LOGARITHMIC FUNCTIONS PRACTICE EXAM

Math 30-2

Name: _____ Date:_____

Multiple Choice & Numerical Response

Identify the choice that best completes the statement or answers the question.

1. The x-intercept of $f(x) = -\frac{1}{3} \ln x$.

   A. -1
   B. $-\frac{1}{3}$
   C. 0
   D. 1

2. Predict the end behavior of $f(x) = 10 \log x$.

   A. curve extends from quadrant I to quadrant II
   B. curve extends from quadrant I to quadrant IV
   C. curve extends from quadrant IV to quadrant I
   D. curve extends from quadrant II to quadrant I

3. Which function will have the slowest increase in the y-values?

   A. $y = \frac{1}{2} \ln x$
   B. $y = 9 \ln x$
   C. $y = \frac{1}{4} \ln x$
   D. $y = 20 \ln x$

4. The approximate value of y in the exponential equation $40 = 10^y$.

   A. 0.6
   B. 1.1
   C. 1.6
   D. 2.1

Use the following information to answer the next question

A student made the following statements about the graph of the exponential function $f(x) = \log_b x$, where $b > 1$.

1. The graph has a vertical asymptote at $x = 0$
2. The graph has a horizontal asymptote at $y = 0$
3. The domain is $y \geq 0$
4. The y-intercept is at 1
5. The x-intercept is at 1
6. The range is $y \in R$.

Numerical Response 1

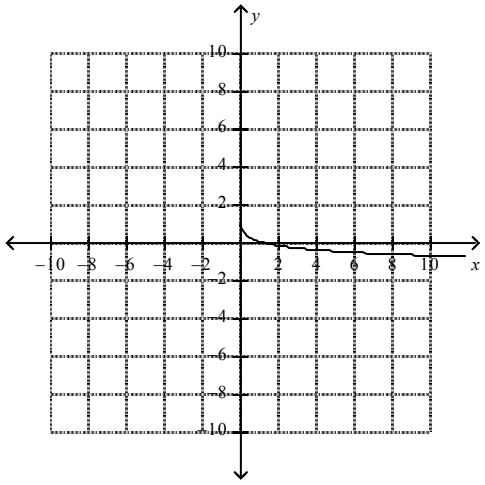
The three statements that are **false** for the graph f(x) are numbers _____, ____ and _____. Record your answer in ascending order (smallest to largest)

5. Evaluate the logarithmic expression $\log_{11} 1331$.
   A. 0
   B. 1
   C. 2
   D. 3

6. Evaluate the logarithmic expression $\log_{\left(\frac{1}{4}\right)} \frac{1}{256}$.
   A. 2
   B. 3
   C. 4
   D. 5

7. Which expression is greatest?
   A. $\log_3 3$
   B. $\log_3 4$
   C. $\log_3 5$
   D. $\log_3 6$

Use the following information to answer the next question



8. Match the following graph with its function.

   A. $y = -\frac{1}{3}\ln x$

   B. $y = 3\log x$

   C. $y = -\frac{1}{3}(3)^x$

   D. $y = 0.3(10)^x$

9. Expressed in exponential form, $-5 = \log_2(\frac{1}{32})$

   A. $32 = 2^5$

   B. $(\frac{1}{32})^{-5} = 2$

   C. $2^{-5} = (\frac{1}{32})$

   D. $5^{-2} = \frac{1}{32}$

10. Which expression is equivalent to $\ln\left(\frac{8}{5}\right)$?

    A. ln 8 – ln 5
    B. ln 5 – ln 8
    C. 8 ln 5
    D. ln 0.625

11. Evaluate: $\log_5\left(\frac{100}{4}\right)+\log_5\left(\frac{4}{100}\right)$

A. -4
B. 4
C. 2
D. 0

12. Which value is the best estimate for x in $7 = 20\left(\frac{1}{3}\right)^x$?

A. -0.05
B. 0.05
C. 0.96
D. 1.05

13. The equation of the logarithmic function that models a data set is $y = 43.9 - 8.7 \ln x$. Determine the value of y when $x = 5.5$.

A. $y = 23$
B. $y = 25$
C. $y = 27$
D. $y = 29$

14. The equation of the logarithmic function that models a data set is $y = 43.9 - 8.7 \ln x$. Determine the range of this function.

