

Risk Aversion in Sovereign Debt and Default

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Winter SED
UTDT, December 2024

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Why risk aversion? Why in sovereign debt?

- In most RBC models, **macro-financial separation** holds
 - Elasticity of intertemporal substitution determines **allocations**
 - Risk aversion determines asset prices
- Sovereign debt literature typically inherits this line of thinking
 - CRRA preferences frequent, typically $\gamma = 2$
- **If** MFS holds in sovereign debt, macro outcomes robust to different preferences
 - In particular, calibration of output/utility costs of default
 - Less clear about welfare effects
 - ... losses from default, debt dilution
 - ... welfare effects of banning debt, introducing state-contingent bonds

Wanting risk prices in sovereign debt

This paper

- Show that macro-financial separation **breaks** in the sovereign debt model
- Understand the impact of preferences consistent with significant risk premia

Model

- Sovereign default model without default [reduces to an income-fluctuations problem]

$$v(b, z) = \max_{b'} u(c) + \beta \mathbb{E} [v(b', z') \mid z]$$

$$\text{subject to } c + \kappa b = q(b', z)(b' - (1 - \rho)b) + y(z)$$

$$\text{with } q(b', z) = \frac{1}{1 + r}$$

- We consider parametrizations of the model to vary risk aversion
 - ... with CRRA preferences $u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}$
 - ... With robustness, $u(c) = \log c$; replace \mathbb{E} with $\mathbb{T}[X \mid \mathcal{F}] = -\frac{1}{\theta} \log (\mathbb{E} [\exp (-\theta X) \mid \mathcal{F}])$

- Start from log-log [$\theta = 0$]: RA moves asset prices and welfare, not the macro

	loglog	$\theta = 1$	$\theta = 2$	$\theta = 3$
Average spread (bps)	0.0276	0.031	0.0406	0.138
Corr. NX,Y (%)	0.00777	0.00916	0.0114	0.0147
Rel. vol. cons (%)	1.59	1.62	1.65	1.66
Risk premium (p.p.)	0.0769	2.03	3.84	5.44
Debt-to-GDP (%)	29.7	29.5	29.2	28.9
Corr. deficit, y (%)	-0.0119	-0.0141	-0.0177	-0.0231
Welfare	1.034	1.008	0.9867	0.971

... welfare in autarky at $\theta = 3$ is 6pp lower than loglog or CRRA

Macro-financial separation without default (cont'd)

- Start from log-log [$\gamma = 1$]: EIS+RA moves mostly macro, not asset prices and welfare

	loglog	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Average spread (bps)	0.0276	0.0273	0.0269	0.0271	0.0285
Corr. NX,Y (%)	0.00777	0.0154	0.0852	0.397	0.668
Rel. vol. cons (%)	1.59	1.56	1.35	0.965	0.727
Risk premium (p.p.)	0.0769	0.227	0.627	1.02	1.67
Debt-to-GDP (%)	29.7	28.8	25.9	19.3	8.75
Corr. deficit, y (%)	-0.0119	-0.0251	-0.162	-0.605	-0.774
Welfare	1.034	1.03	1.021	1.01	0.9918

... in fully Epstein-Zin, move only EIS for even less effect on asset prices and welfare

Models with default

- Option value of default (with small pref. shocks for numerical performance)

$$\mathcal{V}(b, z) = \max\{v_R(b, z) + \epsilon_R, v_D(b, z) + \epsilon_D\}$$

- Similar equation for value of repayment v_R , debt prices reflect default probabilities

$$q(b', z) = \frac{1}{1+r} \mathbb{E} \left[(1 - \mathbb{1}_{D'}) (\kappa + (1 - \rho)q(b'', z')) \mid z \right]$$

- Costs of default

$$v_D(b, z) = u(h(y(z))) + \beta \mathbb{E} \left[\mathbb{1}_R \mathcal{V}(B(b, z'), z') + (1 - \mathbb{1}_R) v_D(b, z') \mid z \right]$$

$$h(y) = y(1 - d_0 - d_1 y)$$

- Risk aversion \implies no-smoothing in default costly \implies no macro-financial separation

