Credibility Dynamics and Inflation Expectations*

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Abstract

We study the optimal design of a disinflation plan by a planner who lacks commitment and has imperfect control over inflation. The government's reputation for being committed to the plan evolves as the public compares realized inflation to the announced targets. Reputation is valuable as it helps curb inflation expectations. At the same time, plans that are more tempting to break lead to larger expected reputational losses in the ensuing equilibrium. Taking these dynamics into consideration, the government announces a plan which balances promises of low inflation with dynamic incentives that make them credible. We find that, despite the absence of inflation inertia in the private economy, a gradual disinflation is preferred even in the zero-reputation limit.

JEL Classification E52, C73

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Introduction

Macroeconomic models give expectations about future policy a large role in the determination of current outcomes. Policy is then generally set under one of two assumptions: commitment to future actions or discretion. Attempts to model policy departing from these extreme cases have found limited success.

However, governments actively attempt to influence beliefs about future policy. Examples include forward guidance and inflation targets but also fiscal rules and the timing of introduction of policies. Such promises rarely constrain future choices, yet they can shift expectations substantially. Standard macroeconomic models cannot capture this idea directly, as expectations of the public are fully determined by the policy chosen with commitment, or with discretion as part of an equilibrium. In both cases the public understands that announcements do not bind the government in any way. Announcements do not grant any additional credibility to the policy maker, as the public is convinced of her course of action.

In this paper we develop a rational-expectations theory of government credibility and apply it to the question of optimal policy announcements. Our notion of credibility is based on the concept of reputation in game theory (Kreps and Wilson, 1982; Milgrom and Roberts, 1982). In our model the government (or central bank, or planner) could be rational and strategic, or one of many possible behavioral types described by a policy that they stubbornly follow. The public is uninformed about the government's type and makes statistical inference about it after observing the government's announcements and actions. This inference is central to our analysis because it turns out to be in the best interest of the rational type to pretend to be one of the behavioral types.

We consider a stylized environment. In the initial period, the government makes an announcement of its policy targets and is then free to choose policy. However, the private sector knows that if the government is behavioral it announced exactly what it will implement. As a consequence, the rational type has an ex-post incentive to stay close to any announced targets, which might earn it a reputation for being committed to them. The incentive exists at any positive level of reputation, though its strength depends on the announced targets. In anticipation of these interactions, the rational type chooses carefully which targets to announce. Our main question concerns the optimal policy announcement in the presence of these reputational concerns.

We set our model of reputation in a modern version of the classic environment of Barro (1986) and Backus and Driffill (1985), where a central bank sets inflation subject to an expectationsaugmented Phillips curve. The monetary authority dislikes inflation but constantly faces an opportunity to engineer surprise inflation, which would deliver output closer to potential. We model these features through the standard, cashless-limit New Keynesian setup for the private economy. To focus on incentives and reputation dynamics, we abstract from an IS curve and let the government control inflation directly.

A natural definition of the government's reputation is the private sector's belief that the government is indeed the behavioral type whose plan was announced. We refer to the total, ex-ante probability of the behavioral types as the government's initial reputation. The credibility of a plan measures the proximity of expected inflation to the targets. While credibility generally increases with reputation, the insights of the reputation literature imply that our notion of credibility need not approach zero as reputation vanishes.

A key assumption we introduce is that the government exerts imperfect control over inflation, perhaps due to underlying shocks to money demand. Imperfect control masks the government's choice of policy: the private sector understands that realized inflation is only an imperfect signal of intended inflation. We consider additive and normally distributed noise which implies that the public can never be certain of the government's action. This assumption distinguishes us in technical terms from the early studies of reputation in monetary policy referenced above, where the public perfectly observes the inflation chosen by the government. But, crucially, it also creates a smooth tradeoff for the government: overshooting the target by more creates, in expectation, a larger boom accompanied by larger reputational losses.

When designing policy, the planner takes into account its own future behavior, which it can influence but not control. 'Future' governments have complete freedom and will only respect promises made at time 0 to the extent that it suits them. Preserving reputation turns out to be a powerful disciplining force for the planner's future self. Crucially, the value of reputation depends on the plan in place. Plans differ in the outcomes they intend to deliver and in how closely they are expected to be followed in the future, their credibility. Both features contribute to current outcomes through the private sector's expectations. These forces lead the planner to weigh a plan's intended outcomes against the reputation dynamics it generates.

Our main result is that the government announces a policy under which inflation starts high and diminishes gradually. Plans with gradual disinflation are more credible: having a higher target for today than tomorrow boosts the gains from sticking to the plan. This slows down the pace of reputational losses sufficiently to offset the negative effect of higher announcements on expected inflation. In contrast, the reputation literature typically considers the limit as the long-lived player becomes arbitrarily patient (Fudenberg and Levine, 1989). In that case the government can obtain a payoff arbitrarily close to its commitment payoff by announcing a static plan with zero inflation in every period.

The gradualism of our optimal policy might lead an outside observer to conclude that there is substantial inflation inertia in the economy and that the government avoids a costly recession when bringing inflation down. However, in our model past inflation does not enter the Phillips curve. Instead, gradual disinflation is a result of the dynamic incentives of the government.

A second result concerns the limit as initial reputation becomes arbitrarily small. At zero initial reputation, the only Markov equilibrium is a repetition of the static Nash equilibrium with high inflation and output at the natural level. However, as is usual in the reputation literature, even a small amount of reputation creates a large departure in behavior from the Nash equilibrium. In particular, we show that the gradualist nature of optimal announcements and the corresponding credibility dynamics are preserved at arbitrarily low levels of initial reputation. The limiting announcement, which can be interpreted as the announcement in a fully rational model where the government has mild credibility concerns, also exhibits gradualism.

Discussion of the Literature. We contribute to a long literature dealing with issues of commitment, imperfect credibility, and reputation. The time-inconsistency of optimal policy (Kydland and Prescott, 1977) has long been recognized by researchers, who have set out to ask whether reputation can be a substitute for commitment.

Barro (1986) and Backus and Driffill (1985) were the first studies of monetary policy to introduce reputation via behavioral types committed to a certain policy. These and many subsequent studies (Cukierman and Liviatan, 1991; Sleet and Yeltekin, 2007; King et al., 2008; Dovis and Kirpalani, 2019) assume the government has perfect control of inflation. Thus, any deviations are detected by the private sector and fully destroy the government's reputation. In contrast, our assumption of imperfect control enables distinct tradeoffs that shape the gradualism of optimal plans.

Another line of research studies monetary policy with imperfect control by considering uncertainty about the preferences of the planner which is distinct from reputation (Cukierman and Meltzer, 1986; Faust and Svensson, 2001; Phelan, 2006). We view reputation as more directly suited to address optimal announcements, which was not the goal of the above papers.

Most closely related is the work of Lu, King, and Pastén (2016) and King and Lu (2020) who consider reputational models with imperfect control. However, their optimizing type has commitment power and the type that lacks commitment follows a fixed rule in Lu, King, and Pastén

(2016), and behaves myopically in King and Lu (2020). This reversal of roles changes the underlying tradeoffs. In these papers the planner announces (and commits to) a plan that promotes separation from the alternative type. In our model, the planner directly chooses the behavioral type at the announcement stage and mimics its policy to convince the public that it is committed to it. This makes the model a natural setting for studying whether reputation-building incentives can substitute for commitment, as well as the credibility of different plans. In addition, Lu, King, and Pastén (2016) and King and Lu (2020) obtain the Ramsey plan in the limit as the planner becomes known to be the optimizing type, whereas the corresponding limiting plan in our model resembles neither commitment nor discretion.

An alternative view of reputation is given by the notion of sustainable plans (Chari and Kehoe, 1990; Phelan and Stacchetti, 2001). This literature considers subgame perfect equilibria in games between the government and the private sector applying the tools of Abreu, Pearce, and Stacchetti (1990). However, this typically generates a large set of equilibria. In fact, reputational models are often used to refine the equilibrium set. Faingold and Sannikov (2011) study a general model of reputation in continuous time which applies to our framework of monetary policy with imperfect control. They find conditions for a unique equilibrium which is Markovian in reputation, informing our restriction to Markovian equilibria. However, their model considers behavioral types with static plans, so it cannot address the gradualism of announcements we are interested in.

Even though the announcements in our model do not constrain the actions of the rational government, they are not cheap talk, as they can be sent by only one of the behavioral types. This distinguishes us from cheap talk models of monetary policy such as Stein (1989) and Turdaliev (2010).

Finally, the gradualism featured by our equilibrium plans is reminiscent of the allocations arising from organizational equilibria described by Bassetto et al. (2018). Those allocations gradually transit from the discretion outcome towards commitment, but do not reach it; in the same way that our equilibrium plans start close to the static Nash and converge to a long-run rate of inflation above the first-best of 0. However, organizational equilibria rely on renegotiation-proofness rather than reputation, as well as on a series of properties used to select the equilibrium. Our results suggest that our framework of reputation with noise may endogenously generate analogues to such properties.

