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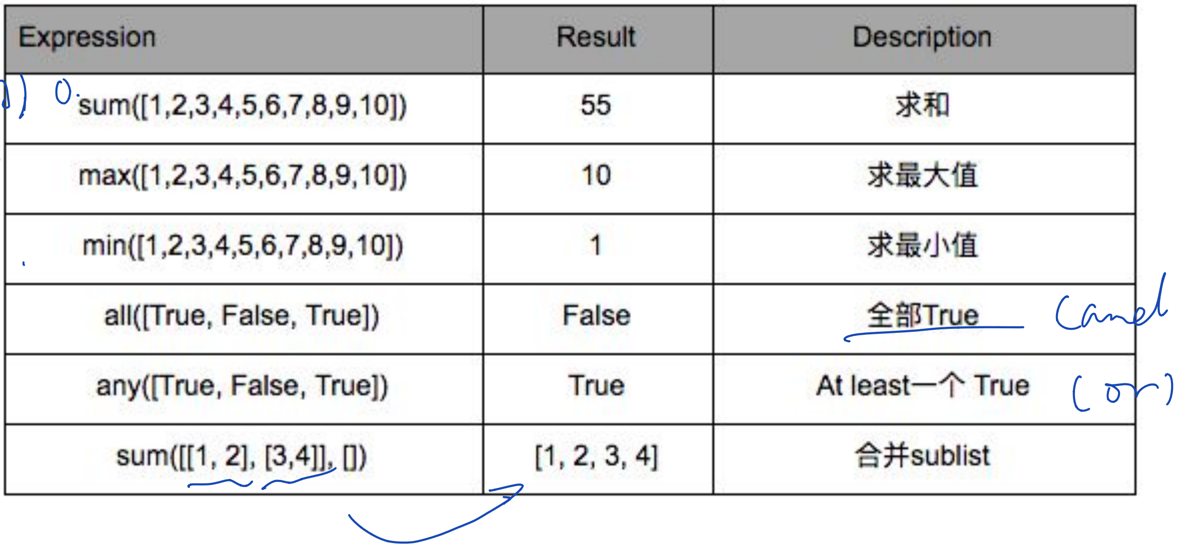
Section 1. Coding Strategy

1. Build-in functions for iterables.



Expression	Result	Description
sum([1,2,3,4,5,6,7,8,9,10])	55	求和
max([1,2,3,4,5,6,7,8,9,10])	10	求最大值
min([1,2,3,4,5,6,7,8,9,10])	1	求最小值
all([True, False, True])	False	全部True
any([True, False, True])	True	At least一个 True
sum([[1, 2], [3,4]], [])	[1, 2, 3, 4]	合并sublist

Sum([]) 0.

不能是空的

(and)

(or).

Warning： Some of them do not work on some types of elements!
For example,You cannot get the sum of a list of string.

2. Type checking.

```
isinstance(x, A) :
   isinstance(object, class-or-type-or-tuple) -> bool

                    检查 x 是否是type A.
   Useful types: int, str, list, tuple, dict, etc……
   Example:
   >>> isinstance(123, int)
   True
```

2. List Comprehension.

Use one line of code to accomplish the construction of new list

G派心法: 表达式在先， 大家都来做， 要是有如果， 符合才通过， 后面接或者，
去做其他活儿，Loop接着放，定义好变量， 筛选很特殊， 条件最后讲！

a) No condition

[表达式 for loop]

b) Filter

[表达式 for loop if 条件]

c) If and else

[表达式 if 条件 else 其他活 for loop]

Example：

```
>>> lst = [1,2,3,4,5,6]
>>> [a for a in lst if a%2 == 0]   # get all even numbers
[2, 4, 6]
>>> lst = [1,2,3,4,5,6]
>>> [1 if a%2==0 else -1 for a in lst]
[-1, 1, -1, 1, -1, 1]
```

Question 1: use one line of code, get the sum of lst

```
>>> lst = [[1,2],[3,4],[5,6]]
>>> sum([sum(L) for L in lst]).
```

Question 2: use one line of code, get a list with all letters in uppercase

```
>>> lst = ['a', 'd', 'C', 'f', 'q', 'A']
>>> [c.upper() for c in lst]
```

Question 3: all string elements wanted!

```
>>> lst = ['e', 'z', 13, '4', 77, (66, 8)]
[item for item in lst if isinstance(item,
                                      str)].
```

