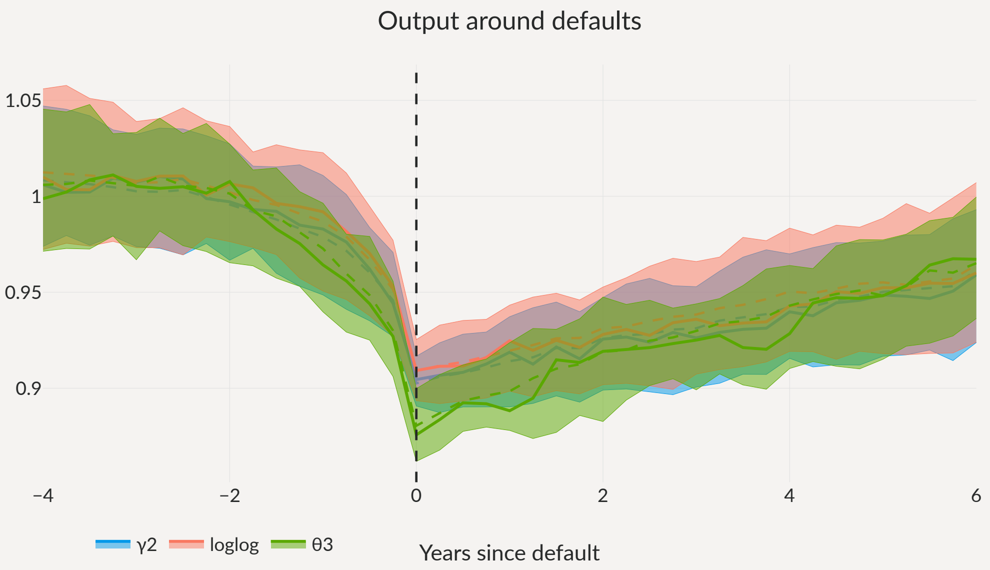
- Increasing RA: less debt tolerance, limited effect on volatilities

	loglog	$\theta = 1$	$\theta = 2$	$\theta = 3$
Avg. spread (bps)	756	1,683	20,929	38,237
Corr. NX,Y (%)	-0.285	-0.227	-0.0903	-0.227
Rel. vol. cons (%)	1.5	1.38	1.26	1.47
Risk premium (p.p.)	0.652	2.92	4.43	7
Debt-to-GDP (%)	16.7	14.2	9.09	9.6
Default freq. (%)	4.4	5.88	3.59	2.51
Std. dev. spreads (bps)	448	2,561	107,449	199,636
Welfare	1.013	0.9848	0.9629	0.9469

