

# Risk Aversion in Sovereign Debt and Default\*

Francisco Roch<sup>†</sup>  
UTDT

Francisco Roldán<sup>‡</sup>  
IMF

September 2024

Preliminary: please do not circulate

## Abstract

We study the interaction of risk-sensitive preferences with sovereign default risk. Macro-financial separation, a pervasive property of real business cycles models, breaks in the context of sovereign debt. Risk aversion crucially affects the strength of the threat of autarky which helps sustain sovereign borrowing. We (re)evaluate the welfare effects of access to capital markets, debt dilution, and post-default negotiations. [We find that...]

## JEL Classification

## Keywords

---

\* The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management.

<sup>†</sup>e-mail: [franroch@gmail.com](mailto:franroch@gmail.com)

<sup>‡</sup>e-mail: [froldan@imf.org](mailto:froldan@imf.org)

## INTRODUCTION

What makes governments repay their debts, thereby sustaining sovereign borrowing? The general consensus points to a combination of direct costs, in terms of utility or foregone output, combined with a period of exclusion from international capital markets. Exclusion is costly as it disrupts consumption smoothing. However, most quantitative studies of sovereign debt and default are specified with constant relative risk aversion (CRRA) for the government's preferences, a feature which renders business-cycle volatility in consumption almost irrelevant (Lucas, 1987).

In this paper we study sovereign default risk under risk-sensitive preferences. We find that risk aversion modulates the welfare costs of fluctuations in consumption and thus the strength of the threat of autarky following default. This leads to a failure of macro-financial separation (Tallarini, 2000) and creates a role for risk aversion in the determination of economic outcomes. We then compare the performance and predictions of our model with the standard CRRA case, both calibrated to the same empirical regularities of emerging-market economies.

**Discussion of the Literature** TBW

## 2. MODEL

We consider a small open economy whose government borrows from competitive international lenders on behalf of its citizens. Debt helps frontload consumption and smooth shocks and takes the form of a long-term, non-contingent, but defaultable bond.

**Resources** The economy receives an exogenous endowment  $y_t$ . While the true stochastic process for  $y_t$  is unknown, an approximating model is that  $y_t$  follows an AR(1) in logs,

$$z_t = (1 - \rho_z)\mu_z + \rho_z z_{t-1} + \epsilon_t^z$$

where  $z_t = \log y_t$  and  $\epsilon \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_z^2)$ .

**Government** The government's objective is to maximize the representative consumer's welfare, given implicitly by the following specification of Epstein-Zin or robust *multiplier* preferences

$$w_t = u(c_t) + \beta \mathbb{T}[w_{t+1}]$$

where  $\mathbb{T}[X | \mathcal{F}] = -\frac{1}{\theta} \log(\mathbb{E}[\exp(-\theta X) | \mathcal{F}])$  is a risk-sensitive expectation.

**Assets** The government borrows from international lenders in the form of a defaultable bond which promises to pay a noncontingent stream of geometrically-decaying coupons as in Leland (1998), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012). A bond issued in period  $t$  pays  $(1 - \rho)^{s-1} \kappa$  units of the good in period  $t + s$ , which effectively makes a one-period-old bond a perfect substitute of  $(1 - \rho)$

units of newly-issued debt. The coupon rate  $\kappa = r + \rho$ , where  $r$  is the international risk-free rate, is chosen so that the price of a bond that is expected to never default is  $q^* = 1$ .

With this structure, the government's budget constraint dictates resources available for consumption in case of repayment and if new debt  $i_t = b'_t + (1 - \rho)b_t$  is issued at price  $q_t$

$$c_t + g + \kappa b_t = y(z_t) + q_t(b'_t + (1 - \rho)b_t)$$

where  $g$  are exogenous, constant government purchases.

