

# Risk Aversion in Sovereign Debt and Default

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## Why risk aversion? Why in sovereign debt?

- In most RBC models, macro-financial separation holds
  - Elasticity of intertemporal substitution determines allocations
  - Risk aversion determines asset prices
- Sovereign debt literature typically inherits this line of thinking
  - CRRA preferences frequent, typically  $\gamma = 2$
- If MFS holds in sovereign debt, macro outcomes robust to different preferences
  - In particular, calibration of output/utility costs of default
  - Less clear about welfare effects
    - ... losses from default, debt dilution
    - ... welfare effects of banning debt, introducing state-contingent bonds

# Wanting risk prices in sovereign debt

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## This paper

- Show that macro-financial separation **breaks** in the sovereign debt model
- Understand the impact of preferences consistent with significant risk premia
- Find that risk aversion affects equilibria in unexpected ways

Model

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- Sovereign default model without default [reduces to an income-fluctuations problem]

$$v(b, z) = \max_{b'} u(c) + \beta \mathbb{E} [v(b', z') \mid z]$$

$$\text{subject to } c + \kappa b = q(b', z)(b' - (1 - \rho)b) + y(z)$$

$$\text{with } q(b', z) = \frac{1}{1 + r}$$

- We consider parametrizations of the model to vary risk aversion
  - ... with CRRA preferences  $u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}$
  - ... With robustness,  $u(c) = \log c$ ; replace  $\mathbb{E}$  with  $\mathbb{T}[X \mid \mathcal{F}] = -\frac{1}{\theta} \log (\mathbb{E} [\exp (-\theta X) \mid \mathcal{F}])$

- Start from log-log [ $\theta = 0$ ]: RA moves asset prices and welfare, not the macro

	loglog	$\theta = 1$	$\theta = 2$	$\theta = 3$
Average spread (bps)	0.0276	0.031	0.0406	0.138
Corr. NX,Y (%)	0.00777	0.00916	0.0114	0.0147
Rel. vol. cons (%)	1.59	1.62	1.65	1.66
Risk premium (p.p.)	0.0769	2.03	3.84	5.44
Debt-to-GDP (%)	29.7	29.5	29.2	28.9
Corr. deficit, y (%)	-0.0119	-0.0141	-0.0177	-0.0231
Welfare	1.034	1.008	0.9867	0.971

... welfare in autarky at  $\theta = 3$  is 6pp lower than loglog or CRRA

## Macro-financial separation without default (cont'd)

- Start from log-log [ $\gamma = 1$ ]: EIS+RA moves mostly macro, not asset prices and welfare

	loglog	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Average spread (bps)	0.0276	0.0273	0.0269	0.0271	0.0285
Corr. NX,Y (%)	0.00777	0.0154	0.0852	0.397	0.668
Rel. vol. cons (%)	1.59	1.56	1.35	0.965	0.727
Risk premium (p.p.)	0.0769	0.227	0.627	1.02	1.67
Debt-to-GDP (%)	29.7	28.8	25.9	19.3	8.75
Corr. deficit, y (%)	-0.0119	-0.0251	-0.162	-0.605	-0.774
Welfare	1.034	1.03	1.021	1.01	0.9918

... in fully Epstein-Zin, move only EIS for even less effect on asset prices and welfare

## Models with default

- Option value of default (with small pref. shocks for numerical performance)

$$\mathcal{V}(b, z) = \max\{v_R(b, z) + \epsilon_R, v_D(b, z) + \epsilon_D\}$$

- Similar equation for value of repayment  $v_R$ , debt prices reflect default probabilities

$$q(b', z) = \frac{1}{1+r} \mathbb{E} \left[ (1 - \mathbb{1}_{D'}) (\kappa + (1 - \rho)q(b'', z')) \mid z \right]$$

- Costs of default

$$v_D(b, z) = u(h(y(z))) + \beta \mathbb{E} \left[ \mathbb{1}_R \mathcal{V}(B(b, z'), z') + (1 - \mathbb{1}_R) v_D(b, z') \mid z \right]$$

$$h(y) = y(1 - d_0 - d_1 y)$$

- Risk aversion  $\implies$  no-smoothing in default costly  $\implies$  no macro-financial separation



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- Risk aversion  $\implies$  no-smoothing in default costly  $\implies$  no macro-financial separation