Section 2. Recursion

Recursion是CSC148的核心内容，中文含义是递归，意指程序在运行当中调用自身，来完成一些复杂和庞大的计算模型。

Let’s compute $\sum_{n=1}^{50} n$

$$= 50 + \sum_{n=1}^{49} n.$$

$$= 50 + 49 + \cdots + \boxed{\sum_{n=1}^{1} n.} \quad 1.$$

$$\sum_{n=a}^{b} n = \begin{cases} a & a=b. \quad \text{\#Base Case,} \\ b + \boxed{\sum_{n=a}^{b-1}} n. & a<b \quad \text{\#Recursive} \end{cases}$$

Recursion 三大要素：

1. Function的作用：根据docstring 搞清function的具体作用, 在写body的时候assume function已经写好啦. input, output

2. Base Case：一般来说是各种数据类型的最小值,
   比如integer的一般为0或1.
   String的一般是empty string, nested list的一般是非list或者empty list.
   Tree: leaf   BT: None.

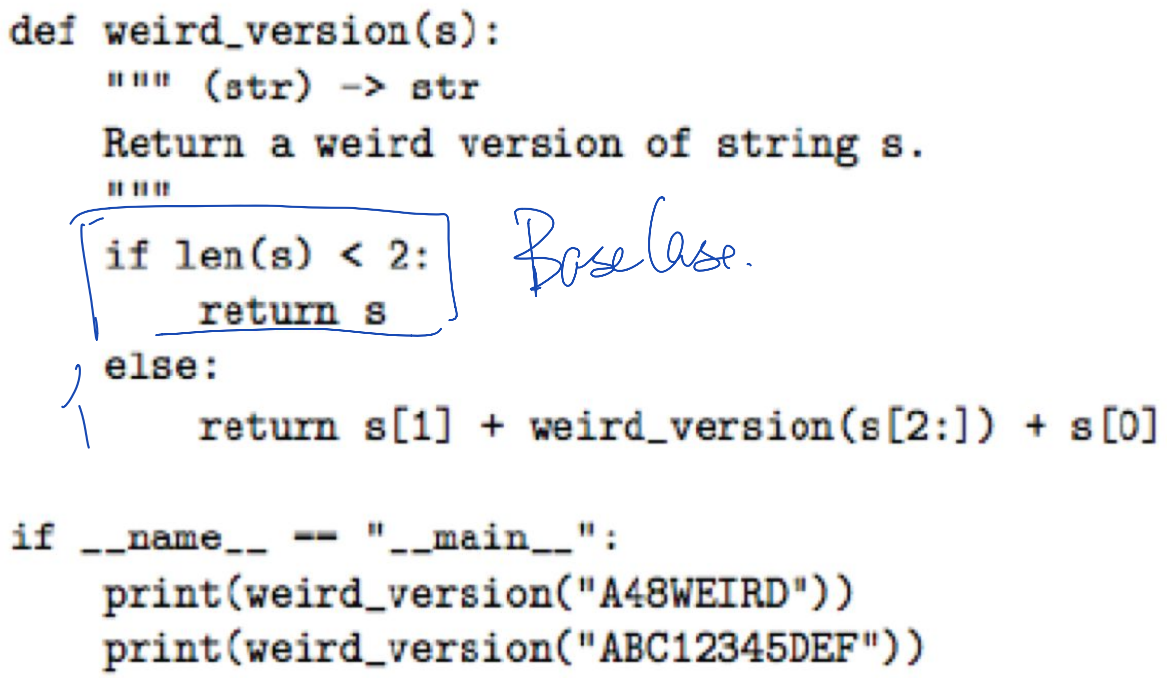
3. Recursive step:

3.1 Divide step： 搞清楚是如何把大问题化小，是如何把当前问题分解成一个相对较小的问题.   partial solution

3.2 Combination step： 搞清楚是如何把子问题的结果组合成原问题的结果.

G派心法: BaseCase先找到， 所有问题都需要，
recursive别多想， step by step找诀窍.

Trace recursion or Compute the result



```
def weird_version(s):
    """ (str) -> str
    Return a weird version of string s.
    """
    if len(s) < 2:
        return s
    else:
        return s[1] + weird_version(s[2:]) + s[0]

if __name__ == "__main__":
    print(weird_version("A48WEIRD"))
    print(weird_version("ABC12345DEF"))
```

Base Case.

4 + weird_version('8WEIRD') + A

4 W I D R, E 8 A

B 1 3 5 E F D 4 2 C A.

