Uncertainty Premia, Sovereign Default Risk, and State-Contingent Debt

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Why do governments borrow noncontingent?

State-contingent debt instruments

- Decrease default risk
- Reduce cyclicality of fiscal policy
- Improve risk-sharing

Why aren't they used?

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Unfavorable prices of state-contingent instruments

- These instruments are heavily discounted by markets
  - Costa, Chamon, and Ricci (2008) compute wide spreads for Argentine GDP-warrants
    - $\sim$300-400bps from default risk of other securities
    - 600-1200bps residual: 'novelty' premium

This paper proposes a framework that

- Rationalizes pricing of SCI + welfare analysis
  - With ingredients from resolutions of the equity premium puzzle
  - Robustness (Hansen and Sargent, 2001; Pouzo and Presno, 2016)
- Links unfavorable prices to common 'threshold' structure
  - Example: Argentina's GDP-warrants, also Ukraine, Greece. . . More

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Main findings

1. Robust lenders dislike repayment structures with thresholds in good times
   - Heavy discounts for these bonds $\implies$ welfare losses
2. Explain most of the 'novelty premium' in Argentina's GDP warrants as ambiguity premia
   - Calibration of robustness from *noncontingent* debt only
3. Characterize the optimal design and how it changes with robustness
   - With high robustness, want to minimize ex-ante and ex-post contingency

Roadmap

- Stylized Model
- Probability Distortions
- Quantitative Implementation
- Concluding Remarks

Stylized Model

The model

We consider a simple two-period model, small open economy

- Uncertain endowment $y(z)$ in the second period
- The government has access to one asset which promises a return $R(z)$.
- A few benchmarks

Noncontingent debt	$R(z)$	$=$	1
Linear indexing	$R^{\alpha}(z)$	$=$	$1+\alpha(y(z)-1)$
Threshold debt	$R^{\tau}(z)$	$=$	$\mathbb{1}(z>\tau)$
Optimal design	$R^{\star}(z;\theta)$		chosen state-by-state

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The government's problem

- The government takes as given the price schedule $q(b)$

$$
\begin{aligned}
\max_b u(c_1^b) &+ \beta_b \mathbb{E}\left[u(c_2^b)\right] \\
\text{subject to } \; c_1^b &= y_1 + q(b)b \\
c_2^b &= y_2(z) - h(z,\Delta)d(b,z) - (1-d(b,z))R(z)b
\end{aligned}
$$

where

$$
h(z,\Delta) = y_2(z)^2\Delta
$$

The lenders' problem

Foreign lenders are less standard and have multiplier preferences

$$
\begin{aligned}
&\max c_1^L + \beta \frac{1}{-\theta} \log \mathbb{E}\left[\exp(-\theta v_2^L)\right] \\
\text{subject to } \; & v_2^L = c_2^L \\
& c_2^L = w_2 + (1 - d(b,z))R(z)b \\
& c_1^L = w_1 - q_1 b
\end{aligned}
$$