Calibrated output costs of default with robustness



Event-study of defaults

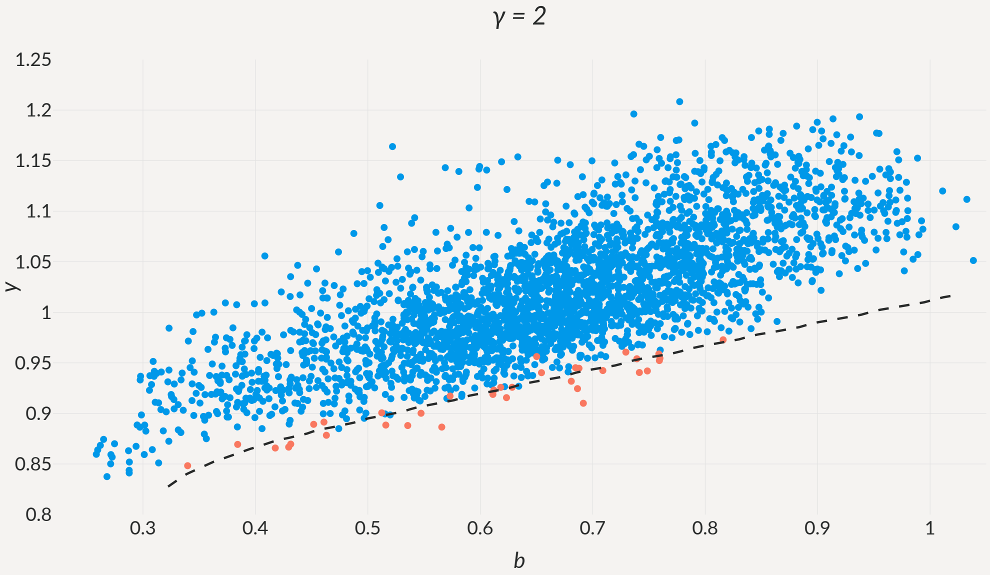


Calibrations with risk aversion

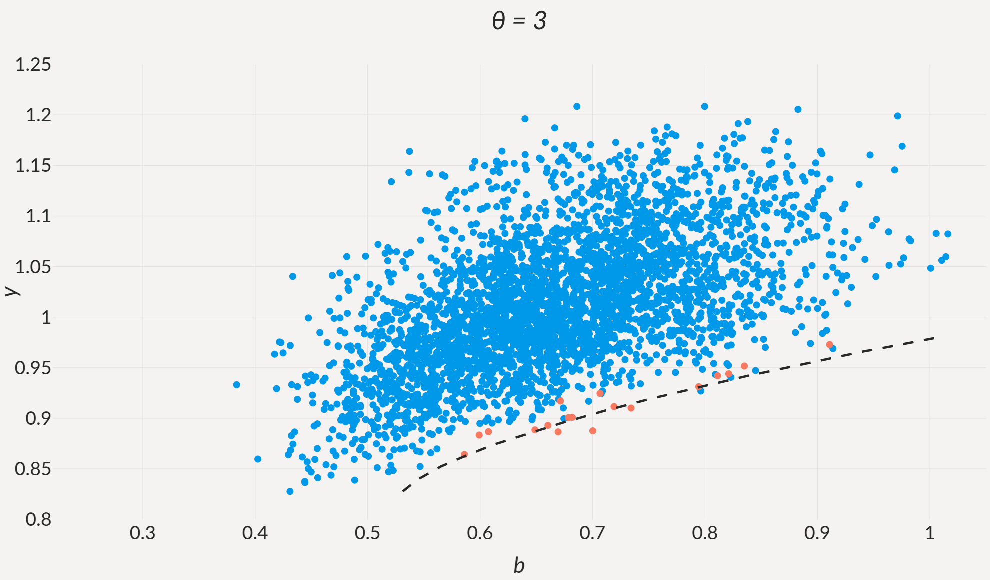
- Calibration with robustness: skewed debt outcomes, small decrease in macro volatility

	Data	$\gamma = 2$	loglog	$\theta = 3$
Avg. spread (bps)	815	754	756	815
Corr. NX,Y (%)	-	-0.314	-0.285	-0.194
Rel. vol. cons (%)	0.94	1.38	1.5	1.35
Risk premium (p.p.)	-	0.778	0.652	5.9
Debt-to-GDP (%)	17.4	16.8	16.7	17.4
Corr. b,y	-	0.343	0.358	0.0985
Default freq. (%)	-	4.21	4.4	1.51
Std. dev. spreads (bps)	443	496	447	2,026

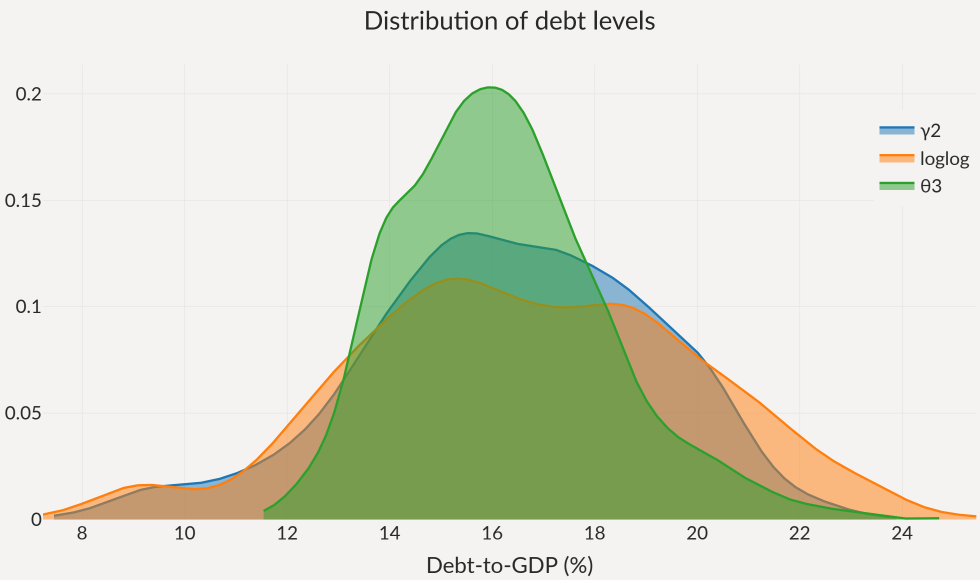
Ergodic distribution for debt in CRRA model



