Fluid Dynamics

Chapter 5

Viscous Flows and Turbulence

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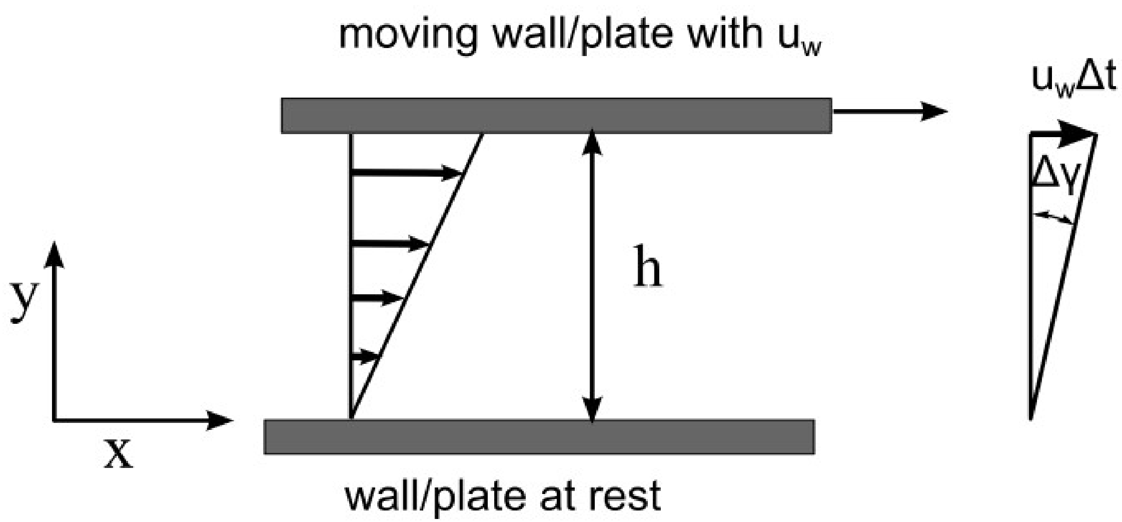
Viscosity

- Now: We start to consider **friction!**
- Important, if friction forces are of the same order of magnitude as inertial forces

  Example:
  Plates with different velocities
  - Upper plate moving with velocity u_w
  - Lower plate at rest

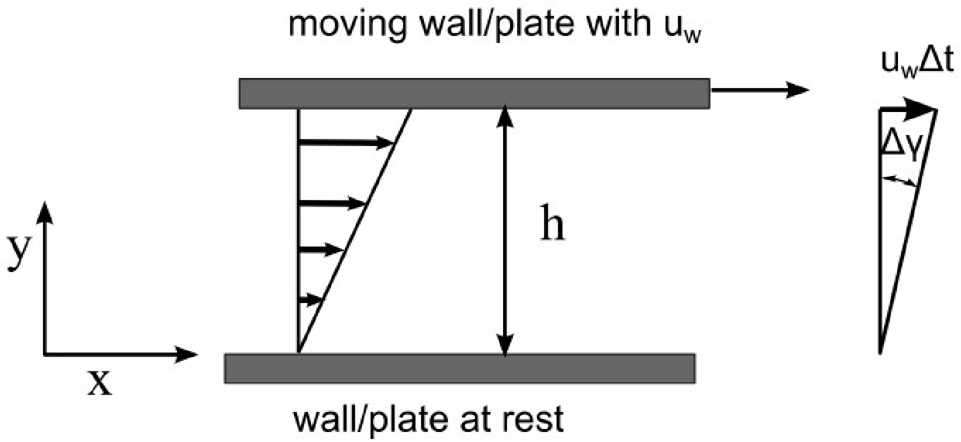


Viscosity

- Fluid elements in direct contact to the wall have the same velocity as the wall: so called *no-slip* boundary condition

  Therefore, the velocities of the fluid are:

  $$\text{lower plate: } u(y=0)=0$$
  $$\text{upper plate: } u(y=h)=u_w$$



- Due to friction between the fluid layers, a linear velocity profile is established (*):

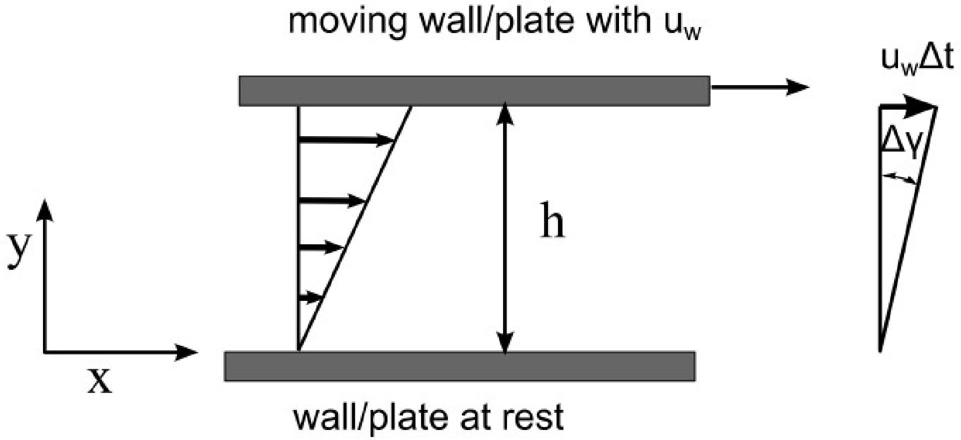
$$u(y)=u_w\frac{y}{h}$$

(*) We check this later.

Viscosity

- The increase of velocity can be described with the angle Δγ (see sketch):

$$\frac{u_w \cdot \Delta t}{h} = \tan(\Delta \gamma)$$
$$\approx \Delta \gamma \quad \text{if} \quad \Delta \gamma \ll 1$$



- The velocity difference causes a shearing of layers in the fluid. The shear rate is given by:

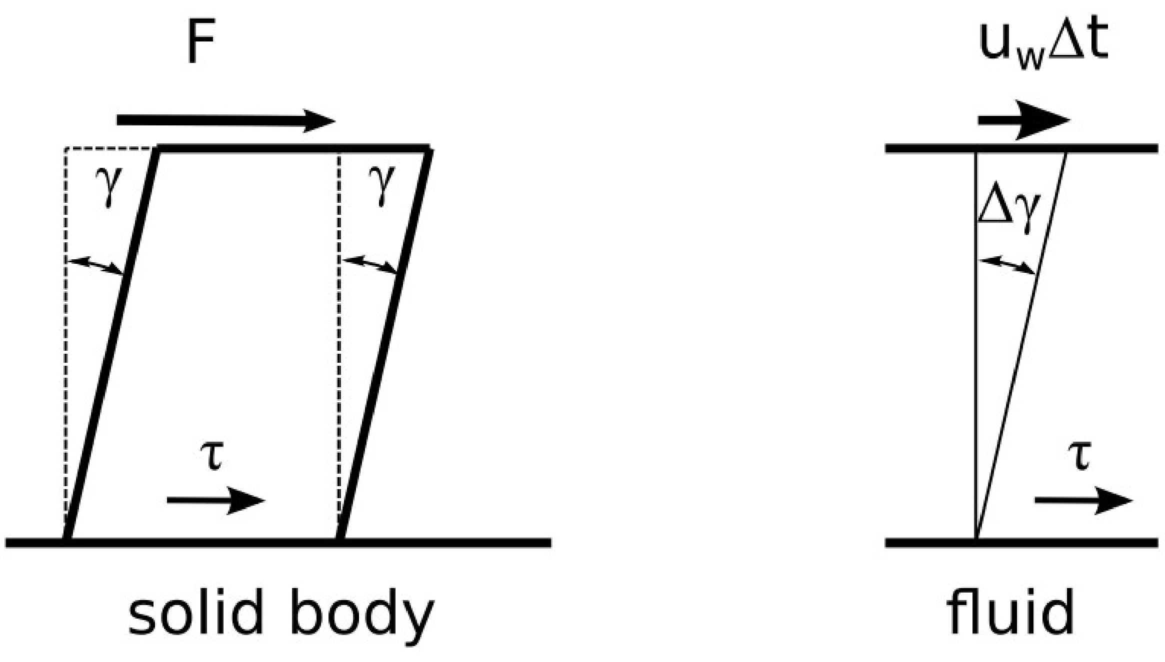
$$\dot{\gamma} = \frac{\Delta \gamma}{\Delta t} = \frac{u_w}{h} = \frac{\mathrm{du}}{\mathrm{dy}}$$

Viscosity

- Analogy: Shear stress in solid bodies and fluids
  - Hooke's law for solid bodies: $\tau = G\,\gamma$

    ⇒ shear angle as influencing parameter
  - Shear stress for Newtonian fluids: $\tau = \mu\,\dot{\gamma} = \mu\,\frac{\mathrm{du}}{\mathrm{dy}}$

    ⇒ Shear rate as influencing parameter



Viscosity

- **Newtonian fluids**:
  Shear stress depends **linearly** on the shear rate:

$$\tau = \mu \dot{\gamma} = \mu \frac{du}{dy}$$

- Factor of proportionality is the material property

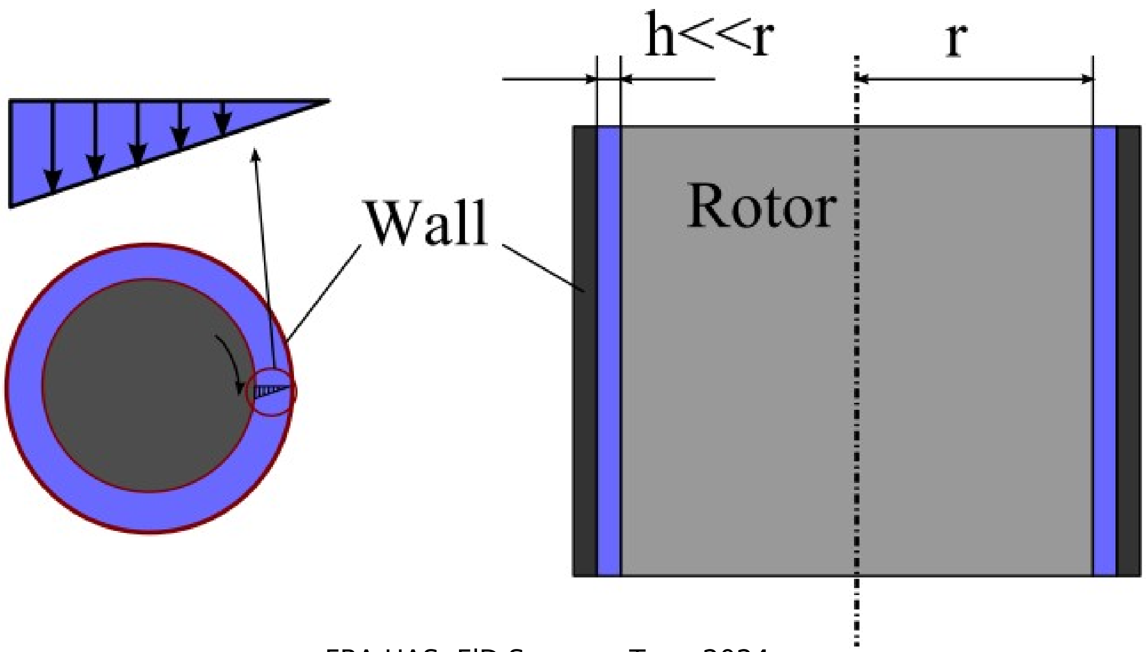
  dynamic viscosity: μ [kg/(ms)] or [Pa s]

- Alternative property:

  kinematic viscosity: $\nu = \mu/\rho$ [m^2/s]

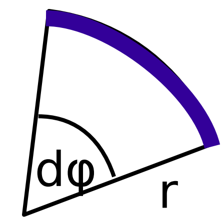
Viscosity Measurement: The Couette Viscosimeter

- Test stand for viscosity measurement
  - Rotation of inner cylinder (height b)
  - Wall around the cylinder at rest. Gap of width h between cylinder and wall
  - Torque is measured as function of rotational speed



Viscosity Measurement: The Couette Viscosimeter

- The viscosity is determined by calculating the shear stress in two different ways
- Consider a fluid layer with radius r and height b. A fluid element in this layer is given by a small angle dφ.



