# Risk Aversion in Sovereign Debt and Default

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# Why risk aversion? Why in sovereign debt?

- In most RBC models, macro-financial separation holds
  - Elasticity of intertemporal substitution determines allocations
  - Risk aversion determines asset prices

- Sovereign debt literature typically inherits this line of thinking
  - CRRA preferences frequent, typically $\gamma = 2$

- If MFS holds in sovereign debt, macro outcomes robust to different preferences
  - In particular, calibration of output/utility costs of default
  - Less clear about welfare effects

    … losses from default, debt dilution

    … welfare effects of banning debt, introducing state-contingent bonds

# Wanting risk prices in sovereign debt

**This paper**

- Show that macro-financial separation breaks in the sovereign debt model
- Understand the impact of preferences consistent with significant risk premia
- Find that risk aversion affects equilibria in unexpected ways
    - Cautious behavior manifests in higher-order moments
    - Convex costs mute post-default volatility

# Model

# Framework

- Sovereign default model without default [reduces to an income-fluctuations problem]

$$v(b,z) = \max_{b'} u(c) + \beta \mathbb{E}\left[v(b',z') \mid z\right]$$

$$\text{subject to} \quad c + \kappa b = q(b',z)(b' - (1-\rho)b) + y(z)$$

$$b' \leq \bar{b}$$

$$\text{with} \quad q(b',z) = \frac{1}{1+r}$$

- We consider parametrizations of the model to vary risk aversion
  … with CRRA preferences $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$
  … with robustness, $u(c) = \log c$; replace $\mathbb{E}$ with $\mathbb{T}\left[X \mid \mathcal{F}\right] = -\frac{1}{\theta}\log\left(\mathbb{E}\left[\exp\left(-\theta X\right) \mid \mathcal{F}\right]\right)$

# Macro-financial separation without default

Autarky

- Start from log-log [$\theta = 0$]: RA moves asset prices and welfare, not the macro

| | loglog | $\theta = 1$ | $\theta = 2$ | $\theta = 3$ |
|---|---|---|---|---|
| Average spread (bps) | 0.0276 | 0.031 | 0.0406 | 0.138 |
| Corr. NX,Y (%) | 0.00777 | 0.00916 | 0.0114 | 0.0147 |
| Rel. vol. cons (%) | 1.59 | 1.62 | 1.65 | 1.66 |
| Risk premium (p.p.) | 0.0769 | 2.03 | 3.84 | 5.44 |
| Debt-to-GDP (%) | 29.7 | 29.5 | 29.2 | 28.9 |
| Corr. deficit, y (%) | −0.0119 | −0.0141 | −0.0177 | −0.0231 |
| Welfare | 1.034 | 1.008 | 0.9867 | 0.971 |

… welfare in autarky at $\theta = 3$ is 6pp lower than loglog or CRRA

# Macro-financial separation without default (cont'd)

- Start from log-log [$\gamma = 1$]: EIS+RA moves mostly macro, not asset prices and welfare

| | loglog | $\gamma = 2$ | $\gamma = 5$ | $\gamma = 10$ | $\gamma = 20$ |
|---|---|---|---|---|---|
| Average spread (bps) | 0.0276 | 0.0273 | 0.0269 | 0.0271 | 0.0285 |
| Corr. NX,Y (%) | 0.00777 | 0.0154 | 0.0852 | 0.397 | 0.668 |
| Rel. vol. cons (%) | 1.59 | 1.56 | 1.35 | 0.965 | 0.727 |
| Risk premium (p.p.) | 0.0769 | 0.227 | 0.627 | 1.02 | 1.67 |
| Debt-to-GDP (%) | 29.7 | 28.8 | 25.9 | 19.3 | 8.75 |
| Corr. deficit, y (%) | −0.0119 | −0.0251 | −0.162 | −0.605 | −0.774 |
| Welfare | 1.034 | 1.03 | 1.021 | 1.01 | 0.9918 |

... in fully Epstein-Zin, move only EIS for even less effect on asset prices and welfare

