

Uncertainty Premia, Sovereign Default Risk, and State-Contingent Debt

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its Executive Board, or its management.

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State-contingent debt instruments

- Decrease default risk
- Reduce cyclicalities of fiscal policy
- Improve risk-sharing

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Unfavorable prices of state-contingent instruments

- These instruments are heavily **discounted** by markets
 - Costa, Chamon, and Ricci (2008) compute wide spreads for Argentine **GDP-warrants**
 - ~300-400bps from default risk of other securities
 - 600-1200bps residual: '**novelty**' premium

This paper proposes a framework that

- Rationalizes **pricing** of SCI + **welfare** analysis
 - With ingredients from resolutions of the equity premium puzzle
 - Robustness (Hansen and Sargent, 2001; Pouzo and Presno, 2016)
- Links unfavorable prices to common 'threshold' structure
 - Example: Argentina's GDP-warrants, also Ukraine, Greece. . .

► More

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Main findings

1. Robust lenders dislike repayment structures with **thresholds** in good times
 - Heavy discounts for these bonds \implies welfare **losses**
2. Explain most of the ‘novelty premium’ in Argentina’s GDP warrants as **ambiguity** premia
 - Calibration of robustness from *noncontingent* debt only
3. Characterize the **optimal** design and how it changes with robustness
 - With high robustness, want to minimize ex-ante and ex-post contingency

Roadmap

- Stylized Model
- Probability Distortions
- Quantitative Implementation
- Concluding Remarks

Stylized Model

The model

We consider a simple two-period model, small open economy

- Uncertain endowment $y(z)$ in the second period
- The government has access to **one** asset which promises a return $R(z)$.
- A few benchmarks

Noncontingent debt	$R(z)$	=	1
Linear indexing	$R^\alpha(z)$	=	$1 + \alpha(y(z) - 1)$
Threshold debt	$R^\tau(z)$	=	$\mathbb{1}(z > \tau)$
Optimal design	$R^*(z; \theta)$	chosen state-by-state	

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The government's problem

- The government takes as given the price schedule $q(b)$

$$\begin{aligned} & \max_b u(c_1^b) + \beta_b \mathbb{E} [u(c_2^b)] \\ \text{subject to } & c_1^b = y_1 + q(b)b \\ & c_2^b = y_2(z) - h(z, \Delta)d(b, z) - (1 - d(b, z))R(z)b \end{aligned}$$

where

$$h(z, \Delta) = y_2(z)^2 \Delta$$

The lenders' problem

Foreign lenders are less standard and have **multiplier preferences**

$$\begin{aligned} & \max c_1^L + \beta \frac{1}{-\theta} \log \mathbb{E} [\exp(-\theta v_2^L)] \\ & \text{subject to } v_2^L = c_2^L \\ & c_2^L = w_2 + (1 - d(b, z))R(z)b \\ & c_1^L = w_1 - q_1 b \end{aligned}$$

Lenders provide us with an **Euler equation** to price the debt

$$\begin{aligned} q(b; R) &= \beta \mathbb{E} \left[\frac{\exp(-\theta v_2^L)}{\mathbb{E} [\exp(-\theta v_2^L)]} (1 - d(b, z))R(z) \right] \\ &= \underbrace{\beta \mathbb{E} [(1 - d)R]}_{= q_{RE}} + \underbrace{(1 - \mathbb{P}(d)) \text{cov}(\beta M, R)}_{= q_\theta^{\text{cont}}} - \underbrace{\mathbb{E} [R] \text{cov}(\beta M, d)}_{= -q_\theta^{\text{def}}} \end{aligned}$$