- Area of fluid element: $dA = r\,b\,d\phi$
- Force = stress times area: $dF = \tau\,dA = \tau\,r\,b\,d\phi$
- Torque = force times lever: $dT = r\,dF = \tau\,r^2\,b\,d\phi$
- … for the total circumference:
$$T = \int_0^{2\pi} \tau\,r^2\,b\,d\phi$$
$$= 2\pi\,\tau\,r^2\,b$$
- Solve for τ:
(Shear stress τ as a function of measured torque T)
$$\tau = \frac{T}{2\pi r^2 b}$$

Viscosity Measurement: The Couette Viscosimeter

- Consider the fluid in the gap h between the inner and outer cylinder
- h≪r => the flow behaves like the flow between horizontal plates:

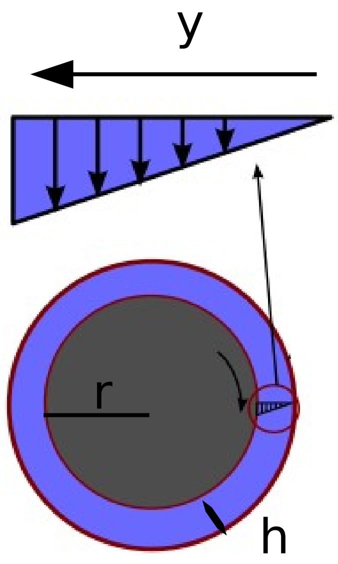
$$u(y)=\frac{u_w}{h}\,y$$

- The wall velocity u_w is a function of the angular velocity ω of the rotating cylinder:

$$u(y)=\frac{u_w}{h}\,y=\frac{r\cdot\omega}{h}\,y \quad\Rightarrow\quad \frac{du}{dy}=\frac{r\cdot\omega}{h}$$

- The shear stress for a Newtonian fluid is:

$$\tau=\mu\frac{du}{dy}=\mu\frac{r\cdot\omega}{h}$$

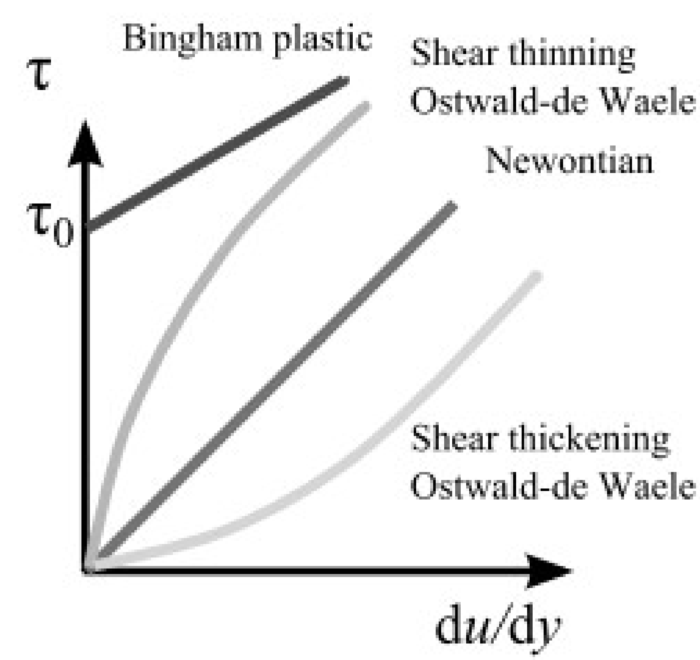


Viscosity Measurement: The Couette Viscosimeter

- Equating both terms for τ, the viscosity of a fluid can be determined:

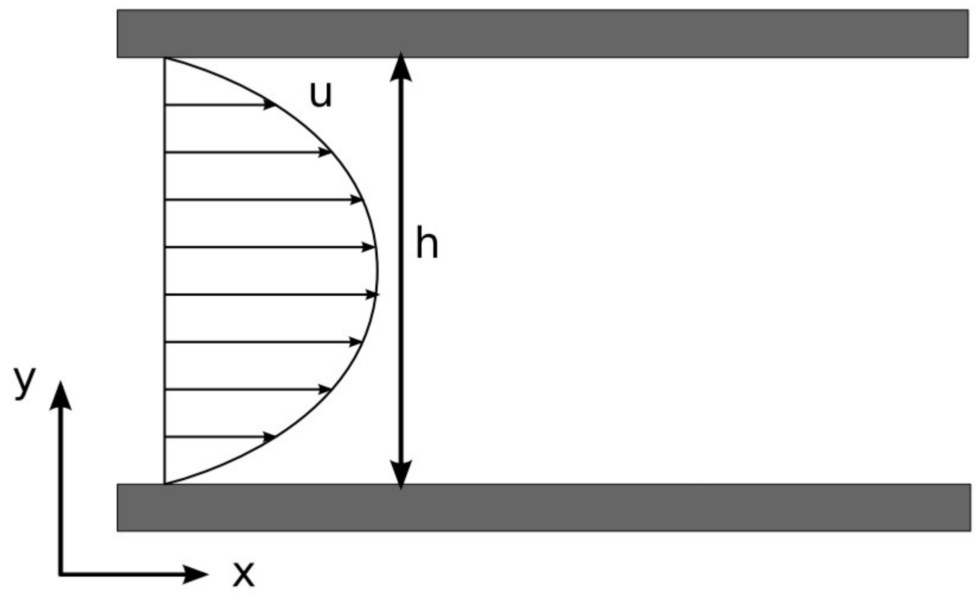
$$\frac{T}{2\pi r^2 b} = \mu \frac{r\,\omega}{h} \quad \Rightarrow \quad \mu = \frac{Th}{2\pi r^3 b\,\omega}$$

- There are not only Newtonian fluids. The shear stress can depend on the shear rate in many ways (example on the right). This is the subject of Rheology.



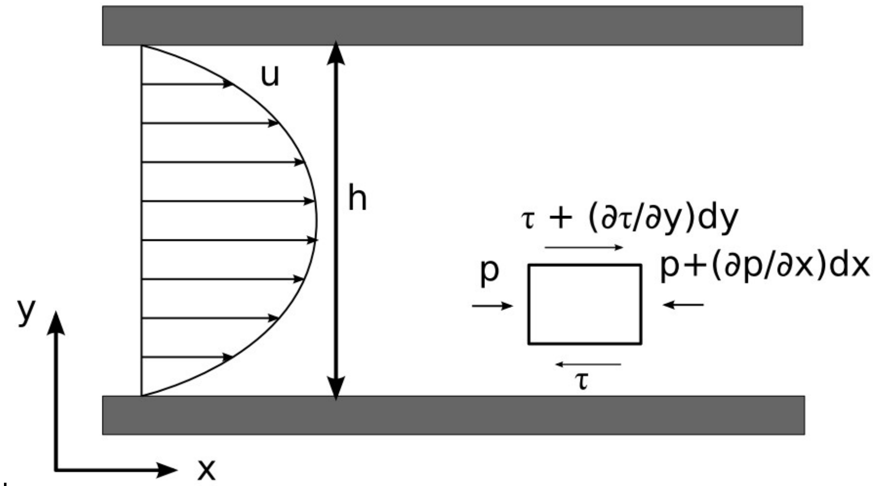
2D Flow Between Two Plates

- Consider parallel plates with a pressure gradient in the x-direction
- Steady state condition
  - Balance of pressure- and friction forces
- Assumption: p = p(x) and v = v(y)
- Since $\tau=\mu \, dv/dy$, tau is a function of y: $\tau=\tau(y)$



2D Flow Between Two Plates

- Consider a moving CV with the dimensions Δx, Δy and Δz in the stream
  - No flow into or out of CV
  - Left boundary of CV at x_0, right boundary at $x_0+\Delta x$
  - Lower boundary of CV at y_0, upper boundary at $y_0+\Delta y$



- Pressure at the right boundary based on Taylor polynomial wrt. x_0:

$$p(x_0+\Delta x)\approx p(x_o)+\frac{\partial p}{\partial x}(x_0+\Delta x-x_0)=p(x_o)+\frac{\partial p}{\partial x}\Delta x$$

- Shear stress at the top from Taylor polynomial wrt. y_0:

$$\tau(y_0+\Delta y)\approx\tau(y_o)+\frac{\partial \tau}{\partial y}(y_0+\Delta y-y_0)=\tau(y_o)+\frac{\partial \tau}{\partial y}\Delta y$$

2D Flow Between Two Plates

- Forces in x-direction on the left and the right:
  - Area left and right:

$$\Delta y \Delta z$$

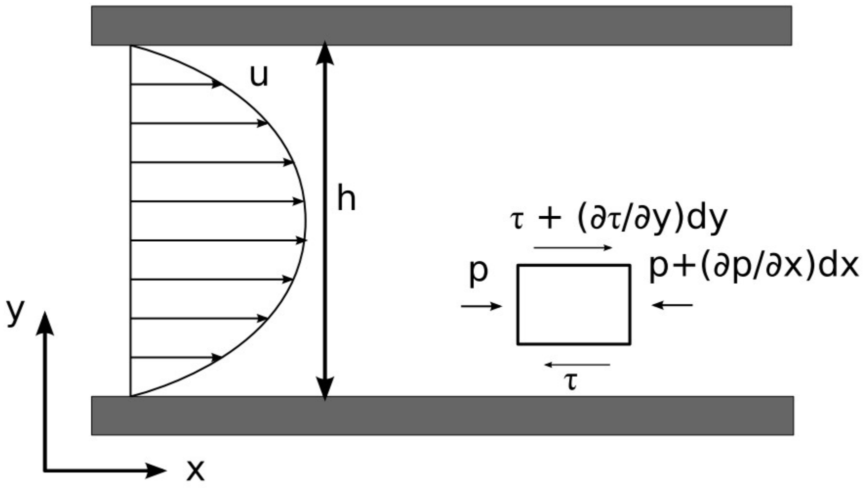
  - Pressure force on the left:

$$p\left(x_0\right)\Delta y \Delta z$$

    - positive x-direction
  - Pressure force on the right:

$$-\left(p\left(x_o\right)+\frac{\partial p}{\partial x}\Delta x\right)\Delta y \Delta z$$

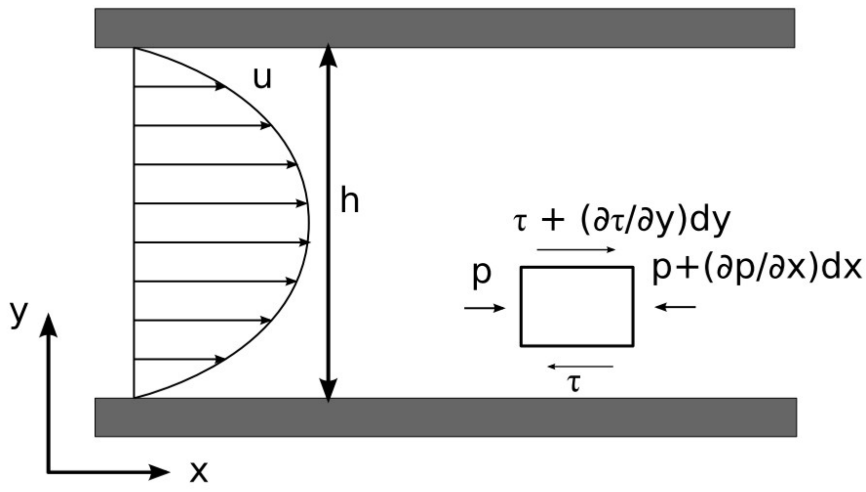
    - negative x-direction!