Lenders provide us with an Euler equation to price the debt

$$
\begin{aligned}
q(b;R) &= \beta \mathbb{E}\left[\frac{\exp(-\theta v_2^L)}{\mathbb{E}\left[\exp(-\theta v_2^L)\right]}(1 - d(b,z))R(z)\right] \\
&= \underbrace{\beta \mathbb{E}\left[(1-d)R\right]}_{=q_{RE}} + \underbrace{(1 - \mathbb{P}(d))\operatorname{cov}(\beta M, R)}_{=q_\theta^{\text{cont}}} - \underbrace{\mathbb{E}\left[R\right]\operatorname{cov}(\beta M, d)}_{=-q_\theta^{\text{def}}}
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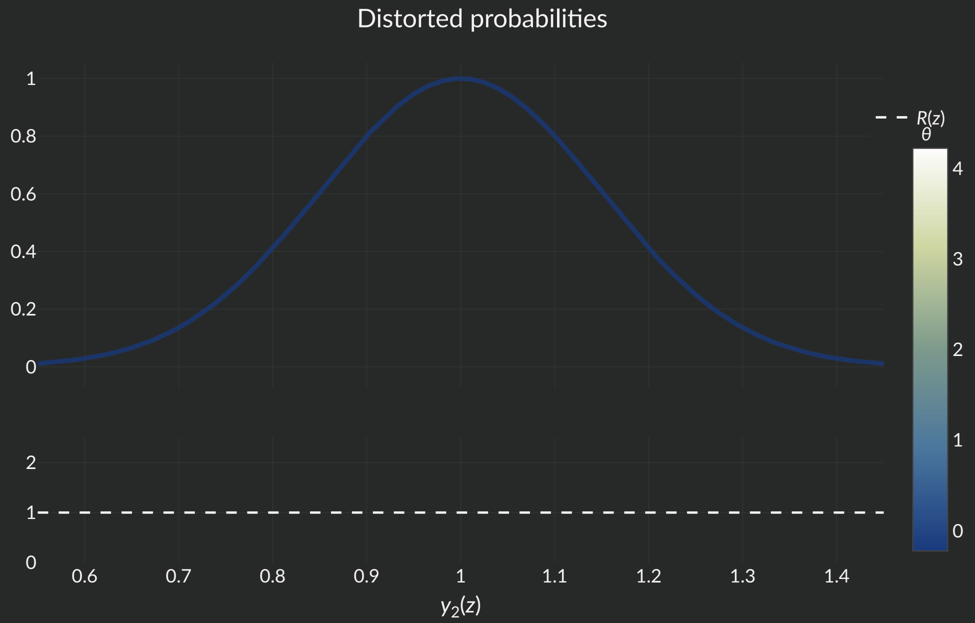
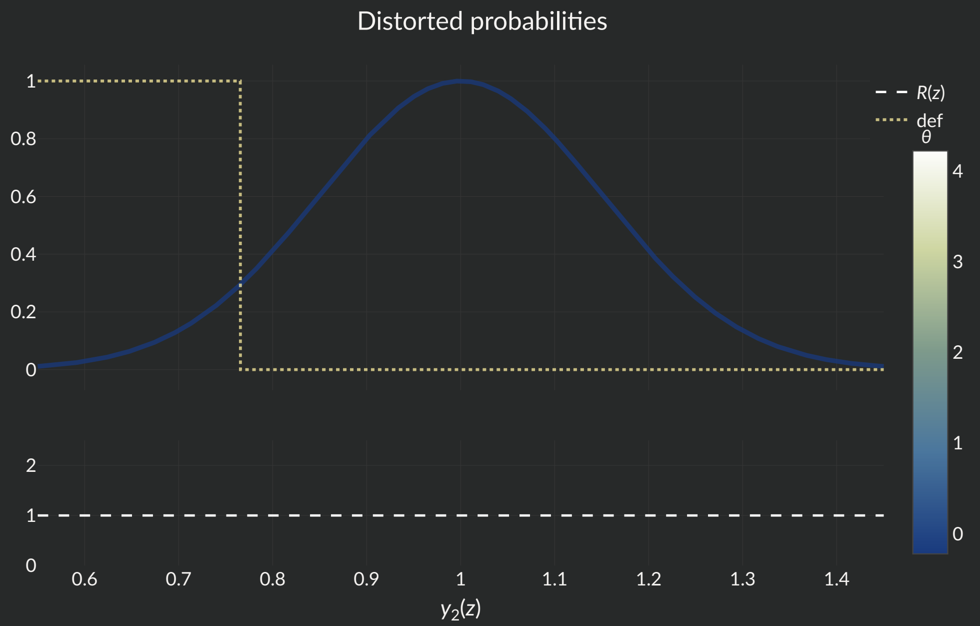
Probability Distortions

