Reserve Accumulation and the Currency Composition of Sovereign Debt*

Francisco Roldán[†]
IMF

César Sosa-Padilla[‡]
University of Notre Dame
and NBER

December 2021

Preliminary: please do not circulate

Abstract

Why do countries which issue debt in local currency accumulate foreign-exchange reserves more than foreign-currency issuers? This fact can be puzzling considering that international reserves are held as insurance against shocks which create recessions at home and increase sovereign spreads. Local currency debt is a two-sided blade: it dampens this risk by enabling lower payments in recessions when the exchange rate depreciates but, at the same time, opens the door for ex-post inefficient policies which affect the exchange rate in order to reduce payments. Holding FX reserves serves as a commitment device not to manipulate the exchange rate in this way when repaying the debt proves difficult.

JEL Classification F32, F34, F41

Keywords International reserves, sovereign default, macroeconomic stabilization, exchange rates.

*The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management.

[†]e-mail: *froldan@imf.org*

[‡]e-mail: *csosapad@nd.edu*

Introduction

Review of the Literature

Layout The remainder of this paper is organized as follows. Section 2 lays out the basic facts about reserve accumulation among countries which issue debt in foreign and local currency. Section 3 introduces our model and Section 4 discusses its equilibrium and calibration. Section 5 presents our main results. Finally, Section 6 concludes.

2. Data

We document the main motivating fact using data on reserve holdings and the currency composition of government debt. FX reserve holdings are obtained from the usual IMF time series. The share of new debt issuances that are made in local currency is from Ottonello and Perez (2019). Figure 1 shows a clear positive association between these two series and captures in a simple way the main motivating fact: countries that borrow more heavily in local currency (*pesos*, for short) tend to hold more reserves than those who borrow predominantly in foreign currency (*dollars*).

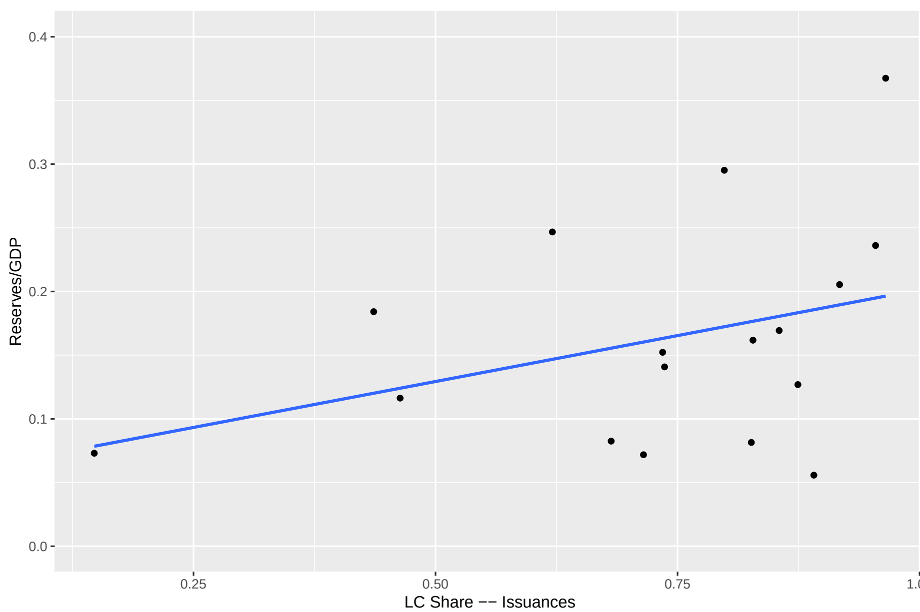

Figure 1: Reserves and currency composition of debt

Note: the horizontal axis shows the issuance of local currency debt as a fraction of all debt issuance. The vertical axis shows the ratio of FX reserves to GDP.

