Sovereign Debt Tolerance with Potentially Permanent Costs of Default*

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Abstract

We investigate the effect of uncertainty about the nature of output costs of sovereign default on debt tolerance. While the theoretical literature assumes output losses lasting until market access is restored, the empirical evidence points to persistent effects, and output may not return to its pre-default trend. We include such uncertainty in a model of sovereign default and find that it can significantly boost equilibrium debt levels. We also consider a government which is averse to this type of uncertainty and seeks robust decision rules. We calibrate the model to match evidence on the output trajectory around debt restructuring episodes and infer output costs of about the size found in the empirical literature, alongside significant uncertainty about their permanence and a strong desire for robustness.

JEL Classification E43, E44, F34, H63

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Introduction

A fundamental question in the sovereign debt literature, and one to which Rogoff made substantial contributions, is what supports debt repayments. In the absence of international enforcement of sovereign debts, costs of default, actual and perceived, physical and reputational, determine how much debt can be sustained as well as the terms at which countries borrow.

This paper focuses on how the possibility of permanent scarring effects of a default heightens the ex-ante costs of restructuring. Motivated by the lack of conclusive empirical evidence one way or the other, we consider uncertainty about how permanent such costs are. We draw implications for the amount of sustainable debt in equilibrium and find that the possibility of permanent costs has a large impact on debt tolerance (Reinhart, Rogoff, and Savastano, 2003), especially when combined with realistic risk attitudes for the debtor country.

In two influential papers, Bulow and Rogoff (1989a) show the limits of reputation and exclusion alone to sustain otherwise unenforceable payments on sovereign debt. Bulow and Rogoff (1989b) propose the inclusion of other costs affecting the economy, such as direct sanctions interfering with trade. The idea that countries sustain debt because defaults entail costs for the real economy still underpins most of the literature on sovereign debt, even if their source has shifted away from sanctions.[1] Costs can also involve broader general reputational spillovers affecting the rest of the economy, as in Cole and Kehoe (1998). Most models in the theoretical literature assume an output cost of default which lasts until credit market access is restored.

It is well documented that economic growth declines around the time of restructurings or defaults (Borensztein and Panizza, 2009) but the exact nature of such costs is not so clear. Subsequent studies with different methodologies (De Paoli et al., 2009; Asonuma et al., 2023; Farah-Yacoub et al., 2022; Cerra and Saxena, 2008) find growth slowdowns following a debt restructuring, some of which are quite persistent.

In practice, countries go to great lengths to remain current on their debts. If anything, they tend to wait until restructurings come "too late" (IMF, 2013), which suggests large perceived costs of restructurings, even though we do not observe much in the way of direct punishments, certainly not in the magnitude of the defaulted debt.

We evaluate the effect of potentially permanent costs within a sovereign default framework à la Eaton and Gersovitz (1981) with trend growth combined with uncertainty about the type of

[1] For surveys of the literature on sustainable public debt and sovereign default, see the handbook chapters by Aguiar and Amador (2014), Aguiar, Chatterjee, Cole, and Stangebye (2016), D'Erasmo, Mendoza, and Zhang (2016), and Martinez, Roch, Roldán, and Zettelmeyer (2023).

costs associated with default. In the model, a government chooses whether to repay its debt without knowing if defaulting would trigger a transitory or permanent decline in output. Moreover, we consider a government who does not fully trust its prior about the probability of a transitory cost and seeks robust decision rules as in Hansen and Sargent (2001).[2]

To obtain decision rules which are robust to different, unidentified misspecifications of its prior on the output cost persistence, the government entertains plausible distortions of it and evaluates each action's payoff under the worst one, the so-called worst-case model. While robustness is in general technically equivalent to risk-sensitive Epstein-Zin preferences (Anderson, Hansen, and Sargent, 2003), we leverage the flexibility of robustness to emphasize aversion towards uncertainty about the permanence of default costs in particular.[3]

These departures from the canonical model allow us to match reduced-form evidence on the trajectory of output following a debt restructuring. Our calibration identifies a degree of robustness and a probability of permanent costs which contribute to an increase of about a third in debt tolerance. Making the costs permanent for sure leads to an increase in debt tolerance of almost a half relative to fully transitory costs. Conceptually, a higher probability of permanent costs and a higher degree of robustness both contribute to debt tolerance, but with different implications for the observed output trajectory following a typical default. Finally, we also find that the robust government sustains higher levels of debt (as if permanent costs were more likely) but does not default less often.

The remainder of the paper is organized as follows. Section 2 discusses some of the possible costs associated with a restructuring and reviews the findings from the empirical literature on the costs of default. Section 3 presents the model and Section 4 describes its calibration. Section 5 discusses the equilibrium and quantitative results of the model. Section 6 concludes.

2. Output losses and other costs around restructurings

The empirical literature estimates sizable impacts of defaults on economic activity, but deciding whether such effects are permanent is a statistically daunting task. [4]. Borensztein and Panizza (2009) estimate a 2.6 percentage point decline in growth in the first year of the episode, but that effect (on growth) is short lived with no statistically significant effect on lagged variables. Levy Yeyati and Panizza (2011) also find in quarterly data that much of the contraction actually takes place

[2]Robustness is also featured in Pouzo and Presno (2016) and Roch and Roldán (2023) in the context of sovereign debt, but on the lender side.

[3]For a discussion of robustness to "structured uncertainty," see Hansen and Sargent (2015)

[4]Following most of the literature we use the term default loosely to refer to any distressed debt restructuring

before the default event in question. Asonuma and Trebesch (2016) show that restructurings that take place preemptively before payments are missed tend to be quicker and have better growth outcomes than those that take place after a default. Asonuma et al. (2023) estimate local projections on the level of log GDP relative to pre-crisis trend to investigate the persistence of a level effect. The adverse effects tend to be mild and temporary for preemptive restructurings, but large and persistent for restructurings involving a default (which historically has been the case in most restructurings). They estimate GDP to remain 4 percent below its pre-crisis trend five years after the default, which suggests these losses are not reversed.[5] Farah-Yacoub et al. (2022) use synthetic controls to estimate the economic and social costs of default on a long historical sample. They estimate a cumulative 8.4 percentage point gap within the first three years, with substantial scarring effects even after a decade.

Most of the literature has focused on external debt restructurings. One notable exception is Reinhart and Rogoff (2011), which show that the output decline after domestic restructurings is much larger than for external ones. Domestic debt restructurings were relatively rare during the 1980s and early 1990s, but have become more common as countries have started to rely more on domestic financing (IMF, 2021). Domestic restructurings can more directly lead to major financial distruption which can by itself entail significant output costs (Pérez, 2018). But following most of the literature, our paper focuses on external debt restructurings.

There is a wide range of outcomes following a debt restructuring. Some countries do experience mild and temporary declines, while for others the impact is deeper and more protracted. A few even experience a growth acceleration. Figure 1 plots the evolution of the log of GDP relative to the pre-restructuring trend for different countries experiencing an external debt restructuring.[6] The left panel covers EMs and the right one, Low-Income Countries (LICs).[7] Each line traces the evolution of a country from 5 years prior to its restructuring to 10 years afterwards. When there are multiple restructurings within that time frame, we code the event based on the first restructuring (e.g. if a country restructures in 1982 and then again in 1987, the lines would plot the time from 1977 to 1992). The log GDP series is detrended using a linear time trend based on the 5 years prior to the restructuring.[8] Hence by construction, the series hovers around zero

[5] Most sources of catch-up growth such as pent-up demand should have already materialized in that horizon

[6] The events are based on the Asonuma and Trebesch dataset which includes both restructurings that take place after payments are missed (default) as well as distressed exchanges that take place preemptively prior to payments being missed. The latter is less frequent, but has become relatively more common over time.