Distorted probabilities – noncontingent debt



Distorted probabilities – noncontingent debt

Reoptimizing



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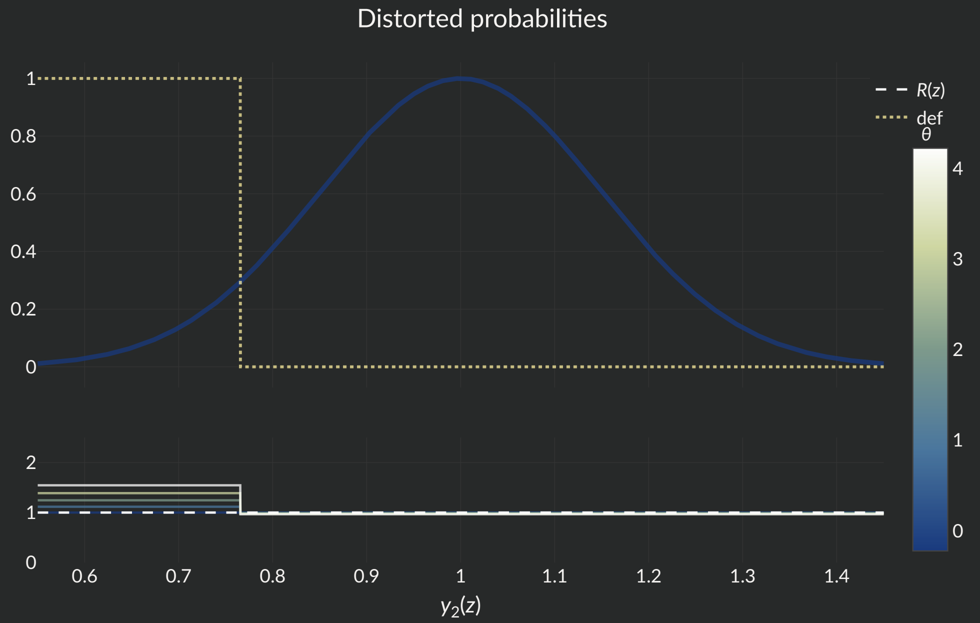