# Models with default

- Option value of default (with small pref. shocks for numerical performance)

$$\mathcal{V}(b,z) = \max\{v_R(b,z) + \epsilon_R, v_D(b,z) + \epsilon_D\}$$

- Similar equation for value of repayment $v_R$, debt prices reflect default probabilities

$$q(b',z) = \frac{1}{1+r}\mathbb{E}\left[(1 - \mathbb{1}_{\mathcal{D}'})\left(\kappa + (1-\rho)q(b'',z')\right) \,\middle|\, z\right]$$

- Costs of default

$$v_D(b,z) = u(h(y(z))) + \beta\mathbb{E}\left[\mathbb{1}_R\mathcal{V}(B(b,z'),z') + (1-\mathbb{1}_R)v_D(b,z') \,\middle|\, z\right]$$
$$h(y) = y(1-d_0-d_1y)$$

- Risk aversion $\implies$ no-smoothing in default costly $\implies$ no macro-financial separation

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# Quantitative properties

# Calibration

- Keep the same discount rate, vary costs of default to match spreads and debt

| | Parameter | $\gamma = 2$ | loglog | $\theta = 3$ |
|---|---|---|---|---|
| Sovereign's discount factor | $\beta$ | 0.9627 | 0.9627 | 0.9627 |
| Sovereign's robustness parameter | $\theta$ | 0 | 0 | 3 |
| Sovereign's EIS | $\gamma$ | 2 | 1 | 1 |
| Default output cost: linear | $d_1$ | –0.2833 | –0.2836 | –0.247 |
| Default output cost: quadratic | $d_2$ | 0.3253 | 0.3228 | 0.3029 |
| Average spread (bps) | 815 | 754 | 756 | 815 |
| Debt-to-GDP ratio (%) | 17.4 | 16.8 | 16.7 | 17.4 |

# Comparative statics: CRRA

- Increasing EIS+RA: Less volatility, procyclical exports, more skewed debt outcomes

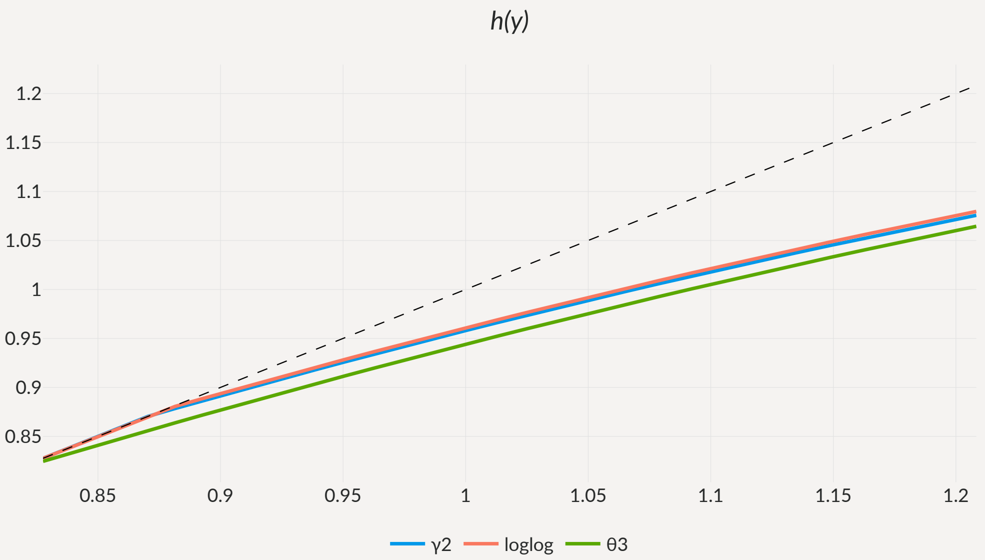
| | loglog | $\gamma = 2$ | $\gamma = 5$ | $\gamma = 10$ | $\gamma = 20$ |
|---|---|---|---|---|---|
| Avg. spread (bps) | 756 | 800 | 912 | 974 | 1,057 |
| Corr. NX,Y (%) | -0.285 | -0.302 | -0.21 | 0.0726 | 0.416 |
| Rel. vol. cons (%) | 1.5 | 1.37 | 1.18 | 1.04 | 0.921 |
| Risk premium (p.p.) | 0.652 | 0.789 | 1.02 | 1.28 | 2.38 |
| Debt-to-GDP (%) | 16.7 | 15.7 | 12.4 | 7.62 | 3.25 |
| Corr. deficit, y (%) | 0.391 | 0.391 | 0.217 | $-0.21$ | $-0.627$ |
| Default freq. (%) | 4.4 | 4.41 | 4.17 | 3.45 | 2.7 |
| Std. dev. spreads (bps) | 448 | 538 | 877 | 1,209 | 1,816 |
| Welfare | 1.013 | 1.01 | 1.002 | 0.9918 | 0.9728 |

