The Perils of Bilateral Sovereign Debt*

Francisco Roldán†
IMF

César Sosa-Padilla‡
University of Notre Dame
and NBER

February 2026

Abstract

We study the interaction between private and official sovereign debts. We develop a quantitative sovereign default model featuring a senior lender with whom borrowing terms are negotiated. We use this model to evaluate implications of the emergence of new official creditors not bound by the Paris Club framework. The dynamics of bilateral bargaining lead the government to issue more market debt, raising default risk and creating welfare losses. This relational overborrowing effect arises due to an endogenous cross-elasticity of bilateral terms to market debt, which can be assessed in practice to evaluate new forms of bilateral sovereign debt.

JEL Classification F34, F41, G15
Keywords Sovereign debt, debt dilution, bilateral bargaining, official debt

*The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management. An earlier version of this paper circulated as "Central Bank Swap Lines as Bilateral Sovereign Debt." For thoughtful comments and suggestions we would like to thank our discussants Saleem Bahaj and Vivian Yue as well as Cristina Arellano, Fernando Arce, Tamon Asonuma, Emine Boz, Marcos Chamon, Giancarlo Corsetti, Pierre-Olivier Gourinchas, Julian Kozlowski, Leonardo Martinez, Andy Neumeyer, Juan Pablo Nicolini, Adrian Peralta Alva, Francisco Roch, Tom Sargent, Mark L. J. Wright and participants at the SED, Banque de France/EUI conference on Macroeconomic, Price, and Debt Stabilization, SEA, UTDT, IMF, McMaster, Waterloo, DebtCon, IEF Workshop, FRB St. Louis, the IMF/IDB conference on Fiscal Policy and Sovereign Debt (Santiago de Chile), and Econometric Society. Caio Dantas and César Ciappa provided excellent research assistance.

†e-mail: *froldan@imf.org*
‡e-mail: *cesarspa@gmail.com*

Introduction

A large fraction of sovereign borrowing in emerging-market economies takes the form of official debt, including loans from other governments, regional development banks, or multilateral institutions such as the IMF and the World Bank. The past few decades have witnessed the rise of new sovereign creditors operating outside of the Paris Club institutional framework (Horn et al., 2021b; Gelpern et al., 2021). The claims to de facto senior creditor status (relative to private creditors) by these new lenders have raised concerns about welfare implications for borrowing countries (Horn et al., 2021a, 2023).

This paper evaluates such concerns in the context of a quantitative sovereign debt and default model, augmented by the presence of a large lender who offers an alternative source of funds. Relative to creditors in international capital markets, the large lender possesses a superior enforcement technology. This assumption allows us to model the potential success of the new sovereign creditors in becoming senior to bondholders in capital markets. In addition, in our baseline setup we assume that borrowing terms with the large lender are determined through bargaining, which gives a crucial role to the dynamics of the bilateral relationship and its surplus.

We focus on the interaction between private markets and the large lender as two possible financing sources.[1] While the availability of bilateral loans from the large lender affects the government's behavior in debt markets, outcomes in debt markets also influence threat points in the bilateral negotiation. The interest rate charged by the large lender is therefore constrained by implicit competition from debt markets. But when default risk pushes up yields on marketable debt, the large lender is also able to charge a premium. Because there is no default risk on this loan, such a premium only reflects the borrower's outside options.

The baseline model yields two main results: the presence of the senior lender leads to a form of overborrowing in debt markets, and this creates welfare losses for the government. While bilateral loans can and are often used on the equilibrium path to avoid costly defaults, the borrowing country is worse off when the large lender is present. One reason for this is that the possibility of borrowing from the large lender while excluded from markets raises the value of default, which increases default incentives and lowers (marketable) debt prices. But there is another, more fundamental reason. Even imposing the constraint that bilateral loans are unavailable during default, welfare for the government is still reduced by the presence of the large lender due to a "relational overborrowing" effect. Borrowing terms with the large lender are determined by splitting the bilateral surplus. This surplus is largest when sovereign risk is present and the government is

[1] As most of the quantitative literature on sovereign debt, we focus on debt as a vehicle for consumption smoothing or frontloading and abstract from project financing or development lending.

paying high spreads on its market debt, which is precisely when the large lender is most needed. Consequently, when the large lender expects the government to face high spreads in the future, it values the relationship more and is willing to invest in it by lending more cheaply. This endogenous elasticity of bilateral loan terms to indebtedness in markets acts as a countervailing force to the market discipline of spreads upon large issuances of debt. The presence of the large lender then leads the government to take on more debt in markets and to delever more slowly. The end result is an increase in the default frequency and therefore in spreads, which ultimately create welfare losses for the government in the model.

The dynamics and welfare effects we describe result from three critical assumptions regarding the large creditor and the funding it offers. Compared to competitive markets, we assume that it is more difficult (or impossible) to default on the large creditor, that its loans are of shorter maturity, and that the terms of borrowing result from bilateral bargaining rather than competition and a zero-profit condition. Central Bank swap lines constitute a prime real-world example, which Horn et al. (2021b) identify as a dominant channel through which official bilateral lending has reemerged since the 2000s.[2] These facilities are typically short-term and involve no recorded defaults (even concurrently with restructuring of market debts). Finally, while their terms are typically strictly confidential, anecdotal evidence suggests that a fair amount of bilateral negotiation is involved in determining amounts and interest rates, rather than market mechanisms.

Other forms of official debt share the seniority and/or duration features emphasized in our baseline model—including for example some types of IMF programs and repo or swap facilities offered by central banks such as the Fed or the ECB. However, unlike the bilateral bargaining framework we assume, these facilities typically feature borrowing terms that are fixed in advance. For instance, IMF lending is subject to surcharges based on access and duration thresholds (IMF, 2024b), while the Fed's swap lines carry a standard 25 basis point spread over policy rates (Bahaj and Reis, 2023) and are publicly reported in real time on the FRBNY's website. To accommodate such institutional designs, we develop an extension in which bilateral loan pricing does not result from bargaining but rather follows fixed rules. This allows us to explore how rules-based lending affects debt dynamics and to compare welfare outcomes across alternative institutional arrangements.

The extension shows in a much simpler setup that when the interest rate on bilateral loans is relatively insensitive to the government's market debt—that is, when the interest rate is fixed or has a low elasticity to market debt—the relational overborrowing channel is muted or disappears altogether. In this case, the dominant effect of the large lender is to help the government

[2]While swap lines are important in our motivation to study the interaction of market debt and (undefaultable) bilateral loans, our model is not geared to represent all aspects of swap lines. For example, we do not include a difference in the currency denomination of bilateral and market debt, conditionality (including requirements on the level of foreign reserves), or collateral requirements.

avoid costly defaults on its marketable debt. As a result, the equilibrium features a lower default frequency and reduced sovereign spreads, improving the welfare of the borrowing country.

A key parameter in the model is the government's bargaining power relative to the large lender. When the government can make take-it-or-leave-it offers (i.e., when it has full bargaining power), the large lender simply provides nondefaultable loans at the risk-free rate, recovering the model of Hatchondo, Martinez, and Önder (2017). In this case, the government's welfare is also higher relative to the equilibrium without the large lender. In our preferred calibration, however, such gains quickly dissipate as the government's bargaining power declines.

In the baseline model, we assume that the large lender is purely motivated by profits. In fact, it shares the objectives, risk attitudes, and intertemporal preferences of the competitive creditors who lend in debt markets. We make this assumption not to represent any specific bilateral lender accurately, but rather to isolate the pure effect of market structure. However, the feedback from debt levels to the bilateral surplus which gives rise to the relational overborrowing effect will be present unless the large lender has a strong motivation to avoid default on the government's marketable debts. In this sense, the core results are robust to alternative descriptions of large bilateral creditors, including those with mixed policy, commercial or financial stability objectives.

A key quantitative finding is that even relatively small bilateral loans can significantly affect sovereign debt outcomes when marketable debt has long maturity. The reason is that debt service on long-term debt is distributed over time, so the amount due in any given period is relatively small. By contrast, bilateral loans—modeled as short-term—must be repaid (or rolled over) every period. As a result, fluctuations in the bilateral loan interest rate have an impact on the current-period budget constraint that is comparable in magnitude to changes in the interest rate on a much larger stock of marketable debt.

The model delivers a simple heuristic for evaluating whether a particular form of bilateral lending is likely to be beneficial for the borrowing country. The key determinant is how the interest rate on bilateral loans responds to the level of the government's market debt: if bilateral loan terms improve as market debt rises, the relational overborrowing channel is likely to be active, as the large lender's willingness to lend cheaply amplifies the government's incentive to accumulate market debt. By contrast, if the terms of bilateral lending become more favorable when debt levels are low, it is more likely to support debt sustainability. These effects operate alongside more familiar channels related to avoiding improvements in the value of default, as recognized in policies like the IMF's Financing Assurances and Sovereign Arrears framework (IMF, 2024a), which aim to limit (bilateral) financing before at least a "credible process for debt restructuring is underway."

Discussion of the Literature We contribute to a nascent literature on the interaction of different types of sovereign debt.[3] Hatchondo, Martinez, and Önder (2017) find that introducing a limited amount of non-defaultable debt improves the government's welfare but only for a short period of time, while Cordella and Powell (2021) show how the preferred creditor status of international financial institutions can arise endogenously. Boz (2011) and Fink and Scholl (2016) study the interaction of market debt with a type of senior debt coming with conditionality. Kirsch and Rühmkorf (2017) and Roch and Uhlig (2018) investigate the role of official lending or bailout agencies in eliminating equilibrium multiplicity, as do Corsetti, Guimarães, and Roubini (2006). The model we consider does not feature this type of equilibrium multiplicity, which would open the door to welfare gains of bilateral loans if they could help rule out unfavorable equilibria. Importantly, we also abstract from conditionality and focus on the impact of market design itself. Also related are Kovrijnykh and Szentes (2007) and Faria-e-Castro et al. (2024) who describe situations featuring strategic behavior of creditors in different contexts, and Dovis and Kirpalani (2023) who propose a design for a lender of last resort that improves the worst equilibrium.

More recently, Arellano and Barreto (2025) combine market and official debt in a model of partial default (Arellano et al., 2023) and find that competitive pricing and longer maturites, like those associated with bilateral lending from the Paris Club, endogenously make official debts less risky despite more favorable restructuring terms. Liu, Liu, and Yue (2025) show how the best subgame-perfect equilibrium of a sovereign debt model with market, bilateral, and multilateral creditors decentralizes the constrained-efficient allocation with imperfect information and moral hazard. We view these papers as emphasizing how certain institutions could potentially improve welfare in a world with both market and official debts, while our results exemplify the perils of bilateral sovereign debt. Also on the risks side, Kondo, Sosa-Padilla, and Swaziek (2025) study a sovereign debt model augmented by exogenous capital flows from a large lender (e.g. China) and focus on the risk that it demands repayment unexpectedly.

Empirically, Perks et al. (2021), and Bahaj and Reis (2021, 2023) document the network of Central Bank swap lines. Cesa-Bianchi et al. (2022) study swap lines among advanced economies, where they can serve a different purpose.

Layout The rest of the paper is structured as follows. Section 2 presents some motivating evidence on the increasing role of official external debt in emerging markets. Section B then introduces our model, starting with the case in which only bilateral loans are available. Section 4 describes the main model in which both types of debt coexist, while Section 5 analyzes its equilibrium, and Section 6 describes the extension with fixed rules for the terms of bilateral loans.

[3] Excellent surveys of the broader literature on sustainable public debt and sovereign default can be found in handbook chapters by Aguiar and Amador (2014), Aguiar, Chatterjee, Cole, and Stangebye (2016), D'Erasmo, Mendoza, and Zhang (2016), and Martinez, Roch, Roldán, and Zettelmeyer (2023).

Finally, Section 7 concludes.

2. Motivating Evidence

Figure 1 summarizes the evolution of official external debt across emerging-market economies, using the World Bank's International Debt Statistics (IDS). The data include all public and publicly guaranteed (PPG) external sovereign debt, measured in constant 2023 USD and aggregated across debtor countries. We distinguish between three broad official creditor categories: multilateral lenders (including the IMF and World Bank), Paris Club bilateral lenders,[4] and other bilateral creditors—primarily non-Paris Club official lenders such as China.

