



武汉大学

WUHAN UNIVERSITY

Wuhan 430072, Hubei, P.R.China 中国·武汉 Tel.(027)

What about if when you have random interactions?

$$\epsilon_p \rightarrow \epsilon_p + \sum_{p'} \bar{f}_{pp'} \delta n_{p'}$$

depends on the occupation number of all the other q -ps

$$\Rightarrow \frac{d\mu}{d\mu} = \frac{N(0)}{1+F_0}$$

Time 54:25

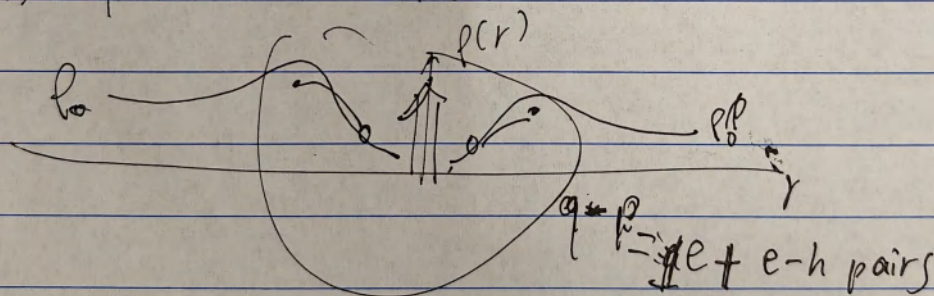
2) Fermi surface survives but not Fermi liquid

FS's only assumption is ^{adiabatic} continuity from the free particle states

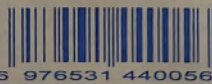
Assume you have some many es system

↳ start from free es \rightarrow turn up int. \rightarrow end up with Fermi liquid states

2) clouds = polarons? Yes!



3) $N \rightarrow \infty$ canonical ensembles grand ... results are the same.



6 976531 440056

第 页



武汉大学

WUHAN UNIVERSITY

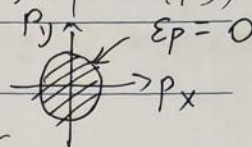
Wuhan 430072, Hubei, P.R. China 电话 027-68752239

[So $|G\rangle$ is a many particle state, like Yu Wang said last night.]

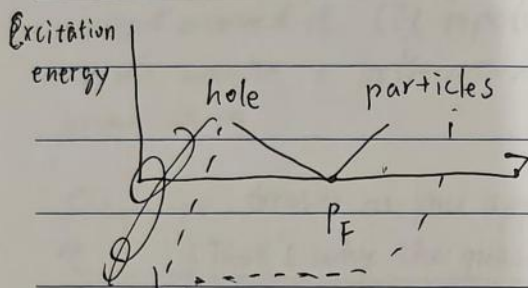
So $|G\rangle = \prod_{\epsilon_p < 0} c_p^\dagger |0\rangle$

introduces Fermi surface (FS)

So FS: is a boundary in the discontinuity in the "Fermion occupation number"



Excited states: $c_p^\dagger |G\rangle$ $\epsilon_p > 0$ (By def., the energy of any excited states must be greater than zero)



(go along the axis along some axis from inside to outside the FS), pick some radial direction)

They both go to zero right at the Fermi p_F

We talk about things near p_F an electron gas, all of the action happens right near p_F , that's where the low energy excitations are.

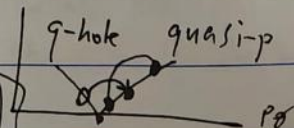
gapless system: can have arbitrarily ~~close~~ excitations close to the $|G\rangle$

Add H int

all that happens when you put interactions is that you just put a [quasi] in front of everything

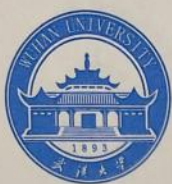


scatter processes near FS



Still a locus in p space where energy goes to 0





武汉大学

WUHAN UNIVERSITY

Wuhan 430072, Hubei, P.R.China 中国·武汉 Tel.(027)

268R Class 1 Fermi Liquid theory

A bunch of particles

$$H = \sum_i \underbrace{-\frac{p_i^2}{2m}}_{\text{kinetic } E} + \sum_i V_c(q_i) + \sum_{i,j} V_{\text{int}}(\vec{q}_i - \vec{q}_j) \quad i=1, \dots, N$$

lattice poten..

Want 1 possible phases? of this type of H.

