The Aggregate-Demand Doom Loop: Precautionary Motives and the Welfare Costs of Sovereign Risk*

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Abstract

I examine the role of households' precautionary savings motive in amplifying and propagating movements in sovereign spreads. I study this mechanism in a model where the government of a small open economy borrows from foreigners but the debt is then partially held by heterogeneous domestic savers. In a calibration to Spain in the 2000s, I find that default risk accounts for about half of the output contraction. More generally, sovereign risk exacerbates volatility in consumption over time and across agents, creating large and unequal welfare costs even if default does not materialize.

JEL Classification E21, F34, H63

Keywords Sovereign risk, default, aggregate demand, precautionary motives, heterogeneous agents

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Introduction

Sovereign debt crises coincide with pronounced output contractions. With sovereign debt levels and yields sharply increasing around the world after the Covid-19 crisis and the return of inflation, there is renewed interest in quantifying the welfare consequences of sovereign risk. I take the Eurozone crisis as an example, when spreads increased steeply for many European countries such as Spain. Spain did not ultimately default on its debts, yet output fell by about 10% of its 2008 peak, unemployment doubled, and private consumption contracted by up to 15% of the pre-crisis maximum. The Spanish experience was not unique. I document that increases in sovereign spreads are associated with large declines in output and consumption in a panel of EU countries during the crisis.

In this paper I develop a model of sovereign debt that rationalizes these large output and consumption contractions in response to sovereign risk. The mechanism relies on the interaction between default risk and the precautionary savings motive of households. When economic conditions worsen and sovereign default becomes more likely, households find it optimal to cut consumption in favor of higher savings, which leads to low aggregate spending. As wages are downwardly rigid, a recession follows. The model then generates a vicious cycle of high spreads and demand shortages causing further increases in debt and spreads. This 'aggregate-demand doom loop' amplifies recessions when sovereign risk emerges. Even if default does not materialize, its possibility endogenously exacerbates the volatility of consumption and output, resulting in large welfare costs of sovereign risk. I calibrate the model to Spain to quantify this amplification mechanism.

The model illustrates how prolonged periods of heightened sovereign risk carry costs in terms of output and employment through the feedback loop between spreads and demand. A lesson for policy is that mechanisms that avoid uncertainty about debt sustainability and delays could enable significant welfare gains, in addition to the better-understood gains from avoiding delays in renegotiation once default has happened or a restructuring is underway (Martinez et al., 2023).

The contribution of this paper relative to the literature is to incorporate private domestic demand conditions to the analysis of sovereign debt sustainability. While there is work on the effect of fiscal policy through the demand side and the tradeoff between stimulus and austerity, most studies assume either households effectively in financial autarky or one-sector models in which all output can be exported, rendering (private) aggregate demand either exogenous or irrelevant. As a result, they cannot account for the role of households' savings decisions in the unfolding of a crisis. In contrast, by explicitly taking households and the government as separate actors, I describe how their interactions can create events in which aggregate spending reacts to the anticipation of the government's actions differently than if those decisions were made in a coordinated way.

Apart from the fiscal policy tradeoff, another line of research highlights bank lending and investments as channels through which sovereign risk affects the economy. In this case sovereign risk is harmful because it affects capital accumulation and the composition of investment, not through the impact of investment on aggregate demand. This mechanism is a complement to the one I emphasize as both rely on de-risking portfolios: while in the investment-channel story agents tilt investment towards safer assets in response to sovereign risk, in this model agents seek to increase (safe) savings at the expense of current consumption. While I leave open the question of how important the investment contraction is for aggregate demand by focusing on a model without capital, I find that the aggregate-demand doom loop generates large welfare costs of sovereign risk, even if default is not ultimately triggered.

I consider a small open economy in which heterogeneous households, subject to uninsurable idiosyncratic income risk, decide both their savings and their exposure to government debt. This Bewley setup results in a wealth distribution which interacts with the government's decision to repay its debts. Rigidities in nominal wage setting combined with a currency peg (the model's representation of the Euro) create an aggregate-demand externality that transmits insufficient spending into increases in unemployment and contractions in output (Schmitt-Grohé and Uribe, 2016). Finally, when the government defaults, it bears a cost in terms of lost TFP and exclusion from capital markets, as is common in the literature. In addition to this effect, as in equilibrium some of the debt is held domestically, defaults redistribute wealth.

These costs of default, which affect the government's repayment strategy, are anticipated by households and induce them to increase their savings during crises. For households, the TFP costs of defaults faced by the government act as volatility in future income, which boosts their precautionary motive. On the other hand, the type of redistribution induced by defaults tends to be progressive: it creates capital losses for holders of debt (who tend to be richer) and lowers tax burdens for everyone. In other words, defaults transfer from agents with low to high marginal propensity to consume (MPC). However, during a crisis, when default is only a potential event in the future, such redistribution is only expected. This reverses the identity of low and high MPC agents, as unconstrained savers react equally to present and future transfers, while constrained agents only react to current income. Section 3 provides more insight into these forces, which underpin the quantitative analysis, by studying two highly stylized models.

To quantify the welfare costs of sovereign risk, I calibrate the model to Spain in the 2000s. In addition to standard moments in the sovereign debt literature, I target the private wealth-to-GDP ratio as well as the Gini index for wealth and the share of sovereign debt held domestically. This ensures that the distributional effects captured by the model are disciplined by the Spanish microdata. I then simulate the model to obtain episodes which resemble the Spanish crisis. I find that the presence of sovereign risk worsens the recession, doubling the output contraction relative to a model in which sovereign risk is absent. Moreover, the presence of sovereign risk amplifies the unconditional volatility of aggregate consumption. This extra volatility is costly for the economy: a move from the ergodic mean of the benchmark model to that of a world in which defaults are impossible is worth about 5.2% of permanent consumption to the average household. The same number grows to about 10% at the height of a crisis. These simple comparisons overstate gains by not including a transition period and by shifting individual agents wealth levels. I also compute the welfare gains of removing default risk, including the transition and keeping agents at their current levels of wealth, and find that the episodes of crisis are extremely costly. At the beginning of a crisis, the average household would give about 3.75% of permanent consumption to ban defaults.

While the main argument of this paper goes through in a representative-agent version of the model, many of the effects of default which underpin the amplification channel are redistributive in nature. This makes inequality quantitatively important, both because of the differential impact of default risk on different agents and because the differential responses of those agents shape the aggregate response. I find that welfare costs during crises are highly unequal and regressive as wealth dampens the impact of ths crisis. The bottom 10% of the distribution experiences a drop in its certainty-equivalent consumption that is on average twice as a large as that experienced by the top 10% during a typical simulated crisis period. Furthermore, the reaction of the consumption of the median household (an approximation of what the representative-agent version of the model would capture) is about 15% weaker than that of aggregate consumption in those same episodes, implying that heterogeneity contributes to about a sixth of the direct aggregate-demand impulse.

Finally, the model ties the presence of sovereign risk to a high volatility of consumption relative to income. Standard models of sovereign debt share this feature on the surface: the real interest rate is countercyclical, which makes consumption procyclical. However, these models refer to the government's borrowing rate, which is affected by default risk, in contexts in which the government chooses private agents' consumption. In a model in which households choose their own consumption, it is important that they face an interest rate with similar features to the data counterpart (although the model would generate even stronger amplification if the interest rate faced by households was countercyclical). During the crisis period, Spanish households were managing an aggregate net worth position of about 100% of GDP (see Figure 2). While private borrowing rates do tend to correlate with the rate on government securities, this is typically not true of saving rates (see Arnold and de Vries-van Ewijk, 2014; Martínez Pagés, 2017, and Figure 26 in the Appendix). The claim that only borrowing rates (in extremis, only the government borrowing rate) matter is therefore problematic. The household's portfolio problem, finally, allows the model to endogenously decide which interest rate is important.

Discussion of the Literature This paper relates to several strands of literature. I build on canonical models of sovereign debt (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008) by considering a benevolent government borrowing without commitment from international creditors. At the core of this literature is the issue of internal costs of default which sustain sovereign borrowing in the first place. From these papers I take the shape and size of default costs, which are exogenous in my model. Mendoza and Yue (2012) argue that domestic firms lose access to some imported inputs after a default, which reduces aggregate productivity. Others such as Gennaioli et al. (2014), Pérez (2018), and Mallucci (2015) argue domestically-held debt disrupts financial intermediation after a default. While in these papers, households save and provide deposits to the financial sector, they are one-sector models with a law of one price and effectively abstract from the aggregate demand effects I emphasize.

I also build on models in which nominal rigidities in wage setting combined with an exchange rate peg create an aggregate demand externality (Schmitt-Grohé and Uribe, 2016, and a large literature). Anzoategui (2020) and Bianchi et al. (2019) combine wage rigidities and default risk to consider policy tradeoffs when austerity depresses aggregate demand but endogenously decreases the probability of a debt crisis. They ask whether nominal rigidities can overturn the result in Cuadra et al. (2010) that lack of commitment and sovereign default risk induces governments to follow procyclical fiscal policies. Bianchi and Sosa-Padilla (2023) show that nominal rigidities create an extra motive for accumulating foreign reserves when default risk is present. Also within this class of models, Bianchi and Mondragon (2021) argue that a fixed exchange rate increases the scope for rollover crises. In a model of a monetary union, de Ferra and Romei (2023) find that sovereign default is deflationary and induces expansionary monetary policy, creating a feedback loop. These papers abstract from the precautionary effects that are at the core of my argument by assuming that domestic households are unable to save.

In a flexible exchange rate context, Arellano et al. (2020) consider a New Keynesian small open economy with an aggregate demand externality where the government chooses its fiscal and default policy. They focus on the case when the Central Bank follows a Taylor rule and the economy endogenously undergoes a real devaluation at the time of default. A similar aggregate demand channel of default is present in that model, but the Central Bank's reaction to it, critically absent in the case of the Eurozone crisis, significantly affects the way it manifests in the economy.

Philippon and Roldán (2018) study the optimal sovereign deleveraging plan in a related but much more stylized setting. They find that the direct contractionary impact of austerity negates the gains from deleveraging in a crisis and argue for a gradual plan. Relative to this work, I investigate in detail the aggregate-demand doom loop in a positive, quantitative way. Corsetti et al. (2013) also present the possibility of expansionary austerity in a setting with equilibrium multiplicity. Romei (2015) considers the distributional impact of different speeds of fiscal consolidation in the absence of aggregate demand effects.

Some studies, like Bocola (2016), Arellano et al. (2017), Arellano et al. (2018), or Balke (2017), explicitly consider anticipation effects in investment from sovereign risk, endogenizing the correlation of interest rates for government borrowing and investment or working capital loans assumed by Neumeyer and Perri (2005). In these papers, when the probability of default increases, banks attach a higher value to safe assets, driving up borrowing costs for firms. Investment drops which depresses growth in a complementary way to the one emphasized here. Because it works through the supply side of the economy, this mechanism cannot by itself account for the savings pattern of households in the crisis. Moreover, this mechanism requires that banks be unable to raise equity, which is correct in the short run but less likely as time passes. I take the opposite stand that the financial sector acts as a veil for the nonfinancial private sector. This also highlights inequality within the private sector as a driver of the output response to sovereign risk.

Part of how sovereign risk affects demand relates to redistribution. In this sense, I relate to models such as Eggertsson and Krugman (2012), Auclert (2017), or Korinek and Simsek (2016), where shocks contract demand because they redistribute from high-MPC to low-MPC agents. This paper features this idea prominently, except that the timing of transfers reverses the identities of low- and high-MPC agents (see Section 3.2).

This paper also relates to studies in which sovereign debt policy responds to distributional concerns, as has been emphasized since Woodford (1990). While distributional concerns are featured in its objective function, the government in my model does not issue debt to help domestic agents, who can save in the international risk-free bond, with their self-insurance (as in Aiyagari and McGrattan, 1998; Shin, 2006). D'Erasmo and Mendoza (2016) build a heterogeneous-agents model of sovereign default and find that levels of debt like those of present day Spain suggest a government with a bias towards favoring its creditors. Ferrière (2016), Ferrière and Navarro (2018), and Deng (2021) argue for a positive link between progressive taxation on the one hand and incentives to repay sovereign debt and fiscal multipliers on the other. Guembel and Sussman (2009), Andreasen et al. (2011), and Dovis et al. (2016) study political economy considerations in sovereign debt policy.

Finally, the setup with heterogeneous households allows for a clean separation of the debts and assets of the government and the private sector. In canonical models of sovereign debt, allowing households access to risk-free borrowing and saving unravels the equilibrium. The reason is simple: if the government has access to lump-sum taxes and the representative household can commit to repay loans, then the government can use its tax policy to effectively have the household borrow on its behalf at the risk-free rate. This has naturally led researchers to study models in which the private sector's financial choices are constrained. An alternative is to constrain the tax instruments at the government's disposal. In my model, even though the government can collect lump-sum taxes and agents can save, it cannot make those taxes agent-specific. This provides a natural constraint on the government's ability to sidestep its lack of commitment.

Layout The remainder of the paper is organized as follows. Section 2 presents some motivating evidence and Section 3 builds intuition by analyzing two simple models. Section 4 describes the quantitative model while Section 5 defines the equilibrium and clarifies the inner workings of the model. Section 6 discusses the calibration and Section 7 summarizes results from the model solution. Section 8 focuses on crises and presents the main results. Finally, Section 9 concludes.

2. Motivating Evidence

Figure 1 plots total GDP and households' consumption for Spain in the 2000s. To show each series in as raw a form as possible, I plot them relative to the value at the start of 2008. Output and consumption strongly contract during the crisis years. Moreover, consumption contracts more than output as the crisis unfolds. Comparing the trough of the crisis to 2010 to isolate the effect of sovereign risk as much as possible, the output and consumption contractions are of the order of 5% and 9%.

The case of Spain in the Eurozone crisis is of particular interest, as a default did not actually happen even though full repayment by the government was uncertain during the period: Figure 1 reveals that a 10-year Spanish government bond paid a significant interest rate spread over a comparable German Bund, peaking at about 500 basis points in late 2012. Figure 23 shows that the share of firms reporting insufficient demand as the main factor limiting their production increased significantly during the crisis.

Moreover, as Figure 2 reveals, Spanish households increased their net worth during the crisis

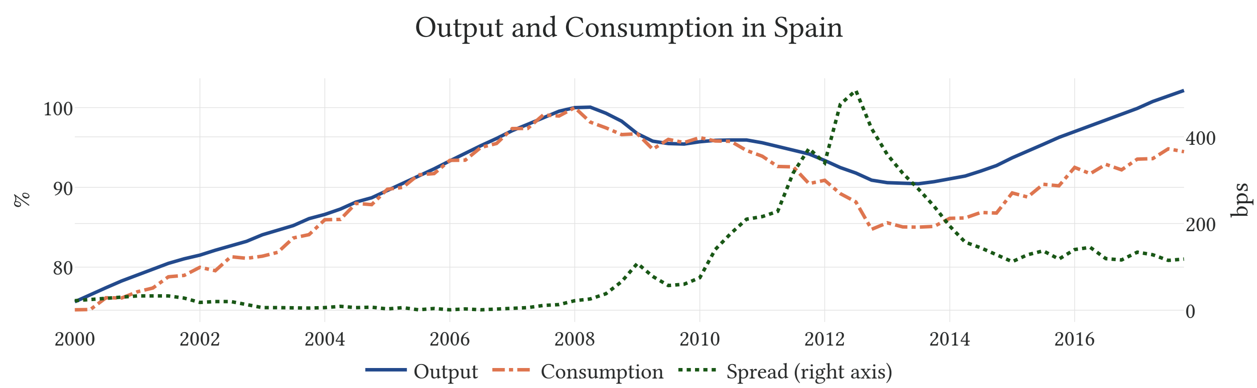

Figure 1: Spanish Output and Consumption in the 2000s

Source: Eurostat

mainly by accumulating more assets. While these are only aggregate, descriptive data, the pattern is consistent with a larger propensity to save when sovereign risk is present. Figure 24 in Appendix C shows that this pattern is also present in levels.

