Counting Methods Practice Exam

Use the following information to answer the next two questions.

Three rows of a pattern are shown below.

Row 1	$1 \times 8 + 1 = 9$
Row 2	$12 \times 8 + 2 = 98$
Row 3	$123 \times 8 + 3 = 987$

1. The **fifth** row of the pattern will be

A. $1\,234 \times 8 + 4 = 9876$
B. $1\,234 \times 8 + 5 = 9876$
C. $12\,345 \times 8 + 4 = 9876$
D. $12\,345 \times 8 + 5 = 98765$

2. If the number 8 in the pattern above is replaced by the number 9 as shown below, **describe** a pattern that could be used to calculate the value of row 7 (*this isn't a multiple choice question)*

Row 1 $1 \times 9 + 1 =$ _____

Row 2 $12 \times 9 + 2 =$ _____

Row 3 $123 \times 9 + 3 =$_____

Use the following information to answer the next question.

The intention of a particular two-player game is to create a line of four adjacent squares using the same letter. To play, each player takes turns placing their first initial somewhere on a six-by-six grid. Margaret and Gerda have started playing this game, as shown on the grid below.

Row \ Column	1	2	3	4	5	6
1	G					
2		G				
3			G	G		
4			M	M	G	
5		M				
6	M	M	G			

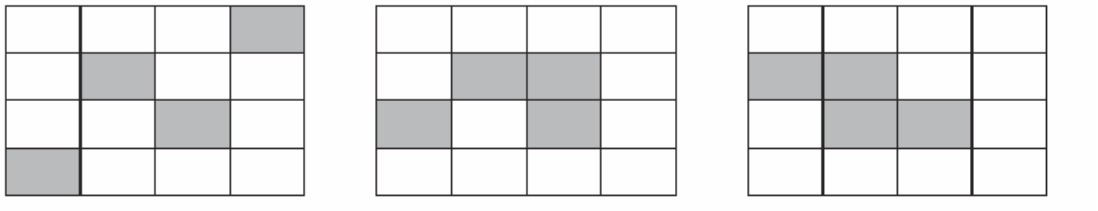
Numerical Response 1

1. It is Margaret's turn, and she determines that she can guarantee a win by placing the letter M in

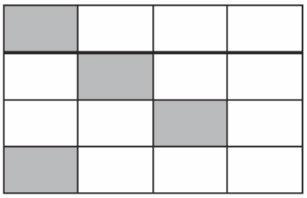
Row _____ (place in the first box of the numerical response)
Column _____ (place in the second box of the numerical response)

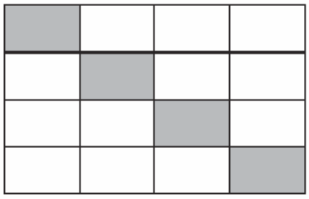
Use the following information to answer the next question

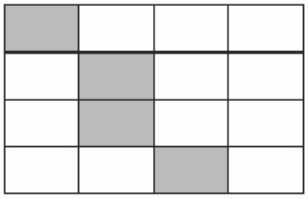
A pattern of pictures is shown below. The first picture is the original. In each subsequent picture, each shaded square has stayed in the same place or moved to a square horizontally, vertically, or diagonally adjacent to its previous location. The shaded square undergoes the same movement in each subsequent step.

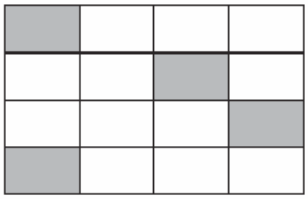


3. Which of the following pictures is next in the pattern?

A. 

B. 

C. 

D. 

4. The number of ways 5 children can be arranged in a row is

A. 5
B. 25
C. 120
D. 24

5. The number of different 5-letter arrangements of all the letters in the word **A P P L E** is

A. 5
B. 10
C. 60
D. 120

6. A new model of a car is available in five colours, three different engine types, and with four different option packages. How many different types of this car are available?

A. $5 \times 3 \times 4$
B. 5!3!4!
C. 5!
D. ${}_5C_3$

7. A fine restaurant provides an evening of dining for one price. The menu lists 6 appetizers, 4 main courses and 5 desserts. If a person must choose one item from each category, the number of ways a meal may be ordered is

A. 15
B. 3
C. 120
D. 40

8. A photographer has 7 people to photograph and wishes to photograph them 3 at a time arranged in a line. How many different arrangements are possible?