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Quantitative properties

Calibration

- Keep the same discount rate, vary costs of default to match spreads and debt

	Parameter	$\gamma = 2$	loglog	$\theta = 3$
Sovereign's discount factor	β	0.9627	0.9627	0.9627
Sovereign's robustness parameter	θ	0	0	3
Sovereign's EIS	γ	2	1	1
Default output cost: linear	d_1	-0.2833	-0.2836	-0.247
Default output cost: quadratic	d_2	0.3253	0.3228	0.3029
Average spread (bps)	815	754	756	815
Debt-to-GDP ratio (%)	17.4	16.8	16.7	17.4

Comparative statics: CRRA

- Increasing EIS+RA: Less volatility, procyclical exports, more skewed debt outcomes

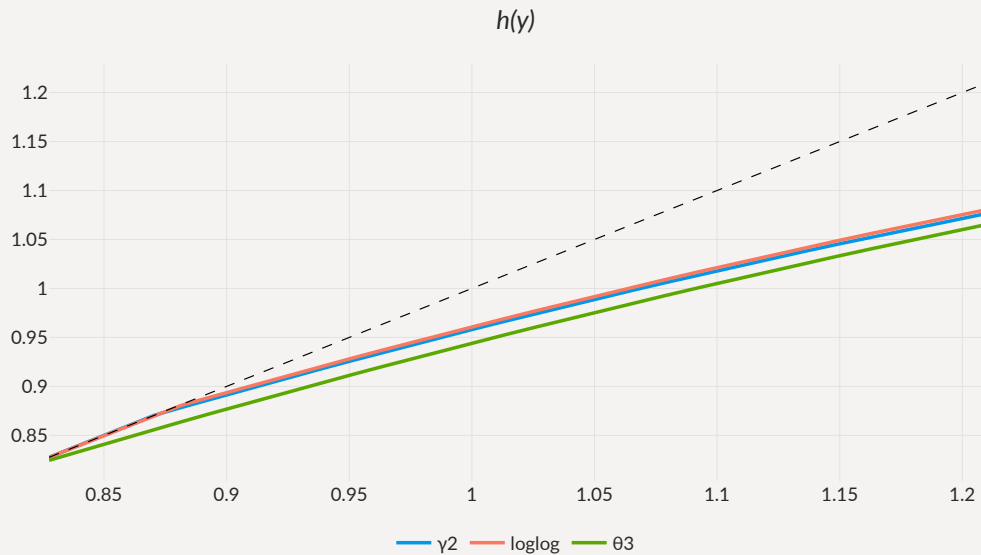
	loglog	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Avg. spread (bps)	756	800	912	974	1,057
Corr. NX,Y (%)	-0.285	-0.302	-0.21	0.0726	0.416
Rel. vol. cons (%)	1.5	1.37	1.18	1.04	0.921
Risk premium (p.p.)	0.652	0.789	1.02	1.28	2.38
Debt-to-GDP (%)	16.7	15.7	12.4	7.62	3.25
Corr. deficit, y (%)	0.391	0.391	0.217	-0.21	-0.627
Default freq. (%)	4.4	4.41	4.17	3.45	2.7
Std. dev. spreads (bps)	448	538	877	1,209	1,816
Welfare	1.013	1.01	1.002	0.9918	0.9728