[7] We define these two groups based on whether they are covered by the IMF-World Bank LIC Debt Sustainability Framework or the one for market access countries. The sample is listed in the Appendix.

[8] A linear trend is preferable to using more sophisticated filters, such as HP, which would use observations after the crisis to inform its trend. One alternative could be to use one-sided filters using only backward-looking data, but for the purposes of this illustration a simple linear trend suffices.

prior to the restructuring, but can deviate significantly after that event.

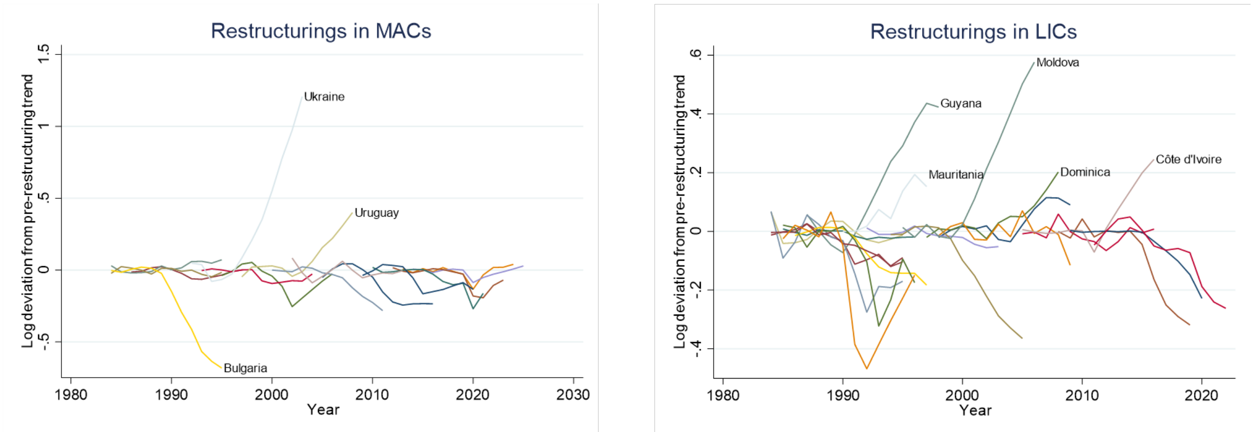

Figure 1: Growth outcomes around debt restructurings

There are fundamental differences between EM and LIC restructurings, especially in the historical sample. The official sector plays a much larger role in LICs, and many of those countries remained deeply insolvent until their debt crises was eventually resolved as part of the Heavily Indebted Poor Countries (HIPC) initiative. HIPC, combined with the Multilateral Debt Relief Initiative (MDRI) provided deep debt writedowns (enough to bring debt down to 150 percent of exports, sometimes even less). This helps explain the weak performance of LICs prior to the late 1990s/early 2000s, and their relatively strong performance afterwards. For that reason, we focus our attention on the sample of EMs, as shown in Figure 1A.

The growth performance of EMs tends to be weaker following restructurings, which were much more common in the 1980s, as expected. There are several cases where GDP remains well below pre-crisis trends, reflecting the severity of that systemic event and the long time it took for its eventual resolution. It is also worth noting that external financial conditions were particularly unfavorable at that time (due to very high U.S. interest rates), and that many of the affected countries had deep structural problems (for example, very closed economies particularly in Latin America). From the 1990s onwards, we see relatively fewer restructuring episodes. We still see quite a few cases where GDP remains below trend, but there are also a number of cases where the lines remain closer to zero (which again, indicates the pre-crisis trend) or where the initial decline starts to revert sooner. There were a number of important innovations in the debt restructuring technology over time, including the adoption of Collective Action Clauses (CACs) on bonds and eventually enhanced CACs which allowed aggregation of votes across series (making it more difficult for a creditor to hold out when there was widespread support from other creditors to a restructuring). IMF (2020) takes stock of developments in sovereign debt restructurings involving the private-sector creditors and notes that compared with previous periods, recent restructurings have generally proceeded smoothly and quicker, and were more likely to be preemptive.

On balance, the output costs are substantial in the historical sample. There are a number of steps a debtor can take to minimize those costs, such as collaboratively engaging with creditors towards restructuring before payments are missed. But there are many things beyond the debtor's control, and restructurings are inherently complicated processes. Thus, even these costs have come down in expectation, there is still a tangible risk that things could go wrong and the costs turn out to be large. That cost can be further amplified by the risk and ambiguity aversion of the debtor, and help sustain more debt in equilibrium.

2.1 *Other costs*

Our focus has been on credit market exclusion combined with output costs, since these are easier to directly map into a model. As Bulow and Rogoff (1989a), Bulow and Rogoff (1989b), and Rogoff (2022) point out, credit exclusion in itself cannot yield a large enough cost to sustain observed levels of debt, which is why researchers tend to combine it with direct costs on productivity or income. But there are other potential sources of costs.

Direct sanctions have been proposed as another channel that could help sustain debt. Such punishments could translate into a decline in output, financial exclusion, and depending on the nature of the sanctions have a direct effect on welfare over and beyond the decline in output. In practice, we do not observe much in the way of such punishments on debtors. Even high profile cases of litigation involving asset seizures involve sums that are nowhere near the order of magnitude of the debt claims. The main example of a successful and disruptive litigation is Argentina vs NML Capital, where a 2014 ruling prevented the debtor from repaying the restructured bonds (from its 2001 default) without repaying the litigants in full. Such litigation can enforce credit market exclusion as a punishment (since it discourages new credit by interfering with repayments to new creditors). The debtor settled with that holdout creditor in 2016. Since then, New York courts have clarified that the decision to repay some creditors but not others in and of itself does not breach the contractual clause that underpinned NML's strategy. And that successful strategy will become increasingly difficult to pursue as enhanced CACs become more widespread. In almost all cases, litigants were not able to seize assets or disrupt ongoing restructurings (IMF, 2020).

It is possible that direct sanctions do not play a significant role because borrowers do follow procedures and norms that have come to be expected during a debt restructuring. Debtors typically ask only for the relief needed to restore sustainability with reasonable buffers, and most restructurings take place in the context of an IMF-supported program to facilitate the adjustment (and where a debt sustainability analysis informs the envelope of relief needed to restore sustainability based on the feasible adjustment.) In practice, even debtors that take a fairly adversarial position with respect to their creditors follow these norms and are eager to bring the restructuring to a close. This is illustrated by the fact that if anything, restructurings tend to deliver "too little" relief (IMF, 2013; Asonuma and He, 2023). EM bond restructurings are rarely associated with significant face-value reductions, with much of the net present value relief coming from the rescheduling of debt at a moderate interest rate (significantly lower than the country would face if it were to issue new debt in the absence of a restructuring).

If countries were to deviate from these norms, e.g. ask for significant relief where it is not needed, they would likely face stronger pressures from the international community. But it is hard to test such hypothesis in the absence of counterfactuals. Even if such considerations play a role in practice, we still have the question of what costs can sustain debt relative to the choice of restructuring following established norms.

3. Model

Resources The economy receives an endowment stream following a stochastic process with trend and cycle components

$$Y_t = \exp(z_t)\Gamma_t \tag{1}$$

with

$$z_t = \rho z_{t-1} + \sigma_z \varepsilon_t^z$$
$$\log(\Gamma_t) = \log(\Gamma_{t-1}) + \log(g_t)$$

where $\varepsilon_t^z \overset{iid}{\sim} \mathcal{N}(0,1)$ and $\log(g_t) \overset{iid}{\sim} \mathcal{N}(0,\sigma_g)$.

We normalize variables by Γ_t and denote normalized values with lowercase. For example, $y_t = Y_t/\Gamma_t = \exp(z_t)$ and the utility function satisfies

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma}\frac{(C/\Gamma)^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma}u(c)$$

which implies a normalization constant $\Gamma_t^{1-\gamma}$ for the value functions, as laid out in more detail in Appendix A. We keep the formulation for the trend Γ as parsimonious as possible since its only purpose is to enable possibly permanent costs of default, which would manifest as ex-post decreases in Γ.