# Comparative statics: robustness

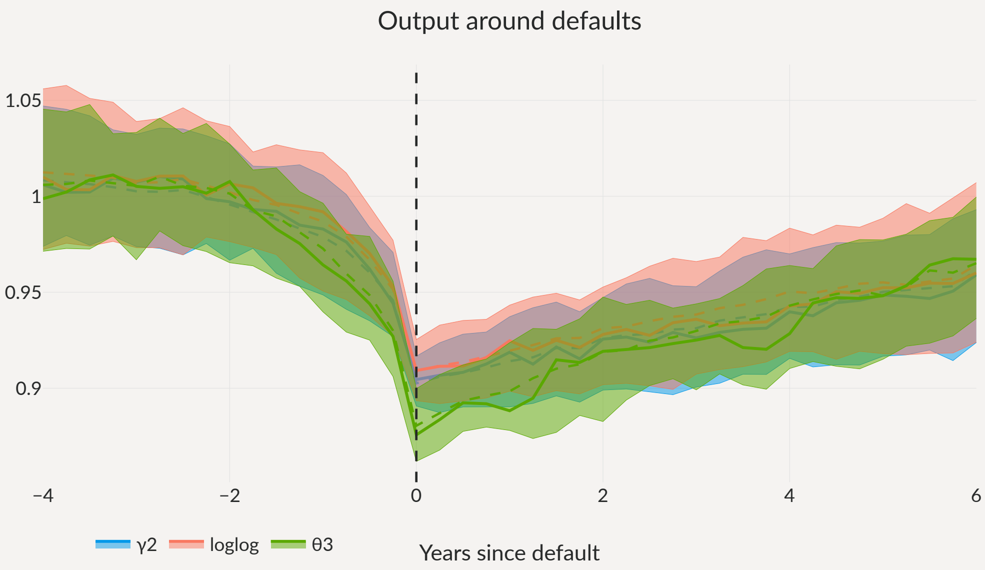
- Increasing RA: less debt tolerance, limited effect on volatilities

| | loglog | $\theta = 1$ | $\theta = 2$ | $\theta = 3$ |
|---|---|---|---|---|
| Avg. spread (bps) | 756 | 1,683 | 20,240 | 36,331 |
| Corr. NX, y (%) | -0.285 | -0.227 | -0.0901 | -0.227 |
| Rel. vol. cons (%) | 1.5 | 1.38 | 1.26 | 1.46 |
| Risk premium (p.p.) | 0.652 | 2.92 | 4.43 | 6.99 |
| Debt-to-GDP (%) | 16.7 | 14.2 | 9.09 | 9.57 |
| Corr. deficit, y (%) | 0.391 | 0.292 | 0.118 | 0.266 |
| Default freq. (%) | 4.4 | 5.88 | 3.57 | 2.47 |
| Std. dev. spreads (bps) | 448 | 2,561 | 103,509 | 189,131 |
| Welfare | 1.013 | 0.9848 | 0.9629 | 0.9469 |