Ergodic distribution for debt with robustness



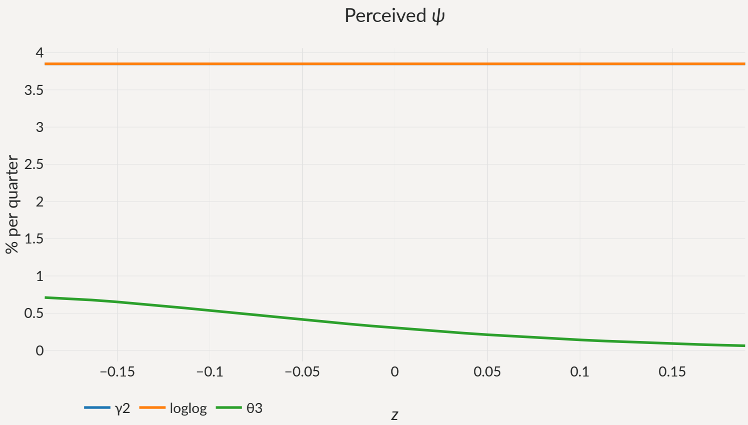
Ergodic distribution for debt



Worst-case models

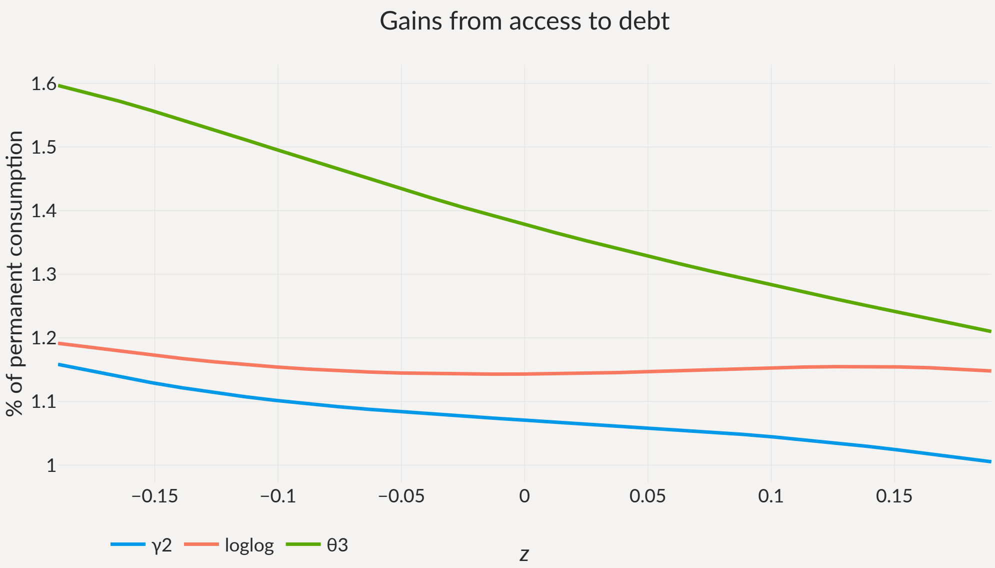
- Distorted expectation of X

$$\tilde{\mathbb{E}}\left[X \mid \mathcal{F}\right] = \mathbb{E}\left[\frac{\exp(-\theta v(s'))}{\mathbb{E}\left[\exp(-\theta v(s')) \mid \mathcal{F}\right]} X \;\middle|\; \mathcal{F}\right]$$