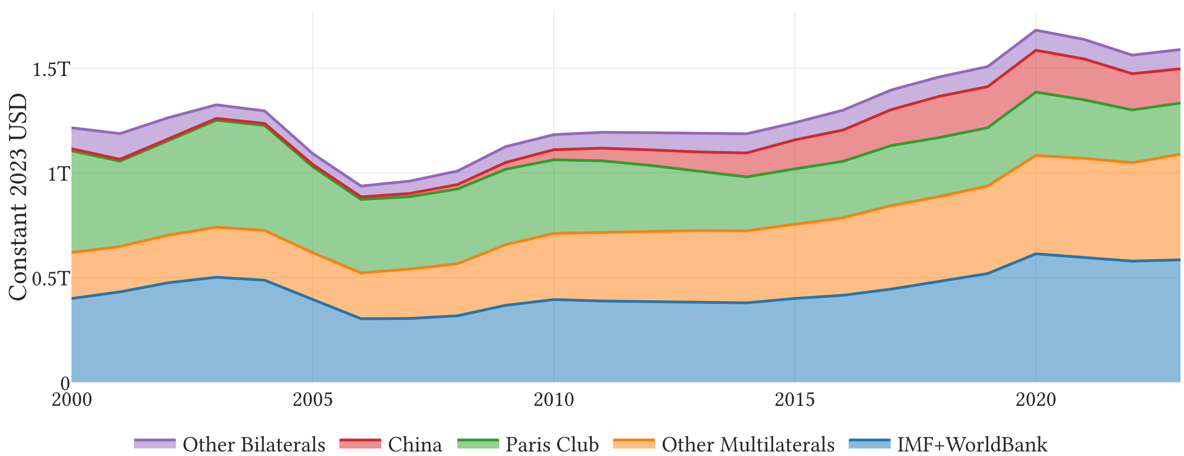

Figure 1: Total Official Debt

Source: International Debt Statistics, The World Bank

The data show a clear shift in the composition of official external debt over the past two decades. While multilateral institutions and Paris Club members accounted for the bulk of official lending in the early 2000s, bilateral creditors outside the Paris Club have grown rapidly in both absolute and relative terms. By 2023, bilateral official lenders collectively account for roughly one-third of the global stock of official external debt to emerging markets, up from less than one-sixth two decades earlier. Among these, China has emerged as the single largest bilateral lender.

While the rise of new bilateral creditors is evident from Figure 1, it only reflects liabilities

[4] As of 2025, the members of the Paris Club are Australia, Austria, Belgium, Brazil, Canada, Denmark, Finland, France, Germany, Ireland, Israel, Italy, Japan, Netherlands, Norway, Russia, South Korea, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

included within the official perimeters of PPG debt and hence reported to the World Bank. Arrangements with Central Banks (notably, swap lines) are often not included in such perimeters and therefore not captured in the figure.[5] This limitation in the data illustrates the opacity of some of these deals (other aspects, like interest rates and maturities, are also typically confidential) which underpins concerns about the welfare implications of these new lenders (Horn et al., 2021a, 2023). Moreover, the scarce data on certain lending channels underscores the need for analytical tools that go beyond reported debt stocks to capture strategic interactions in sovereign borrowing decisions. The rest of our paper addresses these concerns through a theoretical and quantitative evaluation of a model that incorporates a large bilateral lender operating alongside competitive market creditors.

3. Two Minimal Models

We begin our analysis by considering two stylized environments. These are designed to clarify the core mechanisms in the model described in Section 4. The first model describes how the cross-elasticity emerges as the government attempts to improve its bargaining position. The second model takes the cross-elasticity as given and characterizes how it magnifies the incentives for debt dilution, potentially leading to welfare losses.

3.1 A Two-Period Model: How the Cross-Elasticity Emerges from Bargaining

There are two dates, $t = 0, 1$. In the final period, the government receives a stochastic endowment $y(z)$, where $z \sim F$, but wishes to consume in both periods to maximize an expected-utility objective with a strictly concave utility function u.

At the beginning of period $t = 0$, the government owes m to the senior lender. Assume that the government has also issued debt b at price q in markets. The focus of this section is to study comparative statics around the debt price q as well as revenues from debt issuance qb. The cross-elasticity arises whenever higher debt revenues qb lead to better terms on the bilateral loan.

After tapping markets, the government enters into negotiations with the senior lender, who offers a transfer x in exchange for a new loan size $m' = (x + m)(1 + r)$, as in the main model. With a Nash bargaining weight of θ for the lender, the bargaining problem is

$$\max_{x,r} \mathscr{L}(m, x, r)^{\theta} \times \mathscr{B}(b, m, x, r)^{1-\theta}$$

[5] Bahaj and Reis (2021, 2023) compile data on the network of Central Bank swap lines and in many cases are only able to see the total committed size of the swap lines, not drawn or borrowed amounts.

where the lender's surplus is

$$\mathscr{L}(m,x,r) = \underbrace{-x + \beta_L(x+m)(1+r)}_{\text{agreement}} - \underbrace{m}_{\text{threat point}} = (\beta_L(1+r)-1)(x+m)$$

while the government's surplus is

$$\mathscr{B}(b,m,x,r) = \underbrace{u(qb+x) + \beta\mathbb{E}\left[u(y(z)-(x+m)(1+r)-f(b,z))\right]}_{\text{agreement}} - \Big(\underbrace{u(qb-m) + \beta\mathbb{E}\left[u(y(z)-f(b,z))\right]}_{\text{threat point}}\Big)$$

where f is weakly increasing in b and summarizes payments, including functions of future income z such as potential default and default costs, on market debt.

The solution to this bargaining problem is characterized by two first-order conditions: a surplus-sharing rule and an Euler equation

$$u'(qb+x) = \frac{\theta}{1-\theta}\frac{\mathscr{B}(b,m,x,r)}{\mathscr{L}(m,x,r)}$$
$$u'(qb+x) = \frac{\beta}{\beta_L}\mathbb{E}\left[u'(y(z)-(x+m)(1+r)-f(b,z))\right]$$

Together, these conditions generate the following proposition, which describes the cross-elasticity of bargained loans terms to market debt prices.

Proposition 1. *Suppose the utility function u displays enough prudence ($u''' \gg 0$). After a positive debt issuance $b > 0$, higher debt prices q induce lower reliance and better terms when bargaining with the senior lender: both $\frac{\partial x}{\partial q} < 0$ and $\frac{\partial r}{\partial q} < 0$.*

Proof See Appendix C.

Intuitively, higher debt prices depress the government's surplus for any values of (x,m,m'), making it stronger in the negotiation. This leads to lower transfers x and better terms r.

While managing its market debt and bilateral loans, the government obviously does not control q. However, while it stays in the increasing part of the debt Laffer curve, it can affect revenues qb by issuing more debt b. This adds the effect of reducing c_1 via the repayment function $f(b,z)$, which also reduces the government's surplus for given (x,m,m') and makes the government stronger at the bargaining stage.

In the next section, we take as given a cross-elasticity of bilateral loan terms to market debt *issuance* and study how it affects the incentives for debt dilution. In the main model, we show numerically that this type of cross-elasticity emerges around the government's optimal choices under discretion.

3.2 A Three-Period Model: How the Cross-Elasticity Worsens Debt Dilution

There are three dates, $t = 0, 1, 2$. In the final period, the government receives a stochastic endowment $z \sim N(0, \sigma_z)$, but wishes to consume in all periods to maximize an expected-utility objective

$$V = \mathbb{E}\left[\sum_{t=0}^{2} u(c_t)\right] \tag{1}$$

for an increasing and concave utility function u satisfying Inada conditions.

To finance consumption in all three periods, the government issues long-term bonds b at $t = 0$ as well as short-term bonds d and a bilateral loan m at $t = 1$. All three are expressed as promises to repay one unit of the good at $t = 2$. While the loan m is undefaultable, the government faces a choice of whether to repay the bonds, subject to some costs $h(z)$. In other words, consumption in period 2 is given by

$$c_2 = \begin{cases} y(z) - m - (b + d) & \text{if repay} \\ y(z) - m - h(z) & \text{if default} \end{cases}$$

In this finite-horizon example, since the utility function is increasing, default occurs exactly in those states for which the costs of default fall short of the value of the debt. In other words,

$$c_2 = y(z) - m - \min\{b + d, h(z)\} \tag{2}$$

In the middle period, the government issues short-term debt d and loans m. Thanks to competition among risk-neutral investors, the price of debt reflects the probability of default so $q(b, d) = \mathbb{P}\left(b + d < h(z)\right) = p(b + d)$ (notice that the price of short-term debt depends also on the outstanding amounts of long-term debt). We assume an exogenous rule for the pricing of bilateral loans, under which more legacy debt b may induce a lower price but more recently-issued debt d induces higher prices (the cross-elasticity). To keep things simple, we assume

$$\phi(b, d, m) = \alpha_0 + \alpha_b b + \alpha_d d + \alpha_m m$$

where $\alpha_d > 0$, as well as a fixed debt limit $\bar{m}$. Consumption in the middle period is

$$c_1 = dq(b, d) + m\phi(b, d, m) \tag{3}$$

Finally, in the initial period, the government issues long-term debt b at price q, but investors hold beliefs d^e about the issuance of short-term bonds d which will happen later. Initial consumption is

$$c_0 = bq(b, d^e) \tag{4}$$

for the same price function q as above (notice that, since the bilateral loan m does not affect the default set, there is no need to anticipate m to decide q).

We consider different cases for the government's ability to influence beliefs d^e.

Commitment When the government can commit, it chooses $\{b, m, d^e, d\}$ to maximize (1) subject to (2), (3), (4), $m \le \bar{m}$ and $d = d^e$.

The first-order conditions for m (using λ to denote the multiplier on the debt limit constraint) and b are

$$u'(c_1)\left(\phi + m\frac{\partial \phi}{\partial m}\right) = \mathbb{E}\left[u'(c_2)\right] + \lambda \tag{5}$$

$$u'(c_0)\Big(bp'(b+d) + p(b+d)\Big) + u'(c_1)\left(p'(b+d) + m\frac{\partial \phi}{\partial b}\right) = \frac{\partial \mathbb{E}\left[u(c_2)\right]}{\partial b}$$

and the first-order condition for d is

$$u'(c_0)p'(b+d) + u'(c_1)\left(dp'(b+d) + p(b+d) + m\frac{\partial \phi}{\partial d}\right) = \frac{\partial \mathbb{E}\left[u(c_2)\right]}{\partial d} \tag{6}$$

where we note that equal seniority of short and long term debt implies that $\frac{\partial \mathbb{E}[u(c_2)]}{\partial b} = \frac{\partial \mathbb{E}[u(c_2)]}{\partial d}$.

Discretion When deciding sequentially, the government takes as given a value d^e and chooses $\{b, m, d\}$ to maximize (1) subject to (2), (3), (4) and $m \le \bar{m}$. Equilibrium then requires investors to be correct in their assessment, so that the government does choose $d = d^e$. As a result, the first-order conditions for both b and m remain unchanged, but the first-order condition for d is now 'missing' the terms reflecting the manipulation of d^e by the committed government:

$$u'(c_1)\left(dp'(b+d) + p(b+d) + m\frac{\partial \phi}{\partial d}\right) = \frac{\partial \mathbb{E}\left[u(c_2)\right]}{\partial d} \tag{7}$$

How the cross-elasticity worsens debt dilution With commitment and $\bar{m} = 0$, our assumptions (no discounting and the fact that the technology for defaulting both types of bonds b and d is the same) mean that the government would like to set $b = d$ and $c_0 = c_1$: combining the first-order conditions for b and d in the case of commitment when $\bar{m} = 0$ yields

$$u'\Big(\underbrace{p(b+d)b}_{=c_0}\Big)\Big(bp'(b+d) + p(b+d) - p'(b+d)\Big) = u'\Big(\underbrace{p(b+d)d}_{=c_1}\Big)\Big(dp'(b+d) + p(b+d) - p'(b+d)\Big)$$

which highlights the symmetry between both periods and both types of bond.