Layout. The rest of the paper is structured as follows. Section 2 introduces our model of reputation. Notions of equilibrium are defined and discussed in Section 3. Section 4 lays out our main results and Section 5 discusses how optimal plans depend on parameters. Section 6 dissects our main result by studying an extension which highlights the role played by incentives and provides more concrete policy implications. Finally, Section 7 concludes.

2. Model

We consider a government which dislikes inflation as well as deviations of output from a target $y^\star$ according to the loss function

$$L_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[(y^\star - y_t)^2 + \gamma \pi_t^2 \right] \right], \tag{1}$$

where y_t, π_t denote output and inflation at time t, $\gamma \geq 0$ is the relative weight on inflation, and $\beta \in (0,1)$ is a discount factor. A Phillips curve relates current output to current and expected inflation:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \left[\pi_{t+1} \right], \tag{2}$$

where $\mathbb{E}_t$ represents the expectations operator based on information up to time t (including π_t), and $\kappa \geq 0$ is the slope of the Phillips curve. The government has imperfect control over inflation. At time t it chooses g_t and realized inflation is

$$\pi_t = g_t + \sigma \epsilon_t, \tag{3}$$

where $\epsilon_t \stackrel{iid}{\sim} \mathcal{N}(0,1)$ is realized after the government has made its choice and $\sigma > 0$. Let f_ϵ denote the density of the control shock ϵ_t.

2.1 *Announcements and behavioral types*

An announcement is a sequence $(a_t)_{t=0}^{\infty}$ of inflation targets a_t for each time t. Inflation targets are in the interval $A = [0, \pi^N]$, where $\pi^N = y^\star \frac{\kappa}{1-\beta+\kappa^2\gamma}$ is inflation in the unique Nash equilibrium of the static game. Consequently, the space of announcements is $\mathcal{A} = A^\infty$.

The government is either rational or one of many behavioral types indexed by a set $\mathcal{C} \subseteq \mathcal{A}$. A behavioral type $c \in \mathcal{C}$ is committed to making an announcement c and following it. The announcement of each behavioral type c is denoted by $(a_t^c)_{t=0}^{\infty}$ and satisfies

$$a_t^c = \chi + e^{-\omega t} (a_0 - \chi)$$

for some parameters (a_0, ω, χ). These announcements include constant, decreasing, and increasing paths for inflation, demonstrated in Figure 1. Inflation starts from a_0 and converges towards χ with a exponential decay rate of ω. When it does not lead to confusion we identify a plan $c = (a_t^c)_t$ with the triple $(a_0^c, \omega^c, \chi^c)$.

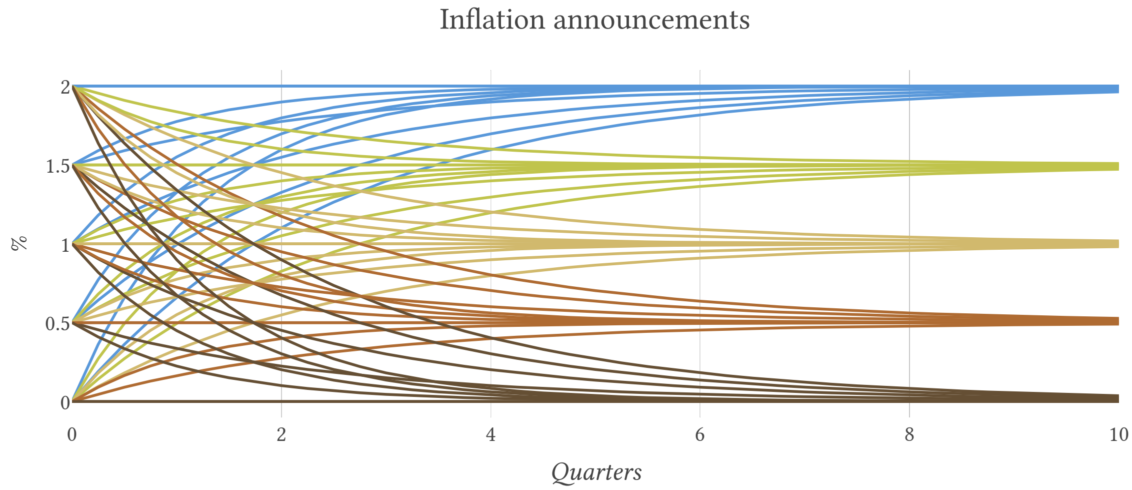


Figure 1: Possible behavioral types' announcements

This parametrization makes $\mathcal{C}$ finite-dimensional and allows us to write each plan recursively as $a_{t+1}^c = \phi_c(a_t^c)$, where

$$\phi_c(a) = \chi + e^{-\omega}(a - \chi).$$

We assume that the government is rational with probability $1 - z$. A probability distribution on $\mathcal{C}$ with density ν describes the distribution of possible behavioral types, which have total probability z. The government's type is private information.

2.2 *Timing of play*

The government begins by announcing an inflation plan $a \in \mathcal{A}$. Then at each $t \geq 0$, the government chooses g_t and inflation π_t is given by (3). After inflation is realized, the private sector's beliefs are updated (see below) and output is determined by (2).

If the government is a behavioral type $c \in \mathcal{C}$, it announces $(a_t^c)_t$ and follows the announcement by setting $g_t = a_t^c$ at each time t. A rational government instead makes a strategic choice of announcement $r \in \mathcal{A}$ and is also free to choose any action g_t at time t based on time-t information, regardless of announced inflation a_t^r.

2.3 Beliefs

We refer to the probability the private sector attaches to the government being a behavioral type as its *reputation*. Reputation evolves as the private sector updates its beliefs via Bayes' rule following the initial announcement of targets at time 0 as well as realized inflation at each time t.

Consider the posterior beliefs following an announcement $a \in \mathcal{A}$. If $a \notin \mathcal{C}$ the private sector infers that the government is rational. If $a \in \mathcal{C}$, the government can be either rational or the behavioral type a. Suppose that in equilibrium the rational type announces a with density $\mu(a)$. Then the posterior probability that the government is behavioral conditional on an announcement a with $\mu(a) > 0$ is given by

$$p_0(a; z, \mu) = \frac{z\nu(a)}{z\nu(a) + (1-z)\mu(a)}. \tag{4}$$

At time t, the private sector's posterior of the government being a behavioral type $c \in \mathcal{C}$ is formed by applying Bayes' rule to the private sector's information. Suppose that inflation π_t is realized at time t. If the government is behavioral of type c, then it must have chosen $g_t = a_t^c$ and the current shock must have been $\epsilon_t = \pi_t - a_t^c$, which has density $f_\epsilon(\pi_t - a_t^c)$. Let $g_t^\star$ denote the rational type's equilibrium strategy. Then if the government is rational, it must have chosen $g_t = g_t^\star$ so the shock must have been $\epsilon_t = \pi_t - g_t^\star$. Therefore, updating from a prior belief of p_t, we have that

$$p_{t+1} = \frac{p_t \cdot f_\epsilon(\pi_t - a_t^c)}{p_t \cdot f_\epsilon(\pi_t - a_t^c) + (1-p_t) \cdot f_\epsilon(\pi_t - g_t^\star)}.$$

It is useful to rewrite this condition as

$$p_{t+1} = p_t + p_t(1-p_t)\frac{f_\epsilon(\pi_t - a_t^c) - f_\epsilon(\pi_t - g_t^\star)}{p_t f_\epsilon(\pi_t - a_t^c) + (1-p_t) f_\epsilon(\pi_t - g_t^\star)}, \tag{5}$$

which shows that reputation moves slowly when it started close to 0 and 1, i.e. when the private sector is almost certain of the government's type. Large increases in reputation occur when realized inflation is much closer to the target than to the rational type's strategy, and large decreases in reputation require the converse. Consequently, large movements in reputation are more likely when the target differs significantly from the expected behavior of the rational type.

