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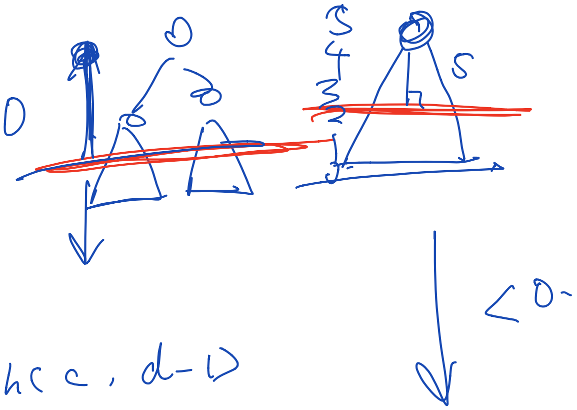
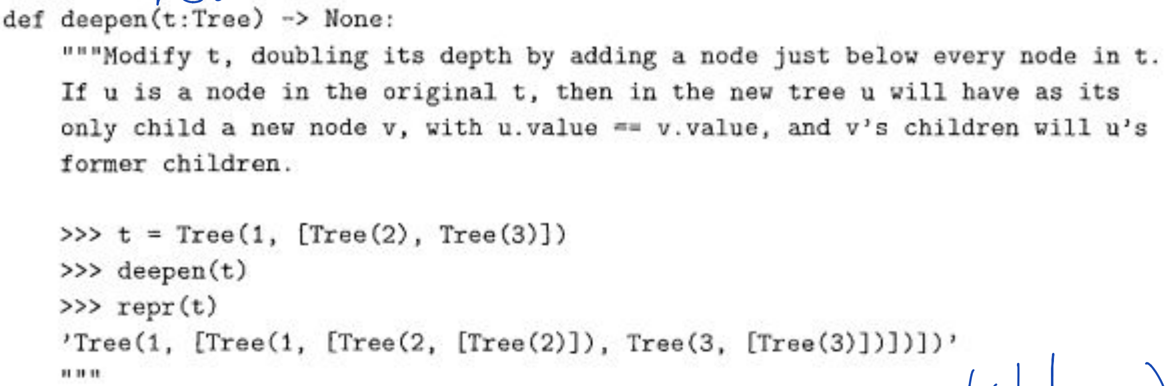
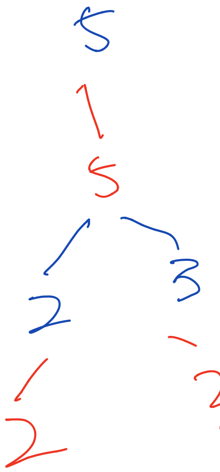
Section 3. Tree

```
def count_at_depth(t: Tree, d) -> int:
    “””Count at the number of node at depth d, assume the root has depth 0.

    >>> t = Tree(‘A’, [Tree(‘B’, [Tree(‘E’)]), Tree(‘C’, [Tree(‘D’)])])
    >>> count_at_depth(t, 1)
    2
    “””
    if d==0:
        return 1

    else:
        acc=0
        for c in t.children:
            acc += count_at_depth(c, d-1)

        return acc
```





```
def deepen(t:Tree) -> None:
    """Modify t, doubling its depth by adding a node just below every node in t.
    If u is a node in the original t, then in the new tree u will have as its
    only child a new node v, with u.value == v.value, and v's children will u's
    former children.

    >>> t = Tree(1, [Tree(2), Tree(3)])
    >>> deepen(t)
    >>> repr(t)
    'Tree(1, [Tree(1, [Tree(2, [Tree(2)]), Tree(3, [Tree(3)])])])'
    """
```



```
t.children=[Tree(t.value, t.children)]
                copy of t.

for c in t.children[0].children:
    deepen(c).
```