## Quantitative properties

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# Calibration

- Keep the same discount rate, vary costs of default to match spreads and debt

	Parameter	$\gamma = 2$	loglog	$\theta = 3$
Sovereign's discount factor	$\beta$	0.9627	0.9627	0.9627
Sovereign's robustness parameter	$\theta$	0	0	3
Sovereign's EIS	$\gamma$	2	1	1
Default output cost: linear	$d_1$	-0.2833	-0.2836	-0.247
Default output cost: quadratic	$d_2$	0.3253	0.3228	0.3029
Average spread (bps)	815	754	756	815
Debt-to-GDP ratio (%)	17.4	16.8	16.7	17.4

## Comparative statics: CRRA

- Increasing EIS+RA: Less volatility, procyclical exports, more skewed debt outcomes

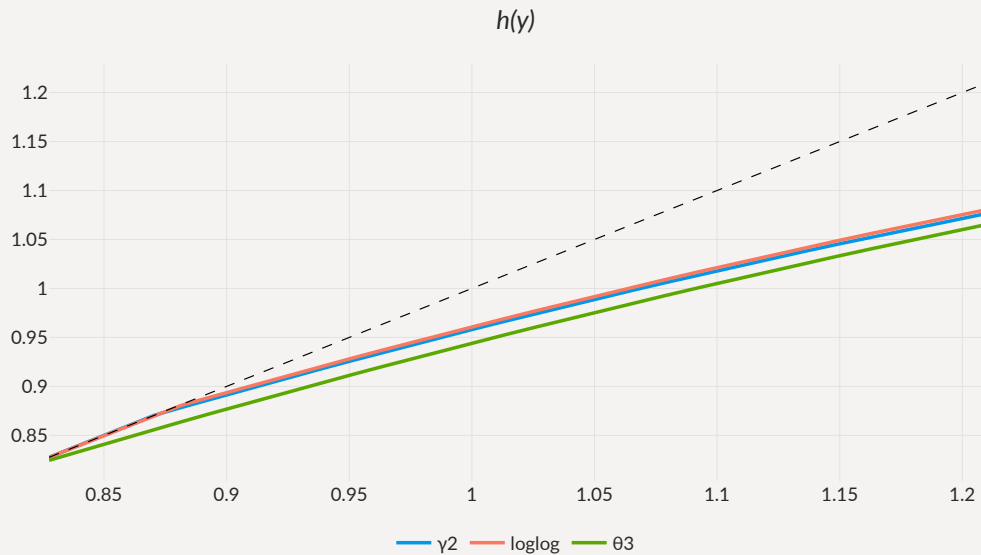
	loglog	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Avg. spread (bps)	756	800	912	974	1,057
Corr. NX,Y (%)	-0.285	-0.302	-0.21	0.0726	0.416
Rel. vol. cons (%)	1.5	1.37	1.18	1.04	0.921
Risk premium (p.p.)	0.652	0.789	1.02	1.28	2.38
Debt-to-GDP (%)	16.7	15.7	12.4	7.62	3.25
Corr. deficit, y (%)	0.391	0.391	0.217	-0.21	-0.627
Default freq. (%)	4.4	4.41	4.17	3.45	2.7
Std. dev. spreads (bps)	448	538	877	1,209	1,816
Welfare	1.013	1.01	1.002	0.9918	0.9728