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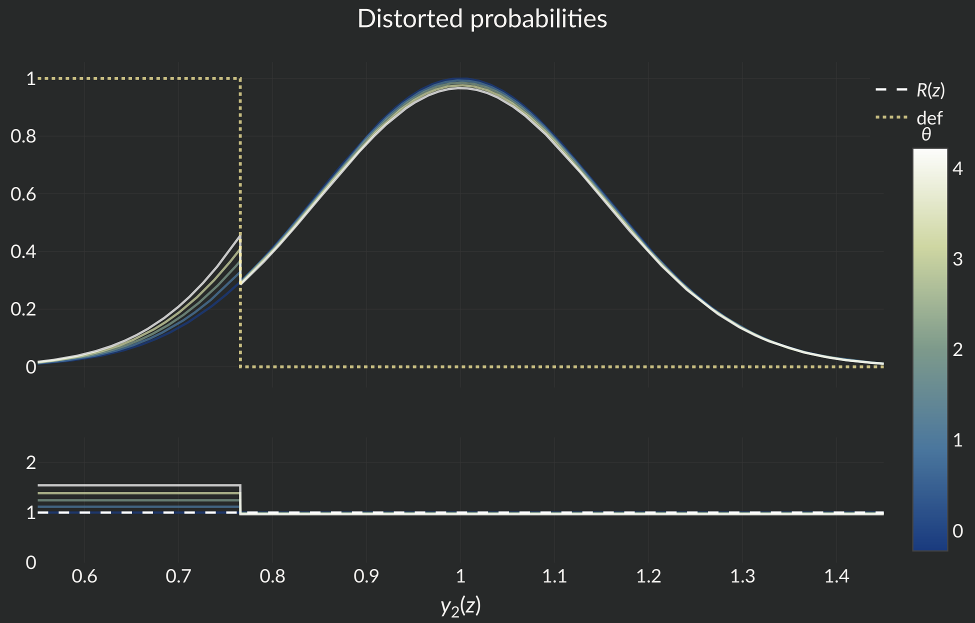
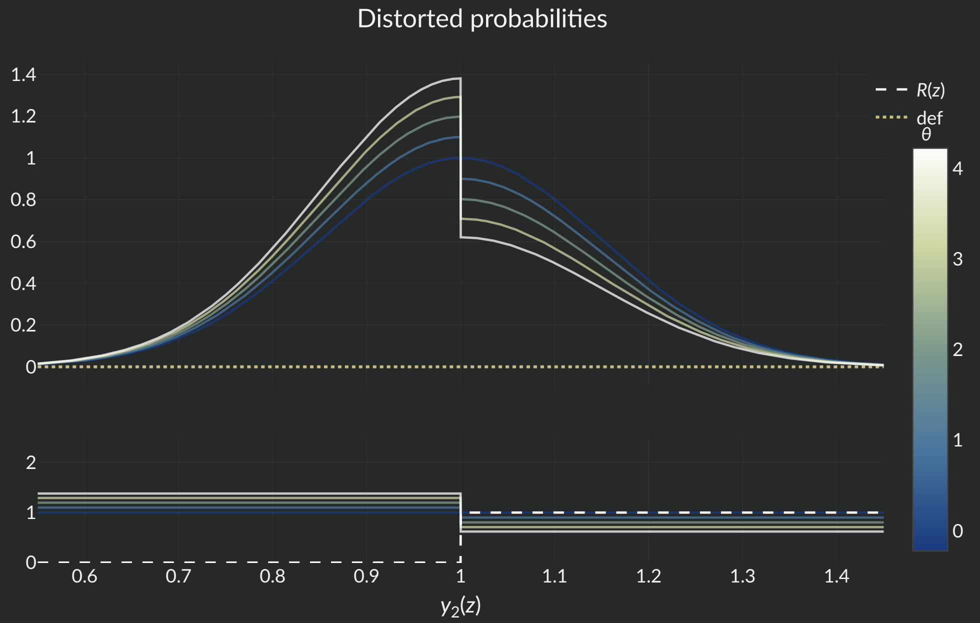
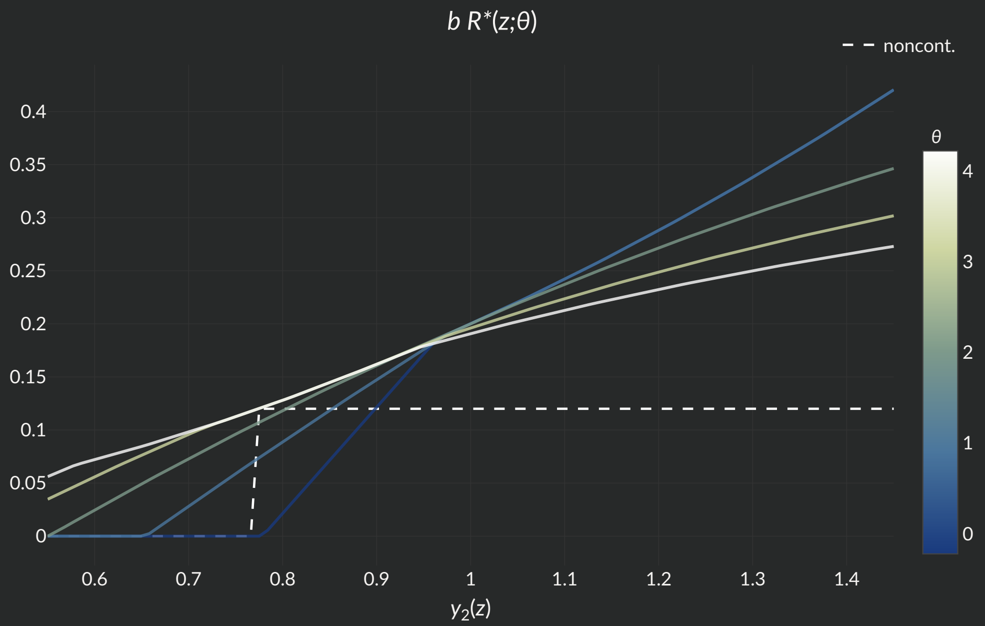