2.4 Bellman equations after an announcement

Given an announcement $c \in \mathcal{C}$, the problem of the rational type is to choose mean inflation g_t in period t to maximize (1) subject to (2), (3), and (5). The time-t government makes its choices taking as given its reputation p_t and the private sector's expectations, including the expected choice $g_t^\star$ of the rational type. We focus on Markovian strategies with $g_t^\star = g^\star(p_t, a_t^c)$. Thus, at time t the private sector expects the behavioral type to choose a_t^c and the rational type to choose $g^\star(p_t, a_t^c)$. This allows us to write the rational government's problem recursively as

$$\mathcal{L}^c(p, a) = \min_g \mathbb{E}\left[(y^\star - y)^2 + \gamma\pi^2 + \beta\mathcal{L}^c(p', \phi_c(a))\right] \tag{6}$$
$$\text{subject to } \pi = g + \epsilon$$
$$\pi = \kappa y + \beta\big[p'\phi_c(a) + (1-p')g^\star(p', \phi_c(a))\big]$$
$$p' = p + p(1-p)\frac{f_\epsilon(\pi - a) - f_\epsilon(\pi - g^\star(p,a))}{pf_\epsilon(\pi - a) + (1-p)f_\epsilon(\pi - g^\star(p,a))}$$

Problem (6) illustrates the government's reputation-building incentives. By controlling current inflation, the rational type can, on average, affect its future reputation p'. This changes the continuation value $\mathcal{L}^c(p', \phi_c(a))$ as well as current output (through the Phillips curve). The effect on output is given by

$$\frac{\partial y}{\partial \pi} = \frac{1}{\kappa}\left[1 - \beta\frac{\partial p'}{\partial \pi}\left(\phi_c(a) - g^\star(p', \phi_c(a)) + (1-p')\frac{\partial g^\star(p', \phi_c(a))}{\partial p'}\right)\right]. \tag{7}$$

Inflation affects current output through three different terms. The first term, $\frac{1}{\kappa}\cdot 1$, describes the direct effect through the Phillips curve.

Inflation also affects output through changes in the posterior belief p'. In our numerical simulations the rational type chooses higher inflation than the announcement, so higher inflation decreases its reputation on average. The second term, $\beta\frac{1}{\kappa}\left(-\frac{\partial p'}{\partial \pi}\right)(\phi_c(a) - g^\star(p', \phi_c(a)))$, captures the fact that a lower posterior belief moves expectations of future inflation away from the target $\phi_c(a)$ and toward the expected choice of the rational type $g^\star(p', \phi_c(a))$. The third term, $\beta\frac{1}{\kappa}\left(-\frac{\partial p'}{\partial \pi}\right)(1-p')\frac{\partial g^\star(p', \phi_c(a))}{\partial p'}$, captures the effect of reputation on the rational type's future action.

At low levels of reputation p, inflation expectations place most of the weight on the rational type's action $g^\star(p', \phi_c(a))$. If future governments are expected to value their reputation and choose inflation close to the target, then the current government has an incentive to conserve its reputation and, therefore, also stay close to its announcement.

3. Equilibrium

3.1 Continuation equilibrium following an announcement

Consider the game immediately following an announcement c. A solution to (6) describes the government's choices $g(p, a)$ as a function of $g^\star$, the private sector's expectations of its actions. As is common with rational expectations, our notions of equilibrium emphasize the dual role played by $g^\star$: as the government's decision rule, and as a description of the private sector's expectations, which restrain the government (Sargent, 1999).

Definition. Given an announcement $c \in \mathcal{C}$, a *continuation equilibrium* consists of a loss function $\mathcal{L}^c : [0, 1] \times A \to \mathbb{R}$ and a policy function $g_c^\star : [0, 1] \times A \to \mathbb{R}$ such that

1. The loss function $\mathcal{L}^c$ satisfies the rational government's Bellman equation (6) taking as given expectations $g_c^\star$.
2. $g_c^\star$ is the policy function that attains the maximum in the solution to (6).

Computation. To find a continuation equilibrium, we take the infinite-horizon limit of the finite-horizon game. Given a continuation value $\mathcal{L}_{t+1}^c$ and policy $g_{t+1}^\star$, we plug these into the objective function and the Phillips curve constraint in (6). This then creates an operator mapping values for g, representing the private sector's expectations in the Bayes' rule constraint, into optimal actions g for the government. For each state (p, a), we find the new $g_t^\star(p, a)$ as the fixed point of this operator, and the new $\mathcal{L}_t^c(p, a)$ as the resulting minimum. Then we iterate (backward in time) until convergence.

A useful property of continuation equilibria follows from close observation of problem (6): given the decay and asymptote parameters, it is equivalent to start the plan at a different initial announcement a or to just have arrived at a current announcement a as the continuation equilibrium unfolded.

Observation. Suppose $(\mathcal{L}, g^\star)$ is a continuation equilibrium for announcement $c = (a_0, \omega, \chi) \in \mathcal{C}$. Then for any b_0, the same pair $(\mathcal{L}, g^\star)$ is a continuation equilibrium for plan $c' = (b_0, \omega, \chi)$.

Another immediate property of equilibrium is that the government cannot design an inflation plan that will increase its reputation over time. If the rational government follows the announcement exactly, i.e. if $g^\star(p, a) = a$, its reputation will stay unchanged because inflation is not a signal of its type. And if the equilibrium strategy calls for deviations from the plan, reputation will decline on average. In other words, rational expectations do not allow the planner to accumulate reputation by consistently delivering on its promises, as such compliance would be anticipated by the private sector. What the planner can do is to design its plan in a way that provides incentives to deliver on it.

Observation. In any continuation equilibrium, the rational type's reputation is a supermartingale:

$$\mathbb{E}\left[p_{t+1} \mid \text{rational}, \mathcal{F}_t\right] \leq p_t,$$

where $\mathcal{F}_t$ denotes information up to time t. Thus, the planner cannot design an announcement that generates expected reputational gains in the ensuing continuation equilibrium.

3.2 *Equilibrium from the announcement stage*

An equilibrium of the entire game an initial announcement, as well as a set of continuation equilibria.

Definition. Given an initial reputation z, a *reputational equilibrium* is a distribution μ_z over $\mathcal{C}$ along with continuation equilibria $\{\mathcal{L}^c, g_c^\star\}_{c\in\mathcal{C}}$ and a posterior reputation $p_0 : \mathcal{C} \to [0,1]$ such that

1. Posterior reputation at each plan c satisfies Bayes' rule (4) given z and the distribution μ_z.
2. The distribution of mimicked types μ_z minimizes the posterior reputation-adjusted loss function

$$\mathcal{L}^\star(\mu_z, z) = \int_{\mathcal{C}} \mathcal{L}^c(p_0(c), a_0^c)\, d\mu_z(c)$$

   taking as given the posterior reputation function p_0.

The second part of the definition implies that the planner is indifferent among plans in the support of μ_z and prefers them to other plans:

$$\begin{aligned}
\mathcal{L}^c(p_0(c), a_0^c) &= \mathcal{L}^{c'}(p_0(c'), a_0^{c'}) && \text{for } c, c' \in \operatorname{supp}(\mu_z) \\
\mathcal{L}^c(p_0(c), a_0^c) &\leq \mathcal{L}^{c'}(1, a_0^{c'}) && \text{for } c \in \operatorname{supp}(\mu_z),\ c' \notin \operatorname{supp}(\mu_z),
\end{aligned}$$

where we highlight the fact that announcements which are not made in equilibrium grant full reputation: $p_0(c) = 1$ for all $c \notin \operatorname{supp}(\mu_z)$.

A lot of our analysis will focus on the limiting reputational equilibrium announcement as initial reputation vanishes:

$$\mu^\star = \lim_{z\to 0} \mu_z.$$

This can be interpreted as a refinement of the game where the government is known to be rational, demonstrating the optimal announcement when the public has little trust in the government's commitment.

An alternative definition of equilibrium follows Kambe (1999) and does away with the initial inference by the private sector. Instead, it corresponds to the case where the government first announces a plan c and subsequently becomes committed to following it with some exogenous probability p_0, independent of c.

Definition. For given $p_0 \in [0, 1]$, a K-*equilibrium* is an announcement $c_K(p_0)$ with an associated continuation equilibrium $\{\mathcal{L}^K, g_K\}$ such that

$$c_K(p_0) = \arg\min_{c\in\mathcal{C}} \mathcal{L}^c(p_0, a_0^c)$$

Similarly to reputational equilibria, we are especially interested in the limiting announcement $c_K(p_0)$ as $p_0 \to 0$.

3.3 *Credibility*

While the government's reputation describes the likelihood it is committed to the plan, it does not reflect how closely the plan is followed on average across both types. To obtain a measure of this, we define the credibility of a plan as the ratio of announced and (expected) realized inflation, normalized by their distance from Nash inflation.

Definition. Given a plan c, its *remaining credibility* in state (p, a) is defined recursively as follows:

$$\begin{aligned} C(p, a; c) &= \mathbb{E}\left[(1-\beta)\frac{\pi^N - \pi}{\pi^N - a} + \beta C(p'_c(p, a), \phi_c(a); c)\right] \\ &= (1-\beta)\frac{\pi^N - [pa + (1-p)g^\star_c(p, a)]}{\pi^N - a} + \beta\mathbb{E}\left[C(p'_c(p, a), \phi_c(a); c)\right] \end{aligned} \tag{8}$$

where π^N is Nash inflation. The *credibility* of a plan in a K-equilibrium is given by

$$C^K(c) = \lim_{p\to 0} C(p, a_0(c); c),$$

while in a reputational equilibrium it is

$$C^\star(c) = \lim_{z\to 0} \int C(p_0(c), a_0(c); c)\, d\mu_z(c).$$

In our simulations $g^\star_c(p, a) \in [a, \pi^N]$ for all (p, a) and all plans c, so credibility lies in $[0, 1]$.