3. A model of reserve accumulation with debt in different currencies

We consider a small open economy which produces traded and nontraded goods subject to nominal rigidities. Its government values consumption of the domestic representative agent and trades two assets with the rest of the world: reserves a and debt b. While reserves are risk-free, the government may choose to default on the debt subject to default costs.

Households The small open economy is populated by a measure one of identical households, whose preferences over streams of consumption c are given by

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(c(t))\right]$$

where $\beta \in (0,1)$ is a discount factor and $\mathbb{E}_t[\cdot]$ denotes an expectation based on time-t information. Let $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ so that γ is the constant coefficient of relative risk aversion.

The consumption good is a composite made of tradable (c_T) and nontradable (c_N) goods with a constant elasticity of substitution (CES) technology

$$c = \mathcal{C}(c_N, c_T) = \left[\varpi_N c_N^{-\eta} + \varpi_T c_T^{-\eta}\right]^{-\frac{1}{\eta}}$$

where $(\varpi_N, \varpi_T) > 0$, $\varpi_N + \varpi_T = 1$, and $\eta > -1$. The elasticity of substitution between the two goods is constant at $\frac{1}{1+\eta}$. An immediate consequence of these preferences is that the relative price of nontradable goods must satisfy

$$\frac{p_N}{p_T} = \frac{\varpi_N}{\varpi_T}\left(\frac{c_T}{c_N}\right)^{1+\eta} \tag{1}$$

In what follows, we will normalize $p_T = 1$.

Finally, while households live in financial autharky and cannot borrow or save, they receive lump-sum transfers (or taxes) from the government who, as is standard in this literature, borrows on their behalf. Consequently, the representative household's consumption of nontradable goods equals production $c_N = y_N$ and its consumption of tradable goods equals production net of the government transfers

$$c_T = y_T + T$$

Resources In each period, the small open economy receives an endowment $y_T = \exp(z)$ of tradable goods following a stationary Markov process given by

$$z' = \rho_z z + (1-\rho_z)\mu_z + \sigma_z \epsilon'$$

where primes denote next-period variables, $|\rho_z| < 1$, and $\epsilon \overset{iid}{\sim} \mathcal{N}(0,1)$.

The economy also produces y_N units of nontradable goods with labor. Output in the nontraded sector is

$$y_N = F(h) = h^{\alpha}$$

where h are labor hours supplied by the representative household. Firms in the nontraded sector maximize profits taking as given the price of nontraded goods p_N as well as the wage rate w, leading to a demand function for labor

$$p_N F'(h) = w \tag{2}$$

Households supply labor inelastically up to $\bar{h}$. However, following Schmitt-Grohé and Uribe (2016), we consider nominal rigidities in the sense that the wage rate w cannot dip below some lower bound $\bar{w}$. Whenever this constraint binds, labor is rationed and demand-determined by (2) evaluated at $w = \bar{w}$.

By combining the equilibrium relative price of nontradables p_N formula (1) with the labor demand equation (2), we obtain the familiar relation between employment in the nontradable sector and consumption of tradables

$$h \leq \mathcal{H}(c_T, \bar{w}) = \left(\frac{\varpi_N}{\varpi_T}\frac{\alpha}{\bar{w}}\right)^{\frac{1}{1+\alpha\eta}} c_T^{\frac{1+\eta}{1+\alpha\eta}} \tag{3}$$

Government The government chooses issuances of long-term bonds b and holdings of risk-free assets a and uses any proceeds to provide lump-sum transfers to households. The government makes its choices sequentially and without commitment. In particular, the government may choose to default on the debt.