Distorted probabilities – threshold debt

▸ Linear+threshold ▸ Reoptimizing



Design of debt



Quantitative Implementation

Quantitative Model

- Infinite horizon, small-open economy
- Robust lenders as before
- Long-term debt, debt issued at t pays coupon at $t+s$

$$\max\left\{0, (1-\delta)^{s-1}(1+\alpha(y_s-1))\mathbb{1}(y_s>\tau)\right\}$$

  - Noncontingent debt: $\alpha = 0, \tau = -\infty$
- Default triggers exclusion + output costs for a random amount of periods $\sim Geo(\psi)$

Robustness in the quantitative model

	Rational Expectations			Benchmark ($\theta = 2.15$)		
Statistic	Noncontingent	Threshold	$\alpha = 1$	Noncontingent	Threshold	$\alpha = 1$
Spread (bps)	893	318	742	842	1636	746
o/w Spread RE	893	318	742	432	2.6	343
Std Spread	439	133	301	376	238	282
Debt-to-GDP (%)	18.3	32.8	17.8	16.7	18.3	17.5
Std(c)/Std(y)	1.4	0.9	1.4	1.3	0.84	1.3
Default Prob (%)	6.0	1.7	5.6	3.2	0.01	2.7
Welfare Gains	-	0.94%	0.22%	-	-1.1%	0.15%
DEP	-	-	-	40.1%	31.4%	39%

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Optimal design of state-contingent debt

▸ Design

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Price of marginal issuances

In reality issuances of state-contingent bonds are small

- Solve the model with noncontingent debt
- Take the lenders' SDF from that equilibrium
- Use it to price another bond

	Noncontingent bond	Linear bond	Threshold bond	Optimal bond
Benchmark	842	845	947	829
Rational Expectations	893	849	367	634

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Concluding Remarks

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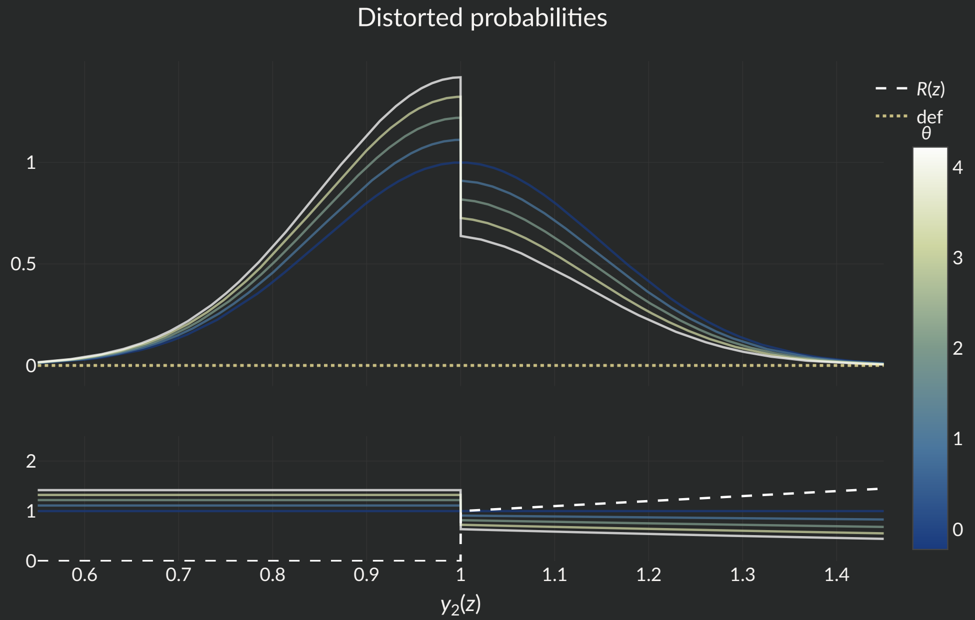
- Standard sovereign debt model augmented with robust lenders
    1. Accounts for spreads on typical threshold SCDIs
    2. Rationalizes part of the ‘novelty’ premium as a premium for ambiguity
    3. Links unfavorable prices to common *threshold* structure
    4. Welfare gains of SCDI decreasing in robustness
        - Both for given instrument and for optimally-designed debt

