Ambiguity and Sovereign Debt Intolerance*

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Abstract

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2. Model

Resources The economy receives an endowment stream following a stochastic process with trend and cycle components

$$Y_t = \exp(z_t)\Gamma_t \tag{1}$$

with

$$z_t = \rho z_{t-1} + \sigma_z \varepsilon_t^z$$
$$\log(\Gamma_t) = \log(\Gamma_{t-1}) + \log(g_t)$$

where $\varepsilon_t^z \overset{iid}{\sim} \mathcal{N}(0,1)$ and $\log(g_t) \overset{iid}{\sim} \mathcal{N}(0,\sigma_g)$.

We normalize variables by Γ_t and denote normalized values with lowercase. For example, $y_t = \exp(z_t)$ and

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma}\frac{(C/\Gamma)^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma}u(c)$$

which translates to a normalization constant $\Gamma_t^{1-\gamma}$ for the value functions, as laid out in more detail in Appendix A.

Assets The government borrows from international lenders in the form of a defaultable bond which promises to pay a noncontingent stream of geometrically-decaying coupons as in Leland (1998); Chatterjee and Eyigungor (2012); Hatchondo and Martinez (2009). A bond issued in period t pays $(1-\rho)^{s-1}\kappa$ units of the good in period $t+s$, which effectively makes a one-period-old bond a perfect substitute of $(1-\rho)$ units of newly-issued debt. The coupon rate $\kappa = r + \rho$, where r is the international risk-free rate, is chosen so that the price of a bond that is expected to never default is $q^\star = 1$.

Upon default, the government loses access to international capital markets and faces a loss of output. There is uncertainty about whether this loss of output is permanent or transitory. Moreover, access to markets is restored with constant probability ψ.

Government The government is benevolent and makes choices on a sequential basis to maximize the utility of a representative household with preferences given by

$$V_t = \mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^s\left(u(C_{t+s}) + \varepsilon_{t+s}\right)\right] = \mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^s\Gamma_{t+s}^{1-\gamma}\left(u(c_{t+s}) + \varepsilon_{t+s}\right)\right] \tag{2}$$

where $\mathbb{E}$ denotes the expectation operator, C_t represents the household's consumption, β is a discount factor, and ε is a preference shock for default or repayment.[1] The utility function is strictly increasing and concave.

While the government is not in default, it chooses whether to repay the debt and attains a value

$$v(b,z,g) = \max\left\{v_R(b,z,g) + \epsilon_R, v_D(z,g) + \epsilon_D\right\} \tag{3}$$

[1] We follow Dvorkin et al. (2021) and introduce preference shocks for repayment and default to improve the numerical convergence of the algorithm used to solve the model.

where the ϵ's follow a Type 1 Extreme Value distribution as in Chatterjee et al. (2018), yielding familiar closed forms for $v(b, z, g)$ and the (ex-post) default probability $\mathcal{P}(b, z, g)$

$$v(b, z, g) = \chi \log\left(\exp(v_D(z, g)/\chi) + \exp(v_R(b, z, g)/\chi)\right)$$
$$\mathcal{P}(b, z, g) = \frac{\exp(v_D(z, g)/\chi)}{\exp(v_D(z, g)/\chi) + \exp(v_R(b, z, g)/\chi)}$$

If the government chooses the repay the debt, it can access capital markets and issue new debt h (because of the normalization, the indebtedness state variable for the next period $b' = h/g'$), so that

$$\begin{aligned} v_R(b, z, g) = {} & \max_h u(c) + \beta \mathbb{E}\left[(g')^{1-\gamma} v(h/g', z', g') \mid z, g\right] \\ \text{subject to} \quad & c + \kappa b = y(z) + q(h, z, g)(h - (1 - \rho)b) \end{aligned} \tag{4}$$