Assets The government borrows from international lenders in the form of a defaultable bond which promises to pay a noncontingent stream of geometrically-decaying coupons as in Leland

(1998), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012). A bond issued in period t pays $(1-\rho)^{s-1}\kappa$ units of the good in period $t+s$, which effectively makes a one-period-old bond a perfect substitute of $(1-\rho)$ units of newly-issued debt. The coupon rate $\kappa = r + \rho$, where r is the international risk-free rate, is chosen so that the price of a bond that is expected to never default is $q^\star = 1$.

Upon default, the government loses access to international capital markets and faces a loss of output. There is uncertainty about whether this loss of output is permanent or transitory, as well as about the length of the exclusion period.

Government The government is benevolent and makes choices on a sequential basis to maximize the utility of a representative household with preferences given by

$$V_t = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(u(C_{t+s}) + \varepsilon_{t+s} \right) \right] = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \Gamma_{t+s}^{1-\gamma} \left(u(c_{t+s}) + \varepsilon_{t+s} \right) \right] \tag{2}$$

where $\mathbb{E}$ denotes the expectation operator, C_t represents the household's consumption, β is a discount factor, and ε is a preference shock for default or repayment.[9]

While the government is not in default, it chooses whether to repay the debt and attains a value

$$v(b, z, g) = \max \left\{ v_R(b, z, g) + \epsilon_R, v_D(z, g) + \epsilon_D \right\} \tag{3}$$

where the ϵ's follow a Type 1 Extreme Value distribution as in Chatterjee et al. (2018), yielding familiar closed forms for $v(b, z, g)$ and the (ex-post) default probability $\mathcal{P}(b, z, g)$

$$\begin{aligned} v(b, z, g) &= \chi \log \left(\exp(v_D(z, g)/\chi) + \exp(v_R(b, z, g)/\chi) \right) \\ \mathcal{P}(b, z, g) &= \frac{\exp(v_D(z, g)/\chi)}{\exp(v_D(z, g)/\chi) + \exp(v_R(b, z, g)/\chi)} \end{aligned}$$

If the government chooses the repay the debt, it can access capital markets and issue new debt h (because of the normalization, the indebtedness state variable for the next period $b' = h/g'$), so that

$$\begin{aligned} v_R(b, z, g) = & \max_h u(c) + \beta \mathbb{E} \left[(g')^{1-\gamma} v(h/g', z', g') \mid z \right] \\ \text{subject to} \quad & c + \kappa b = y(z) + q(h, z, g)(h - (1-\rho)b) \end{aligned} \tag{4}$$

[9] We follow Dvorkin et al. (2021) and introduce preference shocks for repayment and default to improve the numerical convergence of the algorithm used to solve the model. As is also common, we keep their variance small to ensure that they do not modify the quantitative properties of the model. However, plausible stories to microfound a meaningful inclusion of such shocks include shifting political preferences, developments in domestic debt markets or in the banking system, or changes in the external borrowing terms faced by private firms. Fourakis (2023) uses preference shocks to obscure the actions taken by two possible 'types' of the government and thus enable a signalling dimension to government borrowing, default, and renegotiation.

If the government chooses to default, it loses access to international capital markets, which it recovers with a constant hazard ψ. While it is excluded, the borrowing economy suffers a loss of output $\varphi(Y)$. Upon default, a shock $k \in \{T, P\}$ determines whether the loss of output is transitory or permanent, which happen with probability p and $1-p$, respectively, independent of the rest of the state vector. We assume that all debt is destroyed after a default[10] for tractability and to more readily compare with other results in the literature. The expected value of default is

$$v_D(z, g) = p v_D^T(z, g) + (1-p)\left(\frac{g-\varphi}{g}\right)^{1-\gamma} v_D^P(z, g-\varphi) \tag{5}$$

where the normalization constant differs in the case of a permanent default cost and

$$v_D^k(z, g) = u(y(z) - 1_{(k=T)}\varphi) + \beta\mathbb{E}\left[(g')^{1-\gamma}\left(\psi v(0, z', g') + (1-\psi)v_D^k(z', g')\right) \mid z\right] \tag{6}$$

Lenders Bonds issued by the government are purchased by deep-pocketed, risk-neutral foreign investors who equate the expected return of the debt to their cost of funds r, yielding a debt price

$$q(h, z, g) = \frac{1}{1+r}\mathbb{E}\left[(1-\mathcal{D}(h/g', z', g'))(\kappa + (1-\rho)q(h', z', g')) \mid z\right] \tag{7}$$

where $\mathcal{D}$ denotes the government's default policy expected by lenders. We implicitly assume these are private investors, but in principle this could also apply to risk-neutral official creditors.[11]

Observation The dependence of all value functions on g is *only* mediated by the renormalization term $\left(\frac{g-\varphi}{g}\right)^{1-\gamma}$ which appears in the value of default v_D. A particular but fairly standard specification of the default cost allows us to avoid having to carry g as a state variable.

Linear costs We assume that default costs are linear in output so that $\varphi(Y) = \Delta Y$. This reduces the formulation of the value of default to

$$\begin{aligned} v_D(z) &= p v_D^T(z) + (1-p)(1-\Delta)^{1-\gamma} v_D^P(z) \\ v_D^k(z) &= u(y(z)(1 - 1_{(k=T)}\Delta)) + \beta\mathbb{E}\left[(g')^{1-\gamma}\left(\psi v(0, z') + (1-\psi)v_D^k(z')\right) \mid z\right] \end{aligned} \tag{8}$$

[10]In practice, restructurings tend to involve substantial recovery rates. Meyer, Reinhart, and Trebesch (2022) estimate an average recovery above 50 in a long historical sample. Adding a recovery rate of that magnitude would reduce the benefits of restructuring, and hence help support more debt in equilibrium. But a positive recovery rate should not interact with the forces we highlight in this model.

[11]The processes for restructuring private and official bilateral debt are very different, but to the extent that our model assumes no recovery after a default that would not make a difference. For studies exploring the interaction of private and different types of official sovereign borrowing, see Arellano and Barreto (2023) and Roldán and Sosa-Padilla (2023).

Furthermore, in order to have the same magnitude of default costs when they are permanent and transitory, we set costs to $\varphi(Y) = \Delta Y = e^z \Delta \Gamma$ or $y\Delta\Gamma$ when they are transitory and to $\varphi(Y) = \Delta Y = e^z \Gamma_{-1} g \Delta$ when they are permanent. In other words, when costs are transitory we obtain consumption $y(1-\Delta)$ and when they are permanent we obtain $g_T = g(1-\Delta)$. In this case, using also the *iid* assumption for g, the Bellman equations describing the government's problem become

$$\begin{aligned} v_R(b,z) &= \max_h u(c) + \beta \mathbb{E}\left[(g')^{1-\gamma} v(h/g', z') \mid z \right] \\ \text{subject to} \quad & c + \kappa b = y(z) + q(h,z)(h - (1-\rho)b) \\ v_D(z) &= p v_D^T(z) + (1-p)(1-\Delta)^{1-\gamma} v_D^P(z) \\ v_D^k(z) &= u(y(z)(1 - 1_{(k=T)}\Delta)) + \beta \mathbb{E}\left[(g')^{1-\gamma} (\psi v(0,z') + (1-\psi) v_D(z')) \mid z \right] \end{aligned} \tag{9}$$

where the term $(1-\Delta)^{1-\gamma}$ in the definition of v_D reflects the renormalization and also

$$q(h,z) = \frac{1}{1+r} \mathbb{E}\left[(1 - \mathcal{D}(h/g', z'))(\kappa + (1-\rho) q(h', z')) \mid z \right]$$