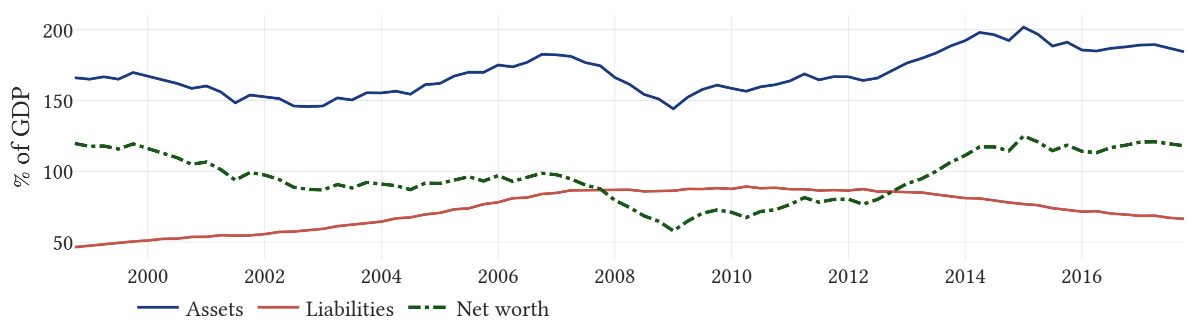

Figure 2: Net Worth of Spanish Households

Source: Eurostat

Finally, the patterns emphasized for Spain in Figure 1 are not a particular feature of the Spanish experience. Using quarterly data for 11 European countries between 2010 and 2013, I consider equations of the type

$$Q_{jt} = \beta Spread_{jt} + \gamma X_{jt} + \mu_j + \delta_t + \epsilon_{jt} \tag{1}$$

for $Q_{jt} = \log Y_{jt}, \log C_{jt}$, where the X_{jt}'s are controls and μ_j and δ_t are country and time fixed effects.

Table 1: Correlation of Spreads and Macroeconomic Outcomes

	$\log Y_t$		$\log C_t$		$\log C_t$	
	(1)	(2)	(3)	(4)	(5)	(6)
Spread_t	-0.007	-0.006	-0.014	-0.009	-0.007	-0.004
	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)
B_t/Y_t		-0.001		-0.002		-0.002
		(0.000)		(0.000)		(0.000)
$\log Y_t$					0.995	0.807
					(0.091)	(0.067)
Country + Time FE	✓	✓	✓	✓	✓	✓
N	143	143	143	143	143	143
Within-R^2	0.274	0.325	0.420	0.677	0.715	0.857

Standard errors in parentheses.

Table 1 summarizes the estimation of equation (1). It shows that countries which saw larger increases in their spreads also saw larger contractions in output and consumption. The coefficients in the first four columns mean that a typical country experiencing the average increase in spreads (about 200bps) saw a fall in output of between 1.2% and 1.4% and a fall in consumption of between 1.8% and 2.7%. The two last columns informally test that the consumption coefficient is indeed larger than the output coefficient, by including output in the consumption regression.

3. Two Minimal Models

I begin by considering two stylized environments. In the first model, expectations of income losses in case of default depress current aggregate demand. In the second model, expected redistribution (e.g. from default with domestic holdings of sovereign debt) also reduces demand. Both these effects are important drivers of the quantitative results later in the paper.

3.1 The costs of default costs

Consider a small open economy with a representative agent who lives for two periods and receives stochastic endowments y_t^T of a tradable good. The representative agent can save $s_1 \geq 0$ in the first period. In the second period, some legacy debt d becomes due to foreigners. If the government fails to repay, output falls by a factor of Δ. Consumption is given by

$$c_2 = \begin{cases} y_2^T - d + s_1 & \text{if the government repays} \\ y_2^T(1-\Delta) + s_1 & \text{if the government defaults} \end{cases}$$

To maximize the representative agent's utility, the government defaults if and only if $y_2^T < d/\Delta$.

In the first period, a nontraded good can be produced with labor, subject to wage rigidities, according to the production function $y_1^N = F(h_1) = h_1^\alpha$, where $\alpha \leq 1$. Consumption is a standard CES aggregate of traded and nontraded goods, so that the prices p_N, p_T satisfy

$$c_N = \left(\frac{\varpi}{1-\varpi}\frac{p_T}{p_N}\right)^{\frac{1}{1+\eta}} c_T$$

where c_T, c_N are quantities consumed. I normalize $p_T = 1$.

This first-order condition, combined with firm profit maximization, implies a labor demand equation

$$h = \mathcal{H}(c_T, w) = \left(\frac{\varpi}{1-\varpi}\frac{\alpha}{w}\right)^{\frac{1}{1+\alpha\eta}} c_T^{1+\eta} \tag{2}$$

I assume nominal rigidities in the form of a wage floor $\bar{w}$, so that either $h = 1$ and the wage satisfies (2) or, when the constraint binds, $w = \bar{w}$ and $h = \mathcal{H}(c_T, \bar{w})$. Crucially, equilibrium employment is an increasing function of traded consumption. A planner would solve

$$\begin{aligned} &\max_{s_1}\, u(c_T, F(h)) + \beta\mathbb{E}\left[u\left(\max\{y_2 - d, y_2(1-\Delta)\} + s_1\right)\right] \\ &\text{subject to } c_T + \frac{s_1}{1+r} = y_1 \\ &h = \min\left\{1, \mathcal{H}(c_T, \bar{w})\right\} \end{aligned} \tag{3}$$

The first-order condition for problem (3) is the planner's Euler equation

$$u_T'(c_T, F(h)) + \mathcal{H}_c'(c_T, \bar{w})\mu = \beta(1+r)\mathbb{E}\left[u'\left(y_2 + s - \min\left\{d, y_2\Delta\right\}\right)\right]$$

where $\mu = u'_N(c_T, F(h))F'(h)$ is the multiplier on the wage floor constraint. The default cost Δ and the debt level d both enter this Euler equation increasing the marginal value of future consumption. An increase in either of them boosts precautionary savings in the first period. The planner, however, understands that more savings decrease labor demand. In a decentralized equilibrium, the household takes as given employment h, which satisfies the constraint only in equilibrium.

Figure 3 summarizes the effect of legacy debt d on the equilibrium, both for the planner and in the decentralized case. It is clear that sovereign risk increases the marginal value of future consumption and consequently boosts the household's precautionary motive, inducing more savings. The planner internalizes the aggregate-demand doom loop and chooses higher consumption.

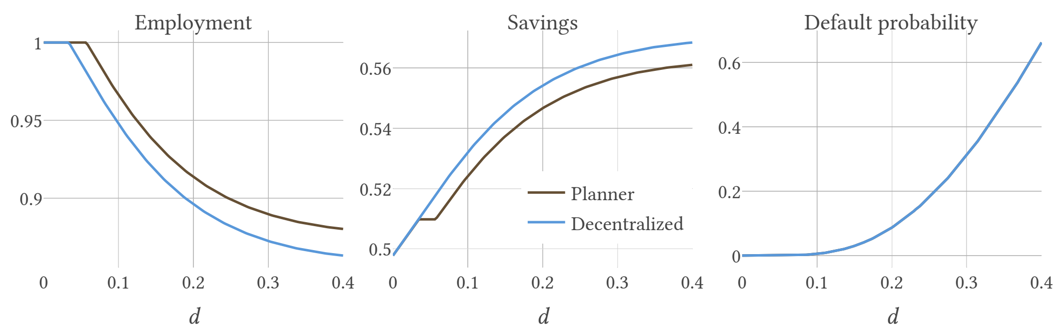

Figure 3: Anticipation of TFP Costs of Default

While it is true that more debt affects aggregate demand even in period 1 by reducing expected future consumption, Figure 16 in the Appendix shows that the results are maintained if all debt is held domestically (i.e. if debt payments do not subtract from consumption).

3.1.1 *Which interest rate?*

If the planner can repurchase the debt instead of having the household save at the risk-free rate, the Euler equation becomes

$$u'_T(c_T, F(h)) + \mathcal{H}'_c(c_T, \bar{w})\mu = \beta \frac{\mathbb{E}\left[u'(y_2 + s_1 - \min\{d, y_2\Delta\})\right]}{q(d - s_1, \Delta) + s_1 q_d(d - s_1, \Delta)}$$

where $q(d, \Delta) = \frac{1 - \mathbb{P}(y_2 < d/\Delta)}{1+r}$ is the price of debt in the first period and $q_d(d, \Delta)$ is its derivative with respect to indebtedness. In this case increases in d (or Δ) still push up the relevant marginal utility of future consumption. Increases in Δ now also push up the debt price, muting the effect. Increases in initial debt d push down the debt price, magnifying the effect of sovereign risk on desired savings.

However, for a given change in expected marginal utility, because debt repurchases push up prices, the reaction of current consumption is muted. The question becomes how much can current savings affect the total stock of debt and future default incentives. An exhaustive discussion of these effects, along with their implications for the ex-ante optimal default cost, requires a less stylized setting and is therefore left as an open question. In the quantitative model, I focus on the reaction of households to sovereign risk itself. The question of whether the government would like to repurchase its debt in such a case is also left as an open question.

3.2 *Propensities to consume and the anticipation of redistribution*

Consider now a closed economy populated by a measure χ of hand-to-mouth agents and $1-\chi$ of 'savers,' each endowed with one unit of labor in each period. The only good is produced with labor with constant returns to scale, $y = h$.

In the second period prices are fully flexible but with probability π there is a transfer of size k from savers to hand-to-mouth agents. In this model the transfer occurs for exogenous reasons but later it will happen as a consequence of default. Consumptions in the second period are

$$(c_2^s, c_2^h) = \begin{cases} (1-k, 1+k) & \text{with probability } \pi \\ (1,1) & \text{with probability } 1-\pi \end{cases}$$

In the first period, all prices are fixed. The wage rate is w, the price of the good is normalized to 1. Savers trade risk-free securities in zero net supply at a real interest rate r. Consumption of savers satisfies the Euler equation

$$u'(c_1^s) = \beta(1+r)\left[\pi u'(1-k) + (1-\pi)u'(1)\right]$$

Assuming CARA utility with absolute risk aversion γ (as in Philippon and Roldán, 2018), the first-period consumption of savers is

$$c_1^s = 1 - \frac{1}{\gamma}\log(\beta(1+r)) - \frac{1}{\gamma}\log\left(1-\pi+\pi e^{\gamma k}\right)$$

The size of the potential transfer k unambiguously reduces savers' consumption. The size of the effect is also increasing in the probability π. The effect of risk aversion parameter is a little more complicated. It both amplifies the effect (γ appears multiplying k inside the exponential) and dampens it. This is because γ controls both risk aversion and the elasticity of intertemporal substitution. Risk aversion makes savers dislike differences in consumption across future states and induces more savings. But it also makes current consumption a worse substitute of future consumption, which reduces the incentive to save (it also reduces the propensity to save in response to interest rates).

Hand-to-mouth agents' consumption, on the other hand, is entirely dictated by their budget constraint, $c_1^h = wh$. For markets to clear, output must equal the total demand of both types, $y = \chi c_1^h + (1-\chi)c_1^s$, or, using that $y = h$,

$$y = \frac{1-\chi}{1-\chi w} c_1^s$$

If nominal rigidities prevent $w \leq \bar{w}$ once more, the presence of hand-to-mouth agents makes output a multiple of the consumption of savers. Figure 4 shows output in percent deviations from a no-transfer benchmark. It illustrates how expected redistribution, both its probability π and its expected size k, depresses saver demand and output.

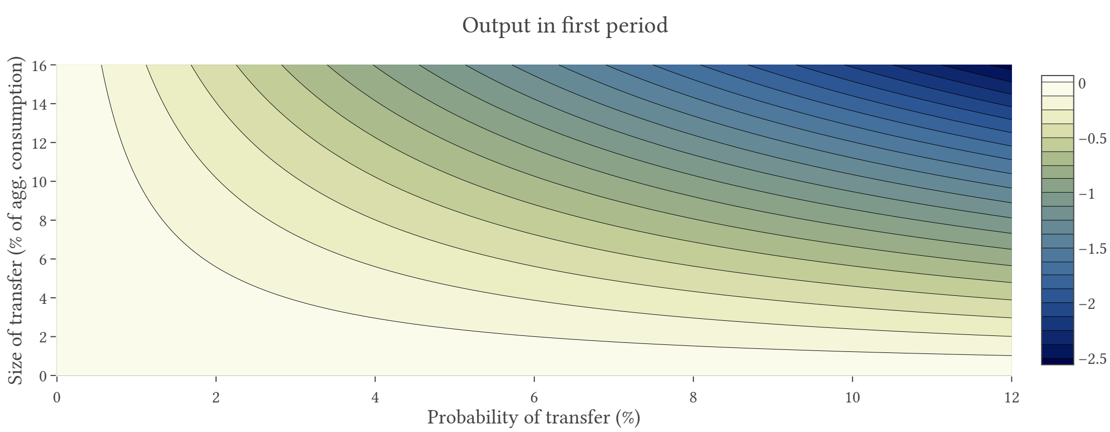

Figure 4: Anticipation of Redistribution in Case of Default

This model illustrates how a potential or future transfer from unconstrained to constrained agents has the opposite effect on the economy than an actual or current one. In this extreme example, hand-to-mouth agents have an MPC of 1 out of current income but of 0 out of future income. Savers, on the other hand, have a small but positive MPC out of income at all dates and are unaffected by the timing of transfers.

4. Quantitative Model

The response of the economy to sovereign risk depends on a variety of endogenous objects: the amount of debt outstanding when spreads increase, its distribution across foreign and domestic agents (as well as among domestic agents), the distribution of MPCs for various current and future transfers. The model presented here aims to provide the necessary elements for a quantitative evaluation of the Spanish crisis.

I consider a small open economy populated by a continuum of heterogeneous households and firms that produce tradable and nontradable goods. A government runs an exogenous, estimated fiscal rule for spending and debt issuance but chooses between default and repayment with discretion. There are incomplete markets and only two assets are traded: a one-period, risk-free private security and a long-term, noncontingent, defaultable government bond.

4.1 Households

There is a continuum of heterogeneous households who differ in the realization of an uninsurable idiosyncratic shock to their effective labor supply, ϵ, as well as in their asset holdings. Let a and b denote holdings of the risk-free asset and of government debt, respectively. Households are limited in their ability to hold negative positions in these assets: it is impossible to short the government, and there is an ad-hoc lower bound $\bar{a}$ on the risk-free asset. Respecting these restrictions, both assets trade at prices q^h and q^g.

Households value the consumption of traded and nontraded goods according to a CES aggregator

$$c = \left[\varpi^{\frac{1}{\eta}} c_N^{\frac{\eta-1}{\eta}} + (1-\varpi)^{\frac{1}{\eta}} c_T^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where η is the elasticity of substitution between both goods. I assume an inelastic labor supply. Households have Epstein-Zin preferences over streams of consumption represented by the value function

$$v_t^{\frac{\psi-1}{\psi}} = (1-\beta)c_t^{\frac{\psi-1}{\psi}} + \beta\mathbb{E}_t\left[v_{t+1}{}^{1-\gamma}\right]^{\frac{\psi-1}{\psi(1-\gamma)}}$$

where β is the discount factor, γ is the coefficient of relative risk aversion and ψ is the inverse elasticity of intertemporal substitution.