2D Flow Between Two Plates

- Friction forces in x-direction at the top and bottom
- Area at top and bottom:

$$\Delta x \Delta z$$



- Area vector at the bottom points against the y-direction
  ⇒ negative sign for friction term

$$-\tau\left(y_0\right)\Delta x \Delta z$$

- Area vector at the top points in the y-direction
  ⇒ positive sign for friction term

$$\left(\tau\left(y_o\right)+\frac{\partial \tau}{\partial y}\Delta y\right)\Delta x \Delta z$$

2D Flow Between Two Plates

- Force equation in x-direction:

$$p\,\Delta y\,\Delta z-\left(p+\frac{dp}{dx}\Delta x\right)\Delta y\,\Delta z-\tau\,\Delta x\,\Delta z+\left(\tau+\frac{d\,\tau}{dy}\Delta y\right)\Delta x\,\Delta z=0$$

$$-\left(\frac{dp}{dx}\Delta x\right)\Delta y\,\Delta z+\left(\frac{d\,\tau}{d\,y}\Delta y\right)\Delta x\,\Delta z=0$$

$$-\frac{dp}{dx}+\frac{d\,\tau}{dy}=0$$

- Observation:
  - Left side: function of x!
  - Right side: function of y!

$$\frac{dp}{dx}=\frac{d\,\tau}{dy}$$

- Conclusion: Equality possible only for a constant.

$$\frac{dp}{dx}=\frac{d\,\tau}{dy}=\text{constant}$$

2D Flow Between Two Plates

- Wanted: Function for u(y)
  - Flow velocity at each point of the cross section
- We get this from integrating $\frac{d\tau}{dy}=\mu\frac{d^2u}{dy^2}$ twice:

$$\mu\frac{\mathrm{d}^2u}{dy^2}=\frac{d\tau}{dy}=\frac{dp}{dx} \qquad \Big|\, integrate$$

$$\Rightarrow \quad \mu\frac{du}{dy}=\int\frac{dp}{dx}dy \qquad \Big|\, \frac{dp}{dx}=constant$$

$$\Rightarrow \quad \frac{du}{dy}=\frac{1}{\mu}\frac{dp}{dx}y+\frac{1}{\mu}c_1 \qquad \Big|\, integrate$$

$$\Rightarrow \quad u(y)=\frac{1}{\mu}\frac{dp}{dx}\int y\,dy+\frac{1}{\mu}c_1\int dy$$

$$\Rightarrow \quad u(y)=\frac{1}{2\mu}\frac{dp}{dx}y^2+\frac{1}{\mu}c_1y+c_2$$

2D Flow Between Two Plates

- Calculation of the constants c_1 and c_2 from boundary conditions:
  - Lower plate at rest: u(0)=0

$$0=\frac{1}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\cdot 0^2+\frac{1}{\mu}c_1\cdot 0+c_2 \quad\Rightarrow\quad c_2=0$$

  - Upper plate with velocity $u(h)=u_w$

$$u_w=\frac{1}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\cdot h^2+\frac{1}{\mu}c_1\cdot h$$

$$\Rightarrow\quad \frac{1}{\mu}c_1\cdot h=u_w-\frac{1}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\cdot h^2$$

$$\Rightarrow\quad c_1=\mu\frac{u_w}{h}-\mu\frac{1}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\cdot h=\mu\frac{u_w}{h}-\frac{1}{2}\frac{\mathrm{d}p}{\mathrm{d}x}\cdot h$$

2D Flow Between Two Plates

- Insert c_1 and c_2 into the equation for u(y):

$$u(y) = \frac{1}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}x}y^2 + \frac{1}{\mu}y\left(\mu\frac{u_w}{h} - \frac{1}{2}\frac{\mathrm{d}p}{\mathrm{d}x}\cdot h\right)$$

$$\Rightarrow \quad u(y) = \frac{1}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\left(y^2 - y\cdot h\right) + \frac{u_w}{h}\cdot y$$

- If both plates are at rest ($u_w = 0$):

$$u(y) = \frac{1}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\left(y^2 - y\cdot h\right)$$

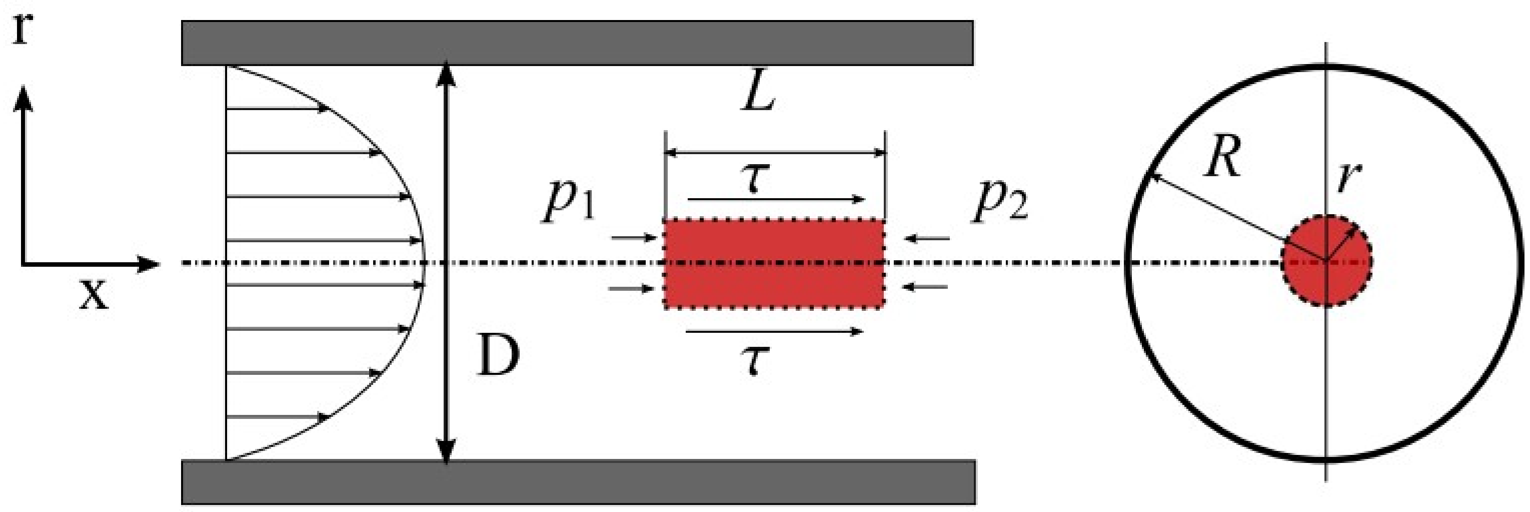
- Without pressure gradient (dp/dx = 0):

$$u(y) = \frac{u_w}{h}\cdot y$$

  – This is the linear velocity profile we used before (see p.3).

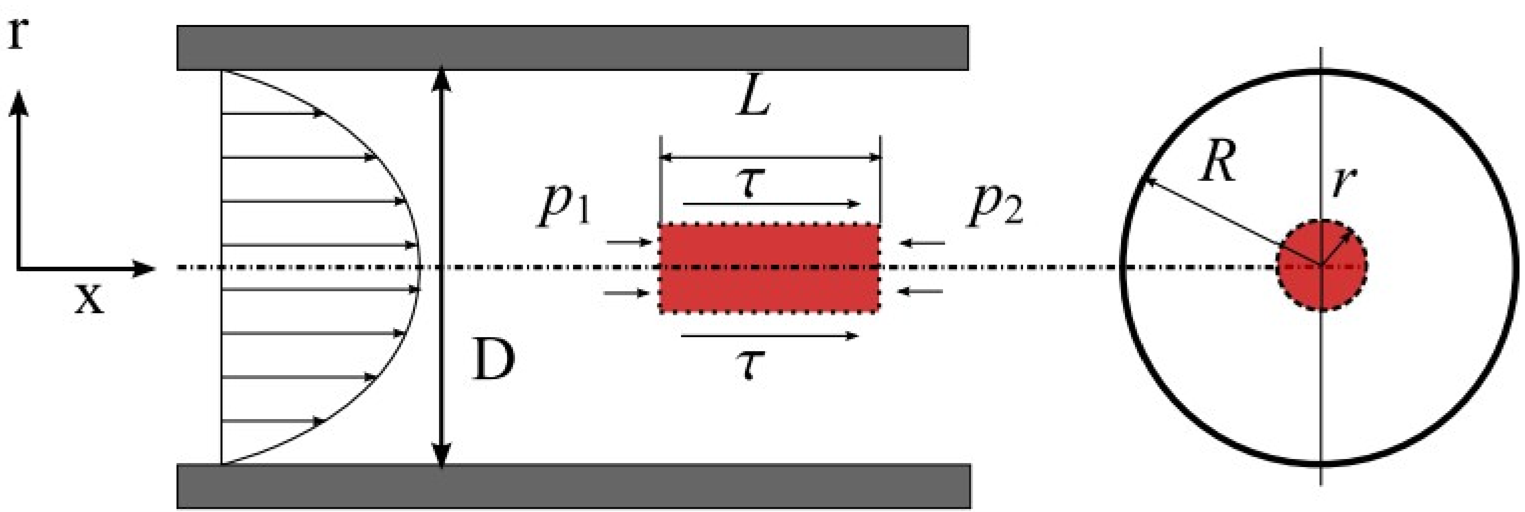
Laminar Pipe Flow

- Steady state, laminar, fully developed flow:
  - Fully developed: velocity profile does not change in flow direction
  - Velocity is only a function of radius r:
    - $u=u(r)$
    - $\tau=\mu\, du/dr \quad \Rightarrow \quad \tau=\tau(r)$