Comparative statics: robustness

- Increasing RA: less debt tolerance, limited effect on volatilities

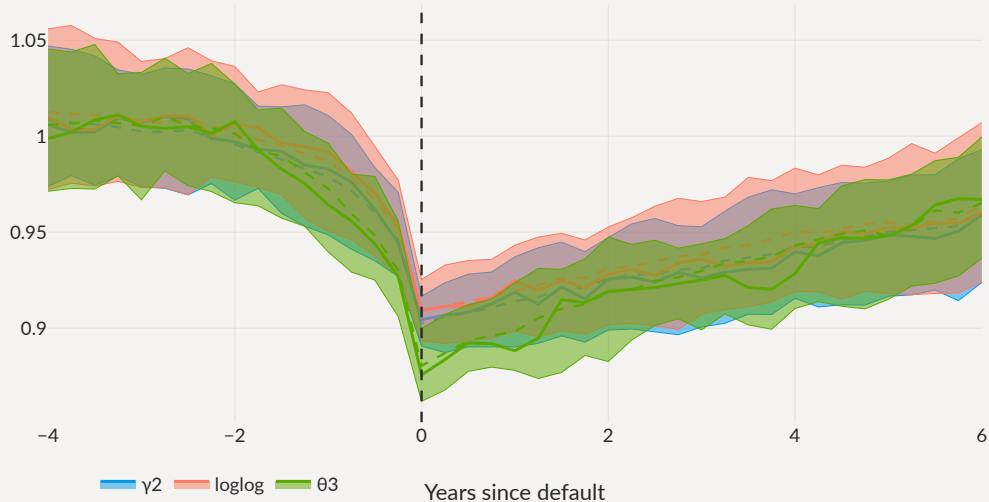
	loglog	$\theta = 1$	$\theta = 2$	$\theta = 3$
Avg. spread (bps)	756	1,683	20,240	36,331
Corr. NX, y (%)	-0.285	-0.227	-0.0901	-0.227
Rel. vol. cons (%)	1.5	1.38	1.26	1.46
Risk premium (p.p.)	0.652	2.92	4.43	6.99
Debt-to-GDP (%)	16.7	14.2	9.09	9.57
Corr. deficit, y (%)	0.391	0.292	0.118	0.266
Default freq. (%)	4.4	5.88	3.57	2.47
Std. dev. spreads (bps)	448	2,561	103,509	189,131
Welfare	1.013	0.9848	0.9629	0.9469