# Calibrated output costs of default with robustness



# Event-study of defaults

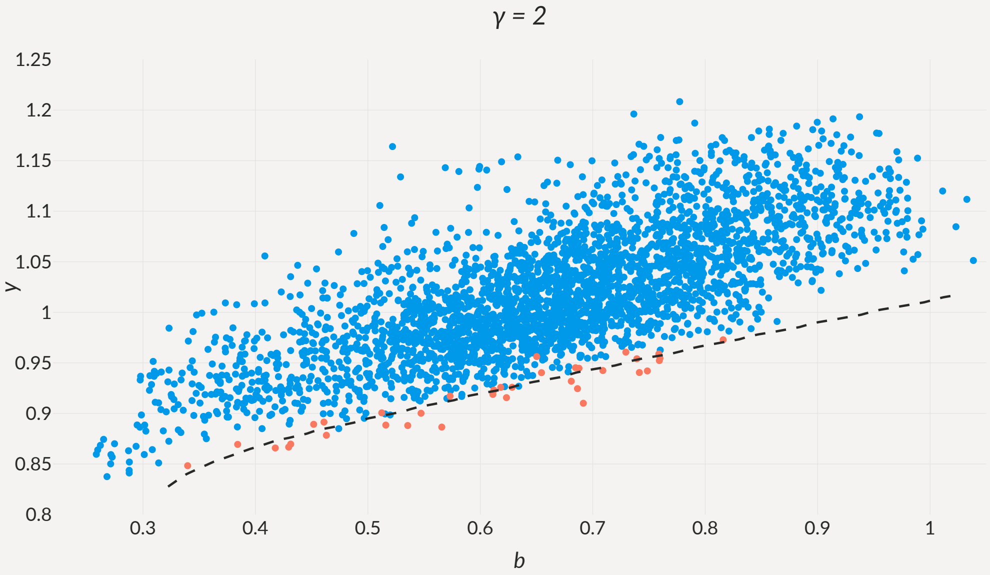


## Calibrations with risk aversion

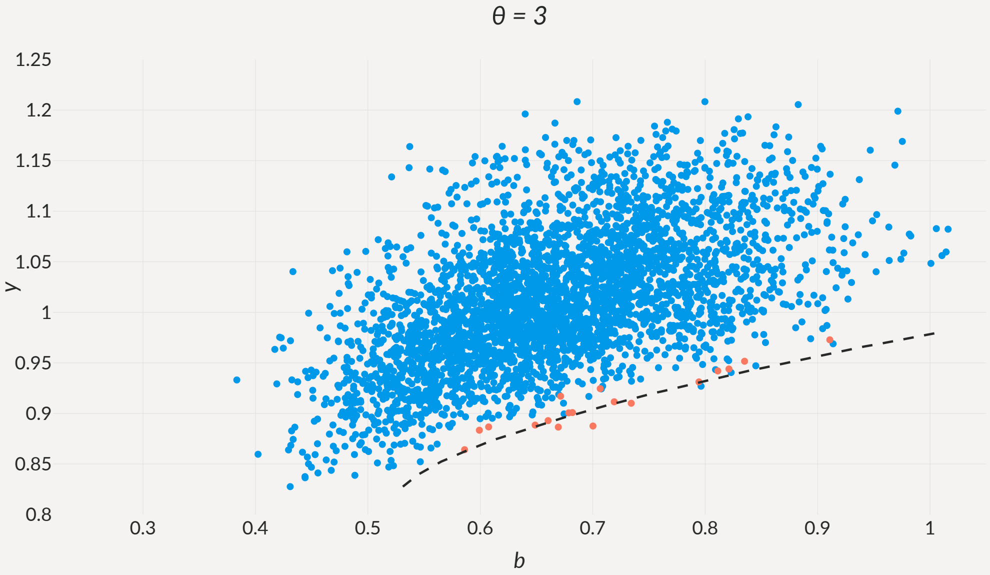
- Calibration with robustness: skewed debt outcomes, small decrease in macro volatility

| | Data | $\gamma = 2$ | loglog | $\theta = 3$ |
|---|---|---|---|---|
| Avg. spread (bps) | 815 | 754 | 756 | 815 |
| Corr. NX,Y (%) | - | -0.314 | -0.285 | -0.194 |
| Rel. vol. cons (%) | 0.94 | 1.38 | 1.5 | 1.35 |
| Risk premium (p.p.) | - | 0.778 | 0.652 | 5.9 |
| Debt-to-GDP (%) | 17.4 | 16.8 | 16.7 | 17.4 |
| Corr. deficit, y (%) | - | 0.405 | 0.391 | 0.207 |
| Default freq. (%) | - | 4.21 | 4.4 | 1.51 |
| Std. dev. spreads (bps) | 443 | 496 | 447 | 2,026 |