In period t, households observe the aggregate state of the economy $\mathbf{S}_t = (B_t, \lambda_t, x_t, d_t, z_t)$, comprised of total government debt outstanding B_t, the current distribution of households over their idiosyncratic states λ_t, the current value of a shock to sovereign spreads x_t, the current state of the country in international credit markets d_t, and the current level of a productivity shock z_t. In equilibrium, this information is enough to recover all aggregate variables, including the relative price of nontraded goods $p_N(\mathbf{S}_t)$, the current wage rate $w(\mathbf{S}_t)$ and employment $L(\mathbf{S}_t)$, as well as the price of government debt $q^g(\mathbf{S}_t)$, lump-sum taxes $T(\mathbf{S}_t)$, firms' profits $\Pi(\mathbf{S}_t)$, and the price of consumption (CPI)

$$p_C(\mathbf{S}_t) = \left[\varpi p_N(\mathbf{S}_t)^{1-\eta} + (1-\varpi)\right]^{\frac{1}{1-\eta}}$$

where the price of traded goods is normalized to $p_T = 1$ (see Section 4.6).

Government debt is a long-term asset, denominated in the traded good, which promises an infinite stream of geometrically decaying coupon payments, as in Leland (1998), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012). While the government is not in default, holders of debt purchased t periods ago receive $\kappa(1-\rho)^{t-1}$. This standard setup makes one unit of debt a perfect substitute of $(1-\rho)$ units of debt issued in the following period. When the government defaults, a haircut $\hbar$ is applied to all outstanding debt and coupon payments are suspended until the government regains market access.

The household's idiosyncratic states are the current level of its labor productivity ϵ_t and the value of its asset portfolio $\omega_t = a'_{t-1} + R^b_{t-1,t}b'_{t-1}$. I adopt the convention that the risk-free asset pays one unit of the traded good while the government bond yields $R^b_{t-1,t}$. Let $\mathbf{s} = (\omega, \epsilon)$ denote the idiosyncratic state vector. Individual labor productivity follows an AR(1) process in logs so $\log \epsilon_{t+1} = \rho_\epsilon \log \epsilon_t + \sigma_\epsilon \nu^\epsilon_{t+1}$, where $\nu^\epsilon_t \overset{iid}{\sim} \mathcal{N}(0,1)$.

Labor supply is inelastic. Because of nominal rigidities (see below), there can be unemployment, in which case I assume proportional rationing so that everyone works the same amount of hours. Thus, a household with current shock ϵ receives labor income equal to $y^L(\mathbf{s}, \mathrm{S}) = w(\mathrm{S})L(\mathrm{S})\epsilon$ at wage w and employment L in state S, of which a fraction τ is paid to the government as labor income taxes. Households also receive income from firm ownership, which I assume to be rebated lump-sum in proportion to the shock ϵ, so $y^\Pi(\mathbf{s}, \mathrm{S}) = \Pi(\mathrm{S})\epsilon$.

The household's problem (4) summarizes the discussion above.

$$
\begin{aligned}
v(\mathbf{s}, \mathrm{S})^{\frac{\psi-1}{\psi}} &= \max_{a',b',c} (1-\beta)c^{\frac{\psi-1}{\psi}} + \beta\mathbb{E}\left[\left(v(a' + R_b(\mathrm{S},\mathrm{S}')b', \epsilon', \mathrm{S}')\right)^{1-\gamma} | \mathbf{s}, \mathrm{S}\right]^{\frac{\psi-1}{\psi(1-\gamma)}} \\
\text{subject to } & p_C(p_N(\mathrm{S}))c + q^h(\mathrm{S})a' + q^g(\mathrm{S})b' = \omega + \ell(\mathrm{S})\epsilon - T(\mathrm{S}) \\
& \ell(\mathrm{S}) = w(\mathrm{S})L(\mathrm{S})(1-\tau) + \Pi(\mathrm{S}) \\
& R_b(\mathrm{S},\mathrm{S}') = 1_{(d'=1)}\kappa + (1-\rho)\left(1 - \hbar 1_{(d=1)\cap(d'\neq 1)}\right) q^g(\mathrm{S}') \\
& b' \geq 0; \qquad a' \geq \bar{a} \\
& \mathrm{S}' = \Psi(\mathrm{S}, x', z', d')
\end{aligned}
\tag{4}
$$

This problem is affected by the presence of sovereign risk in at least three distinct ways. An increase in default risk depresses expected future income, generates capital losses through movements in realized R_b, and worsens the savings technology by making expected R_b more negatively correlated with future income. Section 5.3 discusses these effects in more detail.

The solution to the household's problem consists of policy functions $\varphi_a, \varphi_b, \varphi_c : \mathbf{s} \times \mathcal{S} \to \mathbb{R}$. It is important to notice that the value function $v(\mathbf{s}, \mathrm{S})$ describes a household *after* the government's default decision.

I bypass the financial sector by assuming that households trade the government bonds directly (in reality, banks are the main domestic holders of sovereign debt). In a richer model, holdings of bank equity would trasmit the gains or losses made by banks on their portfolio to their owners and shareholders. In a crisis, bank equity would suffer from capital losses after unexpected negative realizations of R_b and would become a worse savings technology when R_b becomes more negatively correlated with future income, just like government bonds do in this model. Explicitly modelling a banking sector would therefore not dramatically affect the equilibrium.

4.2 *Relative prices and the real exchange rate*

Because of the homotheticity of CES demand, each household consumes traded and nontraded goods in the same proportions. The first-order condition for the composition of consumption can thus be integrated over all agents to produce the aggregate relation

$$p_N(\mathbf{S}) = \frac{\varpi^{1/\eta}}{(1-\varpi)^{1/\eta}} \left(\frac{C_T(\mathbf{S})}{C_N(\mathbf{S})}\right)^{\frac{1}{\eta}} \tag{5}$$

4.3 *Firms*

There are two types of firms that produce traded and nontraded goods. Their technologies are concave in labor, as described by the functions f_i for $i \in \{N, T\}$. TFP depends on the productivity shock z_t and is reduced when the economy is in default. As a benchmark, I consider the case where the shock z_t only affects the production of traded goods

$$Y_{Nt} = f_N(z_t, d_t) L_{Nt}^{\alpha_N} = \left(1 - \Delta 1_{(d_t \neq 1)}\right) L_{Nt}^{\alpha_N} \tag{6}$$

$$Y_{Tt} = f_T(z_t, d_t) L_{Tt}^{\alpha_T} = z_t \left(1 - \Delta 1_{(d_t \neq 1)}\right) L_{Tt}^{\alpha_T} \tag{7}$$

where Δ is the output cost of default and $d = 1$ denotes good standing in international markets.

In equilibrium, firms in both sectors must pay the same wage. However, because of nominal rigidities, the wage w_t cannot fall below $\bar{w}$, as in Bianchi et al. (2019). When the constraint does not bind, the economy operates at full employment; otherwise, workers are rationed. I discuss this way of introducing nominal rigidities in more detail in Section B.7.

4.4 *Fiscal policy*

The government's policy determines four actions: whether to repay its current debt obligations in full, how much new debt to issue, the amount of government spending, and the level of lump-sum transfers it gives to households. While I specify borrowing and spending decisions exogenously (see Section 6.1), default decisions, and consequently lump-sum taxes or transfers, are chosen with discretion.

The government's budget constraint (8) equates resources from (net) debt issuance and labor income taxes to expenditures given by coupon payments, government spending, and lump-sum transfers

$$\underbrace{q_t^g}_{\text{debt price}} \underbrace{(B_t' - (1-\rho)B_t)}_{\text{new debt issued}} + \underbrace{\tau w_t L_t}_{\text{income tax}} = \underbrace{\kappa 1_{(d=1)} B_t}_{\text{coupon}} + \underbrace{g_t}_{\text{spending}} - \underbrace{T_t}_{\text{lump-sum}} \tag{8}$$

This budget constraint means that, given q_t^g, the government's choice of transfers T_t can be obtained as a residual from its issuance B' and spending g policies.

When the government is in default (denoted by $d = 0$), coupon payments are interrupted. However, holders can still trade the bonds in secondary markets. Defaulted debt remains valuable as the government recovers access to markets with probability θ each period. During default, new debt cannot be issued (even if it would out-of-the-equilibrium-path command a positive price), which restricts $B_t' = B_t$ in default states.

Finally, the strategies for spending g and debt issuances B' are exogenous. I estimate them as a function of the whole state vector to match observed correlations with key business cycles statistics (see Section 6). Finally, I assume that the government spends a constant fraction ϑ_N of its expenditures on the nontraded good.

4.5 *Defaults and the evolution of debt*

The repayment strategy of the government $h'(\mathbf{S}_t, x_{t+1}, z_{t+1})$ specifies a repayment probability in each state of the following period. The government makes its default choice in period $t+1$ having observed the exogenous states (x_{t+1}, z_{t+1}) and understanding which aggregate states $\mathbf{S}_{t+1}$ result from repayment and from default. The government also receives an *iid* preference shock ϵ^{def} orthogonal to all other variables, which plays the role of smoothing out the policy for numerical tractability. The mean of the shock also helps to match the average spread by controlling the unconditional default frequency, as discussed in Section 5.1.

If there is a default in period $t+1$, a haircut of $\hbar$ applies to the debt of the government. This means that $B(\mathbf{S}_{t+1}) = (1-\hbar)B'(\mathbf{S}_t)$, whereas $B(\mathbf{S}_{t+1}) = B'(\mathbf{S}_t)$ otherwise. When in default, there is a constant probability θ of reentering financial markets.

The budget constraint (8) captures a particular tradeoff. When resources from tax collections and debt issuance are low (for instance, when spreads are high), the government chooses between default or lump-sum taxes. In this context, one could interpret the second option as a regressive austerity plan.

4.6 Monetary policy

The small open economy defends a pegged exchange rate. Everywhere in the model, this assumption amounts to a normalization of the (constant) price of traded goods $p_T \equiv 1$. Importantly, I assume that the economy does not abandon the peg upon default. Even though, as Na et al. (2018) argue, real devaluations tend to follow defaults, it seems reasonable to think that Spain would remain in the euro even after a default. Moreover, factoring in the uncertainties implicit in such a scenario would likely boost the precautionary motive even more.

4.7 Foreign borrowing and the external sector

I assume that a large quantity of foreigners have access to funds at a fixed international risk-free rate $r^\star$. This immediately implies that $q_t^h = \frac{1}{1+r^\star}$.

Furthermore, if foreigners hold the government's debt in state **S**,

$$q_t^g = \frac{1}{1+r^\star}\mathbb{E}_t\left[\underbrace{1_{(d_{t+1}=1)}\left(1-x_{t+1}\right)\kappa}_{\textit{coupon}} + \underbrace{\left(1-\rho 1_{(d_{t+1}=1)}\right)}_{\textit{depreciation}}\underbrace{\left(1-\hbar 1_{(d_t=1\cap d_{t+1}\neq 1)}\right)}_{\textit{potential haircut}}\underbrace{q_{t+1}^g}_{\textit{resale price}}\right] \tag{9}$$

which reflects that debt is a claim to coupon payments while there is no default, that a default entails the haircut $\hbar$, and that the unmatured fraction $(1-\rho)$ of the bond can be resold in secondary markets.

I assume that foreigners price debt as if the coupon payment was $(1-x')\kappa$, where the stochastic process for x is constrained to remain within the interval $(0,1)$. This assumption artificially depresses the price of government debt in order to induce domestic households to hold the debt and match the home bias in holdings (see Section 5.2 for a discussion).

Equation (9) only holds when foreigners hold some of the debt. I assume that, as in the data, domestic demand for government debt always falls short of the total amount outstanding. I then check in simulation that this is the case.

I measure the implicit (promised) interest rate r^b on a government bond as the discount rate that equalizes the promised payments to the debt price, which comes down to $r^b(\mathbf{S}) = \frac{\kappa}{q^g(\mathbf{S})} - \rho$. The spread on government debt is then the difference between r^b and the risk-free rate $r^\star$. Because of the normalization that $\kappa = r^\star + \rho$, the spread can be easily computed from the debt price as $spr(\mathbf{S}) = \kappa \left(\frac{1}{q^g(\mathbf{S})} - 1 \right)$

When the small open economy is indebted with the rest of world, its consolidated intertemporal budget constraint states that the value of debt obligations must equal the expected discounted value of trade surpluses. If A denotes the total amount of risk-free debt and A^f, A^h that in hands of foreigners and domestic agents, respectively, and the same convention applies to government debt B, net foreign inflows are given by

$$\texttt{NFI}_t = \underbrace{q_t^h A_{t+1}^f + q_t^g \left(B_t'^f - (1-\rho) B_t^f \right)}_{\textit{Capital inflows}} - \underbrace{\left(\kappa B_t^f + A_t^f \right)}_{\textit{Capital outflows}} \tag{10}$$

where resources flow into the small open economy when domestic agents borrow from foreigners and when foreigners purchase government debt. On the other hand, resources flow out when the government makes coupon payments to foreigners and when domestic agents repay their debts or save.

Because the distribution λ does not distinguish holdings of both assets separately, neither A_t nor its components are a function of the state variables $\mathbf{S}_t$. However, some manipulation allows to recast (10) in terms of flows as

$$\begin{aligned} \texttt{NFI}_t &= q_t^g B_t'^f - \left(A_t^f + (\kappa + (1-\rho) q_t^g) B_t^f \right) + q_t^h A_{t+1}^f \\ &= \int \left(\omega - q_t^h \varphi_a - q_t^g \varphi_b \right) d\lambda_t - \kappa B_t + q_t^g (B_t' - (1-\rho) B_t) \end{aligned}$$

where government debt held by foreigners equals $B_t'^f = B_t' - \int \varphi_b d\lambda_t$, private debt held by foreigners equals $A_{t+1}^f = -\int \varphi_a d\lambda_t$, and $\int \omega d\lambda_t = A_t^h + (\kappa + (1-\rho) q_t^g) B_t^h$.

Finally, market clearing requires that

$$Y_{Nt} = C_{Nt} + \frac{\vartheta_N}{p_{Nt}} G_t \qquad \text{and} \qquad Y_{Tt} + \texttt{NFI}_t = C_{Tt} + (1-\vartheta_N) G_t \tag{11}$$

as net foreign inflows must equal the trade deficit.

4.8 Evolution of the distribution

Before defining the equilibrium, I discuss the assumptions that allow me to solve the model parsimoniously. The state vector S contains the distribution of agents across their idiosyncratic states, which is an infinitely-dimensional object. As is usual in heterogeneous-agents models, I solve for a bounded rationality equilibrium where agents only have limited knowledge of the distribution λ.

Specifically, I assume that agents believe the distribution of wealth to be lognormal with

$$\omega_t \sim \log \mathcal{N}(\mu_t, \sigma_t) \tag{12}$$

This assumption allows me to summarize λ_t with (μ_t, σ_t). Furthermore, the agents' policy function imply laws of motion for (μ_t, σ_t) under the erroneous, approximating assumption that the future distribution is also lognormal. Appendix B.8 provides more detail on the procedure and discusses its accuracy.

