Reputation and the Credibility of Inflation Plans*

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Abstract

We study the optimal design of inflation targets by a planner who lacks commitment and exerts imperfect control over inflation. By comparing realized inflation to the targets, the public forms beliefs about the government's commitment. Such reputation is valuable as it helps curb inflation expectations. However, plans that are more tempting to break lead to faster reputational losses in the ensuing equilibrium. The planner's optimal announcement balances low inflation promises with incentives to enhance credibility. We find that, despite the absence of private sources of inflation inertia, a gradual disinflation is preferred even in the zero-reputation limit.

JEL Classification E52, C73

Keywords Imperfect credibility, reputation, optimal monetary policy, time inconsistency

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Introduction

Macroeconomic models give expectations about future policy a large role in the determination of current outcomes. Policy is then generally set under one of two assumptions: commitment to future actions or discretion. Attempts to model policy departing from these extreme cases have found limited success.

However, governments actively attempt to influence beliefs about future policy. Examples include forward guidance and inflation targets but also fiscal rules and the timing of introduction of policies. Such promises rarely constrain future choices, yet they can shift expectations substantially. Standard macroeconomic models cannot capture this idea directly, as expectations of the public are fully determined by the policy chosen with commitment, or with discretion as part of an equilibrium. In both cases the public understands that announcements do not bind the government in any way. In other words, announcements do not grant any additional credibility to the policy maker, as the public is convinced of her course of action.

In this paper we develop a rational-expectations theory of government credibility and apply it to the question of optimal policy announcements. Our notion of credibility is based on the concept of reputation in game theory (Kreps and Wilson, 1982; Milgrom and Roberts, 1982). In our model the policy maker (or government, or central bank) could be rational and strategic, or one of many possible behavioral types described by a policy that they follow stubbornly. The public is uninformed about the policy maker's type and makes statistical inference about it after observing the government's announcements and actions. This inference is central to our analysis because it turns out to be in the best interest of the rational type to pretend to be one of the behavioral types.

We consider a stylized environment. In the initial period, policy targets are announced and the government is then free to choose policy. However, the private sector knows that if the policy maker is behavioral it will implement the announcement. As a consequence, the rational type has an ex-post incentive to stay close to any announced targets, which might earn it a reputation for being committed to them. The incentive exists at any positive level of reputation and its strength depends on the entire announced sequence of targets. In anticipation of these interactions, a planner chooses carefully which targets to announce. Our main question concerns the optimal policy announcement in the presence of these reputational concerns.

We set our model of reputation in a modern version of the classic environment of Barro (1986) and Backus and Driffill (1985), where a central bank sets inflation subject to an expectationsaugmented Phillips curve. The monetary authority dislikes inflation but constantly faces an opportunity to engineer surprise inflation, which would deliver output closer to potential. We model these features through the standard, cashless-limit New Keynesian setup for the private economy. To focus on incentives and reputation dynamics, we abstract from an IS curve and let the government control inflation directly.

A natural definition of the government's reputation is the private sector's belief that the government is indeed the behavioral type whose plan was announced. The credibility of a plan instead measures the proximity of expected inflation to the targets. While credibility generally increases with reputation, the insights of the reputation literature imply that credibility need not vanish as reputation approaches zero.

When the policy maker has perfect control over inflation, the optimal announcement chosen by the planner is the Ramsey plan and both the behavioral and the rational type follow it. If the rational type deviates from the announcement, perfect control implies that the private sector realizes that it is facing the rational type. All reputation is lost and the policy maker faces high inflation expectations for the rest of the game. This makes some plans perfectly credible, including the Ramsey plan due to the forward-looking Phillips curve in our case.

A key assumption we introduce is that the policy maker cannot perfectly control inflation, perhaps due to underlying shocks to money demand. Imperfect control masks the choice of policy: the private sector understands that realized inflation is only an imperfect signal of intended inflation. This induces the rational type not to follow the Ramsey plan if it is announced. In fact, the rational type deviates from any announcement.

We consider additive and normally distributed noise which implies that the public can never be fully certain of the policy maker's action. This assumption distinguishes us in technical terms from the early studies of reputation in monetary policy referenced above, where the public perfectly observes the inflation chosen by the government. But, importantly, imperfect control also creates a smooth tradeoff: overshooting the target by more creates, in expectation, a larger boom accompanied by larger reputational losses.

When designing the announcement, the planner takes into account the degree of compliance by the different types of the policy maker, which it can influence but not control. Preserving reputation turns out to be a powerful disciplining force for the rational type of the policy maker. Crucially, the value of reputation depends on the entire plan in place. Plans differ in the outcomes they intend to deliver and in how closely they are expected to be followed in the future, i.e. their credibility. Both features contribute to current outcomes through the private sector's expectations. These forces lead the planner to weigh a plan's intended outcomes against the reputation dynamics it generates.

Our main result is that the planner announces a sequence of targets under which inflation starts high and diminishes gradually. Plans with gradual disinflation are more credible: having a higher target for today than tomorrow boosts the gains from sticking to the plan. This slows down the pace of reputational losses sufficiently to offset the negative effect of higher announcements on expected inflation. In an extension, we show that the planner also benefits from gradual feedback rules: promises to shift subsequent targets after one has been missed. This modification effectively increases the plan's gradualism and, consequently, its credibility. This second source of gradualism affects equilibrium plans beyond the early stages of the game and constitutes a clear lesson for policy from this model.

The gradualism of our optimal policy might lead an outside observer to conclude that there is substantial inflation inertia in the economy and that the government avoids a costly recession when bringing inflation down. However, in our model past inflation does not enter the Phillips curve. Instead, gradual disinflation is a result of the dynamic incentives of the policy maker.

A second result concerns the limit as initial reputation becomes arbitrarily small. At zero initial reputation, the only Markov equilibrium is a repetition of the static Nash equilibrium with high inflation and output at the natural level. However, as is usual in the reputation literature, even a small amount of reputation creates a large departure in behavior from the Nash equilibrium. In particular, we show that the gradualist nature of optimal announcements and the corresponding credibility dynamics are preserved at arbitrarily low levels of initial reputation. The limiting announcement, which can be interpreted as the announcement in a fully rational model where the public grants the government a shred of credibility, also exhibits gradualism.

Finally, while most of our analysis assumes an exogenous probability that the policy maker is behavioral, we consider an extension to endogenize such initial reputation. We postulate a distribution of behavioral types and postpone the initial announcement until after the planner has observed the policy maker's type, which introduces a signaling component to the announcement. In this extension, the equilibrium features mixing over announcements: plans that are announced often by the rational type incur a reputational discount, which serves to create the indifference required for the rational type to mix. In this case, we also find that, in the limit as the behavioral types vanish, the rational type mostly announces gradual plans and the average plan features gradualism.

Discussion of the Literature. We contribute to a long literature dealing with issues of commitment, imperfect credibility, and reputation. The time inconsistency of optimal policy (Kydland and Prescott, 1977) has long been recognized by researchers, who have set out to ask whether reputation can be a substitute for commitment.

Barro (1986) and Backus and Driffill (1985) were the first studies of monetary policy to introduce reputation via behavioral types committed to a certain policy. These and many subsequent studies (Cukierman and Liviatan, 1991; Sleet and Yeltekin, 2007; King et al., 2008; Dovis and Kirpalani, 2021) assume perfect control of inflation. Thus, any deviations are detected by the private sector and fully destroy the reputation. In contrast, our assumption of imperfect control enables distinct tradeoffs that shape the gradualism of optimal plans. Moreover, the reputation literature typically considers the limit as the long-lived player becomes arbitrarily patient (Fudenberg and Levine, 1989), while we use a fixed discount factor for the planner.

Another line of research studies monetary policy with imperfect control by considering uncertainty about the preferences of the planner which is distinct from reputation (Cukierman and Meltzer, 1986; Faust and Svensson, 2001; Phelan, 2006). We view reputation as more directly suited to address optimal announcements, which was not the goal of the above papers.

Most closely related is the work of Lu, King, and Pastén (2016) and King and Lu (2023) who consider reputational models with imperfect control. However, their optimizing type has commitment power and the type that lacks commitment follows a fixed rule in Lu, King, and Pastén (2016), and behaves myopically in King and Lu (2023). This reversal of roles changes the underlying tradeoffs. In these papers the planner announces (and commits to) a plan that promotes separation from the alternative type. In our model, the planner chooses a behavioral type at the announcement stage and the rational type mimics its policy to convince the public that it is committed to it. This makes the model a natural setting for studying whether reputation-building incentives can substitute for commitment, as well as the credibility of different plans. In addition, Lu, King, and Pastén (2016) and King and Lu (2023) obtain the Ramsey plan in the limit as the planner becomes known to be the optimizing type, whereas the corresponding limiting plan in our model resembles neither commitment nor discretion.

An alternative view of reputation is given by the notion of sustainable plans (Chari and Kehoe, 1990; Phelan and Stacchetti, 2001). This literature considers subgame perfect equilibria in games between the policy maker and the private sector applying the tools of Abreu, Pearce, and Stacchetti (1990). This typically generates a large set of equilibria. In fact, reputational models are often used to refine the equilibrium set. Faingold and Sannikov (2011) study a general model of reputation in continuous time which maps to our framework of monetary policy with imperfect control. They find conditions for a unique equilibrium which is Markovian in reputation, providing justification for our focus on Markov equilibria. Their model restricts to behavioral types with static behavior, so it cannot address the dynamic announcements we are interested in.

Even though the announcements in our model do not constrain the actions of the rational policy maker, they are not cheap talk, as they can be sent by only one of the behavioral types. This distinguishes us from cheap talk models of monetary policy such as Stein (1989) and Turdaliev (2010).

Finally, the gradualism featured by our equilibrium plans is reminiscent of the allocations arising from organizational equilibria described by Bassetto et al. (2018). Over time these allocations move further from the discretion outcome and closer to the commitment outcome without reaching the latter; similarly, our equilibrium plans transition away from the static Nash outcome and converge to a long-run rate of inflation above the first-best rate of 0. Organizational equilibria are based on equilibrium refinements from the renegotiation-proofness literature (Bernheim and Ray, 1989; Farrell and Maskin, 1989; Kocherlakota, 1996). Our work suggests that these dynamics can be generated endogenously by modeling reputational concerns directly.

Layout. The rest of the paper is structured as follows. Section 2 introduces our model of reputation. Section 3 characterizes the Ramsey plan, while Section 4 shows that the Ramsey plan is the optimal announcement in a setting with perfect control. Having established these benchmarks, Section 5 describes post-announcement equilibria and optimal announcements in the main version of our model with imperfect control. Section 6 highlights the role of incentives through an extension, and Section 7 discusses an equilibrium notion that endogenizes initial reputation as a function of different announcements. Section 8 concludes. All proofs are in the Appendix.

2. Model

We set our model of credibility in the standard New Keynesian environment, where the presence of a Phillips curve creates time inconsistency in the choice of inflation. We consider a two-stage setting in which inflation targets are announced initially but the policy maker chooses policy sequentially. We describe equilibrium with an announcement in place before moving on to evaluating different types of announcements in terms of the outcomes they induce, their credibility, and their welfare implications.

2.1 *Setting*

A policy *announcement* $\mathbf{a} = (a_t)_{t=0}^{\infty}$ is made in the first stage. Let $\mathcal{A}$ denote the set of possible announcements. In Sections 3 and 4 we let $\mathcal{A} = \mathbb{R}^{\infty}$. We place some restrictions on $\mathcal{A}$ in Section 5.

In the second (post-announcement) stage a policy maker and the private sector play an infinitely repeated game in discrete time $t = 0, 1, \ldots$. The policy maker can be one of two types. The rational type is free to choose any policy $g_t \in \mathbb{R}$ at time t. The behavioral type is committed to following the announcement $\mathbf{a}$, so it sets $g_t = a_t$. Let p_0 denote the initial probability of the behavioral type, whose possible origins we discuss below.

The policy maker's action g_t influences inflation π_t as follows:

$$\pi_t = g_t + \epsilon_t, \tag{1}$$

where $\epsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$ is a control shock and $\sigma \geq 0$. The policy maker has perfect control of inflation if $\sigma = 0$ and imperfect control if $\sigma > 0$. Let f_ϵ denote the density of ϵ_t.[1]

The private sector forms expectations e_t of next-period inflation π_{t+1} conditional on observed inflation $\pi_0, .., \pi_t$ up to, and including, time t. Output y_t is determined implicitly by a Phillips curve[2]

$$\pi_t = \kappa y_t + \beta e_t, \tag{2}$$

where $\kappa \geq 0$ is the slope of the Phillips curve, and $\beta \in (0, 1)$ is the discount factor of the economy.

The policy maker dislikes inflation as well as deviations of output from a target $y^\star > 0$. The resulting loss in period t is

$$(y_t - y^\star)^2 + \gamma \pi_t^2,$$

where $\gamma \geq 0$ is the relative weight on inflation. It will be convenient to express the loss as a function of π_t and e_t as follows using the Phillips curve (2):

$$l(\pi_t, e_t) = \left(\frac{\pi_t - \beta e_t}{\kappa} - y^\star \right)^2 + \gamma \pi_t^2.$$

[1] When $\sigma = 0$, we set $f_\epsilon(x) = 1_{x=0}$.

[2] Because π_t may contain information useful for the private sector's forecasts, we have to be explicit about our timing assumptions. Here we assume that π_t is used to form e_t. We believe this formulation to be closest to the narrative of the New Keynesian model, in which the policy maker moves first by setting the nominal interest rate. Firms and unions observe this and then update prices if they can, determining price and wage inflation. Finally, output is determined by the demand of each good at these prices. When the central bank follows a Taylor rule, it *targets* inflation by setting the interest rate to react to the inflation it expects to induce.

2.2 *Post-announcement equilibrium*

A time-t history h^t is a sequence of realized inflation $(\pi_0, \ldots, \pi_{t-1})$ up to time t. Let $H^t = \mathbb{R}^t$ denote the set of time-t histories with $t > 0$, and $H^0 = \{h^0\}$ denote the singleton set containing the initial history. Let $H = \cup_{t=0}^{\infty} H^t$ be the set of all histories.

A strategy $\mathbf{e} : H \backslash H^0 \to \mathbb{R}$ for the private sector assigns expectations of time-$t+1$ inflation to each history $(h^t, \pi_t) \in H^{t+1}$ consisting of a history $h^t \in H^t$ of inflation up to time t and inflation π_t observed at time t. A strategy $\mathbf{g} : H \to \Delta\mathbb{R}$ for the policy maker assigns a (potentially random) policy g_t to each history $h^t \in H^t$, where $\Delta\mathbb{R}$ denotes the set of probability distributions over $\mathbb{R}$ with finite support. We will use $\mathbf{g}^\star$ to denote the equilibrium strategy of the rational type and $\mathbf{a}$ to denote the strategy of the behavioral type which follows announcement $\mathbf{a} = (a_t)_{t=0}^{\infty}$, i.e. $\mathbf{a}(h^t) = a_t$.