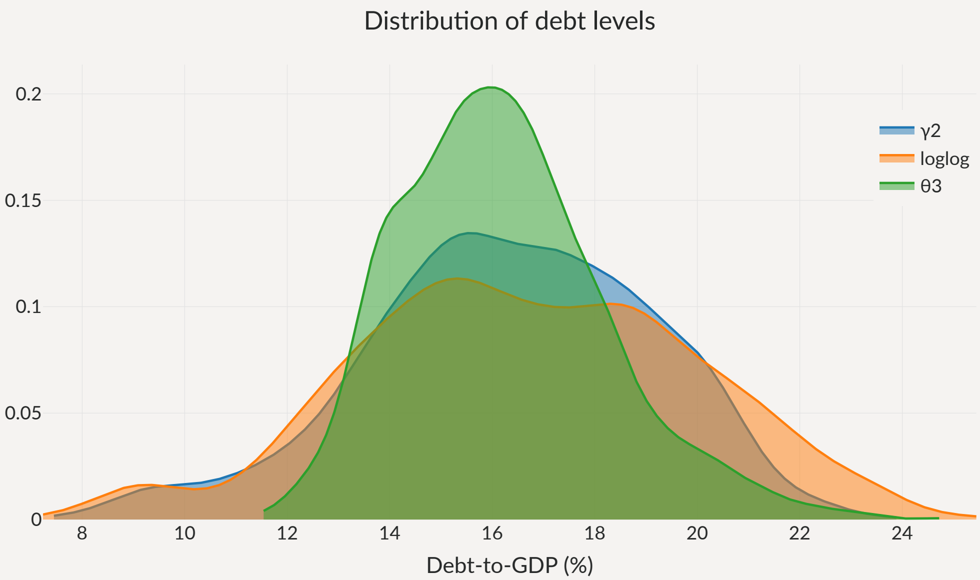
# Ergodic distribution for debt in CRRA model



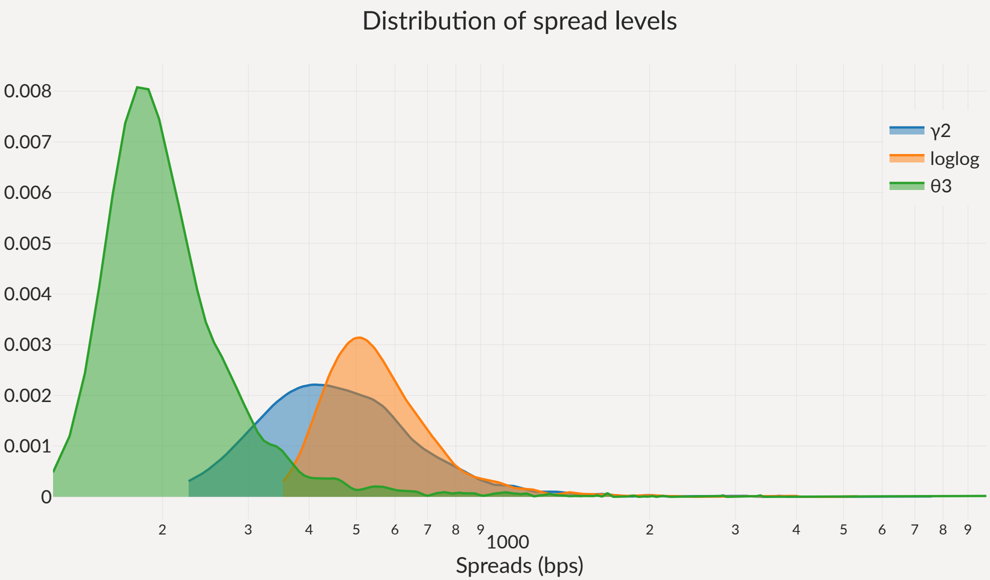
# Ergodic distribution for debt with robustness