4. Analysis and Numerical Results

We solve the model numerically for different announcements $c \in \mathcal{C}$. We parametrize our model following Lu, King, and Pastén (2016). Our preference and technology parameters $\gamma, \kappa, y^\star$ are

TABLE 1: BENCHMARK CALIBRATION

Parameter	Value	Definition	Source / Target
β	0.995	Discount factor	2% real interest rate
γ	60	Inflation weight	Lu, King, and Pastén (2016)
σ	1%	Std of control shock	Lu, King, and Pastén (2016)
κ	0.17	Slope of Phillips curve	Lu, King, and Pastén (2016)
$y^\star$	5%	Output target	Lu, King, and Pastén (2016)

consistent with the planner's objective function and Phillips curve in a standard New Keynesian economy calibrated to US data (Galí, 2015; Galí and Gertler, 1999). Table 1 summarizes our parameter choices. These parameters imply a level for Nash inflation of about 2% annualized.

4.1 *Continuation equilibrium after announcement* c

Figure 2 shows a typical value function $\mathcal{L}^c(p, a)$ for an arbitrary plan c. All plots have current reputation p on the x-axis. Darker lines correspond to a lower current target a, which is measured relative to Nash inflation π^N. We draw three main lessons from this figure.

Firstly, $\mathcal{L}^c$ is decreasing in p. An increase in reputation generally decreases expected inflation leading to higher current output and, therefore, smaller losses.

Secondly, the loss function has a convex-concave shape reflecting the dynamics of reputation. When reputation is close to 0 or 1 the public is confident in its assessment of the government's type, and significant evidence is required to move beliefs. Conversely, near $\frac{1}{2}$, movements in reputation are fickle and can easily be reversed. Thus, the same change in reputation is more valuable the closer p is to the extremes, leading to steepness of the loss function.

Thirdly, at high levels of reputation, a lower target a is preferred. The reason for this is that, as reputation increases, beliefs place a higher weight on the behavioral type who sticks to the announcement. As the rational type's strategy becomes less important, the credibility tradeoff dissipates.

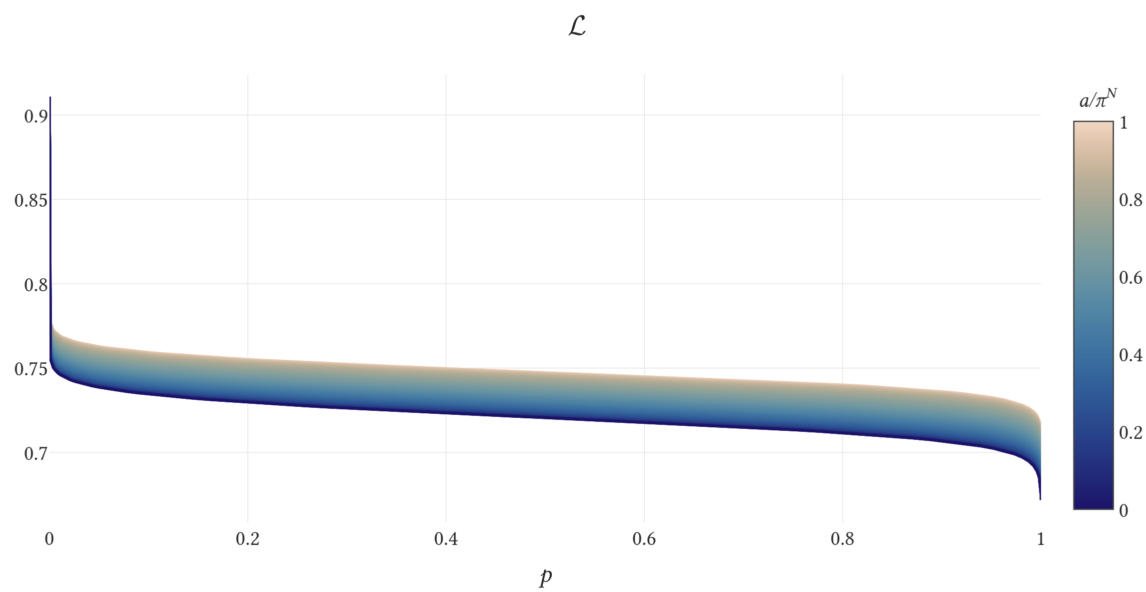


Figure 2: Loss function after announcement c

Finally, the range of values $\mathcal{L}$ across different targets is generally smaller at lower levels of reputation. One reason is that with lower reputation the current target becomes less relevant, as its weight in expected inflation decreases. Another, more nuanced reason is that the tradeoffs between stimulating output and sustaining reputation are more pronounced when the government is seen as less likely to be committed to the target. While lower targets are directly beneficial to inflation expectations, the government finds it more costly to deliver on low targets when reputation is low (see discussion below). Thus, the benefits of ambitious announcements with low targets can be offset more heavily at low levels of reputation.

Figure 3 shows how the current target a and reputation p affect $g^{\star}(p, a) - a$, the rational type's deviation from the current target. The deviation is generally decreasing in reputation, except when reputation becomes large and the government is able to produce surprise inflation largely undetected. There is a discontinuity at zero reputation, where the unique Markov equilibrium exhibits inflation π^N regardless of the announcement, since there are no reputational concerns. In contrast, at small but positive levels of reputation, the government benefits from staying somewhat close to the announcement.

The effect of the current target a on deviations is unambiguous: a lower target a causes the rational type to deviate farther from it. In comparison to an equilibrium with a higher target, the government needs to inflate less to maintain the same evolution of reputation (which would yield broadly the same level of output), holding expectations of future policy fixed. Thus, there

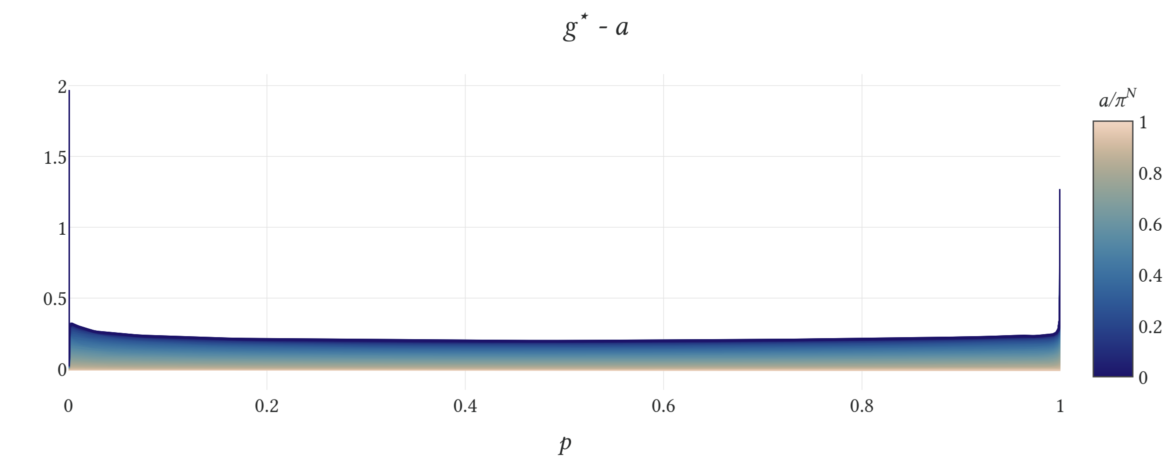


Figure 3: Inflation deviations

are lower costs to creating surprise inflation when the target is low.

Current reputation p affects optimal deviations through several different channels. On the one hand, a larger stock of reputation makes the planner more inclined to spend it by creating substantial surprise inflation. On the other hand, higher reputation anchors expectations more tightly and makes it less costly for the government to preserve its reputation by staying close to the target, especially when the target is high.