- Optimal design
    - With realistic robustness, lower thresholds and flatter indexation than RE
    - With extreme robustness, eliminate contingency ex-ante (*stipulated*) and ex-post (*default*)
    - In general, tradeoff between contingency and risk-sharing

Distorted probabilities – threshold+linear debt

◂ Back



CARA

Back

Euler equations of a rational-expectations agent with CARA preferences and access to a risk-free bond

$$q = \beta \mathbb{E}\left[\frac{u'(c_2)}{u'(c_1)} R\right] = \beta \mathbb{E}\left[\frac{\exp(-\gamma c_2)}{\exp(-\gamma c_1)} R\right]$$

$$\frac{1}{1+r} = \beta \mathbb{E}\left[\frac{u'(c_2)}{u'(c_1)}\right]$$

hence

$$q = \beta \mathbb{E}\left[\frac{\exp(-\gamma c_2)}{\beta(1+r)\mathbb{E}\left[\exp(-\gamma c_2)\right]} R\right]$$

Same as robustness in two periods, in general the robust sdf is

$$q = \beta \mathbb{E}\left[\frac{\exp(-\theta v')}{\mathbb{E}\left[\exp(-\theta v')\right]} R\right]$$

Multiplier preferences

In general,

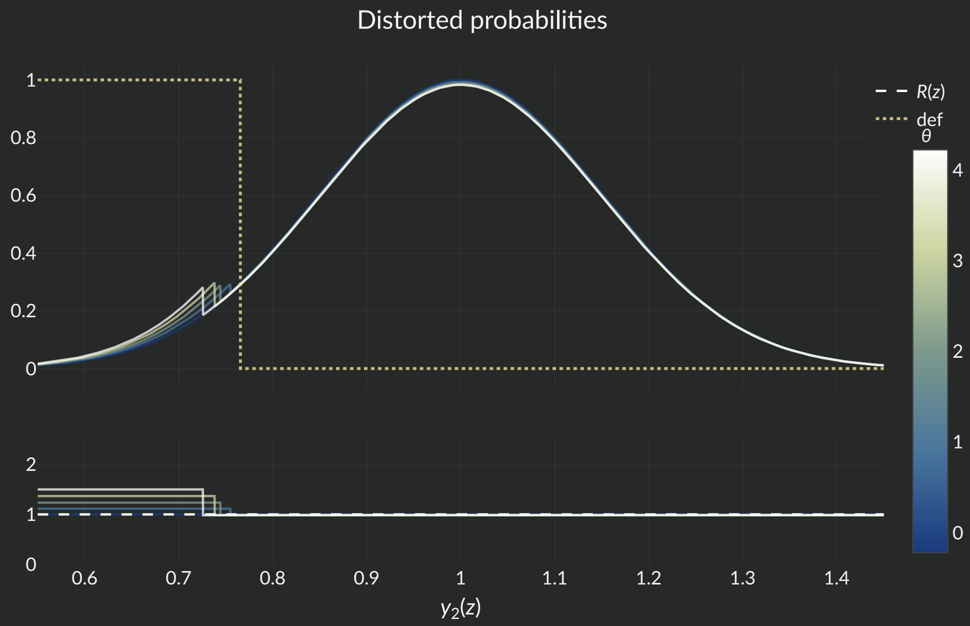
$$\min_{\tilde{p}} \max_{c} u(c) + \beta \int v(a')dp + \frac{1}{\theta}\text{ent}(p, \tilde{p})$$

turns into

$$\max_{c} u(c) - \frac{\beta}{\theta} \log\left(\mathbb{E}\left[\exp(-\theta v(a'))\right]\right)$$