3.1 *Model with robust preferences*

There are four shocks in the model, ε^z and g for the endowment process, the reentry to international markets after a default, and whether the costs of defaulting are permanent or transitory.[12] We assume that, while the true stochastic processes governing these shocks are unknown, all agents in the economy take the descriptions above to be approximating models. While foreign lenders trust the approximating model and use it to compute the expectation in (7), the government does not. The government is instead concerned about potential misspecification of these processes and seeks robust decision rules. We represent such doubts and objectives with *multiplier preferences* (Hansen and Sargent, 2001) which capture ambiguity aversion with a single parameter. In this framework, robust decision rules emerge from confronting the decision maker with an 'evil agent' alter-ego. This fictitious, auxiliary agent chooses a worst-case model to minimize the decision maker's utility for each possible choice of action. Critically, the evil agent faces an entropy cost $\frac{1}{\theta}$ which only allows it to pick distortions from the approximating model (worst-case models) which are difficult to statistically distinguish from it. When the marginal cost of this relative entropy is infinite, we recover the standard rational-expectations decision maker. In what follows, we refer to the inverse of the marginal cost of relative entropy, θ, as the robustness parameter.

[12] The preference shocks (ϵ_R, ϵ_D), which are only introduced for numerical purposes, are assumed not to be subject to potential misspecification.

Moreover, the government potentially places different degrees of trust in its model for the endowment process, which is observed continuously, and in its model for the default costs, which are observed infrequently. We therefore denote as θ_s and θ_c the two robustness parameters for evaluating expectations involving (z', g') and k, respectively. Finally, we assume that the government does not doubt the stochastic process for reentry to markets after a default. While it is difficult to empirically determine whether output eventually catches up to its pre-default trend, exclusion periods are generally better understood (see Cruces and Trebesch, 2013).[13]

With multiplier preferences, the government's problems and value functions are[14]

$$\begin{aligned} v_R(b,z) &= \max_h u(c) - \beta \frac{1}{\theta_s} \log\left(\mathbb{E}\left[\exp\left(-\theta_s (g')^{1-\gamma} v(h/g', z')\right) \mid z\right]\right) \\ &\text{subject to} \quad c + \kappa b = y(z) + q(h,z)(h - (1-\rho)b) \end{aligned} \tag{10}$$

$$v_D(z) = -\frac{1}{\theta_c} \log\left(p \exp\left(-\theta_c v_D^T(z)\right) + (1-p)\exp\left(-\theta_c (1-\Delta)^{1-\gamma} v_D^P(z)\right)\right) \tag{11}$$

and

$$v_D^k(z) = u(y(z)(1 - 1_{(k=T)}\Delta)) - \beta \frac{1}{\theta_s} \log\left(\mathbb{E}\left[\exp\left(-\theta_s (g')^{1-\gamma}\left(\psi v(0,z') + (1-\psi) v_D^k(z')\right)\right) \mid z\right]\right) \tag{12}$$

4. Calibration

4.1 *Output trajectories around restructuring episodes*

We summarize the evidence on output costs of restructuring by computing deviations of output around debt restructuring episodes from a pre-restructuring trend in the Asonuma and Trebesch (2016) dataset. We purposefully abstract from causality. Instead, our approach is to parametrize the model to ensure that it generates the same correlations we find in the data. This allows us to indirectly infer the underlying causal parameters determining the size and persistence of default costs.

We consider a panel of market-access countries (MACs) which have had a sovereign restructuring event in 1990-2020 for which data on external debt, standing in international capital markets, GDP, and spreads are all available for at least 10 years previous to the default event. For each restructuring episode occuring at time t in country i, we construct a pre-restructuring trend

[13] Fourakis (2023) provides a model of signalling to understand the response of interest rates after market access has been recovered.

[14] For more details about how multiplier preferences yield a Bellman equation of the form of (10), see Appendix B.

for output by regressing log output in years $t-j$, for $1 \leq j \leq 6$, against a linear function of time

$$\log Y_{i,t-j} = \alpha_{it} + \beta_{it}(t-j) + \epsilon_{itj} \tag{13}$$

We then use this pre-restructuring trend to obtain fitted values $\hat{Y}_{i,t+k}$ and compare realized output $Y_{i,t+k}$ against this trend at horizon k. We focus on horizons $k = 1$ and 5 and take the median across all episodes. We complement these calculations with values for external debt (as share of GDP) and spreads, taken as averages between $t-10$ and $t-5$.

Table 1 summarizes our results, which we later use as calibration targets for the model presented in Section 3. The output deviation is sensitive to how the trend is defined, as the loss may be spread over many years and some of the decline relative to trend may take place in the run-up to the restructuring. We are therefore careful to estimate the same statistics in the model-generated data. We recover the structural causal cost-of-default parameters by matching the output deviations, while the overall output trajectory can be used for validation of the model. Finally, as all debt in the model is external, we compute a target for the external-debt-to-GDP ratio.[15]

Table 1: Calibration targets

Description	**Calculation**	**Value**
Output deviation, 1-year horizon, %	$Y_{t+1}/\hat{Y}_{t+1} - 1$	8.27
Output deviation, 5-year horizon, %	$Y_{t+5}/\hat{Y}_{t+5} - 1$	7.6
Average external debt-to-GDP ratio, %	$\overline{B_{t-j}/Y_{t-j}}$	23.4
Average spread, bps	$\overline{Spread_{t-j}}$	793

4.2 *Baseline calibration with robustness*

We parametrize our model at a quarterly frequency. We externally set most parameters (the sovereign's risk aversion γ, the risk-free interest rate r, the duration of debt ρ and the reentry probability ψ) to standard values in models of sovereign default. We choose a small value for the preference shock scale parameter χ to minimize the impact of this numerical device on the equilibrium.

We estimate the endowment process parameters $(\rho_z, \sigma_z, \sigma_g)$ for each country in our sample in the period before the first default, starting in 1960, and take averages across the episodes. In

[15] This number is lower than the headline debt-to-GDP ratio at which most countries tend to experience sovereign debt defaults.

each case, we run the log of output through an HP filter and estimate an AR(1) process on the cycle component to obtain (ρ_z, σ_z) and retrieve σ_g as the standard deviation of the residuals of the trend component against a linear time trend.

Finally, we assume that the government places full trust in its approximating model for the income process, so $\theta_s = 0$. This leaves us the sovereign's discount factor β, the level of default costs Δ, the probability of a transitory cost p (under the approximating model), and the robustness parameter θ_c. We set these parameters to match the moments from Table 1: the local projections for output at 1 and 5 years after a default event, relative to a pre-default trend, the average level of spreads and the average debt-to-GDP ratio while in repayment. Table 2 summarizes our parametrization.

Table 2: Baseline parameter values

Externally chosen		
	Parameter	**Value**
Sovereign's risk aversion	γ	2
Preference shock scale parameter	χ	0.01
Risk-free interest rate	r	0.01
Robustness parameter: income shocks	θ_s	0
Duration of debt	ρ	0.05
Reentry probability	ψ	0.0385
Income autocorrelation coefficient	ρ_z	0.9256
Standard deviation of z_t	σ_z	0.0231
Standard deviation of g_t	σ_g	0.0211
Internally calibrated		
	Parameter	**Value**
Sovereign's discount factor	β	0.902
Default cost	Δ	0.0411
Probability of transitory shock	p	0.339
Robustness parameter: default costs	θ_c	7.6

To assess the match to the moments from Table 1, we generate simulated data from the model[16] and filter it through the same procedure used for the sample of EM default episodes. The calibrated model matches those patterns closely, as shown in Table 3. We note that our structural estimate

[16] We take 2000 simulations of 250 years in each case.

for the costs of default ($\Delta = 4.11\%$) falls well within the range of empirical estimates of the causal effect, and quite close to the value found by Asonuma et al. (2023) which controls for a number of variables as well as the endogeneity of the choice of restructuring.