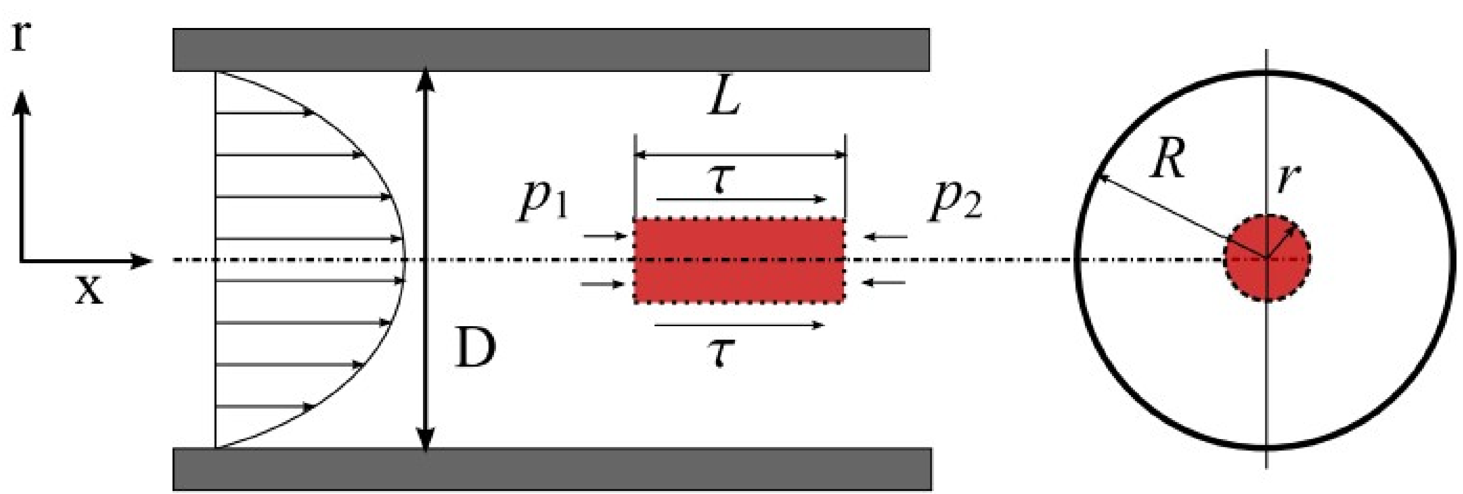
Laminar Pipe Flow

- Steady state, laminar, fully developed flow:
  - assumption: p=p(x) (pressure is function of x)
  - What if p would depend on the radius too? For example, the pressure would be greater in the center compared to outer radius? The fluid would flow from the center to the outer radius. However, then the flow would not be fully developed!



Laminar Pipe Flow

- Force balance for CV (red) in x-direction:
  - p_1 with positive, p_2 with negative contribution
  - What about τ? The boundary of the CV is the surface of a cylinder barrel
    - The area vector points radially to the outside
    - Same direction as the r-coordinate: „plus“



Laminar Pipe Flow

- General mom. equation $\sum \boldsymbol{F} = \sum_{outlets} \rho \dot{V} \boldsymbol{v} - \sum_{inlets} \rho \dot{V} \boldsymbol{v}$
- Axial direction = x-direction

$$\sum F_x = 0 \qquad \text{| in- and outflow cancel}$$

$$\Rightarrow \quad p_1 \pi r^2 - p_2 \pi r^2 + \tau 2 \pi r L = 0 \qquad \text{| solve for } \tau$$

$$\Rightarrow \quad \frac{p_1 - p_2}{2L} r = -\tau = -\mu \frac{du}{dr} \qquad \text{| use } \tau = \mu \frac{du}{dr}$$

$$\Rightarrow \quad \frac{du}{dr} = -\left(\frac{p_1 - p_2}{L} \frac{1}{2\mu} \right) r \qquad \text{| integrate}$$

$$\Rightarrow \quad u(r) = \int \frac{du}{dr} dr = -\left(\frac{p_1 - p_2}{L} \frac{1}{2\mu} \right) \int r \, dr$$

$$\Rightarrow \quad u(r) = -\left(\frac{p_1 - p_2}{L} \frac{1}{4\mu} \right) r^2 + c$$

velocity profile

Laminar Pipe Flow

- Determine c with u(r=R)=0 (no slip at walls)

$$0=-\left(\frac{p_1-p_2}{L}\frac{1}{4\mu}\right)R^2+c$$

$$\Rightarrow \quad c=\left(\frac{1}{4\mu}\frac{p_1-p_2}{L}\right)R^2$$

- In general: velocity of laminar pipe flow depends on radius r

$$u(r)=\left(\frac{1}{4\mu}\frac{p_1-p_2}{L}\right)\left(R^2-r^2\right)$$

- Maximal velocity occurs in the center of the pipe (r=0)

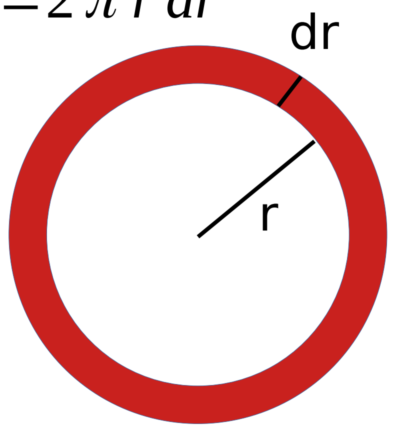
$$u_{max}=\left(\frac{1}{4\mu}\frac{p_1-p_2}{L}\right)R^2$$

Laminar Pipe Flow

- The total volume flow rate follows from integration over the area:

$$d\dot{V} = u(r)\,dA = 2\pi\,u(r)\,r\,dr$$

$$dA = 2\pi\,r\,dr$$



$$\Rightarrow\quad \dot{V} = \int d\dot{V} = \int 2\pi\,u(r)\,r\,dr$$

$$= 2\pi \int_0^R \left(\frac{1}{4\mu}\frac{p_1 - p_2}{L}\right)(R^2 - r^2)\,r\,dr$$

$$= 2\pi \frac{1}{4\mu}\left(\frac{p_1 - p_2}{L}\right)\int_0^R (r \cdot R^2 - r^3)\,d\,r$$

$$= \pi \frac{1}{2\mu}\left(\frac{p_1 - p_2}{L}\right)\left[\frac{r^2 \cdot R^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$\Rightarrow\quad \dot{V} = \frac{\pi}{8\mu}\frac{p_1 - p_2}{L}R^4$$

Hagen-Poiseuille equation

Laminar Pipe Flow

- Average velocity over the cross sectional area:

$$\bar{u} = \frac{\dot{V}}{A_{pipe}} = \frac{\pi\left(\frac{p_1 - p_2}{L}\frac{1}{8\mu}\right)R^4}{\pi R^2} = \left(\frac{p_1 - p_2}{L}\frac{1}{8\mu}\right)R^2 = \frac{1}{2}u_{max}$$

Laminar Pipe Flow
Pressure Losses and Friction Factor

- For pipe flow the pressure losses are important
  - How much pumping power is required?
- Radius of the pipe and volume flow are usually given.
- From Hagen-Poiseuille equation:

$$\dot{V} = \frac{\pi}{8\mu}\left(\frac{p_1 - p_2}{L}\right) R^4 \qquad \Delta p_L = p_1 - p_2 \qquad R = \frac{D}{2}$$

$$\Rightarrow \quad \dot{V} = \pi \left(\frac{\Delta p_L}{128\,\mu\,L}\right) \cdot D^4$$

- Pressure loss for pipe with diameter D:

$$\Delta p_L = \frac{\dot{V}\,128\,\mu\,L}{\pi\,D^4} = 32\,\mu\,L\,\frac{\bar{u}}{D^2} \qquad \dot{V} = \bar{u}\,A = \bar{u}\,\pi\,\frac{D^2}{4}$$

Laminar Pipe Flow
Pressure Losses and Friction Factor

- Helpful: **Dimensionless Coefficients**
  - allows for a comparison and analysis of different flow situations
- Division of the derived equation by kinetic energy density

$$\frac{\Delta p_L}{\frac{\rho}{2}\bar{u}^2} = \frac{64\,\mu\,L}{D^2\,\rho\,\bar{u}} = \frac{L}{D}\,64\,\frac{\mu}{\rho\,\bar{u}\,D} = \frac{L}{D}\,\frac{64}{\mathrm{Re}}$$

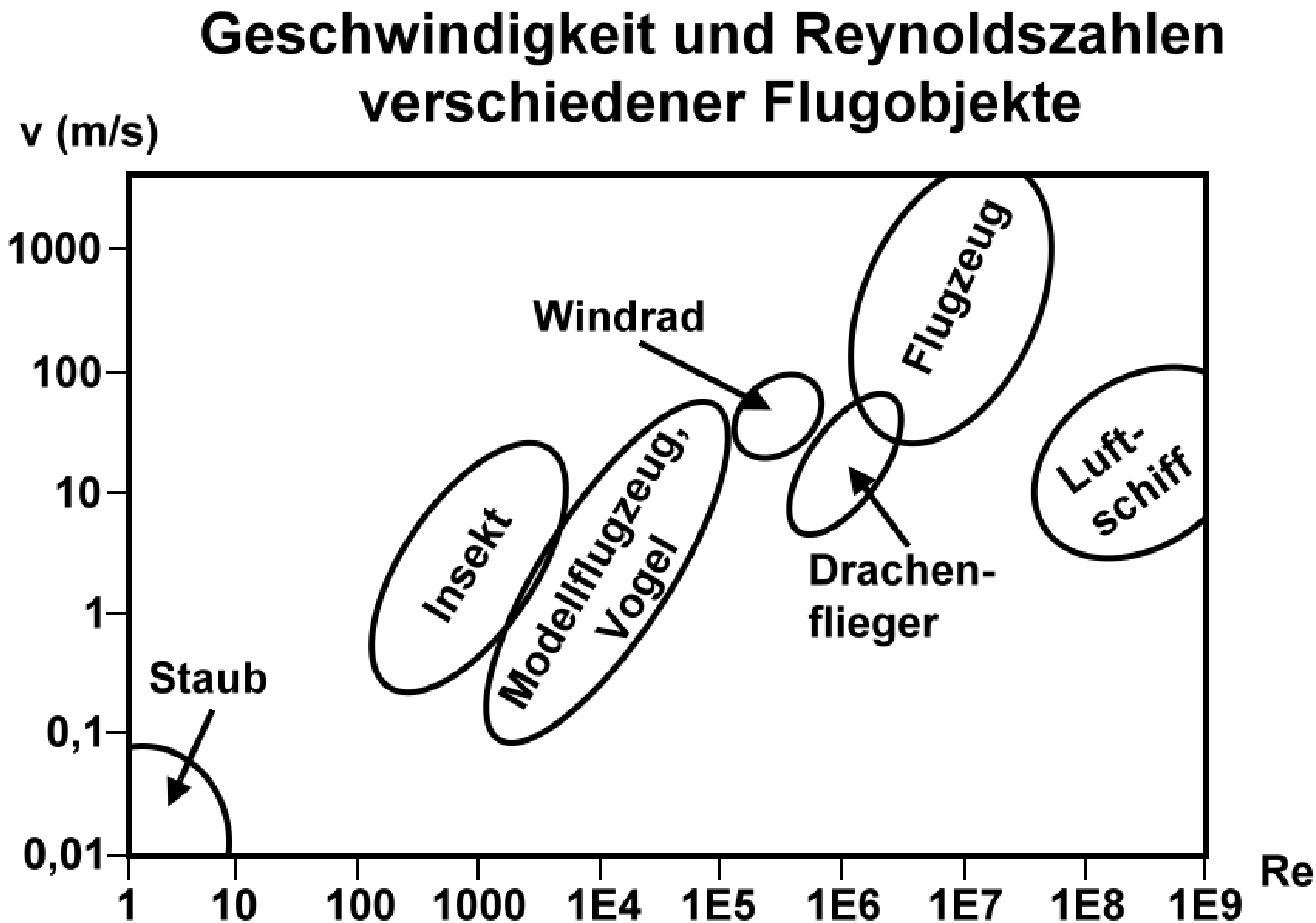
- Thereby, we introduced the first dimensionless number:

$$\mathrm{Re} = \frac{\rho\,\bar{u}\,D}{\mu} = \frac{\bar{u}\,D}{\nu}$$

Reynolds number

$$\nu = \frac{\mu}{\rho}$$ Kinematic viscosity

Typical Reynolds Numbers



https://de.wikipedia.org/wiki/Reynolds-Zahl

Laminar Pipe Flow
Pressure Losses and Friction Factor

- How to determine the pressure loss for a given Reynolds number?