The policy maker's continuation payoff following a history $h^t \in H^t$ when it chooses strategy $\mathbf{g}$ and the private sector follows $\mathbf{e}$ is given by

$$L(h^t|\mathbf{g}, \mathbf{e}) = \mathbb{E}_{h^t}^{\mathbf{g}} \left[\sum_{s=t}^{\infty} \beta^{s-t} l\Big(\pi_s, \mathbf{e}(h^t, \pi_t, .., \pi_s) \Big) \right],$$

where $\mathbb{E}_{h^t}^{\mathbf{g}}$ denotes expectation over inflation $(\pi_s)_{s \geq t}$ from history h^t given the strategy $\mathbf{g}$ and the shocks $(\epsilon_s)_{s \geq t}$. Hence, the rational type's loss is given by $L(h^t|\mathbf{g}^\star, \mathbf{e})$, while the behavioral type's loss is given by $L(h^t|\mathbf{a}, \mathbf{e})$.

We now outline the the equilibrium conditions for the strategies of the policy maker and the private sector. A strategy $\mathbf{g}^\star$ is optimal given expectations $\mathbf{e}$ if it minimizes expected discounted losses from any history $h^t \in H^t$ given the private sector's expectations, i.e. $\mathbf{g}^\star \in \underset{\mathbf{g}}{\operatorname{argmin}}\, L(h^t|\mathbf{g}, \mathbf{e})$.

At the beginning of each period t the private sector has beliefs $\mathbf{p}(h^t)$ representing the probability of the behavioral type as a function of the history h^t of inflation observed so far, where $\mathbf{p}(\emptyset) = p_0$ is the prior belief. After observing inflation π_t, the private sector updates beliefs via Bayes' rule if possible, forming a posterior $\mathbf{p}(h^t, \pi_t)$ as follows. If the policy maker is the behavioral type, observed inflation π_t indicates that the shock was $\epsilon_t = \pi_t - a_t$. If, instead, observed inflation resulted from a policy g_t in the support of the rational type's strategy $\mathbf{g}^\star(h^t)$, the shock must have been $\epsilon_t = \pi_t - g_t$. Hence, the posterior belief is given by

$$\mathbf{p}(h^t, \pi_t) = \frac{\mathbf{p}(h^t) f_\epsilon(\pi_t - a_t)}{\mathbf{p}(h^t) f_\epsilon(\pi_t - a_t) + (1 - \mathbf{p}(h^t)) \mathbb{E}_{(h^t, \pi_t)}^{\mathbf{g}^\star} \left[f_\epsilon(\pi_t - g_t) \right]}, \tag{3}$$

where the expectation is over the stochastic realizations g_t of the rational type's policy when it follows a mixed strategy and π_t is fixed given the history (h^t, π_t). If $\sigma = 0$, $\pi_t \neq a_t$ and π_t is outside the support of $\mathbf{g}^\star(h^t)$, then Bayes' rule does not apply and we set $\mathbf{p}(h^t, \pi_t) = 0$.

We say the expectations $\mathbf{e}$ are rational given $\mathbf{g}^\star$ and $\mathbf{a}$ if

$$\mathbf{e}(h^t, \pi_t) = \mathbf{p}(h^t, \pi_t) a_{t+1} + (1 - \mathbf{p}(h^t, \pi_t)) \mathbb{E}_{h^t}^{\mathbf{g}^\star} [g_{t+1}] \tag{4}$$

for any time t, history $h^t \in H^t$, and observed inflation π_t.

We are now ready to state our equilibrium definition.[3]

Definition 1. A *post-announcement equilibrium* is a strategy $\mathbf{g}^\star$ for the rational type, an announcement $\mathbf{a} \in \mathcal{A}$, and an expectations function $\mathbf{e}$ such that

- $\mathbf{g}^\star$ is optimal given the expectations $\mathbf{e}$
- expectations $\mathbf{e}$ are rational given $\mathbf{g}^\star$ and $\mathbf{a}$

2.3 *Optimal announcements*

So far we have defined an equilibrium from the post-announcement stage of the game. We now turn to the initial stage where the announcement is determined. For most of the paper, we assume that a benevolent planner chooses an announcement in $\mathcal{A}$ to minimize the policy maker's loss. The announcement is chosen without knowledge of the policy maker's type, so it minimizes an average of the expected discounted losses of both types given the prior p_0 shared by the planner and the private sector. This average loss is denoted

$$\tilde{L}(\mathbf{g}^\star, \mathbf{a}, \mathbf{e}) := p_0 L(h^0 | \mathbf{a}, \mathbf{e}) + (1 - p_0) L(h^0 | \mathbf{g}^\star, \mathbf{e}).$$

The potential multiplicity of post-announcement equilibria may create (pre-announcement) equilibria with different average losses. We are interested in the optimal equilibrium and its associated optimal announcement. We can now define an equilibrium of the game from the initial stage prior to the announcement.

Definition 2. A (pre-announcement) *equilibrium* is a collection of strategies $\{\mathbf{g}^{\mathbf{a}}, \mathbf{e}^{\mathbf{a}}\}_{\mathbf{a} \in \mathcal{A}}$ and an announcement $\mathbf{a}^\star$ such that

- $(\mathbf{g}^{\mathbf{a}}, \mathbf{a}, \mathbf{e}^{\mathbf{a}})$ is a post-announcement equilibrium for all $\mathbf{a} \in \mathcal{A}$.

[3]Definition 1 describes a Perfect Public Equilibrium of the game between the policy maker and the private sector (Mailath and Samuelson, 2006).

- $\mathbf{a}^\star \in \underset{\mathbf{a}\in\mathcal{A}}{\operatorname{argmin}}\, \tilde{L}(\mathbf{g}^{\mathbf{a}}, \mathbf{a}, \mathbf{e}^{\mathbf{a}})$.

In each equilibrium, the planner understands which expectations $\mathbf{e}$ the private sector will hold and which policy $\mathbf{g}^\star$ the rational type will follow after every announcement. The equilibrium announcement minimizes the planner's expected loss. In an optimal equilibrium, the announcement on path is followed by a payoff that is maximal among all announcements and all equilibria.

Definition 3. An equilibrium $\{\{\mathbf{g}^{\mathbf{a}}, \mathbf{e}^{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}, \mathbf{a}^\star\}$ is *optimal* if it minimizes $\tilde{L}(\mathbf{g}^{\mathbf{a}^\star}, \mathbf{a}^\star, \mathbf{e}^{\mathbf{a}^\star})$ among all equilibria. An announcement $\mathbf{a}^\star$ is optimal if it is part of an optimal equilibrium $\{\{\mathbf{g}^{\mathbf{a}}, \mathbf{e}^{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}, \mathbf{a}^\star\}$.

One interpretation is that the planner is a government that appoints a policy maker – the central banker. Unlike in Rogoff (1985), the government and the central banker share the same preferences about output and inflation. Rather, the government can choose a central banker who advocates for a specific inflation policy (the announcement), but it does not know whether the central banker is tough, i.e. committed to follow this policy. This interpretation also maps to a model in the spirit of Kambe (1999) where following the announcement the planner becomes committed to it with probability p_0.

3. Ramsey plan

Suppose the policy maker is committed to follow $\mathbf{a} = (a_t)_{t=0}^\infty$. This is a special case of our model where $p_0 = 1$. Then expected time-$t+1$ inflation is simply a_{t+1}, so the average discounted loss is $\mathcal{L}(\mathbf{a}) + \xi(\sigma)$, where

$$\mathcal{L}(\mathbf{a}) := \sum_{t=0}^{\infty} \beta^t l(a_t, a_{t+1}) \tag{5}$$

and $\xi(\sigma)$ captures the contribution of the control shocks ϵ which are terms independent of policy in this linear-quadratic problem. The optimal announcement in this setting is called the Ramsey plan. Proposition 1 below characterizes it using the Bellman equation

$$R(a) = \min_{a'\in\mathbb{R}} l(a, a') + \beta R(a'). \tag{6}$$

Proposition 1. *There exists a function $R : \mathbb{R} \to \mathbb{R}_+$ satisfying (6) and an associated policy function ϕ_R such that the unique Ramsey plan $\mathbf{a}^R = (a_t^R)_{t=0}^\infty$ is given by $a_0^R = \underset{a}{\operatorname{argmin}}\, R(a)$ and $a_{t+1}^R = \phi_R(a_t^R)$. Moreover, $a_0^R = \frac{\kappa y^\star}{1-\beta\omega_R+\gamma\kappa^2} > 0$ and $\phi_R(x) = \omega_R x$, where $\omega_R = \frac{1+\beta+\gamma\kappa^2-\sqrt{(1+\beta+\gamma\kappa^2)^2-4\beta}}{2\beta} \in (0,1)$.*

The Ramsey plan is shown in Figure 1. It starts from a positive level of inflation and decreases over time with a constant decay rate, approaching zero inflation in the long run. This stimulates output in every period at the cost of creating inflation that vanishes in the long run. Note that inflation costs could be minimized if inflation was zero throughout, i.e. $a_t = 0$ for all t. This policy is, however, suboptimal because increasing inflation at time 0 would bring output closer to the desired level $y^\star$ with no effect on expected inflation in previous periods.

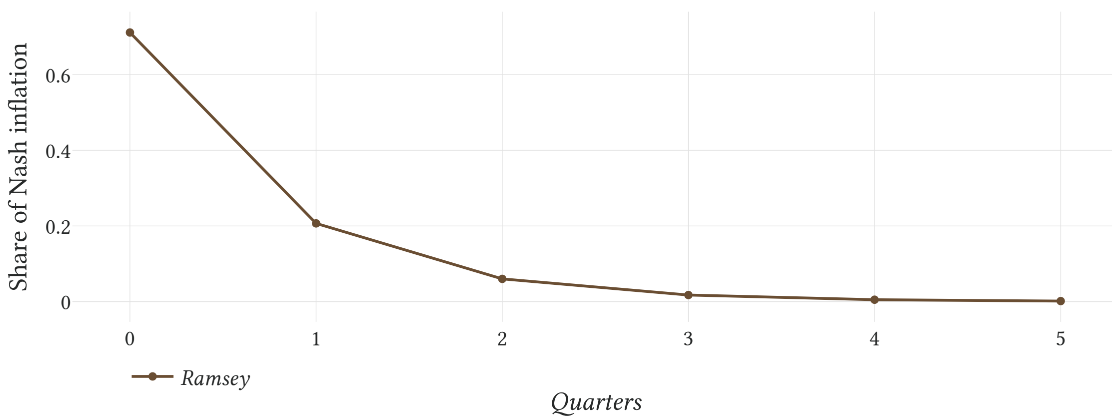


Figure 1: The Ramsey plan

4. Perfect control

We begin by analyzing the case $\sigma = 0$, in which the policy maker is able to perfectly control inflation. Proposition 2 below shows that the Ramsey plan can be implemented in equilibrium. To describe the necessary assumptions, consider a one-shot game where the policy maker chooses π to minimize $l(\pi, e)$ and the private sector simultaneously chooses expectations e to match π. There is a unique Nash equilibrium of this game with inflation π^N satisfying $\pi^N = \operatorname*{argmin}_{\pi} l(\pi, \pi^N)$.[4] The Appendix shows that $\pi^N = \frac{\kappa y^\star}{1-\beta+\gamma\kappa^2}$, from which it is immediate that $a_0^R < \pi^N$. We assume that committing to zero inflation in the one-shot game is better for the policy maker than committing to Nash inflation π^N.

Assumption 1. $l(0,0) \leq l(\pi^N, \pi^N)$

Assumption 1 always holds in models with a traditional Phillips curve $\pi_t = \kappa y_t + e_t$ where output equals zero in equilibrium, making low inflation desirable. However, the Phillips curve

[4] Alternatively, π^N is the inflation in the unique *stationary* equilibrium of the infinitely repeated game.

(2) we consider discounts inflation expectations by β, making output increase with inflation in equilibrium. It is therefore possible that the higher costs of Nash inflation are offset by higher output. The Appendix shows that Assumption 1 corresponds to a parametric condition stating that β is not too low. At our baseline values for κ and γ (see Table 1), Assumption 1 boils down to $\beta > 0.56$ (or 0.1 annualized).

We now state our equilibrium characterization.

Proposition 2. *Suppose that $\sigma = 0$ and Assumption 1 holds. Then both types of the policy maker follow the Ramsey plan with probability 1 in any optimal equilibrium. The Ramsey plan is an optimal announcement; it is the unique optimal announcement when $p_0 > 0$.*

Proposition 2 means that reputation is not beneficial in the case of perfect control over inflation. Regardless of initial reputation, it is optimal to announce the Ramsey plan.[5] On the path of the optimal equilibrium the private sector expects that the Ramsey plan will continue to be followed and reputation never changes. Hence, any initial level of reputation results in the same optimal equilibrium outcome.

Proposition 2 follows from the existence of an equilibrium where the rational type, who lacks commitment, is willing to follow the announced Ramsey plan. Any deviation is perfectly observed by the private sector due to $\sigma = 0$ and is met with expectations of Nash inflation π^N for the rest of the game.

Our timing assumption, consistent with the standard exposition of the New Keynesian model, plays a crucial role for incentives as discussed in Section 2. The private sector sets expectations *after* observing the chosen policy, so a deviation at time t is immediately met with expectations $e_t = \pi^N$, thereby punishing the policy maker at time t. If, in contrast, expectations were set before observing the policy, punishments would not take effect until time $t+1$, allowing deviations to boost output at time t. In these models[6] the Ramsey outcome is attainable only if the policy maker is sufficiently patient so that the deviation gains are offset by the subsequent punishment. Our result also relies on a relatively high β (Assumption 1) but for a different reason. Indeed, the patience of the policy maker does not affect incentives because the rational type is punished instantly after deviating and, therefore, does not face an intertemporal tradeoff. A deviation in the initial period is suboptimal for any β because following the equilibrium strategies results in the Ramsey outcome. However, the nonstationarity of the Ramsey plan described in Proposition 1

[5] If $p_0 = 0$ the announcement is immaterial, so it need not equal the Ramsey plan.