TABLE 3: MODEL FIT

	Data	Model
Output deviation, 1-year horizon, %	8.27	9.75
Output deviation, 5-year horizon, %	7.6	7.99
Average external debt-to-GDP ratio, %	23.4	21.3
Average spread, bps	793	813

Figure 2 plots the entire trajectories for output around default events in the data and model, with a shaded area indicating the interquantile range in the data. The model-generated dynamics match the data counterparts well, especially taking into account that the (annual) data reflects some time-averaging relative to our quarterly model. Both data and model exhibit a slight acceleration of output in the run-up to the default (between years -5 and -2) followed by a series of negative shocks starting around 1.5 years before the default. Both model and data then show a slow recovery in the years following the episode.

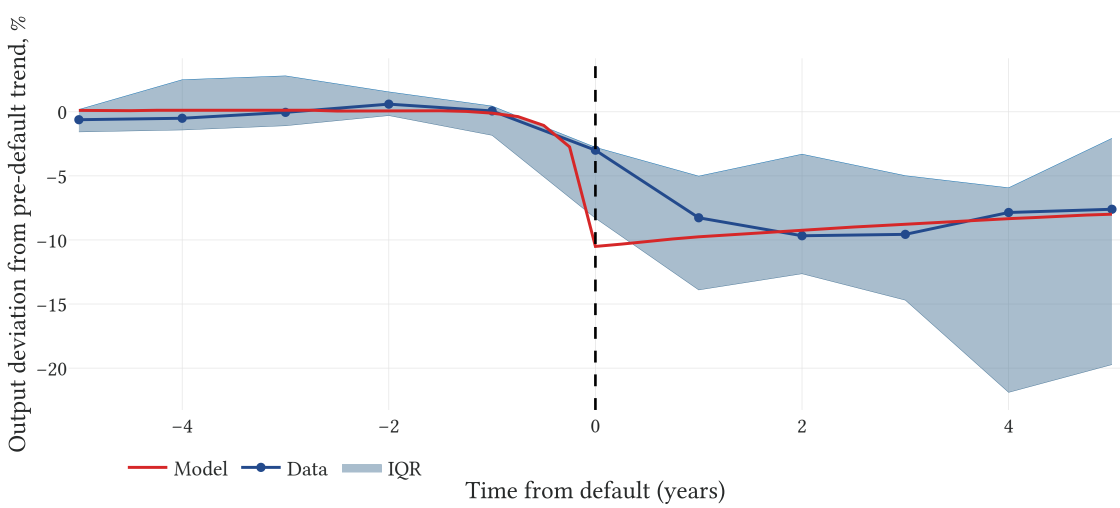

FIGURE 2: OUTPUT TRAJECTORIES AROUND DEFAULTS

5. Equilibrium and Quantitative Results

5.1 *Actual and perceived persistence of default costs*

Figure 3 shows simulation results at the baseline parametrization, varying the probability p that default costs are transitory and the robustness parameter θ_c affecting this particular source of uncertainty. We solve and simulate the economy for each combination of (p, θ_c) and report the average debt-to-GDP ratio B/Y and the default frequency, both in yearly terms.

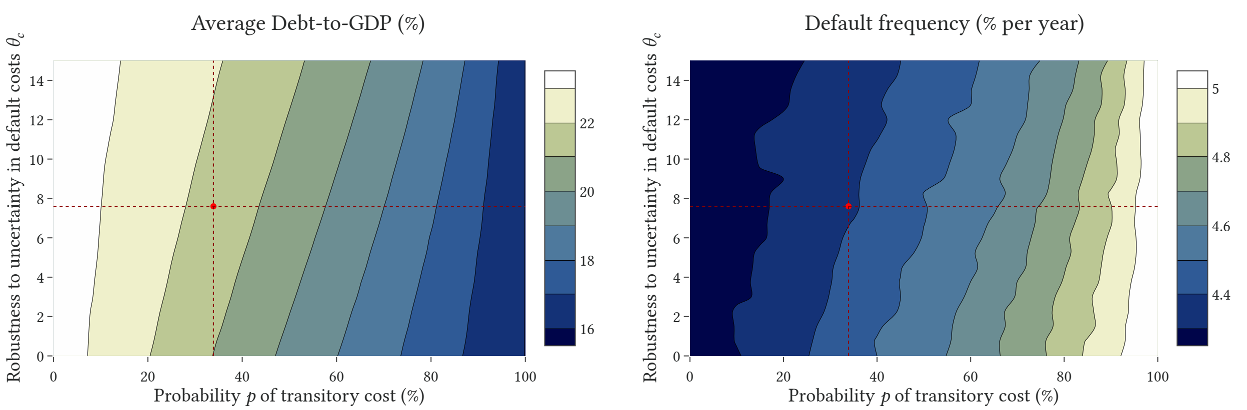

Figure 3: Debt tolerance varying p and θ_c

Note: Red dots denote the baseline calibration.

When the shock is more likely to be permanent, or the government has greater misspecification concerns, the left panel of Figure 3 shows that the economy sustains larger levels of debt. Robustness has a larger impact when the underlying uncertainty about the permanence of costs is greater (i.e. when p is closer to 50%). Conversely, if p is close to 0 or 1, the government may distrust its assessment of p but is quite convinced about the nature of costs, one way or the other. In the left panel of Figure 3, the average debt responds little to changes in θ_c for p close to the extremes and much more for intermediate values of p. Our calibrated value for p is about 35% which is squarely in the range where robustness has more quantitative bite. Compared to a parametrization in which the government is convinced that the cost is transitory ($p = 1$, the standard case in the literature), the average level of debt increases by about a quarter. Making the cost permanent for sure would result in a further increase in average debt of about 15% from the baseline calibration level.

The right panel reveals that the default frequency is less sensitive to either of these parameters, and relatively more insensitive to the degree of robustness.[17] Shifting the cost from cer-

[17] At the calibrated level of p, shifting θ_c moves debt levels but leaves the default frequency relatively unchanged.

tainly permanent to certainly transitory only increases the default frequency by about 17% (or about 0.3p.p.). Thus, most of the increase in perceived costs of default (both through actual persistence and robustness) translates into additional borrowing. The debtor government uses the extra 'credibility' afforded by the higher default costs to sustain higher levels of indebtedness, rather than keeping debt the same and facing the costs of default less frequently.

5.2 *Decomposition of output costs of default*

The calibrated model matches the output trajectory estimated from the data (Figure 2). In the model, we can decompose the output deviations according to[18]

$$\log Y_t - \log \hat{Y}_t = z_t + \log \Gamma_t + \log(1-\Delta)1_{(D_t=1)} - \log \hat{Y}_t \tag{14}$$

where $1_{(D_t=1)}$ refers to whether costs of default are active in period t, which could result from the economy being excluded in the current period or from a permanent cost of default in the current episode. We further normalize Y_t, $\hat{Y}_t$, and Γ_t to the trend level at the start of each episode such that $\Gamma_{t-21} = 1$.