Calibrated output costs of default with robustness



Event-study of defaults

Output around defaults

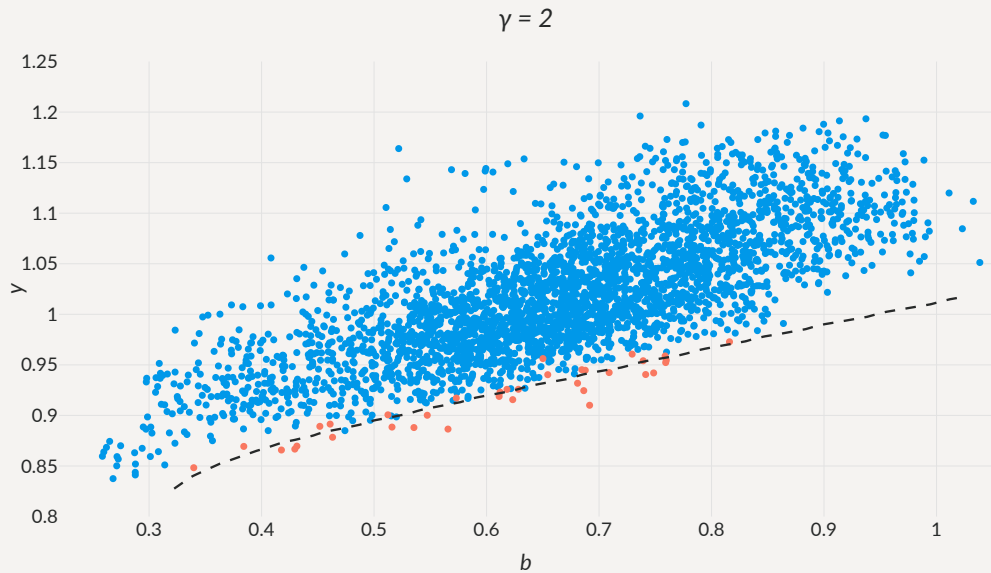


Calibrations with risk aversion

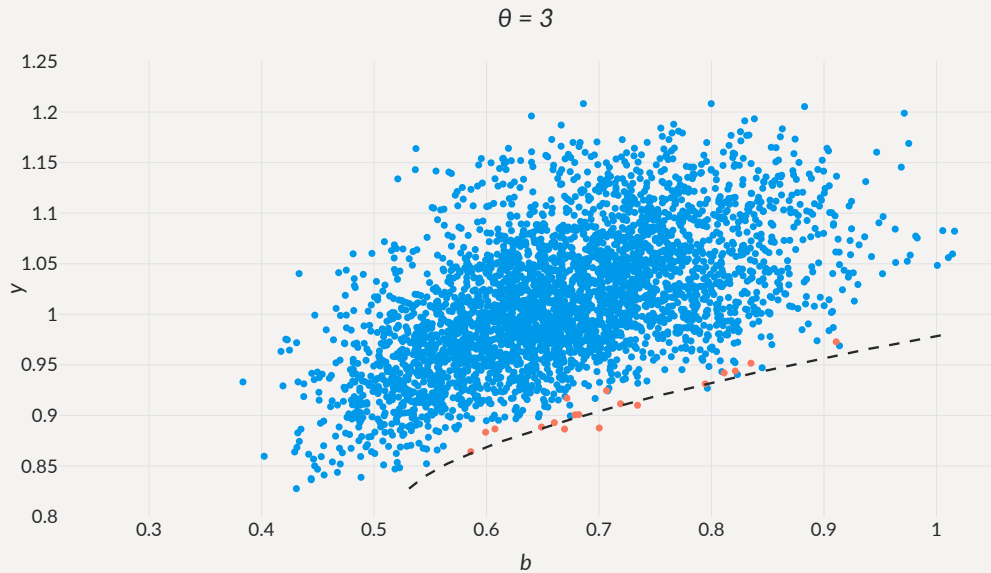
- Calibration with robustness: skewed debt outcomes, small decrease in macro volatility

	Data	$\gamma = 2$	loglog	$\theta = 3$
Avg. spread (bps)	815	754	756	815
Corr. NX,Y (%)	-	-0.314	-0.285	-0.194
Rel. vol. cons (%)	0.94	1.38	1.5	1.35
Risk premium (p.p.)	-	0.778	0.652	5.9
Debt-to-GDP (%)	17.4	16.8	16.7	17.4
Corr. deficit, y (%)	-	0.405	0.391	0.207
Default freq. (%)	-	4.21	4.4	1.51
Std. dev. spreads (bps)	443	496	447	2,026

