

**procura adversarial
ou, simplesmente,
jogos
(determinísticos)**

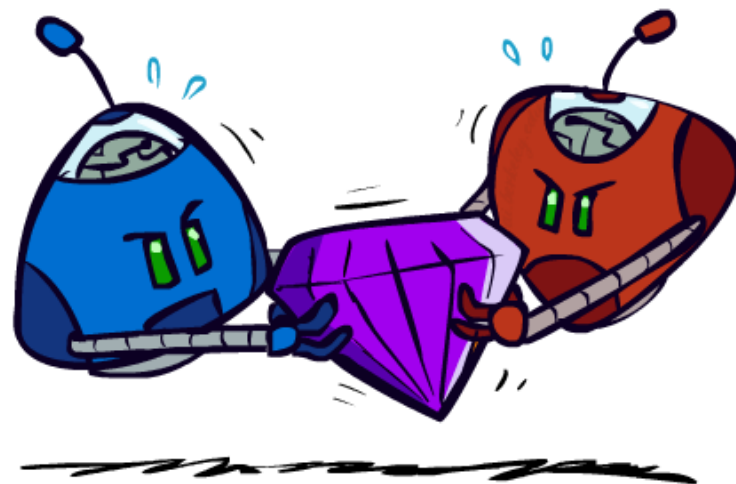
jogos

... há muitos

os que vamos estudar
agora:

- determinísticos
- informação perfeita
- dois jogadores, à vez

e soma zero

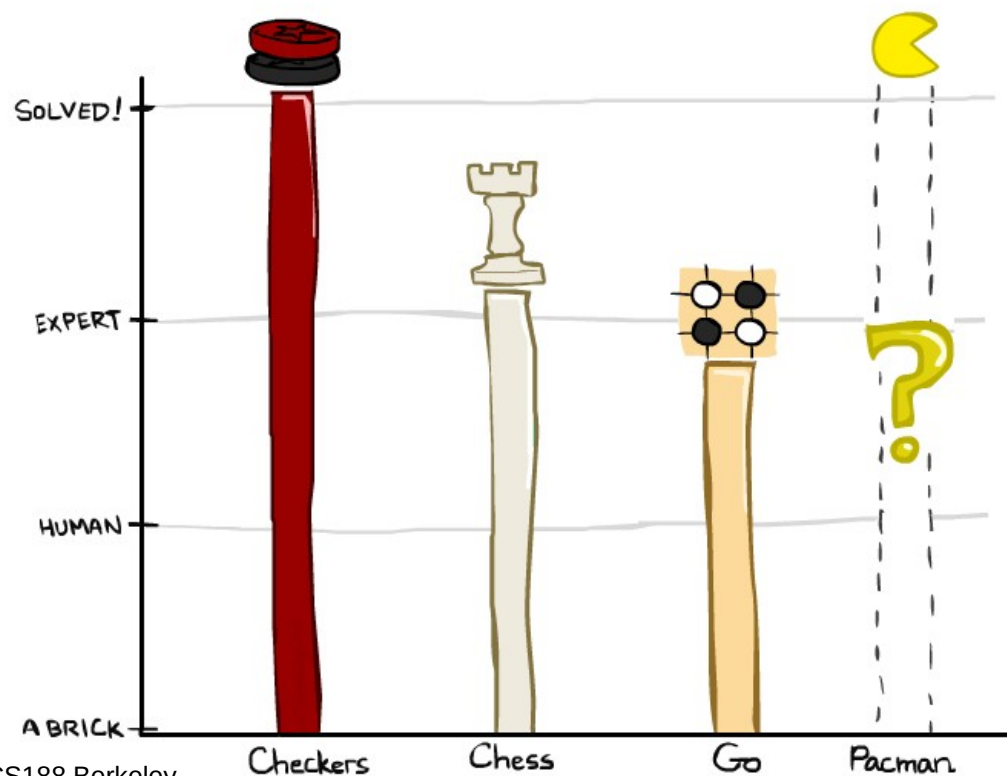


fonte: CS188 Berkeley

o que um ganha o outro perde

jogos soma zero 2 jogadores

- **Damas.** 1950: 1º programa. 1994: Chinook bateu campeão humano (há 40 anos). 2007: resolvido
- **Xadrez.** 1997: Deep Blue vence campeão humano. Programas atuais melhores ainda
- **Go.** 2016: AlphaGo vence campeão humano
- **Pacman.** 2019: MuZero com muito melhor desempenho que humanos e outros progrs. (12h de aprendizagem)



jogos são complexos

xadrez

factor de ramificação, $b \sim 35$

árvore de procura com $\sim 35^{100} \sim 10^{154}$

tempo é determinante!

