**CSC148 term test #2, L0101/L0301**

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Three questions:

Q1: / 5

Q2: / 6

Q3: / 9

1. **(5 pts)** Consider a post-order traversal, i.e. visiting each node in post-order, of a **Binary Search Tree,** with the act function defined as;

```
def act(node):
    print(node.value, end=' ') # prints all on the same line
```

Recall that for a BST, a postorder traversal first visits the left subtree in postorder, then the right subtree in postorder, and finally the root.

Draw a representation of a Binary Search Tree that would produce the following output when traversed as described above.

```
15 10 25 20 50 45 33
```

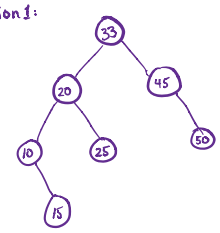
**Hint:** Recall that both postorder traversals and the Binary Search Tree property are defined recursively, so that they apply at the root of the main tree and of each subtree...

**sample solution:** Below are solutions for all three test versions (9/10 a.m., 1 p.m., 6 p.m.)

General question:
Given a post-order traversal of a BST, draw the tree.
Hints: give definition of postorder

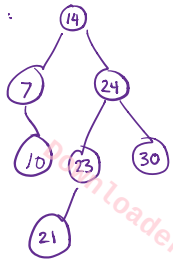
each tree should have 7 nodes, with no left leaf of leftmost interior node.

Version 1:



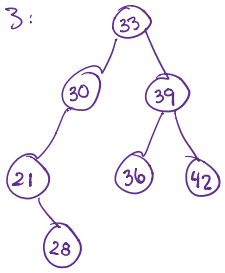

15 10 25 20 50 45 33

Version 2:




10 7 21 23 30 24 14

Version 3:




28 21 30 36 42 39 33

Use the recursive definitions of both postorder traversals and the BST property to help.

2. **(6 pts)** Read the definition of class **Tree** below, from module **tree**. **Notice** that a Tree's children is a list of 0 or more Tree objects, and does not contain any None objects. Also, the only functions or methods you may rely are on are below.

   Implement function **count_at_depth** on the next page. You may implement helper functions, if you wish, or implement it as one function.

```
class Tree:
    """
    A bare-bones Tree ADT that identifies the root with the entire tree.
    """

    def __init__(self, value=None, children=None):
        """
        Create Tree self with content value and 0 or more children

        @param Tree self: this tree
        @param object value: value contained in this tree
        @param list[Tree] children: possibly-empty list of children
        @rtype: None
        """
        self.value = value
        # copy children if not None
        self.children = children.copy() if children is not None else []


    def __str__(self, indent=0):
        """
        Produce a user-friendly string representation of Tree self,
        indenting each level as a visual clue.

        @param Tree self: this tree
        @param int indent: amount to indent each level of tree
        @rtype: str

        >>> t = Tree(17)
        >>> print(t)
        17
        >>> t1 = Tree(19, [t, Tree(23)])
        >>> print(t1)
        19
           17
           23
        >>> t3 = Tree(29, [Tree(31), t1])
        >>> print(t3)
        29
           31
           19
              17
              23
        """
        root_str = indent * " " + str(self.value)
        return '\n'.join([root_str] +
                         [c.__str__(indent + 3) for c in self.children])
```

```
from tree import Tree


def count_at_depth(t, d):
    """ Return the number of nodes at depth d of t.

    @param Tree t: tree to explore --- cannot be None
    @param int d: depth to report from, non-negative
    @rtype: int

    >>> t = Tree(17, [Tree(0), Tree(1, [Tree(4)]), Tree(2, [Tree(5)]), Tree(3)])
    >>> print(t)
    17
       0
       1
          4
       2
          5
       3
    >>> count_at_depth(t, 0)
    1
    >>> count_at_depth(t, 1)
    4
    >>> count_at_depth(t, 2)
    2
    >>> count_at_depth(t, 5)
    0
    """
```



# **Hint**: Any node that is at depth **d** from **t** is at depth **d-1** from **t**'s children.

**sample solution(s)**: All three versions below.



```
    if d < 0:
        return 0
    elif d == 0:
        return 1
    else:
        return sum([count_at_depth(c, d - 1)
                    for c in t.children
                    if c is not None])
```

```
def sum_at_depth(t, d):
    """ Return the sum of node values at depth d of t.

    Assume that node values are integers and that there are no
    None values in any list of children in t or its descendants.

    @param Tree t: tree to explore, cannot be None
```

```
    @param int d: depth to report from, non-negative
    @rtype: int

    >>> t = Tree(17, [Tree(0), Tree(1, [Tree(4)]), Tree(2, [Tree(5)]), Tree(3)])
    >>> print(t)
    17
       0
       1
          4
       2
          5
       3
    >>> sum_at_depth(t, 0)
    17
    >>> sum_at_depth(t, 1)
    6
    >>> sum_at_depth(t, 2)
    9
    >>> count_at_depth(t, 5)
    0
    """
    if d == 0:
        return t.value
    else:
        return sum([sum_at_depth(c, d - 1)
                    for c in t.children
                    if c is not None])


def concatenate_at_depth(t, d):
    """ Return the concatenation of node values at depth d of t.

    Assume that node values are strings and that there are no
    None values in any list of children in t or its descendants.

    @param Tree t: tree to explore, cannot be None
    @param int d: depth to report from, non-negative
    @rtype: str

    >>> t = Tree("a", [Tree("b"), Tree("c", [Tree("d")]), Tree("e", [Tree("f")]), Tree("g")])
    >>> print(t)
    a
       b
       c
          d
       e
          f
       g
    >>> concatenate_at_depth(t, 0)
    'a'
    >>> concatenate_at_depth(t, 1)
    'bceg'
    >>> concatenate_at_depth(t, 2)
    'df'
    >>> concatenate_at_depth(t, 5)
    ''
    """
    if d == 0:
```

```
    return t.value
else:
    return "".join([concatenate_at_depth(c, d - 1)
                for c in t.children
                if c is not None])
```