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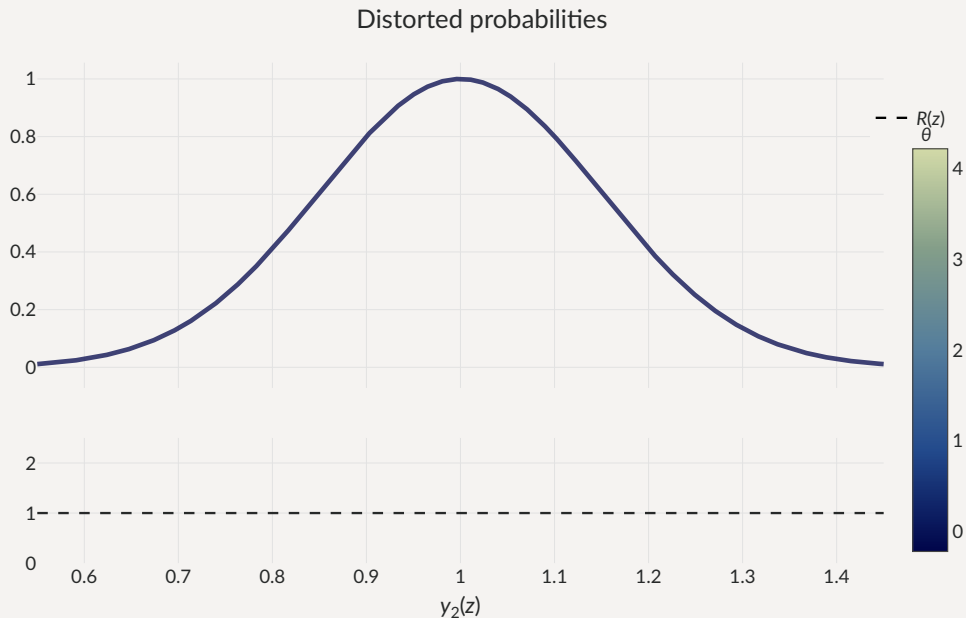
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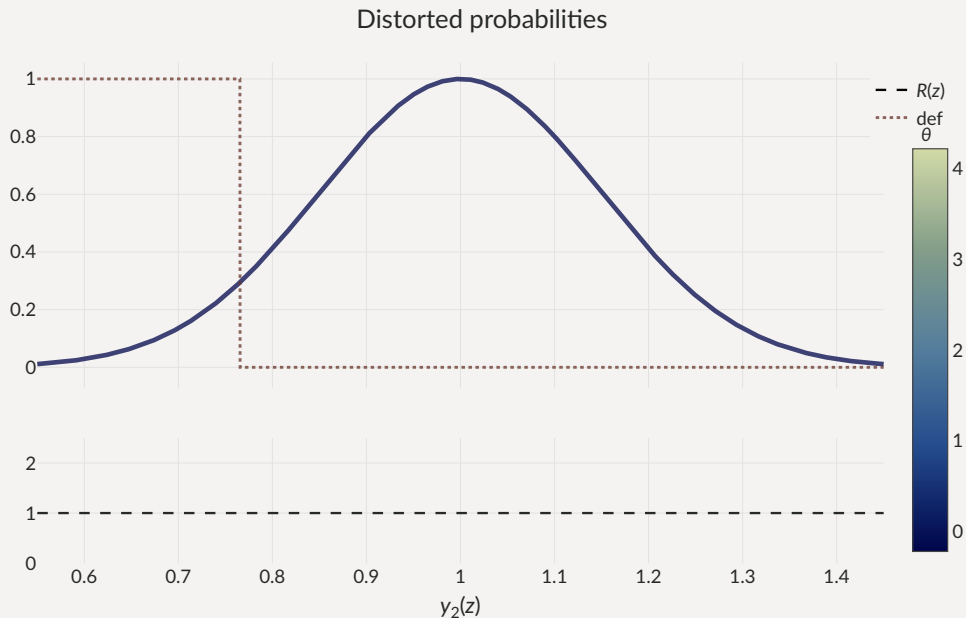
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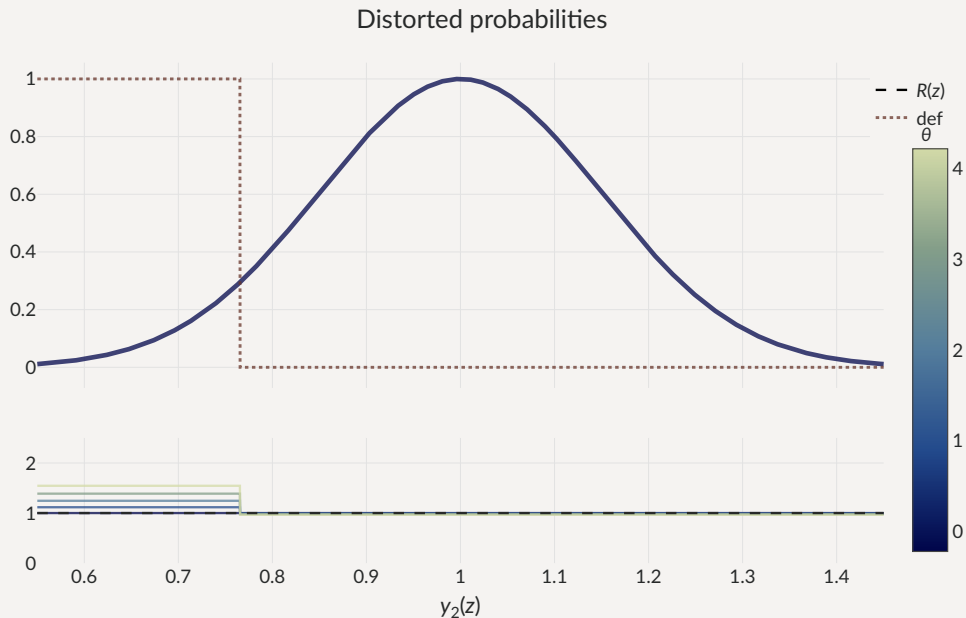