In the case of discretion, the missing time-0 term in (7) induces a desired increase in d at the expense of b, as increasing d increases the default probability and worsens debt prices, which induces lower b.[6] At $\bar{m} = 0$, the difference between d^D and d^C measures the extent of debt dilution coming only from the presence of long-term debt.

[6]This is stabilizing effect as lower b improves prices marginally which may increase d a bit again.

As $\bar{m}$ increases, if it binds and $m^D = \bar{m}$,[7] we would have c_1 increasing in $\bar{m}$. This means that, when comparing (6) with (7), the removal of a cost of short-term debt d remains the same, but the term which is used to compensate is now multiplied by a smaller marginal utility $u'(c_1)$, so a larger compensation is required. This means that the incentive to issue more short-term debt (from the commitment solution) is increasing in $\bar{m}$.

After increasing d, the government would observe a higher default probability, so it would like to respond by decreasing b, which brings a stabilizing effect on prices. If $\alpha_b < 0$, like we obtain in the quantitative version (because more legacy debt makes the government weaker in the bargaining), moving $\bar{m}$ will also affect the commitment solution to begin with. In that case the government would already start to decrease b and increase d as $\bar{m}$ increases.

Figure 2 illustrates how the cross-elasticity worsens dilution. It shows the solutions to the problems of commitment and discretion (as well as the choices of the sequential government if it faced the prices from the problem with commitment) as a function of a maximum amount $\bar{m}$ that can be drawn on the bilateral loan.

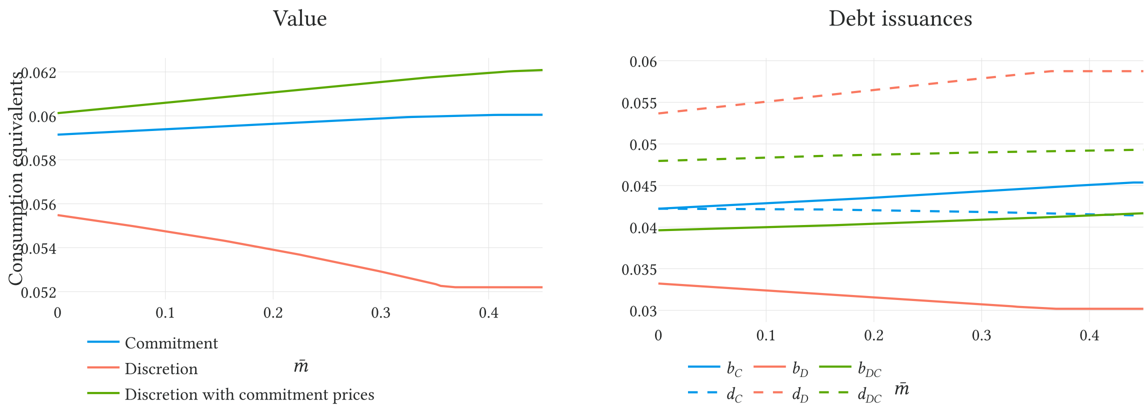

Figure 2: Debt dilution and the cross-elasticity

For every value of $\bar{m}$, as anticipated in our discussion the government chooses a higher debt issuance d_{DC} in the middle period when it reoptimizes but continues to face the prices from the solution with commitment, relative to the commitment solution d_C. The gap between d_C and d_{DC} is also increasing with $\bar{m}$, owing to the fact that the partial-equilibrium incentives in (6) are an increasing function of m. The Figure also shows that in this example, the effect of prices induces further increases in the amount issued under discretion d_D relative to d_{DC}. These forces combine to boost the losses from debt dilution: the gap between the value attained under discretion and commitment, shown in the left panel of Figure 2, clearly increases with $\bar{m}$ while the constraint $m \le \bar{m}$ binds.

[7] If the limit on m does not bind, from (5), a larger d in the case of discretion will push ϕ up, inducing $m^D > m^C$.

4. Model with bilateral loans and defaultable market debt

In this section we present the full version of the model, in which the borrowing government has access to the large lender as well as to a competitive fringe of lenders. Default on the debt b held by competitive lenders is possible, subject to standard output costs of default. However, just as before, bilateral loans m cannot be defaulted.

The timeline of events within a period are illustrated in Figure 3.

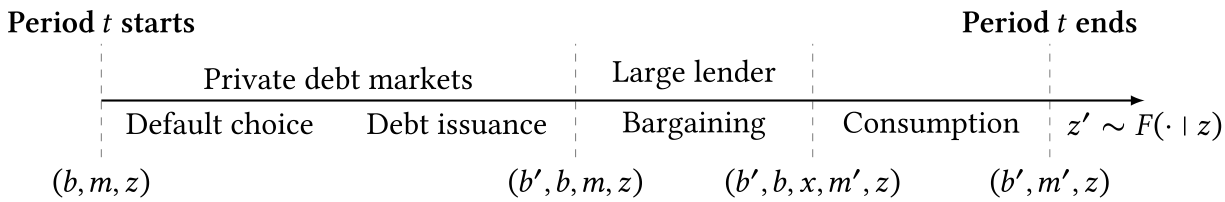

Figure 3: Timeline of events while not in default

At the start of t, the government owes m to the large lender, b to the market, and observes the exogenous state z. Additionally, the economy can be in default ($\zeta = D$) or in repayment ($\zeta = R$). Let $v(b, m, z)$ and $h(b, m, z)$ represent the government's and the large lender's value functions at the beginning of a period, when the government is in good standing with the market.

Private markets In the morning of t, first, the government decides default for the current period if it is in good standing with the market.

$$v(b, m, z) = \max\left\{v_R(b, m, z) + \epsilon_R, v_D(m, z) + \epsilon_D\right\}, \tag{8}$$

where the ϵ's follow a Type 1 Extreme Value distribution, yielding closed forms for $v(b, m, z)$ and the (beginning-of-period) default probability $P(b, m, z)$

$$\begin{aligned} v(b, m, z) &= \chi \log\big(\exp(v_D(m, z)/\chi) + \exp(v_R(b, m, z)/\chi)\big) \\ P(b, m, z) &= \frac{\exp(v_D(m, z)/\chi)}{\exp(v_D(m, z)/\chi) + \exp(v_R(b, m, z)/\chi)} \end{aligned}$$

We assume that defaulting on the market debt fully destroys all bonds outstanding and triggers exclusion from international capital markets. Access to markets is then recovered stochastically with constant probability ψ. In addition, the maximum amount that can be borrowed from the large lender m' while in default on its market debt is constrained by $m' \leq \Gamma(m, z)$. We begin our analysis without restrictions by setting $\Gamma(m, z) = +\infty$ but describe below different possibilities for the value of Γ.

If default is not chosen in the current period, the government issues new debt b' to the fringe of lenders understanding the value of entering negotiations with the monopolist having issued this debt level b',

$$v_R(b,m,z) = \max_{b'} w_R(b',b,m,z)\,, \tag{9}$$

where debt is a perpetuity of geometrically-decaying coupons (Leland, 1998; Hatchondo and Martinez, 2009; Arellano and Ramanarayanan, 2012). A unit of debt issued in period t promises to repay $\kappa(1-\delta)^{s-1}$ units of the tradable good in period $t+s$, effectively making a unit issued at $t-1$ a perfect substitute for $(1-\delta)$ units issued at t. In this sense, δ controls the maturity of debt while κ controls the average size of the coupon payments.

The price faced by the borrower government reflects its lenders' expectations of repayment, discounted with a risk-neutral kernel

$$q(b',b,m,z) = \frac{1}{1+r^*}\mathbb{E}\left[(1-\mathbb{1}_D(b',m',z'))\left(\kappa+(1-\delta)q(b'',b',m',z')\right)\mid z\right] \tag{10}$$

where r^* is the international risk-free rate,[8] $\mathbb{1}_D(b,m,z)$ denotes the government's default policy as perceived by the competitive lenders, $m' = m'_R(b',b,m,z)$ is the expected result of negotiations with the large lender, to happen in the 'afternoon' of period t, and $b'' = g_b(b', m'_R(b',b,m,z), z')$ is the expected debt issuance in the following period.

Bilateral loan In the afternoon of t, the government meets with the large lender to negotiate the bilateral loan. As before, the outcome of their negotiation is a transfer x and new loan size m' which solve the following Nash bargaining problems, depending on default status vis-à-vis the market

$$\begin{gathered}\max_{m',x} \mathscr{L}_R(b',x,m,m',z)^{\theta}\ \mathscr{B}_R(b',b,x,m,m',z)^{1-\theta}\\ \text{or}\\ \max_{m',x} \mathscr{L}_D(x,m,m',z)^{\theta}\ \mathscr{B}_D(x,m,m',z)^{1-\theta}\end{gathered} \tag{11}$$

As before, the large lender's surplus is

$$\begin{aligned}\mathscr{L}_R(b',x,m,m',z) &= -x-m+\beta_L\mathbb{E}\left[h(b',m',z')-h(b',0,z')\mid z\right]\\ \mathscr{L}_D(x,m,m',z) &= -x-m+\beta_L\mathbb{E}\left[\psi\left(h(0,m',z')-h(0,0,z')\right)+(1-\psi)\left(h_D(m',z')-h_D(0,z')\right)\mid z\right]\,,\end{aligned}$$

while the borrower's surplus now also reflects outcomes in debt markets

$$\begin{aligned}\mathscr{B}_R(b',b,x,m,m',z) &= u(y(z)+B(b',b,m,z)+x)-u(y(z)+B(b',b,m,z)-m)\,+\\ &\quad+\beta\mathbb{E}\left[v(b',m',z')-v(b',0,z')\mid z\right]\\ \mathscr{B}_D(x,m,m',z) &= u(y_D(z)+x)-u(y_D(z)-m)\,+\\ &\quad+\beta\mathbb{E}\left[\psi\left(v(0,m',z')-v(0,0,z')\right)+(1-\psi)\left(v_D(m',z')-v_D(0,z')\right)\mid z\right]\end{aligned}$$

[8] We set $\kappa = r^*+\delta$, so that the price of debt equals 1 when the government repays with certainty.

where the function $y_D(z) = y(z) - \xi(z)$ is output in default and $B(b', b, m, z)$ summarizes net transfers from the competitive lenders received in the morning. Our assumptions on debt maturity yield $B(b', b, m, z) \equiv q(b', b, m, z)(b' - (1-\delta)b) - \kappa b$. In default, opportunities to reaccess markets arrive with probability ψ. The bargaining problems yield new terms for the bilateral loan $\{x_R(b', b, m, z), m'_R(b', b, m, z)\}$ and $\{x_D(m, z), m'_D(m, z)\}$ in repayment and in default, respectively.

The most important way in which the presence of debt markets affects the bargaining stage is through the net flows from debt repayment and new issuance, $B(b', b, m, z)$. These flows modulate the government's threat point: after a successful issuance which raises large revenues, the government is in a strong position to negotiate as repaying m is less costly.

Consumption After the negotiation is done and transfers settled, consumption takes place.

$$\begin{aligned} c_R(b', b, m, z) &= y(z) + B(b', b, m, z) + x_R(b', b, m, z) \\ c_D(m, z) &= y(z) - \xi(z) + x_D(m, z) \end{aligned} \tag{12}$$

Given transfers $x_R(b', b, m, z)$ and $x_D(m, z)$, we can now return to the beginning of the period and determine the values of repaying and defaulting

$$\begin{aligned} w_R(b', b, m, z) &= u(c_R(b', b, m, z)) + \beta \mathbb{E}\left[v\left(b', m'_R(b', b, m, z), z'\right) \mid z\right] \quad \text{and} \\ v_D(m, z) &= u(c_D(m, z)) + \beta \mathbb{E}\left[\psi v\left(0, m'_D(m, z), z'\right) + (1-\psi) v_D\left(m'_D(m, z), z'\right) \mid z\right], \end{aligned} \tag{13}$$

while for the large lender we have

$$\begin{aligned} h(b, m, z) &= P(b, m, z) h_D(m, z) + (1 - P(b, m, z)) h_R(b'(b, m, z), b, m, z), \\ h_R(b', b, m, z) &= a - x_R(b', b, m, z) + \beta_L \mathbb{E}\left[h(b', m'_R(b', b, m, z), z') \mid z\right], \text{ and} \\ h_D(m, z) &= a - x_D(m, z) + \beta_L \mathbb{E}\left[\psi h(0, m'_D(m, z), z') + (1-\psi) h_D(m'_D(m, z), z') \mid z\right]. \end{aligned} \tag{14}$$

5. Quantitative results

We parametrize our model at a quarterly frequency following standard strategies in the sovereign default literature. Most parameters are set externally, such as those governing the stochastic process for income, risk-free rates, risk aversion, the length of exclusion after default, and the duration of market debt (Chatterjee and Eyigungor, 2012).[9]

The critical parameter θ, which governs the lender's bargaining power, is set to 0.5 in our baseline but we study comparative statics around it later on. The remaining parameters (the sovereign's discount rate and the default costs) are calibrated internally to match the average

[9] As has also become standard for numerical performance, we include preference shocks for default as described above, but also for the debt issuance choice; both are set to a small variance to avoid impacting the equilibrium.

debt-to-output ratio as well as the average and standard deviation of spreads. Our calibration is based on comparing data for Argentina between 1993q1 and 2001q4 to simulated data from the model. For the simulations, we use 35-period samples that end just before a default event, and condition on no default occurring within those 35 periods or two years prior, as is standard in this literature when assuming no recovery after default (Arellano, 2008; Chatterjee and Eyigungor, 2012; Roch and Roldán, 2023).