**Lenders** Financing to the small open economy is provided by a continuum of deep pocketed foreign investors, whose attitudes towards risk are subject to a global shock  $x_t$ . We model their stochastic discount factor following [Vasicek \(1977\)](#) as is often done in models of sovereign default ([Arellano and Ramarayanan, 2012](#); [Bianchi et al., 2018](#)),

$$M(x, z, z') = \exp \left( -r - x \left( \alpha \epsilon' + \frac{1}{2} \alpha^2 \sigma_z^2 \right) \right) \quad (1)$$

where  $\alpha \geq 0$  and  $\epsilon'$  is the innovation in the stochastic process for  $z$ . The creditor risk-premium shock  $x \in \{0, 1\}$  follows a Markov process with transition probabilities  $(\pi_{LH}, \pi_{HL}) \in [0, 1]$ .

**Default** Each period, the government may choose to default on the debt, which triggers temporary exclusion from international capital markets. As in [Bianchi et al. \(2018\)](#) and [Hatchondo et al. \(2022\)](#), we assume that while excluded, the government (and/or the representative agent) suffers a utility cost  $U^D(y)$ . While excluded, the government faces a constant hazard  $\psi$  of reaching a deal with its bondholders, in which case a share  $N(b, x, z)$  of the defaulted bonds become due again. The value of default for the government is then

$$v_D(b, x, z) = u(y(z)) - U^D(y(z)) + \beta \mathbb{T} [1_R \mathcal{V}(N(b, x', z')b, x', z') + (1 - 1_R) v_D(b, x', z') \mid x, z] \quad (2)$$

where  $1_R$  is an indicator function for the event of market reentry and  $\mathcal{V}$  is the value attained by the government when it has access to markets.

At the beginning of each period in which it has access to markets, the government faces a choice to repay the debt or default, so that

$$\mathcal{V}(b, x, z) = \max \left\{ v_R(b, x, z) + \epsilon_R, v_D(b, x, z) + \epsilon_D \right\}$$

where  $(\epsilon_R, \epsilon_D)$  follow independent Type 1 Extreme Value distributions with scale parameter  $\chi$ . As is well-known ([Chatterjee et al., 2018](#); [Dvorkin et al., 2021](#)), this specification leads the distribution of the difference  $\epsilon_R - \epsilon_D$  to be logistic and yields the closed forms for the value function and the ex-post probability of default

$$\begin{aligned} \mathcal{V}(b, x, z) &= \chi \log \left( \exp \left( \frac{1}{\chi} v_R(b, x, z) \right) + \exp \left( \frac{1}{\chi} v_D(b, x, z) \right) \right) \\ \mathcal{P}(b, x, z) &= \frac{\exp \left( \frac{1}{\chi} v_D(b, x, z) \right)}{\exp \left( \frac{1}{\chi} v_R(b, x, z) \right) + \exp \left( \frac{1}{\chi} v_D(b, x, z) \right)} \end{aligned} \quad (3)$$

**Debt issuances** While it remains current on its obligations, the government can issue new debt  $b'$  on the market<sup>1</sup> and attain a value

$$\begin{aligned} v_R(b, x, z) &= \max_{b'} u(c) + \beta \mathbb{E} [\mathcal{V}(b', x', z') \mid x, z] \\ \text{subject to } c + \kappa b &= y(z) + q(b', x, z)(b' - (1 - \rho)b) \end{aligned} \quad (4)$$

**Debt prices** Bonds are priced so that creditors make zero profits in expectation given their stochastic discount factor (1)

$$\begin{aligned} q(b', x, z) &= \mathbb{E} [M(x, z, z')(1 - 1_{D'}) (\kappa + (1 - \rho)q(g_b(b', x', z'), x', z')) + 1_{D'} q_D(b', x', z') \mid z] \\ q_D(b, x, z) &= \mathbb{E} [M(x, z, z') (\psi N(b, x', z') R(N(b, x', z') b, x', z') b + (1 - \psi) q_D(b, x', z')) \mid x, z] \end{aligned} \quad (5)$$

where  $R(b, x, z) = (1 - 1_D)(\kappa + (1 - \rho)q(g_b(b, x, z))) + 1_D q_D(b, x, z)$  is the value of a restructured bond and  $g_b(b, x, z)$  denotes the government's policy function for debt issuance.