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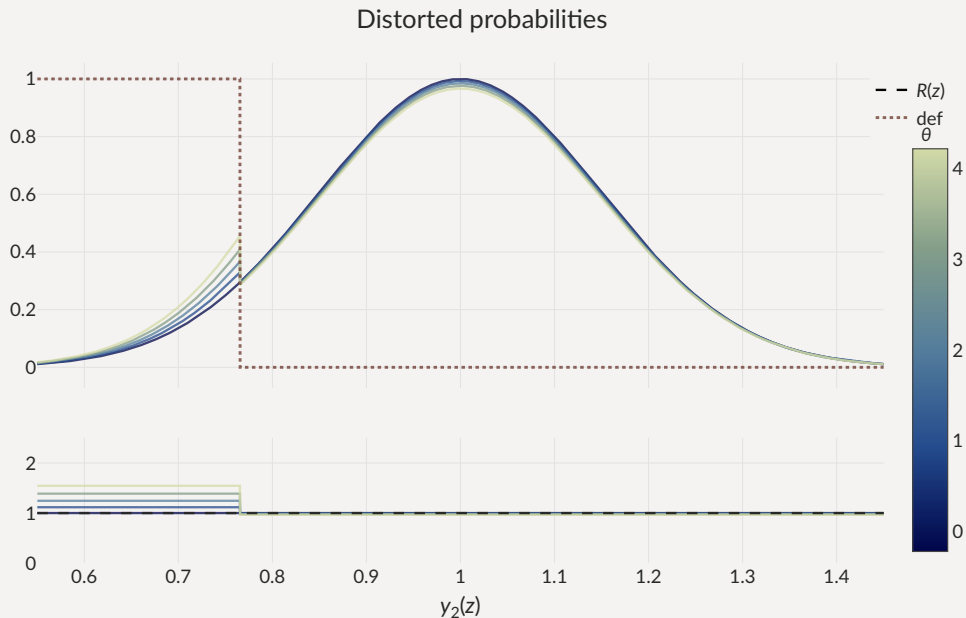
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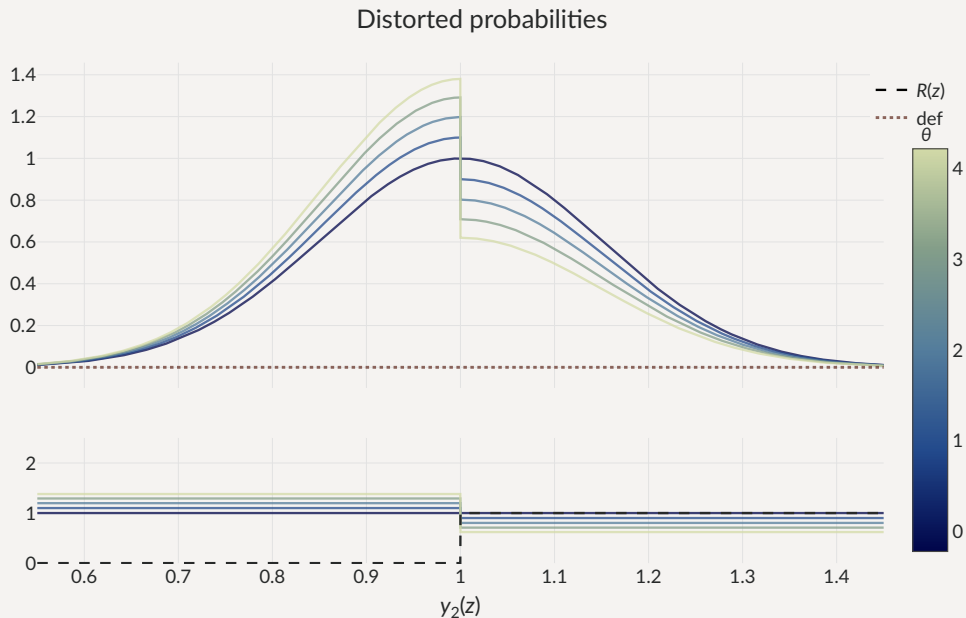
Probability Distortions