Figure 4 shows the average change in reputation $\mathbb{E}[p' - p]$, again as a function of the current reputation p and target a. As previously noted, the government's reputation declines on average:

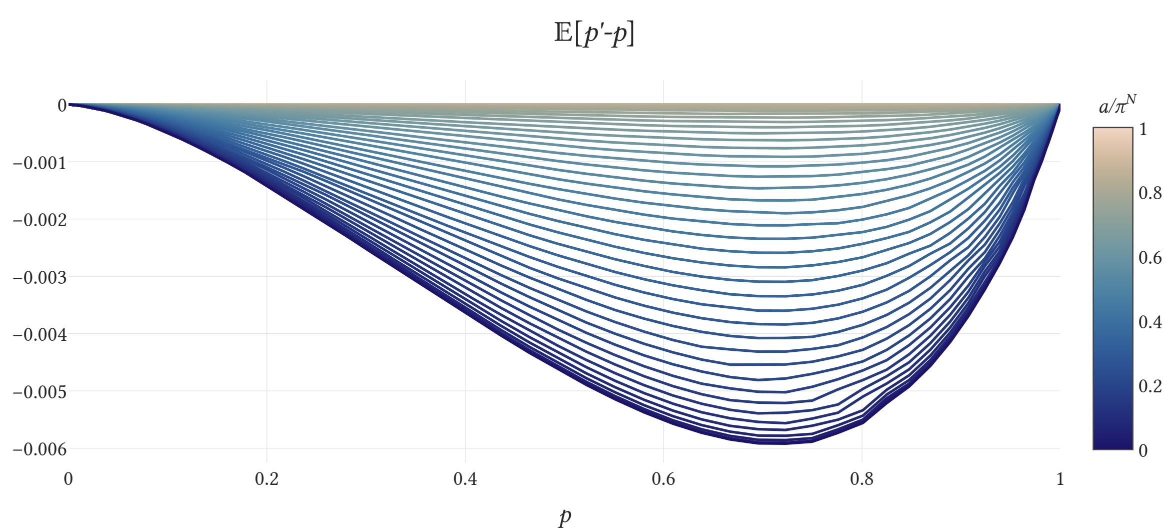


Figure 4: Expected reputation losses

$\mathbb{E}\left[p'\right]$ is never above p. In particular, reputation is preserved when $p' = p$ in all contingencies, which only happens when $p \in \{0, 1\}$ (because no update is possible) or when the current target a equals Nash inflation π^N, so that incentives to create surprise inflation are eliminated. Also, as a consequence of Bayes' rule, changes in reputation are smaller when initial reputation is closer to 0 or 1. Finally, lower announcements are associated with a larger expected reputation loss. Lower, more ambitious targets generate weaker incentives: as the temptation to inflate grows larger, the government prefers to spend more of its reputation to achieve higher output.

4.2 K-*equilibrium announcements*

Figure 5 shows K-equilibrium announcements. Each announcement minimizes the government's loss function conditional on starting with reputation p_0. The top panel shows the decay rate $1 - e^{-\omega}$ (in percent terms), while the bottom panel shows the choice of initial inflation a_0 and asymptote χ.

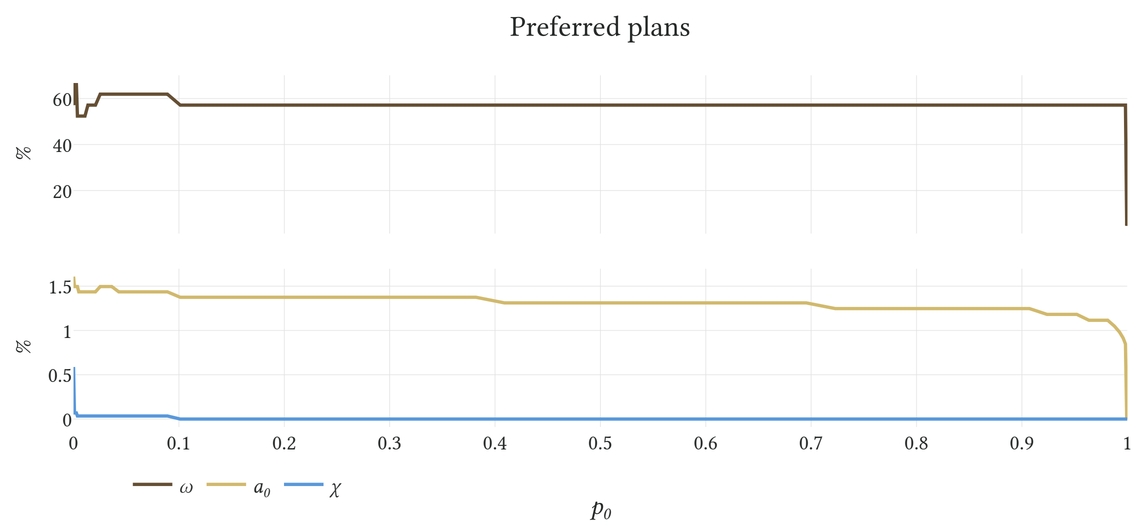


Figure 5: K-equilibrium announcements

At $p_0 = 1$, any announcement is fully believed by the private sector, regardless of expectations about the behavior of the rational type. The government sets expectations at their most advantageous level by promising zero inflation throughout. Since the private sector is convinced the government is committed to the announcement, the rational type has a clear incentive to create surprise inflation. When $p_0 < 1$, such a deviation incurs reputational losses. Thus, the government announces plans with inflation above zero, evidenced by the initial target $a_0 > 0$. Positive inflation targets help the incentives of the government's future selves to stay closer to the announcement, conserving reputation. Since $a_0 > \chi$, the optimal inflation targets decrease over time. This structure of the optimal announcements is preserved even as initial reputation p_0 approaches zero. As p_0 becomes small, the planner raises the level of long-run inflation χ as the lack of credibility of an asymptote of 0 overwhelms its benefits.

Figure 6 shows the determination of the K-equilibrium when p_0 is small. For each initial and long-run level of inflation a_0 and χ we plot the minimized loss function $\min_\omega \mathcal{L}(p_0, (a_0, \omega, \chi))$. The minimum is achieved at a point with $a_0 > \chi > 0$: in the K-equilibrium, the initial planner promises a gradual disinflation which converges to a level above the first-best level of zero inflation.

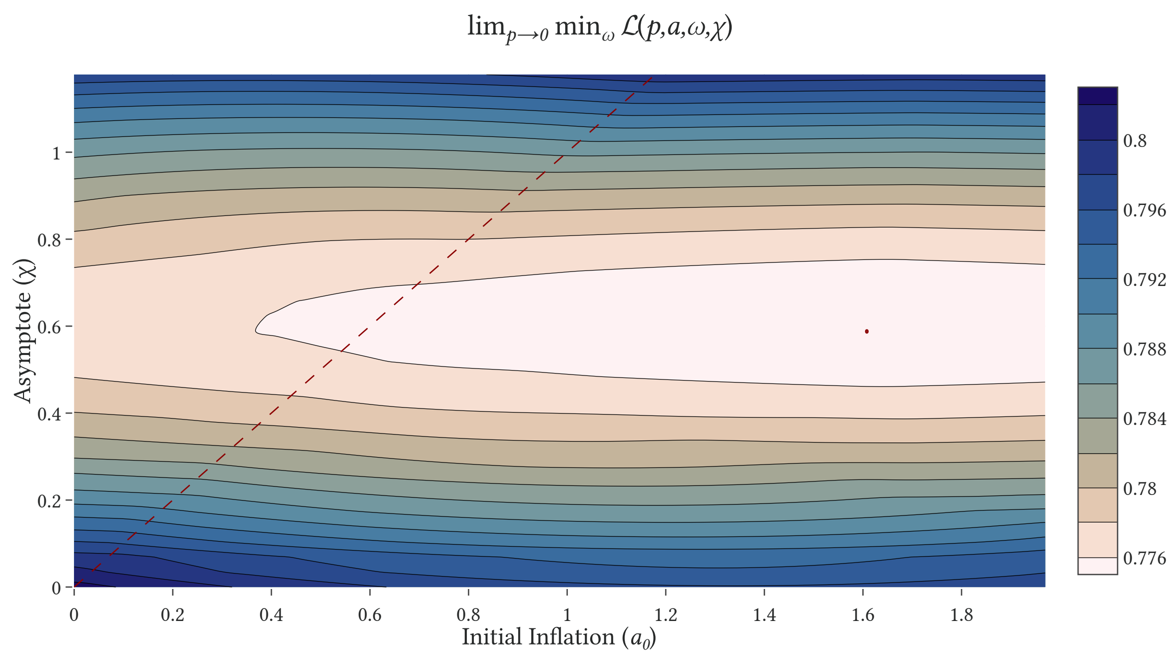


Figure 6: Loss function across announcements

When χ is small, plans eventually call for ambitiously low levels of inflation which are difficult to sustain. Consequently, reputation is quickly lost giving rise to unfavorable continuation values. High levels of long-run inflation, on the other hand, are easier to sustain but provide less benefit. For intermediate levels of long-run inflation χ, the benefit of a higher initial level $a_0 > \chi$ is visible. Such levels of initial inflation are valuable as they enhance the planner's ability to stick to a plan which eventually delivers the right level of inflation. When the long-run level is either too high or too low, the planner is less affected by the starting point.