necessidade de tomar decisões mesmo que não seja viável saber qual é o ótimo

terminologia

S_0 : estado inicial

Player(s): jogador que tem vez no estado s

MAX – 1º a jogar
MIN – 2º a jogar

Actions(s): movimentos possíveis no estado s

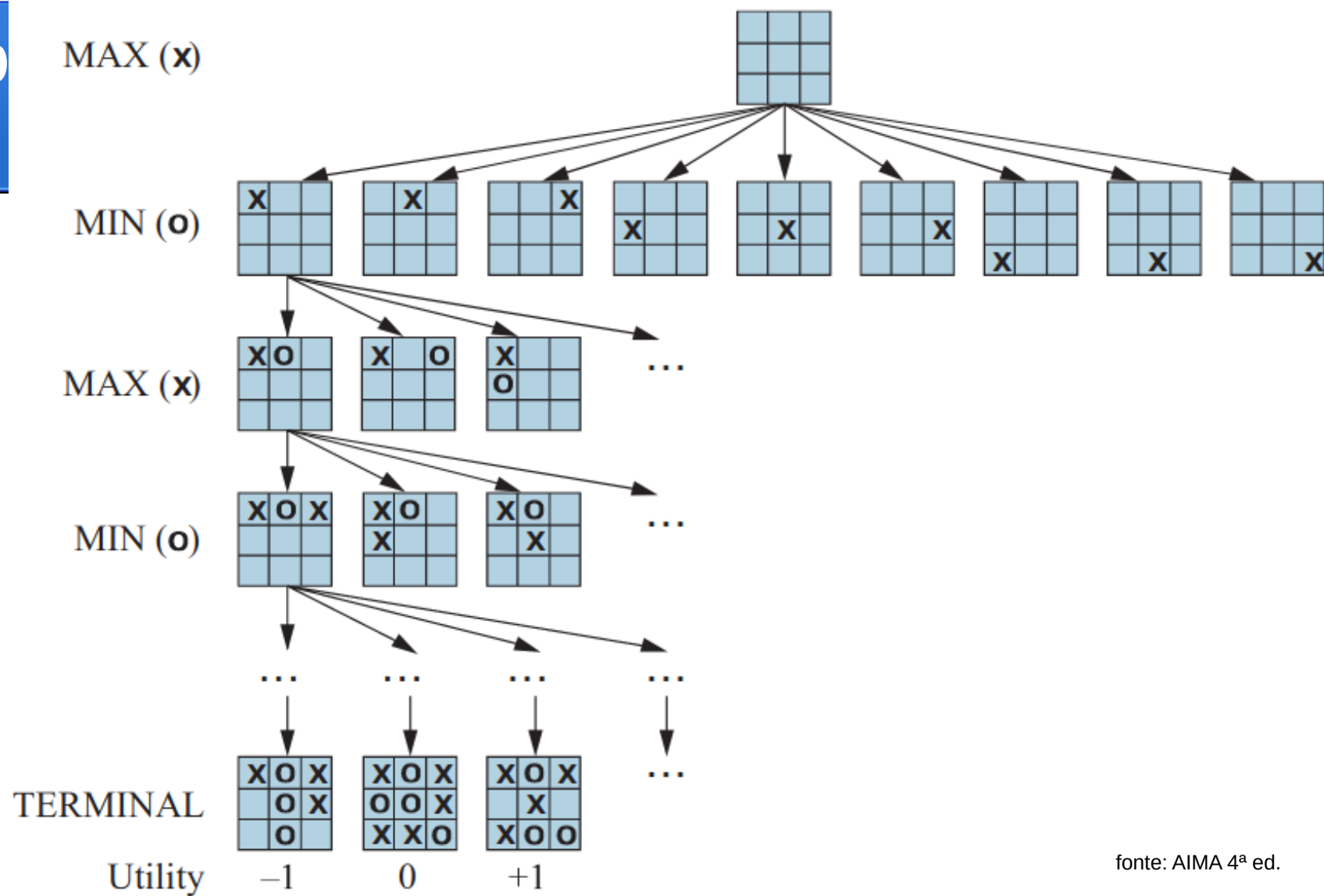
Result(s, a): resultado após movimento a no estado s

TerminalTest(s): teste de fim de jogo. $V \Rightarrow$ estado terminal

Utility(s, p): ganho para o jogador p quando o jogo termina no estado s (ex: no xadrez, 0, $\frac{1}{2}$, +1)

utilidade ou ganho

jogo do galo



nota sobre espaços de procura

	estrutura	ciclos	estados repetidos
xadrez	grafo	sim	não
galo	árvore	não	sim
?	árvore	não	não

árvore de procura

nº de nós (estados) terminais

- jogo do galo: $< 9! = 362.880$
- xadrez: $> 10^{40}$

estratégia ótima produz
resultado pelo menos
tão bom como qualquer
outra, se o adversário
for infalível

árvore de procura usada é um subconjunto
da árvore completa para permitir escolher jogada

estratégia ótima

exemplo com um jogo mais simples

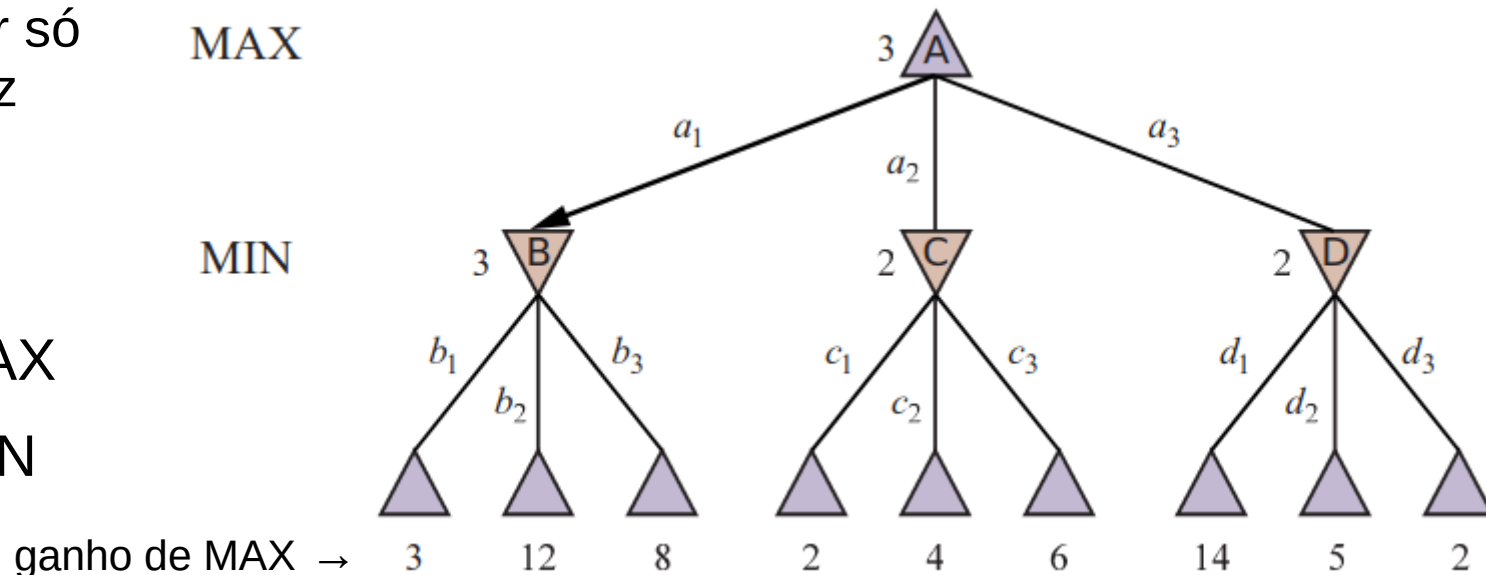
cada jogador só
joga uma vez
(jogo de 1
movimento)