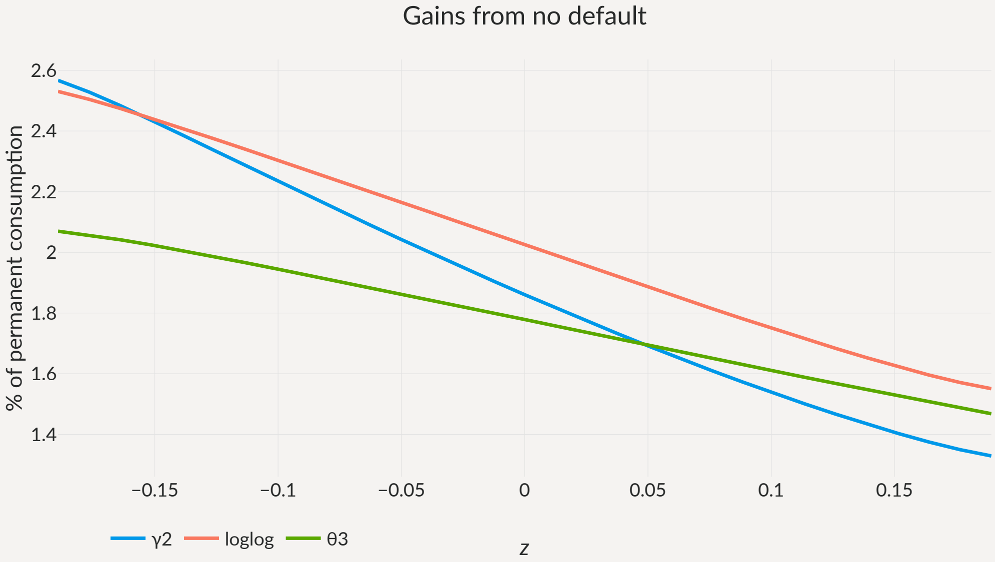


Welfare effects

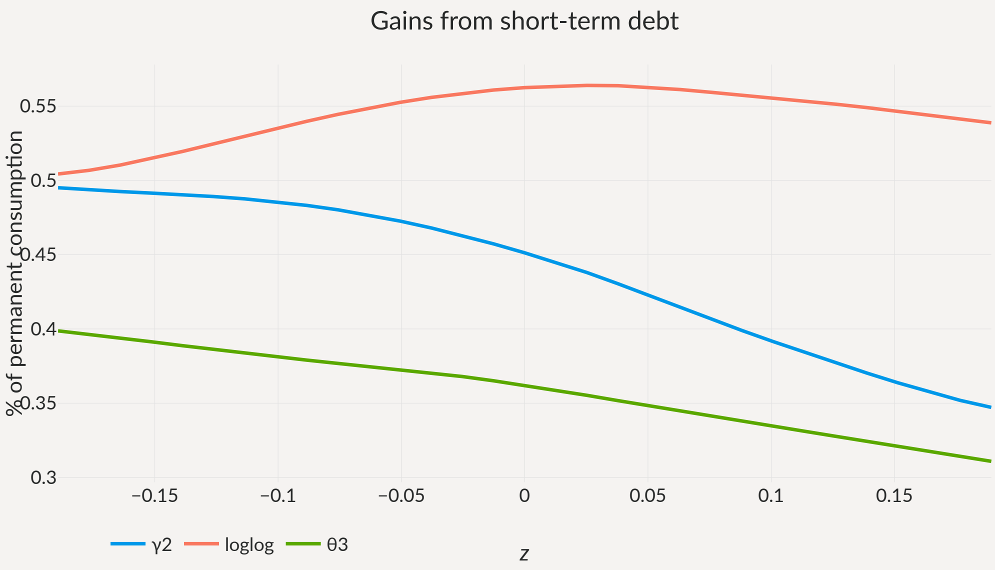
Welfare effects of debt



Welfare effects of banning defaults



Welfare effects of shortening maturity



Takeaways

With preferences consistent with positive risk premia

- *Lower* debt tolerance
  … Larger default costs required

- Less staying at the edge of default
  … More skewness in the distribution of debt and spreads

- More use of the debt for insurance
  … Large gains from debt access, not so much for making debt safe

Welfare gains decomposition

Consumption without default costs c_t^R

$$c^R(b,z) = \mathbb{1}_{\mathcal{D}}(b,z)y(b,z) + (1 - \mathbb{1}_{\mathcal{D}}(b,z))c(b,z)$$

Evaluate value of consuming c^R [instead of c] and removing uncertainty

$$\begin{aligned} V_{NC}(b,z) &= u(c^R(b,z)) + \beta\mathbb{E}\left[V_{NC}(b',z') \mid z\right] \\ V_{NV}(b,z) &= u(c^R(b,z)) + \beta V_{NV}(b',\mathbb{E}\left[z' \mid z\right]) \end{aligned}$$

Welfare gains between models/equilibria with value functions v and $v^\star$

$$\frac{v^\star(b_0,z_0)}{v(b_0,z_0)} = \frac{v^\star(b_0,z_0)/v^\star_{NC}(b_0,z_0)}{v(b_0,z_0)/v_{NC}(b_0,z_0)} \times \frac{v^\star_{NC}(b_0,z_0)/v^\star_{NV}(b_0,z_0)}{v_{NC}(b_0,z_0)/v_{NV}(b_0,z_0)} \times \frac{v^\star_{NV}(b_0,z_0)}{v_{NV}(b_0,z_0)}$$

Welfare gains

	Total gains	From default costs	From volatility	From level
		$\gamma = 2$		
Access to markets	0.622	-0.273	0.218	0.679
No default	1.87	0.274	-0.292	1.89
Short-term debt	0.411	0.255	-0.448	0.606
		loglog		
Access to markets	0.663	-0.294	0.284	0.674
No default	2.04	0.295	-0.345	2.09
Short-term debt	0.519	0.272	-0.439	0.688
		$\theta = 3$		
Access to markets	0.961	-0.25	0.0354	1.18
No default	1.72	0.251	-0.0744	1.54
Short-term debt	0.262	0.233	-0.45	0.481