## Comparative statics: robustness

- Increasing RA: less debt tolerance, limited effect on volatilities

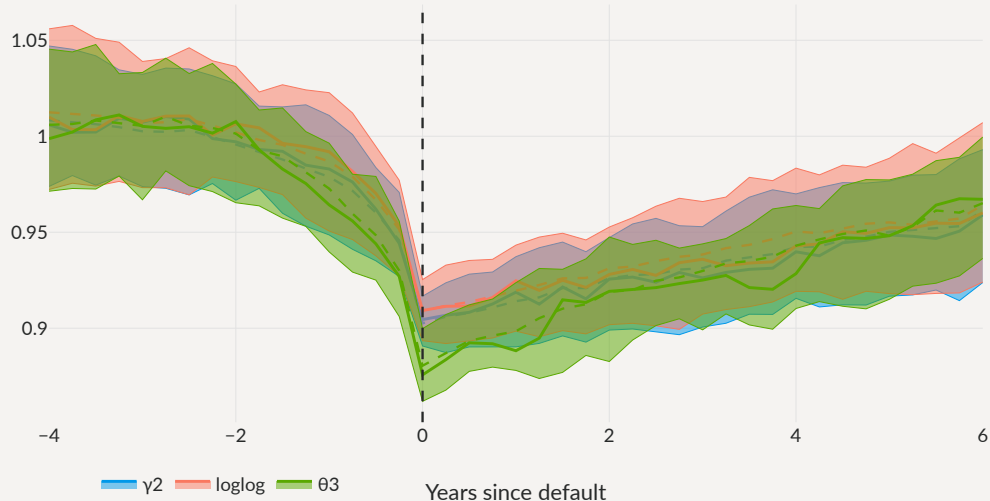
	loglog	$\theta = 1$	$\theta = 2$	$\theta = 3$
Avg. spread (bps)	756	1,683	20,240	36,331
Corr. NX, $y$ (%)	-0.285	-0.227	-0.0901	-0.227
Rel. vol. cons (%)	1.5	1.38	1.26	1.46
Risk premium (p.p.)	0.652	2.92	4.43	6.99
Debt-to-GDP (%)	16.7	14.2	9.09	9.57
Corr. deficit, $y$ (%)	0.391	0.292	0.118	0.266
Default freq. (%)	4.4	5.88	3.57	2.47
Std. dev. spreads (bps)	448	2,561	103,509	189,131
Welfare	1.013	0.9848	0.9629	0.9469

## Calibrated output costs of default with robustness



# Event-study of defaults

Output around defaults



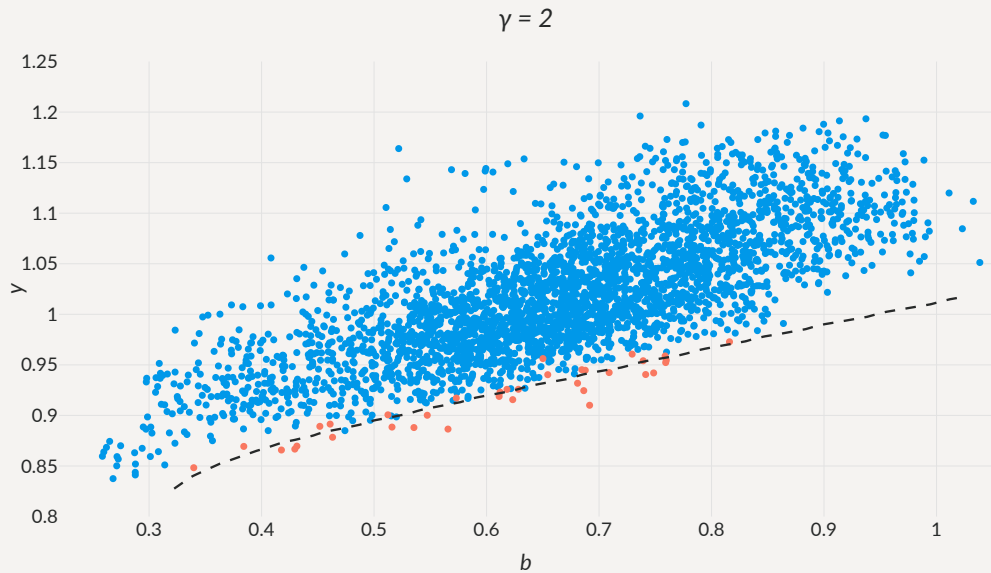
## Calibrations with risk aversion

- Calibration with robustness: skewed debt outcomes, small decrease in macro volatility