Part (b) [3 MARKS]

Suppose we made a new version of the function weird called weirdplus, where we changed the line
`if len(s) < 2:`
to
`if len(s) < 3:`
What can we say about a string s if we know that weird(s) != weirdplus(s)?

len(s) % 2 == 0

Nested List recursion

Nested list recursion是我们在148接触的第一种recursion的考点, 每一个nested list的recursion问题简单来说可以分为两种case:

1. 当前非list的情况.
2. 当前依然是list的情况.

[□, ☆, △]
0 0 0

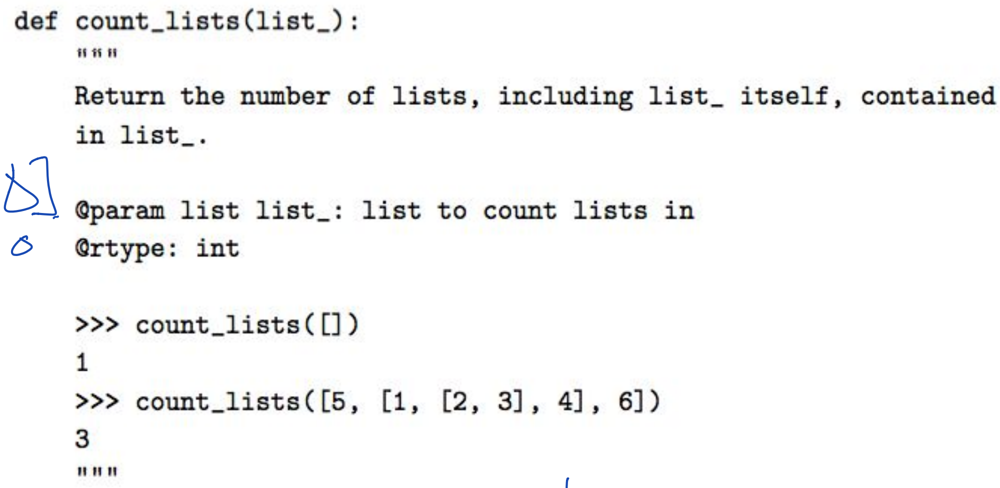
```
def count_items(lst):
    """Return the number of items in the nested list.
    """
    if not isinstance(lst, list):
            return 1.

    else:   acc= 0
            for sublist in lst:
                acc += count_items(sublist)
            return acc.

def flatten_items(lst):
    """Return the number of items in the nested list.
    """
    if not isinstance(lst, list):
        return [lst]
    else:
        acc= []
        for s in lst:
            acc += flatten_items(s)
    return acc
```

在recursion问题中, accumulation (combination)环节会根据题上不同要求存在不同的方法，例如求depth, 每次是取sub problem 的max 加1, 求总和是直接求和或者求 sum etc

[□, ☆, △]
8 7 6.

[[]]

```
def depth(lst):
    “””Return the depth of the nested list.
    “””
    if not isinstance(lst, list):
        return 0.

    else:
        acc=[ 0 ]
        for s in lst:
            acc.append(depth(s).
        return max(acc) + 1.
```

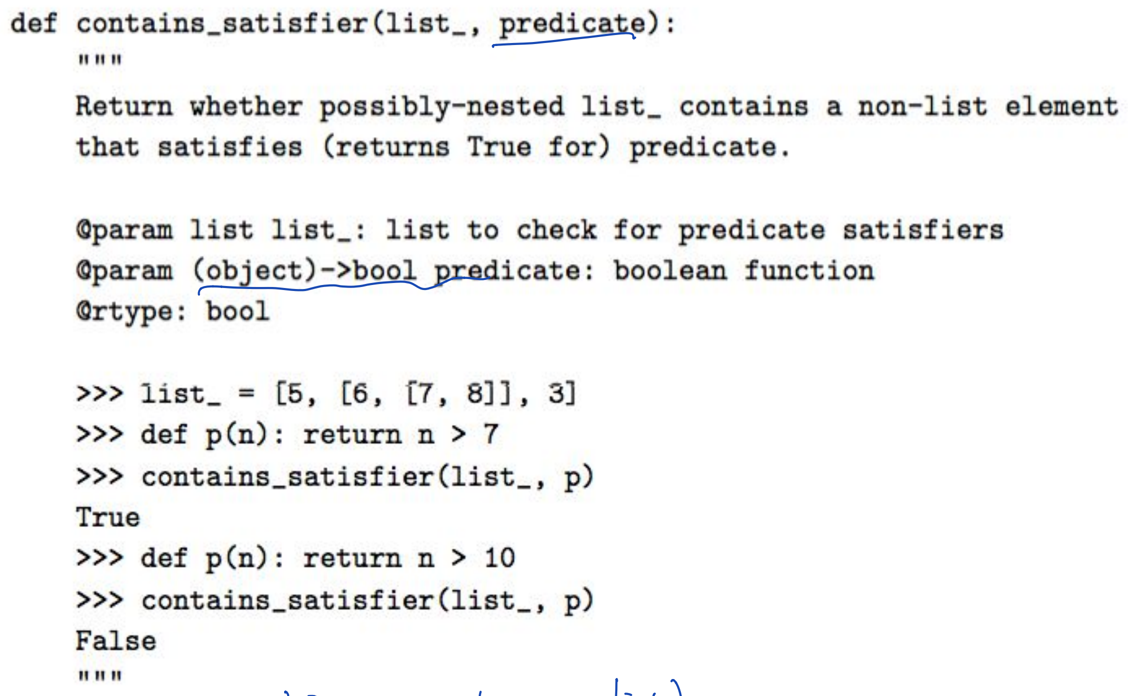


```
def count_lists(list_):
    """
    Return the number of lists, including list_ itself, contained
    in list_.

    @param list list_: list to count lists in
    @rtype: int

    >>> count_lists([])
    1
    >>> count_lists([5, [1, [2, 3], 4], 6])
    3
    """
```