MAX

MIN

△ : ações de MAX

▽ : ações de MIN



fonte: AIMA 4ª ed.

estratégia ótima

minimax (de um nó)

ganho de MAX num nó quando, daí em diante, ambos os jogadores escolhem jogadas ótimas

$$\text{MINIMAX}(S) = \begin{cases} \text{UTILITY}(S) & \text{se } \text{TERMINALTEST}(S) \\ \max_{a \in \text{ACTIONS}(S)} \text{MINIMAX}(\text{RESULT}(S, a)) & \text{se } \text{JOGADOR}(S) = \text{MAX} \\ \min_{a \in \text{ACTIONS}(S)} \text{MINIMAX}(\text{RESULT}(S, a)) & \text{se } \text{JOGADOR}(S) = \text{MIN} \end{cases}$$

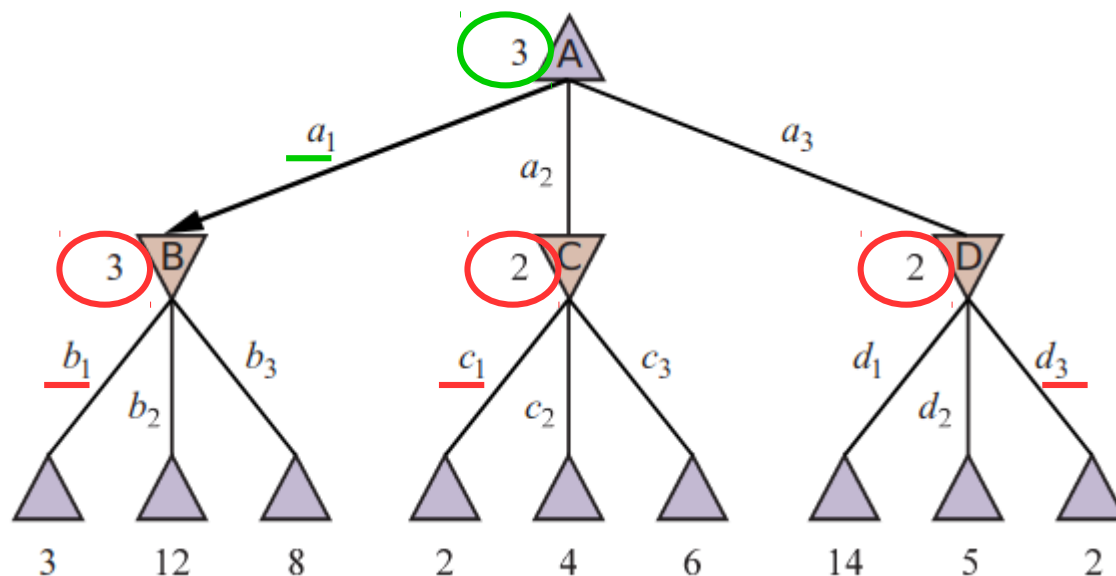
MAX prefere ganho máximo; **MIN** prefere ganho mínimo

ganhos
de MAX

no jogo de profundidade 1

MAX

MIN



MAX – escolhe melhor
dos piores casos
(MIN ótimo)

se MIN não for
ótimo, MAX tem
ganho maior ainda

computação
naturalmente
recursiva

adaptado de AIMA 4ª ed.

complexidade

minimax faz uma pesquisa completa em profundidade

complexidade temporal:

$$O(b^m)$$

complexidade espacial:

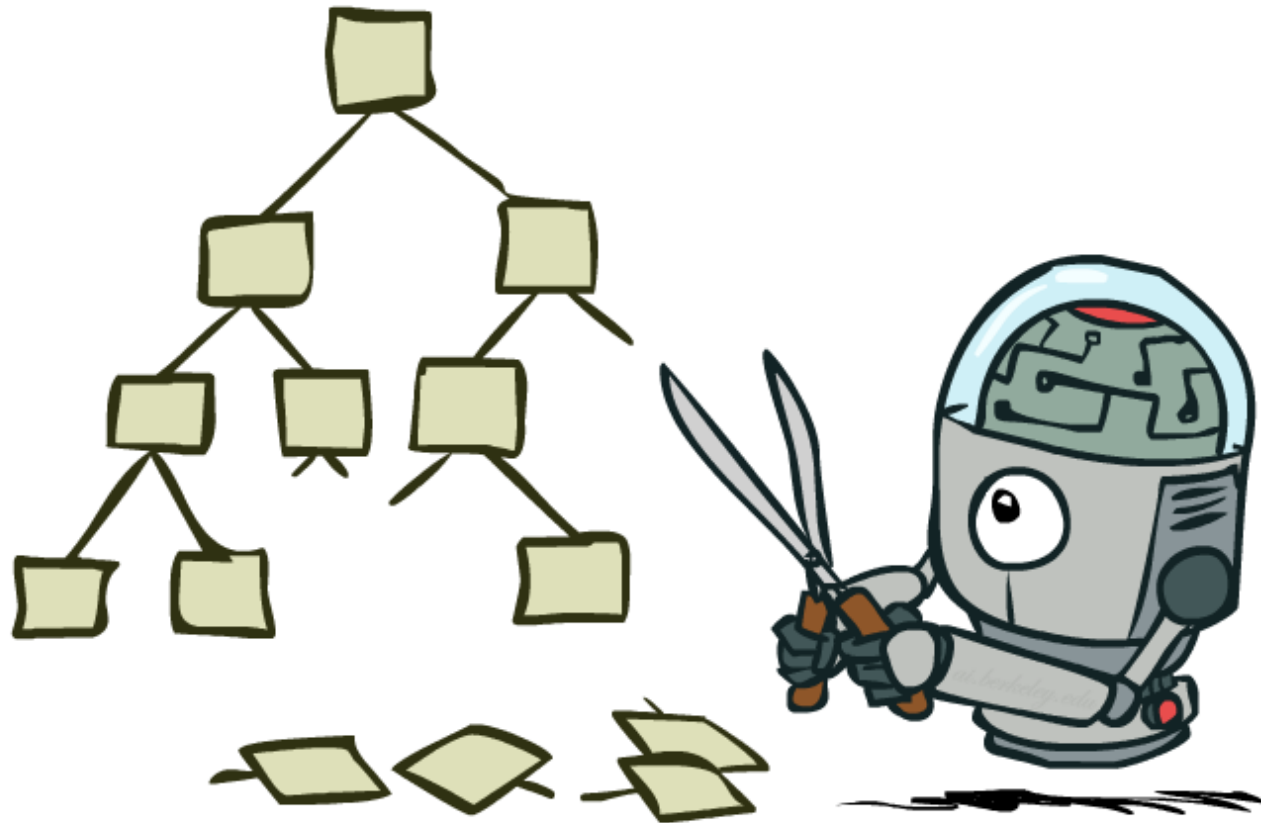
$O(bm)$ - se expande de uma vez todas as ações de um nó