### 3. CALIBRATION

We parametrize our model to match salient features of emerging-market economies. Our calibration strategy follows [Hatchondo et al. \(2022\)](#) and [Bianchi et al. \(2018\)](#) closely.

A period in the model refers to a year. We set the risk-free interest rate  $r$  and the government's discount factor  $\beta$  to standard values in this literature. We use data from Mexico, a common reference for quantitative studies of sovereign default. We set the level of government spending  $g$  at 12% of average income, the inverse maturity of debt  $\rho = 0.286$  to match the Macaulay duration of Mexican debt of about 3 years. We assume that any defaults are resolved within a year ( $\psi = 1$ ), which is on the short end of the range of empirical estimates ([Cruces and Trebesch, 2013](#)).

The parameters governing the two exogenous shock processes are taken from [Bianchi et al. \(2018\)](#) to match the behavior of output ( $\rho_z = 0.66, \sigma_z = 0.034$ ) and the frequency with which the global EMBI+, excluding countries in default, is one standard deviation above its mean. This yields one episode of high risk premia every 20 years with an average duration of 1.25 years, or  $(\pi_{HL}, \pi_{LH}) = (0.8, 0.15)$ . For the 1993–2014 period, the episodes are the Tequila crisis in 1994–95, the Russian default in 1998, and the global financial crisis in 2008. On average, the EMBI+ spread was 200 basis points higher during these times than in normal times. For the debt restructuring function  $N(b, x, z)$ , for the time being we assume a constant haircut to match the average recovery of 63%, following [Bianchi et al. \(2018\)](#).

Table 1 summarizes the parameters we set externally.

In our baseline calibration the utility function displays unit elasticity of intertemporal substitution, so that  $u(c) = \log(c)$ . The utility cost of default is given by  $U^D(y) = \max\{0, \lambda_0 + \lambda_1 y\}$ . These two parameters help match the average levels of debt and spread in the data ([Hatchondo and Martinez, 2017](#)).

<sup>1</sup>In the numerical solution of the model, we discretize the set of choices  $\mathcal{B}$  for debt and include Extreme Value Type 1 preference shocks, similarly to the default choice. This helps smooth out the choice of debt by making the probability of choosing level  $b'$  proportional to  $\exp\left(\frac{1}{\chi_b} (u(c) + \beta \mathbb{E} [\mathcal{V}(b', x', z') \mid x, z])\right)$ . Similarly to the default decision, we choose  $\chi_b$  as small as possible to ensure that it does not affect the equilibrium.

	Parameter	Value
Sovereign's discount factor	$\beta$	0.92
Income autocorrelation coefficient	$\rho_z$	0.66
Standard deviation of $y_t$	$\sigma_z$	0.034
Government consumption	$g$	0.12
Preference shock scale parameter	$\chi$	0.01
Risk-free interest rate	$r$	0.04
Duration of debt	$\rho$	0.286
Reentry probability	$\psi$	1
Haircut upon default	$\hbar$	0.37

Table 1: Externally chosen parameters

For contrast, we also consider an alternative parametrization with expected CRRA utility, where we set  $\theta = 0$  and let the EIS parameter  $\gamma$  adjust. In this version of the model, we only vary the parameters that are calibrated internally and target the same moments in the data as the baseline version.

We calibrate the cost of default parameters  $(\lambda_0, \lambda_1)$ , the government's risk aversion  $\theta$  (or  $\gamma$ ), and the risk-premium  $\alpha$  to match four moments in the Mexican data. Our targets are the average level of spreads (240 bps) and debt-to-GDP ratio (45%), a relative volatility of consumption relative to output of unity, and the average increase in spreads during high risk premium episodes (200 bps). These correspond to observations for Mexico in 1993–2014, reported by [Hatchondo et al. \(2022\)](#).