Model with linear costs

Concluding remarks

Macro-finanical separation with autarky

◂ Back

	loglog	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Corr. NX,Y (%)	−0.00131	−0.00131	−0.00131	−0.00131	−0.00131
Rel. vol. cons (%)	1	1	1	1	1
Risk premium (p.p.)	0.0833	0.251	0.751	1.57	3.05
Welfare	1.002	1	0.9951	0.9868	0.9699

	loglog	$\theta = 1$	$\theta = 2$	$\theta = 3$
Corr. NX,Y (%)	−0.00131	−0.00131	−0.00122	−0.00073
Rel. vol. cons (%)	1	1	1	1
Risk premium (p.p.)	0.0833	2.02	3.81	5.32
Welfare	1.002	0.9769	0.9564	0.9411

Sovereign Default

Option value of defaulting (with small pref. shocks for numerical performance)

$$\mathcal{V}(b,z) = \max\left\{v_R(b,z) + \epsilon_R, v_D(b,z) + \epsilon_D\right\}$$

Value of repayment involves issuing new debt at price $q(b', z)$

$$v_R(b,z) = \max_{b'} u(c) + \beta \mathbb{E}\left[\mathcal{V}(b', z') \mid z\right]$$

$$\text{subject to} \quad c + \kappa b = q(b', z)\left(b' - (1-\rho)b\right) + y(z)$$

Value of default involves lower output and exclusion with constant reentry ψ

$$v_D(b,z) = u(h(y(z))) + \beta \mathbb{E}\left[\mathbb{1}_R \mathcal{V}(B(b,z'), z') + (1 - \mathbb{1}_R) v_D(b, z') \mid z\right]$$

- Traditionally solved with $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$
- With robustness, $u(c) = \log c$; replace $\mathbb{E}$ with $\mathbb{T}\left[X \mid \mathcal{F}\right] = -\frac{1}{\theta} \log\left(\mathbb{E}\left[\exp\left(-\theta X\right) \mid \mathcal{F}\right]\right)$

Sovereign Default

Option value of defaulting (with small pref. shocks for numerical performance)

$$\mathcal{V}(b,z) = \max\left\{v_R(b,z) + \epsilon_R, v_D(b,z) + \epsilon_D\right\}$$

Value of repayment involves issuing new debt at price $q(b',z)$

$$v_R(b,z) = \max_{b'} u(c) + \beta \mathbb{E}\left[\mathcal{V}(b',z') \mid z\right]$$

$$\text{subject to} \quad c + \kappa b = q(b',z)\left(b' - (1-\rho)b\right) + y(z)$$

Value of default involves lower output and exclusion with constant reentry ψ

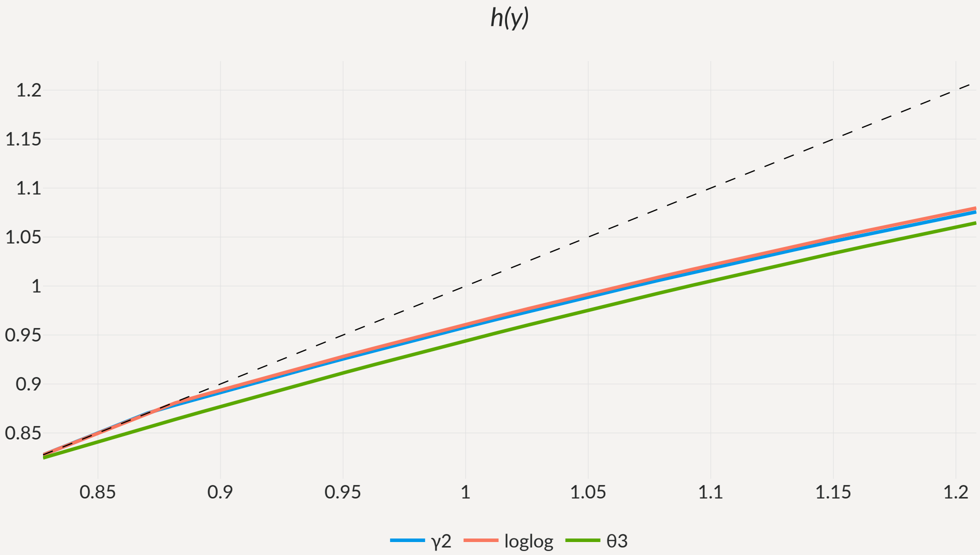
$$v_D(b,z) = u(h(y(z))) + \beta \mathbb{E}\left[\mathbb{1}_R \mathcal{V}(B(b,z'),z') + (1-\mathbb{1}_R) v_D(b,z') \mid z\right]$$