$O(m)$ – se expande à vez cada ação de um nó

inviável em jogos normais

mas é a base para algoritmos práticos para jogos normais

poda



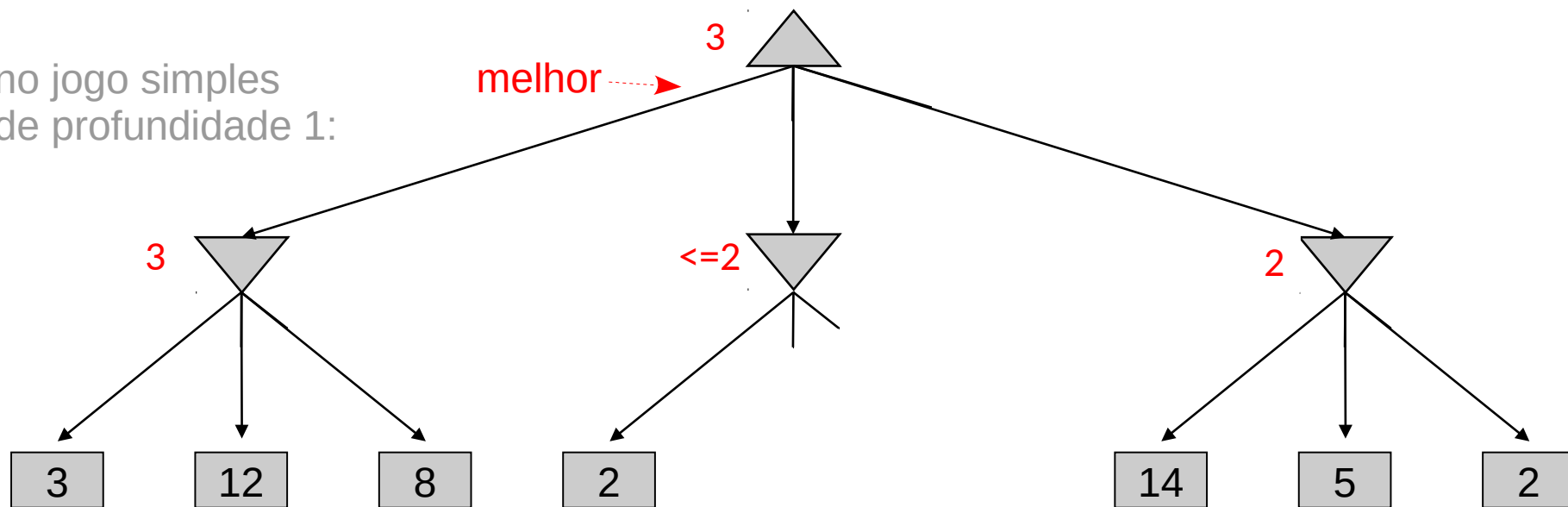
fonte: CS188 Berkeley

poda alfa-beta

retorna o mesmo resultado do minimax

mas não expande caminhos que não podem influir no resultado

no jogo simples
de profundidade 1:

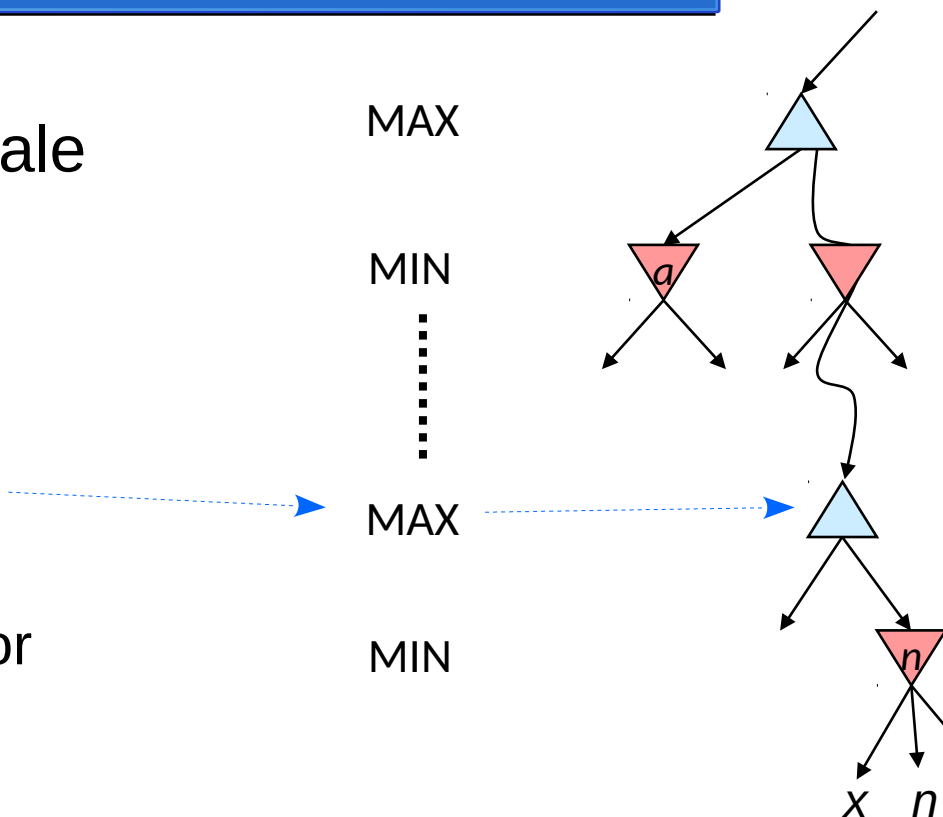


poda pelo menor valor – nó MIN

- se o valor de n for pior (menor) do que a , não vale a pena expandir mais irmãos de x e n

porque MAX, na jogada acima, nunca há-de escolher esse ramo

(comparação com melhor valor de MAX é feita no ciclo do MIN)