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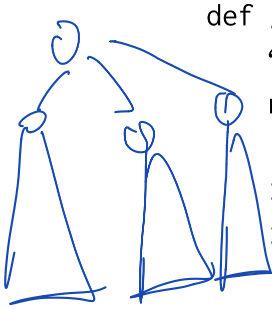
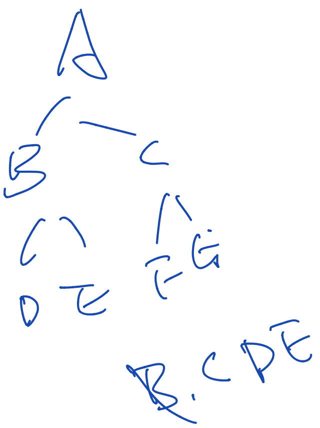


```
def get_longest_path(t: Tree) -> List:
    “”” Return a list contains the values in the right most longest pathfrom
    root to leaf.

    >>> t = Tree(‘A’, [Tree(‘B’, [Tree(‘E’)]), Tree(‘C’, [Tree(‘D’)])])
    >>> get_longest_path(t)
    [‘A’, ‘C’, ‘D’]
    “””
    if not t.children:
        return [t.value]

    else: result=[].
        for c in t.children:
            path= get_longest_path(c).
            if len(path) >=. len(result).
                result = path[:]
        return [t.value] + result.
```

```
def level_order_number_nodes(t):
    “”” Replace the value of all nodes by the sequence of their occurrence in
    the level order

    >>> t = Tree(‘A’, [Tree(‘B’, [Tree(‘E’)]), Tree(‘C’, [Tree(‘D’)])])
    >>> level_order_number_nodes(t)
    >>> repr(t)
    Tree(1, [Tree(2, [Tree(4)]), Tree(3, [Tree(5)])])
    “””
    n=1.
    q=[]
    q.append(t).
    while. len(q) > 0:  # not queue.is_empty().

        temp= q.pop(0)
        temp.value = n.
        n+=1.
        q.extend(temp.children).
```

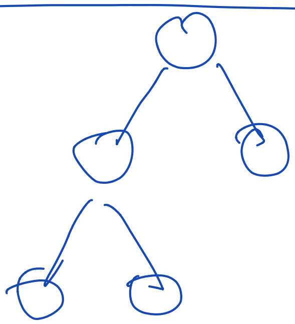


Section 4. Binary Tree

None

Binary Tree, 顾名思义, arity最大只有2的Tree, 但是, 我们的implementation有改变，不再是node, 变成了 left和right.（虽然我们还是可以 for child in [t.left, t.right]).
注意：binary tree并没有规定顺序!
所以说在这里我们可以分成三项.

1. Value
2. left child
3. right child

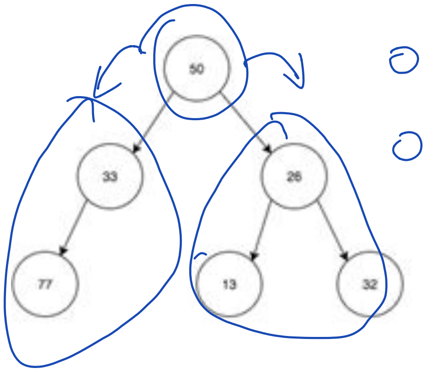


```
class BinaryTree:
    """
    A Binary Tree, i.e. arity 2.
    """
    def __init__(self, value, left=None, right=None):
        """
        Create BinaryTree self with value and children left and right.
        @param BinaryTree self: this binary tree
        @param object value: value of this node
        @param BinaryTree|None left: left child
        @param BinaryTree|None right: right child
        @rtype: None
        """
        self.value, self.left, self.right = value, left, right
```

Pre order: root先. children后.

Traversals:

我们一共接触了三种traversal, pre-order,　in-order和post-order. 这里pre, in和post可以理解为 root放的位置. Pre-order也就是先root再children, in-order为root在左右children中间, post-order为root在最后. 注意,



item　recursion.

○ Pre-order: 中 左右.

50, 33, 77, 26. 13, 32

○ Post-order: 左右中 77, 33, 13, 32, 26, 50

In-order: 左中右

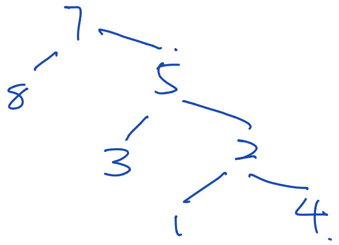
77, 33, 50, 13, 26, 32.