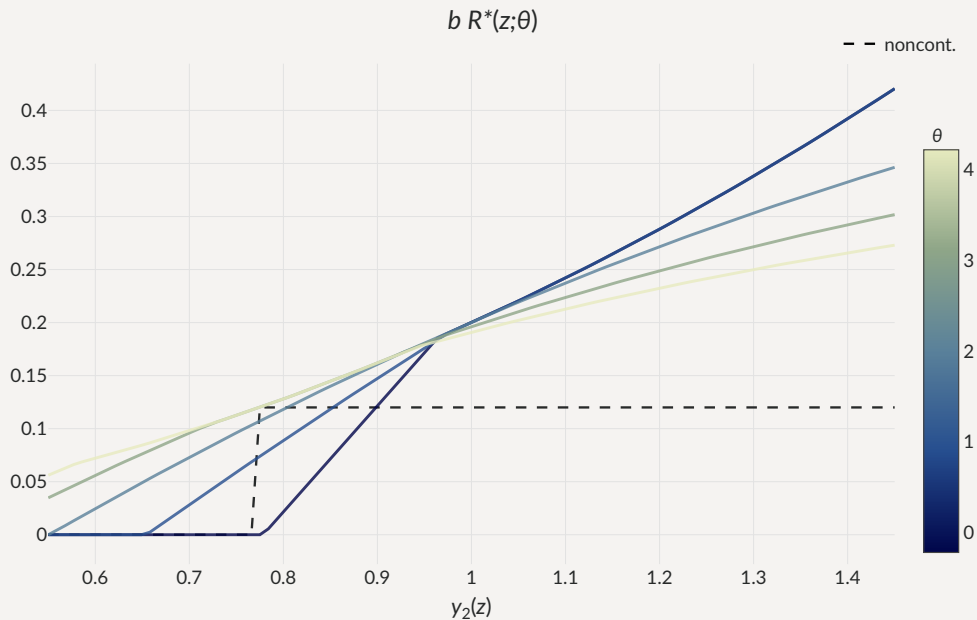








Design of debt



Quantitative Implementation

- Infinite horizon, small-open economy
- **Robust** lenders as before
- Long-term debt, debt issued at t pays coupon at $t + s$

$$\max \{0, (1 - \delta)^{s-1} (1 + \alpha(y_s - 1)) \mathbb{1}(y_s > \tau)\}$$

- Noncontingent debt: $\alpha = 0, \tau = -\infty$
- Default triggers exclusion + output costs for a random amount of periods $\sim \text{Geo}(\psi)$

Robustness in the quantitative model

Statistic	Rational Expectations			Benchmark ($\theta = 2.15$)		
	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread (bps)	893	318	742	842	1636	746
o/w Spread RE	893	318	742	432	2.6	343
Std Spread	439	133	301	376	238	282
Debt-to-GDP (%)	18.3	32.8	17.8	16.7	18.3	17.5
Std(c)/Std(y)	1.4	0.9	1.4	1.3	0.84	1.3
Default Prob (%)	6.0	1.7	5.6	3.2	0.01	2.7
Welfare Gains	-	0.94%	0.22%	-	-1.1%	0.15%
DEP	-	-	-	40.1%	31.4%	39%

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Price of marginal issuances

In reality issuances of state-contingent bonds are **small**

- Solve the model with noncontingent debt
- Take the lenders' **SDF** from that equilibrium
- Use it to price another bond