[□, ☆, △]
△. △ ○

```
    if not isinstance(list_, list):
        return 0

    else:
        acc= 1.
        for s in list_:
            acc += count_lists(s)

        return acc
```

常见 Recursion 问题中 可能存在filter condition来控制base case的结果. 对于这种类型, 我们基本上可以总结成只需要在base case中进行操作



```
def contains_satisfier(list_, predicate):
    """
    Return whether possibly-nested list_ contains a non-list element
    that satisfies (returns True for) predicate.

    @param list list_: list to check for predicate satisfiers
    @param (object)->bool predicate: boolean function
    @rtype: bool

    >>> list_ = [5, [6, [7, 8]], 3]
    >>> def p(n): return n > 7
    >>> contains_satisfier(list_, p)
    True
    >>> def p(n): return n > 10
    >>> contains_satisfier(list_, p)
    False
    """
```

```
    if not isinstane(list_, list):
        return predicate(list_).

    else:
        for s in list_:
            if contains_satisfier(s, predicate):
                return True.
        return False.
```

更多**门神**喜欢的 Recursion 练习:
(要注意使用我们之前提到的三步法呦.)

Referring to the join method for str in python.
>>> str.join('+', ['a', 'b', 'c'])
'a+b+c'

Complete the following function

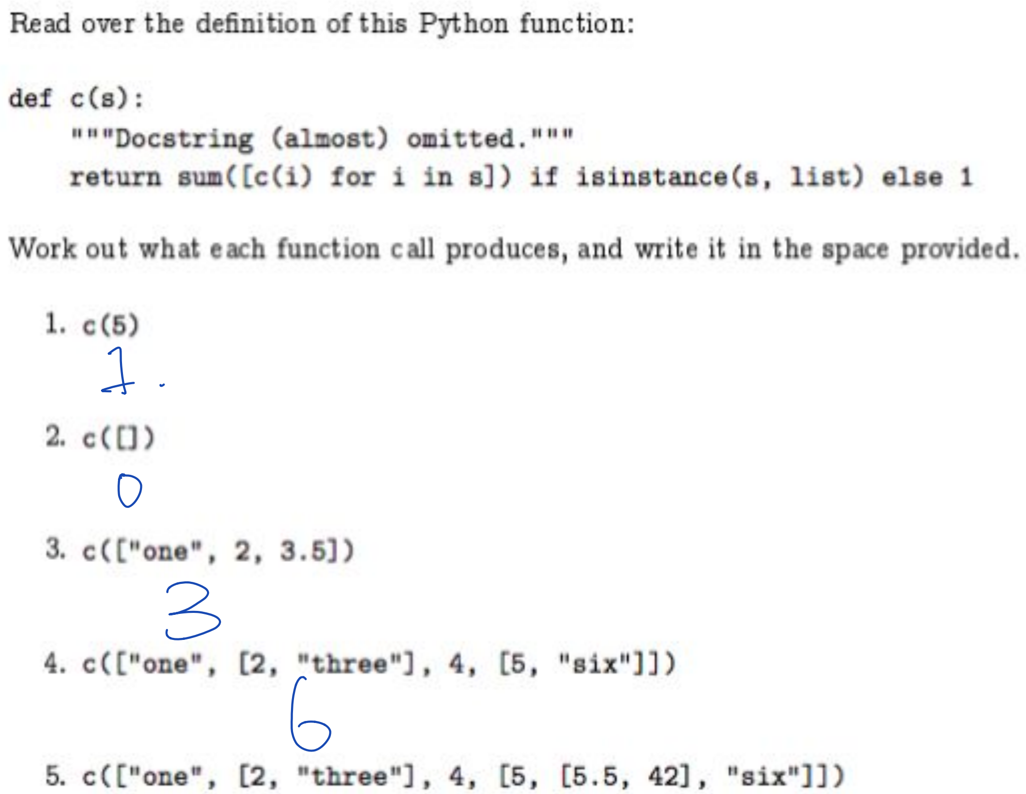
```
def nested_join(s: str, L: list) -> str:
    """ Return the join of nested list of strings L with separator s.

    >>> nested_join(' ', ['hello', ['my', 'boy']])
    'hello my boy'
    """
    if not isinstance(L, list):
        return L.
    else:
        acc = []
        for sublist in L:
            acc.append(nested_join(S, sublist))

        return str.join(S, acc)
```

```
    s  s
[□, ☆, △]
```