Table 1 summarizes our parametrization.

TABLE 1: BASELINE PARAMETER VALUES

	Parameter	Value
Sovereign's discount factor	β	0.9504
Sovereign's risk aversion	γ	2
Preference shock scale parameter	χ	0.025
Lender's bargaining power	θ	0.5
Risk-free interest rate	r^*	0.01
Duration of debt	δ	0.05
Income autocorrelation coefficient	ρ_z	0.9484
Standard deviation of y_t	σ_z	0.02
Reentry probability	ψ	0.0385
Default cost: linear	d_0	-0.24
Default cost: quadratic	d_1	0.3

Table 2 presents some statistics from simulating the model with and without bilateral loans, for different values of the lender's bargaining weight θ. Statistics correspond to pre-default samples of 35 quarters and the term 'loans' refers to bilateral loans m. Welfare gains are reported in equivalent consumption terms and calculated averaging the ergodic distribution of the equilibrium in which only market debt is allowed, conditional on repayment. These welfare gains arise from comparing the economy with only market debt with economies with both types of debt at $m = 0$. The leftmost column corresponds to the equilibrium with only market debt and as such generates the business-cycle properties of Argentina in the years preceding its 2001 default.

The remaining columns show that even with a relatively large bargaining weight for the government, the availability of bilateral loans substantially increases the frequency of default. As a consequence, the government pays higher and more volatile spreads, despite slightly lower market debt levels and relatively modest amounts borrowed from the large lender. As a result, even with $\theta = 0.25$, the government prefers the equilibrium in which the large lender is not present.

Table 2: Business cycle statistics with and without bilateral loans

	Only market	**Unrestricted,** $\theta = 0.25$	**Unrestricted,** $\theta = 0.5$
Avg spread (bps)	714	1,613	2,105
Std spread (bps)	399	927	1,331
$\sigma(c)/\sigma(y)$ (%)	113	109	109
Debt to GDP (%)	22.5	21.7	21.2
Loan to GDP (%)	0	3.4	3.02
Loan spread (bps)	–	-52.5	-429
Corr. loan & spreads (%)	–	61.7	67.5
Default frequency (%)	5.72	11	13
Welfare gains (rep)	–	-0.15%	-0.43%

Note: Statistics are based on model simulations of 35 quarters before a default on market debt. The column marked 'Only market' refers to a version of the model without bilateral loans, while the columns marked 'Unrestricted' refer to versions in which bilateral loans can be used even during default episodes. Default frequencies are computed on the ergodic distribution. Welfare gains are given as equivalent increases in consumption calculated over the ergodic distribution of the model without bilateral loans, conditional on repayment.

Since the large lender keeps a share of the surplus generated by the loan, the borrower government is somewhat reluctant to use it. In a typical simulation path, conditional on no default the amount borrowed bilaterally is 3.3% of annual income with a standard deviation of 1.6%. Figure 4 shows that this changes significantly around default events: the loan size m shoots up around the moment of default. The large lender heavily subsidizes this accumulation of bilateral debt. As before, the large lender provides negative interest rates at first, while bilateral debt is increasing, to then raise them as default grows closer. In this case, the gamble for debt overhang has a twist: when the economy recovers market access, it immediately issues market debt to pay off the bilateral loan (see Figure 12 in the Appendix). The large lender is then gambling that income will not revert and that the exclusion period will be long.

While most of the use of the bilateral loan m occurs during default, Figure 4 shows that default episodes are preceded by bilateral issuance in an effort to avoid or postpone default (Figure 4 does not show the defaults that were avoided as a consequence of the large lender's presence).

Bilateral loans affect the economy in two ways: on the one hand, they provide extra financing when default risk makes borrowing in private markets costly. But they also provide funds in default, which raises the value of being excluded from private markets and consequently spreads.

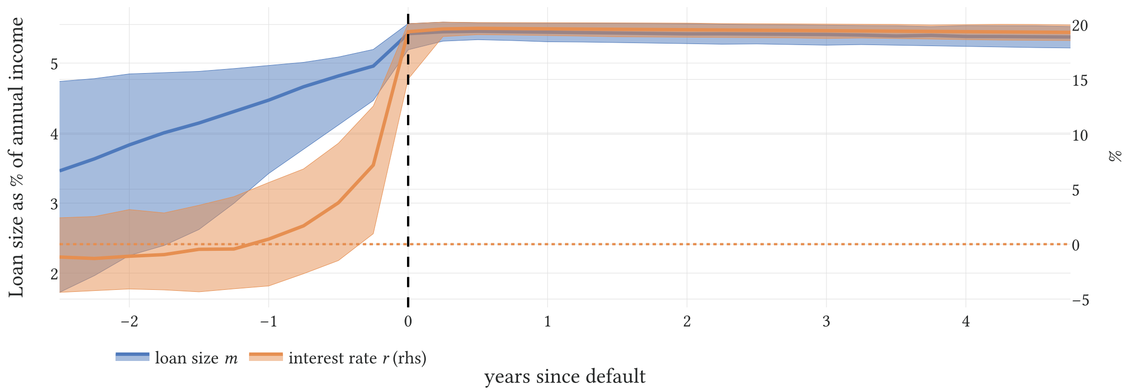

Figure 4: Loans around default events

To disentangle these effects, we consider a variant of the model in which the large creditor is only willing to lend while the economy is in good standing with private debt markets, in other words, a variant where we set $\Gamma(m, z) = 0$. This implies in particular that whenever the government defaults, it must at the same time repay all of its bilateral loans.

Table 3 presents some statistics from simulating the model with and without bilateral loans (now marked 'Unrestricted'), as well as the variant in which bilateral loans are 'Limited.'

In the version with Limited bilateral loans in default, the usage of these loans declines by more than two-thirds on average, as repaying the large creditor after defaulting becomes even more costly (and can even act as an extra cost of default). However, bilateral loans are still mostly used when spreads are high, as shown by the high correlation between spreads and loans. The Limited version still creates welfare losses for the economy, but these are much lower than in the Unrestricted case.

5.1 *Default probabilities and debt prices*

Figure 5 shows ex-post default regions for private debt, when the bilateral loan $m = 0$. Solid lines represent the case without bilateral loans, while dotted and dashed lines correspond to the versions with Unrestricted and Limited bilateral loans. When bilateral loans are Unrestricted, their presence leads to an increase in default as the government is able to sustain lower levels of debt. The Limited loans case has government default policies which are virtually unchanged from the case without bilateral loans.

Figure 5 highlights the negative impact of the bilateral loans. In the Unrestricted version, loans are available during default, which makes market debt repayment less attractive. The higher default probability translates into lower prices for debt, as shown in Figure 6. This effect is muted

Table 3: Business cycle statistics with bilateral loans

	Only market	**Unrestricted,** $\theta = 0.5$	**Limited,** $\theta = 0.5$
Avg spread (bps)	714	2,105	1,038
Std spread (bps)	399	1,331	612
$\sigma(c)/\sigma(y)$ (%)	113	109	113
Debt to GDP (%)	22.5	21.2	22.5
Loan to GDP (%)	0	3.02	1.06
Loan spread (bps)	–	-429	536
Corr. loan & spreads (%)	–	67.5	71.1
Default frequency (%)	5.72	13	7.72
Welfare gains (rep)	–	-0.43%	-0.2%

Note: Statistics are based on model simulations of 35 quarters before a default on market debt. 'Only market' refers to a version of the model without bilateral loans, 'Unrestricted' to a version in which bilateral loans can be used even during default episodes, and 'Limited' to a version in which the bilateral loan must have a balance of 0 during default. Default frequencies are computed on the ergodic distribution, as are welfare gains, which are given as equivalent increases in consumption calculated over the ergodic distribution of the model without bilateral loans, conditional on repayment.

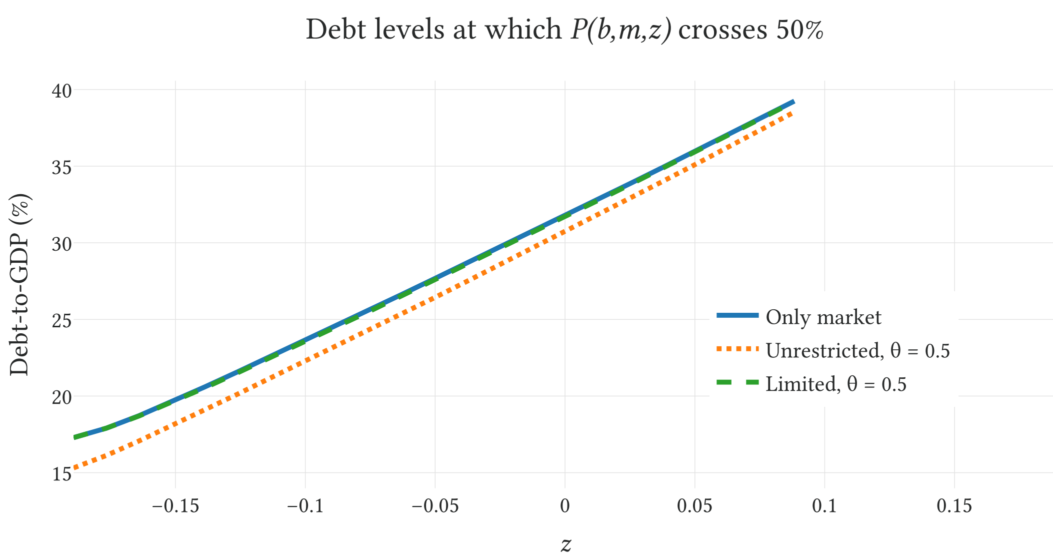

Figure 5: Default regions

in the Limited variant, when bilateral loans are not available in default. However, as shown in Figure 6, prices remain lower in the Limited variant relative to the model without the large creditor, especially when debt is low. This means that even though the one-period-ahead default probability may not increase when bilateral loans are introduced in a limited fashion, policies (notably, future borrowing) are altered in a way that still creates more default risk later on. In other words, the option of bilateral loans, even in the Limited variant, increases debt dilution and therefore lowers equilibrium debt prices.

