

# Uncertainty Premia, Sovereign Default Risk, and State-Contingent Debt

---

Francisco Roch  
IMF

Francisco Roldán  
IMF

August 2021

The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management.

# Why do governments borrow noncontingent?

## State-contingent debt instruments

- Decrease default risk
- Reduce cyclicalities of fiscal policy
- Improve risk-sharing

Why aren't they used?

# Why do governments borrow noncontingent?

## State-contingent debt instruments

- Decrease default risk
- Reduce cyclicalities of fiscal policy
- Improve risk-sharing

Why aren't they used?

# Unfavorable prices of state-contingent instruments

- These instruments are heavily **discounted** by markets
  - Costa, Chamon, and Ricci (2008) compute wide spreads for Argentine **GDP-warrants**
    - ~300-400bps from default risk of other securities
    - 600-1200bps residual: '**novelty**' premium

This paper proposes a framework that

- Rationalizes **pricing** of SCI + **welfare** analysis
  - With ingredients from resolutions of the equity premium puzzle
- Links unfavorable prices to common 'threshold' structure
  - Example: Argentina's GDP-warrants, also Ukraine, Greece. . .

► More

# Unfavorable prices of state-contingent instruments

- These instruments are heavily **discounted** by markets
  - Costa, Chamon, and Ricci (2008) compute wide spreads for Argentine **GDP-warrants**
    - ~300-400bps from default risk of other securities
    - 600-1200bps residual: '**novelty**' premium

This paper proposes a framework that

- Rationalizes **pricing** of SCI + **welfare** analysis
  - With ingredients from resolutions of the equity premium puzzle
- Links unfavorable prices to common 'threshold' structure
  - Example: Argentina's GDP-warrants, also Ukraine, Greece. . .

► More

# A framework for pricing state-contingent debt

- Standard quantitative model of sovereign default with long-term debt
  - Aguiar and Gopinath (2006), Arellano (2008), Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012)
- International lenders with concerns about *model misspecification*
  - Preference for **robustness** Hansen and Sargent (2001), Pouzo and Presno (2016)
- Mechanism: lenders act *as if* the probability of states with low repayment was higher
  - With noncontingent debt, lenders overestimate the default probability
  - Pouzo and Presno (2016) uses robustness to reconcile **spreads** with default **frequencies**
  - In general, probability distortion depends on type and quantity of debt issued

# Main findings

1. Robust lenders dislike repayment structures with **thresholds** in good times
  - Heavy discounts for these bonds  $\implies$  welfare **losses**
2. Explain most of the 'novelty premium' in Argentina's GDP warrants as **ambiguity** premia
  - Calibration of robustness from *noncontingent* debt only
3. Characterize the **optimal** design and how it changes with robustness
  - With high robustness, want to minimize ex-ante and ex-post contingency

# Roadmap

---

- Stylized Model
- Probability Distortions
- Pricing and Welfare
- Quantitative Implementation
- Concluding Remarks



## Stylized Model

---

# The model

We consider a simple two-period model, small open economy

- Uncertain endowment  $y(z)$  in the second period
- The government has access to **one** asset which promises a return  $R(z)$ .
- A few benchmarks

Noncontingent debt	$R(z)$	$=$	1
Linear indexing	$R^\alpha(z)$	$=$	$1 + \alpha(y(z) - 1)$
Threshold debt	$R^\tau(z)$	$=$	$\mathbb{1}(z > \tau)$
Optimal design	$R^*(z; \theta)$		chosen state-by-state

# The model

We consider a simple two-period model, small open economy

- Uncertain endowment  $y(z)$  in the second period
- The government has access to **one** asset which promises a return  $R(z)$ .
- A few benchmarks

Noncontingent debt	$R(z)$	$=$	1
Linear indexing	$R^\alpha(z)$	$=$	$1 + \alpha(y(z) - 1)$
Threshold debt	$R^\tau(z)$	$=$	$\mathbb{1}(z > \tau)$
Optimal design	$R^*(z; \theta)$		chosen state-by-state

# The government's problem

- The government takes as given the **price schedule**  $q(b)$

$$\begin{aligned} & \max_b u(c_1^b) + \beta_b \mathbb{E} [u(c_2^b)] \\ \text{subject to } & c_1^b = y_1 + q(b)b \\ & c_2^b = y_2(z) - h(z, \Delta)d(b, z) - (1 - d(b, z))R(z)b \end{aligned}$$

where

$$h(z, \Delta) = \phi y_2(z)\Delta + (1 - \phi)y_2(z)^2\Delta$$

- In the second period, **default** if

$$\underbrace{u(y_2(z) - h(z, \Delta))}_{\text{v. default}} > \underbrace{u(y_2(z) - R(z)b)}_{\text{v. repayment}}$$

# The government's problem

- The government takes as given the **price schedule**  $q(b)$

$$\begin{aligned} & \max_b u(c_1^b) + \beta_b \mathbb{E} [u(c_2^b)] \\ \text{subject to } & c_1^b = y_1 + q(b)b \\ & c_2^b = y_2(z) - h(z, \Delta)d(b, z) - (1 - d(b, z))R(z)b \end{aligned}$$

where

$$h(z, \Delta) = \phi y_2(z)\Delta + (1 - \phi)y_2(z)^2\Delta$$

- In the second period, **default** if

$$\underbrace{u(y_2(z) - h(z, \Delta))}_{\text{v. default}} > \underbrace{u(y_2(z) - R(z)b)}_{\text{v. repayment}}$$