	Noncontingent bond	Linear bond	Threshold bond	Optimal bond
Benchmark	842	845	947	829
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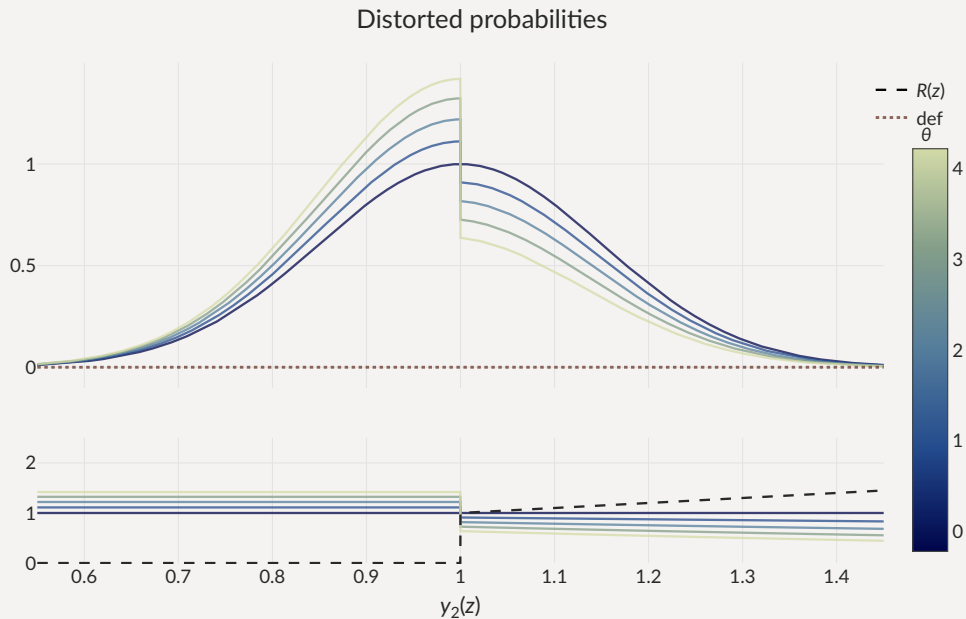
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Concluding Remarks

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- Standard sovereign debt model augmented with robust lenders
 1. Accounts for **spreads** on typical threshold SCDIs
 2. Rationalizes part of the 'novelty' premium as a premium for **ambiguity**
 3. Links unfavorable prices to common *threshold* structure
 4. **Welfare** gains of SCDI decreasing in robustness
 - Both for given instrument and for optimally-designed debt
- Optimal design
 - With realistic robustness, lower thresholds and **flatter** indexation than RE
 - With extreme robustness, eliminate contingency ex-ante (*stipulated*) and ex-post (*default*)
 - In general, tradeoff between **contingency** and **risk-sharing**

Distorted probabilities – threshold+linear debt

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Euler equations of a rational-expectations agent with CARA preferences and access to a risk-free bond

$$q = \beta \mathbb{E} \left[\frac{u'(c_2)}{u'(c_1)} R \right] = \beta \mathbb{E} \left[\frac{\exp(-\gamma c_2)}{\exp(-\gamma c_1)} R \right]$$
$$\frac{1}{1+r} = \beta \mathbb{E} \left[\frac{u'(c_2)}{u'(c_1)} \right]$$

hence

$$q = \beta \mathbb{E} \left[\frac{\exp(-\gamma c_2)}{\beta(1+r) \mathbb{E} [\exp(-\gamma c_2)]} R \right]$$

Same as robustness **in two periods**, in general the robust sdf is

$$q = \beta \mathbb{E} \left[\frac{\exp(-\theta v')}{\mathbb{E} [\exp(-\theta v')]} R \right]$$