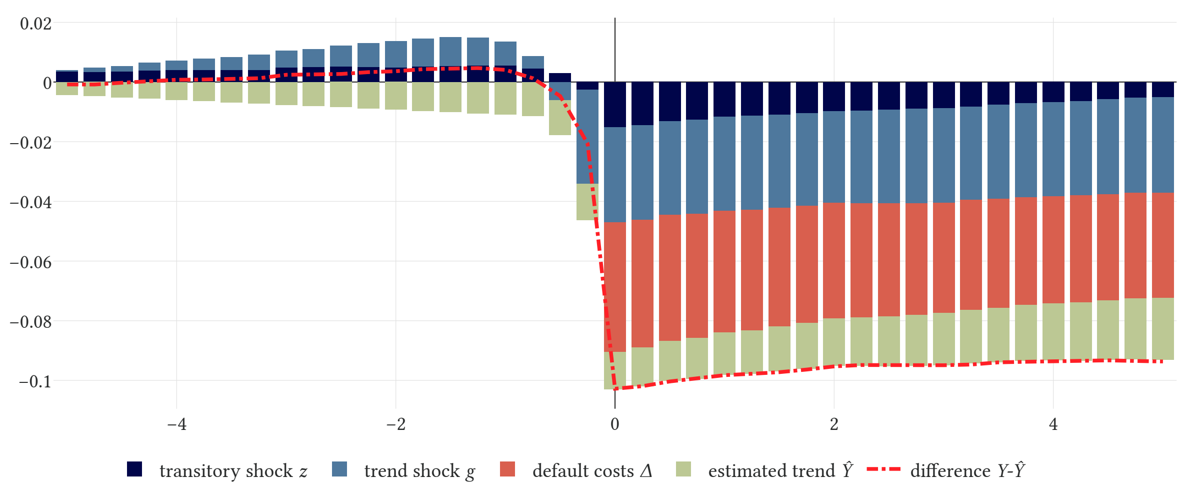

FIGURE 4: DECOMPOSITION OF OUTPUT DEVIATIONS FROM TREND

Figure 4 plots the decomposition and reveals that actual, 'causal' default costs account for about 35% of the output deviation from trend. Another 40% comes from underlying shocks which

[18] To obtain an exact decomposition in this case, we measure the output deviation from trend as log differences, which leads to slightly different headline numbers than in the calibration and Figure 2, where we measured such deviations as ratios.

tend to be negative at the time of default. Finally, about 25% of the deviation comes from the fact that output is growing (more than usual) in the pre-default period, leading to an upward trend later on. By construction, growth tends to be strong prior to the default, since we are drawing from a sample where the restructuring occurs at $t = 0$ but not before, which selects a relatively strong pre-crisis trend. In a similar vein, the growth shocks turn negative as we approach the time of the default event $t = 0$. These considerations explain why it would be hard for output to recover to the pre-crisis trend even if the default cost were to completely disappear, and how simple before-and-after growth comparisons can be misleading.

5.3 *The effect of robustness*

To better understand the effect of robustness on the government's decisions, we compute some expectations under the equilibrium worst-case model, the distortion which represents the government's misspecification concerns. Let the *distorted expectation* of random variable X at information set $\mathcal{F}$ be the objective expectation of the product of X with a likelihood ratio

$$\tilde{\mathbb{E}}[X \mid \mathcal{F}] = \mathbb{E}\left[\frac{\exp(-\theta v)}{\mathbb{E}\left[\exp(-\theta v) \mid \mathcal{F}\right]} X \,\middle|\, \mathcal{F} \right] \tag{15}$$

As compared to an expectation taken with the objective probability measure (the approximating or baseline model), the distorted expectation magnifies the likelihood of states for which the government's utility, measured by the value function v, is low. This procedure results in endogenously pessimistic beliefs which sustain robust decision-making. The role of the robustness parameter θ can also be clearly understood from (15). A larger value for θ means stronger misspecification concerns on the part of the government, and consequently a more distorted expectation. Conversely, as $\theta \to 0$, misspecification concerns vanish and the distorted expectation converges back to the objective one.

We focus on how robustness amplifies the perceived probability of a permanent cost of default. Because this variable $k \in \{P, T\}$ is only realized after a default has taken place, the appropriate value function to use in (15) is v_D^k,[19] with the renormalization factor $(1 - \Delta)^{1-\gamma}$ in case the cost is permanent. Letting $\tilde{p}(z)$ denote the distorted probability that the cost is transitory (i.e. $k = T$) at state z,

$$\tilde{p}(z) = \tilde{\mathbb{E}}[1_{(k=T)} \mid z] = \mathbb{E}\left[\frac{\exp(-\theta v_D^k(z)(1 - 1_{(k=P)}\Delta)^{1-\gamma})}{\mathbb{E}\left[\exp(-\theta v_D^k(z)(1 - 1_{(k=P)}\Delta)^{1-\gamma})\right]} 1_{(k=T)} \,\middle|\, z \right]$$

[19] In other words, the value function at the information set corresponding to the fact that a default has taken place (and the current state is z) is $v_D^k(z)$

Figure 5 shows that the government's misspecification concerns make it act as if the probability of a permanent cost of default was larger by a factor of about a quarter. Moreover, the distortion is larger when current income is lower, as a higher likelihood of default naturally makes the government more concerned about misspecification of the consequences of default. This effect is the key to the differential effect of robustness θ_c and the actual probability of a permanent cost $1-p$: the worst-case probability of a permanent cost increases more in low states, which is where default is more likely. As (perceived) costs become relatively larger in low states, the level of debt at which default happens becomes less dependent on the current income state z. This makes it easier to forecast whether a certain level of debt will be repaid in the future. As a consequence, when default costs increase via robustness, the government is more able to 'extract' the increase in debt tolerance while keeping the default frequency relatively unchanged. When they increase via the actual probability p, the government chooses a lower default frequency as well as a higher debt level (Figure 3).

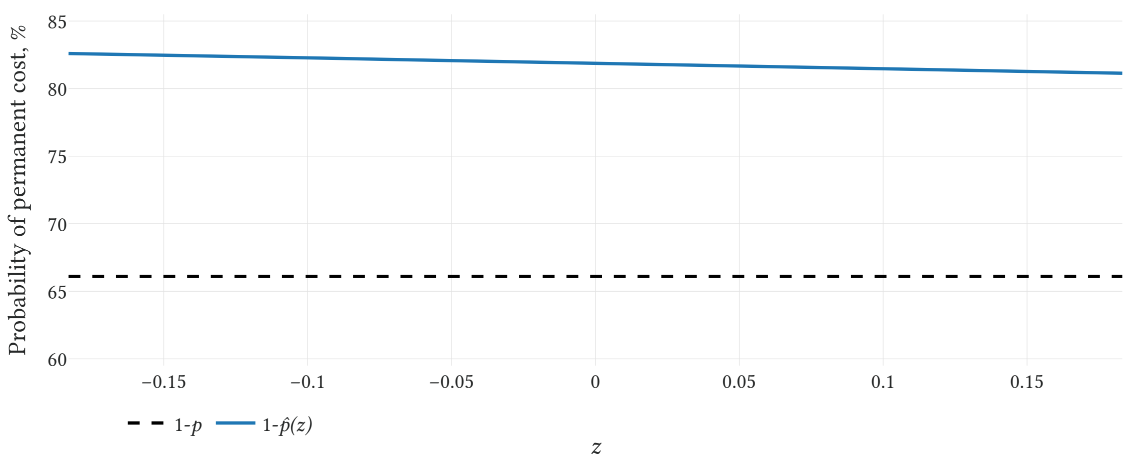

FIGURE 5: WORST-CASE PROBABILITY OF A TRANSITORY COST, $\tilde{p}$

Figure 6 illustrates the relationship between borrowing terms faced by the government and the costs of default, actual and perceived, it faces. For a moderate level of debt (but in the region where spreads start to increase), spreads decrease when the shock is more likely to be permanent. They also decrease when the government has stronger misspecification concerns. As before, the power of robustness is dampened when the government is more certain about the nature of costs (i.e. when p is closer to 0 or 1). The response of spreads becomes very non-linear as we approach high values of p (especially at low values for $theta_c$). At the chosen debt level (19% of mean income), the default probability would be almost 1 at such parameter values. Figure 7 recasts this same information in terms of the debt issuance level at which spreads become very large, which is perhaps easier to read in terms of debt tolerance.