The government issues debt in the form of long-term bonds (Leland, 1994; Hatchondo and Martinez, 2009; Chatterjee and Eyigungor, 2012; Arellano and Ramanarayanan, 2012) which promise a sequence of geometrically-decaying coupons. Specifically, a bond issued in period t promises to repay $(1-\delta)^{j-1}\kappa(\mathbf{s}_{t+j})$ in state $\mathbf{s}_{t+j}$ in period $t+j$, where $\delta \in (0,1)$ is a decay rate and $\kappa(\mathbf{s})$ is a potentially state-contingent coupon rate given by

$$\kappa(\mathbf{s}) = (r+\delta)(\xi + (1-\xi)p_N(\mathbf{s}))$$

We introduce currency of denomination via the parameter $\xi \in [0,1]$. When $\xi = 1$, debt is entirely denominated in units of tradable goods (dollars) and our model reduces to Bianchi and Sosa-Padilla (2020). On the other extreme, when $\xi = 0$, debt is entirely denominated in units of nontradable goods (pesos). In Section 5.2 below, we consider other possibilities for the function $\kappa(\mathbf{s})$ which retain some but not all of the properties of local currency debt. In the baseline case of dollar-denominated debt ($\xi = 1$), we set the coupon rate to $r+\delta$ so that the price of debt is 1 in absence of default.

Reserves are standard and pay one unit of the tradable good. Letting $a, b \geq 0$ denote the government's holdings of reserves and outstanding debt at the beginning of a period (where q_a is the price of reserves), the government's budget constraint is

$$T = \begin{cases} a + q_b(b' - (1-\delta)b) - q_a a' - \kappa(\mathbf{s})b & \text{in repayment} \\ a - q_a a' & \text{in default} \end{cases}$$

where T represent lump-sum transfers to the representative household. We emphasize that, even upon default, the government retains access to its stock of reserves which can be used for consumption (or, potentially, accumulated further). However, while the government remains in default, it is excluded from international capital markets for new borrowing. Default also entails a utility cost which we explain below.

Foreign Lenders The price of bonds is set in a competitive market inhabited by risk-averse foreign lenders. Following Vasicek (1977), we specify the stochastic discount factor of lenders as

$$m(\mathbf{s},\mathbf{s}') = \exp\left(-r - \nu(\psi\epsilon' + 0.5\psi^2\sigma_\epsilon^2)\right)$$

where r is the risk-free rate and $\nu \in \{0,1\}$ is a risk-premium shock with transition probability π_{ij} ($i, j \in \{0,1\}$) which exposes the lenders stochastic discount factors to fluctuations in the small open economy's endowment.

Equations (4) and (5) below reflect that whenever the government defaults, it applies a haircut $\hbar$ to the debt, suspends coupon payments. It also becomes excluded from financial markets and cannot issue new debt or make debt payments. While it remains in the default state, the government may regain market access with a constant exogenous probability θ.

When the government is not in default and issues reserves a' and debt b', the price of debt is

$$q_b(b', a', \mathbf{s}) = \mathbb{E}\left[m(\mathbf{s},\mathbf{s}')\left(\mathbb{1}_{\mathcal{D}'}(1-\hbar)q_b^D(b', a'', \mathbf{s}') + (1-\mathbb{1}_{\mathcal{D}'})(\kappa(\mathbf{s}') + (1-\delta)q_b(b'', a'', \mathbf{s}'))\right)\right] \tag{4}$$

where $\mathbb{1}_{\mathcal{D}}$ is the indicator of default, $\hbar$ is a haircut coefficient and q_b^D is the price of a defaulted bond. Notice that the expectations of future issuances (and holdings of reserves) affect the price today via the expected resale price of bonds.

The price of a defaulted bond is, similarly,

$$\begin{aligned} q_b^D(b', a', \mathbf{s}) = \mathbb{E}\big[m(\mathbf{s},\mathbf{s}')\big(&\theta(1-\mathbb{1}_{\mathcal{D}'})(\kappa(\mathbf{s}') + (1-\delta)q_b(b'', a'', \mathbf{s}')) \\ &+ (1-\theta)q_b^D(b', a'', \mathbf{s}') \\ &+ \theta\mathbb{1}_{\mathcal{D}'}(1-\hbar)q_b^D((1-\hbar)b', a'', \mathbf{s}')\big)\big] \end{aligned} \tag{5}$$

4. Equilibrium and Calibration

The state of the economy is given by the amount of debt b outstanding, the level of reserves a, the (log) tradable endowment z, and the risk aversion shock ν, as well as whether the government is currently in default. We refer to $\mathbf{s} = (z, \nu)$ as the exogenous part of the state vector.