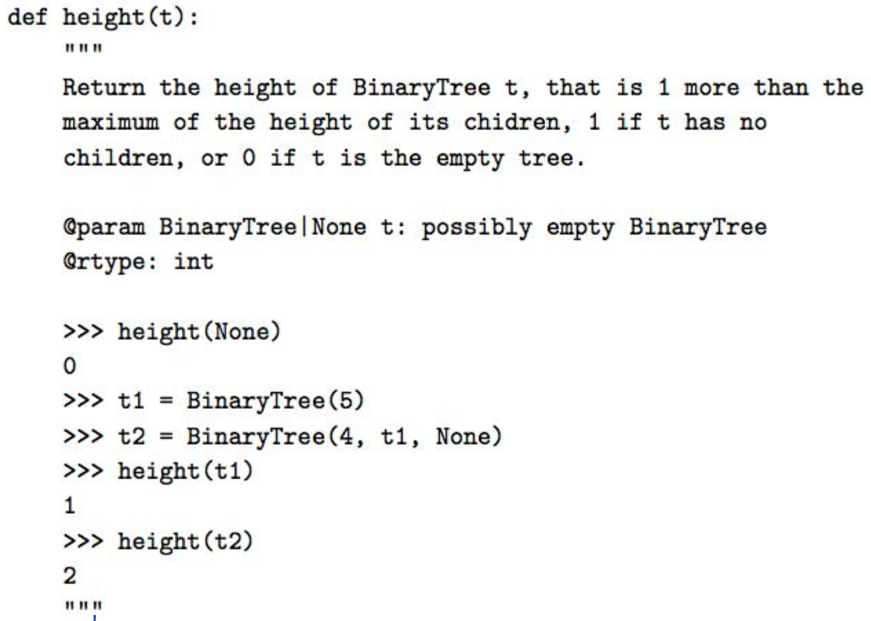
```
def inorder(node):
    if not node :          # if node is None.
        return []

    else:  return.  inorder( node.left) +
                     [node. value.]  +
                    inorder ( node . right).
```

常见题型： 给两种order要求画出原本的binary tree

pre-order: 7 8 5 3 2 1 4

in-order : 8 7 3 5 1 2 4

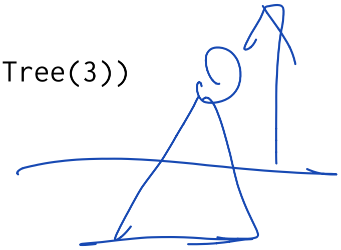




```
def height(t):
    """
    Return the height of BinaryTree t, that is 1 more than the
    maximum of the height of its chidren, 1 if t has no
    children, or 0 if t is the empty tree.

    @param BinaryTree|None t: possibly empty BinaryTree
    @rtype: int

    >>> height(None)
    0
    >>> t1 = BinaryTree(5)
    >>> t2 = BinaryTree(4, t1, None)
    >>> height(t1)
    1
    >>> height(t2)
    2
    """
```

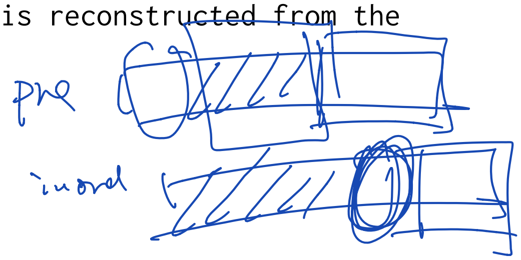
```
    if not t:
        return 0.
    else:
        return max( height(t.left), height(t.right)) + 1.
```

```
def count(node, item) -> int:
    “”” Count the occurrence of item in the tree rooted at node.
    “””
    if not node:
        return 0.

    else:
        if node.value == item:
            return 1 + count(node.left, item) +
                       count(node.right, item).
        else:
            return count(node.left, item) +
                   count(node.right, item).

def get_above(node: BinaryTree, d: int) -> List:
    “”” Get all the value in the tree rooted at node and above the depth d,
    Assume root has depth 0.

    >>> t = BinaryTree(1, BinaryTree(2, BianryTree(5)), BinaryTree(3))
    >>> get_above(t, 2)
    [1, 2, 3]
    “””
    if not node:
        return []
    if d == 0:
        return []

    else:
        return [node.value] + get_above(node.left, d - 1) +
               get_above(node.right, d - 1).
```