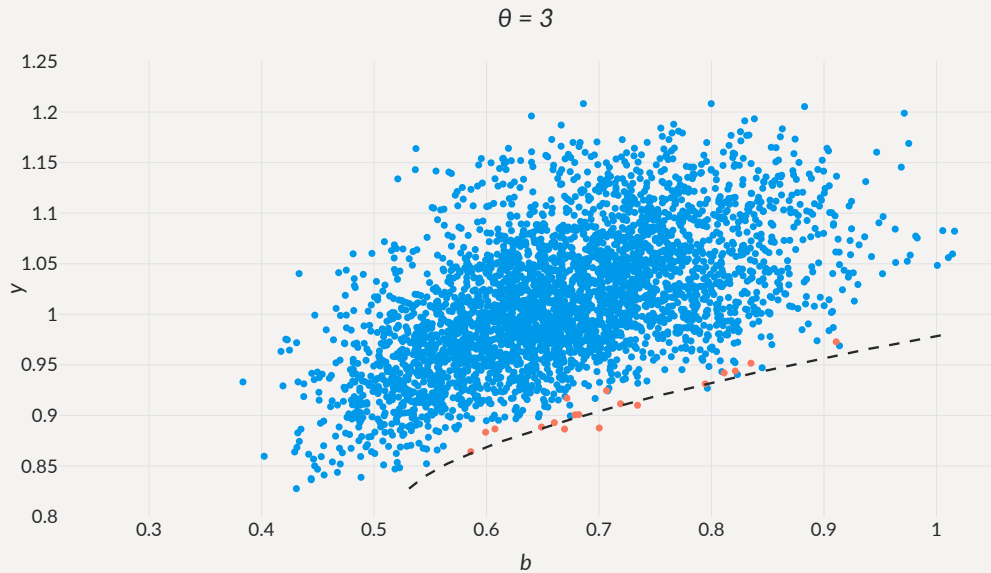
	Data	$\gamma = 2$	loglog	$\theta = 3$
Avg. spread (bps)	815	754	756	815
Corr. NX,Y (%)	-	-0.314	-0.285	-0.194
Rel. vol. cons (%)	0.94	1.38	1.5	1.35
Risk premium (p.p.)	-	0.778	0.652	5.9
Debt-to-GDP (%)	17.4	16.8	16.7	17.4
Corr. deficit, y (%)	-	0.405	0.391	0.207
Default freq. (%)	-	4.21	4.4	1.51
Std. dev. spreads (bps)	443	496	447	2,026