[6] Backus and Driffill (1985); Barro (1986); King et al. (2008); Dovis and Kirpalani (2021) fall into this category as they consider a Phillips curve based on expectations about current inflation.

implies that the policy maker's continuation loss increases over time converging to $l(0,0)/(1-\beta)$ in the long run, while the value of a deviation remains constant. Hence, Assumption 1 is required to rule out profitable deviations in later periods. Thus, the lower bound on β comes essentially from its effect on the Phillips curve (2), not on the planner's patience.[7]

Even though the Ramsey outcome is attainable in equilibrium, it is not obvious that announcing it is optimal. For instance, there exists a post-announcement equilibrium that is preferred by the rational type over the Ramsey outcome whenever $p_0 > 0$. Consider an announcement $\mathbf{a}$ with $a_0 = a_0^R$ and $a_t = 0$ for all $t > 0$, coupled with a strategy for the rational type of implementing the Ramsey plan (setting $g_t = a_t^R$ for all t). This strategy is part of a post-announcement equilibrium for any announcement. However, because the rational type chooses $g_0 = a_0^R$, like the behavioral type, the initial reputation p_0 is preserved in the first period, which leads to higher output than under the Ramsey plan as $a_1 < a_1^R$. From the second period onwards, the rational type continues to implement the Ramsey plan $g_t = a_t^R$ and fully separates from the behavioral type after the private sector observes $\pi_1 = a_1^R$, i.e. $p_t = 0$ for all $t \geq 1$. The level of output induced by the Ramsey plan is still attained as the rational type is expected to continue implementing the Ramsey plan $g_t = a_t^R$ for all t. Compared to the Ramsey outcome, the rational type is better off in the initial period and obtains the same outcomes in subsequent periods. Further welfare improvements can be found, for example by raising the initial period target a_0.

It follows that if the planner wishes to minimize the loss of the rational type, the optimal announcement can differ from the Ramsey plan to allow the rational type to 'spend' any initial reputation. However, any improvement to the rational type's welfare comes at the cost of decreasing the welfare of the behavioral type. Proposition 2 shows that this would be detrimental to average welfare.

In principle, our setup can result in unbounded punishments due to the unbounded nature of the policy space. In particular, any outcome can be supported in equilibrium by threatening deviations with sufficiently high inflation, which itself is incentivized by threats of even higher inflation. We do not consider such equilibria. The equilibrium constructed in Proposition 2 would continue to be optimal when policies are chosen from a (sufficiently large) bounded set.

[7]Stronger punishments (e.g. sustainable plans) can make the Ramsey outcome attainable for a larger set of parameters. However, the Ramsey outcome is not attainable for sufficiently low β.

5. Imperfect control

Suppose now that $\sigma > 0$, so that control over inflation is imperfect. This amounts to a reputational model with imperfect monitoring. These models are rarely tractable analytically outside of the patient limit. Therefore, this section will focus on numerical results.

We restrict the space of announcements $\mathcal{A}$ for computational feasibility. We assume inflation targets for each period are in the interval $A = [0, \pi^N]$. In addition, we assume that $\mathcal{A}$ only contains announcements $\mathbf{a} = (a_t)_{t=0}^{\infty}$ parametrized by (a_0, ω, χ), where

$$a_t = \chi + \omega^t (a_0 - \chi) .$$

These announcements include constant, decreasing, and increasing paths for inflation, demonstrated in Figure 2. Inflation starts from $a_0 \in A$ and converges towards $\chi \in A$ with a decay rate of $\omega \in [0, 1]$. When it does not lead to confusion we identify a plan $(a_t)_{t=0}^{\infty}$ with the triple (a_0, ω, χ).

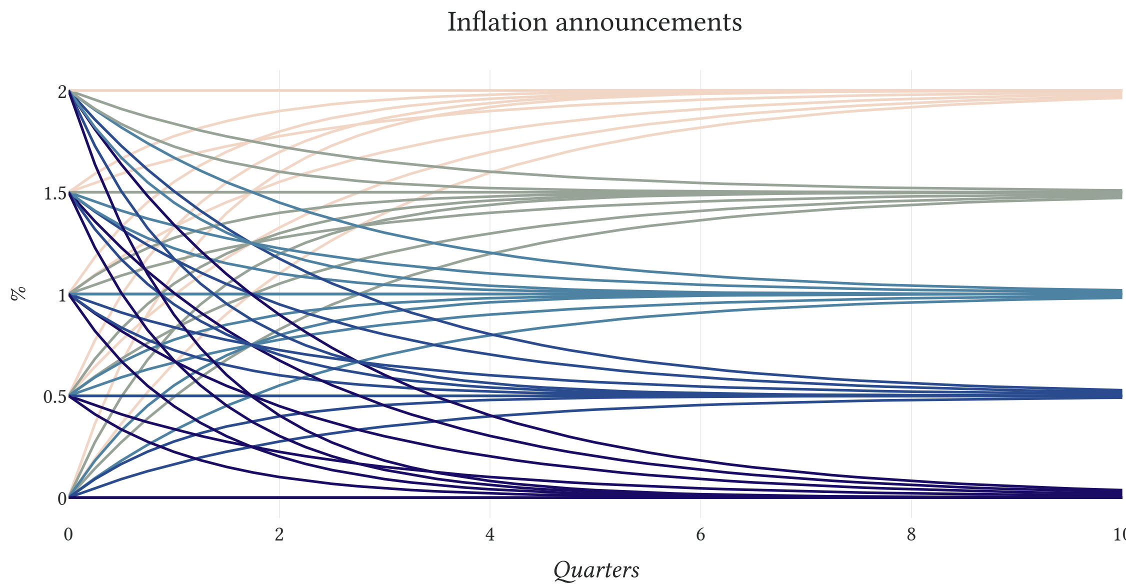


Figure 2: Possible behavioral types' announcements

This parametrization makes $\mathcal{A}$ finite-dimensional and allows us to write each plan recursively as $a_{t+1} = \phi(a_t)$, where

$$\phi(a) = \chi + \omega (a - \chi) .$$

As discussed in Section 3, the Ramsey plan $\mathbf{a}^R$ belongs to this class of announcements. At our baseline parametrization (see Table 1), $\mathbf{a}^R$ has $a_0 = 0.71 \times \pi^N$, $\chi = 0$, and $\omega = 29\%$.

We also restrict attention to pure-strategy Markov equilibria where the state variables are reputation and the announced target for the current period. In the context of Definition 1 this means the distribution $\mathbf{g}^\star(h^t)$ is degenerate and depends only on $\mathbf{p}(h^t)$ and a_t. Dependence on a_t is needed due to the nonstationary announcements we consider. Mixed strategies are often necessary for equilibrium existence when monitoring is perfect because pure strategies create limited reputational dynamics – conditional on the rational type, reputation either stays constant, or goes to 0. This issue does not arise in the context of imperfect monitoring considered here because posterior beliefs are fully supported on $(0, 1)$, even with pure strategies. Faingold and Sannikov (2011) also study pure-strategy Markov equilibria in a reputational model, except that time is continuous and the announcements are constant, so the only state is reputation.

The equilibria we consider (henceforth called Markov) have a recursive structure with state variables (p, a) corresponding to $(\mathbf{p}(h^t), a_t)$. In a Markov equilibrium, the public expects the rational type to choose $g^\star(p, a)$ at state (p, a) and uses it to form inflation expectations as well as to update beliefs via Bayes' rule. Given these beliefs $g^\star$, the rational type's problem is described by the following Bellman equation:

$$\mathcal{L}(p, a) = \min_g \mathbb{E}\left[l(\pi, e) + \beta \mathcal{L}(p', \phi(a))\right] \tag{7}$$

$$\text{subject to } \pi = g + \epsilon \tag{8}$$

$$\pi = \kappa y + \beta e \tag{9}$$

$$p' = \frac{p f_\epsilon(\pi - a)}{p f_\epsilon(\pi - a) + (1 - p) f_\epsilon(\pi - g^\star(p, a))} \tag{10}$$

$$e = p'\phi(a) + (1 - p') g^\star(p', \phi(a)), \tag{11}$$

where the constraints correspond to (1), (2), (3), and (4), respectively. In equilibrium, even though it is free to set policy and does not need to convalidate the public's expectations $g^\star$, those expectations must be such that the rational type finds it optimal to choose $g^\star(p, a)$ at each state (p, a).

Computation To find a post-announcement equilibrium given an announcement $\mathbf{a}$, we take the infinite-horizon limit of the finite-horizon game. The algorithm takes a continuation loss function $\mathcal{L}_{t+1}$ and continuation policy function $g^\star_{t+1}$. Substituting $\mathcal{L}_{t+1}$ for the continuation loss in the RHS of the objective (7) and substituting $g^\star_{t+1}$ for the continuation policy in (11) results in a fixed-point operator mapping $g^\star$ from (10) into the solution g of (7). We find a fixed point $g^\star_t$ and the resulting loss $\mathcal{L}_t$. Then we iterate (backward in time) until convergence.

We parametrize our model following Lu, King, and Pastén (2016). Our preference and technology parameters $\gamma, \kappa, y^\star$ are consistent with the planner's objective function and Phillips curve in a standard New Keynesian economy calibrated to US data (Galí, 2015; Galí and Gertler, 1999). Table 1 summarizes our parameter choices. These parameters imply a level for Nash inflation of about 2% annualized.

TABLE 1: BENCHMARK CALIBRATION

Parameter	Value	Definition	Source / Target
β	0.995	Discount factor	2% real interest rate
γ	60	Inflation weight	Lu, King, and Pastén (2016)
σ	1%	Std of control shock	Lu, King, and Pastén (2016)
κ	0.17	Slope of Phillips curve	Lu, King, and Pastén (2016)
$y^\star$	5%	Output target	Lu, King, and Pastén (2016)

5.1 *Post-announcement equilibrium*

We now discuss the structure of post-announcement equilibria following an announcement $\mathbf{a}$. Under perfect control we showed that the rational type can follow the announcement in equilibrium (Proposition 2). This is no longer true in the case of imperfect control.

Lemma 1. *Suppose $\sigma > 0$ and consider an announcement $\mathbf{a} = (a_t)_{t=0}^{\infty} \in \mathcal{A}$ parametrized by (a_0, ω, χ) with $a_0 < \pi^N$ or $\chi < \pi^N$. There exists no Markov post-announcement equilibrium following $\mathbf{a}$ where the rational type plays a_t at every time t.*

The reason for Lemma 1 is as follows. Suppose the private sector observes inflation π different from the target a. When control is perfect ($\sigma = 0$) this can only be the result of a deviation $g \neq a$ by the rational type. Beliefs are therefore updated to $p' = 0$, which serves to punish the rational type's deviation. In contrast, when monitoring is imperfect ($\sigma > 0$) inflation $\pi \neq a$ can be produced by both the rational and the behavioral type because the shock ϵ is supported on the entire real line. Moreover, if both types are expected to choose the same policy, updating through Bayes' rule (10) prescribes $p' = p$, so every posterior belief equals the prior p. The rational type then receives the same continuation loss regardless of realized inflation, and would want to deviate unless following the announcement by setting $g_t = a_t$ was myopically optimal given expectations a_{t+1}. While a plan could be designed with this feature at a particular time t, the only way to string along a full sequence of myopically optimal targets is by announcing the constant plan π^N.

Lemma 1 implies that imperfect control tempts the rational type to deviate from any announcement in which myopic gains are present. In general, such gains will come from deviating upwards and overshooting the inflation target a. However, if for example the descent of inflation in the announcement is too quick, output would be overstimulated by the expectations and the rational type could deviate downward to obtain gains from lower inflation. Such plans would be costly for the behavioral type and hence would be unlikely to be optimal.

It follows that the rational type deviates from inflation targets in equilibrium. Figure 3 illustrates the size of this deviation $g^\star(p, a)$ as a function of beliefs p and the current target a, normalizing by Nash inflation, for an arbitrary plan $\mathbf{a}$.

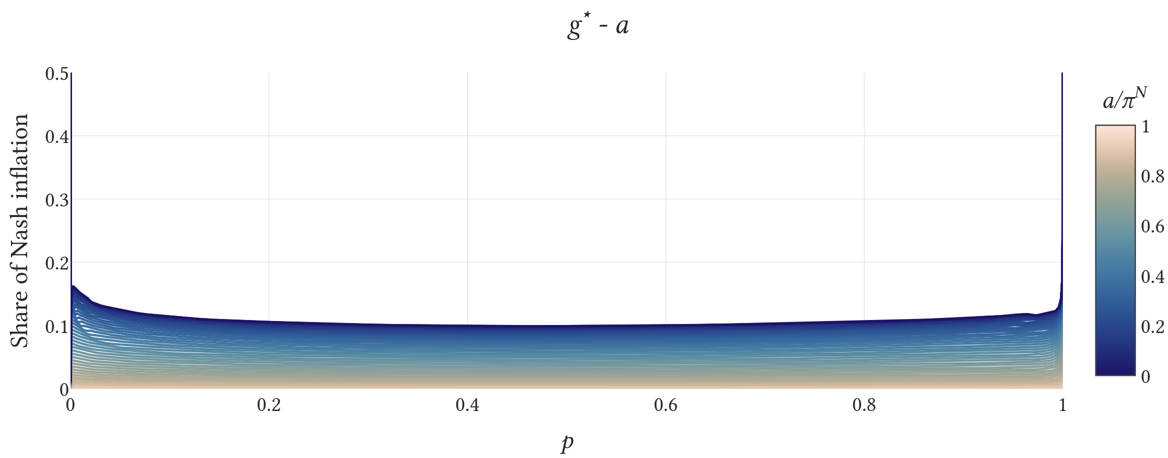


Figure 3: Inflation deviations

A lower target a results in larger deviations. In comparison to a higher target, the policy maker needs to inflate less to maintain the same evolution of reputation (which would yield broadly the same level of output), holding expectations of future policy fixed. Thus, there are lower costs to creating surprise inflation when the target is low. This force seems to dominate the fact that, with a lower target, expectations of the rational type's action $g^\star$ also change.

The deviation is also generally decreasing in reputation, except when reputation becomes large and the policy maker is able to produce surprise inflation largely undetected. There is a discontinuity at zero reputation, where the unique Markov equilibrium exhibits inflation π^N regardless of the announcement, since there are no reputational concerns. In contrast, at small but positive levels of reputation, the policy maker benefits from staying somewhat close to the announcement. Current reputation p affects optimal deviations through several different channels. On the one hand, a larger stock of reputation makes the policy maker more inclined to spend it by creating surprise inflation. On the other hand, higher reputation anchors expectations more tightly and makes it less costly to preserve reputation by staying close to the target, especially when the target is high.

When considering how much to deviate, the policy maker weighs expected reputational losses against the potential to boost current output. With an announcement $\mathbf{a}$ in place, at (p, a) the slope of the Phillips curve (9) is given by

$$\frac{\partial y}{\partial \pi} = \frac{1}{\kappa}\left[1 - \beta \frac{\partial p'}{\partial \pi}\left(\phi_{\mathbf{a}}(a) - g^{\star}(p', \phi_{\mathbf{a}}(a))\right) + (1-p')\,\frac{\partial g^{\star}(p', \phi_{\mathbf{a}}(a))}{\partial p'}\right] \tag{12}$$

Inflation affects current output through three distinct channels, corresponding to three terms in equation (12). The first term, $\frac{1}{\kappa} \times 1$, describes the standard, direct effect of inflation on output, given expected inflation. The second term, $\beta \frac{1}{\kappa}\left(-\frac{\partial p'}{\partial \pi}\right)\left(\phi_{\mathbf{a}}(a) - g^{\star}(p', \phi_{\mathbf{a}}(a))\right)$, describes an expectations-shifting effect by which more inflation reduces the posterior p' and, therefore, moves expectations of future inflation away from the target $\phi_{\mathbf{a}}(a)$ and toward the choice of the rational type $g^{\star}(p', \phi_{\mathbf{a}}(a))$. Finally, the third effect is given by $\beta \frac{1}{\kappa}\left(-\frac{\partial p'}{\partial \pi}\right)(1-p')\,\frac{\partial g^{\star}(p', \phi_{\mathbf{a}}(a))}{\partial p'}$. It describes how more inflation moves reputation and through it the future choice of the rational type.