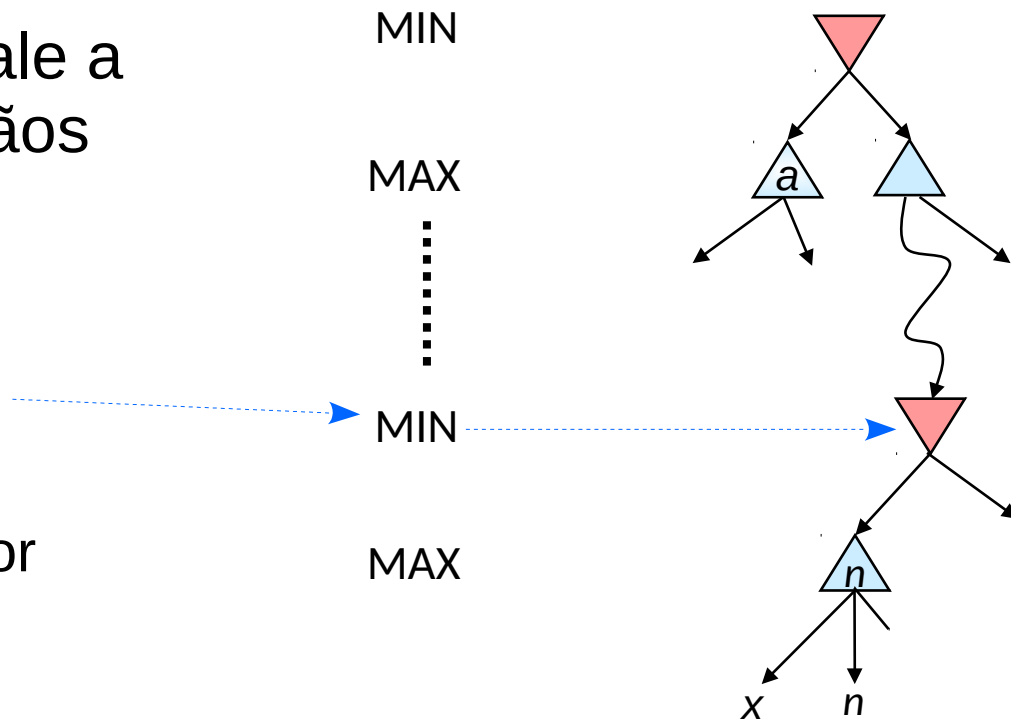


poda pelo maior valor – nó MAX

- se o valor de n for pior (maior) do que a , não vale a pena expandir mais irmãos de x e n

porque MIN, na jogada acima, nunca há-de escolher esse ramo

(comparação com melhor valor de MIN é feita no ciclo do MAX)



o alfa e o beta

- α = valor da melhor escolha (maior valor) do MAX até agora
- β = valor da melhor escolha (menor valor) do MIN até agora

pára a expansão do nó atual assim que o valor deste é pior do que α ou β , conforme jogada de MAX ou de MIN respetivamente

function ALPHA-BETA-SEARCH($state$) **returns** an action
 $v \leftarrow \text{MAX-VALUE}(state, -\infty, +\infty)$
 return the *action* in $\text{ACTIONS}(state)$ with value v

function MAX-VALUE($state, \alpha, \beta$) **returns** a utility value
 if $\text{TERMINAL-TEST}(state)$ **then return** $\text{UTILITY}(state)$
 $v \leftarrow -\infty$
 for each a **in** $\text{ACTIONS}(state)$ **do**
 $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(\text{RESULT}(s, a), \alpha, \beta))$
 if $v \geq \beta$ **then return** v ◀ ponto de poda
 $\alpha \leftarrow \text{MAX}(\alpha, v)$
 return v

function MIN-VALUE($state, \alpha, \beta$) **returns** a utility value
 if $\text{TERMINAL-TEST}(state)$ **then return** $\text{UTILITY}(state)$
 $v \leftarrow +\infty$
 for each a **in** $\text{ACTIONS}(state)$ **do**
 $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(\text{RESULT}(s, a), \alpha, \beta))$
 if $v \leq \alpha$ **then return** v ◀ ponto de poda
 $\beta \leftarrow \text{MIN}(\beta, v)$
 return v

complexidade temporal da poda alfa-beta

se sucessores estiverem por ordem aleatória

$$O(b^{3m/4})$$

mas...