lab3(4)

```
def print_by_level(L):
    """ Print all items in the nested list level by level.
    """
    q = [L].
    while len(q) > 0:
        temp = q.pop(0).
        if not isinstance(temp, list):
            print(temp).
        else: q.extend(temp).
```

[[2, 3], 4, [5]]



Read over the definition of this Python function:

```
def c(s):
    """Docstring (almost) omitted."""
    return sum([c(i) for i in s]) if isinstance(s, list) else 1
```

Work out what each function call produces, and write it in the space provided.

1. `c(5)`

   1 .

2. `c([])`

   0

3. `c(["one", 2, 3.5])`

   3

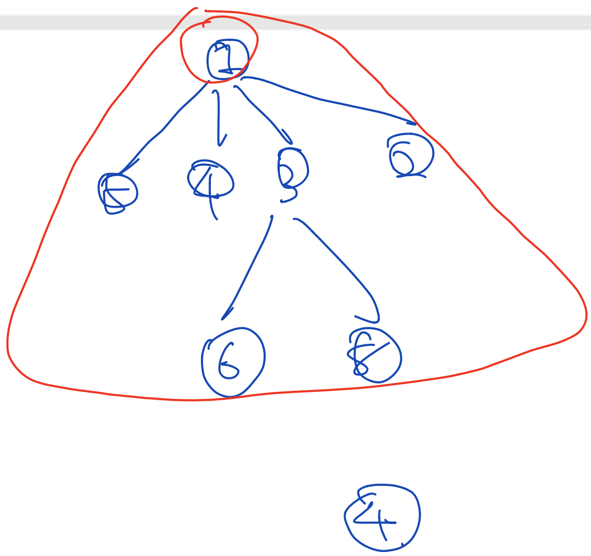
4. `c(["one", [2, "three"], 4, [5, "six"]])`