# The lenders' problem

Foreign lenders are less standard and have **multiplier preferences**

$$\begin{aligned} & \max c_1^L - \frac{\beta}{\theta} \log (\mathbb{E} [\exp(-\theta v_2^L)]) \\ & \text{subject to } v_2^L = c_2^L \\ & c_2^L = w_2 + (1 - d(b, z))R(z)b \\ & c_1^L = w_1 - q_1 b \end{aligned}$$

Lenders provide us with an **Euler equation** to price the debt

$$q(b; R) = \beta \mathbb{E} \left[ \frac{\exp(-\theta c_2^L)}{\mathbb{E} [\exp(-\theta c_2^L)]} (1 - d(b, z))R(z) \right]$$

# The lenders' problem

Foreign lenders are less standard and have **multiplier preferences**

$$\begin{aligned} & \max c_1^L - \frac{\beta}{\theta} \log (\mathbb{E} [\exp(-\theta v_2^L)]) \\ & \text{subject to } v_2^L = c_2^L \\ & c_2^L = w_2 + (1 - d(b, z))R(z)b \\ & c_1^L = w_1 - q_1b \end{aligned}$$

Lenders provide us with an **Euler equation** to price the debt

$$q(b; R) = \beta \mathbb{E} \left[ \frac{\exp(-\theta c_2^L)}{\mathbb{E} [\exp(-\theta c_2^L)]} (1 - d(b, z))R(z) \right]$$

- The lenders' Euler equation explains the sources of the **spreads** they charge
- Call  $M = \beta \frac{\exp(-\theta c_2^L)}{\mathbb{E}[\exp(-\theta c_2^L)]}$  the stochastic discount factor

$$\begin{aligned}
 q(b; R) &= \beta \mathbb{E} \left[ \frac{\exp(-\theta c_2^L)}{\mathbb{E}[\exp(-\theta c_2^L)]} (1 - d(b, z)) R(z) \right] \\
 &= \underbrace{\beta \mathbb{E}[(1 - d)R]}_{= q_{RE}} + \underbrace{(1 - \mathbb{P}(d)) \text{cov}(M, R)}_{= q_{\theta}^{\text{cont}}} - \underbrace{\mathbb{E}[R] \text{cov}(M, d)}_{= -q_{\theta}^{\text{def}}}
 \end{aligned}$$

- The debt price is a rational-expectations price and two sources of **ambiguity** premia



Interpret lenders' stochastic discount factor as **probability distortions**

- For a random variable  $X$

$$\tilde{\mathbb{E}}[X] = \mathbb{E} \left[ \frac{\exp(-\theta v_2^L)}{\mathbb{E}[\exp(-\theta v_2^L)]} X \right]$$

- $\tilde{\mathbb{E}}$  **tilts** probabilities towards *less-favorable* states for lenders
- Obs The tilting is endogenous to the lenders' **outcomes**

Interpret lenders' stochastic discount factor as **probability distortions**

- For a random variable  $X$

$$\tilde{\mathbb{E}}[X] = \mathbb{E} \left[ \frac{\exp(-\theta v_2^L)}{\mathbb{E}[\exp(-\theta v_2^L)]} X \right]$$

- $\tilde{\mathbb{E}}$  **tilts** probabilities towards *less-favorable* states for lenders
- Obs The tilting is endogenous to the lenders' **outcomes**

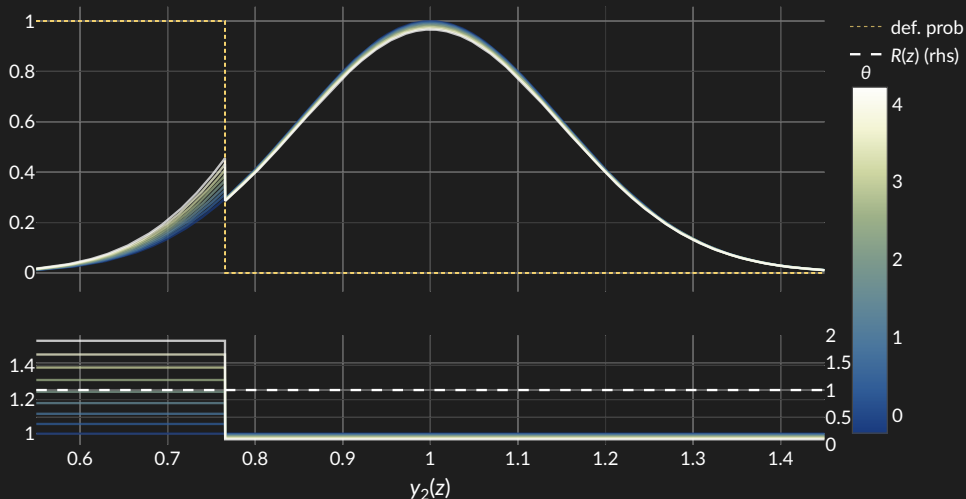
# Probability Distortions

---

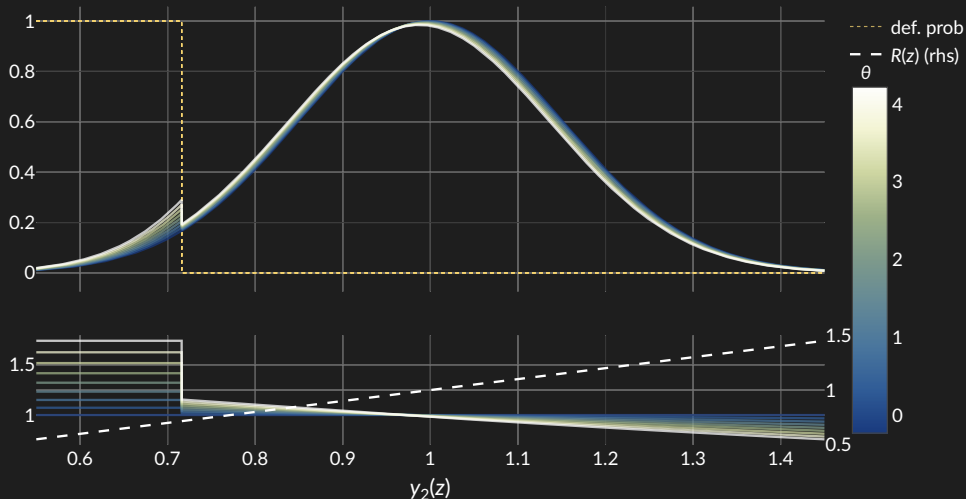
Costa, Chamon, and Ricci (2008) study the GDP-warrants issued by **Argentina**