The government acts on behalf of the representative agent. While the government is in repayment, it attains a value of

$$V^R(b, a, \mathbf{s}) = \max_{b', a', c_T, h} u(c_T, F(h)) + \beta \mathbb{E}\left[\mathcal{V}\left(b', a', \mathbf{s}'\right)\right] \tag{6}$$
$$\text{subject to } c_T + q_a a' + \kappa(\mathbf{s}) b = a + y_T(\mathbf{s}) + q_b(b', a', \mathbf{s})(b' - (1-\delta)b)$$
$$h \leq \mathcal{H}(c_T, \bar{w})$$

where the government understands the general-equilibrium effect of consumption on employment (3). In default, the government solves

$$V^D(b, a, \mathbf{s}) = \max_{a', c_T, h} u(c_T, F(h)) - \varphi(y_T) + \beta \mathbb{E}\left[\theta \mathcal{V}(b, a', \mathbf{s}') + (1-\theta) V^D(b, a', \mathbf{s})\right] \tag{7}$$
$$\text{subject to } c_T + q_a a' = a$$
$$h \leq \mathcal{H}(c_T, \bar{w})$$

where $\varphi(y_T)$ are some reduced-form utility costs of default and the expectation takes into account that the probability of returning to international capital markets is $\theta \in (0, 1]$.

Finally, the value of choosing between default or repayment is

$$\mathcal{V}(b, a, \mathbf{s}) = \max\left\{V^R(b, a, \mathbf{s}) + \epsilon_R, V^D((1-\hbar)b, a, \mathbf{s}) + \epsilon_D\right\}$$

where (ϵ_R, ϵ_D) are *iid* preference shocks for repaying and defaulting the debt. We assume that both follow type-I extreme value distributions with scale parameter σ_V, so that the difference $\epsilon = \epsilon_R - \epsilon_D$ has a logistic distribution

$$F(\epsilon) = \frac{\exp(\epsilon/\sigma_V)}{1 + \exp(\epsilon/\sigma_V)}$$

As is standard (see, for example Espino et al., 2020), this assumption results in closed forms to the probability of default and the value function, respectively given by

$$\mathcal{P}(b, a, \mathbf{s}) = \frac{\exp(V^D((1-\hbar)b, a, \mathbf{s})/\sigma_V)}{\exp(V^D((1-\hbar)b, a, \mathbf{s})/\sigma_V) + \exp(V^R(b, a, \mathbf{s})/\sigma_V)} \tag{8}$$

and

$$\mathcal{V}(b, a, \mathbf{s}) = \sigma_V \log\left[\exp\left(V^R(b, a, \mathbf{s})/\sigma_V\right) + \exp\left(V^D((1-\hbar)b, a, \mathbf{s})/\sigma_V\right)\right] \tag{9}$$

Definition 1. *A* Markov-perfect equilibrium *consists of value functions $\mathcal{V}$, V^R, V^D, policy functions ϕ_b, ϕ_a, ϕ_c, a default probability $\mathcal{P}$, and bond-price schedules q_b, q_b^D such that*