A. 21
B. 35
C. 210
D. 840

9. How many 2-letter arrangements are there using the letters in the word **M A T R I X**?

A. 30
B. 32
C. 36
D. 60

10. To evaluate the expression ${}_{12}C_4$ one could simplify

A. $\frac{12!}{4!}$

B. $\frac{12!}{8!}$

C. $\frac{12!}{8!4!}$

D. 4! 8!

11. A wall has space for five pictures. If there are seven different pictures available to put on the wall, in how many different ways can the pictures be arranged on the wall?

A. 5!

B. 7×5

C. ${}_7C_5$

D. ${}_7P_5$

12. The number of different arrangements of all the letters in the word **T I N N I N G** is

A. 12

B. 112

C. 420

D. 5040

13. How many arrangements of all the letters in the word **O P T I C A L** are there?

A. ${}_7C_7$

B. $\frac{{}_7P_7}{2}$

C. $\frac{{}_7C_7}{2}$

D. ${}_7P_7$

14. The number of arrangements of the letters in the word **PARKING** that begin with **P** is

A. 720
B. 120
C. 240
D. 480

15. Using all the letters in the word **COURTING**, the number of words that begin and end with a vowel are

A. 4320
B. 1440
C. 5040
D. 2520

16. $\dfrac{(n-1)!}{(n-3)!}$ is equal to

A. $(n-3)(n-2)$
B. $n\,(n-1)$
C. $(n-1)(n-2)$
D. $\dfrac{1}{(n-2)(n-3)}$

17. The number of ways a committee of 5 people can be selected from 7 males and 6 females, if the committee must contain <u>at least</u> 4 females, is

A. 2520
B. 105
C. 630
D. 111

18. The value of ${}_nC_3$ is

A. $\dfrac{n(n-1)(n-2)}{6}$
B. $\dfrac{n!}{3!}$
C. $\dfrac{n!}{(n-3)!}$
D. $\dfrac{n-3}{6}$

19. Assume that in a given set of 3-digit area codes, the middle digit of each code is either "0" or "1." Which of the following conditions on the digits would result in exactly 180 area codes?

A. The first digit cannot be zero, and the third digit must be different from the first.
B. There are no restrictions on the possible values.
C. All three digits must be different.
D. The first digit cannot be zero.

Use the following information to answer the next question.

A student makes the following statement.

"VOTE compares to VETO as the number 8570 compares to the number _____."

Numerical Response 2

2. The 4-digit number that completes the statement above is _____.

Use the following information to answer the next question.

A new licence plate in Alberta consists of three letters followed by four numbers. Letters are chosen from a list of 24 acceptable letters that may be repeated.

Maureen wants the first letter on her licence plate to be an M, which is an acceptable letter, and she also wants the four numbers to match the last four digits of her cell phone number.

Numerical Response 3

3. The number of licence plates that will meet Maureen's criteria is _____.

20. What is the number of distinguishable arrangements of the letters in the word **SYLLABLE** if all of the letters must be used in each arrangement?

A. $\frac{8!}{3!}$
B. 8!
C. 6!
D. 8! – 3!

21. In how many ways can a gymnastics class of 15 gymnasts be divided into 3 groups so that 3 gymnasts are in one group, 5 gymnasts are in a second group, and 7 gymnasts are in a third group?

A. 105
B. 2 184
C. 360 360
D. 3 628 800

22. Pat's parents Asked him to complete the following seven chores:

- take out the garbage
- clean his bedroom
- vacuum the living room
- drive his brother to a piano lesson
- wash the car
- do the dishes
- clean the bathroom

They asked Pat to drive his brother to a piano lesson first and then to complete the rest of the chores in any order. In how many different orders could Pat complete the chores?

A. ${}_7C_6$

B. ${}_7P_6$

C. ${}_6C_6$

D. ${}_6P_6$

23. For a certain college program, a student must take an English course. The student must also take any 4 of the following courses: Mathematics, Chemistry, Physics, Psychology, Biology, and French. The number of possible 5-course programs that a student could take to complete this program is

A. ${}_6C_4$

B. ${}_7C_5$

C. ${}_6P_4$

D. ${}_7P_5$

Use the following information to answer the next question.