- The warrant paid if
  - Output **growth** above pre-set level (4.3% initially, later 3%)
  - Output *level* above the compounded cutoff growth
  - There is also a cap on total payments

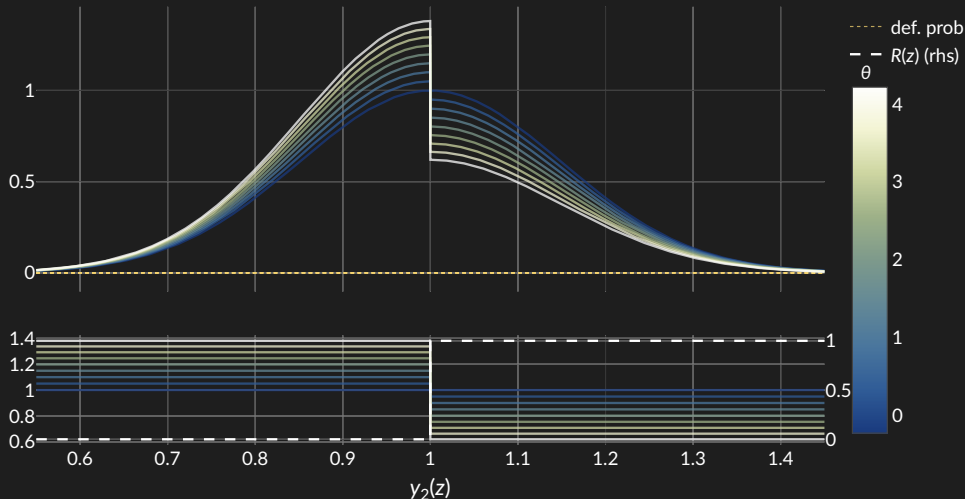
Distorted probabilities



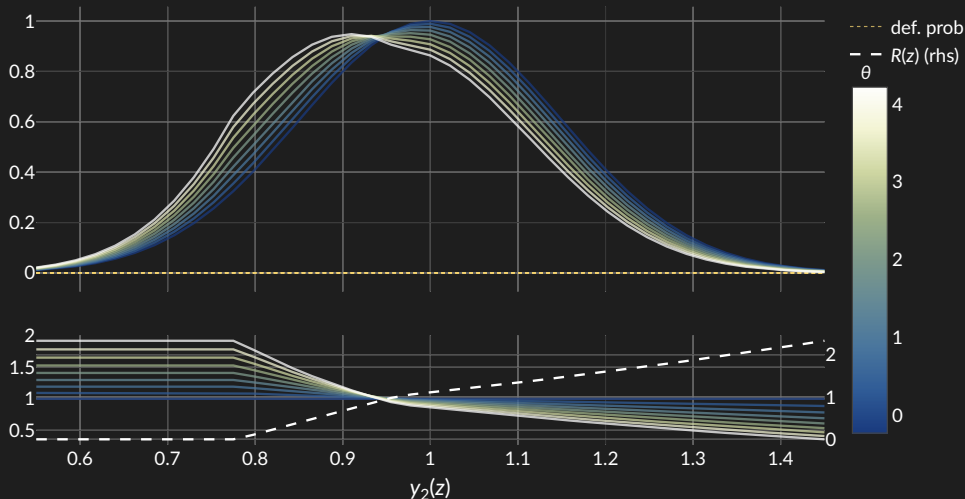
Distorted probabilities



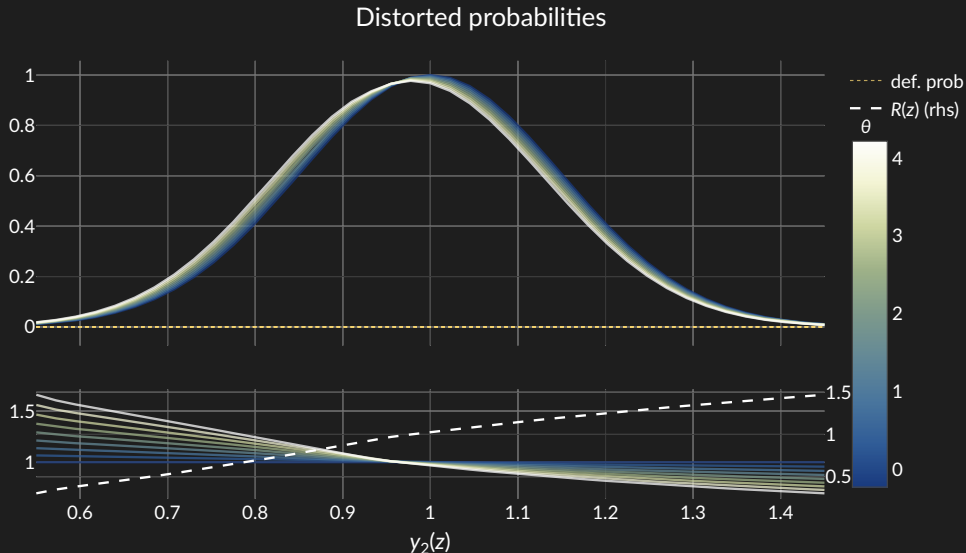
Distorted probabilities



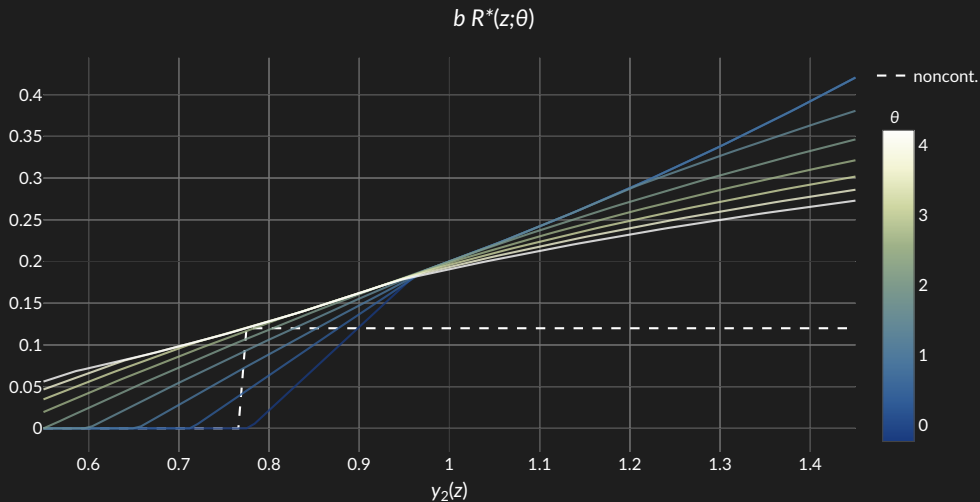
Distorted probabilities







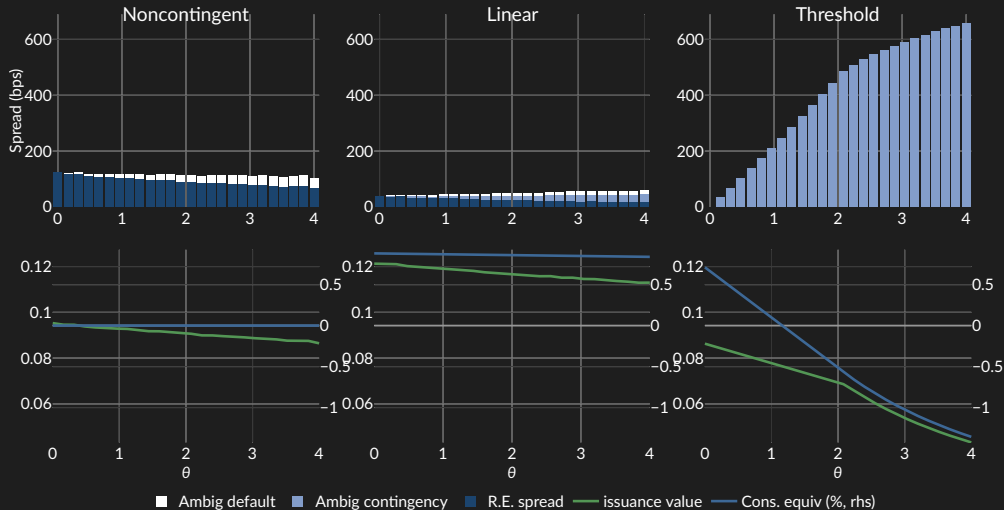
# Design of debt



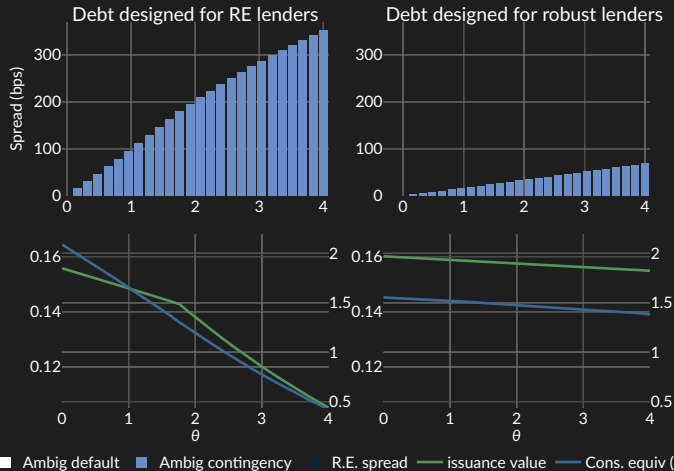
# Pricing and Welfare

---

# Parametric debt types



# Optimal debt designs



# Quantitative Implementation

---

- Infinite horizon, small-open economy
- **Robust** lenders as before
- Long-term debt, debt issued at  $t$  pays coupon at  $t + s$

$$\max \{0, (1 - \delta)^{s-1} (1 + \alpha(y_s - 1)) \mathbb{1}(y_s > \tau)\}$$

- Noncontingent debt:  $\alpha = 0, \tau = -\infty$
- Default triggers exclusion + output costs for a random amount of periods  $\sim \text{Geo}(\psi)$

	Parameter	Chatterjee and Eyigungor (2012)	Pouzo and Presno (2016)
Sovereign's risk aversion	$\gamma$	2	2
Interest rate	$r$	0.01	0.01
Income autocorrelation coefficient	$\rho$	0.9485	0.9484
Standard deviation of innovations	$\sigma_\epsilon$	0.027	0.02
Reentry probability	$\psi$	0.0385	0.0385
Duration of debt	$\delta$	0.05	0.05
Discount factor	$\beta$	0.95402	0.9627
Default cost: linear	$d_0$	-0.18819	-0.255
Default cost: quadratic	$d_1$	0.24558	0.296
Degree of robustness	$\theta$	0	1.62
Linear coupon indexation	$\alpha$	0	0
Coupon repayment threshold	$\tau$	$-\infty$	$-\infty$

Table 1: Parameter values for the baseline parametrizations.