An immediate property of equilibrium is that there is no announcement under which the rational type's reputation would increase over time. If the rational type follows the announcement exactly, i.e. if $g^{\star}(p, a) = a$ (which would not be an equilibrium given Lemma 1), reputation will stay unchanged because realized inflation is not an informative signal about type. But if the equilibrium strategy calls for deviations from the plan, reputation will decline on average. In other words, rational expectations prevent the accumulation of reputation by consistently delivering on promises, as such compliance would be anticipated by the private sector. What the planner can do is design the announcement in a way that provides incentives to deliver on it.

Observation 1. In any post-announcement equilibrium, the rational type's reputation is a supermartingale, i.e. $\mathbb{E}^{\mathbf{g}^{\star}}_{h^t}\left[p_{t+1}(h^t, \pi_t)\right] \leq p_t(h^t)$ for all $h^t \in H^t$. Thus, the planner cannot design an announcement that generates expected reputational gains in the ensuing post-announcement equilibrium.

Figure 12 in the Appendix shows the expected change in reputation, also as a function of the current reputation p and target a. It confirms Observation 1: $\mathbb{E}\left[p' - p\right] \leq 0$ so the rational type's reputation declines on average. Consistent with the sizes of deviations observed before, lower targets a are associated with larger expected reputational losses. More ambitious targets generate weaker incentives: as the temptation to inflate grows larger, the policy maker prefers to spend more of its reputation to achieve higher output.

All these dynamics come together in the value function $\mathcal{L}^{\mathrm{a}}(p, a)$, which is shown in Figure 4.

We draw three main lessons from this figure.

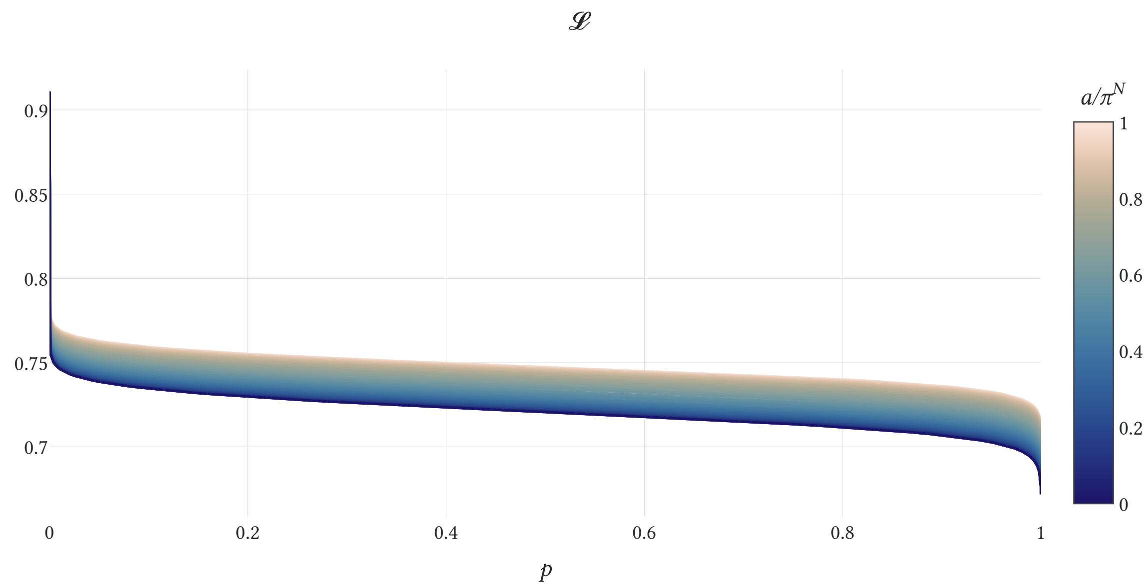


FIGURE 4: LOSS FUNCTION AFTER ANNOUNCEMENT $\mathbf{a}$

Firstly, $\mathcal{L}^{\mathbf{a}}$ is decreasing in p. An increase in reputation generally decreases expected inflation leading to higher current output and, therefore, smaller losses.

Secondly, the loss function has a convex-concave shape reflecting the dynamics of reputation. When reputation is close to 0 or 1 the public is confident in its assessment and significant evidence is required to move beliefs. Conversely, near $\frac{1}{2}$, movements in reputation are fickle and can easily be reversed. Thus, the same change in reputation is more valuable the closer p is to the extremes, leading to steepness of the loss function.

Thirdly, at high levels of reputation, a lower target a is preferred. The reason for this is that, as reputation increases, beliefs place a higher weight on the behavioral type who sticks to the announcement. As the rational type's strategy becomes less important, the credibility tradeoff dissipates.

Finally, the range of values of $\mathcal{L}$ across different targets is generally smaller at lower levels of reputation. One reason is that with lower reputation the current target becomes less relevant, as its weight in expected inflation decreases. Another, more nuanced reason is that the tradeoffs between stimulating output and sustaining reputation are more pronounced when the policy maker is seen as less likely to be committed to the target. While lower targets are directly beneficial to inflation expectations, it is more costly to deliver on low targets when reputation is low. Thus, the benefits of ambitious announcements with low targets can be offset more heavily at low levels of reputation.

5.2 Credibility

While the policy maker's reputation describes the likelihood it is committed to the plan, it does not reflect how closely the plan is followed on average across both types. To obtain a measure of this, we define the credibility of a plan as the ratio of announced and (expected) realized inflation, normalized by their distance from Nash inflation.

Definition 4. Given a plan $\mathbf{a}$, its *remaining credibility* in state (p, a) is defined recursively as follows:

$$\begin{aligned} \mathcal{C}(p, a; \mathbf{a}) &= \mathbb{E}\left[(1-\beta)\frac{\pi^N - \pi}{\pi^N - a} + \beta\mathcal{C}(p'_{\mathbf{a}}(p, a), \phi_{\mathbf{a}}(a); \mathbf{a})\right] \\ &= (1-\beta)\frac{\pi^N - [pa + (1-p)g^{\star}_{\mathbf{a}}(p, a)]}{\pi^N - a} + \beta\mathbb{E}\left[\mathcal{C}\left(p'_{\mathbf{a}}(p, a), \phi_{\mathbf{a}}(a); \mathbf{a}\right)\right] \end{aligned} \tag{13}$$

where π^N is Nash inflation. The *credibility* of a plan is given by

$$\mathcal{C}(\mathbf{a}) = \lim_{p \to 0} \mathcal{C}(p, a_0(\mathbf{a}); \mathbf{a})$$

In our simulations $g^{\star}_{\mathbf{a}}(p, a) \in [a, \pi^N]$ for all (p, a) and all plans $\mathbf{a}$, so credibility lies in $[0, 1]$.

Our setup distinguishes reputation p, the posterior belief that the policy maker is the behavioral type announced at the start of the game, from credibility $\mathcal{C}(p, a; \mathbf{a})$, the expected discounted deviations from plan $\mathbf{a}$ at reputation p and current target a, as defined in (13). Figure 5 plots the credibility of different plans (at vanishingly small reputation), as a function of initial and long-run inflation a_0 and χ, for the corresponding loss-minimizing decay rate ω.

Plans with a lower asymptote are less credible as they eventually imply too low levels of inflation and a quick loss of reputation. Moreover, especially when long-run inflation χ is low, plans with decreasing targets ($a_0 > \chi$) are more credible.

5.3 Optimal announcements

Figure 6 shows the optimal announcements for each level of initial reputation p_0. At $p_0 = 1$, the policy maker is behavioral for sure and the optimal announcement is the Ramsey plan. As initial reputation declines, the planner prefers announcements featuring more initial inflation a_0. At even lower levels of initial reputation p_0, the optimal announcement displays positive long-run inflation χ.[8] For every value of p_0, the planner prefers backloaded or gradual plans which feature

[8] We show optimal announcements for $p_0 \in (0, 1]$ as the announcement is irrelevant at $p_0 = 0$.

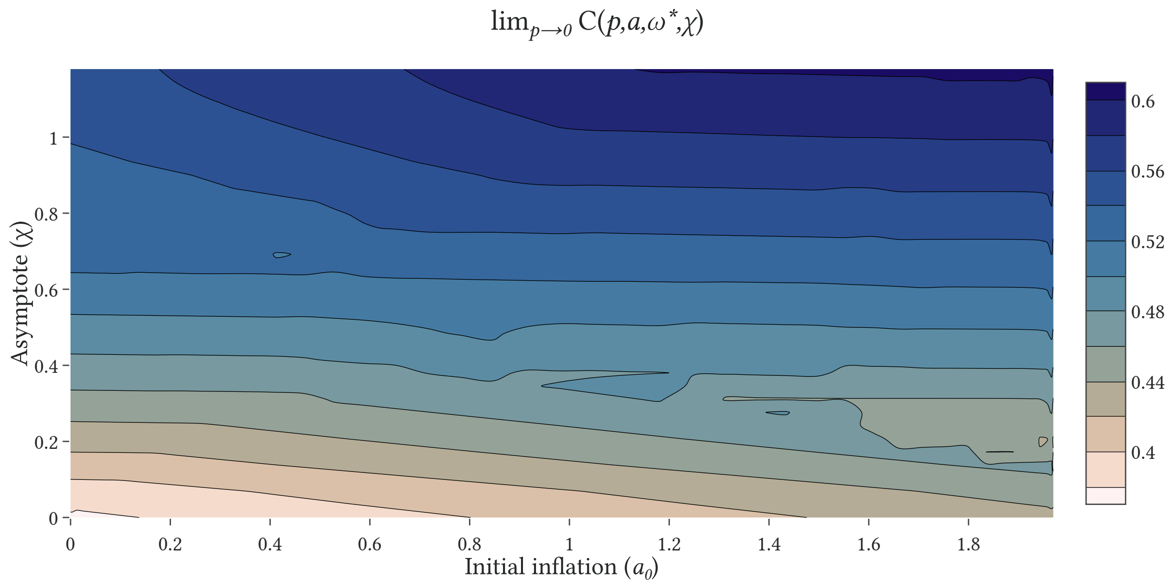


Figure 5: Credibility

a decreasing sequence of targets a_t.

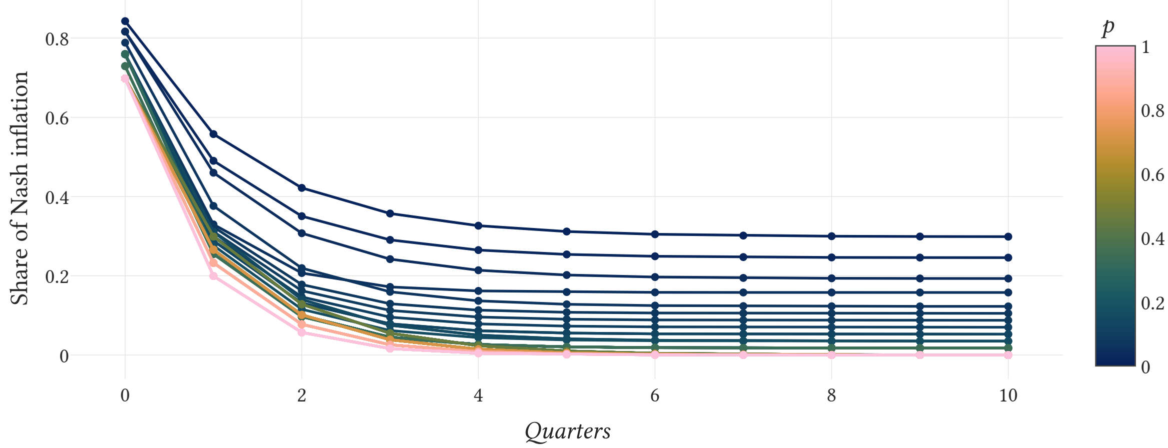


Figure 6: Optimal announcements

Because the rational type deviates upwards (Lemma 1), on average its reputation erodes over time, which is costly for the planner. This leads to optimal inflation targets that are higher than the Ramsey plan when $p_0 < 1$, especially early on.

At small values of p_0, if χ is too small, plans eventually call for ambitiously low levels of inflation which are difficult to sustain. As discussed above, such plans lack credibility $\mathcal{C}$. Consequently, reputation is quickly lost giving rise to unfavorable continuation values as well as high inflation expectations: the promises of such plans are then negated by their lack of credibility. Too high levels of long-run inflation, on the other hand, are easier to sustain but provide less benefit. This intuition is confirmed by Figure 13 in the Appendix, which plots expected equilibrium losses for different announcements at the zero-reputation limit. It shows that an intermediate value of long-run inflation χ is optimal, corresponding to the choice observed in Figure 6. Moreover, the planner is roughly indifferent among initial inflation levels a_0 when long-run inflation χ is too high or too low. When χ is close to its welfare-maximizing value, the benefits of $a_0 > \chi$ become more clear. Such levels of initial inflation are valuable as they enhance the policy maker's ability to stick to a plan which eventually delivers the right level of inflation. When the long-run level is either too high or too low, the planner is less affected by the starting point.

At large p_0, the announcement that maximizes only the rational type's payoff differs from the optimal announcement maximizing the average loss across both types, in particular, by entailing lower inflation throughout. This promotes favorable expectations even as the rational type expects to 'burn through' its initial reputation, which may take long if p_0 is large enough. This intuition is confirmed by Figure 15 in the Appendix, which shows the preferred announcements for the rational type. Similarly to the case of perfect control, boosting the rational type's welfare at the expense of the behavioral type in this way is not optimal.

5.4 *Comparative Statics*

Optimal announcements entail a decreasing path for inflation before stabilizing at a positive long-run level χ. This section investigates how key parameters of the model affect the shape of the optimal announcement.

Proposition 3 states that when control is imperfect, the average loss of the policy maker exceeds that of the Ramsey planner, making the case of $\sigma = 0$ welfare-maximizing. Moreover, Figure 16 in the Appendix shows that welfare is monotonically decreasing in σ, for all values of reputation p.

Proposition 3. *If $\sigma > 0$, the expected loss of the policy maker is strictly larger than $\min_a R(a)$ in any equilibrium.*

The result does not depend on equilibrium incentives. The Ramsey loss can be obtained only if inflation is exactly a_t^R at each time t, which is impossible due to noise, even if the rational type could commit.