   6

5. `c(["one", [2, "three"], 4, [5, [5.5, 42], "six"]])`

   8

Section 3. Tree

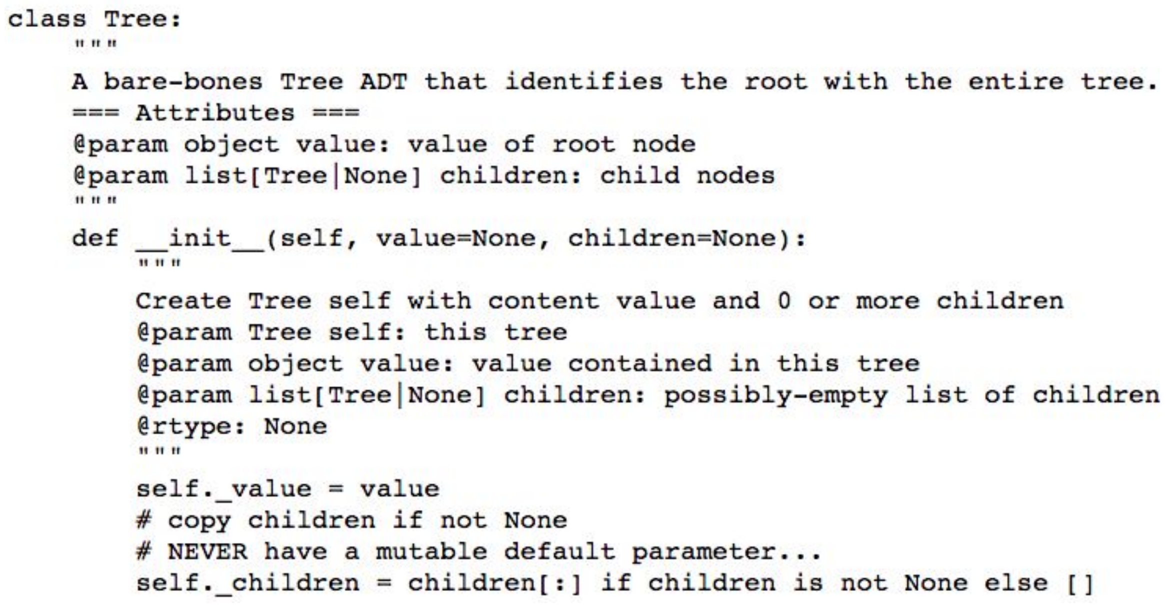


对于Tree有以下几点概念要搞清

- root
- leaf
- internal nodes

- value
- children
- height
- depth
- arity

Tree Node 的implementation



```
class Tree:
    """
    A bare-bones Tree ADT that identifies the root with the entire tree.
    === Attributes ===
    @param object value: value of root node
    @param list[Tree|None] children: child nodes
    """
    def __init__(self, value=None, children=None):
        """
        Create Tree self with content value and 0 or more children
        @param Tree self: this tree
        @param object value: value contained in this tree
        @param list[Tree|None] children: possibly-empty list of children
        @rtype: None
        """
        self._value = value
        # copy children if not None
        # NEVER have a mutable default parameter...
        self._children = children[:] if children is not None else []
```

Tree, 一个倒过来的树, 每一个tree node都有 value和children, 常见的basecase就是在没有children的时候. 如果有children, 那么要对于每个children进行recursion. 这里要注意, 类似于linkedlist, 每一个node即是一个node, 同时也代表着整个subtree.我们对一个tree object做recursion的时候, 我们可以分为两步:

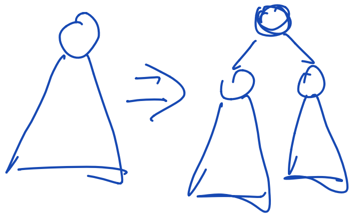
① 看自己.

1. 对node的value进行操作.
2. 对 每一个child node都 recursively 操作.

② 看 subtree.

BC: leaf

Recursive: internal



```
def gather_odd_items(t: Tree) -> list:
    “”” Return all values in the tree in a list.
    “””
    acc = []
    if t.value % 2 == 1:
        acc.append(t.value)

    for c in t.children:
        acc += gather_odd_items(c)

    return acc.
def height(t: Tree) -> int:
    “”” Return the height of the tree.
    “””
    if t.children == []:
        return 1.

    else:
        acc = []
        for c in t.children:
            acc.append(height(c))

        return max(acc) + 1.
```

```
def equivalent(t1: Tree, t2: Tree) -> bool:
    “”” Return True if the tree rooted at t1 and t2 are the same.

    >>> t1 = Tree(‘A’, [Tree(‘B’, [Tree(‘E’)]), Tree(‘C’, [Tree(‘D’)])])
    >>> t2 = Tree(‘A’, [Tree(‘B’, [Tree(‘E’)]), Tree(‘C’, [Tree(‘D’)])])
    >>> equivalent(t1, t2)
    True
    “””
    if t1.value != t2.value and len(t1.children) != len(t2.children):
        return False.

    for i in range(len(t1.children)):
        if not equivalent(t1.children[i], t2.children[i]):
            return False.

    return True..
```

```
def gather_by_depth(t: Tree) -> dict[int, list]:
    “””Gather all items in the Tree by their depth

    >>> t = Tree(‘A’, [Tree(‘B’, [Tree(‘E’)]), Tree(‘C’, [Tree(‘D’)])])
    >>> gather_by_depth(t) = {0: [‘A’], 1:[‘B’, ‘C’], 2: [‘E’, ‘D’]}
    True
    “””
    acc = {0: [t.value]}
    for c in t.children:
        temp = gather_by_depth(c)
        for depth in temp:
            if depth+1 in acc:
                acc[depth+1].extend(temp[depth])
            else:
                acc[depth+1] = temp[depth].

    return acc
```