A. $\{y \mid y \in R\}$
B. $\{y \mid y > 0, y \in R\}$
C. $\{y \mid y < 43.9, y \in R\}$
D. $\{y \mid y > -8.7, y \in R\}$

Numerical Response 2

A partial graph of $y = \log_a x$ passes through the point (8, 1.5). Correct to the nearest whole number, the value of a is _____

Use the following information to answer the next question

The following data set involves logarithmic function.

x	1	2	3	4	5	6
y	32.0	19.5	12.2	7.0	3.0	–0.3

15. Determine the equation of the logarithmic regression function for the data.
    A. $y = -18 - 32 \ln x$
    B. $y = -18 + 32 \ln x$
    C. $y = 32 + 18 \ln x$
    D. $y = 32 - 18 \ln x$

Use the following information to answer the next question

The following data set involves logarithmic growth.

x	1	5	10	20	50	100
y	0.0	0.7	1.0	1.3	1.7	

Numerical Response 3

To the nearest whole number, the missing value is _____

16. What is the sound level of a noise ten times as intense as a conversation at 68 dB?
    Recall that sound level, β, in decibels, is defined by the equation
    $\beta = 10(\log I + 12)$
    where I is the sound intensity in watts per square metre.

    A. 680 dB
    B. 58 dB
    C. 69 dB
    D. 78 dB

17. Evaluate $5 \log 100\,000 - \log \frac{3}{3000}$

    A. 28
    B. 31
    C. 34
    D. 4

18. Which logarithmic expression is **not** equivalent to the others?

A. $\frac{\log 5}{\log 3}$

B. $\log_9 25$

C. $\frac{\log 25}{2\log 3}$

D. $\log_{27} 100$

Numerical Response 4

An 8.5 earthquake compared to an 8.0 earthquake is _____ times more intense. Round your answer to the nearest hundredth.

WRITTEN RESPONSE

Please show all work for full marks. Full algebraic solutions are required (no marks for graphing).

1. Write each of the following logarithmic equations in exponential form.

a. $\log(1000) = 3$

b. $\log_3 9 = 2$

c. $\log_b 25 = 6$

2. Express $\log 12$ in **two** different logarithmic forms.

3. Use the laws of logarithms (algebraically) to determine the value of each of the following.

a. $\log_4 32 + \log_4 2$

b. $\log 1200 - \log 120$

4. Solve each equation (algebraically). Round answer to three decimals.

a. $30 = \left(\frac{1}{2}\right)^{x+2}$

b. $75 = 4(2)^{x}$

5. Describe how to determine the solution of $3^{(x-2)} = 5^{(x+1)}$ graphically.

[BONUS] Determine the solution of $3^{(x-2)} = 5^{(x+1)}$ algebraically.

6. The euphotic zone the upper 200m layer of the oceans. Very little sunlight penetrates deeper than 200, so most plants live in the euphotic zone. As a result, 70% of all photosynthesis on Earth occurs in the euphotic zone of the oceans. The following table gives light penetration data for the location in the Pacific Ocean.

Depth (m)	Penetration of Sunlight (%)
0.01	100
20	54.37
40	29.57
60	16.08
80	8.74
100	4.76

- Determine the equation of the logarithmic regression function that best models the data. Round your values to the nearest hundredth.

- Using your equation, determine the depth with a sunlight penetration of 35%. Express your answer to the nearest tenth of a meter.

- Using your equation, determine the amount of sunlight that penetrates to a depth of 200m at this location. Express your answer to the nearest hundredth of a percent.

Mathematics 30–2 Formula Sheet

Relations and Functions

Graphing Calculator Window Format

x: $[x_{min}, x_{max}, x_{scl}]$

y: $[y_{min}, y_{max}, y_{scl}]$

Exponents and Logarithms

$$y = a^x \leftrightarrow x = \log_a y$$

$$\log_b c = \frac{\log_a c}{\log_a b}$$

Laws of Logarithms

$$\log_b(M \cdot N) = \log_b M + \log_b N$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b(M^n) = n \log_b M$$

Exponential functions

$$y = a \cdot b^x$$

Sinusoidal functions

$$y = a \cdot \sin(bx + c) + d$$

$$\text{Period} = \frac{2\pi}{b}$$

Quadratic equations

For $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Probability

$n! = n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$,
where $n \in N$ and $0! = 1$

$$_nP_r = \frac{n!}{(n-r)!}$$

$$_nC_r = \frac{n!}{(n-r)!r!}$$

$$_nC_r = \binom{n}{r}$$

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

Logical Reasoning

A'	Complement
$\varnothing$	Empty set
$\cap$	Intersection
$\subset$	Subset
$\cup$	Union

Answer Key

1. D
2. C
3. C
4. C
5. D
6. C
7. D
8. A
9. C
10. A
11. D
12. C
13. D
14. A
15. D
16. D
17. A
18. D

NR 1: 234
NR 2: 4
NR 3: 2
NR 4: 3.16

Written Response:
1a) $10^3 = 1000$ b) $3^2 = 9$ c) $b^6 = 25$

2) Answers may vary. $\log 2 + \log 6$ and $\log 24 - \log 2$

3a) 3 b) 1

4a) -6.907 b) 4.229

5) -7.45

6) a. y=59.87-9.61lnx

b. 13.3 m

c. 8.96% visibility