The optimal plan involves an intermediate announcement of long-run inflation, balancing the desire for low expectations with incentives to preserve reputation. The choice of decay rate, shown in Figure 14 in the Appendix, also matters. The planner chooses a slope in its targets that is steep enough to boost incentives in the initial periods when inflation targets are above their long-run levels.

4.3 Credibility

Our setup distinguishes reputation p, the posterior belief that the government is the behavioral type announced at the start of the game, from credibility $C(p, a; c)$, the expected discounted deviations from plan c at reputation p and current target a, as defined in (8). Figure 7 plots the credibility of different plans at vanishingly small reputation, as a function of initial and long-run inflation a_0 and χ, for the correspondig loss-minimizing decay rate ω.

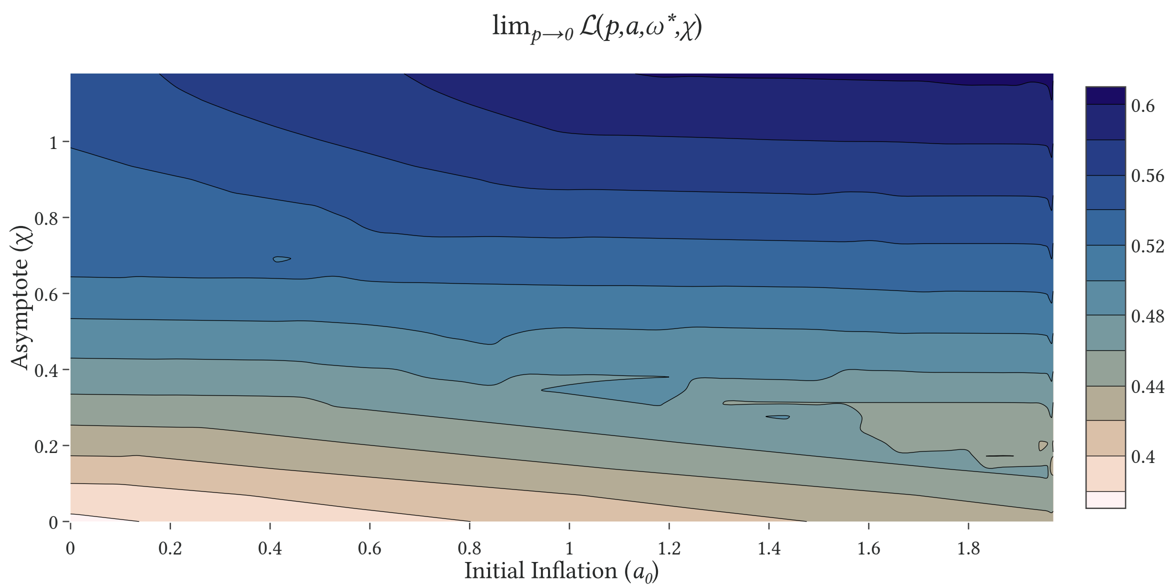


Figure 7: Credibility

Plans with a lower asymptote are less credible as they eventually imply too low levels of inflation and a quick loss in reputation. Moreover, especially when long-run inflation χ is low, plans with decreasing targets ($a_0 > \chi$) are more credible.

4.4 Distribution of announcements in the reputational equilibrium

We now turn to a description of the reputational equilibrium and the associated distributions μ_z of announcements by the rational type. Figure 8 plots the average plan as a function of initial reputation z prior to the announcement.

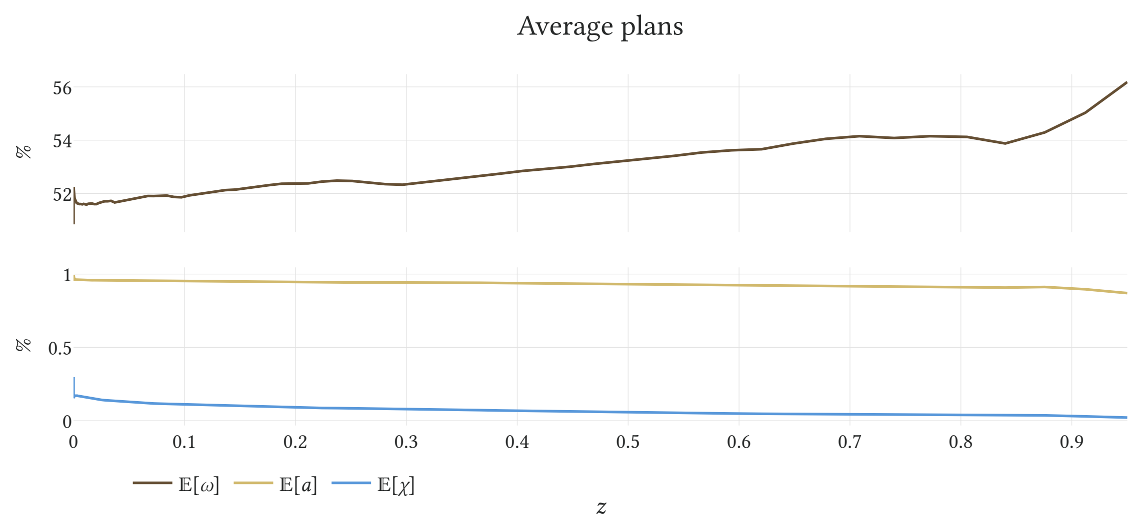


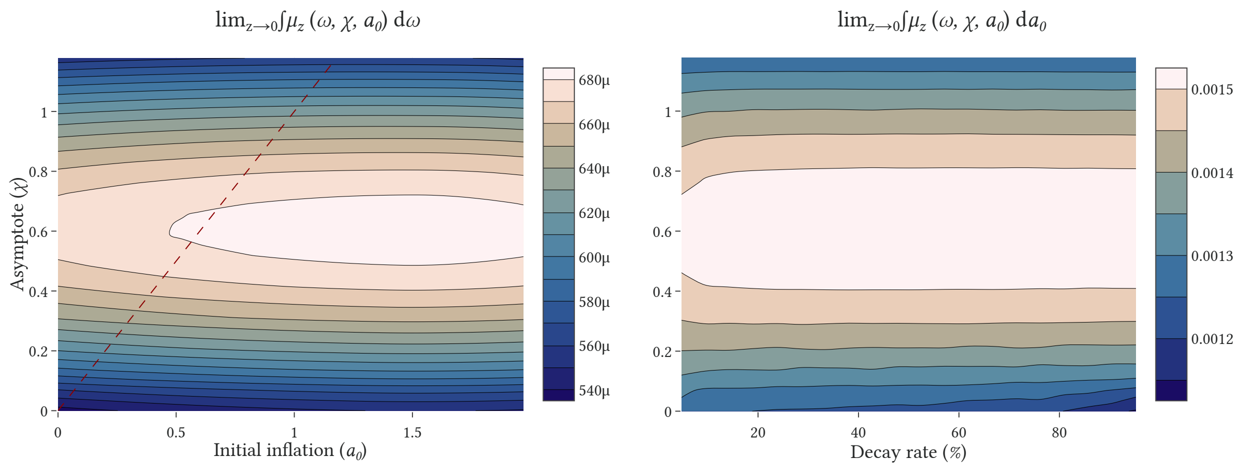
Figure 8: Reputational equilibrium announcements

For intermediate values of initial reputation, the planner chooses (on average) a disinflation path that starts from about half of Nash inflation π^N and converges towards a tenth of it by about half the distance each period. As initial reputation becomes small, the planner starts to put more weight on plans that converge toward a higher asymptote χ, similarly to the optimal K-equilibrium announcement in Figure 5.

Figure 9 on the left shows the limiting distribution $\mu^\star = \lim_{z\to 0} \mu_z$ of announcements as initial reputation vanishes. The left panel shows the distribution of types as a function of the asymptote χ and initial inflation a_0, integrating over the decay rate ω, while the right panel integrates over initial inflation.

Figure 9a reveals that the government tends to choose gradual plans with higher initial inflation a_0 than asymptote inflation χ. This probability is $\mathbb{P}\left(a_0 > \chi\right) = 64.9\%$. Plans with initial inflation of even five times the asymptote are still announced quite often: $\mathbb{P}\left(a_0 > 5\chi\right) = 16.6\%$. While the level of initial inflation has a fairly wide distribution, the asymptote is more precisely set: the density of a_0 and χ in the reputational equilibrium announcement falls sharply for χ away from the optimum, while it stays flat over many more values of initial inflation a_0.