Ergodic distribution for debt in CRRA model

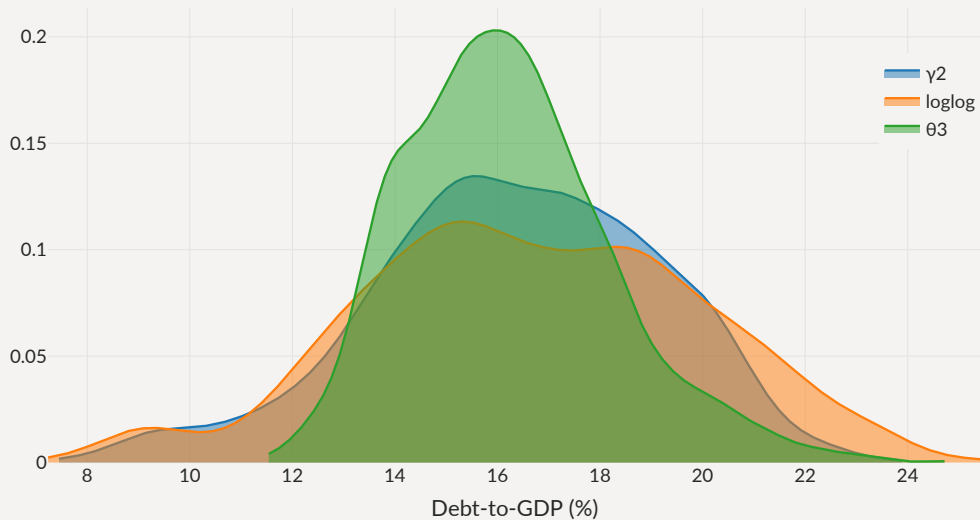


Ergodic distribution for debt with robustness



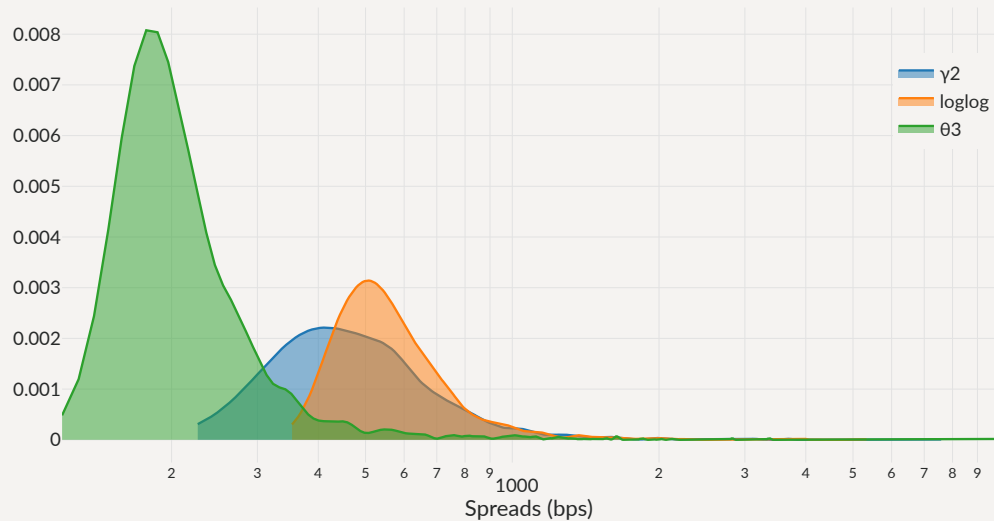
Ergodic distribution for debt

Distribution of debt levels



Ergodic distribution for spreads

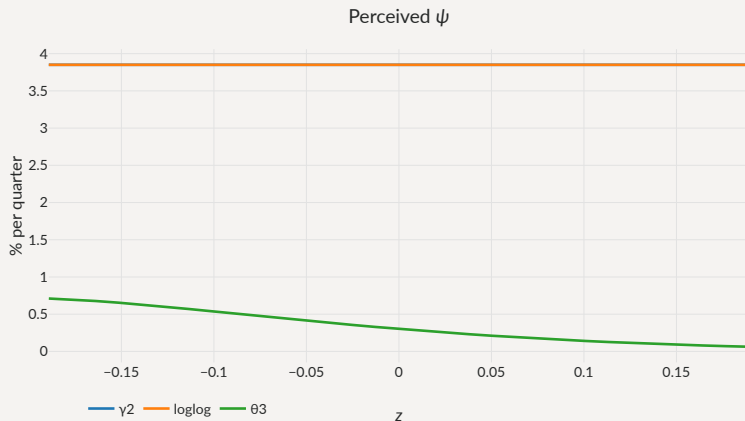
Distribution of spread levels



Worst-case models

- Distorted expectation of X

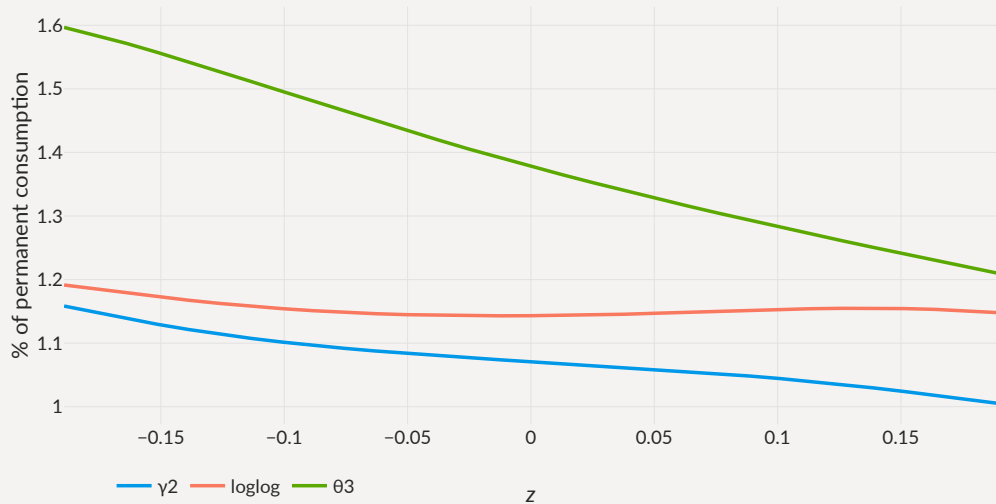
$$\tilde{\mathbb{E}}[X \mid \mathcal{F}] = \mathbb{E} \left[\frac{\exp(-\theta v(s'))}{\mathbb{E}[\exp(-\theta v(s')) \mid \mathcal{F}]} X \mid \mathcal{F} \right]$$