- Traditionally solved with $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$
- With robustness, $u(c) = \log c$; replace $\mathbb{E}$ with $\mathbb{T}\left[X \mid \mathcal{F}\right] = -\frac{1}{\theta}\log\left(\mathbb{E}\left[\exp\left(-\theta X\right) \mid \mathcal{F}\right]\right)$

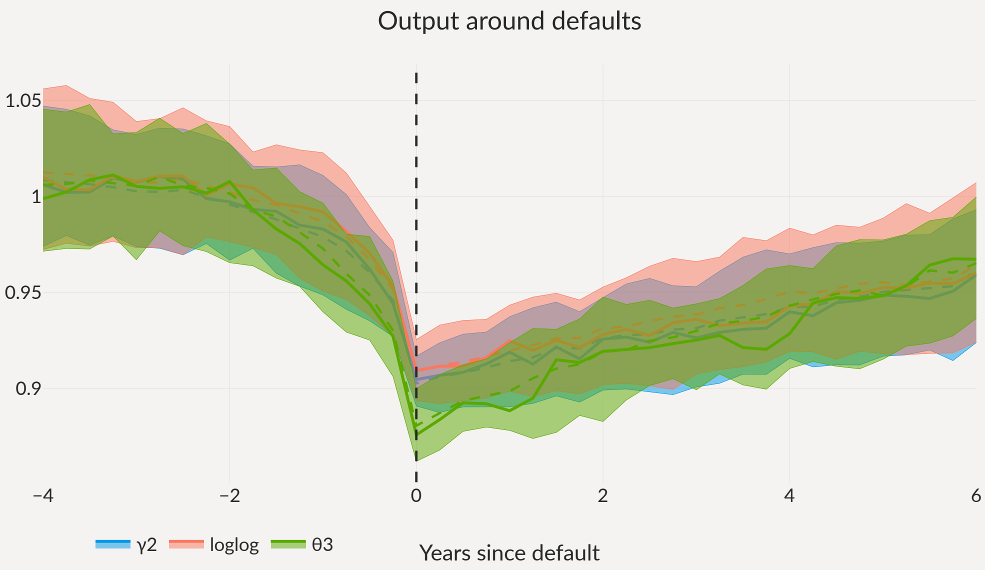
Calibrations

	Parameter	$\gamma = 2$	loglog	$\theta = 3$
Sovereign's discount factor	β	0.9627	0.9627	0.9627
Sovereign's risk aversion	θ	0	0	3
Sovereign's EIS	γ	2	1	1
Default output cost: linear	d_1	–0.2833	–0.2836	–0.247
Default output cost: quadratic	d_2	0.3253	0.3228	0.3029
Average spread (bps)	815	754	756	815
Relative volatility of consumption (%)	0.94	1.38	1.5	1.35
Debt-to-GDP ratio (%)	17.4	16.8	16.7	17.4
Std. deviation of spreads (bps)	443	496	447	2,026