$$\frac{\Delta p_L}{\frac{\rho}{2}\bar{u}^2} = \frac{L}{D}\frac{64}{\mathrm{Re}} = \frac{L}{D}\lambda \qquad \lambda = \frac{64}{\mathrm{Re}} \qquad \mathrm{Re} = \frac{\rho\,\bar{u}\,D}{\mu}$$

- Pipe friction factor λ:
  - Dimensionless variable
  - Depends on the Reynolds number
  - 64/Re for laminar flow

Laminar Pipe Flow
Pressure Losses and Friction Factor

- How to determine the pressure loss for a given Reynolds number?

$$\frac{\Delta p_L}{\frac{\rho}{2}\bar{u}^2} = \frac{L}{D}\frac{64}{\mathrm{Re}} = \frac{L}{D}\lambda \qquad \lambda = \frac{64}{\mathrm{Re}} \qquad \mathrm{Re} = \frac{\rho\,\bar{u}\,D}{\mu}$$

- How to use this?

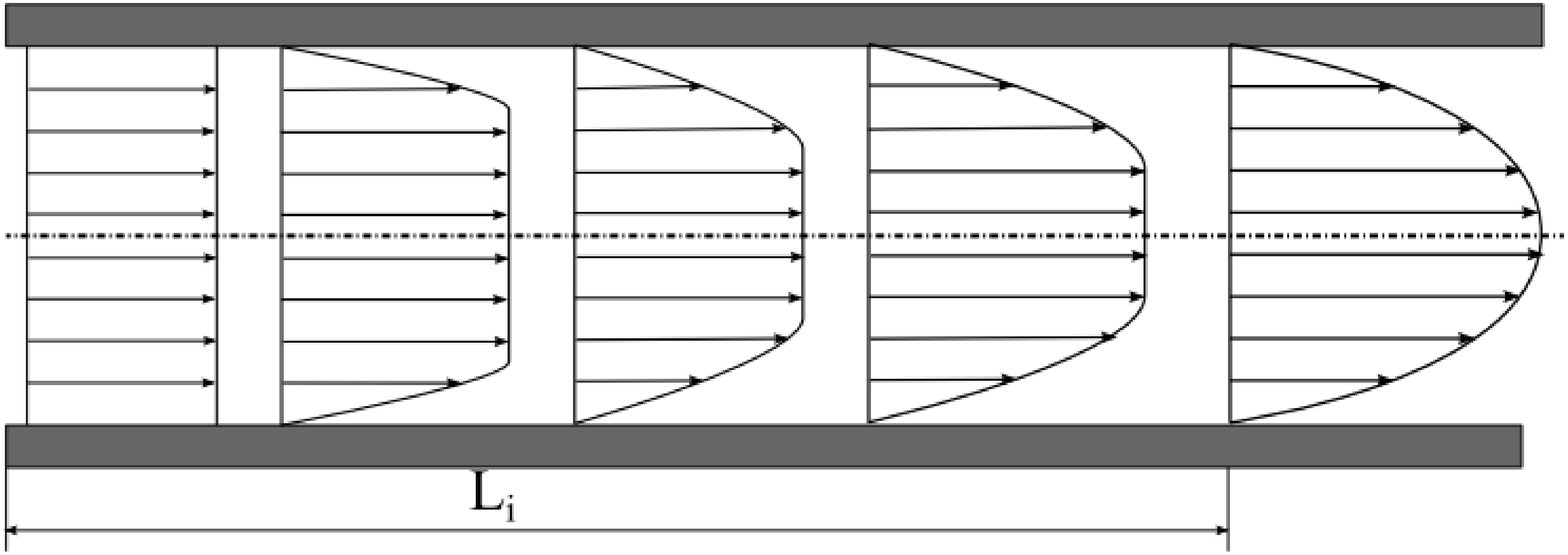
$$\Delta p_L = \frac{L}{D}\frac{64}{\mathrm{Re}}\frac{\rho}{2}\bar{u}^2 = \frac{L}{D}\lambda\frac{\rho}{2}\bar{u}^2 \qquad \lambda = \frac{64}{\mathrm{Re}} \qquad \mathrm{Re} = \frac{\rho\,\bar{u}\,D}{\mu} ¿$$

  – We need length and diameter of the tube
  – We need the density as well as the average velocity and the viscosity of the fluid
  – With these numbers, we can calculate the fluid pressure loss in the tube

Entrance Length and Fully Developed Flow Profile

Example: Fluid flows from a water reservoir into a pipe

- Constant velocity profile at the beginning
- Wall friction affects the flow
- Result: a parabolic profile develops after a certain length



Entrance Length and Fully Developed Flow Profile

- Depending on flow conditions, the flow profile is fully developed after a certain entrance length L_i. It can be roughly determined as:

$$L_i = 0.029 \ldots 0.065 \cdot \mathrm{Re} \cdot D$$

- The pressure losses are increased in the entrance length compared to fully developed flow. This effect is considered by an additional loss constant $\zeta_e = 1.16 \ldots 2.7$

$$\frac{\Delta p_L}{\frac{\rho}{2} \bar{u}^2} = \frac{L}{D} \frac{64}{\mathrm{Re}} = \frac{L}{D} \lambda + \zeta_e$$

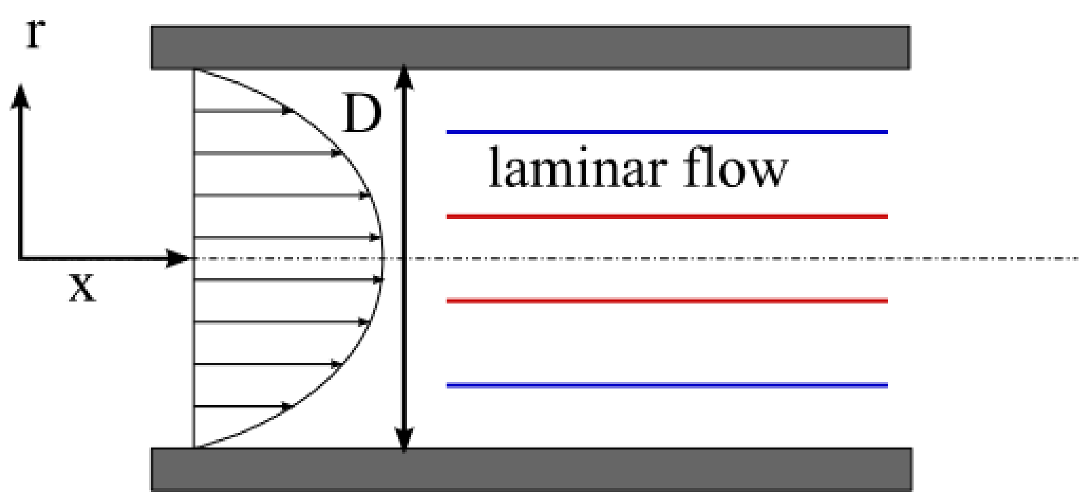
- Note: For long pipes (large L), the first term dominates and ζ_e can be neglected.

Turbulent Flow

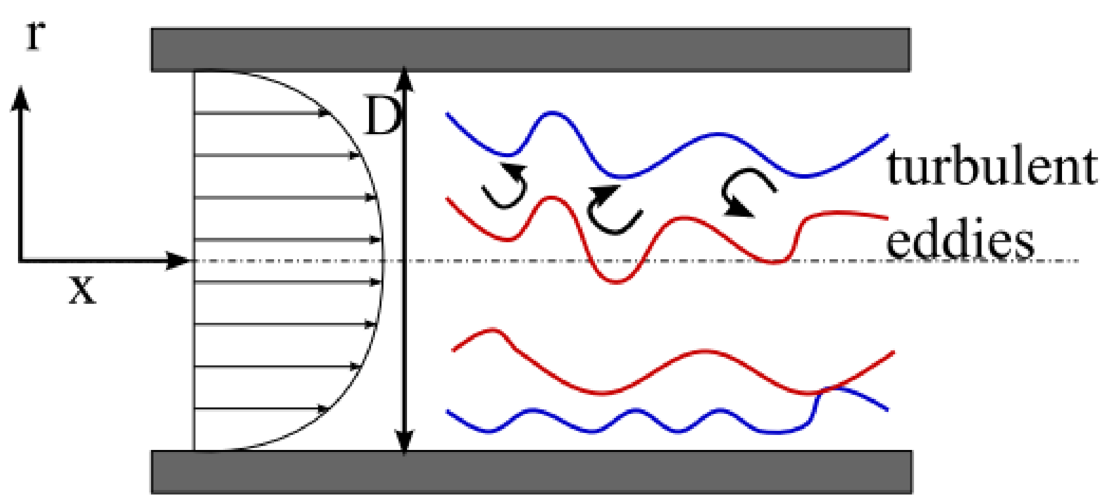
- A distinction is made between laminar and turbulent flow.
- Laminar flows have “separated” flow layers.
- In turbulent flows, the layers mix strongly and the idea of flow layers is not valid any more.
- What does that mean?
- Let us compare laminar and turbulent pipe flow.