## Ergodic distribution for debt in CRRA model

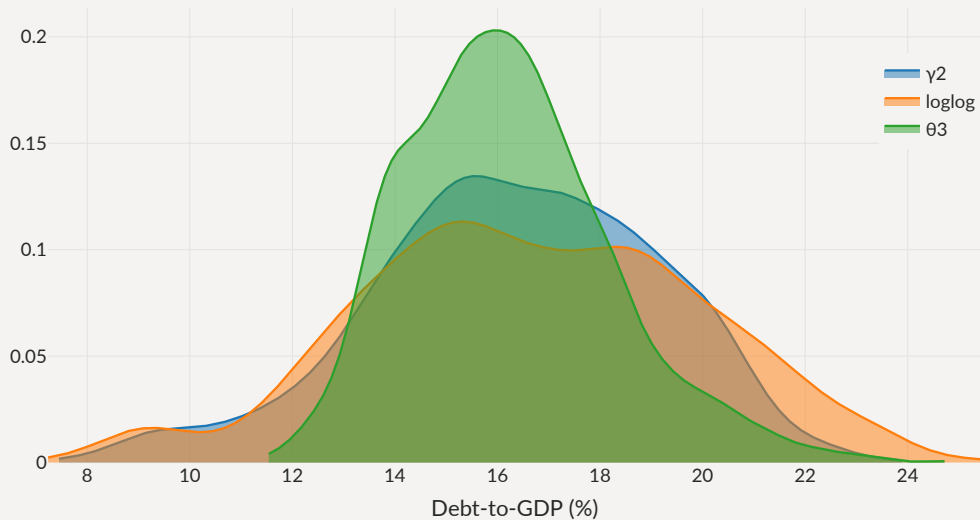


## Ergodic distribution for debt with robustness



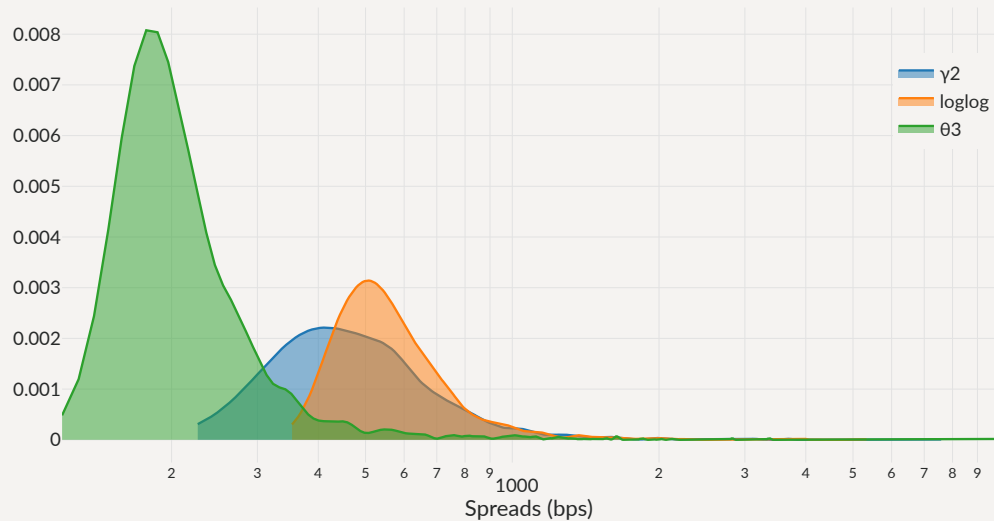
# Ergodic distribution for debt

Distribution of debt levels



# Ergodic distribution for spreads

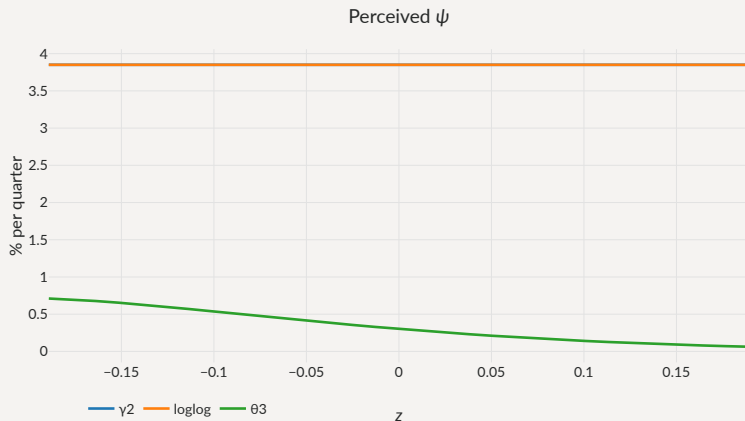
Distribution of spread levels



# Worst-case models

- Distorted expectation of  $X$

$$\tilde{\mathbb{E}}[X \mid \mathcal{F}] = \mathbb{E} \left[ \frac{\exp(-\theta v(s'))}{\mathbb{E}[\exp(-\theta v(s')) \mid \mathcal{F}]} X \mid \mathcal{F} \right]$$