5. Equilibrium

Definition. *Given government policies* $h'(\boldsymbol{S}, x', z')$, $B'(\boldsymbol{S})$, *and* $g(\boldsymbol{S})$, *a* competitive equilibrium *consists of value and policy functions* $\{v, \varphi_a, \varphi_b, \varphi_c\}(\boldsymbol{s}, \boldsymbol{S})$, *aggregates* $L_T(\boldsymbol{S})$, $L_N(\boldsymbol{S})$, $\Pi(\boldsymbol{S})$, $Y_N(\boldsymbol{S})$, $Y_T(\boldsymbol{S})$, *prices* $p_C(\boldsymbol{S})$, $p_N(\boldsymbol{S})$, $w(\boldsymbol{S})$, $q^g(\boldsymbol{S})$, *taxes* $T(\boldsymbol{S})$ *and laws of motion for the distribution parameters* $\{\mu', \sigma'\}(\boldsymbol{S}, x', z', d')$ *such that*

- *The policy functions solve the household's problem* (4) *given prices, aggregates, and the law of motion for the distribution.*
- *The price of nontraded goods* $p_N(\boldsymbol{S})$ *satisfies the household's first-order condition* (5).
- *Labor demands* $L_T(\boldsymbol{S}), L_N(\boldsymbol{S})$ *maximize firms' profits given prices* $w(\boldsymbol{S})$, $p_N(\boldsymbol{S})$ *and the quantities produced* $Y_N(\boldsymbol{S})$, $Y_T(\boldsymbol{S})$ *satisfy the production functions (6, 7).*
- *The lump-sum taxes* $T(\boldsymbol{S})$ *satisfy the government's budget constraint* (8).
- *The debt price* $q^g(\boldsymbol{S})$ *satisfies the no-arbitrage condition (9).*

- *Market clearing in traded and nontraded goods* (11) *and in labor: either* $w(S) = \bar{w}$ *or* $L_T(S) + L_N(S) = \int \epsilon d\lambda_S$.

- *The laws of motion for the distribution parameters satisfy the consistency requirement* (18) *in Appendix B.8.*

5.1 The government's strategy

The government's objective is to maximize current welfare in the economy. I assume that it places equal weights on every agent. In each state and without commitment, the government maximizes

$$\mathcal{W}(\mathbf{S}, h') = \int v(\mathbf{s}, \mathbf{S}) d\lambda_{\mathbf{S}}(\mathbf{s}) + 1_{(d=1)} \left(\mu_g + \sigma_g \epsilon^{def} \right) \tag{13}$$

where $\epsilon^{def} \overset{iid}{\sim} \mathcal{N}(0, 1)$ is a preference shock that serves the numerical purpose of smoothing the default policy. The mean of the shock μ_g helps me discipline the average default frequency in the model as, in contrast to standard models of sovereign default, here the discount factor and the risk aversion parameter are tied to moments of the private wealth distribution. The government is subject to equilibrium conditions and its budget constraint, where the notation $\lambda_{\mathbf{S}}$ emphasizes that the distribution is a part of the aggregate state $\mathbf{S}$. Importantly, the value function and the distribution correspond to the competitive equilibrium that results under the policy h'.

Definition. *A policy* h' *for repayment is a part of an* equilibrium *if, at each* (S, x', z')*, the probability of repayment satisfies*

$$h'(S, x', z') = \mathbb{P}\left(\mu_g + \sigma_g\, \epsilon^{def} \le \underbrace{\mathcal{W}\left(\Psi(S, x', z', d' = 1), h'\right)}_{\text{value under repayment}} - \underbrace{\mathcal{W}\left(\Psi(S, x', z', d' \neq 1, h')\right)}_{\text{value under default}} \right) \tag{14}$$

where $\Psi(S, x', z', d') = S'$ *is the state that ensues when* (x', z') *are realized after* S *and the government chooses a default state* d'.

After observing the realization of x' and z', the government understands which state $\mathbf{S}'$ results if it decides to default or to repay. This includes the level of debt remaining to be paid as well as the distribution induced in each case.

Condition (14) is a rational-expectations restriction: the policy that households, foreigners, and the current government expect of future governments, h', must coincide with the policy that the government would choose if allowed a deviation that did not alter future expectations. In other words, condition (14) insists that the policy h' be part of a Nash equilibrium. The restriction that all policies depend only on the current state $\mathbf{S}$ (and not on the whole history of play) further refines the solution concept to that of recursive equilibrium.

Appendix A describes the computation of a solution in detail.

5.2 *Euler equations and coupon payments*

The Euler equation (15) determines a household's purchases of government bonds:

$$q^g(\mathbf{S}) \geq \beta \mathbb{E}\left[\underbrace{R_b(\mathbf{S}, \mathbf{S}') \frac{p_C(\mathbf{S})}{p_C(\mathbf{S}')}}_{\textit{real repayment}} \underbrace{\underbrace{\left(\frac{\varphi_c(\omega', \epsilon', \mathbf{S}')}{\varphi_c(\omega, \epsilon, \mathbf{S})} \right)^{-\frac{1}{\psi}}}_{\textit{Intertemp. subs.}} \underbrace{\left(\frac{v(\omega', \epsilon', \mathbf{S}')}{\mathbb{E}\left[v(\omega', \epsilon', \mathbf{S}')^{1-\gamma} \mid \mathbf{S} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi} - \gamma}}_{\textit{Risk aversion}}}_{\textit{Stochastic discount factor}} \,\middle|\, \mathbf{S} \right] \tag{15}$$

with equality if the household is purchasing a positive amount of bonds. It is also clear from (15) that $\gamma = 1/\psi$ recovers the standard case of expected CRRA utility. I employ an Epstein-Zin formulation as the main mechanism in the paper stems from an asset-pricing effect (namely, the correlation between the repayment of different assets and the stochastic discount factor). This makes risk-sensitive preferences yielding realistic asset pricing implications a natural choice.

Being risk-averse, the household demands a risk premium to expose itself to the risk of the government. The shock x plays the role of creating such a risk premium in the return of the government bond when compared to the return of the risk-free asset. This allows the model to match the high proportion of sovereign debt held by domestic agents in the data on average. The dynamics of debt holdings, however, are determined endogenously.

5.3 The household's reaction to sovereign risk

There are at least three main ways in which sovereign risk affects the household's problem (4). The first effect concerns the aggregate income losses that happen in case of default. Conditional on default, TFP drops by Δ in both sectors for some time, which puts downward pressure on the market-clearing wage. If the constraint binds, unemployment increases. In either case, other things equal labor income $w(\mathbf{S})L(\mathbf{S})\epsilon$ is lower in default than in repayment. In states with a higher default probability, the household feels poorer and reduces consumption.

Figure 5 shows expected labor income (integrating out heterogeneity in ϵ) as a function of next period's TFP for default and repayment, conditioning in each panel on different levels of debt. Labor income is clearly increasing in TFP and higher in repayment than in default. The effect of government debt on labor income is small in default but large in repayment: higher debt means a higher default probability, hence lower aggregate demand and downward pressure on wages. In the end, when debt is very large, income is much less affected by current default status.

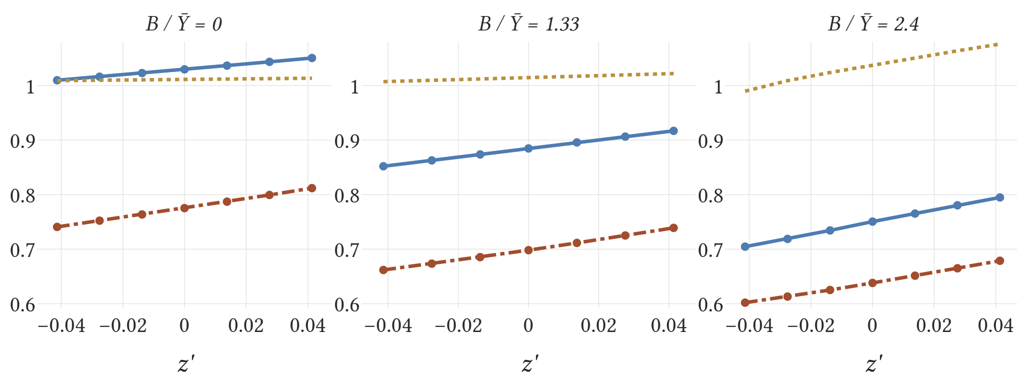

FIGURE 5: LABOR INCOME AND EXPECTED RETURNS

Note: Blue lines plot income in repayment, red dashed lines plot income in default, orange dotted lines plot the return of holding government debt. Debt is expressed as share of mean income.

Secondly, the household cares about the insurance properties of government debt. Sovereign risk makes those very different in normal times and in crisis times. Recall the return of a government bond

$$R_b(\mathbf{S}, \mathbf{S}') = 1_{(d'=1)}\kappa + (1-\rho)\left(1 - \hbar 1_{(d=1\cap d'\neq 1)}\right) q^g(\mathbf{S}')$$

In normal times, the variance of R_b is relatively low (in the leftmost panel of Figure 5, the return is almost flat). Its variation comes mostly from variation in the future resale price $q^g(\mathbf{S}')$. However, as the default probability increases, more and more of the variance of R_b becomes driven by variation in the repayment probability. In Figure 5, the return on debt increases more with z' when debt is higher. This imparts a stronger correlation between returns and income, which is also positively related to z. Hence, the conditional covariance between the bond return and the stochastic discount factor of households tends to be larger in crises. This feature makes the bond a bad hedge always but an even worse one when spreads are high. In the aggregate, this ceteris-paribus force continuously battles distributional forces, as debt holdings are positively associated with wealth.

A third effect operates through the distribution. Shocks that make default more likely also decrease the resale value of bonds and households who purchased these bonds in the past make an inmediate capital loss when q^g drops. In the aggregate, therefore, increases in default probability shift the wealth distribution to the left. The strength of this channel depends critically on the proportion of bonds held by domestic agents, as well as on the level of inequality in domestic bondholdings. On the other hand, while the default probability is positive but the government does not default, holders of debt make excess returns on their portfolios, which shifts the wealth distribution to the right.

Appendix B.3 shows how these effects translate into who fears sovereign default more.

6. Calibration

6.1 *Fiscal rules*

I estimate fiscal rules for government spending and issuances of new debt, using quarterly data for Spain. Data are taken from Eurostat and cover the period 1999Q1 to 2017Q4. In the model, government consumption and net issuances as fractions of GDP depend on the whole state vector, so in the data I regress those against endogenous variables. Notice that, by construction, data generated from the model would match these regressions exactly, both in repayment and, counterfactually, in default. Table 2 summarizes the results. For each dependent variable, the

Table 2: Estimated Fiscal Rules

	G_t/Y_t		$(B'_t - (1-\rho)B_t)/Y_t$	
	(1)	(2)	(3)	(4)
Constant	13.194	14.352	2.680	1.027
	(1.350)	(0.982)	(3.087)	(1.407)
Unemployment$_t$	1.078	0.330	0.410	0.286
	(0.086)	(0.028)	(0.197)	(0.042)
Unemployment$_t^2$	-0.020		-0.003	
	(0.002)		(0.005)	
B_t/Y_t	-0.187	-0.021	-0.099	-0.020
	(0.028)	(0.010)	(0.063)	(0.015)
$(B_t/Y_t)^2$	0.001		0.001	
	(0.000)		(0.000)	
Net Exports$_t$	-0.309	-0.167	0.233	0.212
	(0.070)	(0.096)	(0.162)	(0.138)
Observations	72	72	71	71
R^2	0.916	0.776	0.814	0.808

Standard errors in parentheses.

first column contains the preferred specification. The second one reports a simpler version of the same regression.

The fit for both government consumption and debt issuances is good, with adjusted R^2s at 90% and 80%, respectively. Fiscal policy is countercyclical, with positive responses to unemployment for both spending and issuances. New issuances respond negatively to the debt-to-GDP ratio, consistent with debt stabilization. Figure 25 in the Appendix shows the fitted values for Spain from the preferred specification. The predicted rules track the observed series closely.

6.2 *Model parameters*

The model is able to generate a good match to some standard targets in the literature. Table 3 reports some critical parameter values. Because of the numerical complexity of the model, I rely

Table 3: Parameter Values

Description	Parameter	Value	Source / Target
Risk-free rate	$r^{\star}$	4% ann.	Anzoategui (2020)
Haircut in case of default	$\hbar$	45%	Philippon and Roldán (2018)
TFP loss in case of default	Δ	10%	Philippon and Roldán (2018)
Reentry probability	θ	0.04167	Cruces and Trebesch (2013)
Share of N in production	ϖ	0.7397	Anzoategui (2020)
Labor share in production	α_N, α_T	0.67	Anzoategui (2020)
Share of N in G	ϑ_N	88%	Anzoategui (2020)
Elasticity of substitution	η	0.74	Anzoategui (2020)
Persistence $\log \epsilon_{it}$ (annual)	ρ_ϵ	0.85	D'Erasmo and Mendoza (2016)
Std. deviation $\log \epsilon_{it}$ (annual)	σ_ϵ	0.2498	D'Erasmo and Mendoza (2016)
EIS	ψ	1	Standard
Internally calibrated			
Discount rate of HHs	$1/\beta - 1$	5.31% ann.	Moments in Table 4
Risk aversion	γ	2.9	Moments in Table 4
Progressivity of tax schedule	τ	31%	Moments in Table 4
Wage minimum	$\bar{w}$	1.178	Moments in Table 4
TFP process	ρ_z, σ_z	(0.63, 0.0107)	Moments in Table 4
Mean risk premium	$\bar{\xi}$	0.025%	Moments in Table 4
Risk premium AR(1)	ρ_ξ, σ_ξ	(0.95, 0.0001)	Moments in Table 4
Mean utility cost of default	μ_g	0.0124	Moments in Table 4

on external calibration as much as possible, while internally calibrated parameters are chosen via Simulated Method of Moments. For the supply side of the economy, I closely follow Anzoategui (2020) and Stockman and Tesar (1995) and set preference parameters ϖ and ϑ_N to match the shares of traded and nontraded goods in both private and public consumption, as well as the elasticity η to the elasticity of relative consumption demand.

The risk-free interest rate is set at a standard value in the literature. For the costs of default, I follow Philippon and Roldán (2018) and set the haircut $\hbar$ and conditional TFP losses Δ to a Greek-style default. The probability of reentry is set to give an expected duration of default of 25 quarters, on the lower end of the Cruces and Trebesch (2013) estimation for large haircuts.

As for the household idiosyncratic income shocks process, I follow the estimation of D'Erasmo and Mendoza (2016) based on the Spanish cross-sectional income distribution for the same period that I study.

Table 4 provides details on the fit of the model. Statistics in the Model column are computed by averaging 500 simulations of 200 years each (with a 400-year burn-in). The fit of the model is generally good. The Spanish sample has one crisis for 17 years of data, which is likely higher than the ergodic frequency. For this reason, in the calibration I informally target lower levels of unemployment and debt-to-GDP. One shortcoming is that the Gini coefficient does fall short of its empirical counterpart. While I opted for simplicity, enriching the taxation scheme could help match this target.

Table 4: Model Fit

Target	**Model**	**Data**	**Target**	**Model**	**Data**
AR(1) autocorr. coef $\log(Y_t)$	0.944	0.966	Std Debt-to-GDP	10.7%	23.5%
AR(1) std coef $\log(Y_t)$	0.887%	0.617%	Avg unemployment	14.8%	15.9%
AR(1) autocorr. coef $\log(C_t)$	0.962	0.954	Std unemployment	2.86%	6.09%
AR(1) std coef $\log(C_t)$	1.03%	1.22%	Median domestic holdings	47.2%	56.5%
AR(1) autocorr. coef spread	0.964	0.967	Avg wealth-to-GDP	92.4%	94.5%
AR(1) std coef spread	34.8	30.1	Avg wealth Gini	49.1%	57.5%
Avg Debt-to-GDP	59.3%	64.6%			

Source: All data from Eurostat 2000Q1:2017Q4, except Gini index from Eurostat 2010, private consumption from OECD 2000Q1:2017Q4, domestic holdings from Banco de España, 2004Q1:2017Q4

7. Analysis

7.1 Government policy

Figure 6 shows the government's value function $\mathcal{W}(\Psi(\mathbf{S}, x', z', d'), h')$ of the following period as function of the realization of shocks. Each panel shows welfare as a function of next period's TFP realization z', with different panels showing different levels of debt, increasing from left to right. Higher debt levels decrease the value of repayment relative to default. For intermediate amounts of initial debt, moreover, a higher realization of future TFP raises the relative value of repayment. Higher spreads also marginally raise the relative value of default.