If the government chooses to default, it loses access to international capital markets, which it recovers with a constant hazard ψ. While it is excluded, the borrowing economy suffers a loss of output $\varphi(y)$. Upon default, a shock k determines whether the loss of output is transitory or permanent, which happen with probability p and $1 - p$, respectively, independent of the rest of the state vector. The expected value of default is then

$$v_D(z, g) = p v_D^T(z, g) + (1 - p) \left(\frac{g - \varphi(y(z))}{g}\right)^{1-\gamma} v_D^P(z, g - \varphi(y(z))) \tag{5}$$

where the normalization constant differs in the case of a permanent default cost and

$$v_D^k(z, g) = u(y(z) - 1_{(k=1)}\varphi(y(z))) + \beta \mathbb{E}\left[(g')^{1-\gamma} \left(\psi v(0, z', g') + (1 - \psi) v_D^T(z', g')\right) \mid z, g\right]$$

Lenders Bonds issued by the government are purchased by deep-pocketed, risk-neutral foreign investors who equate the expected return of the debt to their cost of funds r, yielding a debt price

$$q(h, z, g) = \frac{1}{1 + r} \mathbb{E}\left[(1 - \mathcal{D}(h/g', z', g'))(\kappa + (1 - \rho) q(h', z', g')) \mid z, g\right] \tag{6}$$

3. Concluding remarks

References

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A. Normalization details

We normalize all variables by Γ_t, denote normalized values with lowercase, and notice that $y_t = \exp(z_t)$ and

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma}\frac{(C/\Gamma)^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma}u(c)$$

so in a typical Bellman equation we can guess and verify (denoting $h = B'/\Gamma$) forms like $V\Gamma^{\gamma-1} = v$

$$\begin{aligned}
V(B,z,\Gamma) &= \max_{B'} u(C) + \beta\mathbb{E}\left[V(B',z',\Gamma')\right] \\
V(B,z,\Gamma) &= \max_{B'} \Gamma^{1-\gamma}u(c) + \beta\mathbb{E}\left[V(B',z',\Gamma')\right] \\
\Gamma^{\gamma-1}V(B,z,\Gamma) &= \max_{B'} u(c) + \beta\mathbb{E}\left[(\Gamma')^{\gamma-1}\left(\Gamma/\Gamma'\right)^{\gamma-1}V(B',z',\Gamma')\right] \\
v(b,z,g) &= \max_{h} u(c) + \beta\mathbb{E}\left[(g')^{1-\gamma}v(b'(h,g'),z',g')\right] \\
v(b,z,g) &= \max_{h} u(c) + \beta\mathbb{E}\left[(g')^{1-\gamma}v(b'(h,g'),z',g')\right]
\end{aligned}$$

while for the budget constraint we have

$$\begin{aligned}
C + \kappa B &= Y + q(B' - (1-\rho)B) \\
c + \kappa b &= y + q(B'/\Gamma - (1-\rho)b) \\
c + \kappa b &= y + q(b'(\Gamma'/\Gamma) - (1-\rho)b) \\
c + \kappa b &= y + q(b'g' - (1-\rho)b)
\end{aligned}$$

This budget constraint makes it clear that $h = b'g'$ (simply substituting $h = B'/\Gamma$ in line 2) or $b'(h,g') = h/g'$.

For the value of default we have

$$\begin{aligned}
V_D(z,g) &= pV_D^T(z,g) + (1-p)V_D^P(z,g-\varphi(y(z))) \\
(\Gamma)^{\gamma-1}V_D(z,g) &= p(\Gamma)^{\gamma-1}V_D^T(z,g) + (1-p)(\Gamma)^{\gamma-1}V_D^P(z,g-\varphi(y(z))) \\
(\Gamma)^{\gamma-1}V_D(z,g) &= p(\Gamma)^{\gamma-1}V_D^T(z,g) + (1-p)(\Gamma_0 g)^{\gamma-1}V_D^P(z,g-\varphi(y(z))) \\
v_D(z,g) &= pv_D^T(z,g) + (1-p)(\Gamma_0 g_z g/g_z)^{\gamma-1}V_D^P(z,g-\varphi(y(z))) \\
v_D(z,g) &= pv_D^T(z,g) + (1-p)\left(\frac{g}{g_z}\right)^{\gamma-1}(\Gamma_0 g_z)^{\gamma-1}V_D^P(z,g-\varphi(y(z))) \\
v_D(z,g) &= pv_D^T(z,g) + (1-p)\left(\frac{g}{g-\varphi(y(z))}\right)^{\gamma-1}v_D^P(z,g-\varphi(y(z)))
\end{aligned}$$