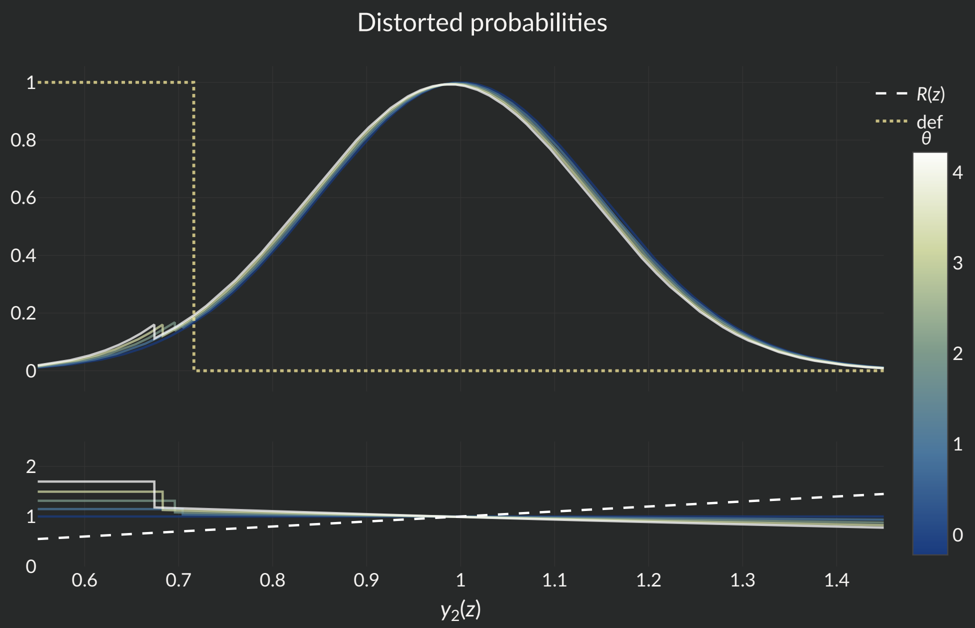
Distorted probabilities – noncontingent debt

◂ Back



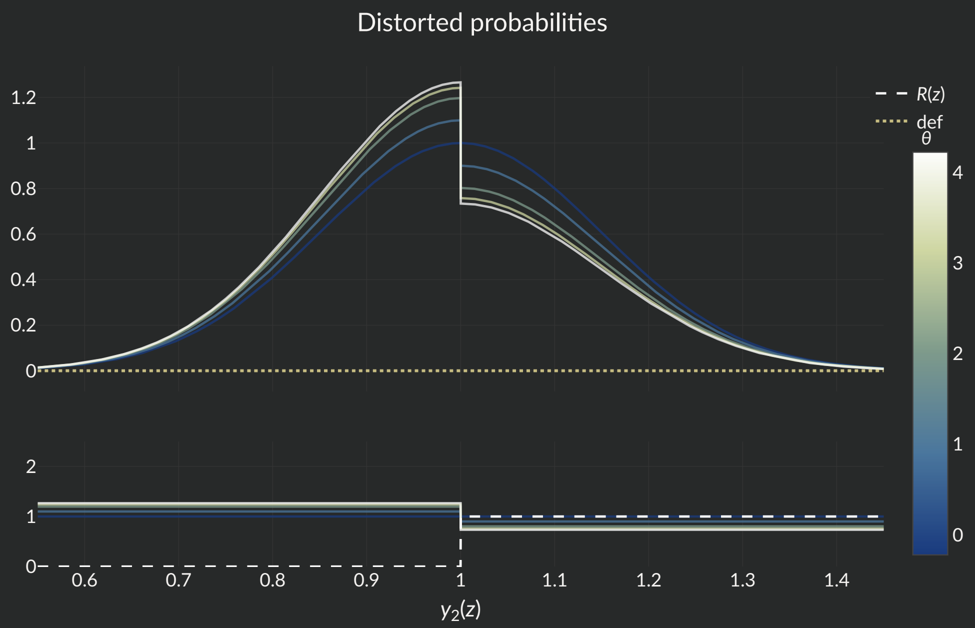
Distorted probabilities – linearly indexed debt