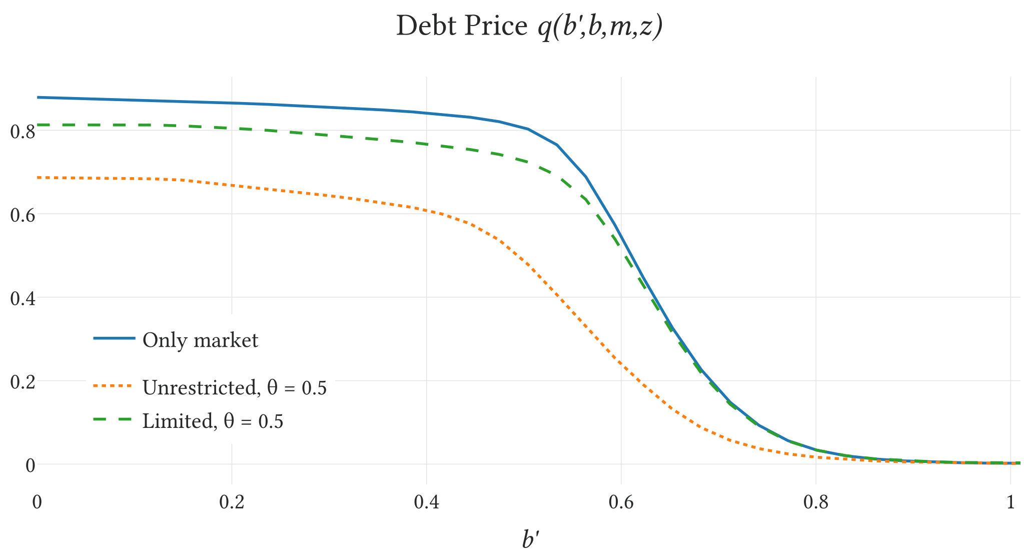

Figure 6: Debt prices

5.2 *Dynamics with bilateral loans*

In the model with bilateral loans, the government issues debt in a riskier manner. Figure 7 presents the ergodic distribution of the debt-to-GDP ratio in simulations of the three models, conditional on repayment. When bilateral loans are available, in both variants, the economy spends more time in the region of the state space where default risk is large.

Figure 8 plots the monopolist's value (or profit) function (14) as a function of debt, keeping $m = 0$ and the endowment level at its mean. In both versions (both when loans are Unrestricted or Limited in default), the large lender's profits are increasing in market debt b, at least when debt is low enough that the one-period-ahead default probability remains contained. In this region, as spreads open up, the presence of the large lender is more valuable to the government, which increases total surplus in the negotiations. As debt continues to grow and default becomes more likely, profits for the large lender decrease in the Limited version, as any bilateral lending m must be repaid at face value upon default, but increase sharply in the Unrestricted case, in which large payments can be extracted from the borrower during the default spell.

When the government has a safe level of debt and pays low spreads, there is little for the

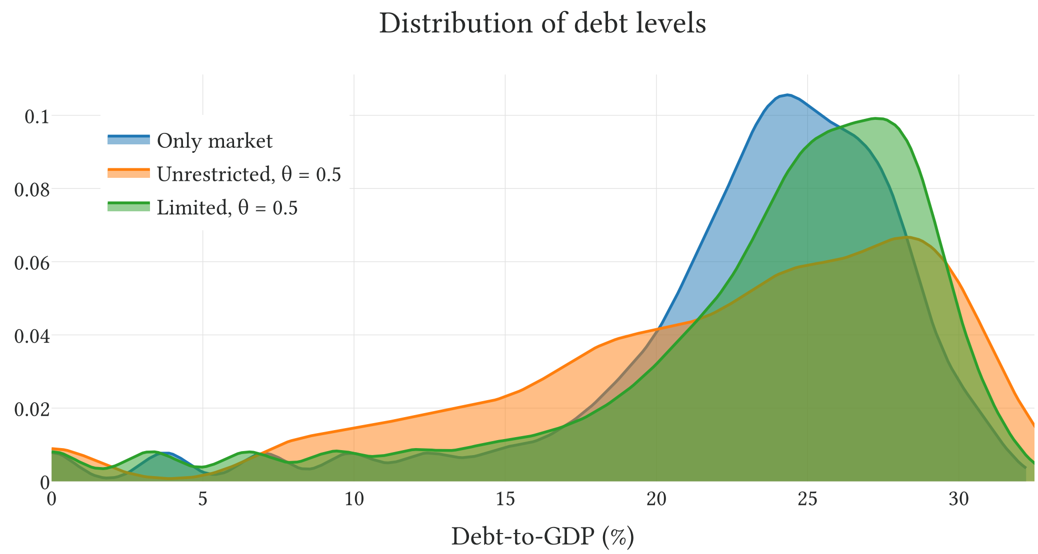

Figure 7: Distribution of debt levels

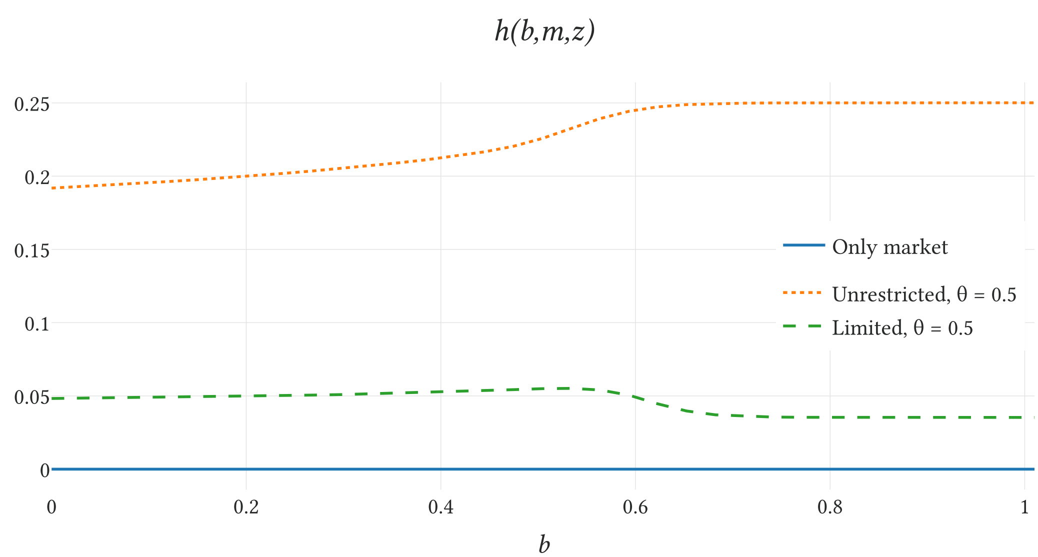

Figure 8: Large lender's profits

monopolist to offer that private markets do not already provide at a competitive price. But when default risk drives up spreads, the monopolist can provide financing through the bilateral loan, which is non-defaultable, and charge an interest rate between the risk-free rate and the rate that the government is paying on its market debt.

Relational overborrowing At the bargaining stage, the choice of market debt b' has already been made by the government. As a result, if bilateral negotiations break down the borrower is left with a consumption level of $y(z) + q(b', b, m, z)(b' - (1-\delta)b) - \kappa b - m$. In periods when the government is trying to reduce market debt b', it enters negotiations with the large lender in a 'weak' position as consumption at the government's threat point is low. This generates large gains (for the borrower) from any transfer x provided by the large lender. Thus, in order to split the surplus such transfers must come at a high interest rate. Conversely, when the government has issued a large amount of bonds b', it expects a high level of consumption even if negotiations break down, which results in a strong position and good borrowing terms with the large lender.

Figure 9 shows the interest rate that results from bilateral negotiations as a function of the choice of b'. It confirms the discussion above: borrowing terms with the large lender respond aggressively to the government's choice of indebtedness in debt markets.

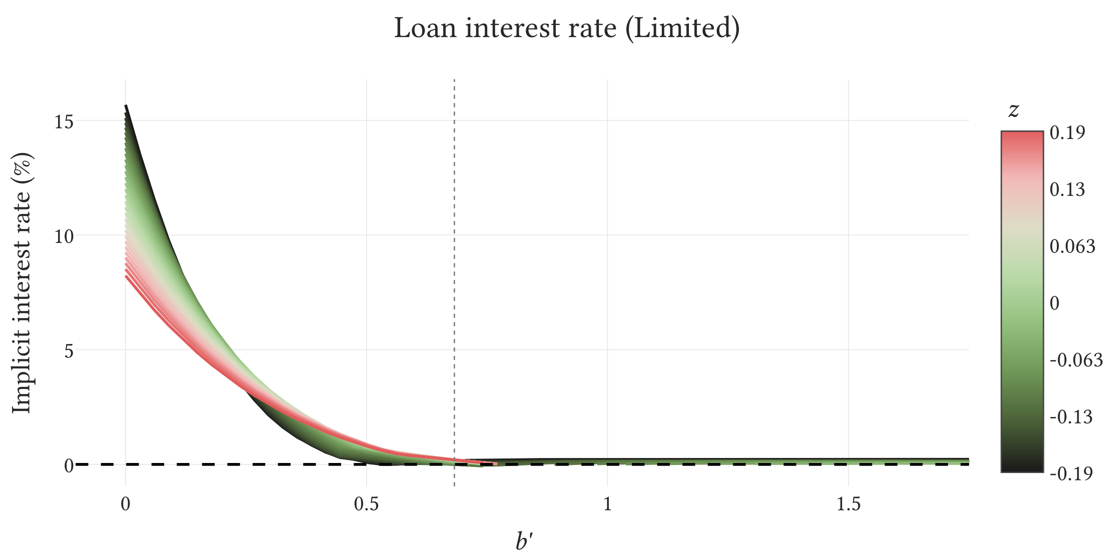

FIGURE 9: INTEREST RATE ON THE BILATERAL LOAN

As a result of this elasticity of loan terms to debt, new terms appear in the government's Euler equation for market debt b':

$$u'(c)\left(q + \frac{\partial q}{\partial b'} i + \frac{\partial x}{\partial b'}\right) = \beta \mathbb{E}\left[u'(c')(1 - \mathbb{1}_D)\left(\kappa + (1-\delta)q' - i'\left(\frac{\partial q'}{\partial b} - \frac{\partial q'}{\partial m}\frac{\partial m'}{\partial b'}\right) - \frac{\partial x'}{\partial b} + \frac{\partial x'}{\partial m}\frac{\partial m'}{\partial b'}\right)\right] \tag{15}$$

where $i = b' - (1-\delta)b$ represents new market debt issuances, $\frac{\partial q'}{\partial b}$ refers to the derivative of the following period's debt price with respect to the beginning-of-period debt level at that point (in other words, b' from the perspective of the current period) and similarly for $\frac{\partial q'}{\partial m}$, $\frac{\partial x'}{\partial b}$, and $\frac{\partial x'}{\partial m}$. Relative to a world with only market debt, there are three new effects arising from the presence of the large lender.

First, issuing debt affects transfers x received from the large lender in the afternoon of the current period. Second, issuing debt also affects the desired amount of loans m' taken from the large creditor. This flows into debt prices q' and transfers x' in the following period through the chain-rule terms. Finally, because debt prices and borrowing terms also depend on the initial level of indebtedness b, issuing new debt affects both the price at which market debt will be placed q' as well as transfers x' in the following period.

The left-hand side of (15) describes a countervailing force to the market discipline of spreads. In a model with only market debt, the benefit of issuing an extra unit of debt is $u'(c)\left(q + \frac{\partial q}{\partial b'} i\right)$: the government understands that issuing more debt may increase spreads (decrease the price q) and this mitigates issuances. With the large lender, the benefit of issuing an extra bond includes additionally the term $\frac{\partial x}{\partial b'}$ as the extra issuance also affects the threat point at the bargaining stage. Notice that, by writing the transfer x as a price $\frac{1}{1+r}$ times the new loan m' (as in (17)), the marginal benefit of debt term becomes $u'(c)\left(q + \frac{\partial q}{\partial b'} i + \frac{1}{1+r}\frac{\partial m'}{\partial b'} + \frac{\partial \frac{1}{1+r}}{\partial b'} m'\right)$, which emphasizes how debt issuances affect revenues from the market and from the large lender in a strictly symmetrical way (notice also that $\frac{\partial i}{\partial b'} = 1$).

5.3 *Welfare effects of bilateral loans*

The forces discussed above combine to produce the welfare effects of bilateral loans. Figure 10 shows the government's value function as a function of debt b when it owes $m = 0$ to the large lender. The government prefers bilateral loans to be Unavailable during default, except of course when a default in the current period is very likely.

6. Programming the large lender

We have shown how the combination of market debt and bilateral loans can create welfare losses for the government through the relational overborrowing effect which incentivizes risk-taking in debt issuances.