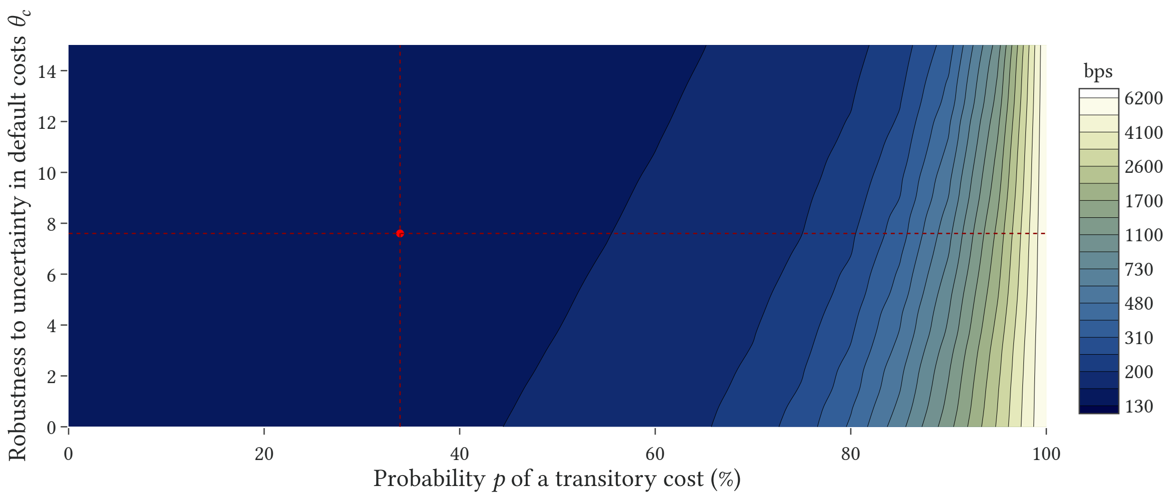

Figure 6: Spreads on government debt

Note: spreads computed at mean income and debt at 19% of income in each case, red dots denote the baseline calibration

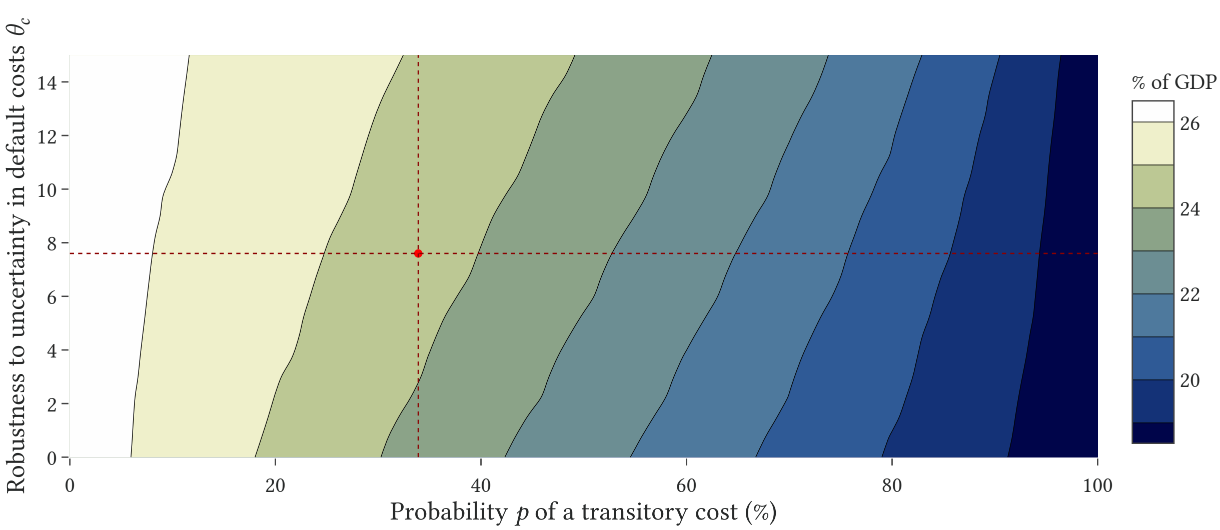

Figure 7: Debt at which spreads cross 1000bps

Note: Red dots denote the baseline calibration.

5.4 *Model detection-error probabilities*

As is common in the robustness literature, we compute model detection-error probabilities (DEP) to help inform and discipline our choice of θ_s and θ_c.[20] When the government follows robust decision rules, it does so by computing an auxiliary worst-case model. The DEP captures the probability that an agent, with a limited amount of data, mistakes data generated by one of the models as coming from the other one. A DEP of 50% means that the baseline and worst-case models are observationally equivalent while a DEP of 0 means that an agent can perfectly distinguish both models. A high DEP suggests that the amount of misspecification implicit in the distorted model is plausible and validates the decision maker's desire for robustness. Typically, in the robust control literature, acceptable values for DEP are of 20% or above (Barillas et al., 2009). Our choices of θ_c stay well within this constraint suggesting we rely on reasonable amounts of probability distortions.[21]

Figure 8 plots the model detection-error probability for different values of the objective probability p of a transitory cost, as a function of the choice of the robustness parameter θ_c.

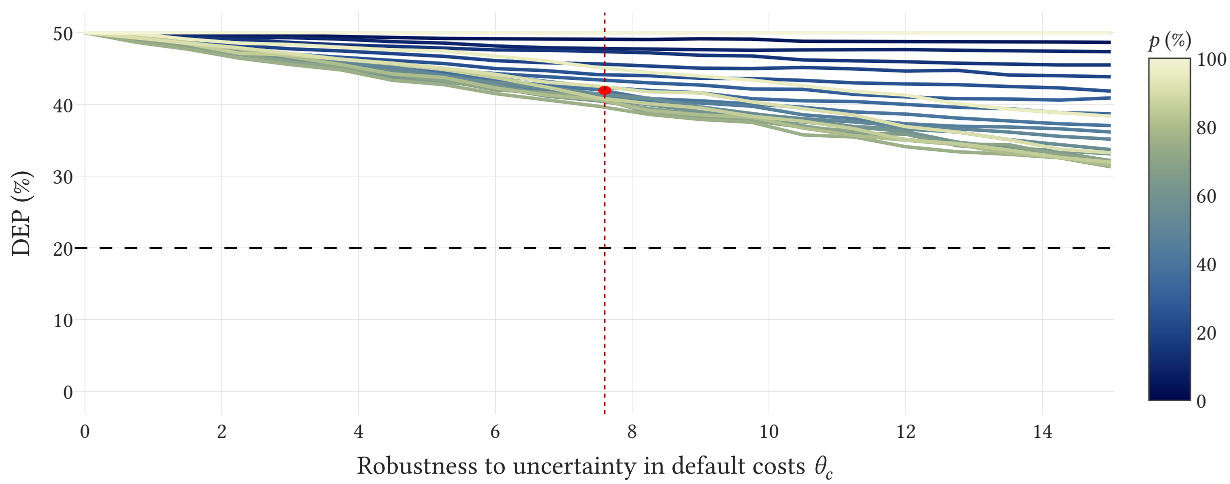

FIGURE 8: DEBT AND DEPS FOR DIFFERENT PRIOR PROBABILITIES OF A TRANSITORY COST

For a given baseline probability p of a permanent shock, increasing robustness leads to higher

[20] Although as of this writing we only consider changes in θ_c.