Figure 9a bears a close resemblance to Figure 6 which plotted the K-equilibrium loss function at low p_0. Announcements with a lower loss in the K-equilibrium are good for the planner, so they are chosen more often in the reputational equilibrium. Hence, the government starts with lower reputation in those plans, which lowers their value in the equilibrium. This initial update of reputation (from z to p_0) makes the planner indifferent across all equilibrium announcements, which ultimately justifies the mixed strategy. Similarly to the K-equilibrium, there is not much



(A)

(B)

FIGURE 9: DISTRIBUTION OF TYPES

variation in the value of announcements along the decay rate dimension, evidenced by the nearly flat density $\mu^{\star}$. One exception is that very small decay rates are significantly less desirable, in contrast to the K-equilibrium.

On the right, Figure 9b shows that, given the 'right' asymptote, the planner chooses decay rates almost uniformly (in the background, different distributions of initial inflation a_0 adjust the choice for each decay rate).

5. COMPARATIVE STATICS

Figure 10 shows the average plan announced in the reputational equilibrium as a function of the variance of the control shock σ around its baseline value of 1%. More noise in the control makes deviations from targets harder to detect. Therefore, the level of adherence to plans is decreasing in σ. This makes the planner choose less ambitious plans when the control over inflation is less tight. These plans have higher inflation throughout, as they feature a higher asymptote χ, a marginally higher initial inflation a_0, and slower decay ω.

Figure 11 repeats the exercise varying the discount factor β and the slope of the Phillips curve κ. It reveals some subtleties in the manipulation of the three parameters that describe our plans. Figure 11a shows the average plan as a function of the discount rate $1/\beta - 1$ (whose benchmark value is 2% in annual terms). As the planner becomes more impatient, average plans start higher but converge to lower inflation, with a faster decay rate. With more impatience, the public expects

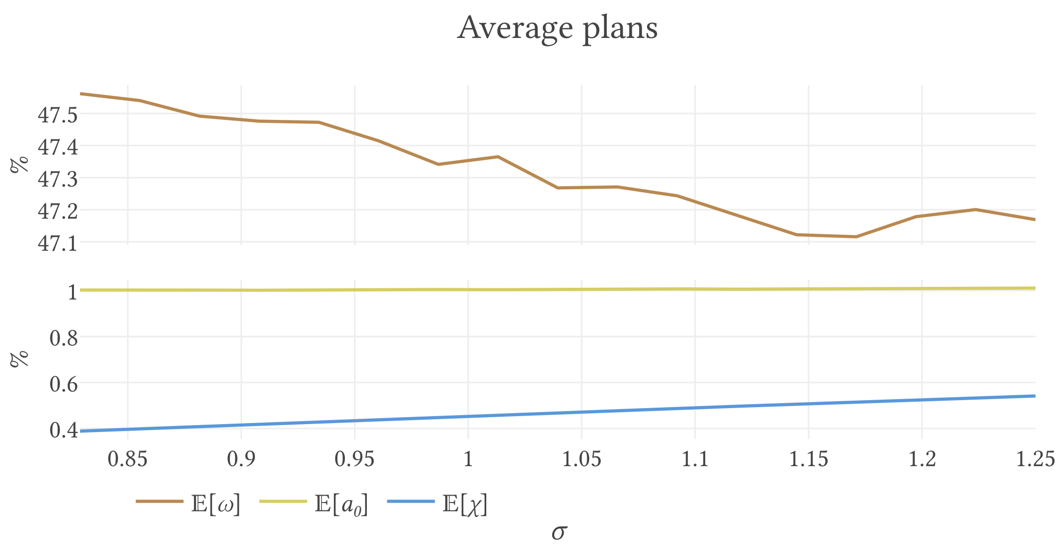
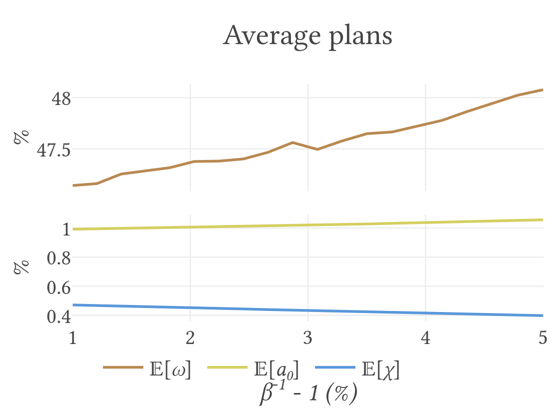


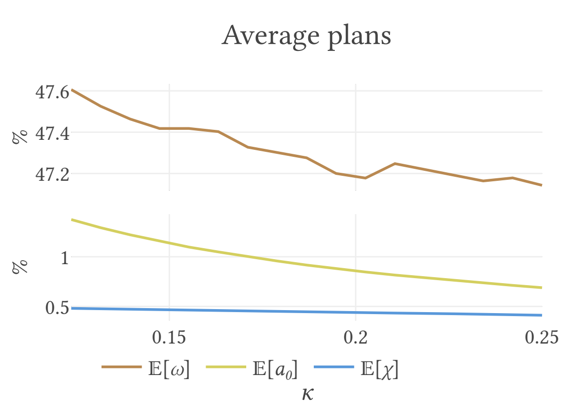
Figure 10: Average plans and the control shock variance

a larger inflation bias. For this reason, the planner tends to choose plans that are more resilient. Increasing initial inflation makes the plan easier to keep, while decreasing asymptotic inflation makes it more costly to deviate early on. A steeper descent of inflation targets contributes to both objectives.

Figure 11b shows that when the slope of the Phillips curve is increased from its baseline value of 0.17, the planner announces lower inflation throughout. When the Phillips curve is steeper, there are weaker incentives to create surprise inflation, as it results in a smaller output boom. Thus, the planner lowers expectations through lower targets without increasing reputational losses.



(a) Varying the discount factor



(b) Varying the slope of the Phillips curve

Figure 11: Average plans

6. Inspecting the Mechanism

Our benchmark model of reputation with imperfect control yields gradual disinflation plans. We dissect this result to by comparing our model to the Ramsey plan, which yields a gradual disinflation for very different reasons.

6.1 The Ramsey plan

We refer to choosing an entire path $\{g_t\}_t$ with commitment to minimize (1) subject to (2) and (3) as the *Ramsey* plan. In this case there are no announcements: with commitment, the planner chooses the public's expectations directly, which coincide with its own future actions. The linear-quadratic structure of this problem alows us to disregard the issue of imperfect control because the shocks ϵ_t only affect the planner through a variance term independent of policy.

Standard techniques allow us to use Lagrange multipliers of each time-t constraint to write the problem recursively for $t > 0$ as

$$v^R(\theta) = \max_{\theta'} \min_{y,\pi} (y - y^\star)^2 + \gamma\pi^2 + \theta'(\pi - \kappa y) - \theta\pi + \beta v^R(\theta') \tag{9}$$

Problem (9) produces policy functions $g^R_\pi(\theta), g^R_y(\theta), g^R_\theta(\theta)$ that describe the planner's actions for each period. At time 0, the planner does not carry any multipliers from the past and attains a value

$$\mathcal{J}^R = v^R(0) \tag{10}$$

To reconstruct the plan, we recursively apply the policy functions g^R

$$\theta_0 = 0, \qquad \pi_t = g^R_\pi(\theta_t), \qquad \text{and} \qquad \theta_{t+1} = g^R_\theta(\theta_t) \tag{11}$$

As is well known, time inconsistency manifests itself in the fact that $g^R_\theta(0) \neq 0$: a free planner wishes its future self to face multipliers that curtail the choice of inflation.

Figure 12 plots the Ramsey plan against the average announcement in the reputational equilibrium, as well as the announcement in the K-equilibrium. The power of commitment enables a complete disinflation: Ramsey inflation is essentially zero a year and a half after the announcement of the plan. The Ramsey plan also starts with a high initial target, about three quarters of the way to Nash inflation. This is because inflation in period t costs the planner through expected inflation in $t-1$, but this tradeoff is absent in the initial period. Inflation then comes down as the planner smooths the benefits of initial inflation on output over a few of the initial periods.