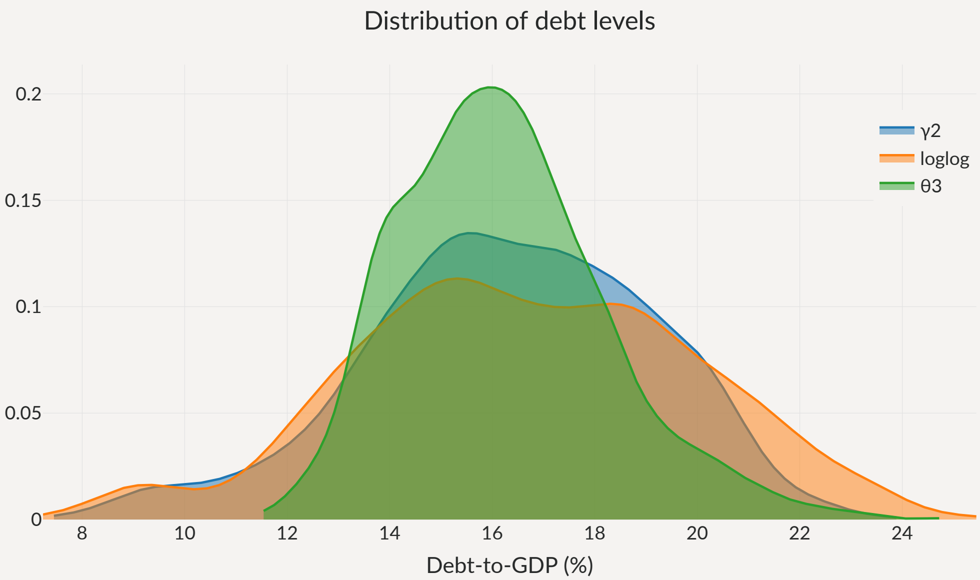
Costs of default



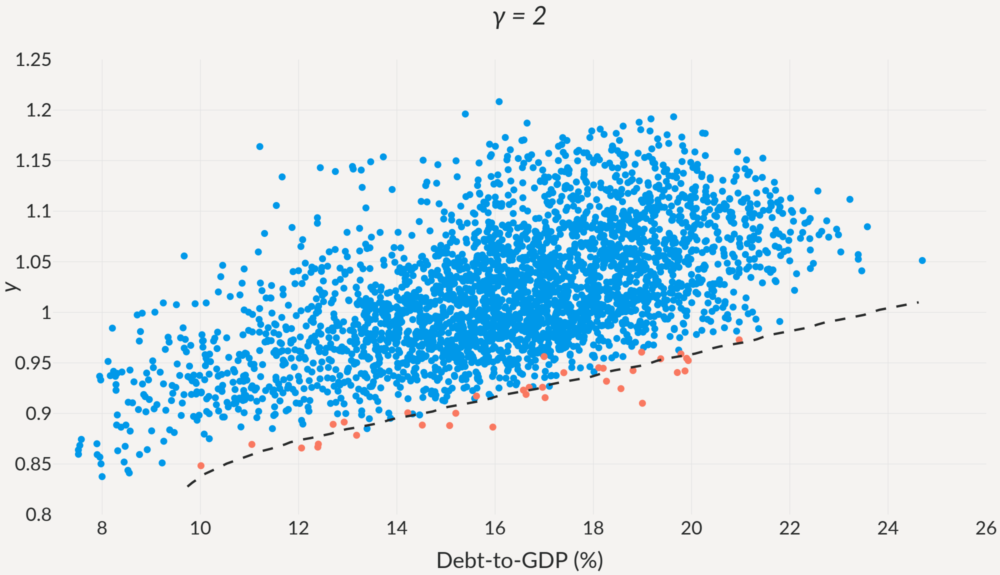
Output paths around defaults



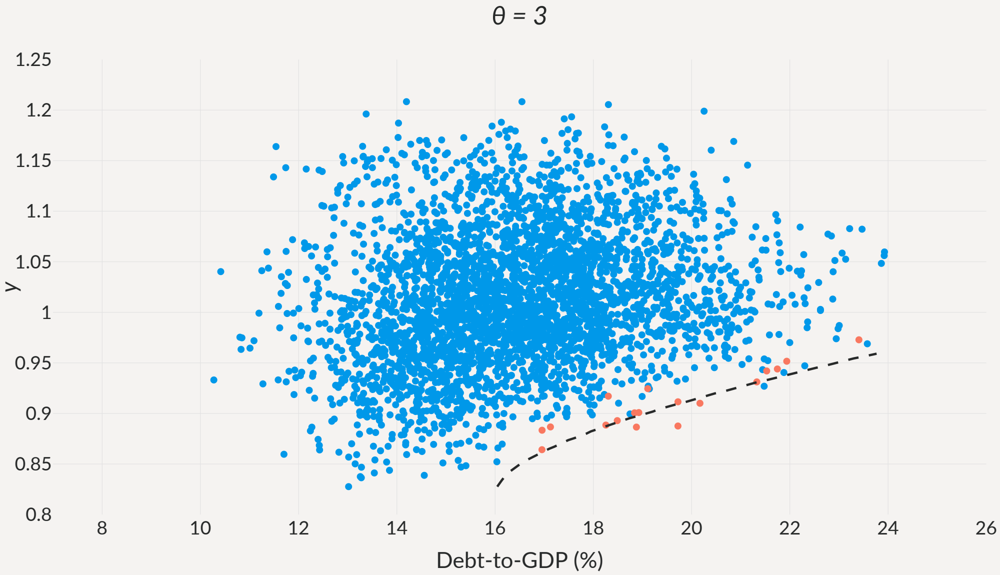
Debt levels and defaults



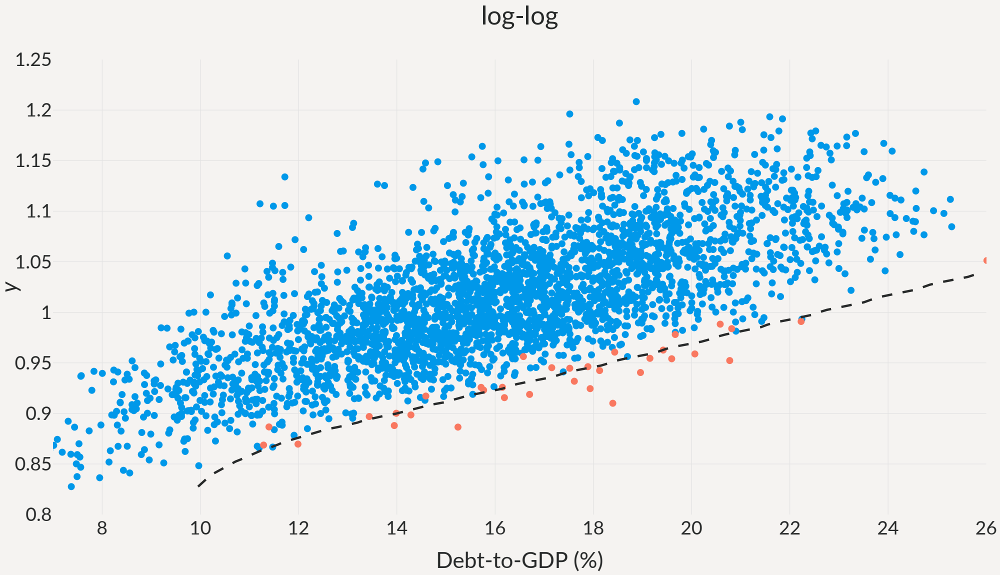
Debt levels and defaults



Debt levels and defaults



Debt levels and defaults



Welfare gains decomposition

Consumption without default costs c_t^R and its expectation $\bar{c}_t^R(b_0, z_0)$

$$\begin{aligned} c_t^R &= \mathbb{1}_{\mathcal{D}}(b_t, z_t) y(b_t, z_t) + (1 - \mathbb{1}_{\mathcal{D}}(b_t, z_t)) c(b_t, z_t) \\ \bar{c}_t^R(b_0, z_0) &= \mathbb{E}\left[c_t^R \mid b_0, z_0\right] \end{aligned}$$

Evaluate value of consuming c^R and $\bar{c}^R$

$$\begin{aligned} V_{ND}(b_0, z_0) &= \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t^R) \mid b_0, z_0\right] \\ V_{NV}(b_0, z_0) &= \sum_{t=0}^{\infty} \beta^t u\left(\bar{c}_t^R(b_0, z_0)\right) \end{aligned}$$

Welfare gains between models/equilibria with value functions v and $v^\star$

$$\frac{v^\star(b_0, z_0)}{v(b_0, z_0)} = \frac{v^\star(b_0, z_0)/v_{ND}^\star(b_0, z_0)}{v(b_0, z_0)/v_{ND}(b_0, z_0)} \times \frac{v_{ND}^\star(b_0, z_0)/v_{NV}^\star(b_0, z_0)}{v_{ND}(b_0, z_0)/v_{NV}(b_0, z_0)} \times \frac{v_{NV}^\star(b_0, z_0)}{v_{NV}(b_0, z_0)}$$