Multiplier preferences

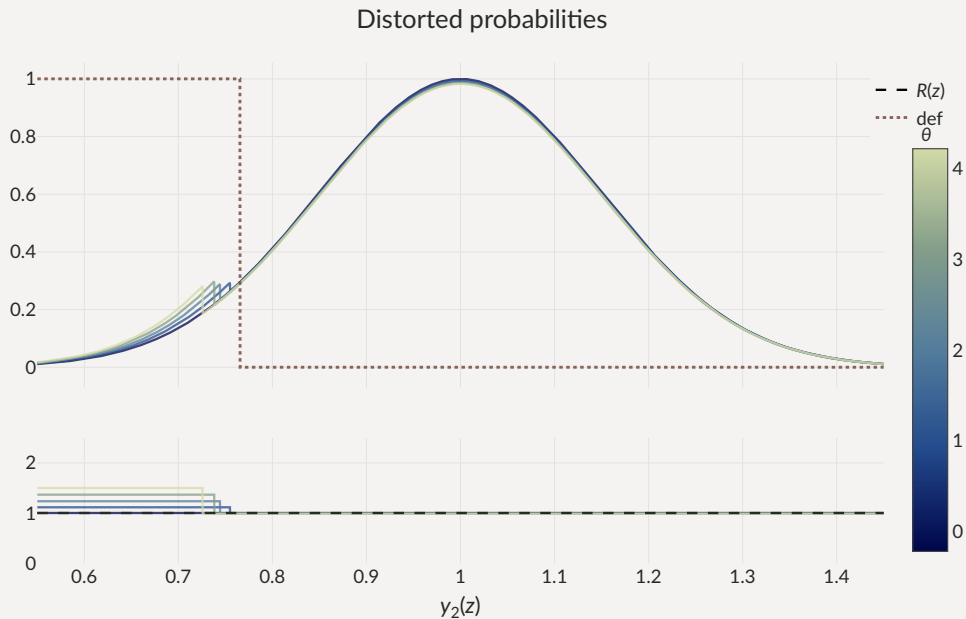
In general,

$$\min_{\tilde{p}} \max_c u(c) + \beta \int v(a') dp + \frac{1}{\theta} \text{ent}(p, \tilde{p})$$