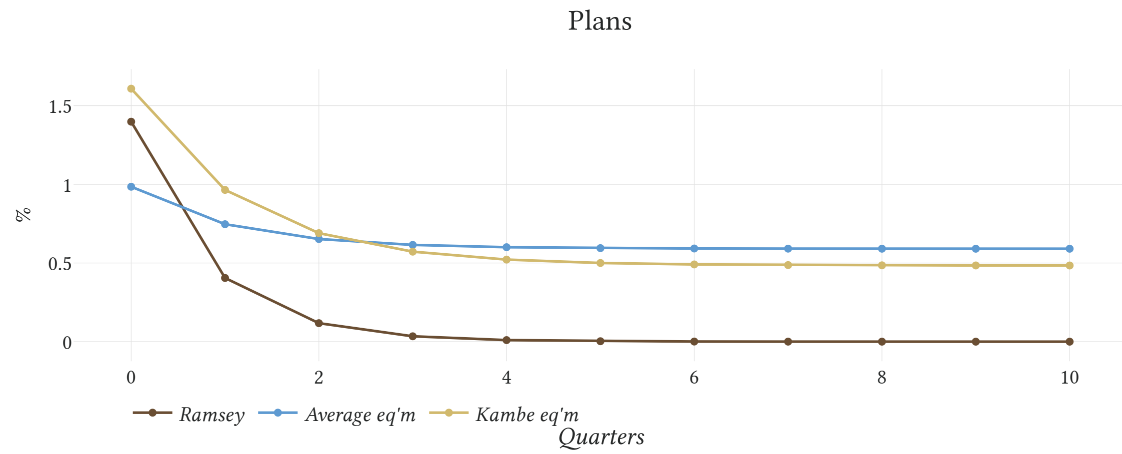


Figure 12: The Ramsey plan and equilibrium announcements

In contrast, the equilibrium announcements do not converge to zero. Even in a reputational equilibrium where the planner randomizes between announcements, the probability of plans with low long-run inflation, i.e. $\chi \approx 0$, is low (Figure 9a). Such plans are avoided due to their low credibility. The K-equilibrium announcement that minimizes the loss conditional on starting from a very low level of reputation, starts above the Ramsey plan. It needs high initial inflation to create the descent that fuels incentives. In a reputational equilibrium, some plans start above the Ramsey and others below, resulting in average plans starting above or below the Ramsey, depending on parameters. As discussed above, plans chosen more often get a lower starting reputation, which lowers their value, until the planner is indifferent among plans with a positive choice probability: this makes more desirable plans chosen more often.

6.2 *Reacting to shocks*

Our notions of equilibrium with reputation yield optimal plans which start from a high initial level of inflation and gradually decrease towards positive long-run values. However, the Ramsey plan, for which incentives are irrelevant, also exhibits 'gradualism.'

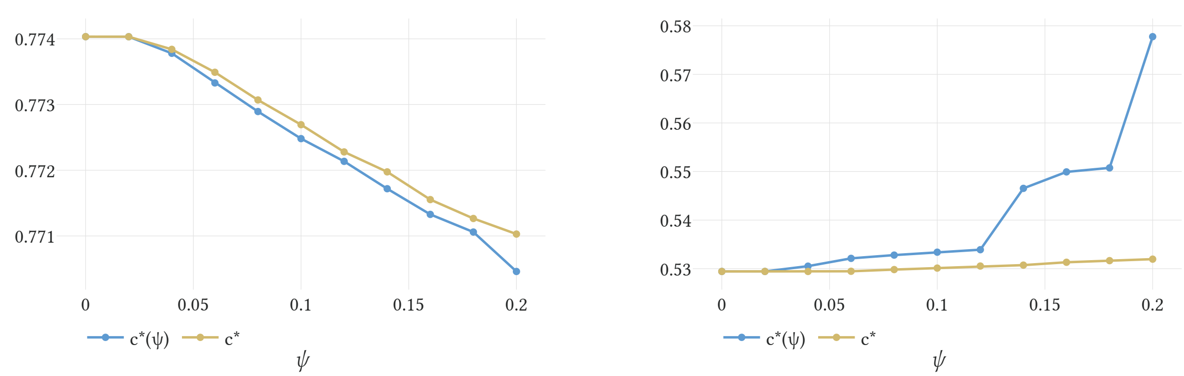
To investigate the importance of incentives in our result, we augment the space of behavioral types to include gradual feedback rules. These types allow plans in which future targets respond to deviations of inflation from the current target

$$a' = \chi + e^{-\omega}(a-\chi) + \psi(\pi - a) \tag{12}$$

so that whenever realized inflation π differs from its current target a, a share ψ of the difference is embedded in the target for the following period.

This extension illuminates the role of gradualism in shaping incentives, as the Ramsey planner does not condition on this type of deviations. Even beyond quadratic utility, the Ramsey plan starts high and converges to zero because the benefits of high initial inflation do not come at a cost in terms of expected inflation in the past (this is also why the Ramsey planner sets $\pi_{t+1} \leq \pi_t$). Gradual feedback rules such as (12) induce paths under which, after a high control shock ϵ, target inflation only comes down gradually. The Ramsey planner avoids these rules because they put costs on path. However, in our notions of reputational equilibrium, gradual feedback rules could be useful for the planner if they enhanced credibility.

Figure 13 confirms this intuition by showing the value function $\mathcal{L}$, on the left panel, and the initial credibility $\mathcal{C}$, on the right, for continuation equilibria. Both are shown as a function of the target updating parameter ψ, either reoptimizing the plan for each ψ (labeled $c^\star(\psi)$) or fixing the parameters (ω, χ, a_0) of the original K-equilibrium plan at $\psi = 0$ (labeled $c^\star$). Gradual feedback rules indeed enhance credibility, thus helping the planner.



(A) Loss function $\mathcal{L}$ (B) Credibility $\mathcal{C}$

Figure 13: Gains from gradual feedback rules

7. Concluding Remarks

This paper addresses an old question: can reputation be a substitute for commitment? We find that a simple model of reputation combined with imperfect control on the part of the government creates incentives for staying close to announced targets. The optimal policy after a plan has been announced trades off the benefits of surprise inflation against the possibility that a deviation becomes known to the public. In this way, the government's reputation becomes an important state variable in the problem of optimal policy under discretion.

Various characteristics of announced plans come to bear when determining the value of reputation. We find that a pervasive feature of optimal plans is gradualism. In anticipation of the continuation equilibrium, the planner finds it desirable to set itself up in situations where keeping its reputation is both easy and valuable. These are situations where announced inflation for the current period is higher than in the future. The resulting gradualism is therefore an artifact of incentives and not a reflection of inflation inertia. Understanding how the presence of sources of true inertia might interact with our results is one of our goals going forward.

The gradualist property of optimal plans holds at positive levels of reputation and also in the limit as initial reputation vanishes. We interpret this limit case as a sensible refinement of the game between a rational government and the private sector.

Finally, we show that a target-updating rule can improve the performance of the optimal plan in a reputational equilibrium. By letting future targets respond to deviations of inflation, the plan reallocates more challenging tasks to the states in which reputation has increased. This property increases the overall credibility of the plan, improves expectations and, through them, outcomes. This new source of gradualism, which does not vanish after the first few periods and continues to affect equilibrium plans even in the long run, constitutes a potential lesson for policy.

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A. More results

Figure 14 shows that, at $a_0 = \chi^\star$ (about 0.6 in this example), the planner is indifferent between decay rates, as the decay rate matters more when a_0 is farther away from χ. Starting from the optimal a_0 (about 1.6), it prefers an intermediate decay rate: too slow would negate the incentives from rapidly decaying targets and make the plan target high inflation for too long, but too fast would not create incentives for long as the plan would rapidly become flat.

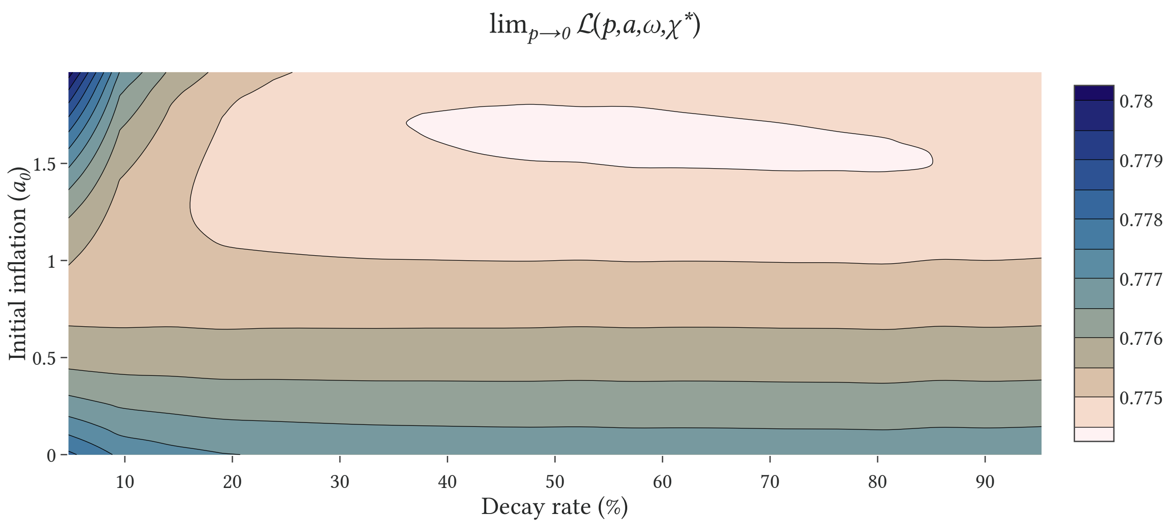

Figure 14: Loss function across announcements