	Parameter	Chatterjee and Eyigungor (2012)	Pouzo and Presno (2016)
Sovereign's risk aversion	$\gamma$	2	2
Interest rate	$r$	0.01	0.01
Income autocorrelation coefficient	$\rho$	0.9485	0.9484
Standard deviation of innovations	$\sigma_\epsilon$	0.027	0.02
Reentry probability	$\psi$	0.0385	0.0385
Duration of debt	$\delta$	0.05	0.05
Discount factor	$\beta$	0.95402	0.9627
Default cost: linear	$d_0$	-0.18819	-0.255
Default cost: quadratic	$d_1$	0.24558	0.296
Degree of robustness	$\theta$	0	1.62
Linear coupon indexation	$\alpha$	0	0
Coupon repayment threshold	$\tau$	$-\infty$	$-\infty$

Table 1: Parameter values for the baseline parametrizations.

Statistic	Rational Expectations			$\theta = 1.6155$ (benchmark)		
	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread	8.1	0.36	7.2	8.15	11.1	7.1
Std Spread	4.5	0.23	3.7	4.6	1.58	3.6
Debt	48.7	116.5	50.8	44.0	67.6	46.1
Std(c)/Std(y)	1.24	0.82	1.22	1.25	0.84	1.23
Default Prob	5.5	0.3	5.3	3.0	0.0	2.6
Welfare Gains	-	1.19	0.09	-	-0.37	0.07

Table 2: Statistics based on Pouzo and Presno (2016)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with  $\alpha = 1$ .

Statistic	Rational Expectations			$\theta = 1.6155$ (benchmark)		
	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread	8.1	0.36	7.2	8.15	11.1	7.1
Std Spread	4.5	0.23	3.7	4.6	1.58	3.6
Debt	48.7	116.5	50.8	44.0	67.6	46.1
Std(c)/Std(y)	1.24	0.82	1.22	1.25	0.84	1.23
Default Prob	5.5	0.3	5.3	3.0	0.0	2.6
Welfare Gains	-	1.19	0.09	-	-0.37	0.07

Table 2: Statistics based on Pouzo and Presno (2016)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with  $\alpha = 1$ .

Statistic	Rational Expectations			$\theta = 1.6155$ (benchmark)		
	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread	8.1	0.36	7.2	8.15	11.1	7.1
Std Spread	4.5	0.23	3.7	4.6	1.58	3.6
Debt	48.7	116.5	50.8	44.0	67.6	46.1
Std(c)/Std(y)	1.24	0.82	1.22	1.25	0.84	1.23
Default Prob	5.5	0.3	5.3	3.0	0.0	2.6
Welfare Gains	-	1.19	0.09	-	-0.37	0.07

Table 2: Statistics based on Pouzo and Presno (2016)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with  $\alpha = 1$ .

Statistic	Rational Expectations			$\theta = 1.6155$ (benchmark)		
	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread	8.1	0.36	7.2	8.15	11.1	7.1
Std Spread	4.5	0.23	3.7	4.6	1.58	3.6
Debt	48.7	116.5	50.8	44.0	67.6	46.1
Std(c)/Std(y)	1.24	0.82	1.22	1.25	0.84	1.23
Default Prob	5.5	0.3	5.3	3.0	0.0	2.6
Welfare Gains	-	1.19	0.09	-	-0.37	0.07

Table 2: Statistics based on Pouzo and Presno (2016)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with  $\alpha = 1$ .

Statistic	Rational Expectations			$\theta = 1.6155$ (benchmark)		
	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread	8.1	0.36	7.2	8.15	11.1	7.1
Std Spread	4.5	0.23	3.7	4.6	1.58	3.6
Debt	48.7	116.5	50.8	44.0	67.6	46.1
Std(c)/Std(y)	1.24	0.82	1.22	1.25	0.84	1.23
Default Prob	5.5	0.3	5.3	3.0	0.0	2.6
Welfare Gains	-	1.19	0.09	-	-0.37	0.07

Table 2: Statistics based on Pouzo and Presno (2016)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with  $\alpha = 1$ .

Statistic	Rational Expectations			$\theta = 1.6155$ (benchmark)		
	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread	8.1	0.36	7.2	8.15	11.1	7.1
Std Spread	4.5	0.23	3.7	4.6	1.58	3.6
Debt	48.7	116.5	50.8	44.0	67.6	46.1
Std(c)/Std(y)	1.24	0.82	1.22	1.25	0.84	1.23
Default Prob	5.5	0.3	5.3	3.0	0.0	2.6
Welfare Gains	-	1.19	0.09	-	-0.37	0.07

Table 2: Statistics based on Pouzo and Presno (2016)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with  $\alpha = 1$ .

Statistic	Rational Expectations			$\theta = 1.6155$ (benchmark)		
	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread	8.1	0.36	7.2	8.15	11.1	7.1
Std Spread	4.5	0.23	3.7	4.6	1.58	3.6
Debt	48.7	116.5	50.8	44.0	67.6	46.1
Std(c)/Std(y)	1.24	0.82	1.22	1.25	0.84	1.23
Default Prob	5.5	0.3	5.3	3.0	0.0	2.6
Welfare Gains	-	1.19	0.09	-	-0.37	0.07

Table 2: Statistics based on Pouzo and Presno (2016)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with  $\alpha = 1$ .