1. *given bond prices, the policy functions solve problems (6-7),*

2. *the probability of default and value function satisfy (8-9)*

Parameter	Description	Value
β	Discount factor	0.9
γ	Risk aversion	2.273
$1/(1+\eta)$	Intratemp. elasticity of substitution	0.44
α	Labor share in non-tradables	0.75
$\bar{h}$	Time endowment	1
ρ_z	Autocorrelation of z	0.84
σ_z	Std. dev. of innovations to z	0.045
μ_z	Unconditional mean of z	$-\frac{1}{\sigma_z^2}$
δ	Coupon decay rate	0.2845
$\hbar$	Haircut in case of default	1
θ	Reentry probability	1
r	Risk-free rate	0.04
π_{LH}	Prob. of transitioning to high risk-aversion	0.15
π_{HL}	Prob. of transitioning to low risk-aversion	0.8
ψ	Pricing kernel parameter	15
ϖ_T	Relative share of tradables	0.4
φ_0	Default cost parameter	3.6
φ_1	Default cost parameter	22
$\bar{w}$	Wage rigidity parameter	0.98

Table 1: Baseline parameters

3. *given the policy functions for debt and reserves and the default probability, the bond price schedules satisfy (4-5)*

Our baseline calibration is standard and follows the literature

We follow Bianchi et al. (2018) and assume that the utility cost of default takes the form

$$\varphi(y_T) = \varphi_0 + \varphi_1 \log(y_T)$$

which, as is well-known since Chatterjee and Eyigungor (2012), allows enough flexibility to match the dynamics of the spread.

5. Quantitative results

5.1 *Comparative statics*

Figure 4 shows the government's value function across equilibria with different shares of dollar- and peso-denominated debt. To have a fair comparison, since the level of debt b might mean different indebtedness when ξ changes, we evaluate the value function at zero debt and zero reserves (and the mean level of tradable endowment and low risk aversion).

Price of debt when varying ξ.