A student is classifying the following contexts that require the use of either permutations or combinations.

Context A	Dialing a 10-digit telephone number with distinct digits
Context B	Choosing 5 people for a committee
Context C	Selecting 4 fruits to put in a salad
Context D	Opening a lock with a 3-number combination

Numerical Response 4

4. For each context, use a 1 to indicate if it should be solved using a permutation and use a 2 to indicate if it should be solved using a combination.

Context A would be solved using a _____ (Record in the **first** column)

Context B would be solved using a _____ (Record in the **second** column)

Context C would be solved using a _____ (Record in the **third** column)

Context D would be solved using a _____ (Record in the **fourth** column)

Use the following information to answer the next question.

A committee of 3 girls and 2 boys is to be chosen from a group of 9 girls and 7 boys. The total number of different committees that can be formed can be expressed in the form

$$ {}_{w}C_{x} \; {}_{y}C_{z} $$

where ${}_{w}C_{x}$ represents the number of possible choices of girls for the committee and ${}_{y}C_{z}$ represents the number of possible choices of boys for the committee.

Numerical Response 5

5. The values of *w, x, y,* and *z* are _____, _____, _____, and _____, respectively.

Numerical Response 6

6. A dance class has 14 students. From these, 3 dancers are to be chosen to do a demonstration. The number of different groups of 3 that can be formed is _____.

Written Response:

1. At a private school, each student must wear a school uniform that includes a dress shirt and pants. The dress shirt can be white, gray, or light blue. The pants can be navy or black. Use a graphic organizer to show the different possible variations of the uniform.

2. A hotel offers free breakfast to its guests. One morning the hotel has 3 different kinds of juice, 4 different kinds of cereal, and 2 different types of pastries available. If a guest can choose one kind of juice, one kind of cereal and one type of pastry, how many different possible breakfasts can be ordered?

3. Determine the number of six-digit odd numbers that can be created using the digits 0 to 9 without repetition. Describe any restrictions that exist.

4. Determine the number of arrangements of all the letters in the word TATTOO.

5. Determine the number of 3-letter arrangements of the letters of the word DIPLOMA.

6. A 7-player volleyball team must stand in a straight line for a picture.

a. Determine the number of different arrangements that can be made for the picture.

b. Determine the number of arrangements that can be made for the picture if the tallest player must stand in the middle.

7. Ralph knows that there are 15 distinguishable possibilities when 2 people are chosen to form a committee from a particular group of n people.

a. Describe what values of n would be admissible in this problem.

b. Determine the number of people in the larger group, n.

Answer Key

1. D
2. The row number tells you the number of ones that occur in the answer followed by a single 0. Therefore, the value of row 7 would be seven ones followed by a single 0; i.e., 11 111 110.
3. A
4. C
5. C
6. A
7. C
8. C
9. A
10. C
11. D
12. C
13. D
14. A
15. A
16. C
17. D
18. A
19. D
20. A
21. C
22. C
23. A

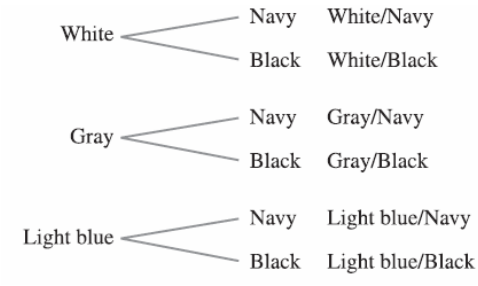
NUMERICAL RESPONSE:

1. 42
2. 8075
3. 576
4. 1221
5. 9372
6. 364

Written Response

Shirts \ Pants	Navy	Black
White	White/Navy	White/Black
Gray	Gray/Navy	Gray/Black
Light blue	Light blue/Navy	Light blue/Black

1)



2) 24
3) 8 • 8 • 7 • 6 • 5 • 5 = 67 200 The sixth digit is restricted if the number is going to be odd. The first digit is also restricted – it cannot be 0, nor can it be the same digit as the sixth digit.
4) 60
5) 210
6) A) 5040 B) 720
7) A) The value of n represents the number of people in the larger group. This must be a positive number, as mathematically you cannot have a negative number of people. Also, n must be greater than 2 because it is impossible to select two objects from a group smaller than what is needed.
B) $n = 6$ since $n \neq -5$. Therefore, the larger group contains 6 people.