Propositions 2 and 3 imply that $\sigma = 0$ is preferable to any $\sigma > 0$ when Assumption 1 holds. In other words, perfect control dominates imperfect control from the perspective of welfare. We take σ to represent actual (physical or informational) constraints in the control of inflation, and therefore σ determines not only the precision of control over inflation, but also the precision with which the private sector observes the chosen policy. In contrast, Atkeson, Chari, and Kehoe (2007), who focus on choosing among different monetary policy instruments in the context of sustainable plans, call the former *tightness* and the latter *transparency*. They find that both tightness and transparency are beneficial. The optimality of $\sigma = 0$ in our model agrees with this finding because lower σ represents more tightness and more transparency.

It is also possible to argue that given perfect control of inflation it is optimal to have full transparency. More precisely, if the policy maker sets inflation directly but the private sector observes inflation imperfectly, then the rational type cannot be incentivized to follow the announcement in any equilibrium for reasons similar to Lemma 1. This makes the Ramsey outcome unattainable, so full transparency is optimal. In contrast, Dovis and Kirpalani (2021) find that with perfect control the optimal level of transparency varies with initial reputation. This is because the Ramsey outcome is not attainable in equilibrium and lower transparency can help preserve the reputation of the rational type. We observe a similar force in our model with $\sigma > 0$, where a highly credible plan is closely followed by the rational type, reducing the equilibrium inference made by the private sector and thereby mitigating reputational losses.

Turning to the effect of the variance of the control shock σ itself, Figure 7 shows the optimal announcement (as $p \to 0$) as a function of σ around its baseline value of 1%. Notice that σ does not enter the formula for Nash inflation π^N or any of the tradeoffs which shape the Ramsey plan. However, more noise in the control makes deviations from targets harder to detect. Therefore, the level of adherence to plans is decreasing in σ. This makes the planner choose less ambitious plans when control over inflation is less tight. These plans have higher inflation throughout, as they feature a higher asymptote χ, a marginally higher initial inflation a_0, and slower decay ω.

Figure 8 repeats the exercise varying the discount factor β and the slope of the Phillips curve κ. It reveals some subtleties in the manipulation of the three parameters that describe our plans. Figure 8a shows the average plan as a function of the discount factor β (whose benchmark value is 2% in annual terms). As the planner becomes more impatient, average plans start higher but display a faster decay rate. With more impatience, the public expects a stronger inflation bias. For this reason, the planner tends to choose plans that are more resilient. Increasing initial inflation makes the plan easier to follow. A steeper descent of inflation targets contributes to both objectives.

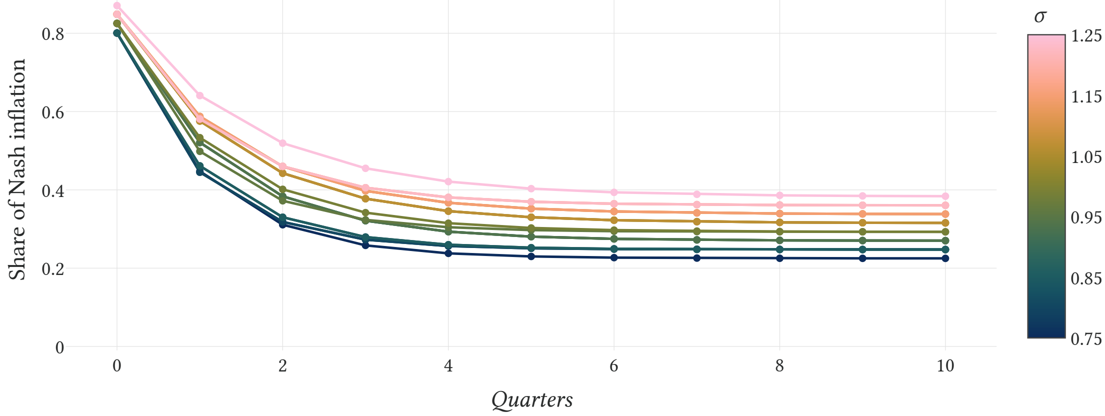
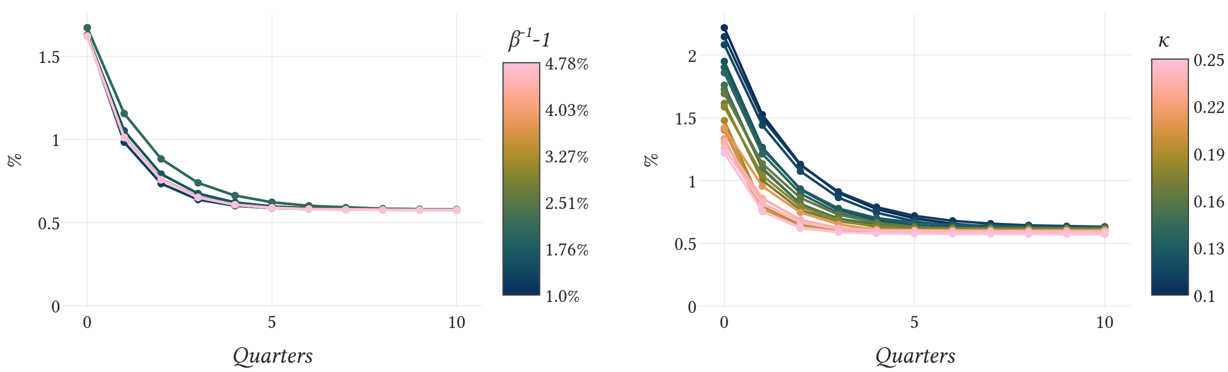


Figure 7: Optimal announcements and the control shock variance

Figure 8b shows that when the slope of the Phillips curve is increased from its baseline value of 0.17, the planner announces lower inflation throughout. When the Phillips curve is steeper, there are weaker incentives to create surprise inflation, as it results in a smaller output boom. Thus, the planner lowers expectations through lower targets without increasing reputational losses.



(a) Varying the discount factor

(b) Varying the slope of the Phillips curve

Figure 8: Optimal announcements

6. Feedback rules

Our notion of equilibrium yields optimal announcements that start from a high level of initial inflation and gradually decrease towards positive long-run values. The Ramsey plan shares this feature (Proposition 1). In this section we demonstrate that gradualism is driven by equilibrium incentives that the Ramsey planner does not face.

To investigate the importance of incentives, we augment the space of behavioral types to include gradual feedback rules. These types follow plans under which future targets respond to deviations of inflation from the current target

$$a' = \chi + \omega(a - \chi) + \psi(\pi - a) \tag{14}$$

so that whenever realized inflation π differs from its current target a, a share ψ of the difference is embedded in the target for the following period.

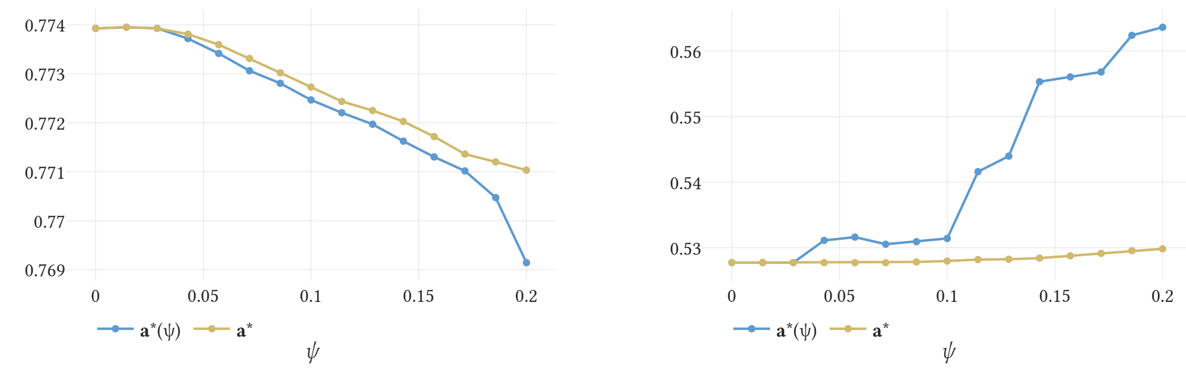
As shown in Section 3, the Ramsey planner does not benefit from conditioning on deviations from the targets. Even beyond quadratic utility (which makes the control shocks separable), the reason why the Ramsey plan starts high and converges to zero is because the benefits of high initial inflation do not come at a cost in terms of expected inflation in the past (this is also why the Ramsey planner sets $\pi_t > \pi_{t+1}$). With gradual feedback rules, the public's expectations incorporate the possibility that shocks will shift future targets. Gradual feedback rules which create benefits at time $t > 0$ also involve costs on the equilibrium path for all times $0 \leq j < t$, which is conceptually why the Ramsey planner does not use them.

In a post-announcement equilibrium, however, gradual feedback can be helpful. Figure 9 shows the value function $\mathcal{L}$ on the left panel along with initial credibility $\mathcal{C}$ on the right. Both are plotted as a function of the target updating parameter ψ, either reoptimizing the plan for each ψ (labeled $\mathbf{a}^\star(\psi)$) or fixing the parameters (ω, χ, a_0) of the original optimal announcement at $\psi = 0$ (labeled $\mathbf{a}^\star$). Gradual feedback rules induce paths where inflation targets come down gradually after a high control shock ϵ. This increases the credibility of the announcement and is hence preferred by the planner.

Figure 10 provides more detail into how the introduction of gradual feedback rules affects the optimal plan. Gradual feedback complements other forms of gradualism: as ψ increases, the planner chooses plans which converge more slowly from the initial to the long-run targets. There is a also a marginal shift up of the entire plan, since both a_0 and χ increase slightly with ψ.

7. Distribution of announcements in a reputational equilibrium

So far we have focused on the optimal announcement for a planner who shares the private sector's uncertainty (and beliefs) about the commitment of the policy maker. This can be rationalized as



(A) Loss function $\mathcal{L}$ (B) Credibility $\mathcal{C}$

Figure 9: Gains from gradual feedback rules

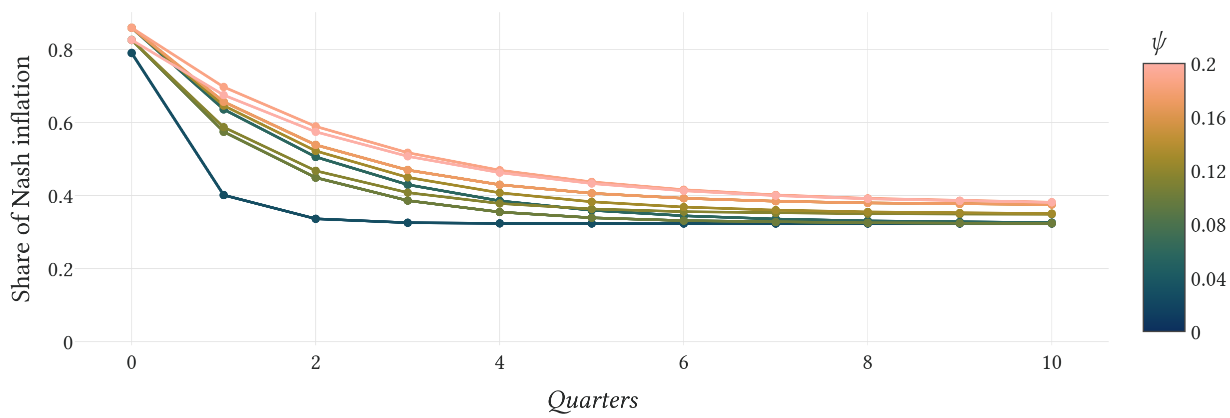


Figure 10: Implied plans with gradual feedback rules

coming from delegation to a policy maker (Dovis and Kirpalani, 2021), or equivalently from a government who may become committed to the plan after its announcement (Kambe, 1999).

We now turn to the description of a different equilibrium notion. In this version, we assume that the planner knows about the commitment of the policy maker or in other words that the announcement is made *after* observing the policy maker's type. The private sector continues to face uncertainty about the policy maker's type and believes that it is behavioral with probability z. There are multiple behavioral types: one for each plan in $\mathcal{A}$. Let ν denote the probability distribution of behavioral types over $\mathcal{A}$. Each behavioral type announces the plan it is committed to implement.[9] When instead the policy maker is the rational type, the planner chooses an

[9]More precisely, the planner announces plan $\mathbf{a}$ when it observes that the policy maker is behavioral and com-

announcement according to an equilibrium distribution μ. We refer to μ as the rational type's announcement distribution.

The private sector uses the announcement to infer whether it faces the rational or one of the behavioral types. After an announcement $\mathbf{a}$, the private sector forms beliefs p_0 using Bayes' rule. When μ and ν admit densities f_μ and f_ν, the updated beliefs are

$$p_0(\mathbf{a}; z, \mu, \nu) = \frac{z f_\nu(\mathbf{a})}{z f_\nu(\mathbf{a}) + (1 - z) f_\mu(\mathbf{a})}. \tag{15}$$

This initial stage can be viewed as a way to endogenize the prior p_0 which has been a parameter thus far. Importantly, unless $\mu = \nu$, $p_0(\mathbf{a}; z, \mu, \nu)$ will not be constant across announcements $\mathbf{a}$.

The intuition of (15) is clear: plans that are announced frequently by the rational type carry a reputational discount. At the same time, all else equal, plans that are more likely under ν obtain higher values for p_0 as they are announced more often by the behavioral types. Because the distribution ν is arbitrary, such a reputational boost would be purely exogenous. In what follows, we assume ν is the uniform distribution over $\mathcal{A}$.

Definition 5. A *reputational equilibrium* is a distribution μ over $\mathcal{A}$ and a collection of strategies $\{\mathbf{g}^{\mathbf{a}}, \mathbf{e}^{\mathbf{a}}\}_{\mathbf{a} \in \mathcal{A}}$ such that

- For all $\mathbf{a} \in \mathcal{A}$, $(\mathbf{g}^{\mathbf{a}}, \mathbf{a}, \mathbf{e}^{\mathbf{a}})$ is a post-announcement equilibrium of the game with $p_0 = p_0(\mathbf{a}; z, \mu, \nu)$.
- The distribution of mimicked types μ minimizes

$$\mathcal{L}_r(\mu, z) = \int_{\mathcal{A}} \mathcal{L}^{\mathbf{a}}(p_0(\mathbf{a}; z, \mu, \nu), a_0(\mathbf{a})) d\mu(\mathbf{a}),$$

  where $\mathcal{L}^{\mathbf{a}}$ is the equilibrium loss function (7) of the rational type given announcement $\mathbf{a}$.

It is clear that the distribution μ cannot have mass points. If, for example, μ calls for announcing plan $\mathbf{a}^\star$ with positive probability then, since ν has no mass points, it follows that $p_0(\mathbf{a}^\star; z, \mu, \nu) = 0$. This means that all reputation is lost after $\mathbf{a}^\star$ is announced, which results in inflation π^N in each period "which is the outcome of the unique Markov equilibrium with $p = 0$. The planner can improve welfare by choosing another announcement for the rational type that results in positive reputation and a smaller loss.

An important part of finding a reputational equilibrium is determining which plans are announced with positive probability and which ones are not. If a plan $\mathbf{a}$ is outside the support of

mitted to $\mathbf{a}$.