# Optimal design of state-contingent debt

Statistic	Rational Expectations	Robustness
	$\tau = 0.875, \alpha = 7$	$\tau = 0.875, \alpha = 5$
Spread	0.1	2.8
Std Spread	0.04	0.13
Debt	79.3	65.9
Std(c)/Std(y)	0.76	0.96
Default Prob	0.1	0.23
Welfare Gains	1.79	0.79

Table 3: Statistics based on Pouzo and Presno (2016) under the optimal state-contingent bond with and without robust lenders.

# Optimal design of state-contingent debt

Statistic	Rational Expectations	Robustness
	$\tau = 0.875, \alpha = 7$	$\tau = 0.875, \alpha = 5$
Spread	0.1	2.8
Std Spread	0.04	0.13
Debt	79.3	65.9
Std(c)/Std(y)	0.76	0.96
Default Prob	0.1	0.23
Welfare Gains	1.79	0.79

Table 3: Statistics based on Pouzo and Presno (2016) under the optimal state-contingent bond with and without robust lenders.

# Optimal design of state-contingent debt

Statistic	Rational Expectations	Robustness
	$\tau = 0.875, \alpha = 7$	$\tau = 0.875, \alpha = 5$
Spread	0.1	2.8
Std Spread	0.04	0.13
Debt	79.3	65.9
Std(c)/Std(y)	0.76	0.96
Default Prob	0.1	0.23
Welfare Gains	1.79	0.79

Table 3: Statistics based on Pouzo and Presno (2016) under the optimal state-contingent bond with and without robust lenders.

# Optimal design of state-contingent debt

Statistic	Rational Expectations $\tau = 0.875, \alpha = 7$	Robustness $\tau = 0.875, \alpha = 5$
Spread	0.1	2.8
Std Spread	0.04	0.13
Debt	79.3	65.9
Std(c)/Std(y)	0.76	0.96
Default Prob	0.1	0.23
Welfare Gains	1.79	0.79

Table 3: Statistics based on Pouzo and Presno (2016) under the optimal state-contingent bond with and without robust lenders.

# Optimal design of state-contingent debt

Statistic	Rational Expectations	Robustness
	$\tau = 0.875, \alpha = 7$	$\tau = 0.875, \alpha = 5$
Spread	0.1	2.8
Std Spread	0.04	0.13
Debt	79.3	65.9
Std(c)/Std(y)	0.76	0.96
Default Prob	0.1	0.23
Welfare Gains	1.79	0.79

Table 3: Statistics based on Pouzo and Presno (2016) under the optimal state-contingent bond with and without robust lenders.

## Concluding Remarks

---

# Concluding Remarks

- Standard sovereign debt model augmented with robust lenders
  1. accounts for **spreads** on typical threshold SCDIs
  2. rationalizes part of the 'novelty' premium as a premium for **ambiguity**
  3. links unfavorable prices to common *threshold* structure
  4. **Welfare** gains of SCDI decreasing in robustness
    - Both for given instrument and for optimally-designed debt
- Optimal design
  - With realistic robustness, lower thresholds and **flatter** indexation than RE
  - With extreme robustness, eliminate contingency ex-ante (*stipulated*) and ex-post (*default*)
  - In general, tradeoff between **contingency** and **risk-sharing**

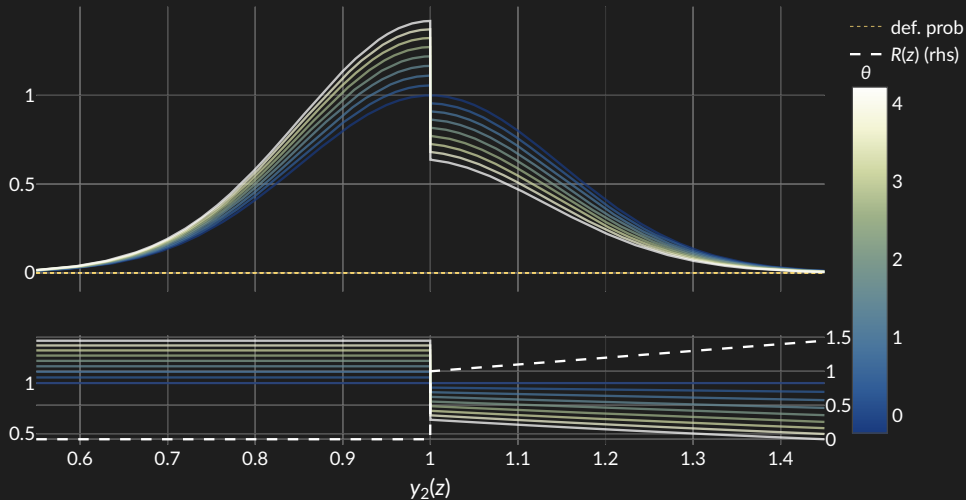




# Distorted probabilities – threshold+linear debt

[◀ Back](#)

Distorted probabilities



Statistic	Rational Expectations (benchmark)			$\theta = 1.6155$		
	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread	8.5	0.6	6.8	8.4	15.5	7.1
Std Spread	4.3	0.4	3.0	4.4	2.3	3.1
Debt	69.9	159.6	74.4	62.6	87.7	67.2
Std(c)/Std(y)	1.24	0.83	1.21	1.25	0.82	1.22
Corr(y,c)	0.98	0.53	0.98	0.98	0.94	0.98
Corr(y,tb/y)	-0.7	0.52	-0.62	-0.67	0.58	-0.6
Corr(y,spread)	-0.77	-0.87	-0.78	-0.75	-0.61	-0.77
Default Prob	5.8	0.56	5.3	2.3	0.12	1.8
Welfare Gains	-	1.86	0.27	-	-0.87	0.2

Table 4: Statistics based on Chatterjee and Eyigungor (2012)

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with  $\alpha = 1$ .

