Risk Aversion in Sovereign Debt and Default*

Francisco Roch† Francisco Roldán‡

UTDT IMF

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Abstract

We study the interaction of risk-sensitive preferences with sovereign default risk. Macro-financial separation, a pervasive property of real business cycles models, breaks in the context of sovereign debt. Risk aversion crucially affects the strength of the threat of autarky which helps sustain sovereign borrowing. We (re)evaluate the welfare effects of access to capital markets, debt dilution, and post-default negotiations. [We find that...]

JEL Classification

Keywords

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†e-mail: *franroch@gmail.com*

‡e-mail: *froldan@imf.org*

Introduction

What makes governments repay their debts, thereby sustaining sovereign borrowing? The general consensus points to a combination of direct costs, in terms of utility or foregone output, combined with a period of exclusion from international capital markets. Exclusion is costly as it disrupts consumption smoothing. However, most quantitative studies of sovereign debt and default are specified with constant relative risk aversion (CRRA) for the government's preferences, a feature which renders business-cycle volatility in consumption almost irrelevant (Lucas, 1987).

In this paper we study sovereign default risk under risk-sensitive preferences, which disentangle risk aversion from the elasticity of intertemporal substitution. We find that risk aversion modulates the welfare costs of fluctuations in consumption and thus the strength of the threat of autarky following default. This leads to a failure of macro-financial separation (Tallarini, 2000) and creates a role for risk aversion in the determination of economic outcomes. We then compare the performance and predictions of our model with the standard CRRA case, both calibrated to the same empirical regularities of emerging-market economies.

Discussion of the Literature TBW

2. Model

We consider a small open economy whose government borrows from competitive international lenders on behalf of its citizens. Debt helps frontload consumption and smooth shocks and takes the form of a long-term, non-contingent, defaultable bond.

Resources The economy receives an exogenous tradable endowment y_t whose evolution follows an AR(1) process in logs,

$$z_t = (1-\rho_z)\mu_z + \rho_z z_{t-1} + \epsilon_t^z$$

where $y_t = \exp(z_t)$ and $\epsilon \overset{iid}{\sim} \mathcal{N}(0, \sigma_z^2)$.

Government The government's objective is to maximize the representative consumer's welfare, given implicitly by the following specification of Epstein-Zin or robust *multiplier* preferences

$$w_t = u(c_t) + \beta \mathbb{T}\left[w_{t+1}\right]$$

where $\mathbb{T}\left[X \mid \mathcal{F}\right] = -\frac{1}{\theta}\log\left(\mathbb{E}\left[\exp\left(-\theta X\right) \mid \mathcal{F}\right]\right)$ is a risk-sensitive expectation.

Assets The government borrows from international lenders in the form of a defaultable bond which promises to pay a noncontingent stream of geometrically-decaying coupons (Leland, 1998; Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012). A bond issued in period t pays $(1-\rho)^{s-1}\kappa$ units of the good in period $t+s$, which effectively makes a one-period-old bond a perfect substitute of $(1-\rho)$ units of newly-issued debt. The coupon rate $\kappa = r + \rho$, where r is the international risk-free rate, is chosen so that the price of a bond that is expected to never default is $q^\star = 1$.

With this structure, the government's budget constraint dictates resources available for consumption in case of repayment and if new debt $i_t = b_t' + (1-\rho)b_t$ is issued at price q_t

$$c_t + g + \kappa b_t = y(z_t) + q_t(b_t' + (1-\rho)b_t)$$

where g are exogenous, constant government purchases.

Lenders Financing to the small open economy is provided by a continuum of deep pocketed foreign investors, whose attitudes towards risk are subject to a global shock x_t. We model their stochastic discount factor following Vasicek (1977) as is often done in models of sovereign default (Arellano and Ramanarayanan, 2012; Bianchi et al., 2018),

$$M(x,z,z') = \exp\left(-r - x\left(\alpha\epsilon' + \frac{1}{2}\alpha^2\sigma_z^2\right)\right) \tag{1}$$

where $\alpha \geq 0$ and ϵ' is the innovation in the stochastic process for z. The creditor risk-premium shock $x \in \{0,1\}$ follows a Markov process with transition probabilities $(\pi_{LH}, \pi_{HL}) \in [0,1]$. When $x = 0$ (normal times), the creditors use a risk-neutral kernel to price financial assets, but become risk-averse when the event $x = 1$ enables correlation between their stochastic discount factor and the endowment z received by the small open economy.

Default Each period, the government may choose to default on the debt, which triggers temporary exclusion from international capital markets. As in Bianchi et al. (2018) and Hatchondo et al. (2022), we assume that while excluded, the government (and/or the representative agent) suffers a utility cost $U^D(y)$. While excluded, the government faces a constant hazard ψ of reaching a deal with its bondholders, in which case a share $N(b,x,z)$ of the defaulted bonds become due again. The value of default for the government is then

$$v_D(b,x,z) = u(y(z)) - U^D(y(z)) + \beta\mathbb{T}\left[1_R\mathcal{V}(N(b,x',z')b,x',z') + (1-1_R)v_D(b,x',z') \mid x,z\right] \tag{2}$$

where 1_R is an indicator function for the event of market reentry and $\mathcal{V}$ is the value attained by the government when it has access to markets.