μ, the private sector expects $\mathbf{a}$ to only be announced by a behavioral type, so that it grants full reputation, i.e. $p_0(\mathbf{a}) = 1$. It follows that $\mathbf{a}$ can only be outside the support of μ only if its expected loss at full reputation is greater than that the loss from plans chosen in equilibrium.

Computation To find a reputational equilibrium, we proceed as follows. Given $k \in \mathbb{R}$, we partition the space of plans according to whether

$$\mathcal{L}^{\mathbf{a}}(1, a_0(\mathbf{a})) \lesseqgtr k$$

Plans $\mathbf{a}$ which imply a loss greater than k at initial reputation $p_0 = 1$ are assigned probability zero: $\mu(\mathbf{a}) = 0$. For the remainder, we find a probability p_0 that delivers loss k by solving $\mathcal{L}^{\mathbf{a}}(p_0(\mathbf{a}), a_0(\mathbf{a})) = k$. Inverting Bayes' rule (15), we find $\mu(\mathbf{a})$ that is consistent with the initial reputation $p_0(\mathbf{a})$ required for $\mathbf{a}$ to deliver a loss of k. Finally, we integrate the resulting non-negative announcement function μ. This operation describes an operator mapping k to the integral of μ. Since μ must be a probability distribution, the last step is to pin down k, the expected loss, by requiring that μ integrates to 1 over the set of possible plans $\mathcal{A}$.

Figure 11 shows the limiting distribution $\mu^\star = \lim_{z\to 0} \mu$ of announcements for the rational type as the initial probability of the behavioral types z vanishes. It plots the announcement distribution as a function of the asymptote χ and initial inflation a_0, integrating over the decay rate ω.

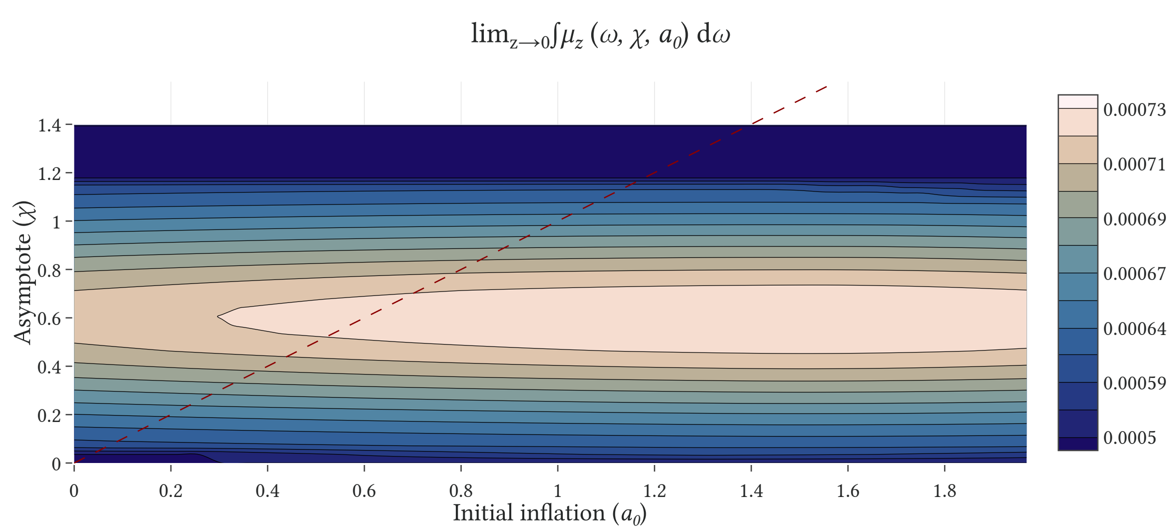


Figure 11: Distribution of announcements

Figure 11 reveals that the planner tends to choose gradual plans with higher initial inflation a_0 than long-run inflation χ. This probability is $\mathbb{P}(a_0 > \chi) = 65\%$. Plans with initial inflation of even five times the asymptote are still announced quite often: $\mathbb{P}(a_0 > 5\chi) = 15.6\%$. While the level of initial inflation has a fairly wide distribution, the asymptote is more precisely set: the density of a_0 and χ in the reputational equilibrium announcement falls sharply for χ away from the optimum, while it stays flat over many more values of initial inflation a_0. In fact, plans with a too high asymptote are announced with probability 0. Finally, notice that plans with inflation increasing over time are also sometimes announced. They are announced infrequently enough that they result in a large reputation p_0 that boosts their remaining credibility at time zero $\mathcal{C}(p_0(\mathbf{a}), a_0(\mathbf{a}); \mathbf{a})$.

Figure 11 bears a close resemblance to Figure 13 in the Appendix which plots the loss function at exogenous but low p_0. Announcements with a lower loss (for the same p_0) are good for the planner, so they are chosen more often in the reputational equilibrium. Hence, the policy maker starts with lower reputation when announcing these plans, making them less valued in equilibrium. This initial update of reputation (from z to p_0) makes the planner indifferent across all equilibrium announcements, which ultimately justifies the mixed strategy.

8. Concluding Remarks

This paper addresses an old question: can reputation be a substitute for commitment? We find that a simple model of reputation combined with imperfect control on the part of the policy maker creates incentives for staying close to announced targets. The optimal policy after a plan has been announced trades off the benefits of surprise inflation against the possibility that a deviation becomes known to the public. In this way, the policy maker's reputation becomes an important state variable in the problem of optimal policy under discretion.

Various characteristics of announcements come to bear when determining the value of reputation. We find that a pervasive feature of optimal plans is gradualism. In anticipation of the post-announcement equilibrium, the planner finds it desirable to set up situations in which conserving reputation is both easy and valuable. One such case is when announced inflation for the current period is higher than in the future. The resulting gradualism is therefore an artifact of incentives and not a reflection of underlying inflation inertia. Understanding how the presence of sources of true inertia might interact with our results is left as an open question.

The gradualist property of optimal plans holds at positive levels of reputation and also in the limit as initial reputation vanishes. We interpret this limit case as a refinement of the game between a rational government and the private sector.

Finally, we show that a target-updating rule can improve the performance of the optimal plan in equilibrium. By letting future targets respond to deviations of inflation, the plan reallocates more challenging tasks to the states in which reputation has increased. This property increases the overall credibility of the plan, improves expectations and, through them, outcomes. This new source of gradualism, which does not vanish after the first few periods and continues to affect equilibrium plans even in the long run, constitutes a potential lesson for policy. Designers of stabilization plans often fear having to change their inflation targets after the fact. The lesson from gradual feedback plans is that it can actually be advantageous to set up a rule for changing targets. Of course, such a rule must be part of the original announcement.

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A. Omitted Proofs

The proofs rely on the solution to the Ramsey problem of minimising $\mathcal{L}(\mathbf{a})$ in (5) over announcements $\mathbf{a} \in \mathbb{R}^\infty$. This is a nonstandard problem due to the unboundedness of the action space resulting in the unboundedness of per-period losses l. Nevertheless, dynamic programming arguments can be applied because l is bounded below. We now follow Hernández-Lerma and Muñoz de Ozak (1992) in defining a Markov control model corresponding to the Ramsey problem.[10]

Let $X = \mathbb{R}$ be the state space, $A = \mathbb{R}$ be the action set, and $A(x) = \mathbb{R}$ for all $x \in X$ be the correspondence of feasible actions. The transition law $Q(\cdot|x, \alpha)$ given state x and action α is a probability measure on X placing probability 1 on $x' = \alpha$. Let $\mathbb{K} = \{(x, \alpha)|x \in X, \alpha \in A(x)\} = \mathbb{R}^2$ be the set of admissible state-action pairs. A deterministic policy is a sequence of functions $(g_t)_{t=0}^\infty$, where $g_t(x_0, \alpha_0, .., x_{t-1}, \alpha_{t-1}, x_t) \in A(x_t) = \mathbb{R}$. Let $\mathcal{G}$ be the set of deterministic policies. A policy g is stationary if g_t depends only on x_t; in this case we write $g(x_t)$ instead of $g_t(x_0, \alpha_0, .., x_t)$. Consider the control problem $\Gamma(x)$ given by

$$\begin{aligned} &\min_{g \in \mathcal{G}} V(x, g) := \sum_{t=0}^{\infty} \beta^t l(x_t, \alpha_t) \\ &\text{s.t. } x_0 = x \\ &\qquad \alpha_{t+1} = x_{t+1} = g(x_0, \alpha_0, .., x_t). \end{aligned}$$

Consider the operator T associated with the Bellman equation (6):

$$Tv(x) = \min_{\alpha \in \mathbb{R}} l(x, \alpha) + \beta v(\alpha). \tag{16}$$

Let $v_0 = 0$ and $v_n = Tv_{n-1}$ for all $n \in \mathbb{N}$.

Lemma 2. *Each function v_n is well-defined and $v_n \leq v_{n+1}$. The function $R(x) = \lim_{n\to\infty} v_n(x)$ satisfies $R = TR$. There exists a stationary policy g that solves $\Gamma(x)$ for any x and satisfies*

$$R(x) = l(x, g(x)) + \beta R(g(x))$$

Proof. This follows from Theorem 4.2. in Hernández-Lerma and Muñoz de Ozak (1992) provided that the following assumptions are satisfied:

(a) l is nonnegative and lower semicontinuous, and $\{\alpha \in A(x)|l(x, \alpha) \leq r\}$ is compact for all $x \in X, r \in \mathbb{R}$.

[10] See Chapter 4 of Hernández-Lerma and Lasserre (2012) for similar results.

(b) For any continuous and bounded function $u : X \to \mathbb{R}$, the map $(x, \alpha) \mapsto \int_X u(y) Q(dy|x, \alpha)$ is continuous on $\mathbb{K}$.

(c) The correspondence A is lower hemicontinuous.

(d) There exists $g \in \mathcal{G}$ such that $V(x, g) < \infty$ for all $x \in X$.

Towards (a), it is immediate that l is nonnegative and continuous. For any $x \in X, r \in \mathbb{R}$ the set $\{\alpha \in \mathbb{R} | l(x, \alpha) \leq r\}$ is closed due to the continuity of l and bounded because $l(x, \alpha) \to \infty$ as $\alpha \to \infty$ or $\alpha \to -\infty$.

Towards (b), note that the map in question is $(x, \alpha) \mapsto u(\alpha)$. It is, therefore, continuous whenever u is continuous.

Towards (c), the correspondence A is constant and, therefore, continuous.

Finally, condition (d) holds by taking the stationary policy $g = 0$. □

Henceforth, R and g refer to the functions described in Lemma 2. We now use it to describe the solution to the Ramsey problem.

Lemma 3. *The function R is strictly convex and differentiable, and it satisfies* (6). *It satisfies* $R(x) = \min\{\mathcal{L}((a_t)_{t=0}^{\infty}) | a_0 = x\}$ *for all* $x \in \mathbb{R}$.

Proof. Since $R = TR$, R satisfies (6). It is straightforward to verify that the operator T maps convex functions to strictly convex functions. Since $v_0 = 0$ is convex, it follows that each v_n is convex. Therefore, R is convex, being the pointwise limit of convex functions. Since $R = TR$, it also follows that R is strictly convex. Differentiability now follows from the Benveniste-Scheinkman theorem. The final property follows by observing from (5) that the problem of minimising $\mathcal{L}(\mathbf{a})$ given a_0 is the same as the problem $\Gamma(a_0)$. □

Lemma 4. *g satisfies* $|g(a)| \leq |a|$ *for all* $a \in \mathbb{R}$.

Proof. Since R is strictly convex (Lemma 3), it follows that $l(a, a') + \beta R(a')$ is strictly convex. Hence, the policy function g satisfies the FOC

$$2\left(y^\star - \frac{1}{\kappa}(a - \beta g(a))\right)\frac{\beta}{\kappa} + \beta R'(g(a)) = 0, \tag{17}$$

where we have used differentiability of R (Lemma 3). The envelope condition is

$$R'(a) = -2\left(y^\star - \frac{1}{\kappa}(a - \beta g(a))\right)\frac{1}{\kappa} + 2\gamma a \tag{18}$$

Combining (17) and (18) yields

$$2\gamma a = R'(a) - R'(g(a)). \tag{19}$$

The strict convexity of R implies that R' is strictly increasing, so $g(0) = 0$ follows from (19). It follows from (17) that $R'(0) = -\frac{2}{\kappa}y^\star$. If $a > 0$, then (19) implies that $g(a) < a$.

$$2\left(y^\star - \frac{1}{\kappa}a_t^R\right)\frac{1}{\kappa} + R'(0) < 0,$$

so (17) implies that $g(a) > 0$. Hence, $0 < g(a) < a$. A similar argument obtains $a < g(a) < 0$ whenever $a < 0$. Hence, $|g(a)| \leq a$, as required. □

Proof of Proposition 1

Let $\lambda = \gamma\kappa^2$ and $\omega = \frac{1+\beta+\lambda-\sqrt{(1+\beta+\lambda)^2-4\beta}}{2\beta}$. The latter is well-defined because

$$(1+\beta+\lambda)^2 - 4\beta = \lambda^2 + 2\lambda(1+\beta) + (1-\beta)^2 = (1-\beta+\lambda)^2 + 4\lambda\beta \geq 0$$

Clearly, $\omega > 0$. Moreover,

$$1 - \omega = \frac{1}{2\beta}\left[-(1-\beta+\lambda) + \sqrt{(1-\beta+\lambda)^2 + 4\lambda\beta}\right] > 0.$$

Hence, $\omega \in (0,1)$. Note that ω solves the quadratic equation

$$\beta\omega^2 - (1+\beta+\lambda)\omega + 1 = 0,$$

which can be rearranged as

$$(1-\beta\omega)(1-\omega) = \lambda\omega. \tag{20}$$

Further algebra obtains

$$\begin{aligned}
\frac{1}{\kappa^2}(1-\beta\omega)((1-\beta\omega^2) - (1-\beta\omega)) &= \frac{1}{\kappa^2}(1-\beta\omega)(\beta\omega - \beta\omega^2) = \gamma\beta\omega^2 \\
\frac{1}{\kappa^2}(1-\beta\omega)^2 + \gamma &= \frac{1}{\kappa^2}(1-\beta\omega)(1-\beta\omega^2) - \gamma\beta\omega^2 + \gamma \\
\frac{1}{\kappa^2}(1-\beta\omega)^2 + \gamma &= \left(\frac{1}{\kappa^2}(1-\beta\omega) + \gamma\right)(1-\beta\omega^2).
\end{aligned} \tag{21}$$

We now show that R equals the function $R^\star : \mathbb{R} \to \mathbb{R}$ given by

$$R^\star(a) = \frac{(y^\star)^2}{1-\beta} - a\frac{2}{\kappa}y^\star + a^2\left(\frac{1}{\kappa^2}(1-\beta\omega) + \gamma\right).$$

with derivative

$$(R^\star)'(a) = -\frac{2}{\kappa}y^\star + 2a\left(\frac{1}{\kappa^2}(1-\beta\omega)+\gamma\right).$$

It follows from $\omega \in (0,1)$ that $R^\star$ is strictly convex.