↳ well-known classes

Assume Fermions first

2) Want 2 find the ground state wave function

Hatrick-fork (mean field approx.)

idea: Pick some set of single particle orbitals:

$$\varphi_a(r) \quad a=1, \dots, N$$

next: the wave functions of the n particles:

$$\psi(n_i) = \det[\varphi_a(r_j)] \quad \text{Slater det...}$$

$$\text{if } \begin{matrix} (2 \text{ orbitals}) \\ n=2 \\ (2 \text{ particles}) \end{matrix} \quad \begin{vmatrix} \varphi_a(r_1) & \varphi_b(r_1) \\ \varphi_a(r_2) & \varphi_b(r_2) \end{vmatrix} \quad \left(\begin{array}{l} \text{orbital index runs over columns} \\ \text{particle index runs over rows} \end{array} \right)$$

(bosons = det \rightarrow perm)

Task: ~~the~~ picked orbitals \rightarrow minimize (variational prin.)

$\langle \psi | H | \psi \rangle$ over your variational para.!

↳ $\varphi_a(r)$



第 页

6 976531 440056



武漢大學

WUHAN UNIVERSITY

Wuhan 430072, Hubei, P.R.China 中国·武汉 Tel.(027)

So the problem is simpler, $\begin{cases} \rightarrow \text{before: one function of } n \text{ variables} \\ \rightarrow \text{now: } n \text{ functions of } 1 \text{ coordinate.} \end{cases}$

\hookrightarrow determine the best possible $\Psi_a \rightarrow$ DFT!

Solid states physics until 1980s \leftarrow $\begin{cases} \text{symm. breaking} \\ \text{pairing (SCP's BCS)} \end{cases}$

\hookrightarrow fails! many particles wave function \neq just the product $\overline{\text{anti-symm.}}$ of single-particles states.

Excited states: N DOFs in a ladder 2^N states

\times write down wave func.

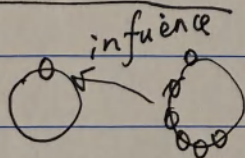
But low energy states \Rightarrow combinations (sums) of quasi-particles

Some excited states $E = \sum_{\alpha} \epsilon_{\alpha} n_{\alpha}$ (Norman Yao had said it)

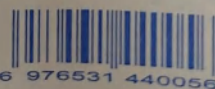
each one of these single orbitals excitation is what we call a quasi-particle

\rightarrow tells whether that particular orbital or state is occupied or not $n_{\alpha} = 0, 1$ (fermions)

But

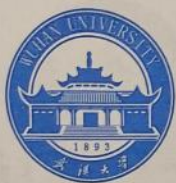


$$\text{So } E_{\text{excited state}} = \sum_{\alpha} \epsilon_{\alpha} n_{\alpha} + \sum_{\alpha, \beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \text{higher order terms} \dots$$



6 976531 440056

第 页



武汉大学

WUHAN UNIVERSITY

Wuhan 430072, Hubei, P.R.China 中国·武汉 Tel.(027)

$\epsilon_\alpha, \epsilon_\beta$ are what we want to determine (295b....)
 \downarrow \searrow
 N orbitals N^2 the ~~new~~ NO. of para. is polynomial not 2^N
 poly(N)

So very very large number of states can be written in terms of
 a much smaller number of para.

low T is \sim

But some quasi-particles are complicated, e.g. fractions of an e.
 (of course serve as a basis for many-body states)

$\left\{ \begin{array}{l} e \\ e-h \text{ pair} \\ e-e \text{ pair} \\ \text{fractions of } e \end{array} \right.$

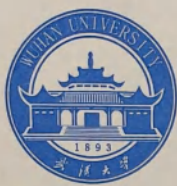
Free electrons $H = \sum_p \left(\frac{\hbar^2 k^2}{2m} - \mu \right) c_p^\dagger c_p$... move in free space and have dispersion
 ... grand canonical ensemble
 $= \sum_p \epsilon_p c_p^\dagger c_p$ [you have some freely available reservoir that can add and remove e]

What is ground state?
 occupy "-" ϵ_p
 leave empty "+" ϵ_p



6 976531 440056

第 页



武汉大学

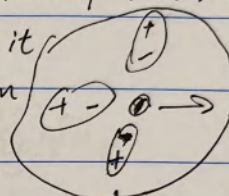
WUHAN UNIVERSITY

Wuhan 430072, Hubei, P.R.China 中国·武汉 Tel.(027)

The phase space for this process becomes smaller and smaller, the closer you get to FS. As you approach FS, it's not possible to scatter anything and conserve energy because of the exclusion principle.