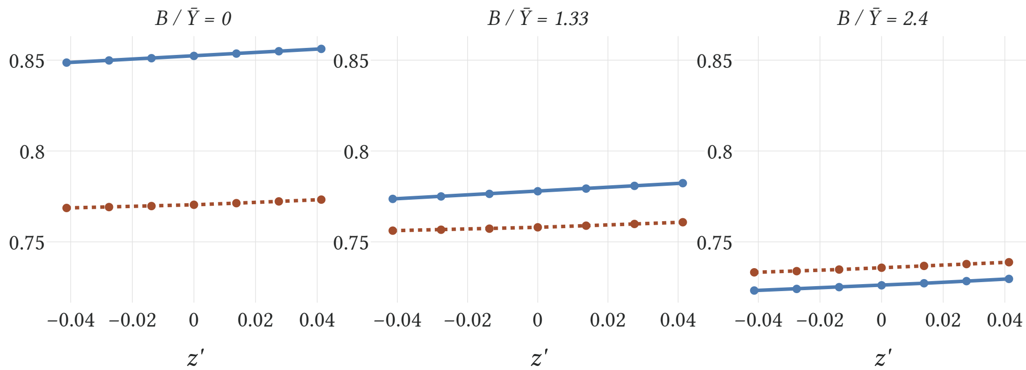

Figure 6: Welfare Functions

Note: Blue lines plot the value of repayment, red dashed lines plot the value of default. Debt is expressed as share of mean income.

Figure 7 shows the price of debt $q^g(\mathbf{S})$ at different states $\mathbf{S}$. The left panel shows that spreads rise (the price of debt falls) with the stock of debt and decrease with productivity. The right panel shows the impact of the distribution (for fixed values of B and z when the economy is not in default). Higher spreads occur when the economy is poorer and more unequal. This is because

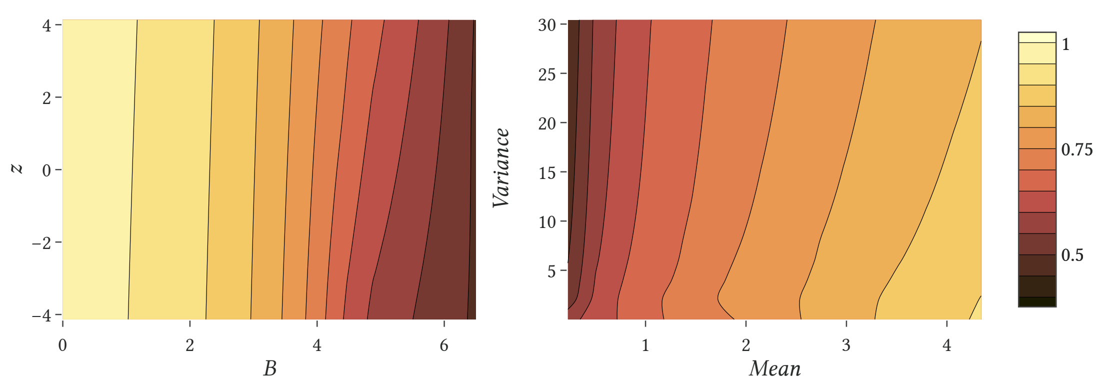

Figure 7: Price of Debt

the value of autarky depends strongly on how rich the economy is: with lower aggregate wealth, more agents are close to their borrowing limit and would suffer from the loss in TFP (and hence wages and employment) that follows a default. The effect of variance goes through the value of repayment: when inequality is greater, the increase in lump-sum taxes required to repay becomes more regressive and hence even less desirable. Defaults avoid this redistribution.

The unemployment rate is shown in Figure 8. Unemployment decreases with productivity and increases with government debt. The right panel shows that unemployment is related negatively to total wealth and positively to inequality.

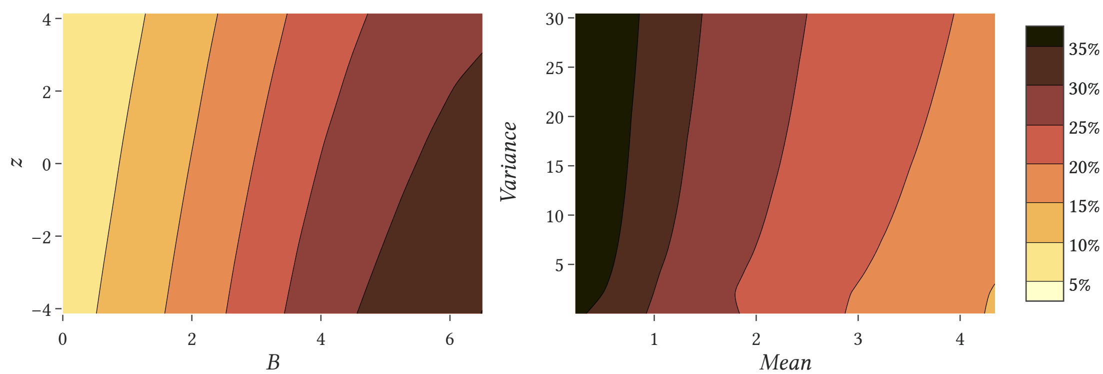

FIGURE 8: UNEMPLOYMENT

7.2 *The impact of default risk*

To understand how default risk affects the economy, I also solve the same model, with the same parameter values, for a government policy h stipulating that debt is always repaid. Table 5 shows statistics from simulation of both models. It illustrates how sovereign risk affects the economy even in normal times. The volatilities of output and consumption are 42% and 58% lower without the possibility of default. Spreads are obviously absent from the no-default simulation (they are only positive because of the positive mean of the risk premium x). Unemployment and government debt are both lower in the economy without default. Because private wealth is not recurrently affected by default, the average agent is marginally wealthier. At the same time, the Gini coefficient decreases as there is less desire to accumulate large amounts of wealth to cover against the possible imapct of default, which makes the right tail of the wealth distribution thin-

Table 5: Models

Moment	Benchmark	No default
AR(1) std coef $\log(Y_t)$	0.887%	0.511%
AR(1) std coef $\log(C_t)$	1.03%	0.43%
AR(1) std coef spread	34.8	0.0539
Avg Debt-to-GDP	59.3%	41.6%
Avg unemployment	14.8%	9.28%
Median dom holdings	47.2%	237%
Avg wealth-to-GDP	92.4%	91.3%
Avg wealth Gini	49.1%	48.7%
Default frequency	1.51%	0%
Welfare in repayment	0.887	0.934

ner. As a result of these differences, the average household attains a value in the world without default which is equivalent to an increase of 5.2% in permanent consumption in the benchmark economy.

The results shown above quantify the impact of default risk on the economy while keeping fixed the distributional assumptions (given by the estimated stochastic process for idiosyncratic risk ϵ). A natural question would be how much does heterogeneity in income and wealth matter for these differences. However, solving a representative-agent version of the model does not provide a simple comparison. The main reason is that removing the idiosyncratic shock takes away much of the normal-times precautionary motives of households. This results in lower levels of wealth, which in turn induce the government to default too often. While quantifying the unconditional effect of heterogeneity is challenging, I discuss the effect of the distribution on crisis events themselves in Section 8.1.

Table 6 summarizes the welfare gains of eliminating sovereign risk for different quantiles of the distribution, averaging over all time periods. The first two columns report the average of the welfare function for each percentile of the wealth distribution in the benchmark and no-default models, while the last column shows the welfare gains of moving from the benchmark to the no-default models keeping constant the position in the wealth distribution. Welfare gains decline with wealth and range between 5% and 5.5% of permanent consumption.

Table 6: The Welfare Costs of Sovereign Risk

Moment	Benchmark	No default	Gains
$p10$	0.818	0.863	5.49%
$p25$	0.844	0.889	5.37%
$p50$	0.883	0.93	5.27%
$p75$	0.926	0.974	5.16%
$p90$	0.964	1.01	5.01%
Average	0.887	0.934	5.24%

8. Crises

Using the simulated series, I focus on episodes of crisis. I define an episode of high spreads as a period of 11 quarters (to match the Spanish experience of 2010-late 2012) at the end of which the spread surpasses 400bps but a default does not happen. Even though the spread volatility is a calibration target, an average crisis in the model does not feature such sharp accelerations of the spread as the 2010 crisis. For this reason, I further condition on lower spreads at the start of the episode, subject to keeping a reasonable number of episodes in the sample.

Figure 9 plots endogenous variables around episodes of high spreads, projecting about a year into the past for context. Time is shown in years on the x-axis. TFP, output, and consumption are measured in percent deviation from their ergodic means. During crisis episodes, both output and consumption are significantly below their normal-times values. The government's finances have deteriorated, pushing lump-sum taxes up. Furthermore, tax collections worsen with higher unemployment. In terms of dynamics during the crisis, high private savings push up the mean of the wealth distribution. The Gini coefficient falls by about two points right before the crisis but picks up marginally during it. Output hits a minimum of about 6% below its long-run mean, as in the data, while TFP varies between 0.5% and -1% at the trough of the episode. Consumption also drops below its long-run mean by almost 7%, more than output but a bit short of the 10% in the data. Table 7 in the Appendix shows that samples with higher inequality at the beginning of

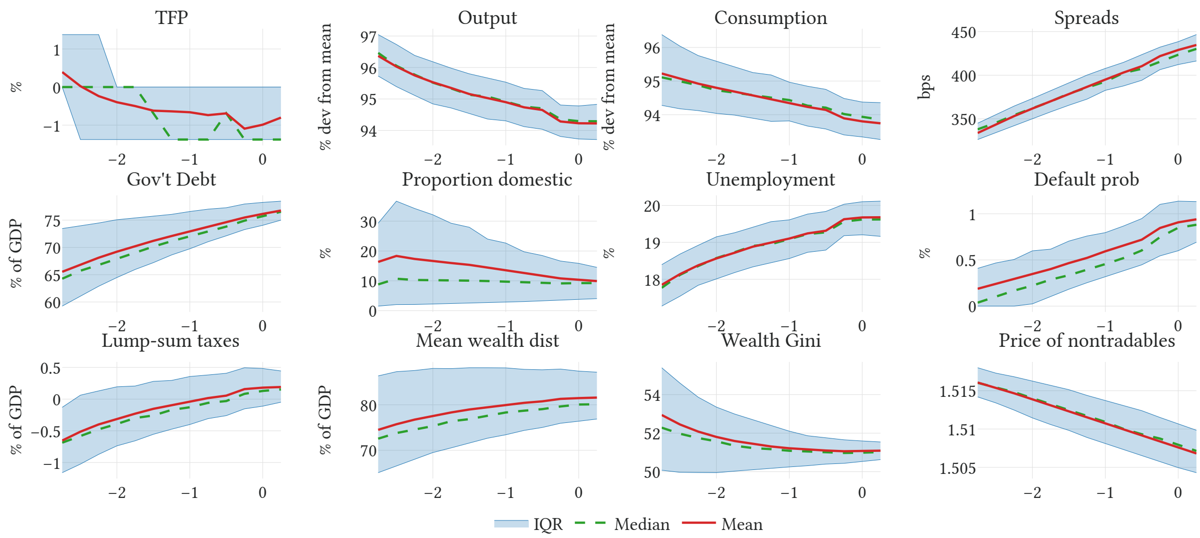

Figure 9: Times of High Spreads

the crisis suffered more pronounced contractions

While the model matches the contractions in output and consumption fairly well and generates an increase in the aggregate saving rate at the same time as sovereign risk appears, the crisis dynamics it generates are too slow, as shown in Figure 10, which superimposes the Spanish data for 2010Q1 through 2013Q1 to the output, consumption, spreads, and debt series obtained from the model.

In Spain, the output and consumption drop occured in about two years, while in the simulations underlying Figure 9, these contractions had started much earlier. The dynamics of the spread are equally slow. In the model, spreads are already significant two years before the peak of the crisis. This leads mechanically to a sharper acceleration of debt in the data. Section 8.3 below computes impulse-responses to an unexpected increase in sovereign risk, taking the interpretation that default risk was perceived as negligible for an advanced economy until the Greek crisis of 2010.

More of the dynamics at play can be seen in Figure 9. There is a significant fiscal contraction: a sustained increase in taxes along with a slight fall in government spending (not pictured), which balances reacting to unemployment and stabilizing the debt. Government debt accumu-

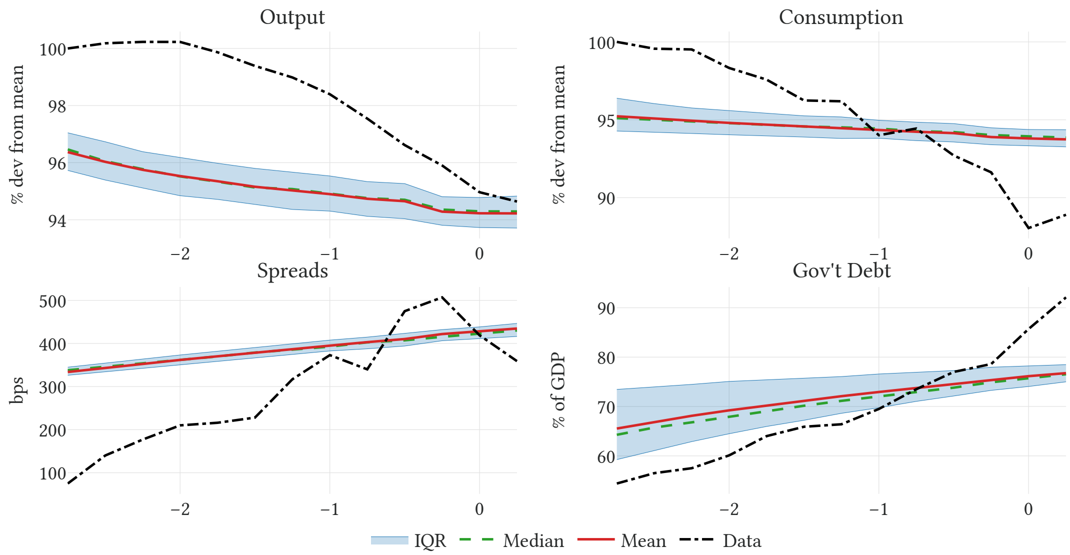

Figure 10: Crisis dynamics in model and data

Note: Data refers to Spain 2010Q1:2013Q1. Output and consumption are normalized to the 2010Q1 level.

lates rapidly as a consequence of high spreads. Consistent with findings in Bianchi and Mondragon (2021), the price of nontraded goods falls but the resulting real depreciation falls short of the amount required to boost aggregate demand back to its normal level.

8.1 *Amplification forces*

I now consider the benchmark economy and its episodes of high spreads and compare it with an alternate economy in which the government follows a policy of always repaying the debt.

Figure 11 presents a comparison between the times in which the benchmark economy is in crisis and the same time periods of a simulation of the no-default economy (with the same shocks). I show output, consumption, the price of nontradables, and government debt as percent deviations from their initial levels. The contractions in output and consumption are muted in the no-default economy. In the no-default economy, output and consumption only fall because of the underlying shock to TFP. Moreover, in the benchmark economy the dynamics (as well as the higher level) of spreads induce an acceleration of the debt-to-GDP ratio. All in all, the no-default economy suffers an output and consumption contraction of between a half and a third relative to the benchmark.