Let $n \in \mathbb{N}$ and let $f_{[-n,n]}$ be the restriction of a function f to the interval $[-n,n]$. Lemma 4 implies that

$$R(a) = \min_{a' \in [-n,n]} l(a,a') + \beta R(a'),$$

so $R_{[-n,n]}$ solves the Bellman equation

$$T^n v(a) = \min_{a' \in [-n,n]} l(a,a') + \beta v(a'). \tag{22}$$

on the space of bounded functions $v : [-n,n] \to \mathbb{R}$. Standard dynamic programming arguments show that (22) has a unique solution. We now show that $R^\star_{[-n,n]}$ solves (22) and its associated policy function is the restriction of $\phi(a) := \omega a$ to $[-n,n]$. To this end,

$$\begin{aligned}
\frac{\partial}{\partial a'}[l(a,a') + \beta R^\star(a')]_{a'=\omega a} &= 2\left(y^\star - \frac{1}{\kappa}(1-\beta\omega)a\right)\frac{\beta}{\kappa} + \beta\left(-\frac{2}{\kappa}y^\star + 2\omega a\left(\frac{1}{\kappa^2}(1-\beta\omega)+\gamma\right)\right)\\
&= 2\beta a\left[-\frac{1}{\kappa^2}(1-\beta\omega) + \frac{1}{\kappa^2}\omega(1-\beta\omega) + \gamma\omega\right]\\
&= \frac{2\beta a}{\kappa^2}\left[-(1-\beta\omega)(1-\omega) + \lambda\omega\right] = 0,
\end{aligned}$$

where the last line uses (20). It follows from the strict concavity of $R^\star$ and $\omega \in (0,1)$ that $\phi_{[-n,n]}(a) = \omega a$ is the unique policy function associated with $R^\star_{[-n,n]}$.

That $R^\star_{[-n,n]}$ satisfies (22) follows from

$$\begin{aligned}
T^n R^\star(a) &= \left(y^\star - \frac{1}{\kappa}a(1-\beta\omega)\right)^2 + \gamma a^2 + \beta R^\star(\omega a)\\
&= (y^\star)^2 - \frac{2}{\kappa}a(1-\beta\omega)y^\star + a^2\left(\frac{1}{\kappa^2}(1-\beta\omega)^2 + \gamma\right)\\
&+ \beta\frac{(y^\star)^2}{1-\beta} - \beta\omega a\frac{2}{\kappa}y^\star + \beta\omega^2 a^2\left(\frac{1}{\kappa^2}(1-\beta\omega)+\gamma\right)\\
&= \frac{(y^\star)^2}{1-\beta} - a\frac{2}{\kappa}y^\star(1-\beta\omega+\beta\omega) + \beta\omega^2 a^2\left(\frac{1}{\kappa^2}(1-\beta\omega)+\gamma\right)\left(1-\beta\omega^2+\beta\omega^2\right) = R^\star(a),
\end{aligned}$$

where the last line follows from (21).

Since n is arbitrary, it follows from $R^\star_{[-n,n]} = R_{[-n,n]}$ and $g_{[-n,n]} = \phi_{[-n,n]}$ that $R = R^\star$ and the unique policy function associated with R is $\phi(a) = \omega a$. It follows from Lemma 3 that the unique

Ramsey plan is given by $a_0^R = \operatorname*{argmin}_a R(a)$ and $a_{t+1}^R = \omega a_t^R$ for all t. The strict convexity of $R^\star$ implies that $R'(a_0^R) = 0$, i.e.

$$a_0^R = \frac{\frac{1}{\kappa} y^\star}{\frac{1}{\kappa^2}(1-\beta\omega)+\gamma} = \frac{\kappa y^\star}{1-\beta\omega+\lambda}.$$

Suboptimality of randomization

Lemma 5 below states that the ex-ante expected loss (unconditional on the policy maker's type) in any equilibrium exceeds the Ramsey planner's loss whenever the rational type's behavior differs from the announcement, or control over inflation is imperfect. The result uses the fact that the Ramsey planner does not benefit from randomization, and it is therefore suboptimal that the policy maker behaves stochastically from the ex-ante perspective. The lack of need for randomization is standard in many stochastic control problems including the problem $\Gamma(x)$ considered above (Theorem 4.2. in Hernández-Lerma and Muñoz de Ozak (1992)). However, randomization is different in our setting due to forward-looking expectations. To see this, consider a Ramsey planner who commits to randomize equally between $\mathbf{a}$ and $\mathbf{a}'$. The resulting loss need not be $0.5\mathcal{L}(\mathbf{a})+0.5\mathcal{L}(\mathbf{a}')$. Indeed, if $a_0 = a_0'$ and $a_1 \neq a_1'$, then the private sector expects $0.5a_1+0.5a_1'$ following a_0, so the period-0 loss equals $l(a_0, 0.5a_1+0.5a_1')$. This is less than $0.5l(a_0,a_1)+0.5l(a_0,a_1')$ by Jensen's inequality. Our argument, instead, makes use of the convexity of the value function to show that randomization is not needed.

Lemma 5. *If $(\mathbf{g}^\star, \mathbf{a}, \mathbf{e})$ is a post-announcement equilibrium, then*

$$\tilde{L}(\mathbf{g}^\star, \mathbf{a}, \mathbf{e}) \geq \min_a R(a) \tag{23}$$

If $p_0 > 0$, (23) holds at equality iff (i) $\sigma = 0$, (ii) $\mathbf{g}^\star(a_0^R, .., a_{t-1}^R)$ puts probability 1 on a_t^R for all t, and (iii) $\mathbf{a} = \mathbf{a}^R$. If $p_0 = 0$, (23) holds at equality iff (i) and (ii) hold.

Proof. Let $\boldsymbol{\pi}$ be the strategy representing the distribution over realised inflation $\pi_t = g_t + \epsilon_t$ at each history $h^t \in H^t$ anticipated by the private sector given the equilibrium strategies. Then

$$\tilde{L}(h^t) = \mathbb{E}_{h^t}^{\boldsymbol{\pi}}\left[\sum_{s=t}^{\infty} \beta^{s-t} l\Big(\pi_s, \mathbf{e}(h^t, \pi_t, .., \pi_s)\Big)\right],$$

where $\mathbb{E}_{h^t}^{\boldsymbol{\pi}}$ denotes expectation over inflation $(\pi_s)_{s\geq t}$ induced by $\boldsymbol{\pi}$ conditional on h^t. Let $\bar{\pi}(h^t) = \mathbb{E}_{h^t}^{\boldsymbol{\pi}}[\pi_t]$ be the expected inflation following history h^t.

We begin by showing that $\tilde{L}(h^t) \geq R(\bar{\pi}(h^t))$ for all t, h^t. To this end, let $\tau > t$ and consider the statement $\tilde{L}(h^s) \geq v_{\tau-s}(\bar{\pi}(h^s))$ for all $t \leq s \leq \tau$ and $h^s \in H^s$. The proof is by induction. The initial step is $\tilde{L}(h^\tau) \geq 0 \equiv v_0(\bar{\pi}(h^\tau))$. Suppose the statement holds for $s+1, s+2, .., \tau$. Then

$$\begin{aligned}
\tilde{L}(h^s) &= \mathbb{E}^{\boldsymbol{\pi}}_{h^s}[l(\pi_s, e(h^s, \pi_s)) + \beta \tilde{L}(h^s, \pi_s)] \\
&\geq \mathbb{E}^{\boldsymbol{\pi}}_{h^s}[l(\pi_s, \bar{\pi}(h^s, \pi_s)) + \beta v_{\tau-(s+1)}(\bar{\pi}(h^s, \pi_s))] \\
&\geq \mathbb{E}^{\boldsymbol{\pi}}_{h^s}[v_{\tau-s}(\pi_s)] \geq v_{\tau-s}(\bar{\pi}(h^s)), \qquad (24)
\end{aligned}$$

where the first inequality follows from the inductive hypothesis, the second inequality follows from (16), and the last inequality follows from the convexity of $v_{\tau-s}$ (since T preserves convexity and v_0 is convex) and Jensen's inequality. It follows that $\tilde{L}(h^t) \geq \lim_{\tau\to\infty} v_{\tau-t}(\bar{\pi}(h^s)) = R(\bar{\pi}(h^s))$.

Note that $\tilde{L}(\mathbf{g}^\star, \mathbf{a}, \mathbf{e}) \equiv \tilde{L}(h^0) \geq R(\bar{\pi}(h^0)) \geq \min_a R(a)$. To complete the proof, we consider two cases.

Case 1: Suppose there exists $(\hat{\pi}_t)_{t=0}^\infty$ such that $\boldsymbol{\pi}(\hat{\pi}_0, .., \hat{\pi}_{t-1})$ puts probability 1 on $\hat{\pi}_t$ for all t. Then it must be that $\sigma = 0$ because noise creates a non-degenerate distribution of inflation. The policy maker's payoff is $\tilde{L}(h^0) = \mathcal{L}((\hat{\pi}_t)_{t=0}^\infty)$. By Proposition 1 it satisfies $\tilde{L}(h^0) \geq \min_x R(x)$ and this holds at equality iff $\hat{\pi}_t = a_t^R$ for all t. Finally, if $p_0 > 0$, the degeneracy of $\boldsymbol{\pi}$ implies that $\hat{\pi}_t = a_t = a_t^R$.

Case 2: Suppose there exists a time t and $(\hat{\pi}_0, .., \hat{\pi}_{t-1})$ such that $\boldsymbol{\pi}(\hat{\pi}_0, .., \hat{\pi}_{s-1})$ puts probability 1 on $\hat{\pi}_s$ for all $s < t$ but $\boldsymbol{\pi}(\hat{\pi}_0, .., \hat{\pi}_{t-1})$ is nondegenerate. It suffices to show that $\tilde{L}(h^0) > R(\bar{\pi}(h^0))$. Let $h^s = (\hat{\pi}_0, .., \hat{\pi}_{s-1})$ for all $s \leq t$. Then

$$\begin{aligned}
\tilde{L}(h^t) &\geq \mathbb{E}^{\boldsymbol{\pi}}_{h^t}[l(\pi_t, \bar{\pi}(h^t, \pi_t)) + \beta R(\bar{\pi}(h^t, \pi_t))] \\
&\geq \mathbb{E}^{\boldsymbol{\pi}}_{h^t}[R(\pi_t)] > R(\bar{\pi}(h^t))
\end{aligned}$$

by a similar argument to (24), where the strict inequality follows from the nondegeneracy of $\boldsymbol{\pi}(h^t)$, the strict convexity of R (Lemma 3) and Jensen's inequality. It follows that

$$\begin{aligned}
\tilde{L}(h^t) &= \sum_{s=0}^{t-1} \beta^s l(\hat{\pi}_s, \hat{\pi}_{s+1}) + \beta^t \tilde{L}(h^t) \\
&> \sum_{s=0}^{t-1} \beta^s l(\hat{\pi}_s, \hat{\pi}_{s+1}) + \beta^t R(\bar{\pi}(h^t)) \\
&\geq T^{(t)} R(\bar{\pi}(h^t)) \geq \min_x R(x),
\end{aligned}$$

where $T^{(t)}$ denotes the t-fold composition of the operator T and we have used $TR = R$ (Lemma 3).

□

Proof of Proposition 2

Consider a strategy $\mathbf{g}^R$ for the rational type and expectations $\mathbf{e}$ for the private sector such that

- $\mathbf{e}(a_0^R, .., a_{t-1}^R) = \pi^N$ and $\mathbf{g}^R(a_0^R, .., a_{t-1}^R)$ puts probability 1 on π^N for all t
- $\mathbf{e}(h^t) = a_t^R$ and $\mathbf{g}^R(h^t)$ puts probability 1 on a_t^R for all t and $h^t \in H^t$ with $h^t \neq (a_0^R, .., a_{t-1}^R)$

We now show that $(\mathbf{g}^R, \mathbf{a}^R, \mathbf{e})$ is a post-announcement equilibrium. It is immediate that the expectations $\mathbf{e}$ are rational given $\mathbf{g}^R$ and $\mathbf{a}^R$. In what follows we show that the rational type cannot profitably deviate from $\mathbf{g}^\star$ at any history h^t. Without loss of generality, we suppose the deviating strategy differs from $\mathbf{g}^\star$ at h^t (it may also differ at subsequent histories).[11]

If the rational type deviates from $\mathbf{g}^R$ at any history $h^t \in H^t$, the private sector expects inflation π^N for the rest of the game. Hence, if the deviation results in inflation π_s at each time $s \geq t$, the rational type's loss is

$$\sum_{s=t}^{\infty} \beta^t l(\pi_s, \pi^N) \geq \frac{l(\pi^N, \pi^N)}{1-\beta}. \tag{25}$$

Since this is true for any $(\pi_s)_{s \geq t}$, it follows that the rational type's loss from deviating at any history is at least $\frac{l(\pi^N, \pi^N)}{1-\beta}$.

Following a history $h^t \in H^t$ with $h^t \neq (a_0^R, .., a_{t-1}^R)$, the rational type obtains $\sum_{t=0}^{\infty} \beta^t l(\pi^N, \pi^N)$ if he does not deviate, which is no larger than his deviation loss.

Following a history $(a_0^R, .., a_{t-1}^R)$, the rational type obtains $\sum_{s=t}^{\infty} \beta^{s-t} l(a_s^R, a_{s+1}^R) = R(a_s^R)$ by following $\mathbf{g}^R$. The strict convexity of R (Lemma 3) and $0 < a_t^R < a_0^R$ (Proposition 1) imply that $R(a_t^R) < R(0)$. Lemma 4 implies that $R(0) = \frac{l(0,0)}{1-\beta}$. Hence, Assumption 1 implies that the rational type has no profitable deviation.

We have now shown that $(\mathbf{g}^R, \mathbf{a}^R, \mathbf{e})$ is a post-announcement equilibrium. The average loss is $\tilde{L}(\mathbf{g}^R, \mathbf{a}^R, \mathbf{e}) = R(a_R^0) \equiv \min_a R(a)$. Lemma 5 implies that $\mathbf{a}^R$ is an optimal announcement and the rational type follows the Ramsey plan in any optimal equilibrium. In addition, if $p_0 > 0$, then the announcement of any optimal equilibrium is $\mathbf{a}^R$.

[11] The one-shot deviation principle does not apply due to the unboundedness of the action space.