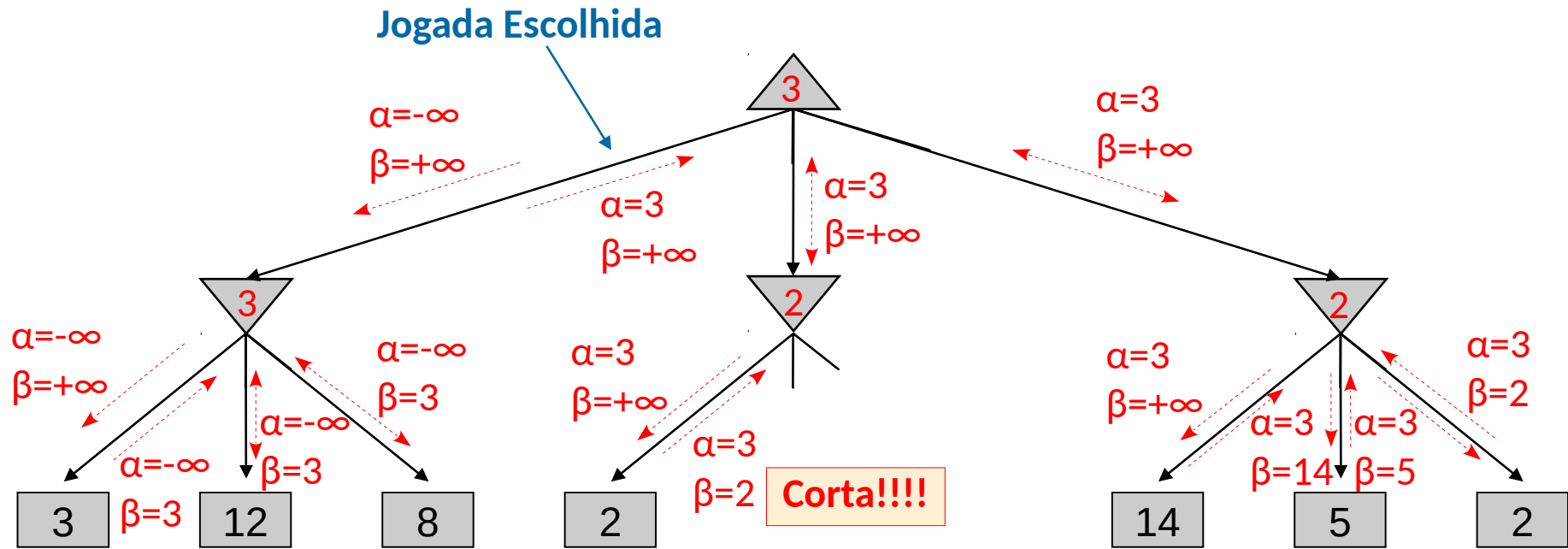
se os sucessores estiverem ordenados (melhores 1º)

$$O(b^{m/2})$$

ex: no xadrez pode preferir-se captura-ameaça-avanço-recuo
o que aproxima a cerca 2x esse valor

extra

valores de alfa e beta no jogo simples

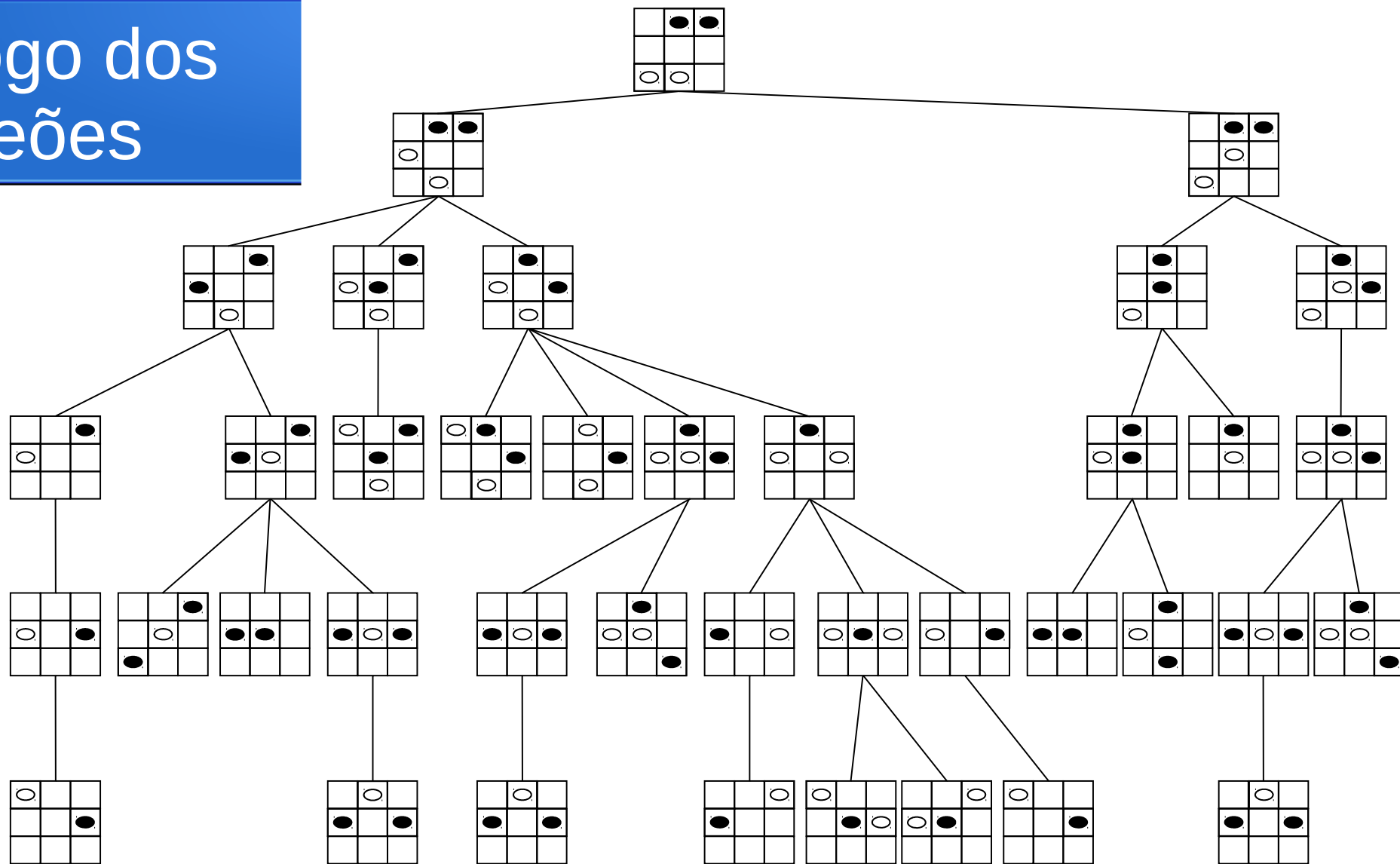


jogo dos peões (slides seguintes)

jogar peões de
xadrez
ganha o 1º a atingir
o lado oposto

- fazer a análise da poda alfa-beta
 - marcar os valores de alfa e beta na descida (chamada) e no retorno
- verificar qual é a ordenação da árvore de procura de modo a que a poda alfa-beta corte o máximo de nós possível

