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What about if when you have Landau interactions?

$$\epsilon_p \rightarrow \epsilon_p + \sum_{\mathbf{p}'} f_{pp'} \delta n_{p'}$$

depends on the occupation number of all the other q-p.s

$$\Rightarrow \frac{dn}{d\mu} = \frac{N(0)}{1 + F_0}$$

Time 54:25

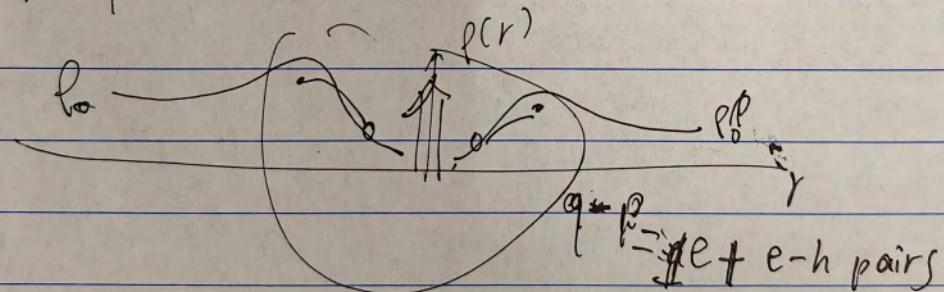
Fermi surface survives ; but not Fermi liquid

FS's only assumption is adiabatic continuity from the free particle states

Assume you have some many es system

↳ start from free es \rightarrow turn up int. \rightarrow end up with Fermi liquid states

2) clouds = polarons? Yes!



3) $N \rightarrow \infty$ canonical ens
grand ... results are the same.



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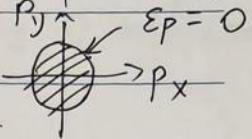
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[So $|G\rangle$ is a many-particle state like Yu Wang said last night. (27)]

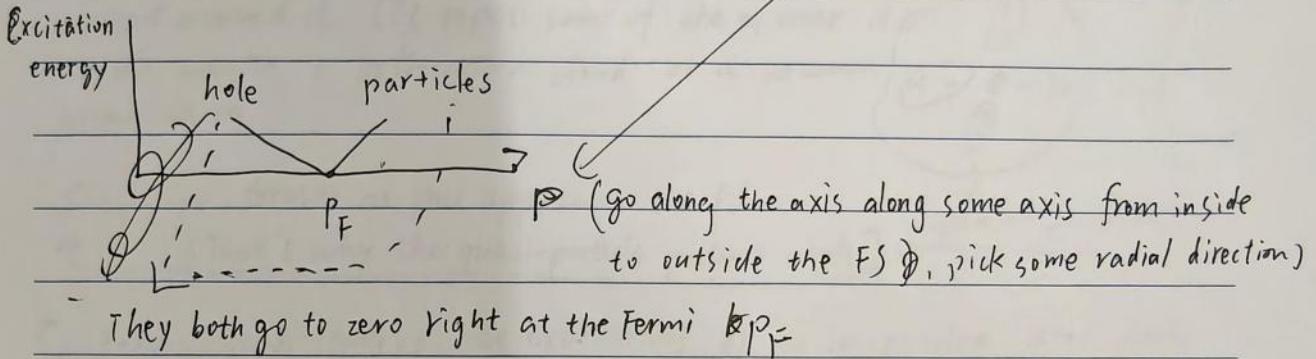
$$S_0 |G\rangle = \prod_{\mathbf{p} < 0} c_{\mathbf{p}}^+ |\rangle$$

introduces Fermi surface (FS)



So FS is a boundary in the discontinuity
in the "Fermion occupation number"

Excited states: $c_{\mathbf{p}}^+ |G\rangle \quad \epsilon_{\mathbf{p}} > 0$ (By def., the energy of any excited states must be greater than zero)

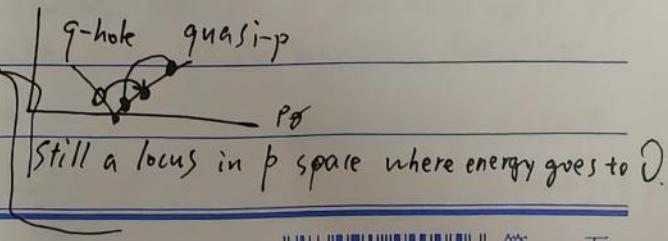
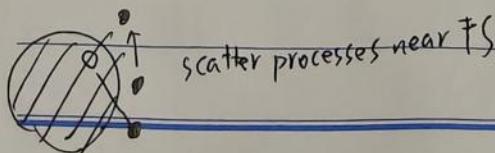