Parametric restrictions for Assumption 1

The problem $\min_\pi l(\pi, \pi^N)$ has FOC

$$2\left(y^\star - \frac{1}{\kappa}(\pi - \beta\pi^N)\right)\left(-\frac{1}{\kappa}\right) + 2\gamma\pi = 0,$$

which is sufficient because $l(\cdot, \pi^N)$ is strictly convex. Hence, $\pi = \pi^N$ satisfies the above FOC and we obtain

$$\pi^N = y^\star \frac{\kappa}{1 - \beta + \gamma\kappa^2}.$$

Hence,

$$\begin{aligned} l(\pi^N, \pi^N) &= \left(y^\star - \frac{1}{\kappa}(1-\beta)y^\star \frac{\kappa}{1-\beta+\gamma\kappa^2}\right)^2 + \gamma\left(y^\star \frac{\kappa}{1-\beta+\gamma\kappa^2}\right)^2 \\ &= (y^\star)^2\left[\left(1 - \frac{1-\beta}{1-\beta+\gamma\kappa^2}\right)^2 + \gamma\left(\frac{\kappa}{1-\beta+\gamma\kappa^2}\right)^2\right] = (y^\star)^2 \frac{(\gamma\kappa^2)^2 + \gamma\kappa^2}{(1-\beta+\gamma\kappa^2)^2} \end{aligned}$$

Since $l(0,0) = (y^\star)^2$, Assumption 1 holds iff

$$\begin{aligned} \lambda(1+\lambda) &\geq (1 - \beta + \lambda^2)^2 \\ \beta &\geq 1 + \lambda - \sqrt{\lambda(1+\lambda)}, \end{aligned}$$

where $\lambda := \gamma\kappa^2$. For the choice of parameters in Table 1 this corresponds to $\beta \geq 0.56$.

Proof of Lemma 1

Suppose, towards a contradiction, that the rational type plays a_t at every time t. This implies that beliefs are equal to p_0 at every history. Hence, the rational type receives the same continuation loss regardless of realised inflation. The optimality of the rational type's strategy then implies that $g_t = a_t$ minimises $\mathbb{E}[l(g_t, e_t)] = l(g_t, e_t) + \frac{\sigma^2}{\kappa^2}$, where $e_t = a_{t+1}$. The FOC is $a_t = f(a_{t+1})$, where

$$f(e) = \frac{\kappa y^\star + \beta e}{1 + \gamma\kappa^2}$$

It is easily verified that $f(e) > e$ if $e < \pi^N$ and $f(e) = e$ if $e = \pi^N$. The continuity of f and $a_t \to \chi$ imply that $f(\chi) = \chi$, so $\chi = \pi^N$. Hence, $a_0 < \pi^N$ by asssumption. It follows that $a_t < a_{t+1} < \pi^N$ for all t, which contradicts $a_t = f(a_{t+1}) > a_{t+1}$.

Proof of Proposition 3

This is a direct consequence of Lemma 5.

B. More results

Figure 12 shows the expected reputational gains for the same plan underlying Figures 3 and 4.

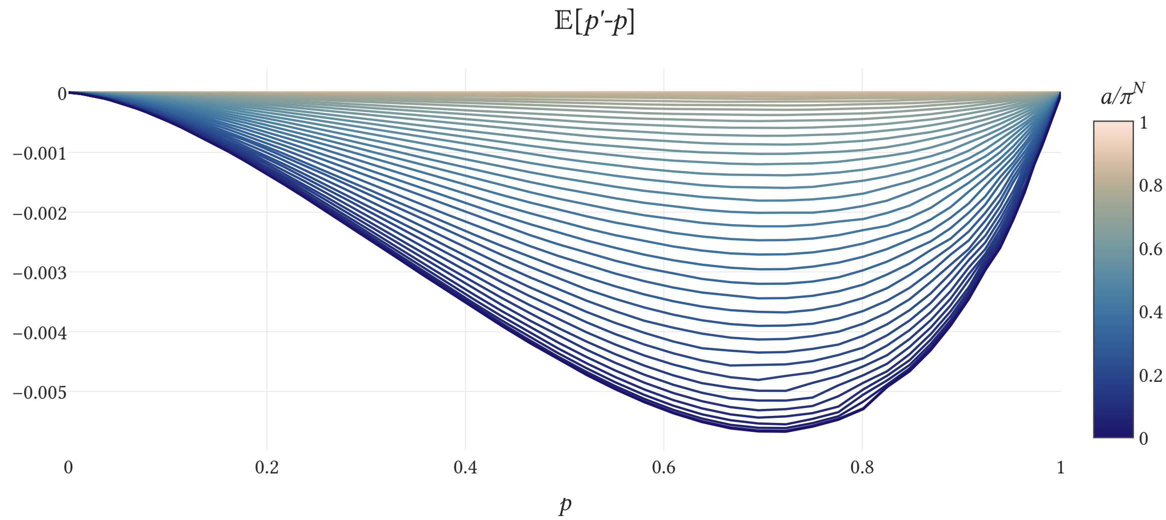

Figure 12: Expected reputation losses

As noted in Observation 1, the policy maker's reputation declines on average: $\mathbb{E}[p']$ is never above p. In particular, reputation is preserved when $p' = p$ in all contingencies, which happens when $p \in \{0, 1\}$ (because no update is possible) or when the current target a equals Nash inflation π^N, so that incentives to create surprise inflation are eliminated. Also, as a consequence of Bayes' rule, changes in reputation are smaller when initial reputation is closer to 0 or 1.

Figure 13 shows the planner's objective function when p_0 is small. For each initial and long-run level of inflation a_0 and χ we plot the minimized loss function $\min_\omega \mathcal{L}(p_0, (a_0, \omega, \chi))$. The minimum is achieved at a point with $a_0 > \chi > 0$ corresponding to the optimal announcement with lowest p_0 from Figure 6.

Figure 14 shows that, at $a_0 \simeq \chi^\star$ (about 0.6 in this example), the planner is indifferent between decay rates, as the decay rate matters more when a_0 is farther away from χ. Starting from the optimal a_0 (about 1.6), it prefers an intermediate decay rate: too slow would negate the incentives from rapidly decaying targets and make the plan target high inflation for too long, but too fast would not create incentives for long as the plan would rapidly become flat. The planner chooses a slope in its targets that is steep enough to boost incentives in the initial periods when inflation targets are above their long-run levels.

Figure 15 shows announcements which maximize the payoff of the *rational* type only.

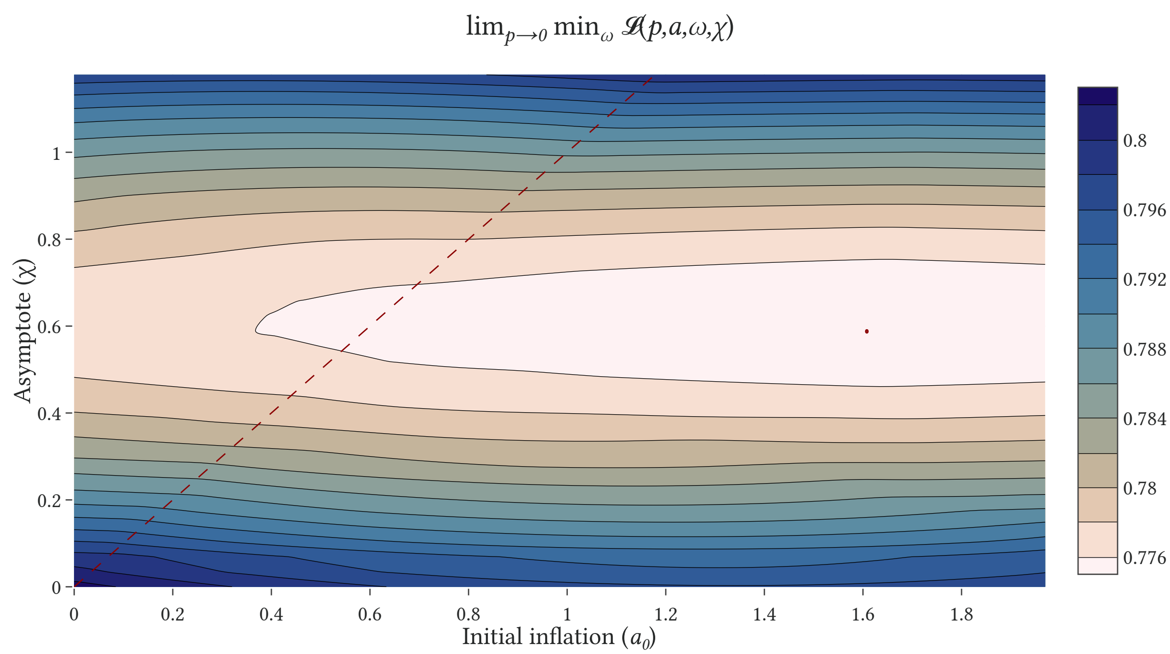

Figure 13: Loss function across announcements

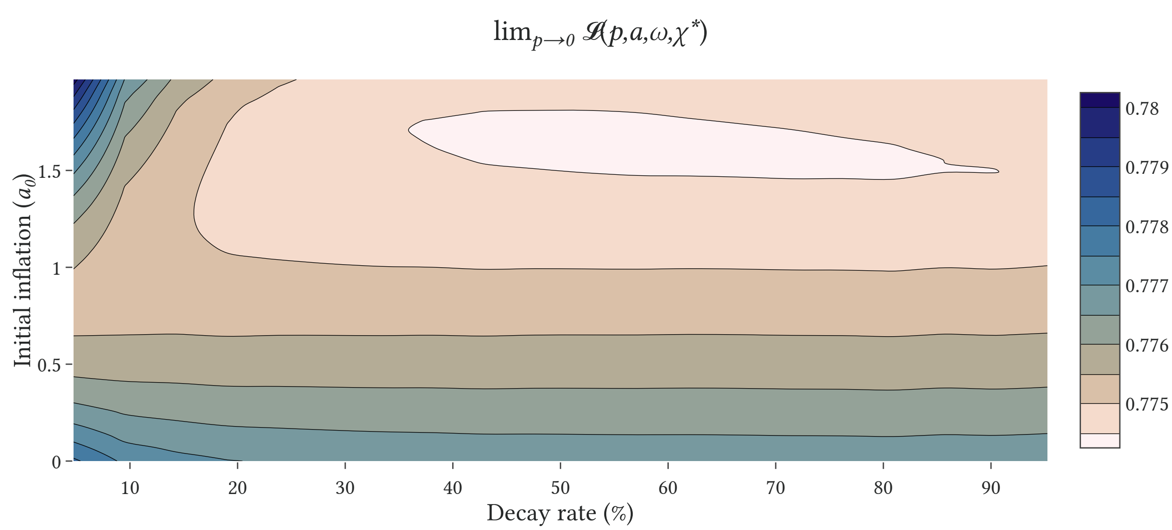

Figure 14: Loss function across announcements

At $p_0 = 1$, any announcement is fully believed by the private sector, regardless of expectations about the behavior of the rational type. The planner sets expectations at their most advantageous level by promising zero inflation throughout. Since the private sector is convinced the policy maker is committed to the announcement, the rational type has a clear incentive to create surprise inflation (illustrating Lemma 1). When $p_0 < 1$, such a deviation incurs reputational losses. Thus, the planner announces plans with positive inflation, especially early on. Positive inflation targets

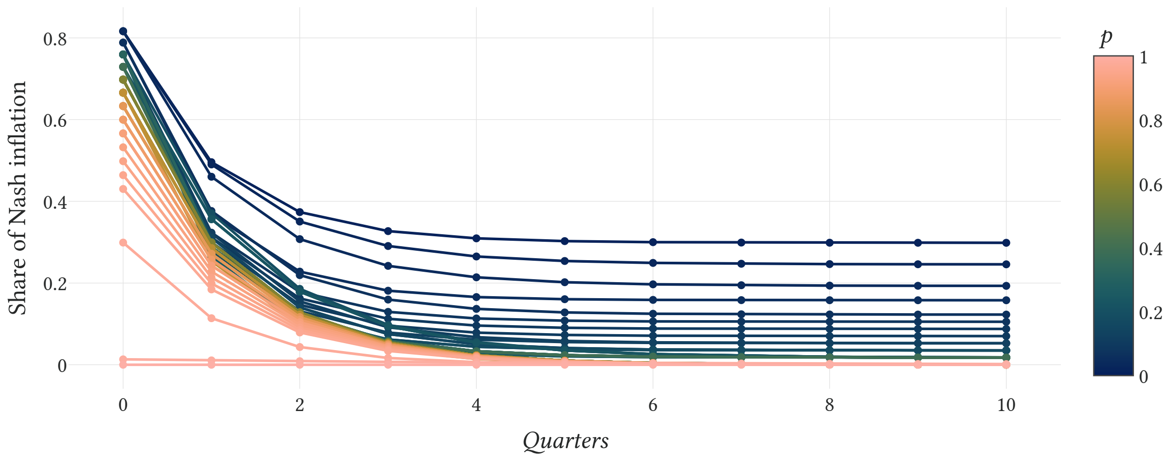

Figure 15: Optimal announcements (for the rational type)

help the incentives of the policy maker to stay closer to the announcement, conserving reputation. As $p_0 \to 0$, the optimal announcements converge to those shown in Figure 6 as the rational type's importance in the objective function grows.

Figure 16 shows the average loss function at the optimal announcement, as function of reputation p and the control shock variance σ. Lower noise is monotonically preferred.

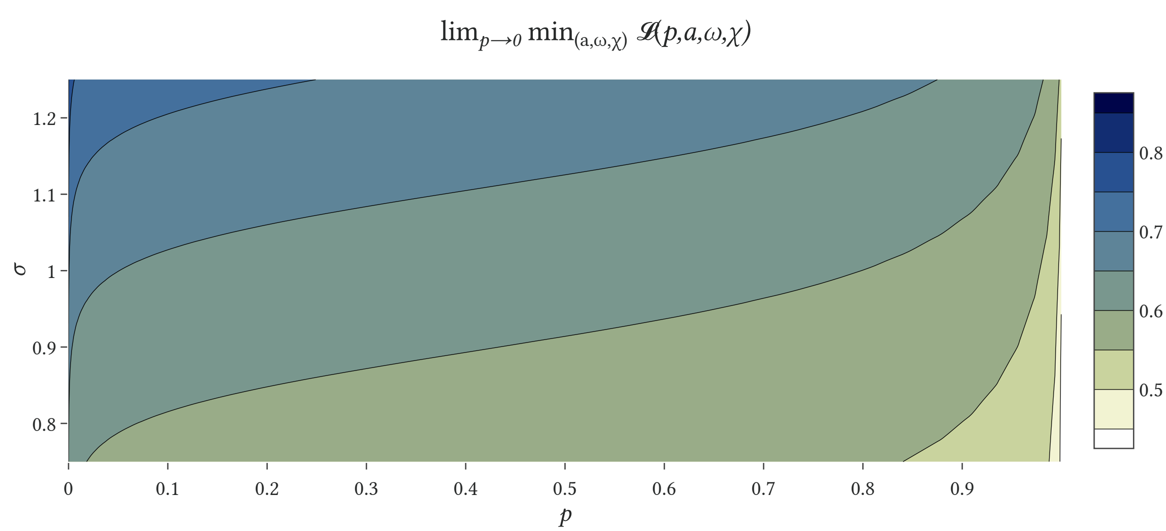

Figure 16: Minimum loss and the control shock variance