jogo dos peões



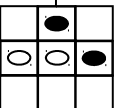
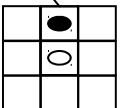
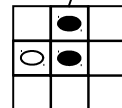
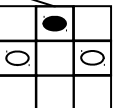
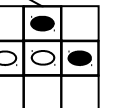
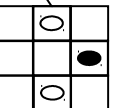
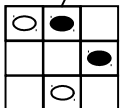
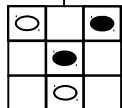
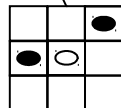
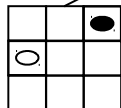
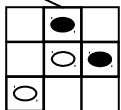
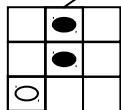
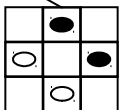
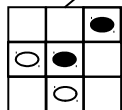
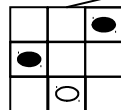
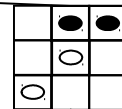
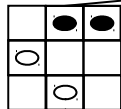
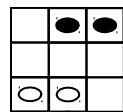
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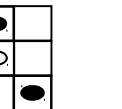
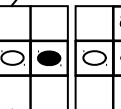
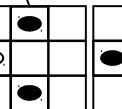
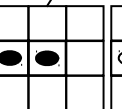
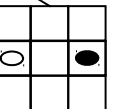
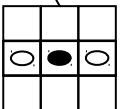
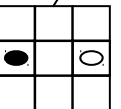
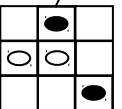
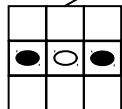
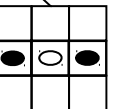
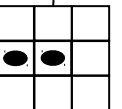
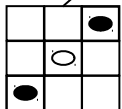
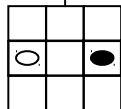