Table 2 contains the values of the parameters that we set by calibration, for both versions of the model.

	Parameter	Baseline	CRRA
Sovereign's risk aversion	$\theta$	7.854	0
Sovereign's EIS	$\gamma$	1	2
Risk-premium shock	$\alpha$	37.79	37.79
Default cost: linear	$\lambda_0$	0.7118	0.7074
Default cost: quadratic	$\lambda_1$	5.402	5.357

Table 2: Calibrated parameters

Table 3 shows that both models succeed in replicating the main features of the data.

### 3.1 *Untargeted moments*

TBW

	Data	Baseline	CRRA
Average spread (bps)	240	243	TBC
$\Delta$ spreads in high risk premium (bps)	200	192	TBC
Relative volatility of consumption (%)	1	1.02	TBC
Debt-to-GDP ratio (%)	43	41.8	TBC

Table 3: Fit of the models

### 3.2 Asset pricing implications

TBW

## 4. QUANTITATIVE ANALYSIS

To study the differential welfare implications of policies, for each model we compute the constant level of consumption  $c^*$  yielding the same utility as the equilibrium, starting at 0 debt and averaging over the ergodic distribution for the exogenous states,

$$c^*(b, x, z) = u^{-1}(\mathcal{V}(b, x, z)(1 - \beta)) \quad (6)$$

### 4.1 Welfare effects of access to markets

### 4.2 Welfare effects of sovereign risk

### 4.3 Welfare effects of debt dilution

### 4.4 The effect of risk aversion on post-default negotiations

## 5. CONCLUDING REMARKS

TBW

## REFERENCES

- ARELLANO, C. AND A. RAMANARAYANAN (2012): “Default and the Maturity Structure in Sovereign Bonds,” *Journal of Political Economy*, 120, 187–232.
- BIANCHI, J., J. C. HATCHONDO, AND L. MARTINEZ (2018): “International Reserves and Rollover Risk,” *American Economic Review*, 108, 2629–70.
- CHATTERJEE, S., D. CORBAE, J.-V. RIOS-RULL, AND K. DEMPSEY (2018): “A Theory of Credit Scoring and the Competitive Pricing of Default Risk,” 2018 Meeting Papers 550, Society for Economic Dynamics.

- CHATTERJEE, S. AND B. EYIGUNGOR (2012): “Maturity, Indebtedness, and Default Risk,” *American Economic Review*, 102, 2674–99.
- CRUCES, J. J. AND C. TREBESCH (2013): “Sovereign Defaults: The Price of Haircuts,” *American Economic Journal: Macroeconomics*, 5, 85–117.
- DVORKIN, M., J. M. SÁNCHEZ, H. SAPRIZA, AND E. YURDAGUL (2021): “Sovereign Debt Restructurings,” *American Economic Journal: Macroeconomics*, 13, 26–77.
- HATCHONDO, J. C. AND L. MARTINEZ (2009): “Long-duration bonds and sovereign defaults,” *Journal of International Economics*, 79, 117–125.
- (2017): “Calibrating the Cost of Defaulting in Models of Sovereign Defaults,” in *Rethinking Fiscal Policy after the Crisis*, Cambridge University Press, 414–440.
- HATCHONDO, J. C., L. MARTINEZ, Y. K. ÖNDER, AND F. ROCH (2022): “Sovereign Cocos,” Working Papers 139, Red Nacional de Investigadores en Economía (RedNIE).
- LELAND, H. E. (1998): “Agency Costs, Risk Management, and Capital Structure,” *Journal of Finance*, 53, 1213–1243.
- LUCAS, R. (1987): *Models of Business Cycles*, Yrjö Jahnsson lectures, Basil Blackwell.
- TALLARINI, T. D. (2000): “Risk-sensitive real business cycles,” *Journal of Monetary Economics*, 45, 507–532.
- VASICEK, O. (1977): “An equilibrium characterization of the term structure,” *Journal of Financial Economics*, 5, 177–188.