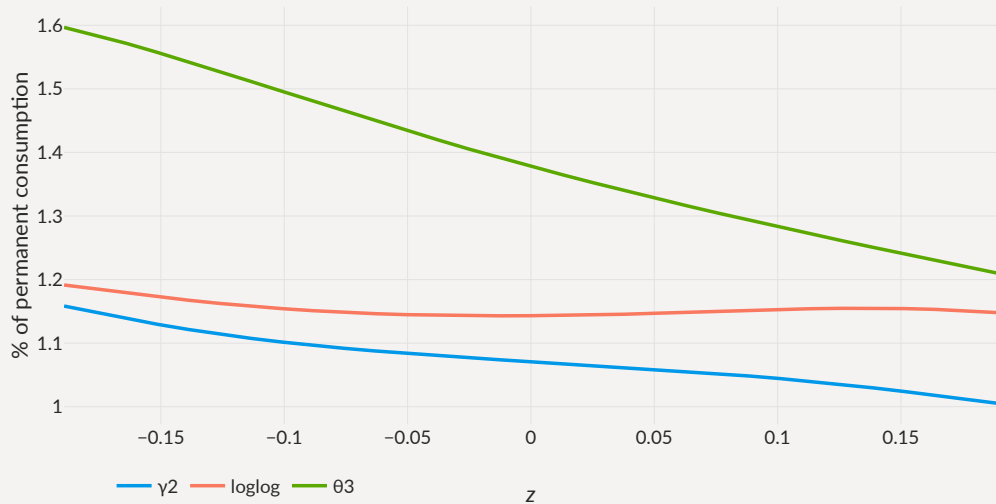


## Welfare effects

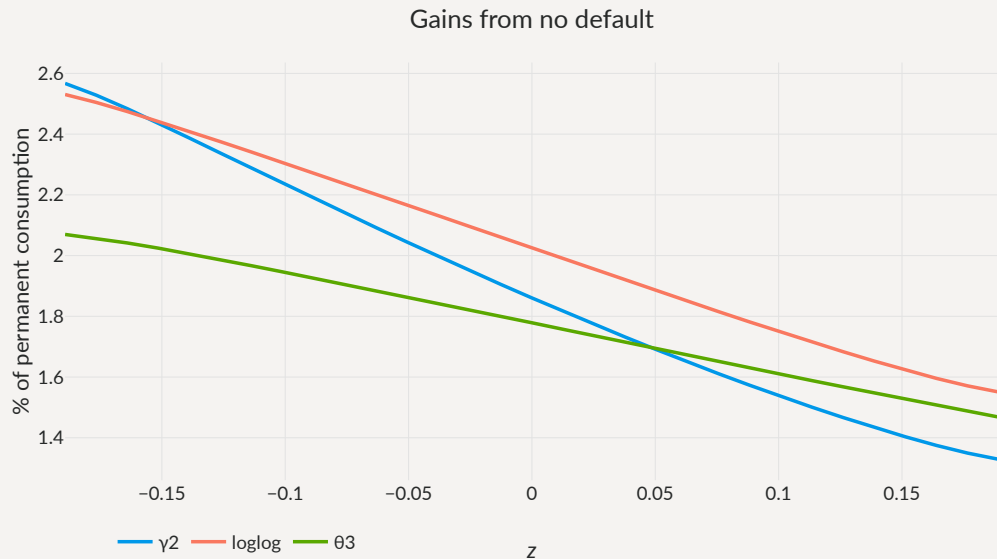
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# Welfare effects of debt

## Gains from access to debt

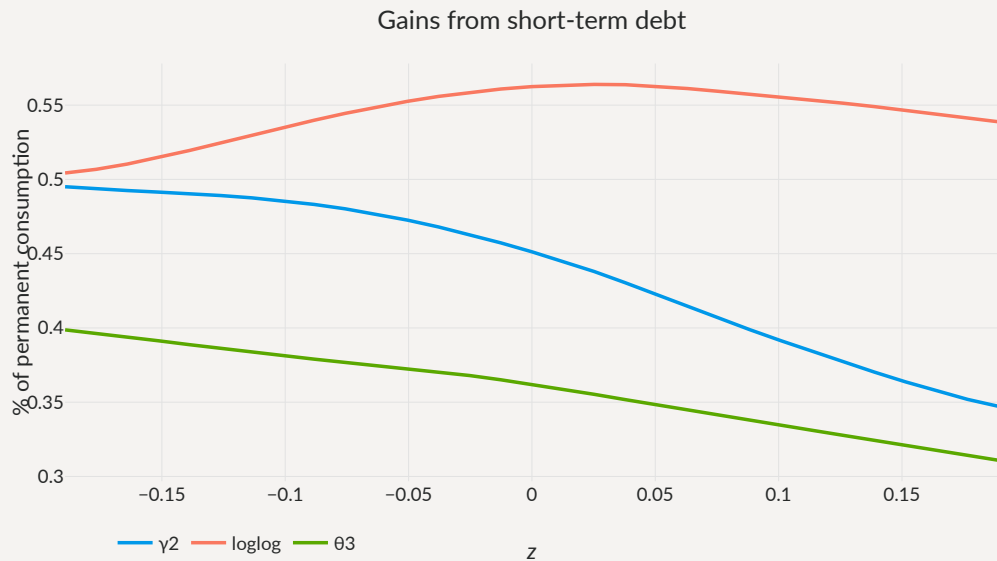


# Welfare effects of banning defaults





# Welfare effects of shortening maturity



## With preferences consistent with positive risk premia

- *Lower* debt tolerance
  - ... Larger default costs required
- Less staying at the edge of default
  - ... More skewness in the distribution of debt and spreads
- More use of the debt for insurance
  - ... Large gains from debt access, not so much for making debt safe

# Welfare gains decomposition

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Consumption without default costs  $c_t^R$

$$c^R(b, z) = \mathbb{1}_{\mathcal{D}}(b, z)y(b, z) + (1 - \mathbb{1}_{\mathcal{D}}(b, z))c(b, z)$$