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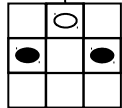
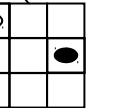
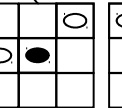
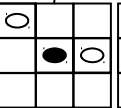
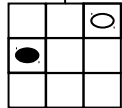
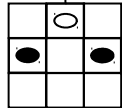
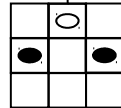
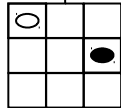
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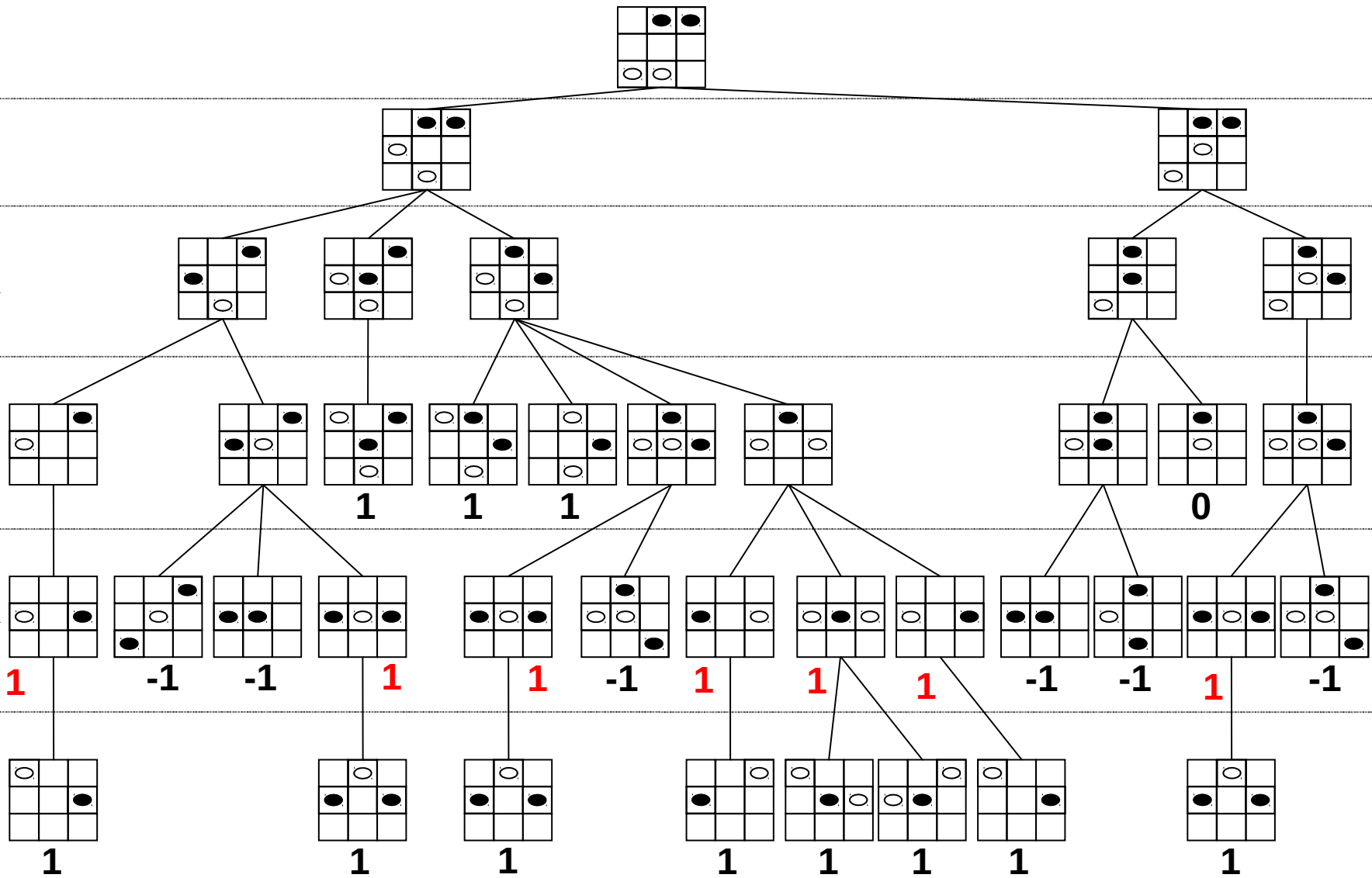
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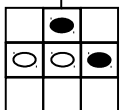
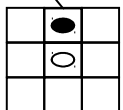
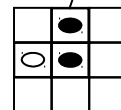
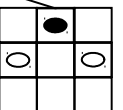
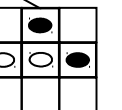
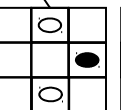
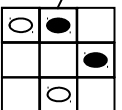
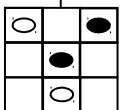
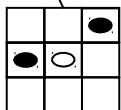
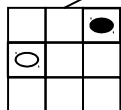
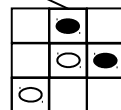
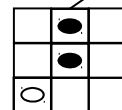
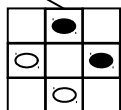
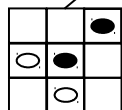
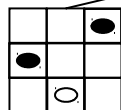
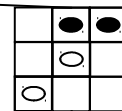
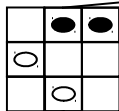
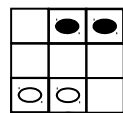
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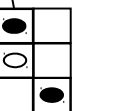
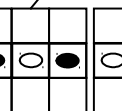
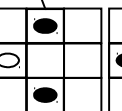
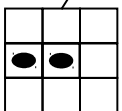
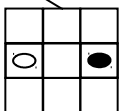
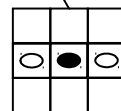
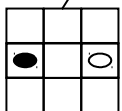
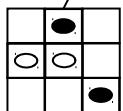
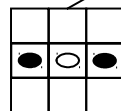
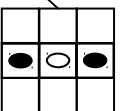
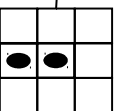
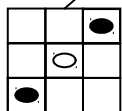
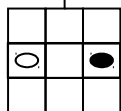
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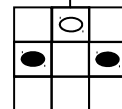
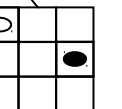
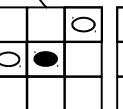
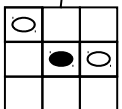
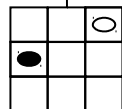
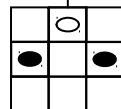
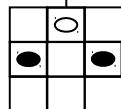
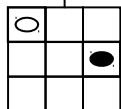
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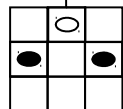
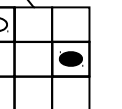
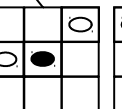
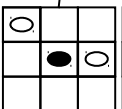
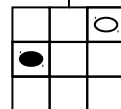
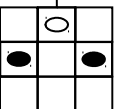
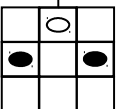
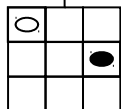
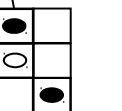
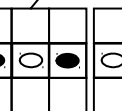
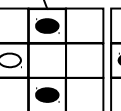
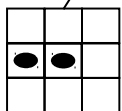
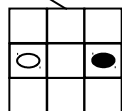
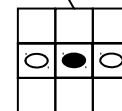
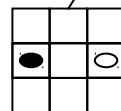
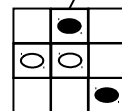
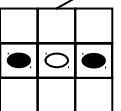
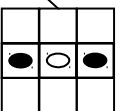
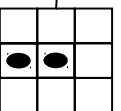
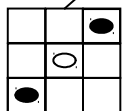
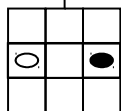
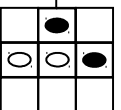
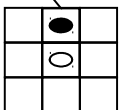
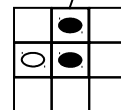
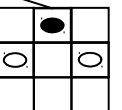
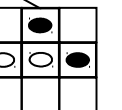
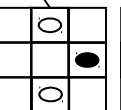
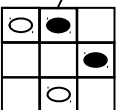
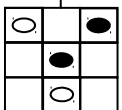
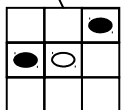
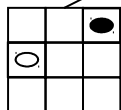
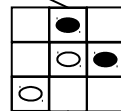
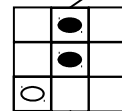
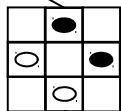
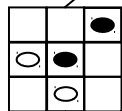
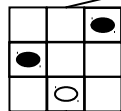
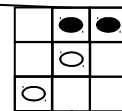
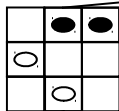
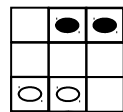
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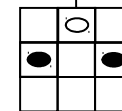
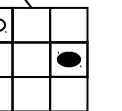
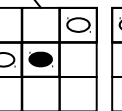
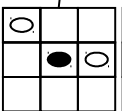
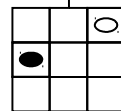
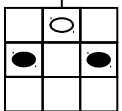
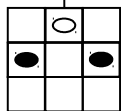
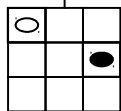
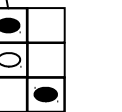
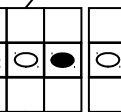
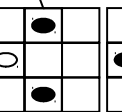
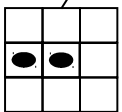
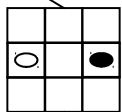
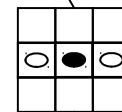
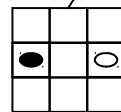
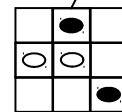
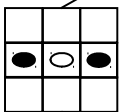
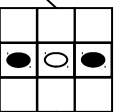
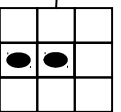
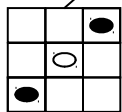
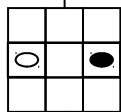
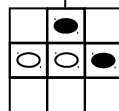
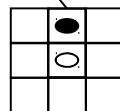
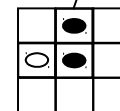
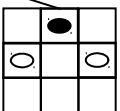
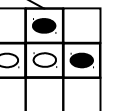
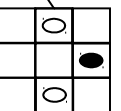
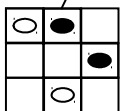
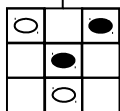
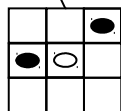
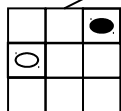
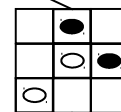
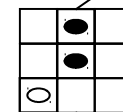
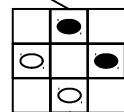
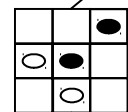
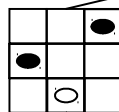
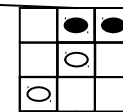
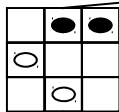
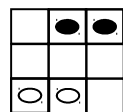
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