Turbulent Pipe Flow

- Laminar flow
  (as until now)

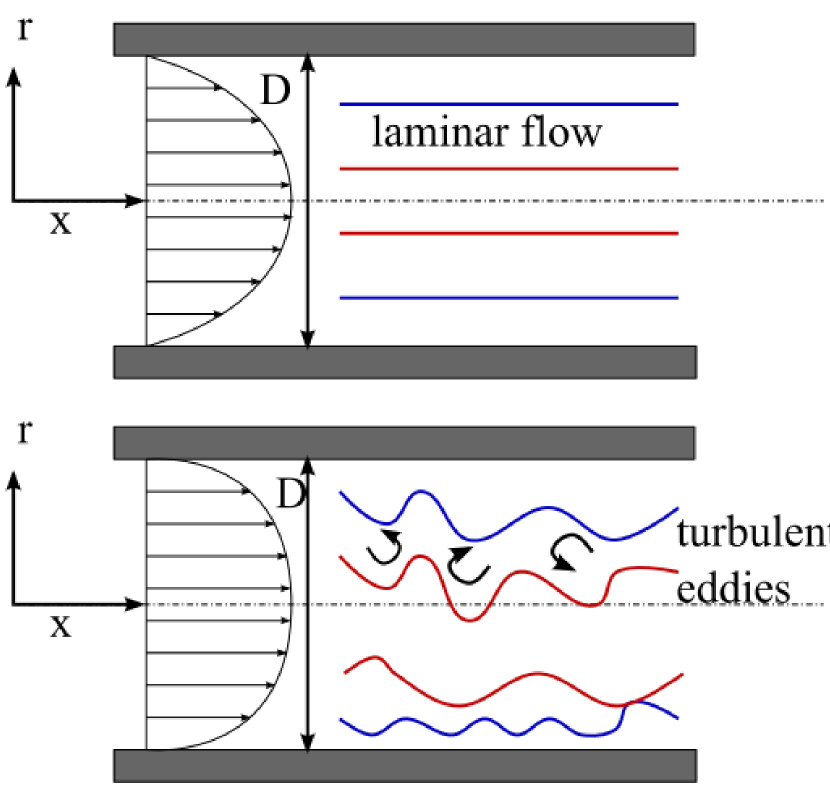


- Turbulent flow
  with a lot of eddies
  (deutsch: Wirbel)

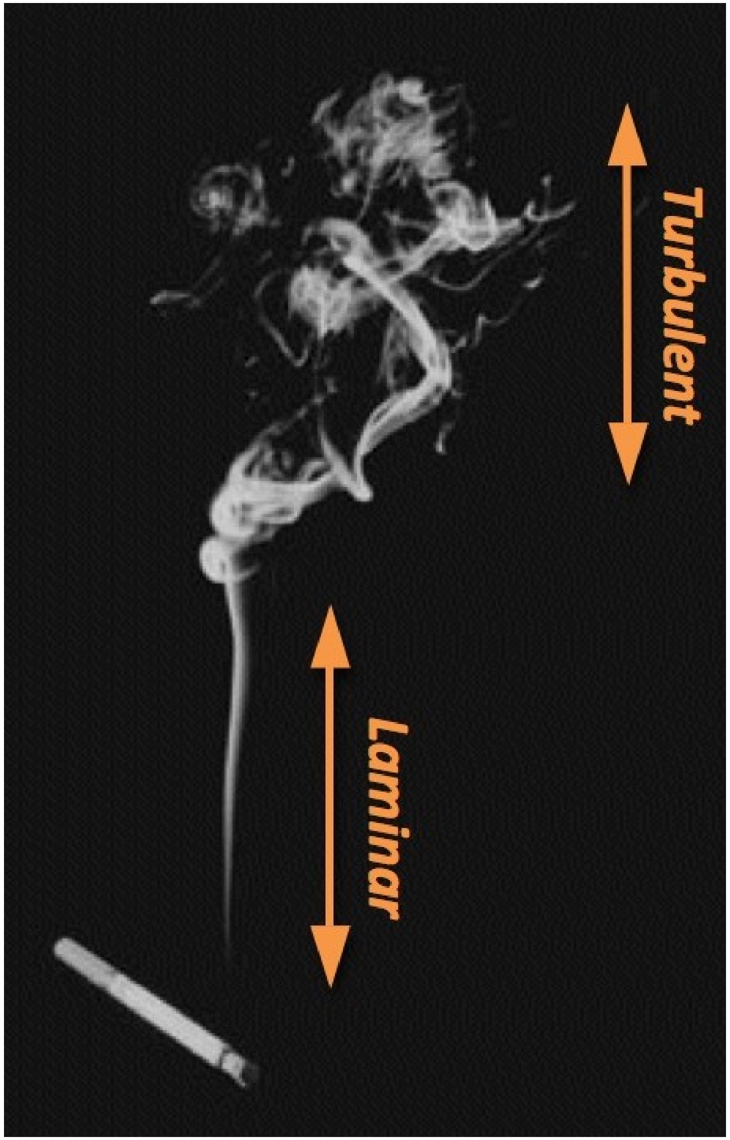


Turbulent Pipe Flow

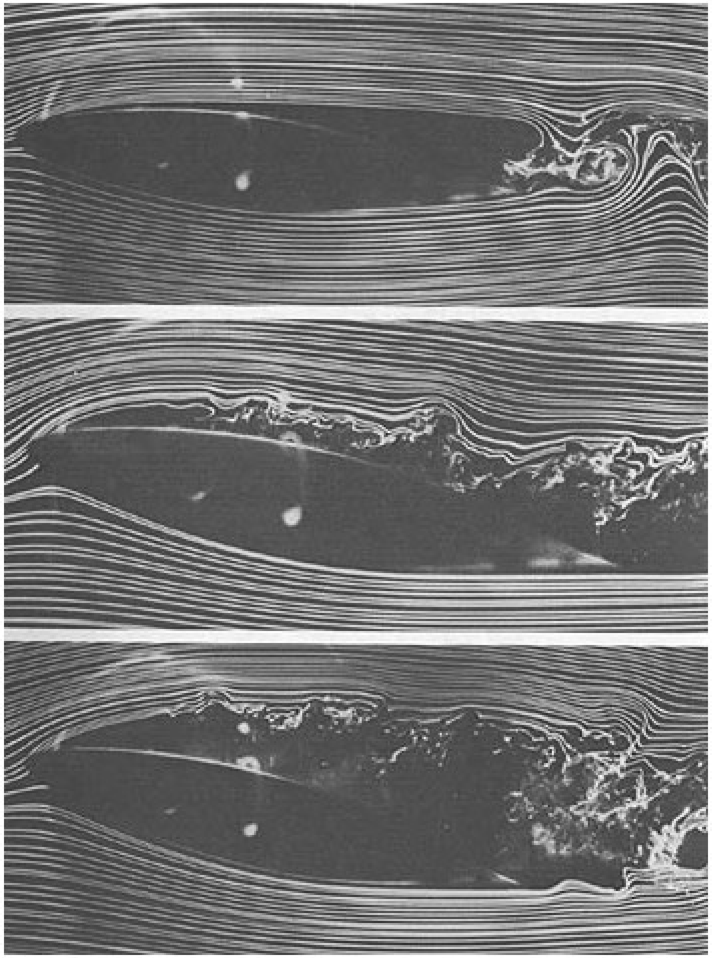
- Characterization of a pipe flow by the Reynolds number Re
  - Pipe diameter as characteristic length
- Re > 2300:
  - Significant change of pressure losses
  - The previous law (Hagen-Poiseuille) no longer applies
- Reason: Fluctuations of speed in radial and axial direction: many eddies



Laminar and Turbulent Flow



http://aerospaceengineeringblog.com/skin-friction-drag/



https://history.nasa.gov/SP-4103/app-f.htm

Turbulent Pipe Flow

- Turbulent flow description according to Reynolds
- Decomposition of the axial velocity u into
  - Average value $\bar{u}$
  - Fluctuating value u'

$$u(r,\phi,x,t)=\bar{u}(r)+u'(r,\phi,x,t)$$

- Fluctuation in radial direction is instantaneous fluctuation because there is no average flow in radial direction

$$v(r,\phi,x,t)=v'(r,\phi,x,t)$$

- Average values are often of interest
- Turbulence increases e.g. mixing and heat transfer

Turbulent Pipe Flow
Pressure Losses and Friction Factor

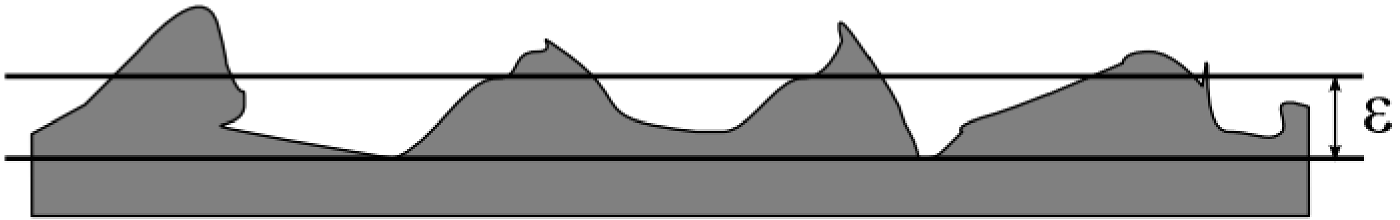
- For laminar flows we had found:

$$\Delta p_L = \lambda \frac{L}{D} \frac{\rho}{2} \bar{u}^2 \qquad \lambda = \frac{64}{\mathrm{Re}} \qquad \mathrm{Re} = \frac{\rho \, \bar{u} \, D}{\mu}$$

- Idea: The first formula with the relationship between λ and Δp should still apply
- How can λ be determined?
- Much more complex for turbulent pipe flow!
- At first we have to introduce the concept of wall roughness

Turbulent Pipe Flow
Pressure Losses and Friction Factor

- Detailed microscopic view of a rough pipe surface:



- These “bumps” cause velocities in radial direction and disrupt the laminar axial flow
- If the axial velocity near the wall is high, the disturbances become so large that visible large-scale and many small-scale eddies occur

Turbulent Pipe Flow
Pressure Losses and Friction Factor

- ε is therefore an average height of the bumps
- ε=0 is also referred to as „hydraulically smooth"
  - e.g. for hydraulically smooth pipes, the friction factor can be calculated as:

    $\lambda = 0.316 \, Re^{-1/4}$ für $5*10^3 < Re < 10^5$.
  - However, in technical applications, the walls are usually not perfectly smooth
- The relative roughness of technical devices is defined as