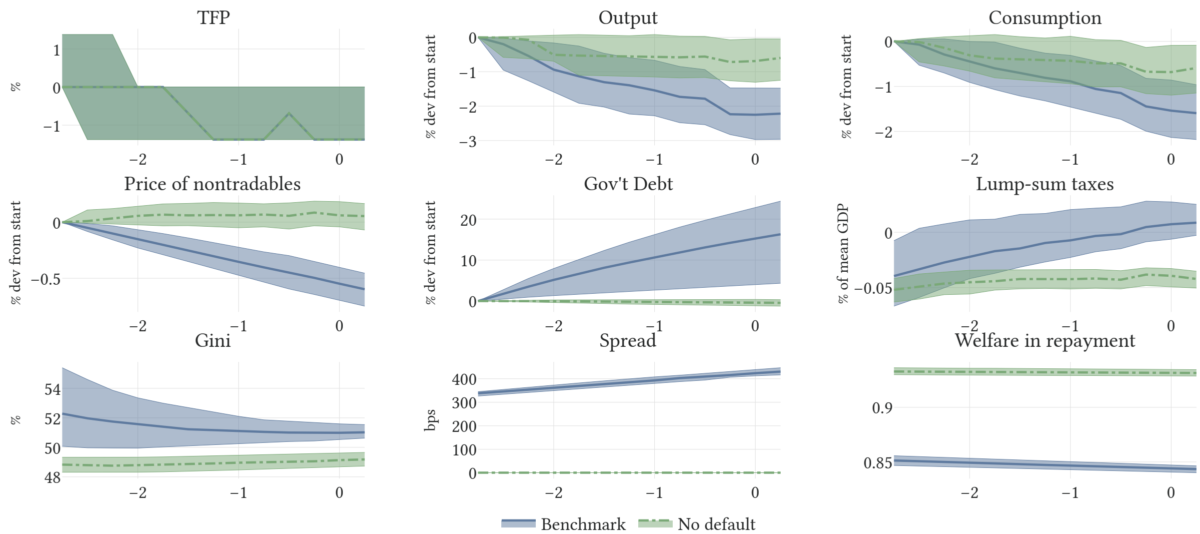

FIGURE 11: CRISES

Note: Interquartile ranges shaded

In this sense, sovereign risk explains about 60% of the recession.

The Gini index is mostly flat in both models, increasing marginally at the height of the crisis after decreasing somewhat in its earlier phase. This happens as the government's budget constraint tightens, forcing lower lump-sum transfers while capital losses made by richer agents tend to impoverish them earlier on, contributing to lower inequality. This comparison reveals that the dynamic interaction between sovereign risk and inequality is not large. However, as shown in Section 8.3, this does not mean that the dynamics of the crisis are not shaped by inequality.

Fiscal policy also differs across both models. The benchmark economy is forced to a larger increase in lump-sum transfers, while the no-default economy can keep a fiscal policy stance closer to neutral.

The differences in welfare are substantial. In this specification of preferences, the value function equals the level of permanent consumption (a constant amount which would yield the same utility). At the height of a crisis, the average household would give up as much as 10.5% of consumption to be able to move to the economy with no default.

8.2 *Amplification and inequality*

Figure 12 provides more details into the distributional aspect of the events studied so far. The left panel plots consumption by each percentile of the wealth distribution (as well as the average). The right panel shows the value attained by households at those quantiles, both as shares of their values at the start of the episode.

The left panel shows that consumption inequality increases during crises as poorer agents cut consumption by more. At the same time, crises also affect the dispersion in values (which coincide with the level of permanent consumption to which the household is indifferent). These events are about twice as costly for the bottom 25% of the distribution as they are for the top 25%.

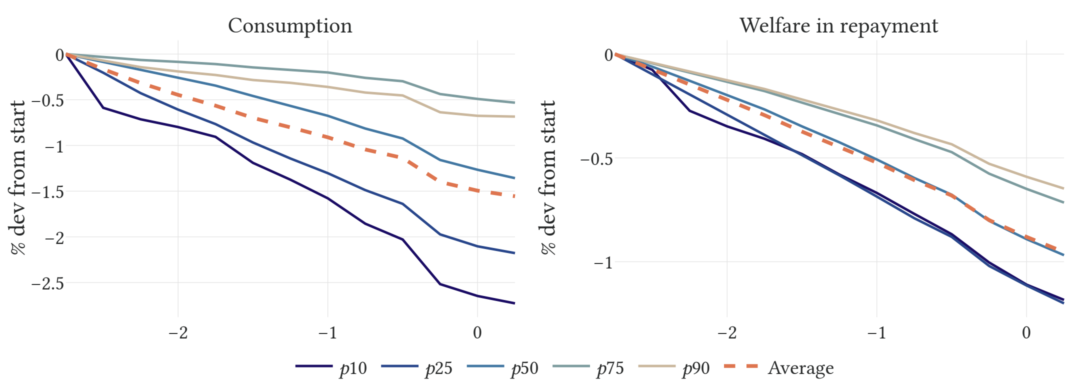

FIGURE 12: CRISES ACROSS THE DISTRIBUTION

The median household's consumption reacts differently than the average. Informally, the response of the median household approximates that of a representative-agent version of the same model, while avoiding all the general-equilibrium complications discussed above. While aggregate consumption falls by 1.56% during the crisis, the median household is only cutting by 1.36% in the same periods. This comparison between an agent in the middle of the distribution and the aggregate agent suggests that heterogeneity is responsible for about 15% of the consumption response.

Figure 19 in the Appendix shows the impact of sovereign risk on the distribution of assets.

8.3 A default-risk IRF

The comparison between crisis episodes in the benchmark and no-default models ignores that average debt levels and the wealth distribution are different in both models. It also does not map directly to the Spanish experience in the crisis, as spreads take longer to rise in the model than in the data. For this reason, I consider the following experiment to complement the results above.

Suppose that Spain did not face any default risk before the crisis (as shown in Figure 1, Spanish spreads were consistently below 100bps before 2010), that after the Greek crisis in early 2010, default was suddenly regarded as possible and, finally, that after Mario Draghi's famous "whatever it takes" speech, the economy reverted to the no-default equilibrium (Figure 1 reveals that spreads came down after 2012, although gradually).

Motivated by this approximation, I consider the following impulse-response exercise. I simulate the no-default economy for 100 years to obtain a draw from its ergodic distribution. I then set the debt level to about 65% of GDP (the observed level for 2010) and, after one quarter, unexpectedly switch the economy to its benchmark equilibrium with endogenous default risk. I simulate the economy forward for 11 quarters and then revert it back, also unexpectedly, to the no-default case. I repeat this experiment 5000 times and keep the paths under which default did not materialize. To match the Spanish experience in the crisis, I further condition on paths that saw an output contraction of at least 5% (targeting an average output contraction of 6% in line with the data).

Figure 13 shows the distribution of outcomes, compared with a simulation where default risk is never activated (but the same shocks are realized). This comparison captures two effects: the direct effect of aggregate demand and the fact that the government engages in a quick deleveraging when borrowing costs are low. For this reason, I add a third comparison point: a simulation of the economy without default risk but that uses the same debt issuances as the benchmark one. Figure 20 in the Appendix superimposes the data and shows that the Spanish crisis unfolds more smoothly than this exercise.

The no-default economies act as counterfactuals for Spain if expectations of default risk had not been triggered. The green line (labeled No default) is a pure counterfactual, while the orange

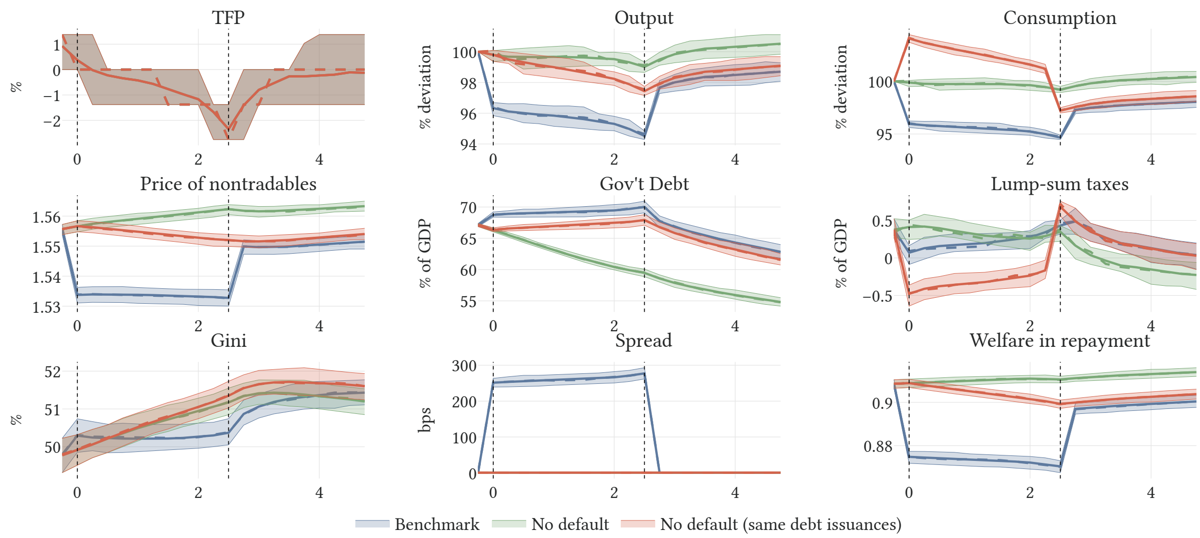

FIGURE 13: DEFAULT-RISK IRF

Note: Vertical dashed lines indicate switch from no-default to benchark economy and viceversa

line, which keeps debt issuances as in the benchmark, reflects how much of the crisis is explained by default risk. It implies that output would have fallen by 2.6% in the absence of default risk, leaving default risk to account for about 53% of the recession.

The bottom-right panel provides an estimate of the value of negating default risk in the crisis. The average household (or the government) would have given up 3.75% (red minus blue at time 0) of permanent consumption to make defaults impossible. A similar calculation (blue in the first period after the crisis minus blue in the last period of the crisis) places the value of the 'Whatever it takes' speech at 3.1% of permanent consumption from the last period of the crisis forward, or 2.5% discounted to a time-0 basis. Lastly, the loss from spending 11 quarters with default risk can be computed by taking the period-0 loss from switching to the benchmark economy forever and subtracting the discounted gain from switching back to the no-default economy at the end of the crisis. This calculation yields a cost of sovereign risk of 1.2% of permanent consumption. Section B.2 in the Appendix explores the distributional aspect of this exercise.

The interaction between inequality and default risk can be seen in the bottom left panel. While Gini increases slightly during the crisis, it does so much less than it would have done for the same shocks in the no-default economies. Inequality has therefore a mitigating effect on the dynamics of the crisis. However, the response of the economy is still influenced by inequality. Figure 14 shows compares output and consumption at the trough of the crisis with the Gini index at time 0 just before default risk was turned on. On average, the output contraction is 0.1 percentage points larger for each extra point in the Gini index. Consumption exhibits a weaker negative correlation with initial inequality.

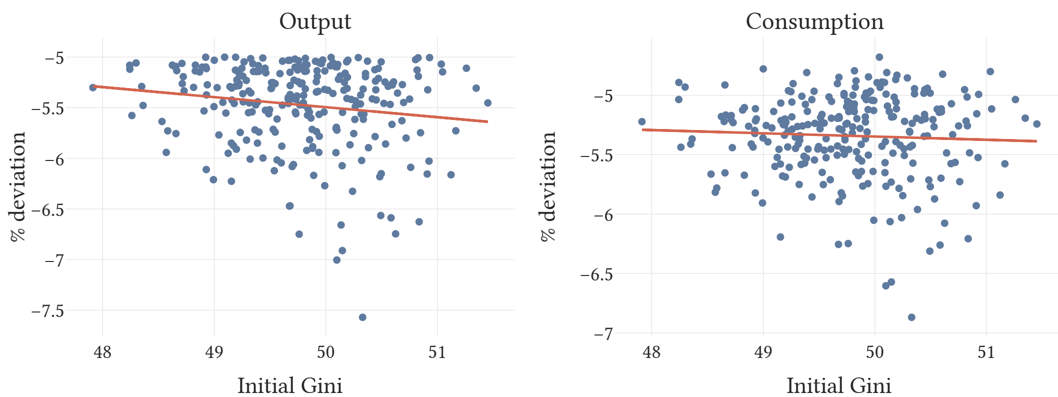

Figure 14: Outcomes conditional on inequality

Note: Blue dots correspond to each simulation. Output and consumption are measured in deviations from their pre-crisis values. A simple regression line is shown in red.

To further study the role of the inequality and redistributive policies in shaping these dynamics, Figure 15 combines the previous exercise with changes to the income tax parameter τ at the same time as default risk is activated. To isolate the effect of inequality itself, in each case I change the value of the income tax parameter τ and solve the private domestic part of the economy, leaving the government's default policy as well as the lenders' pricing function unchanged, although some feedback takes place as these are functions of the underlying state of the economy, which evolves differently.

Increasing the slope parameter τ enables the government to provide more lump-sum transfers, on net making the tax structure more progressive. Figure 15 shows that the Gini index increases during the crisis when τ takes a lower value than the benchmark of 0.31. At the same time, output and consumption are boosted by progressivity: for each point in the Gini at the peak of the crisis, the output (consumption) contraction is boosted by 0.4 (0.6) percentage points at the peak of the

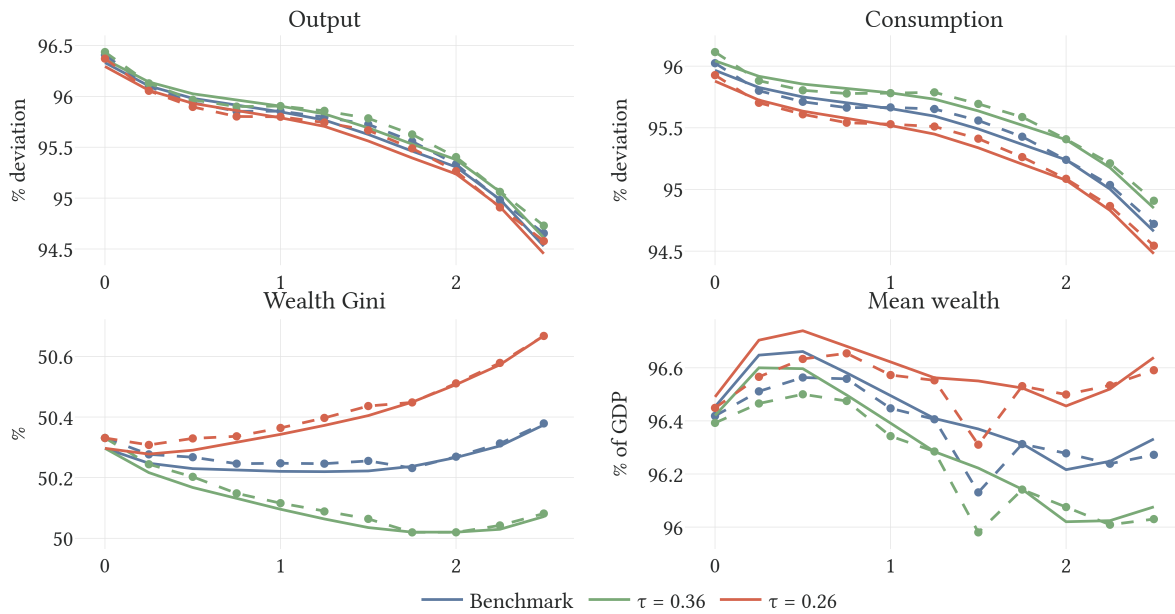

Figure 15: Default-risk IRFs and tax progressivity

crisis, of which 0.2 (0.3) percentage points occur after default risk is switched on.

9. Concluding Remarks

Inspired by events in the Eurozone crisis, this paper analyzes a model in which households' consumption demand is negatively affected by the presence of sovereign risk. The mechanisms in the model generate substantial amplification of underlying shocks even if the risk of default does not materialize.