In this section we replace the bargaining protocol with fixed rules for the terms of bilateral loans, with two objectives. First, to better understand the dynamics of the relational overborrow-

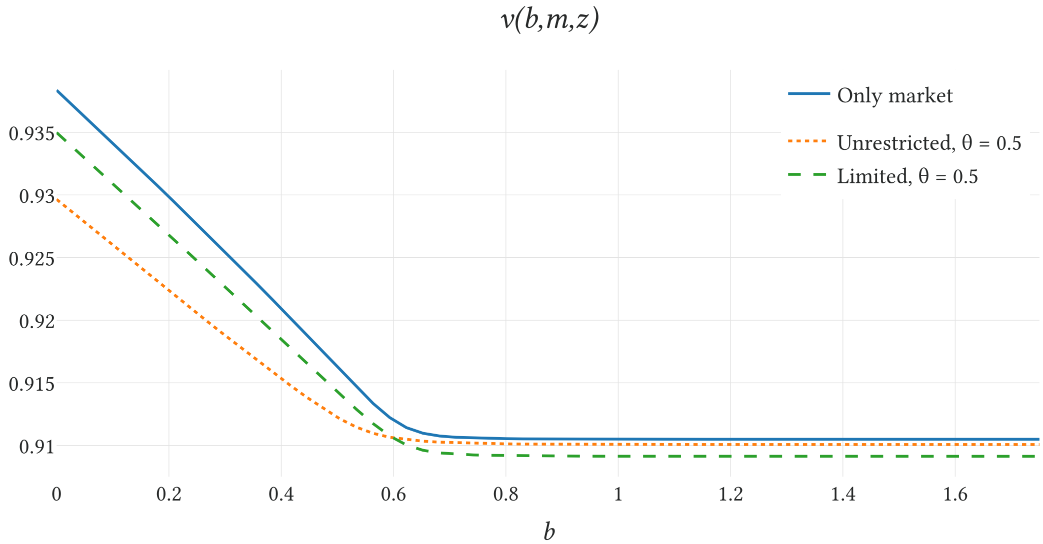

FIGURE 10: VALUE FUNCTIONS

ing effect in a simpler case. Second, to explore design questions surrounding bilateral loans. Can rules that contain spreads on market debt by curbing sovereign default risk be found? Are there designs that can create welfare gains for the government? What is the optimal design of bilateral loans? Are there possible Pareto improvements, under which the government attains a higher value than without bilateral loans and also the large lender obtains profits?

To investigate these questions, we remove the bargaining protocol for the bilateral loans and, instead, consider rules of the form

$$r(b', m') = \max\left\{r^*,\ \alpha_0 + \alpha_b b' + \alpha_m m'\right\}$$

With these rules, the government now faces the exogenous interest rate $r(b', m')$ (on the bilateral debt) when it chooses market debt b' and bilateral loan m'. Notice that because the bilateral loan always costs weakly more than the risk-free rate r^*, the large lender makes non-negative profits.

At this moment, we consider a 'risk-inducing' rule with $\alpha_0 > 0, \alpha_b < 0, \alpha_m = 0$, which replicates the main properties of the equilibrium with bargaining, and a size-dependent rule which does not load on indebtedness in markets, with $\alpha_0 > 0, \alpha_b = 0, \alpha_m > 0$. Table 4 summarizes our findings.

The risk-inducing rule creates similar dynamics to the equilibrium with bargaining and, thus, welfare losses for the government. However, the size-dependent rule exemplifies the possibility of rules which induce lower default and spreads, improving the government's welfare, while also generating profits for the large lender.

Table 4: Outcomes with exogenous rules for bilateral loans

	Only market	Size dependent r	Risk inducing r	Limited, $\theta = 0.5$
Avg spread (bps)	714	623	921	1,038
Std spread (bps)	399	315	552	612
$\sigma(c)/\sigma(y)$ (%)	113	115	115	113
Debt to GDP (%)	22.5	23.5	22.8	22.5
Loan to GDP (%)	0	0.71	0.972	1.06
Loan spread (bps)	–	682	1,264	536
Corr. loan & spreads (%)	–	62.5	48.1	71.1
Default frequency (%)	5.72	5.13	6.92	7.72
Welfare gains (rep)	–	0.21%	-0.079%	-0.2%

Note: Statistics are based on model simulations of 35 quarters before a default on market debt. 'Only market' refers to a version of the model without bilateral loans, while 'Limited' refers to the version in which biltaral loan balances must be 0 while in default. The middle columns denote the versions with exogenous terms for the bilateral loan: 'Size dependent r' has the bilateral interest rate increase with the balance in the bilateral loan, while 'Risk inducing r' has the bilateral interest rate decrease with the amount of market debt. Default frequencies are computed on the ergodic distribution. Welfare gains are given as equivalent increases in consumption calculated over the ergodic distribution of the model without bilateral loans, conditional on repayment.

7. Concluding remarks

Inspired by the rise of new bilateral lenders outside the Paris Club and with claims to de facto senior status relative to private creditors, we investigate the interaction between marketable debt and a particular type of bilateral debt. We develop a model of sovereign debt in which the government can borrow from a large lender as well as from a competitive fringe of private creditors. We find that both sources of funds are linked by strong interactions, even when quantitatively the amounts borrowed from the large lender remain an order of magnitude smaller than market debts.

The model features a 'relational overborrowing' effect which arises in the presence of the large lender. This effect is due to an endogenous cross-elasticity of bilateral borrowing terms to outcomes in debt markets which erodes the market discipline of spreads. Our model of bargaining with the large lender generates this cross-elasticity with three key ingredients: loans from the large lender are more costly (or impossible) to default, their terms result from bargaining, and they are of shorter maturity than marketable debts. These ingredients capture the defining features of

Central Bank swap lines, which Horn et al. (2021b) identify as a key instrument of senior bilateral sovereign lending. The simpler version of the model with exogenous bilateral terms shows that the cross-elasticity is in itself sufficient to incentivize the government to overborrow, face more sovereign risk, and ultimately attain lower levels of welfare.

The model with exogenous bilateral terms also illustrates a broader point. Policy discussions surrounding bilateral debts often focus on the price of the loans and on seniority itself. Our results emphasize that welfare losses are likely to come from a different source. In particular, we show how welfare losses can arise even when the government is free not to borrow from the bilateral lender if the terms offered are not advantageous enough relative to the market. We also provide examples of pricing schedules for the senior debt, like the size-dependent rules, which improve welfare for the government without losses for the large lender. Rather, our main result is that the perverse incentives created by the cross-elasticity, which have been more overlooked, are crucial to generate the welfare losses we find. Our main model then describes a set of plausible assumptions, especially sequential bargaining over borrowing terms, under which the cross-elasticity could be expected to arise, thus creating perils of bilateral sovereign debt.

Our results suggest that increasing the amount of sources of indebtedness is not necessarily beneficial for the borrowing government. While bilateral loans can in some cases help a government fend off default, they can also make it more likely: either through reducing the effective costs of default (when it is possible to borrow from the large lender while excluded from markets) or through the relational overborrowing effect.

The welfare impact of the large lender's presence raises important policy questions and challenges. Limiting the use of bilateral debt during defaults is a clear welfare-enhancing policy in this model. Because the relational overborrowing effect operates through increased default risk, the gains of fiscal rules that constrain market borrowing should be larger for countries with access to the type of bilateral debts we describe.

The model implies a simple test to gauge the likely effects of a new bilateral creditor or instrument in practice. Bilateral loans whose interest rate is expected to be strongly decreasing in the amount (or spreads) of marketable debt will induce relational overborrowing and are thus likely to hurt welfare. More generally, it highlights the benefits of transparent rules for the terms of bilateral debts.

References

Aguiar, M. and M. Amador (2014): "Sovereign Debt," *Handbook of International Economics*, 4, 647–87.

Aguiar, M., S. Chatterjee, H. Cole, and Z. Stangebye (2016): "Quantitative Models of Sovereign Debt Crises," *Handbook of Macroeconomics*, 2B, 1697–1755, chapter 21.

Arellano, C. (2008): "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 98, 690–712.

Arellano, C. and L. Barreto (2025): "Official Sovereign Debt," Mimeo.

Arellano, C., X. Mateos-Planas, and J.-V. Ríos-Rull (2023): "Partial Default," *Journal of Political Economy*, 131, 1385–1439.

Arellano, C. and A. Ramanarayanan (2012): "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 120, 187–232.

Bahaj, S. and R. Reis (2021): "Central Bank Swap Lines: Evidence on the Effects of the Lender of Last Resort," *The Review of Economic Studies*, 89, 1654–1693.

Bahaj, S. and R. Reis (2023): "The workings of liquidity lines between central banks," in *The Research Handbook of Financial Markets*, ed. by R. S. Gürkaynak and J. H. Wright, Edward Elgar Publishing, chap. 5, 102–124.

Boz, E. (2011): "Sovereign default, private sector creditors, and the IFIs," *Journal of International Economics*, 83, 70–82.

Cesa-Bianchi, A., F. Eguren-Martin, and A. Ferrero (2022): "Dollar Shortages and Central Bank Swap Lines," Mimeo.

Chatterjee, S. and B. Eyigungor (2012): "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 102, 2674–99.

Cordella, T. and A. Powell (2021): "Preferred and non-preferred creditors," *Journal of International Economics*, 132, 103491.

Corsetti, G., B. Guimarães, and N. Roubini (2006): "International lending of last resort and moral hazard: A model of IMF's catalytic finance," *Journal of Monetary Economics*, 53, 441–471.

D'Erasmo, P., E. Mendoza, and J. Zhang (2016): "Quantitative Models of Sovereign Debt Crises," *Handbook of Macroeconomics*, 2B, 2493–2597, chapter 32.

Dovis, A. and R. Kirpalani (2023): "On the Design of a Robust Lender of Last Resort," Mimeo.

Faria-e-Castro, M., P. Paul, and J. M. Sánchez (2024): "Evergreening," *Journal of Financial Economics*, 153, 103778.

Fink, F. and A. Scholl (2016): "A quantitative model of sovereign debt, bailouts and conditionality," *Journal of International Economics*, 98, 176–190.

Gelpern, A., S. Horn, S. Morris, B. Parks, and C. Trebesch (2021): "How China lends: A rare look into 100 debt contracts with foreign governments," Working Paper Series WP21-7, Peterson Institute for International Economics.

Hatchondo, J. C. and L. Martinez (2009): "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 79, 117–125.

Hatchondo, J. C., L. Martinez, and Y. K. Önder (2017): "Non-defaultable debt and sovereign risk," *Journal of International Economics*, 105, 217–229.

Horn, S., B. C. Parks, C. M. Reinhart, and C. Trebesch (2023): "China as an International Lender of Last Resort," Working Paper 31105, National Bureau of Economic Research.

Horn, S., C. M. Reinhart, and C. Trebesch (2021a): "China's overseas lending," *Journal of International Economics*, 133, 103539.

Horn, S., C. M. Reinhart, and C. Trebesch (2021b): "Coping with Disasters : Two Centuries of International Official Lending," Policy Research Working Paper Series 9612, The World Bank.

IMF (2024a): "Guidance Note On The Financing Assurances And Sovereign Arrears Policies And The Fund's Role In Debt Restructurings," Policy Paper No. 2024/053, International Monetary Fund, Washington, DC.

IMF (2024b): "Review Of Charges And The Surcharge Policy—Reform Proposals," Policy Paper No. 2024/046, International Monetary Fund, Washington, DC.

Kirsch, F. and R. Rühmkorf (2017): "Sovereign borrowing, financial assistance, and debt repudiation," *Economic Theory*, 64, 777–804.

Kondo, I., C. Sosa-Padilla, and Z. Swaziek (2025): "Borrowing in the Shadow of China," Mimeo.

Kovrijnykh, N. and B. Szentes (2007): "Equilibrium Default Cycles," *Journal of Political Economy*, 115, 403–446.

Leland, H. E. (1998): "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance*, 53, 1213–1243.

Liu, Q., Z. Liu, and V. Yue (2025): “A Theory of International Official Lending,” Mimeo.

Martinez, L., F. Roch, F. Roldán, and J. Zettelmeyer (2023): “Sovereign Debt,” in *The Research Handbook of Financial Markets*, ed. by R. S. Gürkaynak and J. H. Wright, Edward Elgar Publishing, chap. 17, 378–405.