3. **(9 pts)** Read the declaration of the **LinkedList** and **LinkedListNode** classes below, from module **node**. **Notice** that we use **property** and **_get_value** to make sure the values of these LinkedListNodes are immutable: they cannot be changed after initialization!

   On page 7 implement the function **reverse_list**. You may create new local names (variables) to refer to existing nodes (if you need to), **but** you may **not** create any new objects (LinkedLists, LinkedListNodes, or Python lists, etc.).

```
class LinkedListNode:
    """
    Node to be used in linked list
    === Attributes ===
    @param LinkedListNode next_: successor to this LinkedListNode
    @param object value: data this LinkedListNode represents
    """
    def __init__(self, value, next_=None):
        """
        Create LinkedListNode self with data value and successor next_.

        @param LinkedListNode self: this LinkedListNode
        @param object value: data of this linked list node
        @param LinkedListNode|None next_: successor to this LinkedListNode.
        @rtype: None
        """
        self._value, self.next_ = value, next_

    def _get_value(self):
        # to show value
        return self._value
    # no way to set value!
    value = property(_get_value)

    def __str__(self):
        """
        Return a user-friendly representation of this LinkedListNode.

        @param LinkedListNode self: this LinkedListNode
        @rtype: str

        >>> n = LinkedListNode(5, LinkedListNode(7))
        >>> print(n)
        5 -> 7 ->|
        """
        s = "{} ->".format(self.value)
        current_node = self.next_
        while current_node is not None:
            s += " {} ->".format(current_node.value)
            current_node = current_node.next_
        assert current_node is None, "unexpected non_None!!!"
        s += "|"
        return s
```

```
class LinkedList:
    """
    Collection of LinkedListNodes
    === Attributes ==
    @param: LinkedListNode front: first node of this LinkedList
    @param LinkedListNode back: last node of this LinkedList
    @param int size: number of nodes in this LinkedList
                        a non-negative integer
    """
    def __init__(self):
        """
        Create an empty linked list.

        @param LinkedList self: this LinkedList
        @rtype: None
        """
        self.front, self.back, self.size = None, None, 0

    def __str__(self):
        """
        Return a human-friendly string representation of
        LinkedList self.

        @param LinkedList self: this LinkedList

        >>> lnk = LinkedList()
        >>> print(lnk)
        I'm so empty...
        """
        if self.front is None:
            assert self.back is None and self.size is 0, "ooooops!"
            return "I'm so empty..."
        else:
            return str(self.front)

    def prepend(self, value):
        """
        Insert value before LinkedList self.front.

        @param LinkedList self: this LinkedList
        @param object value: value for new LinkedList.front
        @rtype: None

        >>> lnk = LinkedList()
        >>> lnk.prepend(0)
        >>> lnk.prepend(1)
        >>> lnk.prepend(2)
        >>> str(lnk.front)
        '2 -> 1 -> 0 ->|'
        >>> lnk.size
        3
        """
        new_node = LinkedListNode(value, self.front)
        self.front = new_node
        if self.size == 0:
            self.back = new_node
        self.size += 1
```

```
from node import LinkedList, LinkedListNode


def reverse_list(list_):
    """ Reverse the order of the nodes in list_.

    @param list_ LinkedList: linked list to modify
    @rtype: None

    >>> lnk = LinkedList()
    >>> lnk.prepend(1)
    >>> lnk.prepend(3)
    >>> lnk.prepend(5)
    >>> print(lnk)
    5 -> 3 -> 1 ->|
    >>> reverse_list(lnk)
    >>> print(lnk)
    1 -> 3 -> 5 ->|
    """

    # Hint: draw pictures.
```



**sample solution(s):** All three versions below...

```
def reverse_list(list_, arg=None):
    current = list_.front
    prev = None
    tail = list_.front
    while current:
        next_ = current.next_
        current.next_ = prev
        prev = current
        current = next_
        # Or in one line:
        # current.next_, current, prev = prev, current.next_, current

    list_.front, list_.back = prev, tail


def reverse_list_to_value(list_, value):
    """ Does not update size, discards other nodes. """
    current = list_.front
    prev = None
    tail = list_.front

    while current and (prev is None or prev.value != value):
        next_ = current.next_
        current.next_ = prev
        prev = current
        current = next_
        # Or in one line:
        # current.next_, current, prev = prev, current.next_, current
```

```
    list_.front, list_.back = prev, tail


def reverse_list_after_value(list_, value):
    """ Does not update size, discards other nodes. """
    current = list_.front

    while current and current.value != value:
        current = current.next_

    if current:
        prev = None
        tail = current
        while current:
            next_ = current.next_
            current.next_ = prev
            prev = current
            current = next_
            # Or in one line:
            # current.next_, current, prev = prev, current.next_, current

        list_.front, list_.back = prev, tail
```

This page is available for answers that don't fit elsewhere.