◂ Back



Distorted probabilities – threshold debt

Back



Parametrization

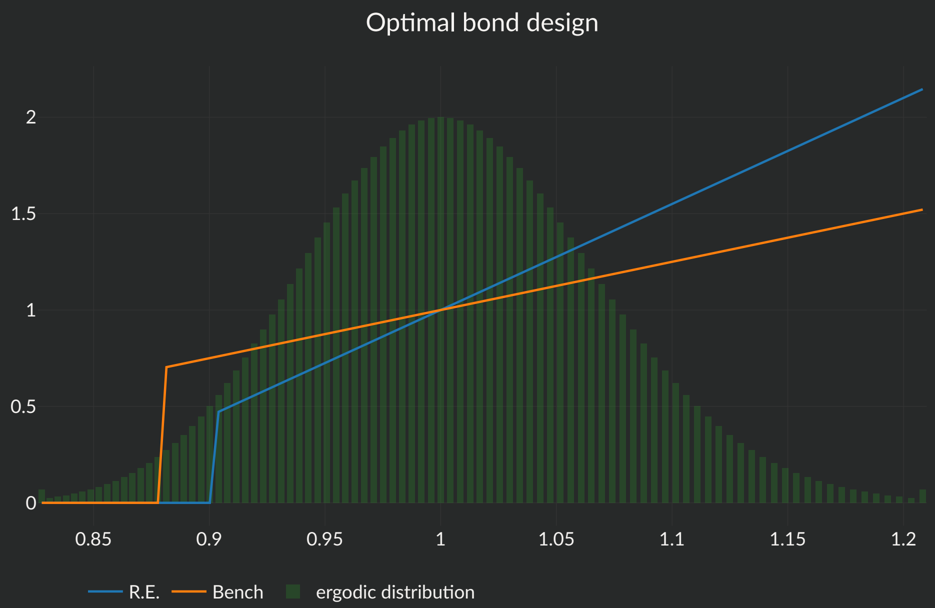
Back

We represent this bond with threshold debt, one period = five years, and

Parameter	Target	Value
β_b	Borrower's discount rate	6% ann.
β	Risk-free rate	3% ann.
γ	Borrower's risk aversion	2
Δ	Output cost of default	20%
g	Expected growth rate	8% ann.
k	Threshold for repayment	50%

Optimal bond design

Back



Decomposition of spreads

◂ Back

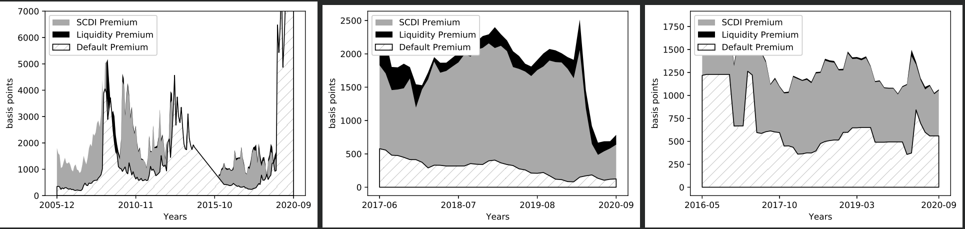


Figure 1: GDP-linked security premia.

The figure shows the estimated spread decomposition in Igan and Kim (2021) for the GDP-warrants issued by Argentina (left), Greece (middle) and Ukraine (right).