Also can be virtual processing, which describe in time-independent perturbation theory:

As this particle moving around with some momentum p , it's creating a cloud around it. It repels some of the e_s near it so it creates a polarization cloud. or a vacuum polarization.



They live forever as you approach the FS.

(That's why the quasi-particle picture works)

quasi-particles

So Fermi liquid theory: the excitations of an interacting metal have quasi-particles and q-holes. The energy of the excitations vanishes on the FS

$E \propto v \rightarrow 0$
(where energy goes to zero, p where the holes turn into particles)
where the lifetime goes to ∞ as you approach FS)

do cal. 1st: technical change ^{from} keep track of holes and particles are nuisance

to (math trick: postulate the $|G\rangle$ is also made up of q-p with $n(p)$ given by step function.

$n(p)$ p p_F true in free e picture
not really true in many-e pic.



6 976531 440056

第 页



武汉大学

WUHAN UNIVERSITY

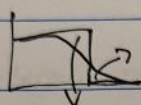
Wuhan 430072, Hubei, P.R.China 中国·武汉 Tel.(027)

Excitations: — as changes in the quasi-p. number

$$\delta n(p) = n(p) - n_0(p)$$

n.o. of q-p. $n_0(\epsilon)$ at momentum p

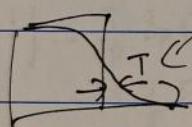
we are interested in the changes in the distribution function



$\delta n(p) \approx \pm 1$, or holes

Can Write [excitation energy] $E[\delta n(p)] = \sum_p \epsilon_p \delta n_p$

but Landau \rightarrow interaction $\left\{ \begin{array}{l} \text{induce } \frac{p^2}{2m^*} - \mu \text{ heavier} \\ \text{② } \frac{1}{2} \sum_{p,p'} F_{pp'} \delta n_p \delta n_{p'} \text{ (interaction correction)} \end{array} \right.$



$\sum_p \epsilon_p \delta n(p)$ is order T
order T

Landau para.

for a metal at low T with a FS, this's the complete theory for the low energy q-p
(always assuming all the action is happening in this little region)

Q: μ get re-normalized by interactions?

$$\mu = \frac{\partial E}{\partial N} \Big|_{T=0} \text{ it certainly does change}$$

what's 反直觉 is in QFT, particles's dispersion go to be min. at $p=0$
But not true in the metal, whose energy is (on a sphere)
excitation



6 976531 440056

第 页



武汉大学

WUHAN UNIVERSITY

Wuhan 430072, Hubei, P.R.China 中国·武汉 Tel.(027)

Specific heat: heat up the system \rightarrow free e_s $C_v \propto T$

cuz the excitations are e_s
 are excited over μ with of order T

typical energy is also of order T

$$\text{So } T^2 \quad C_v = \frac{\partial E}{\partial T}$$

Ask: Landau interactions will change the C_v ?

No! $\sum_p \delta n_p \rightarrow 0$

~~$A \equiv A_s$~~ $A_s = A_s$

$$C_v = \frac{\pi^2 k_B^2}{3} N(0) T, \quad N(0) \text{ is DOS in the Fermi level}$$

Compressibility: change μ a little bit

$$H = \sum_p \epsilon_p c_p^\dagger c_p$$

$$\rightarrow \sum_p \epsilon_p c_p^\dagger c_p - \delta \mu \sum_p c_p^\dagger c_p$$

Fermi function

$$\delta n(p) = \frac{\partial}{\partial \mu} \langle c_p^\dagger c_p \rangle \delta \mu = \frac{\partial}{\partial \mu} f(\epsilon_p - \mu) \delta \mu$$

$$= \delta \mu \left(-\frac{\partial f}{\partial \epsilon_p} \right) = \delta \mu \delta(\epsilon_p)$$

$$\frac{\partial}{\partial \epsilon} \left[\frac{1}{e^{\epsilon/T} + 1} \right] \xrightarrow{T \rightarrow 0} -\delta(\epsilon)$$

Next

You want the change in density due to change in μ :

$$\sum_p \delta n(p) = \delta \mu N(0)$$

the DOS at the Fermi level

$$\boxed{\frac{dn}{d\mu} = N(0)}$$



第 页

6 976531 440056