# Ergodic distribution for debt



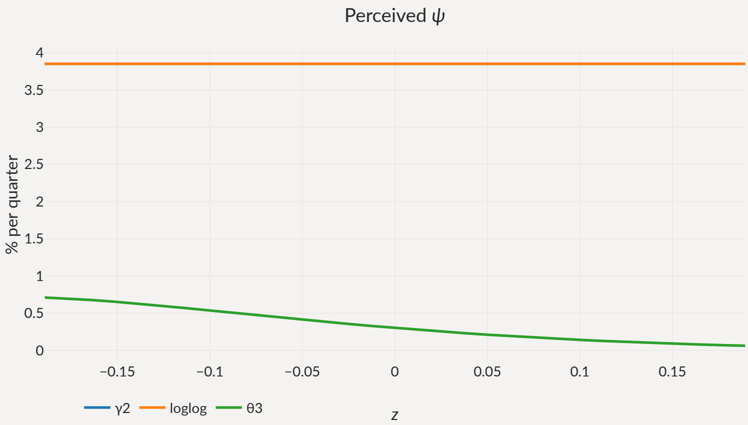
# Ergodic distribution for spreads



# Worst-case models

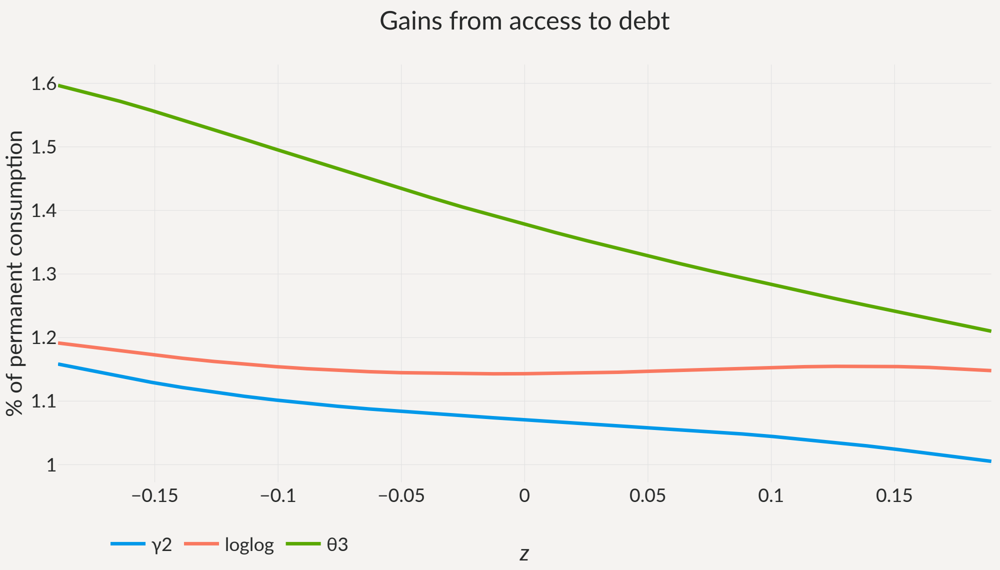
- Distorted expectation of $X$

$$\tilde{\mathbb{E}}\left[X \mid \mathcal{F}\right] = \mathbb{E}\left[\frac{\exp(-\theta v(s'))}{\mathbb{E}\left[\exp(-\theta v(s')) \mid \mathcal{F}\right]} X \;\middle|\; \mathcal{F}\right]$$