Perks, M., Y. Rao, J. Shin, and K. Tokuoka (2021): “Evolution of Bilateral Swap Lines,” IMF Working Papers 2021/210, International Monetary Fund.

Roch, F. and F. Roldán (2023): “Uncertainty Premia, Sovereign Default Risk, and State-Contingent Debt,” *Journal of Political Economy Macroeconomics*, 1, 334–370.

Roch, F. and H. Uhlig (2018): “The dynamics of sovereign debt crises and bailouts,” *Journal of International Economics*, 114, 1–13.

A. More Results

Increasing the borrower's bargaining power Figure 11 shows that when the borrower holds all the bargaining power, the loan interest rate is constant at β_L^{-1}. In this example with $\beta = \beta_L$, the upper bound on how large the loan m does not bind, but in the quantitative version (where market and bilateral debt coexist) with $\beta < \beta_L$, which at $\theta = 0$ recovers the model of Hatchondo, Martinez, and Önder (2017), the borrower prioritizes bilateral loans as a source of funding and thus quickly reaches the upper bound.

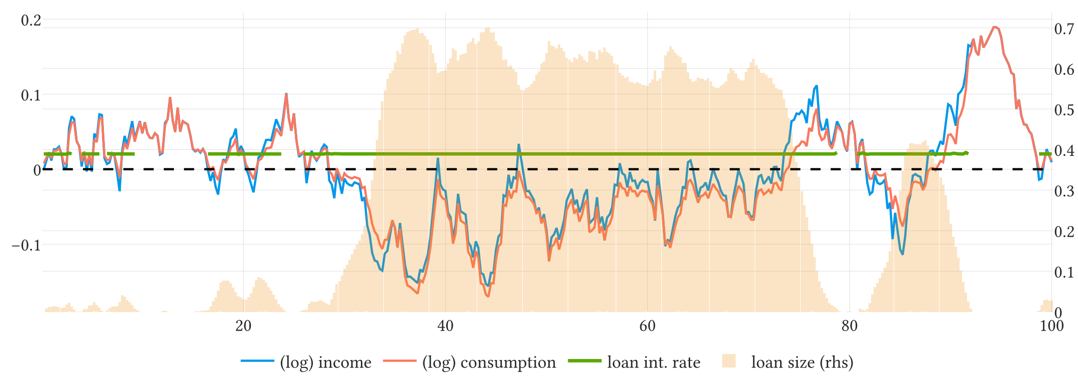

Figure 11: Simulated path, $\theta = 0$

Dynamics of loans around defaults Figure 12 shows that, further conditioning on an exclusion period of 2 years, the economy issues debt in the market in order to pay off the loan as soon as it recovers market access.

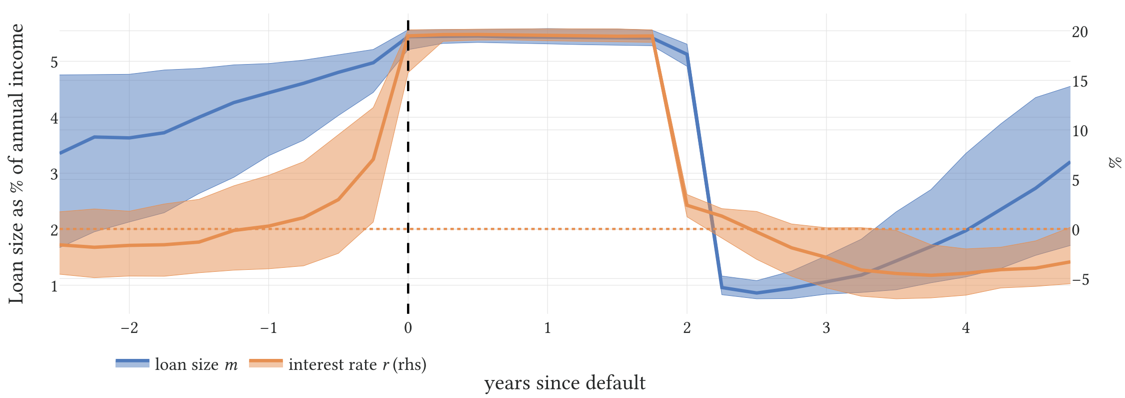

Figure 12: Loans around default events

B. Model with bilateral loans only

We study a simple model in which only bilateral loans are available. This model serves to clarify the dynamics of bargaining in bilateral lending and the strategy through which the large lender extracts surplus: subsidized terms while debt accumulates, combined with high interest rates when the debt stock becomes large and the borrower reduces leverage.

We model a small open economy borrowing from a large lender, who acts as a monopolist. The economy receives an endowment stream $y(z)$ where the state z follows an AR(1) process. Loans are short-term and therefore effectively continuously renegotiated. At the beginning of period t, let $v(m,z)$ represent the value attained by the government (or sovereign, or borrower) at income state z and owing m to the monopolist. Similarly, the large lender attains a value $h(m,z)$.

At the beginning of period t, borrower and lender negotiate over the terms of the loan. Payment of the full amount m extinguishes any debts and serves as a natural threat point. We use a simple Nash bargaining framework and set θ as the lender's bargaining power. The outcome of this negotiation is a transfer x and a new loan size m' determined by the solution to

$$\max_{x,m'} \mathscr{L}(x,m,m',z)^{\theta} \times \mathscr{B}(x,m,m',z)^{1-\theta} \tag{16}$$

where $\mathscr{L}$ and $\mathscr{B}$ represent the lender and borrower surplus functions, respectively. It will be useful to keep track of the implicit interest rate r of the loan which satisfies

$$x = \frac{1}{1+r}m' - m \tag{17}$$

After negotiations are concluded and transfers settled, consumption takes place. The large lender finances the net transfer x with a constant endowment a and thus consumes $c_L = a - x$. Conversely, the borrower receives the transfer so $c = y(z) + x$. Under risk neutral preferences for the lender,

$$\begin{aligned}\mathscr{L}(x,m,m',z) &= a - x + \beta_L \mathbb{E}\left[h(m',z') \mid z\right] - \left(a + m + \beta_L \mathbb{E}\left[h(0,z') \mid z\right]\right) \\ &= -x - m + \beta_L \mathbb{E}\left[h(m',z') - h(0,z') \mid z\right]\end{aligned}$$

and similarly

$$\mathscr{B}(x,m,m',z) = u(y(z)+x) - u(y(z)-m) + \beta \mathbb{E}\left[v(m',z') - v(0,z') \mid z\right]$$

where β_L and β are the discount factors of the lender and borrower, respectively. The borrower's utility function u is increasing and concave.

Notice that the choice of m' only affects continuation values v and h, while the choice of x only affects flow payoffs. Given a choice of m', the first-order condition for x is

$$\mathscr{B}(x,m,m',z)\theta = \mathscr{L}(x,m,m',z)u'(y(z)+x)(1-\theta)$$

Given the solution $\{x(m,z), m'(m,z)\}$ to (16), the value functions satisfy

$$\begin{aligned} v(m,z) &= u(y(z) + x(m,z)) + \beta \mathbb{E}\left[v(m'(m,z),z') \mid z\right] \\ h(m,z) &= a - x(m,z) + \beta_L \mathbb{E}\left[h(m'(m,z),z') \mid z\right] \end{aligned} \tag{18}$$

Finally, we normalize $a = 0$, which allows us to interpret $h(m,z)$ as the expected present discounted value of transfers along the equilibrium path, or the lender's total expected profits.

B.1 Equilibrium with bilateral loans only

We solve the model with bilateral loans only with a parametrization that illustrates the forces at play. Most parameters are set to standard values, Section 4 discusses calibration of the full version of the model with both types of debt. We choose $\theta = 0.5$ so the surplus is split equally between borrower and lender; we also set $\beta = \beta_L$ to isolate consumption smoothing and bargaining from the front-loading motive that would result if the borrower was relatively impatient, which in sovereign debt models tends to be the relevant case.

Figure 13 summarizes the terms of the new loan, for each level of income z and initial loan size m. Unsurprisingly, the borrower economy delevers in high-income states and receives positive transfers in low-income ones. The large lender makes intense use of the interest rate to extract surplus. When both debt and income are low, the lender offers subsidized and even negative rates. The benefit of incurring this cost is to induce high levels of debt, which make the borrower's future threat point more costly to exercise. Once the loan size is large, repaying it in full becomes difficult and the lender is able to charge much higher interest rates.

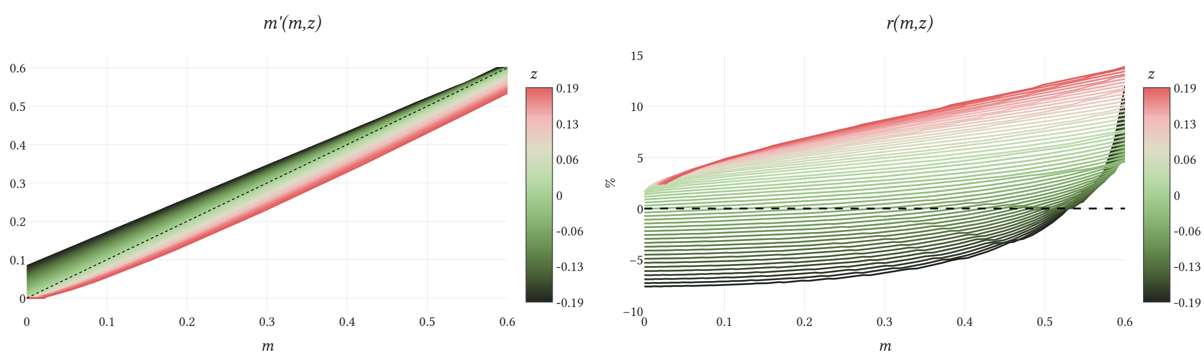

FIGURE 13: BILATERAL LENDING TERMS WITH $\theta = 0.5$

Figure 14 shows the value functions v and h for borrower and lender, respectively. As indebtedness m increases, the borrower's threat point becomes less credible. This increases the total surplus available, as new transfers are more valuable, but also makes the lender 'stronger' in the

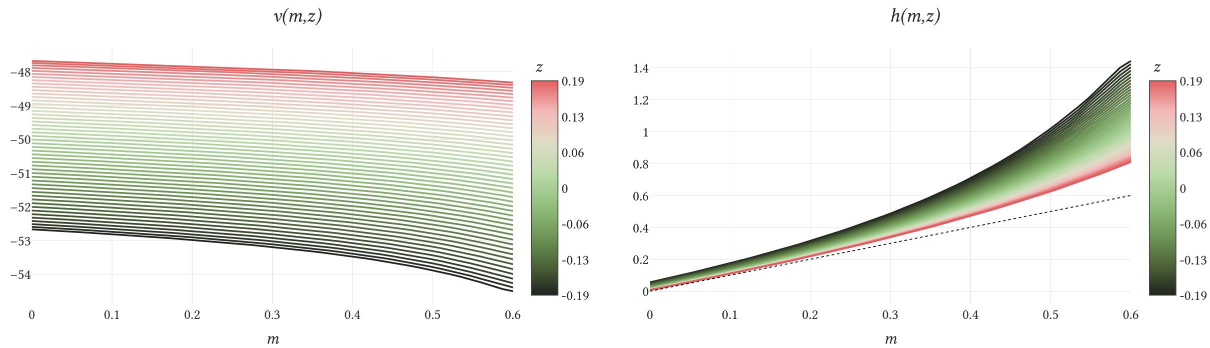

FIGURE 14: VALUE FUNCTIONS, $\theta = 0.5$

negotiation, leading to higher interest rates. This effect creates convexity in the lender's profits and, hence, in the value function h.

Convexity in the lender's value function implies endogenously risk-loving behavior. In equilibrium, the (risk-neutral) lender gambles for debt overhang. Subsidizing new loans in order to induce high indebtedness only pays off if the borrower's income takes a long time to recover. If the borrower receives a favorable income shock quickly, the loan is repaid before the lender has had an opportunity to raise rates and collect profits.

Figure 15 shows a simulation path, which further clarifies the lender's strategy. The loan is subsidized on the way up and, once debt has accumulated, the interest rate can increase to extract profits from the borrower. The borrower government anticipates these dynamics: the relationship between the initial subsidy and the expected high rates later on is disciplined by the requirement to deliver part of the surplus to the borrower, depending on the value of θ.