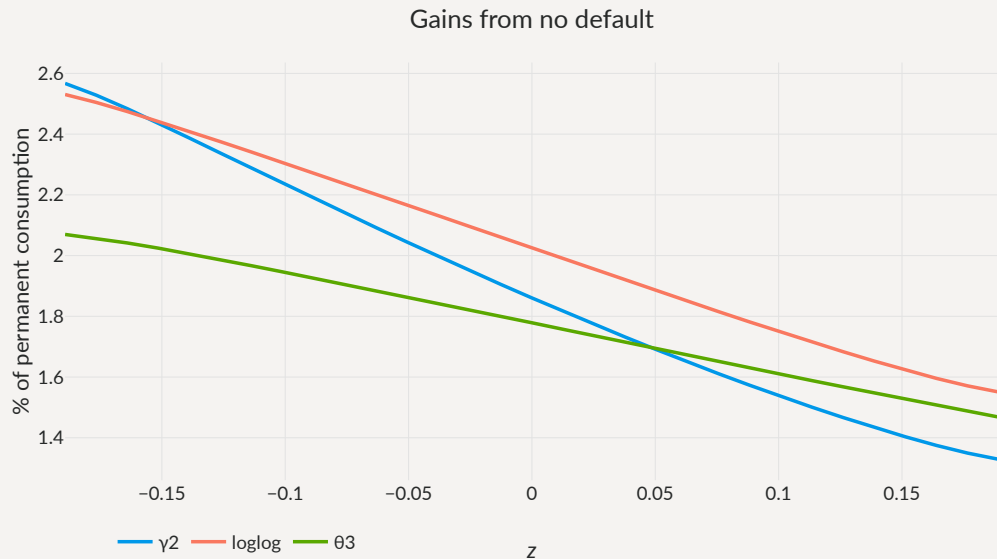
Welfare effects

Welfare effects of debt

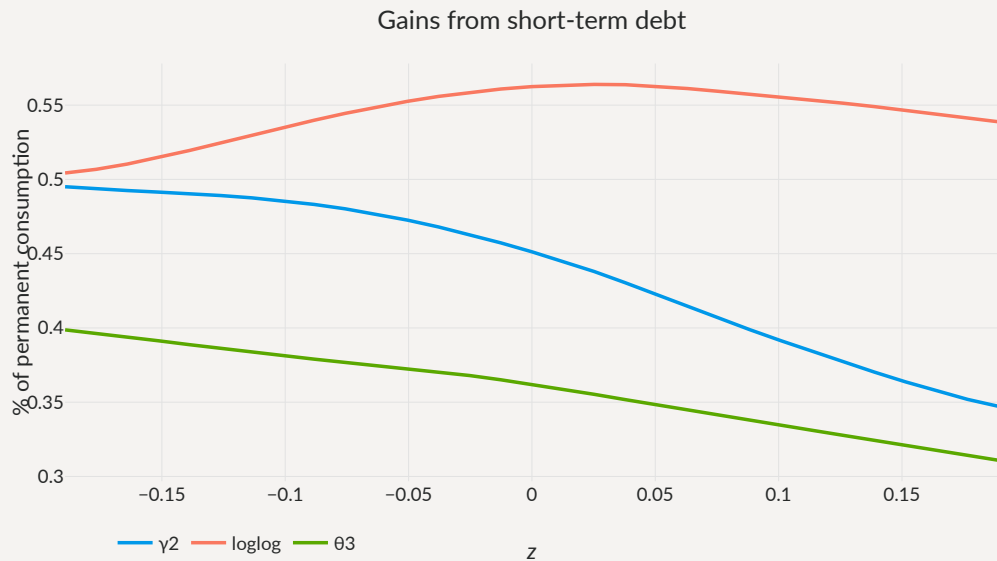
Gains from access to debt



Welfare effects of banning defaults



Welfare effects of shortening maturity



With preferences consistent with positive risk premia

- *Lower debt tolerance*
 - ... Larger default costs required
- *Less staying at the edge of default*
 - ... More skewness in the distribution of debt and spreads
- *More use of the debt for insurance*
 - ... Large gains from debt access, not so much for making debt safe

Welfare gains decomposition

Consumption without default costs c_t^R

$$c^R(b, z) = \mathbb{1}_{\mathcal{D}}(b, z)y(b, z) + (1 - \mathbb{1}_{\mathcal{D}}(b, z))c(b, z)$$