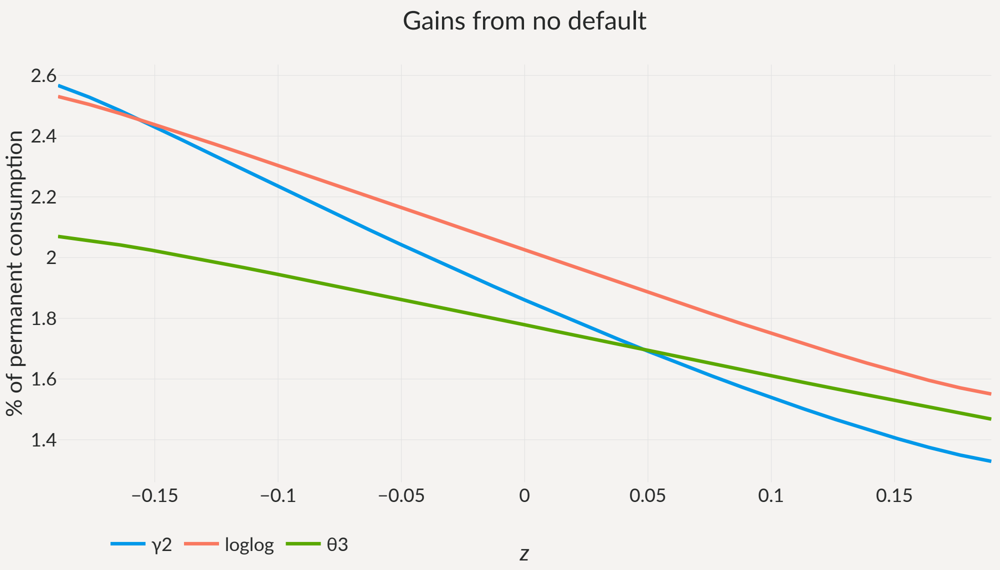


# Welfare effects

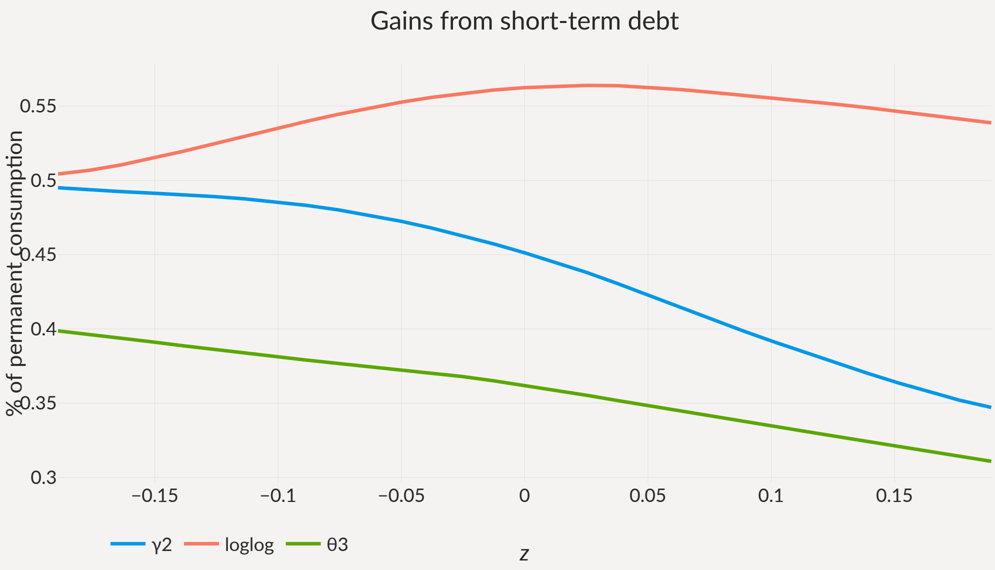
# Welfare effects of debt



# Welfare effects of banning defaults



# Welfare effects of shortening maturity



# Takeaways

**With preferences consistent with positive risk premia**

- *Lower* debt tolerance
  … Larger default costs required
- Less staying at the edge of default
  … More skewness in the distribution of debt and spreads
- More use of the debt for insurance
  … Large gains from debt access, not so much for making debt safe

# Welfare gains decomposition

Consumption without default costs $c_t^R$

$$c^R(b,z) = \mathbb{1}_{\mathcal{D}}(b,z)y(b,z) + (1 - \mathbb{1}_{\mathcal{D}}(b,z))c(b,z)$$

Evaluate value of consuming $c^R$ [instead of c] and removing uncertainty

$$\begin{aligned} V_{NC}(b,z) &= u(c^R(b,z)) + \beta\mathbb{E}\left[V_{NC}(b',z') \mid z\right] \\ V_{NV}(b,z) &= u(c^R(b,z)) + \beta V_{NV}(b',\mathbb{E}\left[z' \mid z\right]) \end{aligned}$$