At the beginning of each period in which it has access to markets, the government faces a choice to repay the debt or default, so that

$$\mathcal{V}(b,x,z) = \max \left\{ v_R(b,x,z) + \epsilon_R, v_D(b,x,z) + \epsilon_D \right\}$$

where (ϵ_R, ϵ_D) follow independent Type 1 Extreme Value distributions with scale parameter χ. As is well-known (Chatterjee et al., 2018; Dvorkin et al., 2021), this specification leads the distribution of the difference $\epsilon_R - \epsilon_D$ to be logistic and yields the closed forms for the value function and the ex-post probability of default

$$\begin{aligned} \mathcal{V}(b,x,z) &= \chi \log \left(\exp\left(\frac{1}{\chi} v_R(b,x,z)\right) + \exp\left(\frac{1}{\chi} v_D(b,x,z)\right) \right) \\ \mathcal{P}(b,x,z) &= \frac{\exp\left(\frac{1}{\chi} v_D(b,x,z)\right)}{\exp\left(\frac{1}{\chi} v_R(b,x,z)\right) + \exp\left(\frac{1}{\chi} v_D(b,x,z)\right)} \end{aligned} \tag{3}$$

Debt issuances While it remains current on its obligations, the government can issue new debt b' on the market[1] and attain a value

$$\begin{aligned} v_R(b,x,z) = &\max_{b'} u(c) + \beta \mathbb{T}\left[\mathcal{V}(b',x',z') \mid x,z\right] \\ \text{subject to} \quad & c + \kappa b = y(z) + q(b',x,z)(b' - (1-\rho)b) \end{aligned} \tag{4}$$

Debt prices Bonds are priced so that creditors make zero profits in expectation given their stochastic discount factor (1)

$$\begin{aligned} q(b',x,z) &= \mathbb{E}\left[M(x,z,z')(1-1_{\mathcal{D}'})(\kappa + (1-\rho)q(g_b(b',x',z'),x',z')) + 1_{\mathcal{D}'} q_D(b',x',z') \mid z\right] \\ q_D(b,x,z) &= \mathbb{E}\left[M(x,z,z')\left(\psi N(b,x',z')R(N(b,x',z')b,x',z')b + (1-\psi)q_D(b,x',z')\right) \mid x,z\right] \end{aligned} \tag{5}$$

where $R(b,x,z) = (1-1_{\mathcal{D}})(\kappa + (1-\rho)q(g_b(b,x,z))) + 1_{\mathcal{D}} q_D(b,x,z)$ is the value of a restructured bond and $g_b(b,x,z)$ denotes the government's (potentially stochastic) policy function for debt issuance.

2.1 *Risk-sensitive preferences*

The government uses the $\mathbb{T}$ operator to form expectations. One interpretation is that the government and/or the representative agent have inherently different attitudes towards volatility over

[1] In the numerical solution of the model, we discretize the set of choices $\mathcal{B}$ for debt and include Extreme Value Type 1 preference shocks, similarly to the default choice. This helps smooth out the choice of debt by making the probability of choosing level b' proportional to $\exp\left(\frac{1}{\chi_b}\left(u(c) + \beta \mathbb{T}\left[\mathcal{V}(b',x',z') \mid x,z\right]\right)\right)$. Similarly to the default decision, we choose χ_b as small as possible to ensure that it does not affect the equilibrium.

time and across states, which are represented by risk-sensitive Epstein-Zin preferences. In this interpretation, the creditors are risk-neutral both over time and across states, which leads them to evaluate outcomes according to their expected payoffs.

Another interpretation of the $\mathbb{T}$ operator is that the government and/or the representative agent have unit elasiticity of substitution both over time and across states, but do not fully trust the stochastic processes specified for the shocks ϵ, x, and the return to markets after a default. Instead, they seek decision rules that would be robust to potential misspecification of such statistical models. The multiplier-preferences model of Hansen and Sargent (2001) then yields the $\mathbb{T}$ operator as a way to achieve robustness.[2] In this case, the creditors do trust the shock processes. However, there is no learning from other agents' models: the government knows that the creditors trust their models, and viceversa. The government might view the creditors as reckless, while they might view it as too cautious.

In both interpretations, gains from trade occur from confronting risk-neutral lenders with a risk-averse borrower.

3. Calibration

We parametrize our model to match salient features of emerging-market economies. Our calibration strategy follows Hatchondo et al. (2022) and Bianchi et al. (2018) closely.