Euler equations of a rational-expectations agent with CARA preferences and access to a risk-free bond

$$q = \beta \mathbb{E} \left[ \frac{u'(c_2)}{u'(c_1)} R \right] = \beta \mathbb{E} \left[ \frac{\exp(-\gamma c_2)}{\exp(-\gamma c_1)} R \right]$$
$$\frac{1}{1+r} = \beta \mathbb{E} \left[ \frac{u'(c_2)}{u'(c_1)} \right]$$

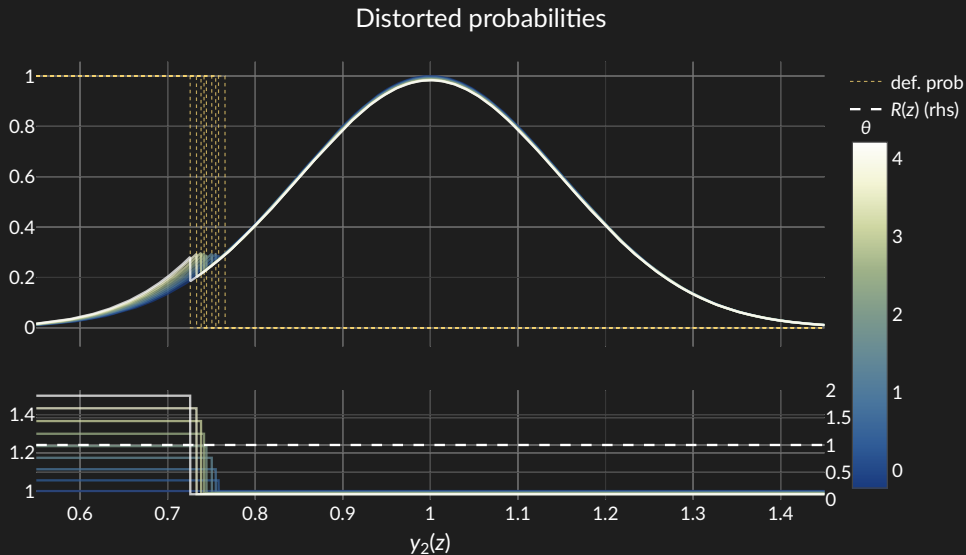
hence

$$q = \beta \mathbb{E} \left[ \frac{\exp(-\gamma c_2)}{\beta(1+r) \mathbb{E} [\exp(-\gamma c_2)]} R \right]$$

Same as robustness in two periods, in general the robust sdf is

$$q = \beta \mathbb{E} \left[ \frac{\exp(-\theta \mathbf{v}')}{\mathbb{E} [\exp(-\theta \mathbf{v}')] } R \right]$$

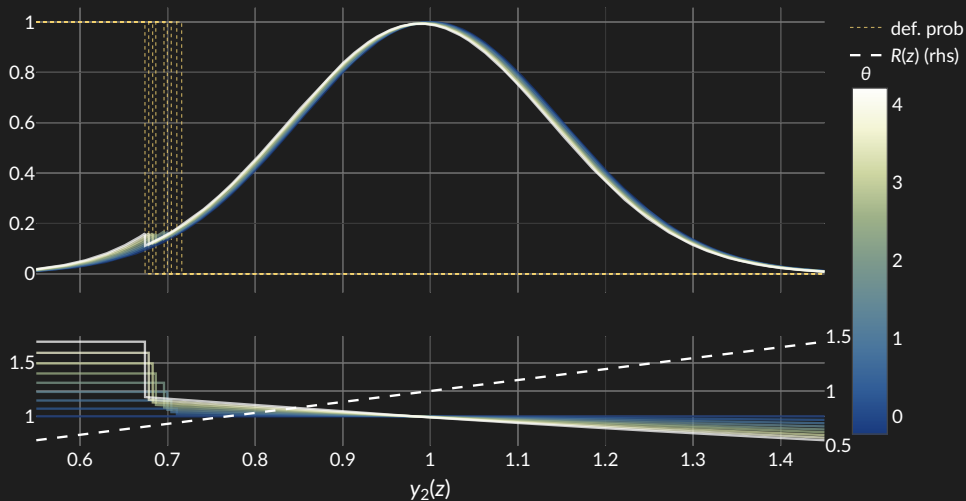
# Distorted probabilities – noncontingent debt

[◀ Back](#)

# Distorted probabilities – linearly indexed debt

[◀ Back](#)

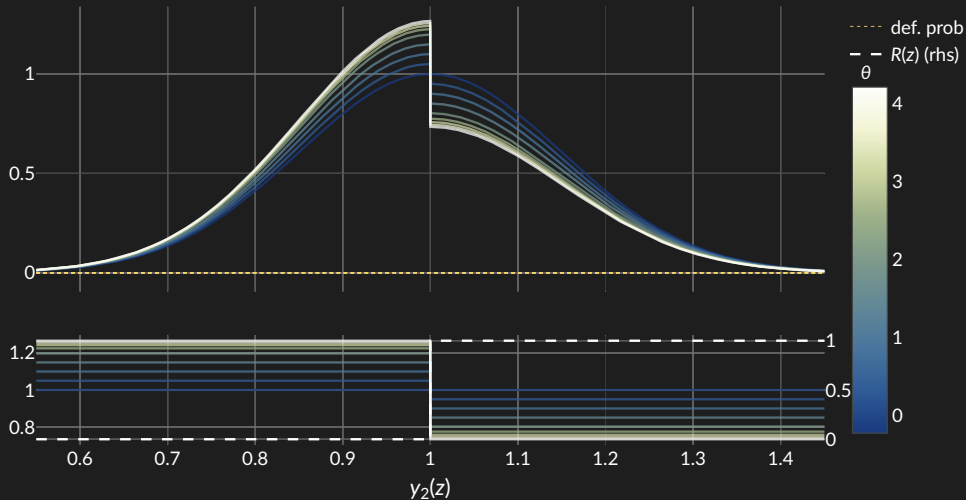
Distorted probabilities



# Distorted probabilities – threshold debt

[◀ Back](#)