The amplification mechanism relies on the precautionary motives of households, which are magnified by sovereign risk. Sovereign risk creates endogenous shifts in demand conditions which exacerbate the equilibrium volatility of aggregate consumption. I find large and regressive welfare costs of sovereign risk, which range from about 5.2% of permanent consumption in normal times to almost 10% at the height of a crisis. The default-risk IRF exercise suggests that the average household would have given up 3.75% of consumption in order to avoid the crisis, of which about 1.2% were recovered 11 quarters later when the crisis ended.

While heterogeneity is not required for the qualitative result, the dynamics of the economy itself are also shaped by inequality. The default-risk IRF exercise shows that higher inequality, obtained via a change in tax progressivity when default risk emerges, implies additional contractions of output and consumption of about 0.4–0.6 percentage points for each extra percentage point in the Gini that such taxes induce.

A common argument in policy circles during debt crises is that a lack of 'confidence' causes aggregate demand to fall. This paper addresses this argument as a rational, although inefficient, response to the evolution of fundamentals. Without commitment to future policies, the government's ex-post default incentives act during crises as large-scale increases in uncertainty (Bloom, 2009). More broadly, this type of amplification helps explain why emerging economies exhibit high volatility of consumption relative to output 'as if' they were subject to trend shocks, especially on the downside (Aguiar and Gopinath, 2007)

While I calibrate the model to Spain, the mechanism can help explain patterns in emerging-market business cycles, which also exhibit sovereign risk as a feature. Both the relative volatility of consumption to output and the volatility of output itself are typical calibration targets in the sovereign debt literature when applied to emerging-market economies. The setup presented here offers a more complete explanation of these phenomena by explicitly considering the saving behavior of private agents as well as the interest rate they face. The amplification mechanism and the welfare costs of sovereign risk are both natural consequences of this more granular description when private agents are net savers. This is the case of Spain in the 2000s but also of salient episodes in emerging markets, when the private sector's international investment position is positive even as the government is in debt.

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A. Solution method

The algorithm follows closely the definition of equilibrium: to solve for an equilibrium, I solve a series of nested problems. Given government policies, I find a competitive equilibrium by finding functions of the aggregate state that describe the aggregates in the economy, in such a way that they are consistent with the household problem and policy functions.

Given a policy for the government, I

1. Guess a law of motion for the distribution.
2. For each state $\mathbf{S}$
   (a) Compute $q^g(\mathbf{S})$ from the foreigners' sdf (9).
   (b) Guess a relative price of nontradables p_N
      - Get the wage rate w as well as total labor demand L^d and profits of the firms Π.
      - Compute lump-sum taxes T from gov't budget constraint (with τwL in hand).
      - Solve the household's problem at prices w, p_N, profits Π, and transfers T.
      - Check market clearing (11) for nontraded goods.

   (c) Iterate on the function $p_N(\mathbf{S})$ to convergence
3. Iterate on the law of motion for the distribution using the households' policy functions.

Finally, I update the government's policy according to (14) and iterate until a policy that respects it is found.

This algorithm has many advantages relative to standard methods for solving heterogeneous-agents models with aggregate shocks. First, it does not rely on a solution-simulation iteration like Krusell and Smith (1998). Having to simulate the model in each step is computationally expensive. Moreover, the explicit bounded rationality solution also ensures that the government shares the same beliefs as the agents about future and counterfactual variables. Second, it allows for a global solution which does not linearize the aggregate shocks, as popular perturbation-based algorithms do. Here, because the agents project the future distribution of assets onto a family of lognormals, the state space is actually finite-dimensional.

B. More model results

B.1 Minimal model with domestic debt

In this case the model is amended to avoid the direct effect of debt on aggregate demand. To achieve this, I keep the government's default policy unchanged but assume that the debt is held domestically, that is, that debt payments do not subtract from consumption. In other words,

$$c_2 = \begin{cases} y_2^T + s_1 & \text{if the government repays} \\ y_2^T(1-\Delta) + s_1 & \text{if the government defaults} \end{cases}$$

The solution still has the planner and household saving in response to sovereign risk, although less than in the original case.

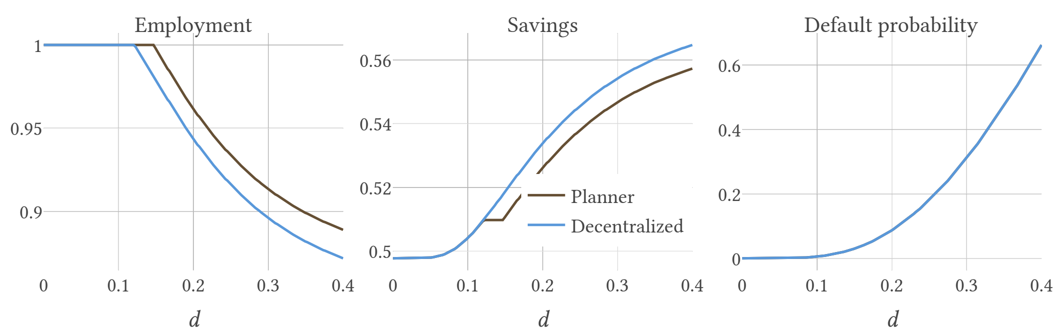


Figure 16: Anticipation of TFP Costs of Default

B.2 The distributional impact of sovereign risk

Figure 17 plots the welfare costs of sovereign risk across the wealth distribution during the same paths studied earlier. The costs from triggering sovereign risk are heterogeneous, increasing in wealth, and range from 0.83% (1.01%) for the bottom 10% (25%) of the distribution to 2.41% (2.06%) for the top 10% (25%). The median (average) household would give up 1.5% (1.6%) of permanent consumption to stay in the no-default economy.

While all measures of costs of sovereign risk have been regressive so far, in this case the welfare cost is higher for the rich. In the previous experiments, by comparing equilibria with and

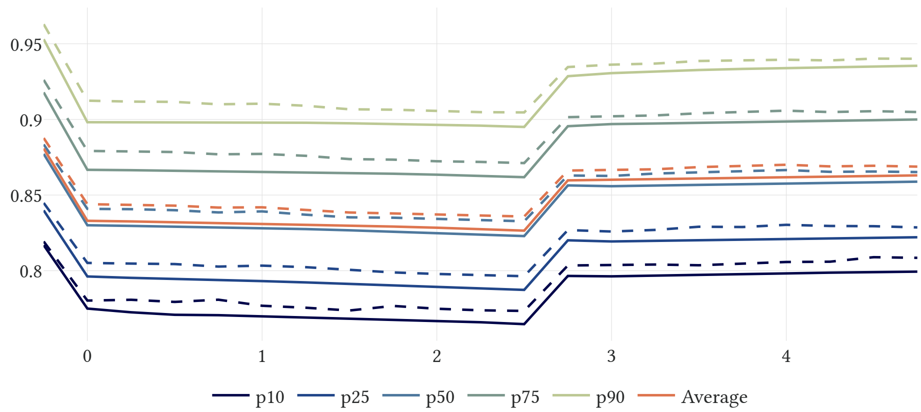


Figure 17: Value Functions in the Crisis

without sovereign risk, I allowed the wealth distribution and portfolios to change. Therefore, rich agents could adjust the level and composition of their savings to the prevailing conditions. At the bottom of the distribution, such adjustment is at least more limited in scope. This makes the welfare costs of sovereign risk regressive. In the IRF experiment, on the other hand, I do not give agents time to adjust and they have to go through the crisis with their no-default-risk portfolios.

B.3 *Who fears sovereign default? A robustness-based perspective*

The household's Euler equation offers insights into how sovereign risk affects different types of households. I calibrate the model with a unitary elasticity of intertemporal substitution. With this parameterization, households act 'as if' they had logarithmic preferences combined with concerns about model misspecification (Maenhout, 2004; Hansen and Sargent, 2001; Tallarini, 2000). In this reinterpretation, the risk aversion parameter maps into a robustness parameter. I define the *subjective expectation*, taken by an agent in state $(\mathbf{s}, \mathbf{S})$, of a random variable X as

$$\tilde{\mathbb{E}}\left[X \mid \mathbf{s}, \mathbf{S}\right] = \mathbb{E}\left[\frac{v(\omega', \epsilon', \mathbf{S}')^{1-\gamma}}{\mathbb{E}\left[v(\omega', \epsilon', \mathbf{S}')^{1-\gamma} \mid \mathbf{S}\right]} X \mid \mathbf{s}, \mathbf{S}\right] \tag{16}$$

The subjective expectation twists expectations by attaching more weight to states in which the household's value function is lower. It overstates events feared by the household. Figure 18 shows the twisted probability of default for each household, computed setting X to the indicator of a default in the next period in (16). The computation is conditional on a state of crisis. Figure 18 also shows the actual probability of default for comparison. Richer and higher-income households fear default, while poorer and low-income households fear the prolongation of the crisis.

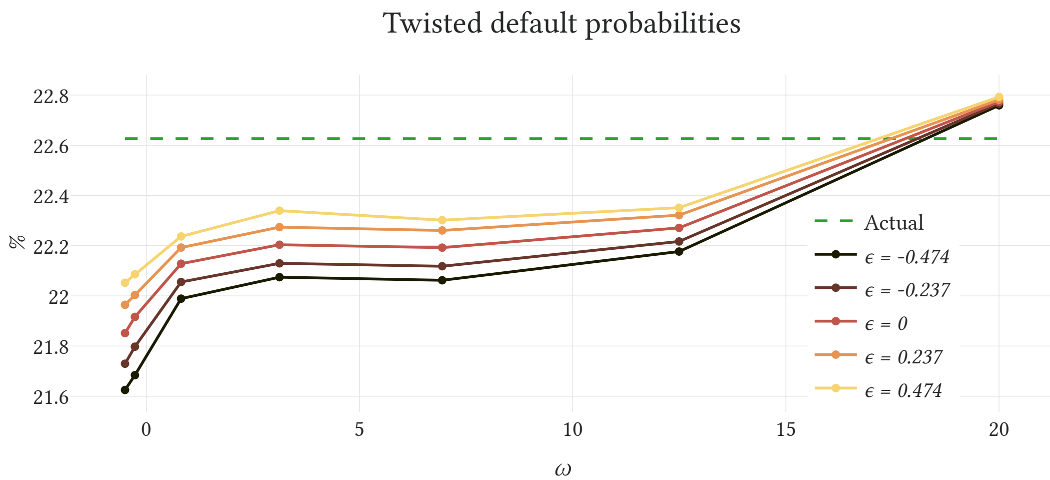


Figure 18: Subjective Probabilities of Default

B.4 The impact of inequality early in the crisis

Table 7 shows the results of regressing Y_T, output at the peak of the crisis, on the Gini coefficient, controlling for Y_{t_0}, output at the beginning of the crisis. The first column shows that samples starting with more inequality saw sharper contractions, while the next two columns show that samples in which inequality ended up being higher experienced sharper contractions. The final column uses the Gini early in the crisis as an instrument for the Gini later on.

B.5 Sovereign risk and the distribution of risk

Figure 19 shows the effect of rising sovereign risk on the distribution of risk portfolios across households. The left panel shows the distribution of holdings of the risk-free bond, while the right panel shows that of government bonds, averaging across crisis episodes. On average, agents cut consumption (see Figure 12), move away from holding government debt, and direct their savings towards the risk-free bond. For the poorest agents, in bottom 10% of the distirbution, income losses are such that they disinvest in both assets as well as cut consumption. Agents in the 25%

Table 7: Output in the crisis and pre-crisis conditions

	Y_T		
(Intercept)	1.282	1.719	1.559
	(0.059)	(0.075)	(0.112)
Y_{t_0}	0.114	0.016	0.063
	(0.037)	(0.035)	(0.043)
Gini_{t_0}	-0.065		
	(0.017)		
Gini_T		-0.645	-0.466
		(0.068)	(0.116)
Estimator	OLS	OLS	IV
N	696	696	696
R^2	0.065	0.154	0.146

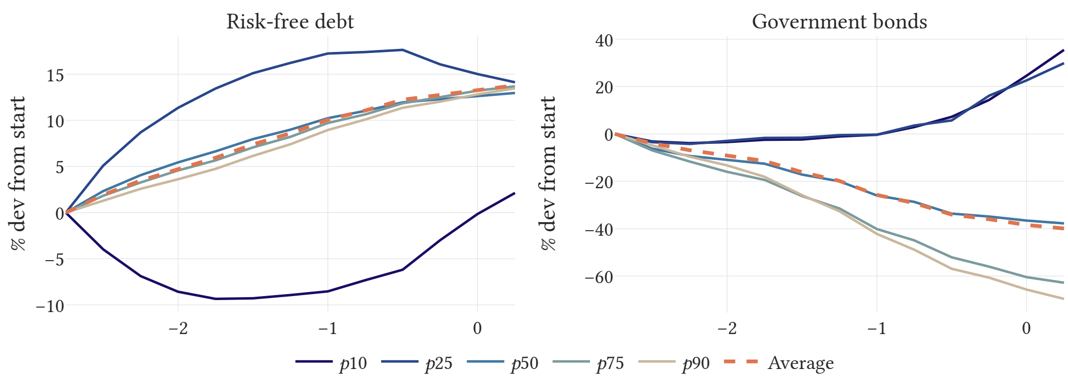


Figure 19: Crises across the Distribution

experience the largest flight to quality as they sharply increase their holdings of risk-free bonds. The richest agents sell most of their bonds (to foreigners) and retrench toward the risk-free asset.

B.6 *Data and the default-risk IRF*

Figure 20 superimposes the Spanish data to the default-risk IRF. The dynamics of the spread are

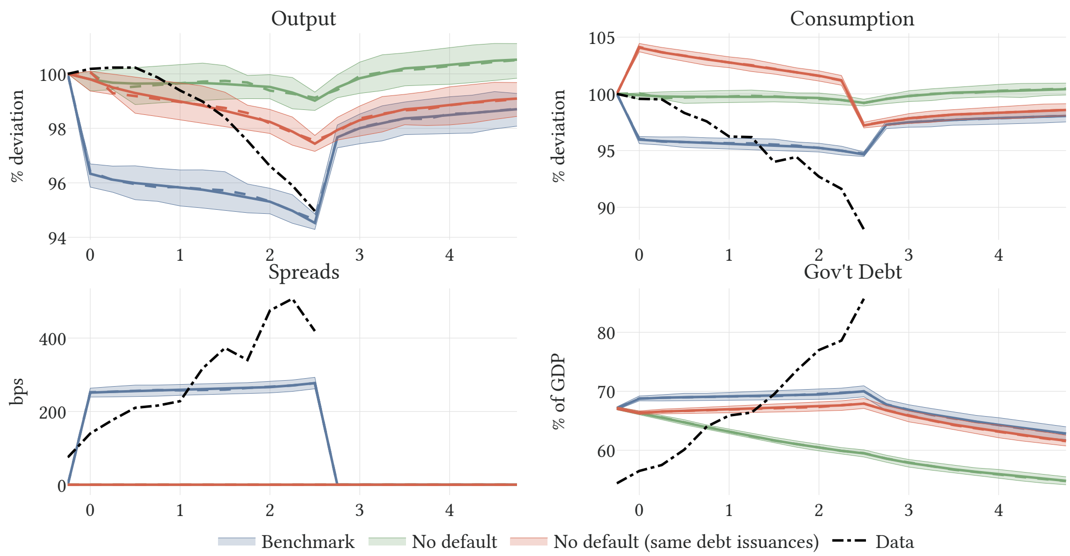


Figure 20: Default-risk IRF and Spanish data

more muted in the data, suggesting that it took some time for agents in the economy to harbor real doubts about debt repayment. The acceleration of spreads mechanically leads to a rapid increase in debt and consequently lump-sum taxes, weighing down on consumption. A version of this exercise in which the no-default economy faces a certain probability of transitioning to the benchmark one should replicate these dynamics much better.