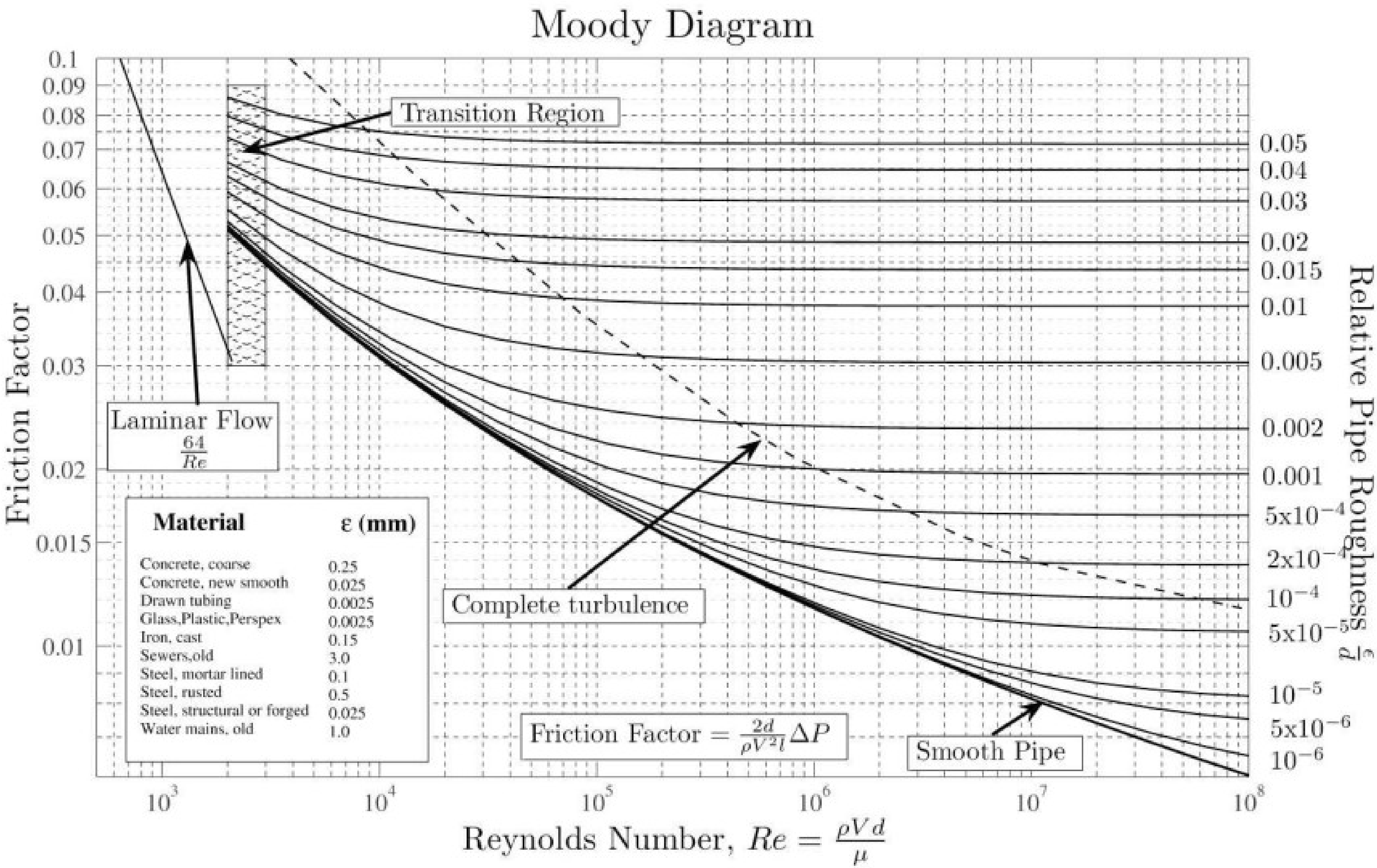
$$k = \frac{\epsilon}{D}$$

- ε can be determined by experiments
- D is the pipe diameter

Turbulent Pipe Flow
Pressure Losses and Friction Factor

- Determination of λ using the Moody diagram
  - You need the Reynolds number of the flow
  - The roughness k of the pipe is required
    - either k is given directly
    - or you have ε and divide it by D
  - In the Moody diagram, find the curve with the appropriate value of k (y-axis on the right side of diagram)
  - Then find the appropriate Reynolds number on the x-axis
  - Find the point where Reynolds number and the k-curve meet
  - Read the friction factor λ on the y-axis on the left of the diagram

Turbulent Pipe Flow
Pressure Losses and Friction Factor



Turbulent Pipe Flow
Pressure Losses and Friction Factor

- Hints for using the Moody Diagram:
  - Friction Factor = Reibungskoeffizient
  - Relative Pipe Roughness = Relative Rauhigkeit
  - Smooth Pipe = hydraulisch glattes Rohr
  - The axes are <u>logarithmic</u>!
    This is especially important for the Reynolds number. If you make a mistake in Re, λ is usually completely wrong.

Turbulent Pipe Flow
Pressure Losses and Coefficient of Friction

- When you have determined λ with the Moody diagram, you use

$$\Delta p_L = \frac{L}{D} \lambda \frac{\rho}{2} \bar{u}^2$$

  to calculate the pressure drop *Δp*.

- Additional note:

  Inlet length L_i for turbulent flows

$$L_i = 0.693 \,\mathrm{Re}^{-\frac{1}{4}} \cdot D$$

Turbulent Pipe Flow
Pressure Losses in non-circular tubes

- The following applies to non-circular pipes
  - Average velocity $$\bar{u}=\frac{\dot{V}}{A}$$
  - hydraulic diameter $$D_h=\frac{4\,A}{s}$$

    with *A* as the cross-sectional area and *s* as the circumference.

- This definitions can be used to determine the Reynolds number, the relative roughness and the pressures losses for non-circular pipes in a similar way as for circular pipes.

Energy Losses in Pipe Flows

- Bernoulli's equation is expanded to include friction
- Friction causes...
  - Loss in kinetic energy
  - an increase in internal energy and temperature (negligible at moderate velocity)
- The Bernoulli equation is expanded by a loss term Δp_L

$$\frac{\rho}{2} v_1^2 + p_1 + \rho \, g \, z_1 = \frac{\rho}{2} v_2^2 + p_2 + \rho \, g \, z_2 + \Delta \, p_L$$

- The pressure losses can be determined for example with a manometer

Energy Losses in Pipe Flows

- Alternative 1: Bernoulli equation…
  in the unit of height [m]

$$\frac{v_1^2}{2g}+\frac{p_1}{\rho\, g}+z_1=\frac{v_2^2}{2g}+\frac{p_2}{\rho\, g}+z_2+\Delta h_L \qquad \Delta h_L=\frac{\Delta p_L}{\rho\, g}$$

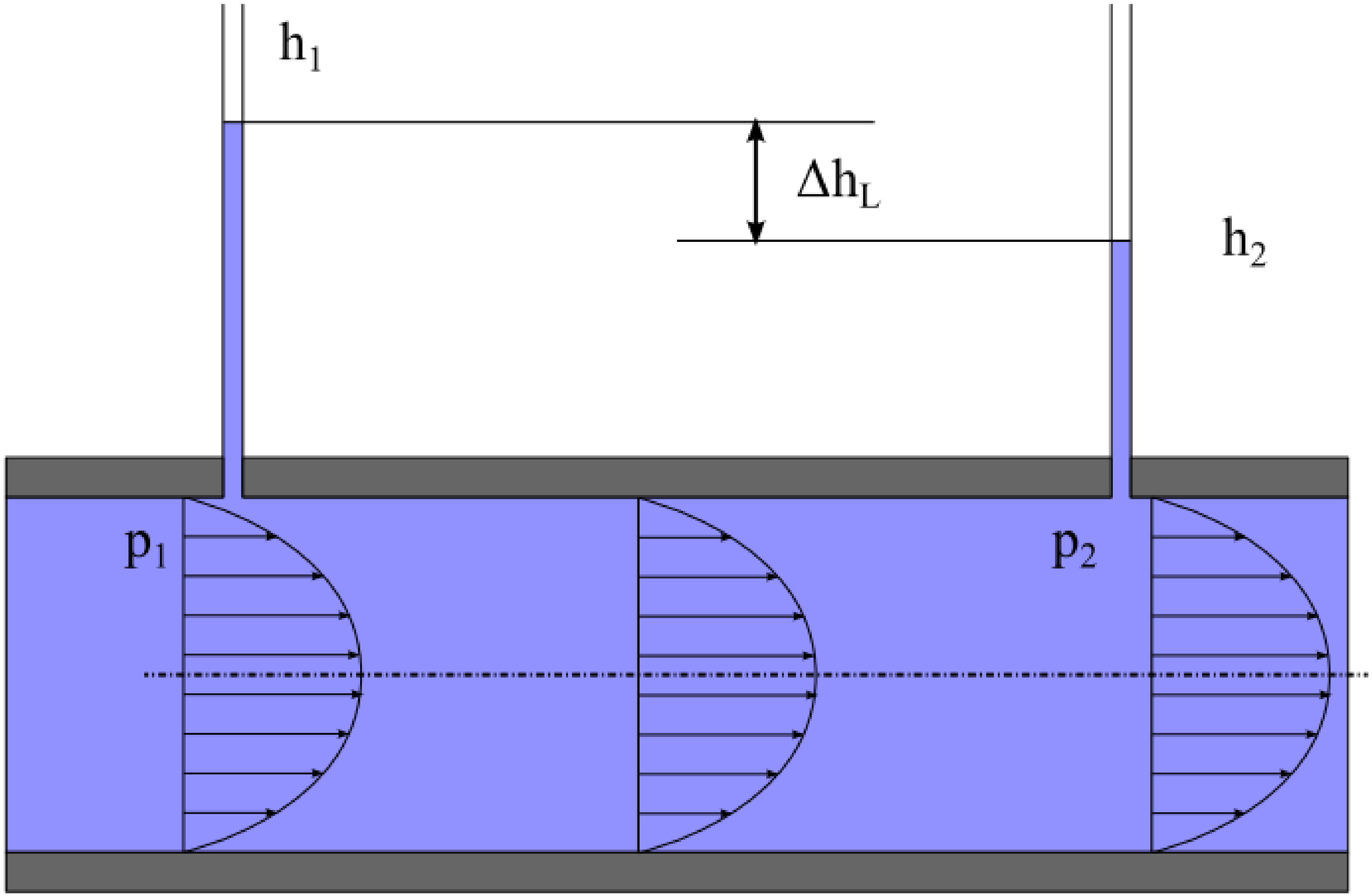
- Alternative 2: Bernoulli equation…
  in the unit of specific energy [m²/s²]

$$\frac{v_1^2}{2}+\frac{p_1}{\rho}+g\, z_1=\frac{v_2^2}{2}+\frac{p_2}{\rho}+g\, z_2+\Delta e_L \qquad \Delta e_L=\frac{\Delta p_L}{\rho}$$