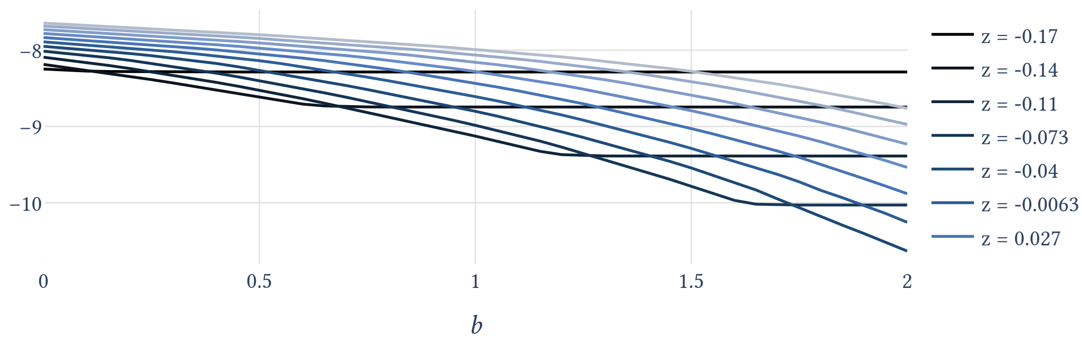

Figure 2: Value function

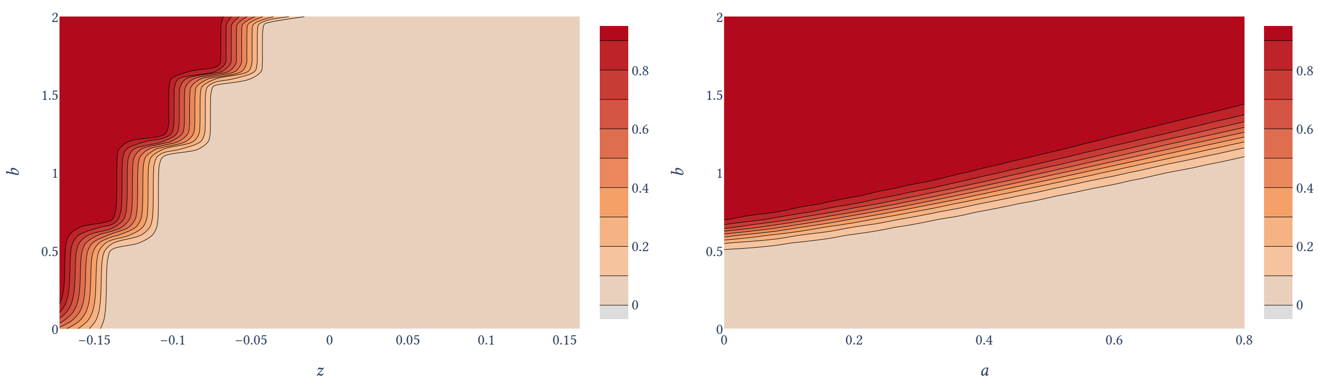

Figure 3: Default probabilities

Figure 4: Value functions for different currency compositions

Figure 5: Debt prices and currency of denomination

Price of nontradables

Figure 6: Price of nontradables

Two uses of reserves with LC debt

5.2 *Synthetic local-currency debt*

To disentangle the advantages of local-currency debt from its potential for ex-post manipulation, we consider the equilibrium of our baseline economy with a state-contingent bond. After solving for the equilibrium when debt is entirely denominated in dollars ($\xi = 1$), we record coupon payments as a function of the state if the debt had been denominated in local currency:

$$\kappa^{synth}(b, a, z, \nu) = (r + \delta) p_N(b, a, z, \nu)$$

We then resolve the model with debt whose coupon rate is now $\kappa^{synth}(\mathbf{s})$ instead of $\kappa(\mathbf{s})$. We refer to this new economy as the model with *fully state-contingent debt*.

In this model, there is still a potential for manipulation of the exchange rate, as the government could use its debt and reserves policies to influence κ^{synth} ex-post. To deal with this, we also define another coupon rate (still at the dollar-denominated debt equilibrium) as those payments made in each state, but when debt and reserves were zero. That is, let

$$\kappa^{exog}(b, a, z, \nu) = (r + \delta) p_N(0, 0, z, \nu)$$

We then resolve the model in the same way and refer to the resulting economy as the model with *exogenously state-contingent debt*.

Table 2 shows statistics for 10000-period simulations of the equilibrium of our model with different types of debt: dollar debt ($\xi = 1$, payments denominated in tradable goods), peso debt ($\xi = 0$, payments denominated in nonntradable goods) as well as the two types of state-contingent debt. It shows the average reserves-to-GDP, debt-to-GDP, interest payments-to-GDP, reserves-to-interest payments, the correlation between consumption and output, the relative volatility of consumption, the default frequency, and the value of the government's value function $\mathcal{V}$.

Table 2 reveals three important facts. First, the government prefers state-contingent debt. These bond structures allow the insurance implicit in peso-denominated debt without the costs from time-inconsistency.

Second, when the government issues peso-denominated debt, although it holds less reserves than with dollar debt, it holds significantly more than with the state-contingent bonds. On the one hand, the insurance diminishes the amount of risk faced by the economy and reduces the desire for reserves. With peso-denominated debt,

Table 2: Comparison of models at baseline parameters with different bond structures

Model	mean a/y	mean b/y	mean ds/y	mean a/ds	corr (c,y)	σ_c/σ_y	default freq	$\mathcal{V}$
Dollar debt	0.0593939	0.187944	0.132074	0.460286	0.754013	0.214547	0.017	-7.99227
Peso debt	0.0319449	0.095705	0.187065	0.171039	0.26348	0.185978	0.028	-7.9795
Exogenous SCI	0.00334786	0.047216	0.13598	0.0241389	0.907407	0.190221	0.013	-7.90971
Full SCI	0.00271302	0.0525733	0.130467	0.0195197	0.922437	0.15255	0.01	-7.85801

reserves become valuable as a way to avoid manipulating the exchange rate, which is why the government holds an order of magnitude higher reserves with peso-debt than with the state-contingent bonds.