Evaluate value of consuming c^R [instead of c] and removing uncertainty

$$V_{NC}(b, z) = u(c^R(b, z)) + \beta \mathbb{E} [V_{NC}(b', z') \mid z]$$

$$V_{NV}(b, z) = u(c^R(b, z)) + \beta V_{NV}(b', \mathbb{E} [z' \mid z])$$

Welfare gains between models/equilibria with value functions v and v^*

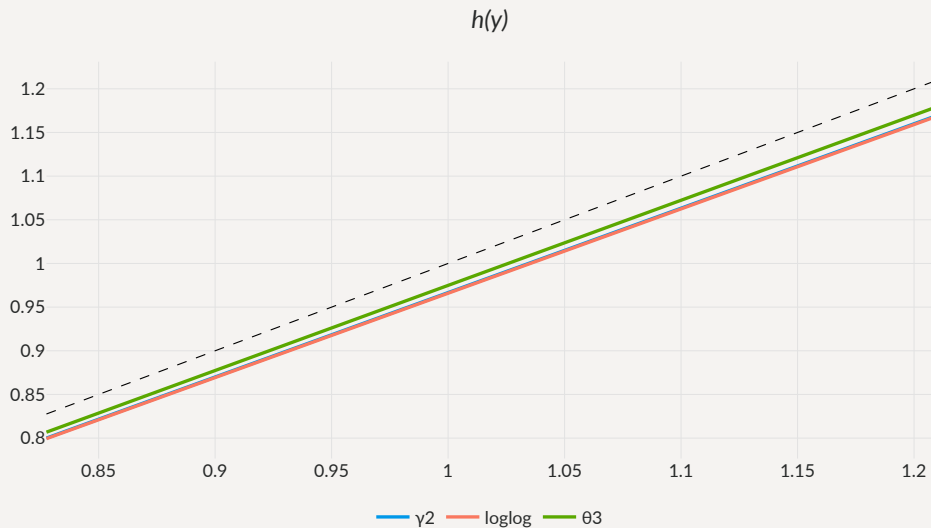
$$\frac{v^*(b_0, z_0)}{v(b_0, z_0)} = \frac{v^*(b_0, z_0)/v_{NC}^*(b_0, z_0)}{v(b_0, z_0)/v_{NC}(b_0, z_0)} \times \frac{v_{NC}^*(b_0, z_0)/v_{NV}^*(b_0, z_0)}{v_{NC}(b_0, z_0)/v_{NV}(b_0, z_0)} \times \frac{v_{NV}^*(b_0, z_0)}{v_{NV}(b_0, z_0)}$$

Welfare gains

	Total gains	From default costs	From volatility	From level
$\gamma = 2$				
Access to markets	0.622	-0.273	0.218	0.679
No default	1.87	0.274	-0.292	1.89
Short-term debt	0.411	0.255	-0.448	0.606
loglog				
Access to markets	0.663	-0.294	0.284	0.674
No default	2.04	0.295	-0.345	2.09
Short-term debt	0.519	0.272	-0.439	0.688
$\theta = 3$				
Access to markets	0.961	-0.25	0.0354	1.18
No default	1.72	0.251	-0.0744	1.54
Short-term debt	0.262	0.233	-0.45	0.481

Model with linear costs

- Convex costs lower income volatility **during** defaults



Concluding remarks

Risk aversion in the sovereign debt model

- We evaluate preferences consistent with risk premia in the sovereign default model
 - ... mostly possible to match standard calibration targets with robustness
- Effect of robustness concentrated at higher-order moments
 - ... makes crises look like more abrupt events
- Innocent-looking features of the standard model weigh against large risks/distortions
 - ... convex costs of default mute post-default uncertainty

	loglog	$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Corr. NX,Y (%)	-0.00131	-0.00131	-0.00131	-0.00131	-0.00131
Rel. vol. cons (%)	1	1	1	1	1
Risk premium (p.p.)	0.0833	0.251	0.751	1.57	3.05
Welfare	1.002	1	0.9951	0.9868	0.9699

	loglog	$\theta = 1$	$\theta = 2$	$\theta = 3$
Corr. NX,Y (%)	-0.00131	-0.00131	-0.00122	-0.00073
Rel. vol. cons (%)	1	1	1	1
Risk premium (p.p.)	0.0833	2.02	3.81	5.32
Welfare	1.002	0.9769	0.9564	0.9411