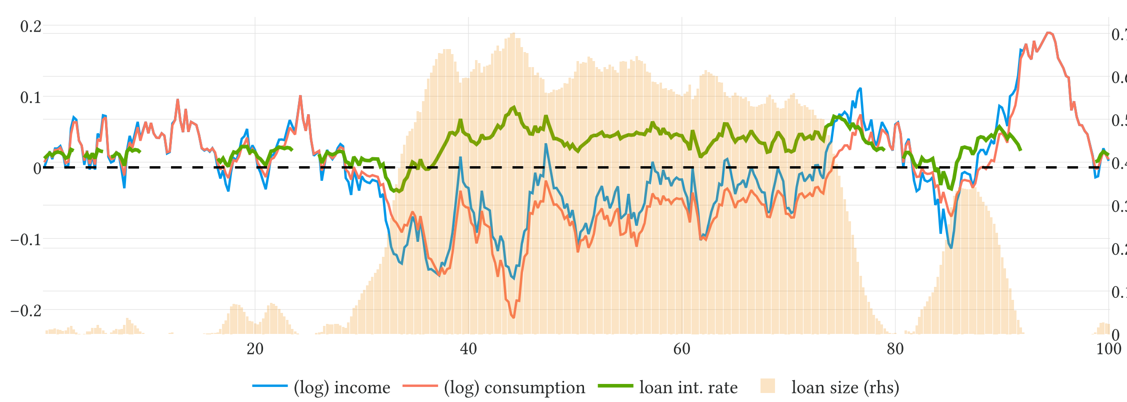

FIGURE 15: SIMULATED PATH, $\theta = 0.5$

Figure 11 in the Appendix, which simulates a model with $\theta = 0$, shows that when the borrower holds all the bargaining power, it is able to borrow at rate β_L^{-1} at all times. Because rates do not go up once the loan is large, they cannot be negative when it is still small. This effectively recovers an income fluctuations problem at the risk-free rate without default.

The model with only bilateral loans available generates two main takeaways. First, bargained interest rates can increase strongly when the borrower's threat point is costly to exercise. In this version of the model this happens when income z is low and indebtedness m is high. Enforcement is not necessary for this result; even if it were possible to default on the loan, as long as the value of the threat point was decreasing in m, similar dynamics would obtain as the borrower government would be 'weak' in the same states. Second, anticipation of higher profits creates surplus for the lender and induces lower interest rates, which support accumulation of larger loan balances m. In the full model with both types of debt, marketable debt modulates the value of the government's threat point, as we describe below.

C. Omitted Proofs

Proof of Proposition 1 **In Progress** Call F_1 the surplus-sharing condition and F_2 the Euler equation.

$$F_1 = u'(c_0) - \frac{\theta}{1-\theta}\frac{\mathscr{B}}{\mathscr{L}}$$
$$F_2 = u'(c_0) - \frac{\beta}{\beta_L}\mathbb{E}\left[u'(c_1)\right]$$

where $c_0 = qb + x$ and $c_1 = y(z) - (x+m)(1+r) - f(b,z)$ and recall that

$$\mathscr{B} = u(c_0) + \beta\mathbb{E}\left[u(c_1)\right] - u(qb-m) - \beta\mathbb{E}\left[u(y(z) - f(b,z))\right]$$
$$\mathscr{L} = \left(\beta_L(1+r) - 1\right)(x+m)$$

The solution to the Nash bargaining problem defines (x, r) as a function of q as the solution to $(F_1, F_2)(x, r, q) = 0$. The Implicit Function Theorem implies that

$$\frac{\partial x}{\partial q} = \frac{\frac{\partial F_1}{\partial r}\frac{\partial F_2}{\partial q} - \frac{\partial F_2}{\partial r}\frac{\partial F_1}{\partial q}}{\det(J)} \qquad \text{and} \qquad \frac{\partial r}{\partial q} = \frac{\frac{\partial F_2}{\partial x}\frac{\partial F_1}{\partial q} - \frac{\partial F_1}{\partial x}\frac{\partial F_2}{\partial q}}{\det(J)}$$

where the jacobian J is

$$J = \begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial r} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial r} \end{bmatrix}$$

We will proceed by showing that $\det(J) > 0$ and that both numerators are negative.

Notice that the following configuration of signs for the derivatives holds:

$$
\begin{aligned}
\frac{\partial F_1}{\partial q} &= u''(c_0)b - \frac{\theta}{1-\theta}\frac{\frac{\partial \mathscr{B}}{\partial q}}{\mathscr{L}} && \gtrless 0 \qquad & \frac{\partial F_2}{\partial q} &= u''(c_0)b && < 0 \\
\frac{\partial F_1}{\partial r} &= -\frac{\theta}{1-\theta}\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial r} && > 0 \qquad & \frac{\partial F_2}{\partial r} &= \frac{\beta}{\beta_L}\mathbb{E}\left[u''(c_1)\right](x+m) && < 0 \\
\frac{\partial F_1}{\partial x} &= u''(c_0) - \frac{\theta}{1-\theta}\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial x} && \gtrless 0 \qquad & \frac{\partial F_2}{\partial x} &= u''(c_0) + \frac{\beta}{\beta_L}\mathbb{E}\left[u''(c_1)\right](1+r) && < 0
\end{aligned}
$$

Most of the signs follow from strict concavity of u so that u'' is always negative. To see that $\frac{\partial F_1}{\partial r} > 0$, notice that $\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial r}$ is negative:

$$
\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial r} = \frac{\frac{\partial \mathscr{B}}{\partial r}\mathscr{L} - \mathscr{B}\frac{\partial \mathscr{L}}{\partial r}}{\mathscr{L}^2} = \frac{-\beta\mathbb{E}\left[u'(c_1)\right](x+m)\mathscr{L} - \mathscr{B}\beta_L(x+m)}{\mathscr{L}^2} < 0
$$

Also notice that, using the definition of $\mathscr{L} = (x+m)(\beta_L(1+r)-1)$, the surplus-sharing condition $\frac{\mathscr{B}}{\mathscr{L}} = \frac{1-\theta}{\theta}u'(c_0)$, and the Euler equation $\beta_L u'(c_0) = \beta\mathbb{E}\left[u'(c_1)\right]$, we can rewrite $\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial r}$ as

$$
\begin{aligned}
\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial r} &= -\beta\mathbb{E}\left[u'(c_1)\right]\frac{x+m}{\mathscr{L}} - \beta_L\frac{x+m}{\mathscr{L}}\frac{\mathscr{B}}{\mathscr{L}} = -\left(\beta_L u'(c_0) + \beta_L\frac{1-\theta}{\theta}u'(c_0)\right)\frac{x+m}{\mathscr{L}} \\
&= -\frac{1}{\theta}\frac{\beta_L u'(c_0)}{\beta_L(1+r)-1}
\end{aligned}
$$

A similar manipulation of $\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial x}$ yields

$$
\begin{aligned}
\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial x} &= \frac{\frac{\partial \mathscr{B}}{\partial x}\mathscr{L} - \mathscr{B}\frac{\partial \mathscr{L}}{\partial x}}{\mathscr{L}^2} = \frac{\left(u'(c_0) - (1+r)\beta\mathbb{E}\left[u'(c_1)\right]\right)\mathscr{L} - \mathscr{B}\left(\beta_L(1+r)-1\right)}{\mathscr{L}^2} \\
&= \frac{\left(u'(c_0) - (1+r)\beta_L u'(c_0)\right)\mathscr{L} - \mathscr{B}(\beta_L(1+r)-1)}{\mathscr{L}^2} = -(\beta_L(1+r)-1)\frac{u'(c_0)\mathscr{L} + \mathscr{B}}{\mathscr{L}^2} \\
&= -\frac{\mathscr{L}}{x+m}\frac{u'(c_0)\mathscr{L} + \mathscr{B}}{\mathscr{L}^2} = -\frac{1}{x+m}\left(u'(c_0) + \frac{\mathscr{B}}{\mathscr{L}}\right) = -\frac{u'(c_0)}{x+m}\left(1 + \frac{1-\theta}{\theta}\right) = -\frac{1}{\theta}\frac{u'(c_0)}{x+m}
\end{aligned}
$$

These permit rewriting $\frac{\partial F_1}{\partial r}$ and $\frac{\partial F_1}{\partial x}$ as

$$
\begin{aligned}
\frac{\partial F_1}{\partial x} &= u''(c_0) - \frac{\theta}{1-\theta}\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial x} = u''(c_0) + \frac{u'(c_0)}{(1-\theta)(x+m)} \\
\frac{\partial F_1}{\partial r} &= \frac{1}{1-\theta}\frac{\beta_L u'(c_0)}{\beta_L(1+r)-1}
\end{aligned}
$$

We are now ready to tackle $\det(J) = \frac{\partial F_1}{\partial x}\frac{\partial F_2}{\partial r} - \frac{\partial F_1}{\partial r}\frac{\partial F_2}{\partial x}$:

$$
\begin{aligned}
\frac{\partial F_1}{\partial x}\frac{\partial F_2}{\partial r} &= \left(u''(c_0) + \frac{u'(c_0)}{(1-\theta)(x+m)}\right)\frac{\beta}{\beta_L}\mathbb{E}\left[u''(c_1)\right](x+m) \\
\frac{\partial F_1}{\partial r}\frac{\partial F_2}{\partial x} &= -\frac{\theta}{1-\theta}\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial r}\left(u''(c_0) + \frac{\beta}{\beta_L}\mathbb{E}\left[u''(c_1)\right](1+r)\right)
\end{aligned}
$$

so

$$\det(J) = \frac{\beta}{\beta_L}\frac{\mathbb{E}\left[u''(c_1)\right]}{1-\theta}\left(u'(c_0) + \theta\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial r}(1+r)\right) + u''(c_0)\left(\frac{\beta}{\beta_L}\mathbb{E}\left[u''(c_1)\right](x+m) + \frac{\theta}{1-\theta}\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial r}\right)$$

where $A = \frac{\beta}{\beta_L}\mathbb{E}\left[u''(c_1)\right](x+m) + \frac{\theta}{1-\theta}\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial r} < 0$ so $u''(c_0)A > 0$. Because of strict concavity of u, $\mathbb{E}\left[u''(c_1)\right] < 0$, so $\det(J) > 0$ if $u'(c_0) + \theta(1+r)\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial r} < 0$. Now,

$$\begin{aligned} u'(c_0) + \theta(1+r)\frac{\partial(\mathscr{B}/\mathscr{L})}{\partial r} &= u'(c_0) - \theta(1+r)\frac{1}{\theta}\frac{\beta_L u'(c_0)}{\beta_L(1+r)-1} = u'(c_0)\left(1 - \frac{\beta_L(1+r)}{\beta_L(1+r)-1}\right) \\ &= u'(c_0)\left(\frac{\beta_L(1+r) - 1 - \beta_L(1+r)}{\beta_L(1+r)-1}\right) = -u'(c_0)\frac{1}{\beta_L(1+r)-1} < 0 \end{aligned}$$

This means that $\det(J) > 0$. **Fine until here**

Can prove $\frac{\partial r}{\partial q} < 0$ for fixed x and $\frac{\partial x}{\partial q} < 0$ for fixed r easily. Question is whether at higher q, since there is less surplus, punishing with lower quantities x is so hurtful that you end up with better prices r.

With CRRA, $m = 0$, $b = 0$, no uncertainty over y_1, there is a condition on θ and γ.

In general, there is a condition on $u'''(c_0)$ but it also involves r.

Finally, $\frac{\partial x}{\partial q} = -\frac{\frac{\partial F_1}{\partial q}\frac{\partial F_2}{\partial r} - \frac{\partial F_1}{\partial r}\frac{\partial F_2}{\partial q}}{\det(J)}$ and

$$\frac{\partial r}{\partial q} = -\frac{\frac{\partial F_1}{\partial x}\frac{\partial F_2}{\partial q} - \frac{\partial F_1}{\partial q}\frac{\partial F_2}{\partial x}}{\det(J)}$$

We need to sign those two numerators but it seems to require some more assumptions. □