Finally, while the level of indebtedness of the economy (measured by the interest expenses-to-output ratio) is broadly similar with state-contingent debt and with dollar debt, it is significantly higher with peso-debt (which exposes the country to more default risk). This ultimately erodes most of the gains from indexation.

6. Concluding remarks

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Table 3: Comparison of models for different parameters

ψ	δ	share dollars	mean a/y	mean b/y	mean a/b	corr c,y	σ_c/σ_y	default freq	v
0.0	0.02845	0.0	0.0230752	0.600873	0.120526	0.789841	0.616268	0.235	-10.4874
0.0	0.02845	0.5	0.0287748	0.639256	0.167965	0.803109	0.545569	0.216	-10.4168
0.0	0.02845	1.0	0.0360758	0.731079	0.25022	0.819536	0.463339	0.2	-10.5713
0.0	0.2845	0.0	0.127683	0.436755	0.687398	0.161639	0.28902	0.034	-8.23124
0.0	0.2845	0.5	0.171318	0.467381	1.0532	0.474824	0.262198	0.042	-8.13418
0.0	0.2845	1.0	0.223161	0.640995	1.73529	0.770908	0.236708	0.049	-8.21042
0.0	0.8	0.0	0.143218	0.371743	0.763846	-0.157586	0.22997	0.023	-7.93435
0.0	0.8	0.5	0.124214	0.37542	0.756872	-0.0788002	0.162211	0.029	-7.92991
0.0	0.8	1.0	0.199983	0.617388	1.70964	0.820723	0.199218	0.029	-7.89053
15.0	0.02845	0.0	0.0416284	0.0885028	0.225951	0.604405	0.242674	0.012	-8.23086
15.0	0.02845	0.5	0.039978	0.0928569	0.246835	0.686545	0.238333	0.017	-8.22085
15.0	0.02845	1.0	0.0514774	0.13899	0.375936	0.850062	0.322003	0.023	-8.32241
15.0	0.2845	0.0	0.0273802	0.0788722	0.145933	0.36983	0.161135	0.025	-7.95509
15.0	0.2845	0.5	0.0362766	0.125913	0.220781	0.425868	0.208714	0.024	-8.01471
15.0	0.2845	1.0	0.0527427	0.174663	0.414793	0.793082	0.21886	0.027	-7.99338
15.0	0.2845	1.0	0.0626772	0.210983	0.48358	0.792366	0.240278	0.027	-8.04693
15.0	0.8	0.0	0.0625004	0.179571	0.335293	0.0228206	0.229681	0.028	-7.97438
15.0	0.8	0.5	0.0911233	0.242833	0.561861	0.203744	0.230015	0.02	-7.97536
15.0	0.8	1.0	0.126919	0.411984	1.05156	0.777702	0.237888	0.015	-7.96694
30.0	0.02845	0.0	0.0375466	0.0369303	0.197794	0.532276	0.167243	0.015	-7.90652
30.0	0.02845	0.5	0.0440975	0.0441448	0.265229	0.663604	0.166993	0.013	-7.88131
30.0	0.02845	1.0	0.0458985	0.050841	0.323134	0.771442	0.194331	0.01	-7.92234
30.0	0.2845	0.0	0.0343766	0.0583306	0.179057	0.412933	0.181884	0.014	-7.9104
30.0	0.2845	0.5	0.0370128	0.0609175	0.221345	0.576983	0.175455	0.014	-7.87472
30.0	0.2845	1.0	0.0469784	0.0886274	0.335344	0.664345	0.220305	0.014	-7.90611
30.0	0.8	0.0	0.0305522	0.0881235	0.15805	0.179677	0.199266	0.016	-7.88467
30.0	0.8	0.5	0.0657825	0.154684	0.395134	0.312359	0.221422	0.019	-7.92495
30.0	0.8	1.0	0.0680905	0.247772	0.537375	0.737032	0.257552	0.014	-7.92346