Evaluate value of consuming  $c^R$  [instead of  $c$ ] and removing uncertainty

$$V_{NC}(b, z) = u(c^R(b, z)) + \beta \mathbb{E} [V_{NC}(b', z') \mid z]$$

$$V_{NV}(b, z) = u(c^R(b, z)) + \beta V_{NV}(b', \mathbb{E} [z' \mid z])$$

Welfare gains between models/equilibria with value functions  $v$  and  $v^*$

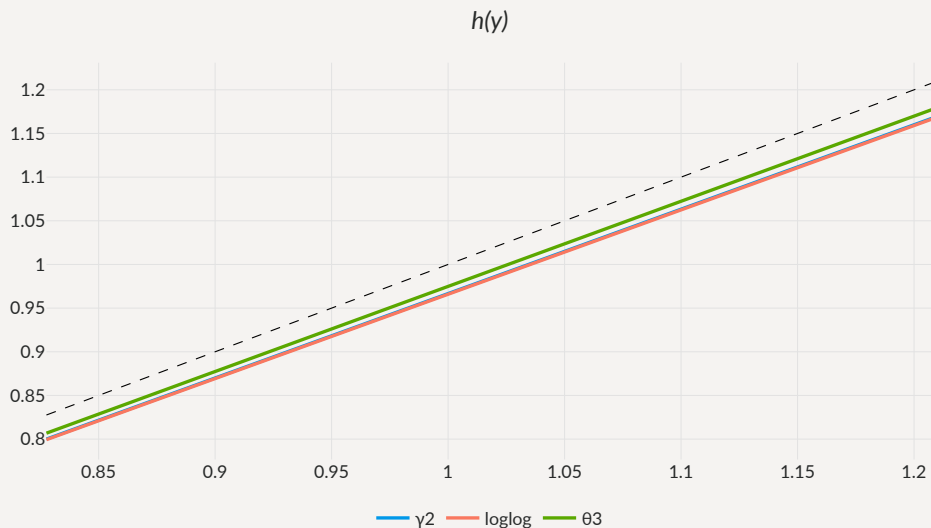
$$\frac{v^*(b_0, z_0)}{v(b_0, z_0)} = \frac{v^*(b_0, z_0)/v_{NC}^*(b_0, z_0)}{v(b_0, z_0)/v_{NC}(b_0, z_0)} \times \frac{v_{NC}^*(b_0, z_0)/v_{NV}^*(b_0, z_0)}{v_{NC}(b_0, z_0)/v_{NV}(b_0, z_0)} \times \frac{v_{NV}^*(b_0, z_0)}{v_{NV}(b_0, z_0)}$$

## Welfare gains

	Total gains	From default costs	From volatility	From level
$\gamma = 2$				
Access to markets	0.622	-0.273	0.218	0.679
No default	1.87	0.274	-0.292	1.89
Short-term debt	0.411	0.255	-0.448	0.606
loglog				
Access to markets	0.663	-0.294	0.284	0.674
No default	2.04	0.295	-0.345	2.09
Short-term debt	0.519	0.272	-0.439	0.688
$\theta = 3$				
Access to markets	0.961	-0.25	0.0354	1.18
No default	1.72	0.251	-0.0744	1.54
Short-term debt	0.262	0.233	-0.45	0.481

## Model with linear costs

- Convex costs lower income volatility **during** defaults



## Concluding remarks

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## Risk aversion in the sovereign debt model

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- We evaluate preferences consistent with risk premia in the sovereign default model
  - ... mostly possible to match standard calibration targets with robustness
- Effect of robustness concentrated at higher-order moments
  - ... makes crises look like more abrupt events
- Innocent-looking features of the standard model weigh against large risks/distortions
  - ... convex costs of default mute post-default uncertainty





	loglog	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Corr. NX,Y (%)	-0.00131	-0.00131	-0.00131	-0.00131	-0.00131
Rel. vol. cons (%)	1	1	1	1	1
Risk premium (p.p.)	0.0833	0.251	0.751	1.57	3.05
Welfare	1.002	1	0.9951	0.9868	0.9699

	loglog	$\theta = 1$	$\theta = 2$	$\theta = 3$
Corr. NX,Y (%)	-0.00131	-0.00131	-0.00122	-0.00073
Rel. vol. cons (%)	1	1	1	1
Risk premium (p.p.)	0.0833	2.02	3.81	5.32
Welfare	1.002	0.9769	0.9564	0.9411