We talk about things near p_F in an electron gas, all of the action happens right near p_F , that's where the low energy excitations are.

gapless system: can have arbitrarily ~~large~~ excitations close to the $|G\rangle$

Add H int

all that happens when you put interactions is that you just put a quasi in front of everything





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268R Class 1 Fermi Liquid theory

A bunch of particles

$$H = \sum_i -\frac{\nabla_i^2}{2m} + \sum_i V_c(q_i) + \sum_{i < j} V_{int}(\vec{q}_i - \vec{q}_j) \quad i=1, \dots, N$$

kinetic E lattice poten..

① Want¹ possible phases? of this type of H .

↳ well-known classes

Assume Fermions first

② Want² find the ground state wave function

Hartree-Fock (mean field approx.)

Idea: Pick some set of single particle orbitals:

$$\varphi_a(r) \quad a=1, \dots, N$$

next: the wave functions of the n particles:

$$\Psi(n) = \det [\varphi_a(r_j)] \quad \text{Slater det...}$$

$$\text{if } \begin{cases} (2 \text{ orbitals}) \\ n=2 \\ (2) \end{cases} = \begin{vmatrix} \varphi_a(r_1) & \varphi_b(r_1) \\ \varphi_a(r_2) & \varphi_b(r_2) \end{vmatrix} \quad \begin{matrix} \text{orbital index runs over columns} \\ \text{particle index runs} \end{matrix}$$

(bosons = det \rightarrow perm)

Task: ~~picked~~ picked orbitals \rightarrow minimize (Variational prin.)

$$\langle \Psi | H | \Psi \rangle \text{ over your variational para.!} \\ \hookrightarrow \varphi_a(r)$$





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So the problem is simpler, \rightarrow before: one function of n variables
 \rightarrow now: n functions of 1 coordinates.

\hookrightarrow determine the best possible $\Psi_0 \rightarrow$ DFT!

Solid states physics until 1980s \leftarrow symm. breaking
pairing (SC) & BCS

\checkmark fails! many particles wave function \times as just the product anti-symm.
of single-particles states.

Excited states: N DOFs in a ladder 2^N states

\times write down wave func.

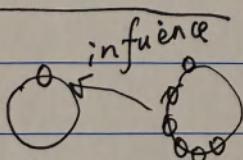
But low energy states \Rightarrow combinations (sums) of quasi-particles

some excited states $E = \sum_{\alpha} E_{\alpha} n_{\alpha}$ (Norman Yao had said it)

each one of these single orbitals excitation is what we call a quasi-particle

\hookrightarrow tells whether that particular orbital or state is occupied or not $n_{\alpha} = 0, 1$ (Fermions)

But



$$\text{So } E_{\text{excited state}} = \sum_{\alpha} E_{\alpha} n_{\alpha} + \sum_{\alpha, \beta} F_{\alpha \beta} n_{\alpha} n_{\beta} + \text{high order terms ...}$$





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$\Sigma \alpha, \bar{\alpha}, \beta$ are what we want to determine (295b....)

\downarrow N orbitals $\rightarrow N^2$ the no. of para. is polynomial not 2^N poly(N)

So very very large number of states can be written in terms of
a much smaller number of para.

low T is \sim

But some quasi-particles are complicated, e.g. fractions of an e.
(of course serve as a basis for many-body states)

$\begin{cases} e \\ e-h \text{ pair} \\ e-e \text{ pair} \\ \text{fractions of } e \end{cases}$

Free electrons $H = \sum_p \left(\frac{\hbar^2 k^2}{2m} - \mu \right) \dots$ move in free space and have dispersion

$$\begin{aligned} & \dots \text{grand canonical ensemble} \\ & \left(C_p^+ C_p \right) \quad \left[\text{you have some freely available reservoir that can add and remove } e \right] \\ & = \sum_p \epsilon_p C_p^+ C_p \end{aligned}$$

What is ground state?

occupy " \downarrow " ϵ_p
leave empty " \uparrow " ϵ_p



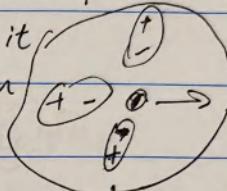
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The phase space for this process becomes smaller and smaller, the closer you get to FS. As you approach FS, it's not possible to scatter anything and conserved energy because of the exclusion principle.