  - Δe_L are the energy losses

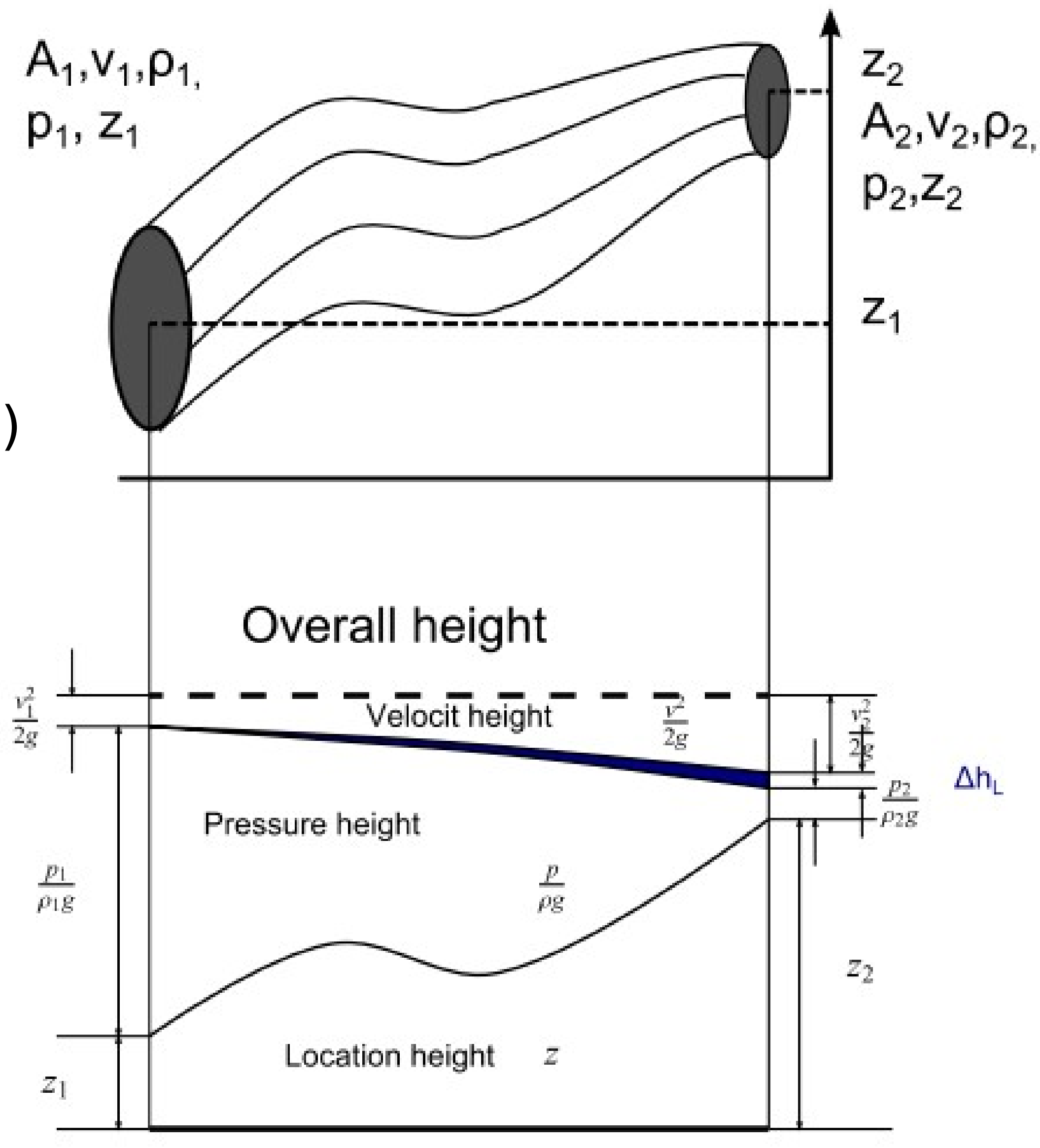
Energy Losses in Pipe Flows

- Graphical representation of the height loss Δh_L of a viscous pipe flow:



Energy Losses in Pipe Flows

- Flow tube with energy levels in diagram below
- Energy is transformed from one type into another along the flow length
- Without friction: The total energy is constant (dashed line)
- With friction: The kinetic energy is reduced by the energy losses (reduced volume flow rate)
- Comparing inlet and outlet, the sum of kinetic, pressure and potential energy is no longer constant



Energy Losses in Pipe Flows
Additional Pressure Losses Due to Components

- Additional losses due to
  - Valves
  - Tube bends
  - Branches
  - any changes in flow direction
  - Changes in cross-sectional area
- Major reason for pressure losses: recirculation zones. Result: kinetic energy dissipates in heat
- Example: sudden change in the cross-sectional area of a pipe

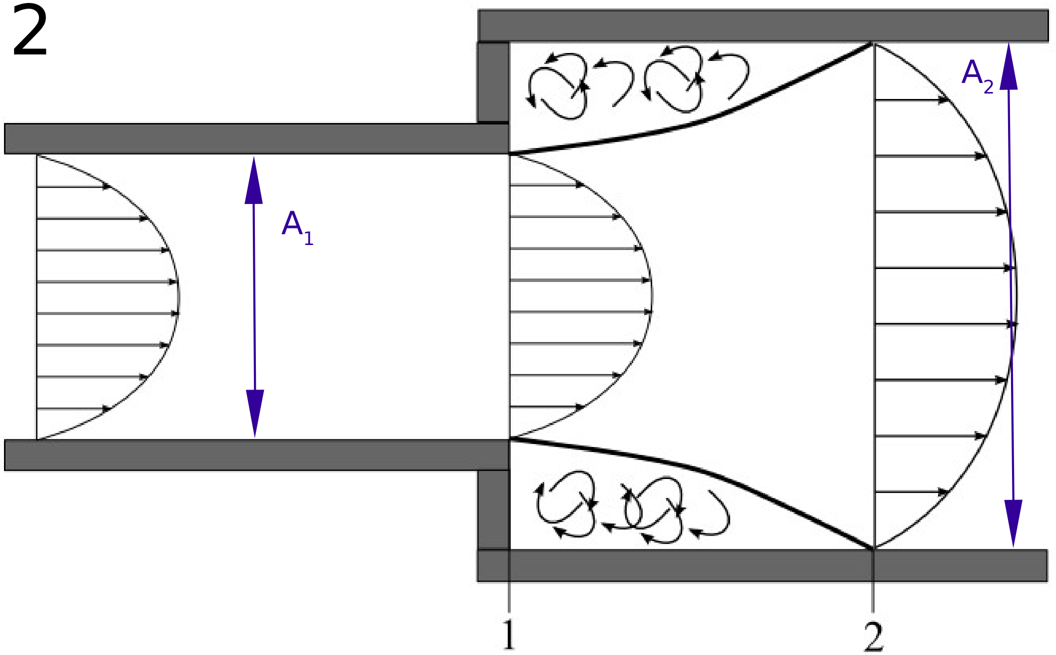
Energy Losses in Pipe Flows
Pressure Losses due to Sudden Pipe Expansion

- Pipe friction is neglected
- Losses due to re-circulation
- CV between point 1 and 2 (large Area)
- Continuity equation

$$v_1 A_1 = v_2 A_2$$

- Bernoulli equation

$$\Delta p_L = p_1 + \frac{\rho}{2} v_1^2 - \left(p_2 + \frac{\rho}{2} v_2^2 \right)$$



- Momentum equation

$$\rho v_2^2 A_2 - \rho v_1^2 A_1 = \left(p_1 - p_2 \right) A_2$$

Note: pressure p1 acts at position 1 over the entire cross section A2 on the CV in positive direction

Energy Losses in Pipe Flows
Pressure Losses due to Sudden Pipe Expansion

- For the pressure losses follows:

$$\Delta p_L = p_1 + \frac{\rho}{2} v_1^2 - \left(p_2 + \frac{\rho}{2} v_2^2\right)$$

$$= p_1 - p_2 + \frac{\rho}{2}\left(v_1^2 - v_2^2\right) \qquad |\text{Continuity equ.}$$

$$= p_1 - p_2 + \frac{\rho}{2}\left(v_1^2 - \left(\frac{A_1}{A_2}\right)^2 v_1^2\right) \qquad |\text{Momentum equ.}$$

$$= \rho v_2^2 - \rho v_1^2 \frac{A_1}{A_2} + \frac{\rho}{2} v_1^2 \left(1 - \left(\frac{A_1}{A_2}\right)^2\right) \qquad |\text{Continuity equ.}$$

$$= \rho v_1^2 \left(\frac{A_1}{A_2}\right)^2 - \rho v_1^2 \frac{A_1}{A_2} + \frac{\rho}{2} v_1^2 \left(1 - \left(\frac{A_1}{A_2}\right)^2\right)$$

$$\boxed{\Delta p_L = \frac{\rho}{2} v_1^2 \left(1 - \frac{A_1}{A_2}\right)^2}$$

Pressure losses due to a sudden expansion

Energy Losses in Pipe Flows
Additional Pressure Losses Due to Components

- General approach for components: Introduction of an additional pressure loss with the loss factor ζ from the data sheet for the component

$$\Delta p_L = \zeta \frac{\rho}{2} \bar{u}^2$$

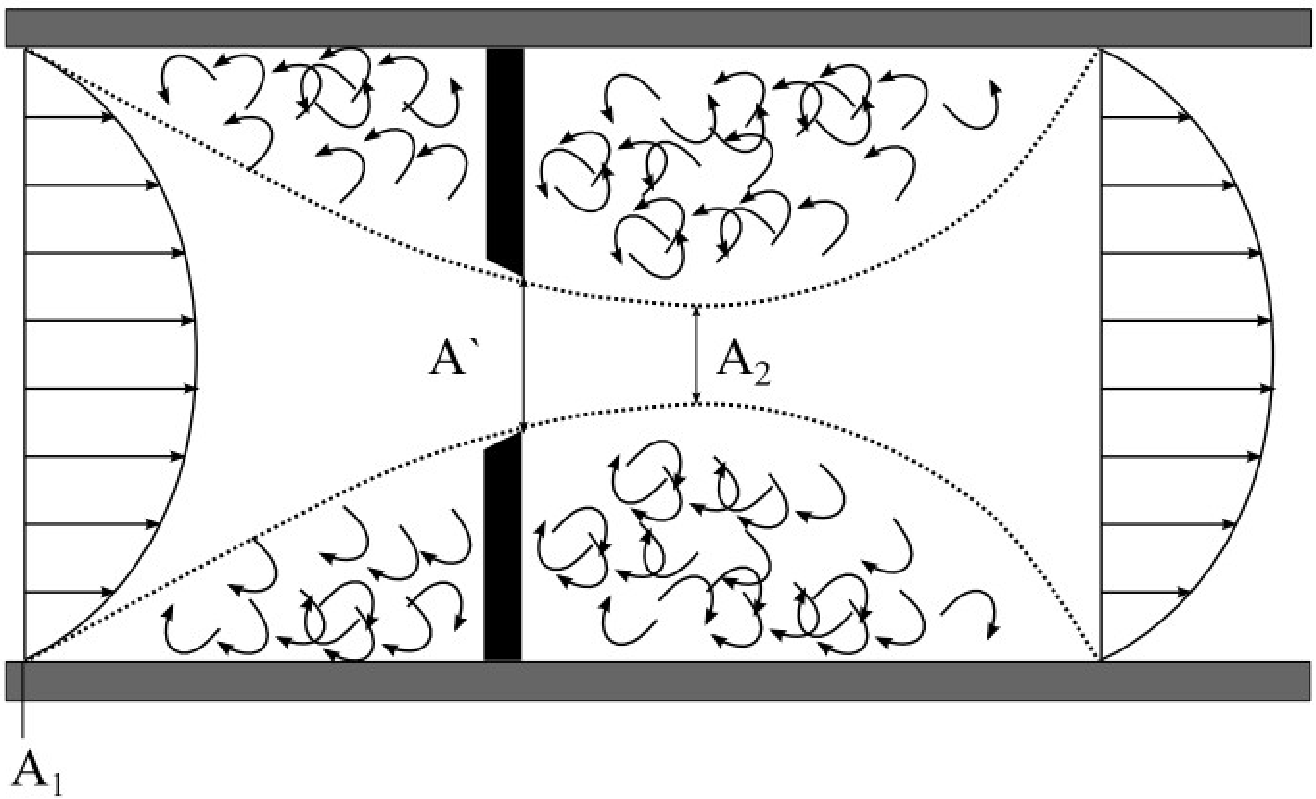
- For the sudden pipe expansion we derived

$$\zeta = \frac{\Delta p_L}{\frac{\rho}{2} v_1^2} = \left(1 - \frac{A_1}{A_2}\right)^2$$

- Important: For what speed range is the loss factor valid?
- Often: Loss factor is defined for velocity before the cross-sectional change
- Speed is the average velocity of the cross-sectional area

Energy Losses in Pipe Flows
Pressure Losses Due to an Aperture

- Opening of an aperture
  - Opening: cross-section A‘
  - Smallest cross-sectional area of the flow: cross-section A_2



Energy Losses in Pipe Flows
Pressure Losses Due to an Aperture

- Contraction coefficient ψ

$$\Psi = \frac{A_2}{A'}$$

- Analogous to the expansion example

$$\zeta = \frac{\Delta p_L}{\frac{\rho}{2} v_1^2} = \left(\frac{1-\Psi}{\Psi}\right)^2$$

- ζ numbers for various flow devices can be found in tables or graphs in the literature