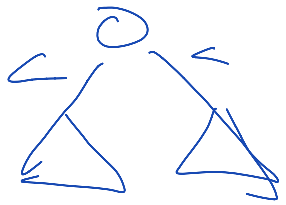
```
def swap_node(node: BinaryTree) -> None:
    """ Swap all the left and child node in the binary tree rooted at node.

    >>> t = BinaryTree(1, BinaryTree(2, None, None), BinaryTree(3, None, None))
    >>> swap_node(t)
    >>> t
    ~~t =~~ BinaryTree(1, BinaryTree(3, None, None), BinaryTree(2, None, None))
    """
    if node:
        node.left, node.right = node.right, node.left.
        swap_node(node.left)
        swap_node(node.right).
```

```
def get_all_path(node: BinaryTree) -> ~~None~~ list:
    """ Get all path from root to leaf in the Binary Tree rooted at node.

    >>> t = BinaryTree(1, BinaryTree(2), inaryTree(3))
    >>> get_all_path(t)
    [[1, 2], [1, 3]]
    """
    if not node:
         return []

    else:
        if node.left is None and node.right is None:
              return [[node.value]]

        else: acc=[]
              for c in [node.left, node.right]:
                   for path in get_all_path(c).
                        acc.append([node.value] + path).

        return acc.
    def reconstruct_tree(preorder: list, inorder: list) -> BinaryTree:
        """ Return the root of new binary tree that is reconstructed from the
given                           inorder and preorder list.
        """
        if len(preorder) == 0:
            return None
        root = preorder[0]
        index = inorder.index(root).
        left_subtree = reconstruct_tree(preorder[1: index+1],
                                        inorder[0: index]).
        right_subtree = reconstruct_tree(preorder[index+1: ],
                                         inorder[index+1: ])
        return BinaryTree(root, left_subtree,
                          right_subtree).
```

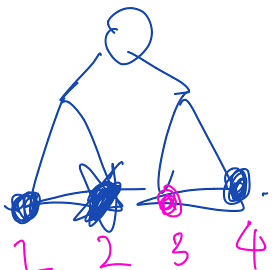


Section 5. Binary Search Tree

Binary Search Tree 简单来说是在 binary tree的基础上增加大小关系, 在binary search tree当中遵循着左边subtree一定小于root, root一定小于右边subtree的定式。所以在binary search tree当中, 如果我们需要找到一个node, 根据性质, 我们只需要检查单边就可以, 这样可以大大缩减running time.



✓ 1) 在BST中, inorder traversal可以返回一个sorted list.
2) 在常规BST中, 一般不存在duplicate value.
3) 对BST的任何操作都不能违反BST左小右大的原则.
4) 左边subtree 的最右是左边subtree中仅小于root的node.
5) 右边subtree的最左是右边subtree中仅大于root的node.



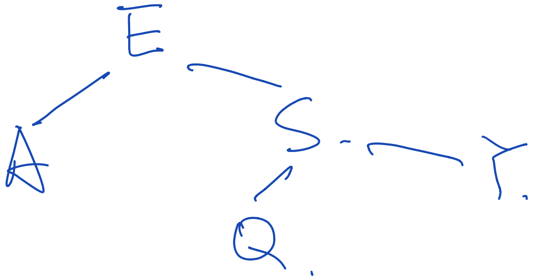
BST的insert

会从root开始按照左小右大原则向下寻找，加到leaf. 如果出现等于, 那么什么都不发生.

Draw the BST that results when you insert items with keys

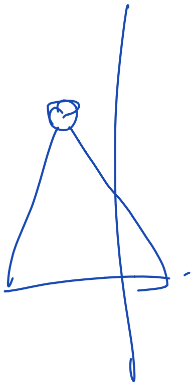
E A S Y Q U E S T I O N

in that order into an initially empty tree.