Welfare gains

	Total gains	From default costs	From volatility	From level
		$\gamma = 2$		
Access to markets	1.07	0.253	-0.09	0.906
No default	1.86	0.265	0.0412	1.55
Short-term debt	0.451	0.233	-0.0811	0.299
		loglog		
Access to markets	1.14	0.273	-0.0642	0.933
No default	2.03	0.275	0.0256	1.72
Short-term debt	0.563	0.25	-0.0637	0.376
		$\theta = 3$		
Access to markets	1.38	0.284	-0.0566	1.15
No default	1.78	0.306	0.0128	1.45
Short-term debt	0.362	-0.0244	-0.0249	0.411

Not much success so far

- Calibrations with CRRA and robustness yield similar moments
  - With similar implied relative volatility of consumption
  - With similar discount rates

    ... version with $\theta = 2$ still has too much debt and too volatile spreads
  - With larger physical default costs for models with robustness

- Gains of various policies still overwhelmingly explained by levels, not (eqm.) volatilities

- Still to do
  - Only move θ starting from loglog for understanding
  - Price of a Lucas tree
  - Re-do calibrations with Epstein-Zin, use EIS and RA, add consumption volatility and/or return on Lucas tree to targets
  - Add growth to unleash long-run risk?
  - Recovery, either exogenous or endogenous with renegotiation