[21] We compute model detection-error probabilities as the probability of misclassifying a 60-year sample (which is more than the length of quarterly data for GDP for most EMs), after a 500-period burn-in to draw from the ergodic distribution, over 2000 repetitions. We compute in each repetition the likelihood $L(x \mid M)$ of data x being generated by model M for the approximating model A and for the (endogenous) worst-case model W. We then compute the probability of misclassification as the average between $\mathbb{P}(L(x \mid A) > L(x \mid W) \mid W)$ and $\mathbb{P}(L(x \mid W) > L(x \mid A) \mid A)$.

levels of debt sustained in equilibrium. The government sustains these higher levels of debt because the worst-case model arising from the government's robustness concerns features a larger probability of permanent costs (Figure 3). This feature also makes it more easily distinguishable from the baseline model. Indeed, Figure 8 shows that the DEP decreases with the robustness parameter θ_c. Moreover, as p grows towards 1, there is more scope to distort beliefs in a pessimistic way. The more the cost looks transitory, the more there is to fear that it is permanent. This leads to lower values for the DEP when p is large, holding robustness constant.

6. Concluding remarks

This paper shows how the potential permanence of output costs following a debt restructuring can significantly amplify a country's willingness to repay its debt and, hence, its debt capacity. Debt tolerance with fully permanent output costs can be almost one half larger than with purely transitory costs. In practice, decision makers are unlikely to have a clear picture of how permanent these costs are, similarly to researchers in sovereign debt. Aversion to this ambiguity boosts debt tolerance, by making the government treat costs as perhaps being more persistent. While these channels have received little attention in the literature, our results suggest that they may be key drivers of debt tolerance.

We find that the persistence of output costs, and the uncertainty surrounding it, has a sizable effect on debt tolerance, but perhaps not as much as one would have expected. We parametrize the model to an exclusion period of 6.5 years on average, following widely used estimates from Cruces and Trebesch (2013). The additional impact of making the cost permanent after the exclusion period is limited by the discounting of a relatively impatient borrower, which is also a common feature of calibrated sovereign debt models.[22] With a more patient borrower (or a counterfactually shorter exclusion period), permanent default costs would boost debt tolerance even more. This interaction could be a key driver of what explains differences in debt tolerance across countries (e.g. "graduation" effects).

The focus of this paper, like much of the literature, has been on external debt. Domestic debt restructurings can be much more disruptive, given the implications for domestic financial stability, which can potentially lead to much larger and protracted output costs relative to an external debt restructuring. Modeling domestic debt introduces complications to the model, but would be an interesting direction for future work, and where both channels proposed could have an even larger amplifying impact on debt tolerance.

[22]Such low discount factors can be interpreted as representing political-economy considerations.

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A. Normalization details

We normalize all variables by Γ_t, denote normalized values with lowercase, and notice that $y_t = \exp(z_t)$ and

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma}\frac{(C/\Gamma)^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma}u(c)$$

so in a typical Bellman equation we can guess and verify (denoting $h = B'/\Gamma$) forms like $V\Gamma^{\gamma-1} = v$

$$\begin{aligned}
V(B,z,\Gamma) &= \max_{B'} u(C) + \beta\mathbb{E}\left[V(B',z',\Gamma')\right] \\
V(B,z,\Gamma) &= \max_{B'} \Gamma^{1-\gamma}u(c) + \beta\mathbb{E}\left[V(B',z',\Gamma')\right] \\
\Gamma^{\gamma-1}V(B,z,\Gamma) &= \max_{B'} u(c) + \beta\mathbb{E}\left[(\Gamma')^{\gamma-1}\left(\Gamma/\Gamma'\right)^{\gamma-1}V(B',z',\Gamma')\right] \\
v(b,z,g) &= \max_{h} u(c) + \beta\mathbb{E}\left[(g')^{1-\gamma}v(b'(h,g'),z',g')\right] \\
v(b,z,g) &= \max_{h} u(c) + \beta\mathbb{E}\left[(g')^{1-\gamma}v(b'(h,g'),z',g')\right]
\end{aligned}$$

while for the budget constraint we have

$$\begin{aligned}
C + \kappa B &= Y + q(B' - (1-\rho)B) \\
c + \kappa b &= y + q(B'/\Gamma - (1-\rho)b) \\
c + \kappa b &= y + q(b'(\Gamma'/\Gamma) - (1-\rho)b) \\
c + \kappa b &= y + q(b'g' - (1-\rho)b)
\end{aligned}$$

This budget constraint makes it clear that $h = b'g'$ (simply substituting $h = B'/\Gamma$ in line 2) or $b'(h,g') = h/g'$.

For the value of default we have, for $\mathrm{g}^C(g,z) = g - \varphi$ and $\Gamma^C = \Gamma_0\mathrm{g}^C(g,z)$, where $\Gamma = \Gamma_0 g$

$$\begin{aligned}
V_D(z,\Gamma) &= pV_D^T(z,\Gamma) + (1-p)V_D^P(z,\Gamma^C) \\
(\Gamma)^{\gamma-1}V_D(z,\Gamma) &= p(\Gamma)^{\gamma-1}V_D^T(z,\Gamma) + (1-p)(\Gamma)^{\gamma-1}V_D^P(z,\Gamma^C) \\
(\Gamma)^{\gamma-1}V_D(z,\Gamma) &= p(\Gamma)^{\gamma-1}V_D^T(z,\Gamma) + (1-p)(\Gamma_0 g)^{\gamma-1}V_D^P(z,\Gamma^C) \\
v_D(z,g) &= pv_D^T(z,g) + (1-p)(\Gamma_0\mathrm{g}^C(g,z)g/\mathrm{g}^C(g,z))^{\gamma-1}V_D^P(z,\Gamma^C) \\
v_D(z,g) &= pv_D^T(z,g) + (1-p)\left(\frac{g}{\mathrm{g}^C(g,z)}\right)^{\gamma-1}\underbrace{(\Gamma_0\mathrm{g}^C(g,z))}_{=\Gamma^C}{}^{\gamma-1}V_D^P(z,\Gamma^C) \\
v_D(z,g) &= pv_D^T(z,g) + (1-p)\left(\frac{g}{g-\varphi}\right)^{\gamma-1}v_D^P(z,g-\varphi)
\end{aligned}$$

B. Robustness

For a technical exposition of robustness, see Hansen and Sargent (2001). Less formally, consider a decision maker facing a state (vector) x with a (trusted) law of motion $\mu(x' \mid x, c)$ and a Bellman equation of the form

$$v(x) = \max_c u(c) + \beta \int v(x')\mu(x' \mid x, c)dx'$$

To include doubts about the law of motion μ and guard against misspecification, the decision maker seeks an action that would perform reasonably well if the true law of motion was instead *any* μ'. The alternative is left unidentified but could be any probability distribution that is difficult to distinguish statistically from μ. To obtain an action that performs well under all possible μ', the decision maker enlists the help of an auxiliary evil agent who chooses a probability distortion m that minimizes the utility given the action c chosen. By behaving in this pessimistic way (each action is evaluated according to the distortion that yields the lowest utility), the decision maker obtains lower bounds for each action.

Finally, in order to only allow distortions that are difficult to identify from data, the evil agent faces a cost for the relative entropy of the original model μ and the distorted one, the so-called worst-case model $m\mu$. The inverse marginal cost of relative entropy, θ, determines how robust the resulting decision will be. The decision maker takes into account the reaction of the evil agent and solves

$$\begin{aligned} v(x) &= \max_c \min_{m \geq 0} u(c) + \beta \int v(x')m(x')\mu(x' \mid x, c)dx' + \frac{1}{\theta}\mathrm{ent}(m\mu, \mu) \\ \text{subject to} \quad &\int m(x')\mu(x' \mid x, c)dx' = 1 \end{aligned}$$

The solution to the minimization problem yields a worst-case model $\hat{m}(x') \propto \exp(-\theta v(x'))$. As Hansen and Sargent (2001) show, plugging in the worst-case model yields the robust Bellman equation

$$v(x) = \max_c u(c) + \frac{1}{-\theta}\beta \log\left(\int \exp\left(-\theta v(x')\right)\mu(x' \mid x, c)dx'\right)$$