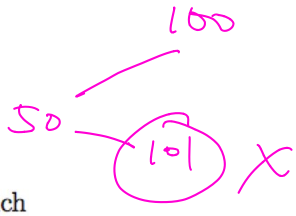


BST的search

def BST_contains(node: BTnode, value:object) -> bool:



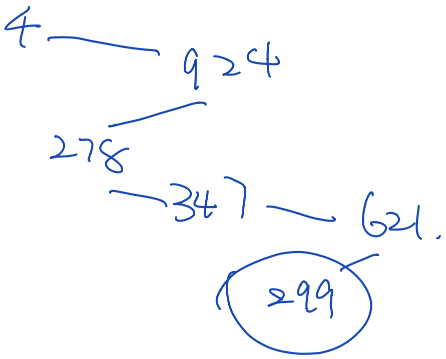
```
  if not node:
      return False
  if node.value == value:
      return True.
  elif: node.value < value:
      return BST_contains(node.right, value).
  else: return BST_contains(node.left.value).
```



Suppose we have int values between 1 and 1000 in a BST and search for 363. Which of the following cannot be the sequence of keys examined.

d

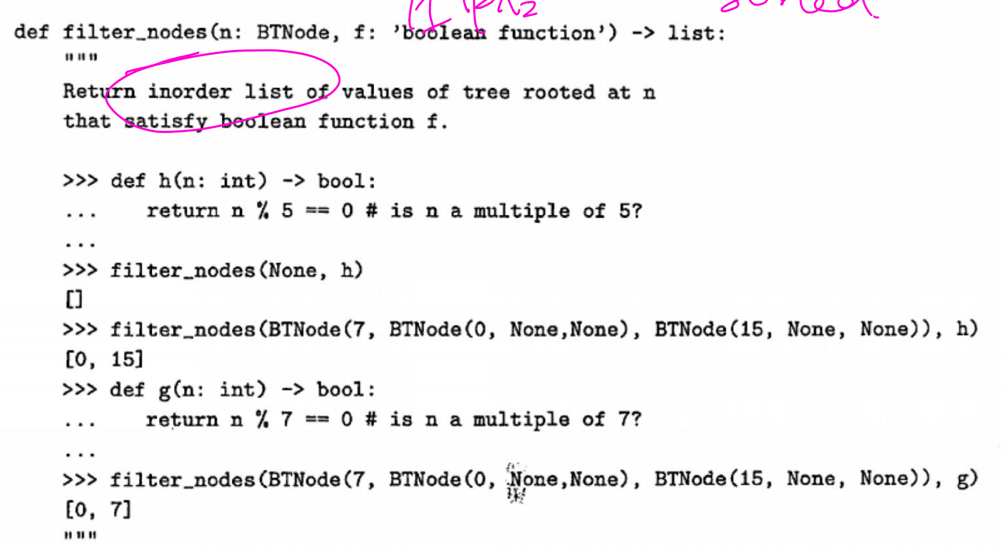
(a) 2 252 401 398 330 363
(b) 399 387 219 266 382 381 278 363
(c) 3 923 220 911 244 898 258 362 363
(d) 4 924 278 347 621 299 392 358 363
(e) 5 925 202 910 245 363



BST的Traversal

在BST中, 做题一定要使用到左小右大的性质, 并且记住 inorder 永远会得到一个sorted list, 做题过程中一旦遇到要求返回sorted list, 一定要使用inorder的顺序.

左中右 → sorted



```
def filter_nodes(n: BTNode, f: 'boolean function') -> list:
    """
    Return inorder list of values of tree rooted at n
    that satisfy boolean function f.

    >>> def h(n: int) -> bool:
    ...     return n % 5 == 0 # is n a multiple of 5?
    ...
    >>> filter_nodes(None, h)
    []
    >>> filter_nodes(BTNode(7, BTNode(0, None,None), BTNode(15, None, None)), h)
    [0, 15]
    >>> def g(n: int) -> bool:
    ...     return n % 7 == 0 # is n a multiple of 7?
    ...
    >>> filter_nodes(BTNode(7, BTNode(0, None,None), BTNode(15, None, None)), g)
    [0, 7]
    """
```

```
if not n:
    return [].

acc = filter_nodes(n.left, f)

if f(n.value):
    acc.append(n.value)
acc += filter_nodes(n.right, f)
return acc
```

```
def get_greater_than(n: BTnode, item: int) -> List[int]:
    “”” Return all items in the BST rooted at n that are greater than item in a sorted
order.
    “””
    if not n:
        return []
    if n.value > item:
        return getgreater_than(n.left, item) + [n.value] +
            get_greater_then (n.right, item)
    else: # n.value <= item
        return getgreater_than (n.right, item)
```