B.7 *Wage rigidities and aggregate demand*

When sovereign risk increases, the demand for consumption is likely to fall. This feeds back to the rest of the economy mainly through the market for nontraded goods.

In the market for traded goods, firms can supply whatever quantities they produce at the international price. Therefore, for a given wage rate w_t prevailing in the economy, traded goods-producing firms observe the current level of TFP and choose employment accordingly.

The market for nontraded goods features more action, which is summarized by its supply curve. To trace it out, suppose a decrease in the relative price of nontraded goods. According to their first-order condition (17), firms respond to this decrease by cutting down production.

$$L_N^d = \left(\alpha_N \frac{p_N}{\max\{w, \bar{w}\}} \right)^{\frac{1}{1-\alpha_N}} \tag{17}$$

When firms in the nontraded sector retract their production they expell workers. This pushes down wages. In normal times, wages fall so some of these workers reallocate to the traded goods sector. At the same time, some others 'return' to work in the nontraded sector. When the constraint is binding, however, these second-round effects cannot happen: the fall in the price of nontradables results in an increase in unemployment and in a larger fall in the production of nontraded goods.

Figure 21 makes this point by showing that the supply curve is flatter when the constraint on wages is binding. This means that when demand falls, quantities fall more and prices fall less than in normal times. Wage rigidities create price stickiness.

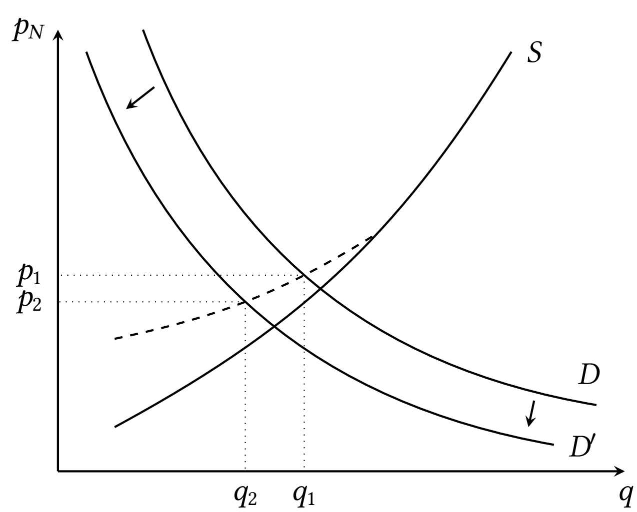


Figure 21: Market Clearing in the Nontradable Sector

The introduction of wage rigidities in this paper departs from the traditional approach of Schmitt-Grohé and Uribe (2016). In it, the wage in period t is constrained be no less than $\gamma_w w_{t-1}$, where $\gamma_w \leq 1$ is a parameter. I follow instead Bianchi et al. (2019) and set a constant lower bound $\bar{w}$ on nominal wages. While both assumptions are similar, in this context there are some advantages to the second formulation.

The first obvious advantage is that not having to carry the previous period wage saves one state variable. But there is a second advantage: in the traditional formulation good TFP shocks can be welfare-decreasing if they push the current wage rate too high and generate future unemployment. This is the 'overborrowing' externality emphasized by Schmitt-Grohé and Uribe (2016): individual households do not internalize that their consumption pushes up wages. In a scenario like this, where defaults also artificially depress TFP, a benevolent government might want to default on its debt only to suppress the overconsumption externality. This would lead to counterfactually many defaults in good times. If the government was allowed to choose spending and debt issuances, it could use fiscal policy to curtail the boom instead of defaulting. However, I am constraining the government to follow the estimated fiscal rules. Hence, the admittedly less realistic constant lower bound on wages is preferred.

B.8 Handling the distribution

I assume that agents believe the distribution of wealth to be always lognormal with mean μ_t and standard deviation σ_t. In practice, this means taking two approximations. The first approximation is that I only solve the model in a two-dimensional subspace, the lognormal distributions, of the infinite-dimensional space of all possible distributions. The second, more important approximation, is that I assume that agents believe next period's distribution to also be lognormal, even when their policy functions do not imply that λ_{t+1} is exactly lognormal even if λ_t is. The second assumption allows me to compute laws of motion for the parameters (μ_t, σ_t) by effectively projecting the exact λ_{t+1} onto the space of lognormal distributions.

Given all functions of the state (including the households' policy functions) and the current distribution, substituting λ_t for the corresponding lognormal in (18) yields a system for the joint evolution of the parameters of the distribution as well as the price of debt, which depends on the future distribution through the government's default incentives (i.e. because $\mathbf{S}_{t+1}$ contains

$(\mu_{t+1}, \sigma_{t+1}))$

$$\begin{cases} R_b(\mathbf{S}_{t+1}) & = & 1_{(d_{t+1}=1)}\kappa + (1-\rho)q^g(\mathbf{S}_{t+1}) \\ \int \omega d\lambda_{t+1} & = & \int \varphi_a(\mathbf{s}_t, \mathbf{S}_t) + R_b(\mathbf{S}_{t+1})\varphi_b(\mathbf{s}_t, \mathbf{S}_t) d\lambda_t \\ \int \omega^2 d\lambda_{t+1} & = & \int \left[\varphi_a(\mathbf{s}_t, \mathbf{S}_t) + R_b(\mathbf{S}_{t+1})\varphi_b(\mathbf{s}_t, \mathbf{S}_t)\right]^2 d\lambda_t \end{cases} \tag{18}$$

These approximations allow me to solve for the equilibrium of the model without the simulation step usual in models following Krusell and Smith (1998). Instead, I check in simulation that the agents' forecasting rule accurately predicts the dynamics of relevant variables.

To evaluate the accuracy of the approximation to the actual distribution of households across their idiosyncratic states, in each simulation I conduct the following tests. In period t, given the aggregate state variables in $\mathbf{S}_t = (B_t, \mu_t, \sigma_t, x_t, d_t, z_t)$, I compute the 'theoretical' value of some endogenous variables x_t from the model solution at this state. I then compare them with the 'actual' values which results from market clearing taking into account the actual distribution λ_t, for a histogram is available during the simulation. Table 8 reports the average of the absolute value of the relative discrepancies and shows that assuming a lognormal distribution does not result in large errors.

Table 8: Discrepancies in Simulation

Variable	**Avg. relative discrepancy**
Price of nontraded goods	0.22%
Consumption	0.92%

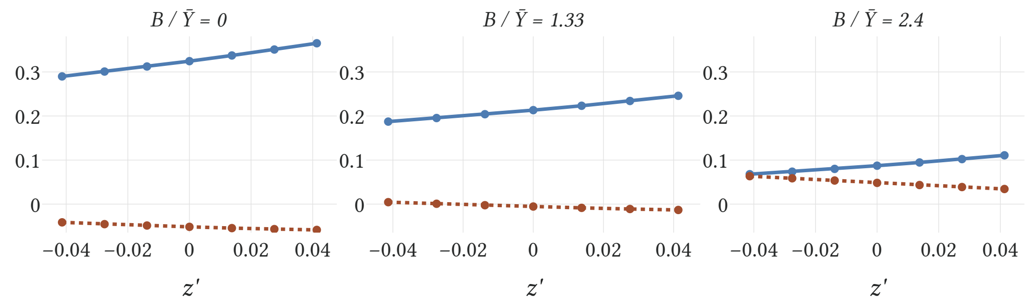


Figure 22: Transfers

Note: Blue lines plot transfers in repayment, red dashed lines plot transfers in default. Debt is expressed as share of mean income.

C. Evidence

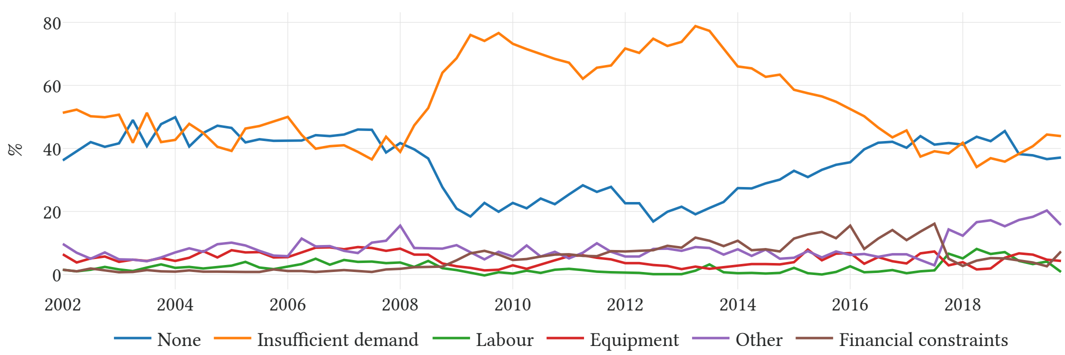


Figure 23: Factors Limiting Production

Source: Eurostat

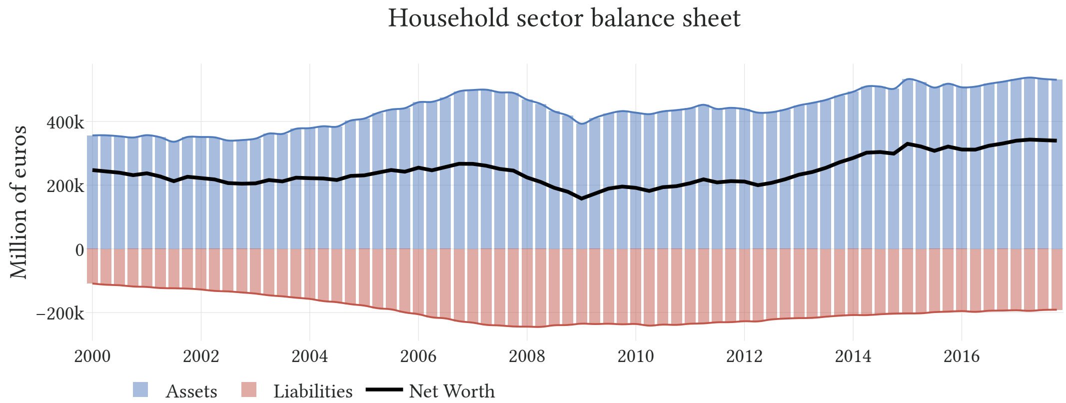


Figure 24: Net Worth of Spanish Households

Source: Eurostat

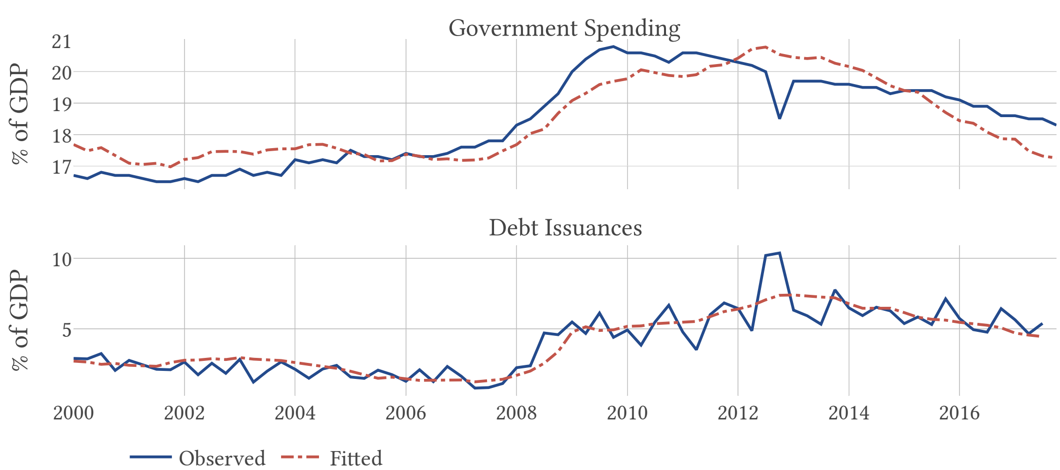
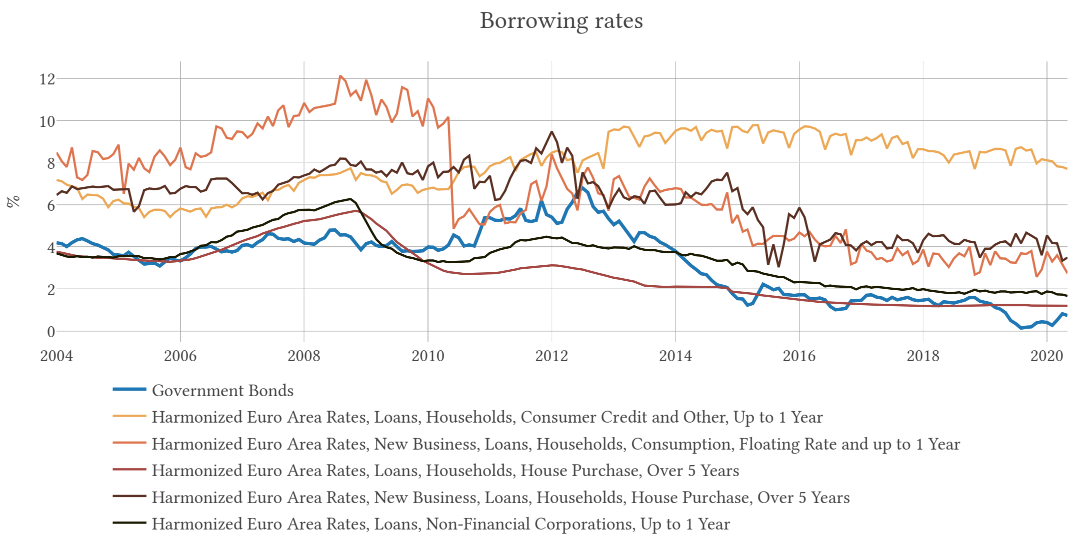


Figure 25: Estimated Fiscal Rules



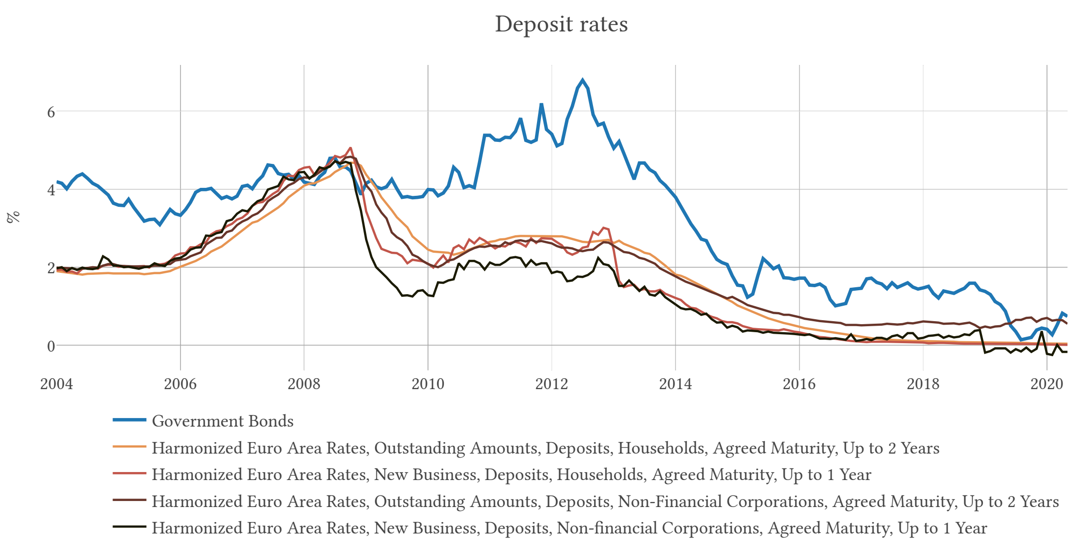


FIGURE 26: INTEREST RATES IN SPAIN

Source: International Financial Statistics, IMF