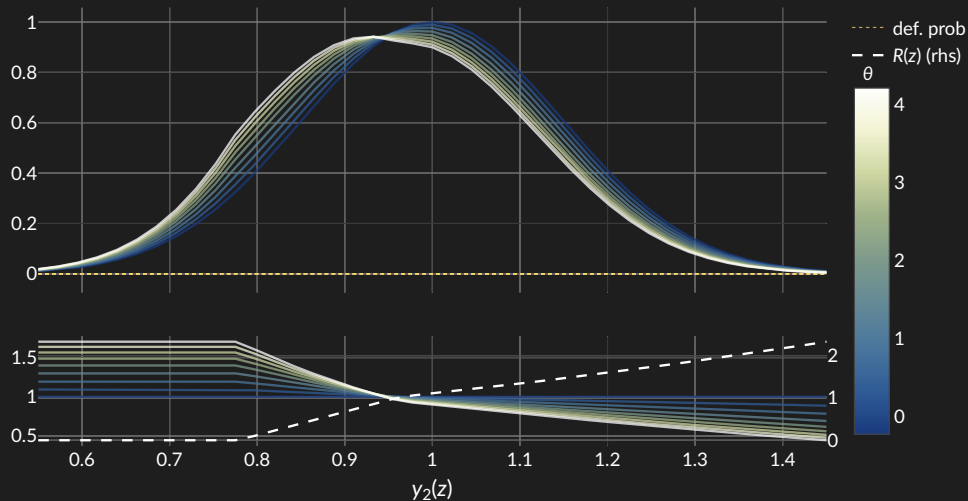
Distorted probabilities



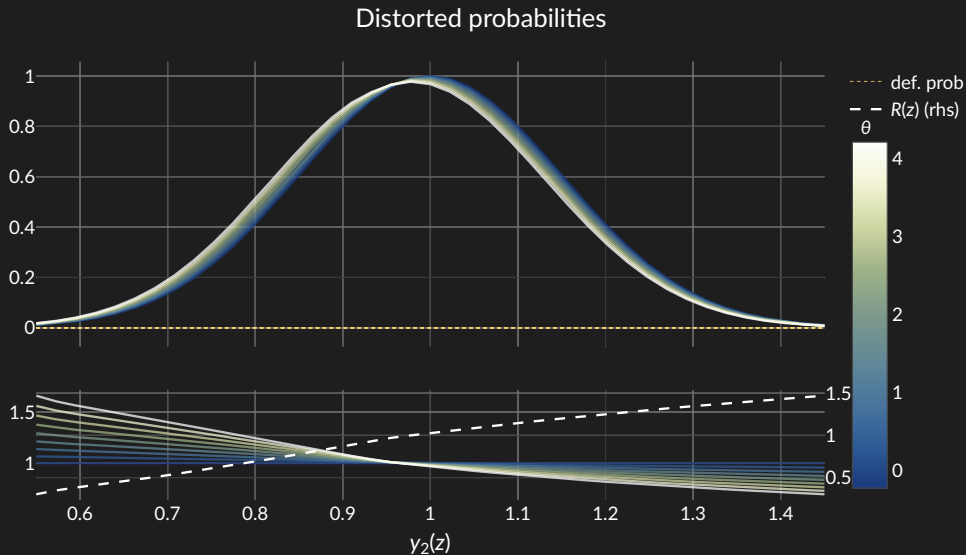
# Distorted probabilities – debt for RE lenders

[◀ Back](#)

Distorted probabilities



# Distorted probabilities – debt for robust lenders

[◀ Back](#)



We represent this bond with threshold debt, one period = five years, and

Parameter	Target	Value
$\beta_b$	Borrower's discount rate	6% ann.
$\beta$	Risk-free rate	3% ann.
$\gamma$	Borrower's risk aversion	2
$\Delta$	Output cost of default	20%
$g$	Expected growth rate	8% ann.
$k$	Threshold for repayment	50%

# Decomposition of spreads

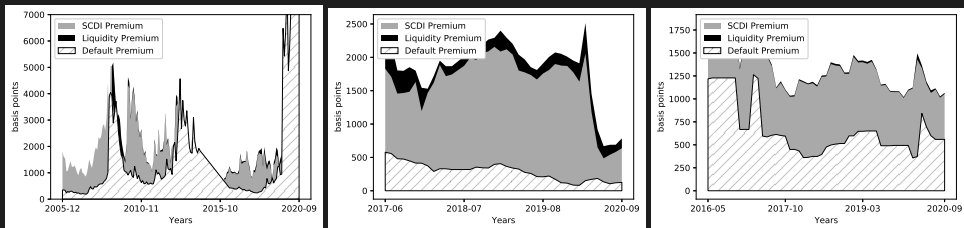


Figure 1: GDP-linked security premia.

The figure shows the estimated spread decomposition in Igan and Kim (2021) for the GDP-warrants issued by Argentina (left), Greece (middle) and Ukraine (right).