A period in the model refers to a year. We set the risk-free interest rate r and the government's discount factor β to standard values in this literature. We use data from Mexico, a common reference for quantitative studies of sovereign default. We set the level of government spending g at 12% of average income, the inverse maturity of debt $\rho = 0.286$ to match the Macaulay duration of Mexican debt of about 3 years. We assume that any defaults are resolved within a year ($\psi = 1$), which is on the short end of the range of empirical estimates (Cruces and Trebesch, 2013).

The parameters governing the two exogenous shock processes are taken from Bianchi et al. (2018) to match the behavior of output ($\rho_z = 0.66, \sigma_z = 0.034$) and the frequency with which the global EMBI+, excluding countries in default, is one standard deviation above its mean. This yields one episode of high risk premia every 20 years with an average duration of 1.25 years, or $(\pi_{HL}, \pi_{LH}) = (0.8, 0.15)$. For the 1993–2014 period, the episodes are the Tequila crisis in 1994–95,

[2] Robustness in sovereign debt models is also featured in Chamon and Roldán (2024), or in Pouzo and Presno (2016) and Roch and Roldán (2023) but on the creditor side.

the Russian default in 1998, and the global financial crisis in 2008. On average, the EMBI+ spread was 200 basis points higher during these times than in normal times. For the debt restructuring function $N(b, x, z)$, for the time being we assume a constant haircut to match the average recovery of 63%, following Bianchi et al. (2018).

Table 1 summarizes the parameters we set externally.

	Parameter	**Value**
Sovereign's discount factor	β	0.92
Income autocorrelation coefficient	ρ_z	0.66
Standard deviation of y_t	σ_z	0.034
Government consumption	g	0.12
Preference shock scale parameter	χ	0.01
Risk-free interest rate	r	0.04
Duration of debt	ρ	0.286
Reentry probability	ψ	1
Haircut upon default	$\hbar$	0.37

Table 1: Externally chosen parameters

In our baseline calibration the utility function displays unit elasticity of intertemporal substitution, so that $u(c) = \log(c)$. The utility cost of default is given by $U^D(y) = \max\{0, \lambda_0 + \lambda_1 y\}$. These two parameters help match the average levels of debt and spread in the data (Hatchondo and Martinez, 2017).

For contrast, we also consider an alternative parametrization with expected CRRA utility $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$, where we set $\theta = 0$ (or equivalently replace the $\mathbb{T}$ operator with an unadjusted expectation) and let the EIS parameter γ adjust. In this version of the model, we only vary the parameters that are calibrated internally and target the same moments in the data as the baseline version.

We calibrate the cost of default parameters (λ_0, λ_1), the government's risk aversion θ (or γ), and the risk-premium α to match four moments in the Mexican data. Our targets are the average level of spreads (240 bps) and debt-to-GDP ratio (45%), a relative volatility of consumption relative to output of unity, and the average increase in spreads during high risk premium episodes (200 bps). These correspond to observations for Mexico in 1993–2014, reported by Hatchondo et al. (2022).

Table 2 contains the values of the parameters that we set by calibration, for both versions of the model.

	Parameter	Baseline	CRRA
Sovereign's risk aversion	θ	17.24	n.a.
Sovereign's EIS	γ	1	1.623
Risk-premium shock	α	47.26	47.05
Default cost: constant	λ_0	0.7649	0.9936
Default cost: linear	λ_1	5.037	6.631

Table 2: Calibrated parameters

Table 3 shows that both models succeed in replicating the main features of the data.

	Data	Baseline	CRRA
Average spread (bps)	240	242	225
Δ spreads in high risk premium (bps)	200	198	201
Relative volatility of consumption (%)	1	1.01	1.07
Debt-to-GDP ratio (%)	43	43	42.1

Table 3: Fit of the models

3.1 *Untargeted moments*

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3.2 *Asset pricing implications*

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4. Quantitative Analysis

To study the differential welfare implications of policies, for each model we compute the constant level of consumption $c^{\star}$ yielding the same utility as the equilibrium $\Theta = (\gamma, \theta, \alpha, \lambda_0, \lambda_1)$, starting at 0 debt and averaging over the ergodic distribution $\mu(x, z)$ for the exogenous states,

$$c^{\star}(\Theta) = \int u^{-1}\left(\mathcal{V}_{\Theta}(0, x, z)(1-\beta)\right) d\mu(x, z) \tag{6}$$

4.1 Welfare effects of access to markets

4.2 Welfare effects of sovereign risk

4.3 Welfare effects of debt dilution

To evaluate debt dilution and its welfare effects, we solve our model with the assumption that the government can commit to state-contingent borrowing plans $b'(b, x, z)$, but must respect the sequential decision-making process for default. We leverage the Marcet and Marimon (2019) approach introduced by Hatchondo et al. (2020).

For comparison, we also solve the model in its original sequential timing, but replacing the fixed-coupon with floating-rate debt, as in Aguiar et al. (2023).

4.4 The effect of risk aversion on post-default negotiations

5. Concluding Remarks

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