Also can be virtual processing, which describe in time-independent perturbation theory:

As this particle moving around with some momentum p , it's creating a cloud around it. (it repels some of the e_s near it) so it creates a polarization cloud. or a vacuum polarization.



They live forever as you approach the FS.

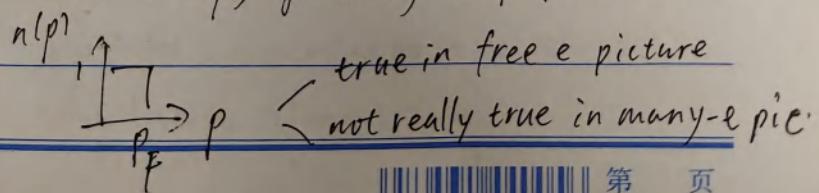
(That's why the quasi-particle picture works)

So Fermi liquid theory: the excitations of an interacting metal have quasi-particles and q -holes. The energy of the excitations vanishes on the FS

$E \uparrow V \rightarrow$
(where energy go to zero, p where the holes turn into particles)
where the lifetime goes to ∞
as you approach FS)

do cal. 1st: technical change $\xrightarrow{\text{from}}$, keep track of holes and particles are nuisance

to math trick: postulate the $|G\rangle$ is also made up of q -p with $n(p)$ given by step function.





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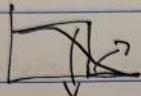
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Excitations: as changes in the quasi-p. number

$$\delta n(p) = n(p) - n_0(p)$$

No. of q-p. No. [e^s] at momentum p

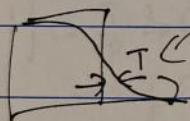
We are interested in the changes in the distribution function



$$\delta n(p) = \text{holes}$$

Can Write Excitation energy $E[\delta n(p)] = \sum_p (\epsilon_p) \delta n_p$

but Landau \rightarrow interactions $\left\{ \begin{array}{l} \text{Induce } \frac{p^2}{2m^*} - \mu \text{ heavier} \\ \text{② } + \frac{1}{2} \sum_{pp'} (F_{pp'}) \delta n_p \delta n_{p'} \text{ (interaction correction)} \end{array} \right.$



$$\sum_p \epsilon_p \delta n(p) \text{ is order } T^2$$

Landau para.

for a metal at low T with a FS, this's the complete theory for the low energy q-p (always assuming all the action is happening in this little region)

Q: μ get re-normalized by interactions?

$$\mu = \frac{\partial E}{\partial N} \Big|_{T=0} \text{ it certainly does change}$$

) what's 反直觉 is in QFT, particles' dispersion go to be min. at $p=0$

But not true in the metal, whose energy is [on a sphere] excitation





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Specific heat: heat up the system \rightarrow freees $C_V \propto T$

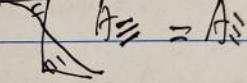
$C_V \propto T$ the excitations are ϵ_s

are excited over a width of order T

typical energy is also of order T

$$So T^2 \quad C_V = \frac{\partial E}{\partial T}$$

Ask: Landau interactions will change the C_V ?

No! $\sum_p \delta n_p \rightarrow 0$ 

$$C_V = \frac{\pi^2 k_B^2}{3} N(0) T, \quad N(0) \text{ is DOS in the Fermi level}$$

Compressibility: change μ a little bit

$$H = \sum_p \epsilon_p c_p^\dagger c_p$$

$$\rightarrow \sum_p \epsilon_p c_p^\dagger c_p - \delta \mu \sum_p c_p^\dagger c_p$$

Fermi function

$$\delta n(p) = \frac{\partial}{\partial \mu} \langle c_p^\dagger c_p \rangle \quad f(\mu) = \frac{\partial}{\partial \mu} f(\epsilon_p - \mu) \delta \mu$$

$$= \delta \mu \left(-\frac{\partial f}{\partial \epsilon_p} \right) = \delta \mu \delta(\epsilon_p)$$

$$\frac{\partial}{\partial \epsilon} \left[\frac{1}{e^{2\epsilon/T} + 1} \right] \stackrel{T \rightarrow 0}{=} -\delta(\epsilon)$$

Next

You want the change in density due to change in μ :

$$\sum_p \delta n(p) = \delta \mu N(0)$$

the DOS at the Fermi level

$$\boxed{\frac{dn}{d\mu} = N(0)}$$



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