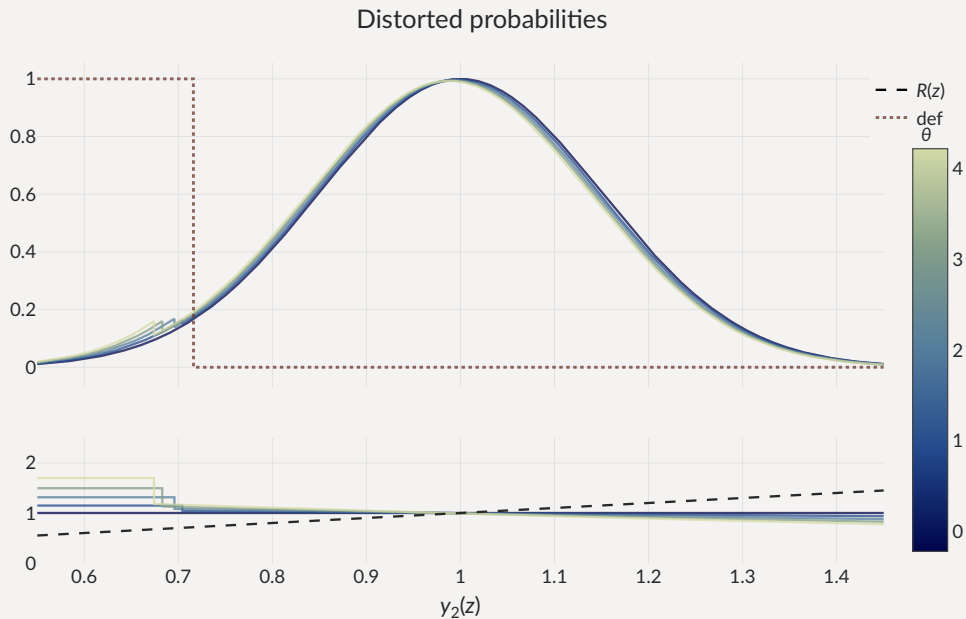
turns into

$$\max_c u(c) - \frac{\beta}{\theta} \log (\mathbb{E} [\exp(-\theta v(a'))])$$

Distorted probabilities – noncontingent debt

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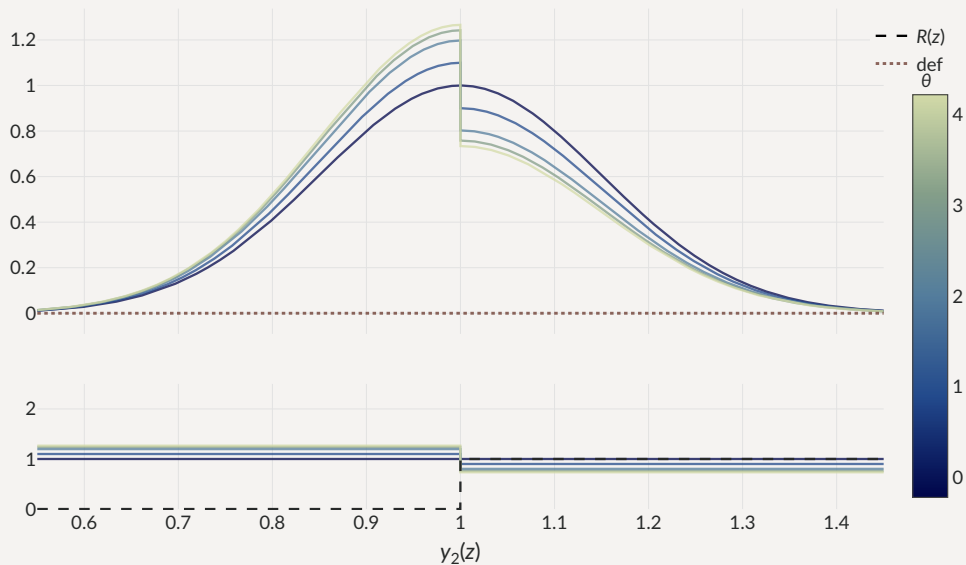
Distorted probabilities – linearly indexed debt

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Distorted probabilities – threshold debt

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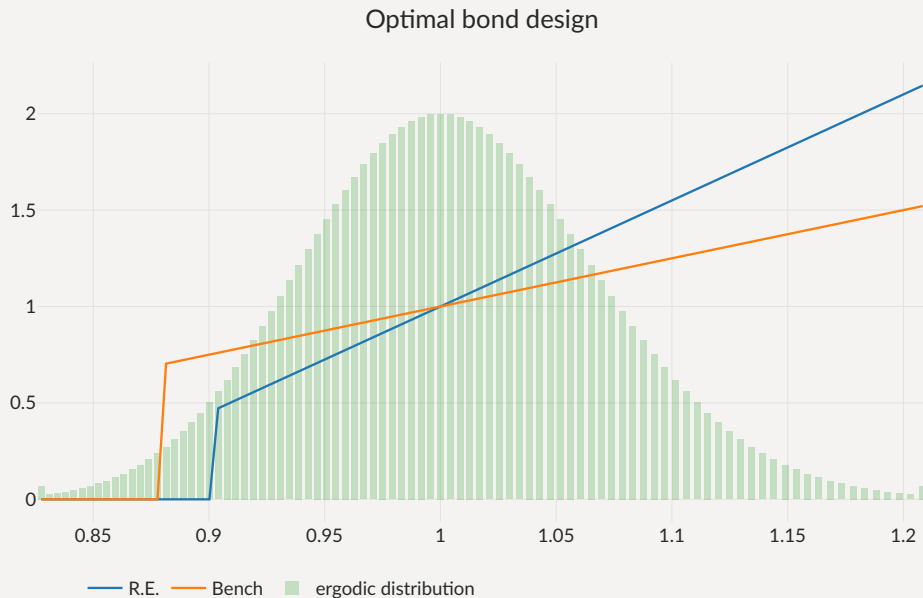
Distorted probabilities



We represent this bond with threshold debt, one period = five years, and

Parameter	Target	Value
β_b	Borrower's discount rate	6% ann.
β	Risk-free rate	3% ann.
γ	Borrower's risk aversion	2
Δ	Output cost of default	20%
g	Expected growth rate	8% ann.
k	Threshold for repayment	50%

Optimal bond design

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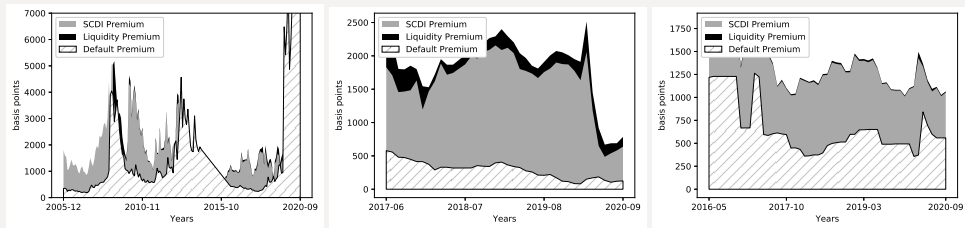


Figure 1: GDP-linked security premia.

The figure shows the estimated spread decomposition in Igan and Kim (2021) for the GDP-warrants issued by Argentina (left), Greece (middle) and Ukraine (right).