Welfare gains between models/equilibria with value functions $v$ and $v^\star$

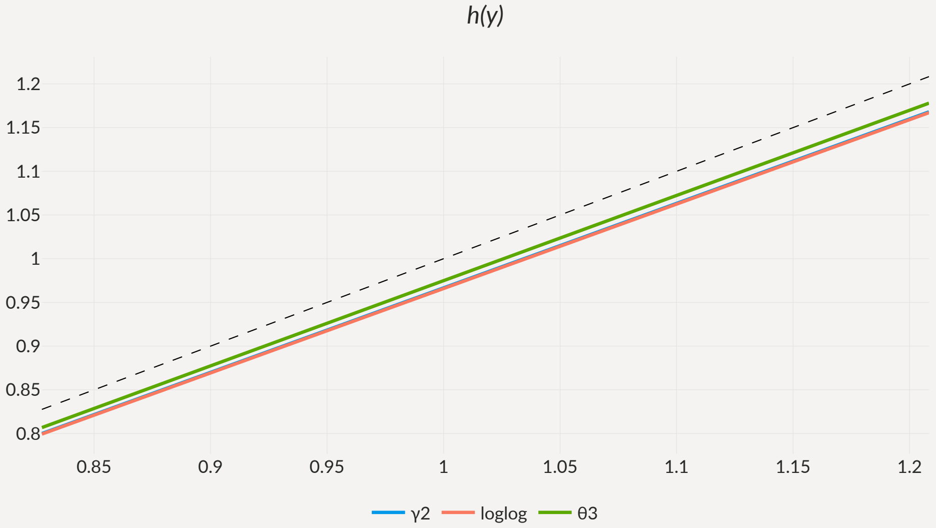
$$\frac{v^\star(b_0,z_0)}{v(b_0,z_0)} = \frac{v^\star(b_0,z_0)/v^\star_{NC}(b_0,z_0)}{v(b_0,z_0)/v_{NC}(b_0,z_0)} \times \frac{v^\star_{NC}(b_0,z_0)/v^\star_{NV}(b_0,z_0)}{v_{NC}(b_0,z_0)/v_{NV}(b_0,z_0)} \times \frac{v^\star_{NV}(b_0,z_0)}{v_{NV}(b_0,z_0)}$$

# Welfare gains

| | Total gains | From default costs | From volatility | From level |
|---|---|---|---|---|
| | | $\gamma = 2$ | | |
| Access to markets | 0.622 | -0.273 | 0.218 | 0.679 |
| No default | 1.87 | 0.274 | -0.292 | 1.89 |
| Short-term debt | 0.411 | 0.255 | -0.448 | 0.606 |
| | | loglog | | |
| Access to markets | 0.663 | -0.294 | 0.284 | 0.674 |
| No default | 2.04 | 0.295 | -0.345 | 2.09 |
| Short-term debt | 0.519 | 0.272 | -0.439 | 0.688 |
| | | $\theta = 3$ | | |
| Access to markets | 0.961 | -0.25 | 0.0354 | 1.18 |
| No default | 1.72 | 0.251 | -0.0744 | 1.54 |
| Short-term debt | 0.262 | 0.233 | -0.45 | 0.481 |

# Model with linear costs

- Convex costs lower income volatility during defaults



# Concluding remarks

# Risk aversion in the sovereign debt model

- We evaluate preferences consistent with risk premia in the sovereign default model

  … mostly possible to match standard calibration targets with robustness

- Effect of robustness concentrated at higher-order moments

  … makes crises look like more abrupt events

- Innocent-looking features of the standard model weigh against large risks/distortions

  … convex costs of default mute post-default uncertainty

# Macro-finanical separation with autarky

Back

| | loglog | $\gamma = 2$ | $\gamma = 5$ | $\gamma = 10$ | $\gamma = 20$ |
|---|---|---|---|---|---|
| Corr. NX,Y (%) | $-0.00131$ | $-0.00131$ | $-0.00131$ | $-0.00131$ | $-0.00131$ |
| Rel. vol. cons (%) | 1 | 1 | 1 | 1 | 1 |
| Risk premium (p.p.) | 0.0833 | 0.251 | 0.751 | 1.57 | 3.05 |
| Welfare | 1.002 | 1 | 0.9951 | 0.9868 | 0.9699 |

| | loglog | $\theta = 1$ | $\theta = 2$ | $\theta = 3$ |
|---|---|---|---|---|
| Corr. NX,Y (%) | $-0.00131$ | $-0.00131$ | $-0.00122$ | $-0.00073$ |
| Rel. vol. cons (%) | 1 | 1 | 1 | 1 |
| Risk premium (p.p.) | 0.0833 | 2.02 | 3.81 | 5.32 |
| Welfare | 1.002 | 0.9769 | 0.9564 | 0.9411 |