Task 2.6

Bouyancy: $F_G = F_B$

$$m_{Balloon}\,\cancel{g} = g\, m_{Displaced\ Air} \quad = DA$$

$$= \cancel{g}\, \rho_{DA}(z)\, V_{Balloon}$$

$$\rho_{DA}(z) = \rho_0\, e^{-\frac{zg}{R_i T_0}}$$

cf. lecture notes

$$\rho_0 = \frac{p_0}{R_i T_0}$$

$$\Rightarrow R_i T_0 = \frac{p_0}{\rho_0} = \ldots\ldots \qquad \text{(Zahlenwert)}$$

$$\Rightarrow m_{Balloon} = \rho_0\, e^{-\frac{zg}{R_i T_0}}\, V_{Balloon} \qquad \Big|\, V_{Balloon}$$

$$\Rightarrow \rho_{Balloon} = \frac{m_{Balloon}}{V_{Balloon}} = \rho_0\, e^{-\frac{zg}{R_i T_0}} \qquad \Big|\, : \rho_0$$

$$\frac{\rho_{Balloon}}{\rho_0} = e^{-\frac{zg}{R_i T_0}} \qquad \Big|\, \ln$$

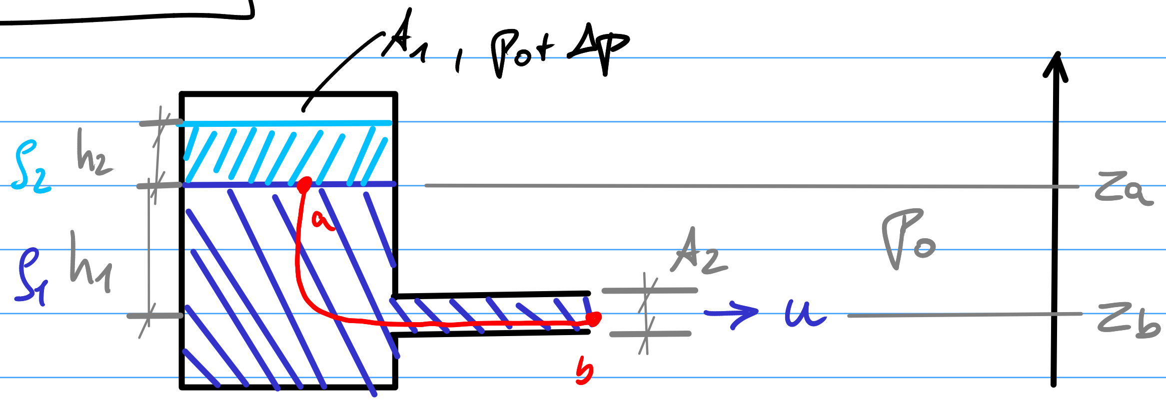
$$\ln \frac{\rho_{Balloon}}{\rho_0} = -\frac{zg}{R_i T_0} \qquad \Big|\, \cdot \left(-\frac{R_i T_0}{g}\right)$$

s.o. $\rightarrow$

$$-\frac{R_i T_0}{g} \ln \frac{\rho_{Balloon}}{\rho_0} = z$$

$$\Rightarrow z = \ldots\ldots \qquad \text{(Taschenrechner)}$$

Task 3.1



Bernoulli-Eq gives relation between pressures and velocities.

But: Bernoulli-Eq. requires constant density!

$\Rightarrow$ Streamline for Bernoulli-Eq has to stay within a single liquid!
(no different densities allowed)

$$p_a + \frac{\rho_1}{2} v_a^2 + g\rho_1 z_a = p_b + \frac{\rho_1}{2} v_b^2 + g\rho_1 z_b \quad \Big| -g\rho_1 z_b$$

$$\underset{?}{p_a} + \underset{\text{see below}}{\cancel{\frac{\rho_1}{2}}} \underset{?}{v_a^2} + \underbrace{g\rho_1 z_a - g\rho_1 z_b}_{g\rho_1 h_1} = \underset{= p_0 \text{ at exit}}{p_b} + \frac{\rho_1}{2} v_b^2$$

u: to be calculated

$v_a = ?$ $\qquad \dot{V}_a = \dot{V}_b$

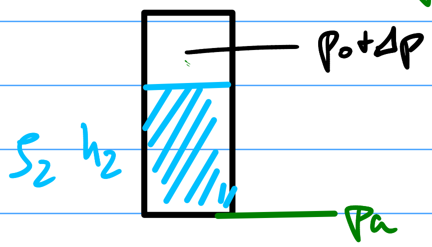
$$A_1 v_a = A_2 \cdot u \Rightarrow v_A = \underbrace{\frac{A_2}{A_1}}_{\ll 1 \text{ (cf. task)}} u$$

$\Rightarrow \frac{\rho_1}{2} v_A^2 \ll \frac{\rho_1}{2} v_b^2 \Rightarrow$ <u>neglect $\frac{\rho_1}{2} v_A$!</u>

Task 3.1 (continued)

$p_a = ?$ $\Rightarrow$ Treat as static, because v_a is small.

$\Rightarrow$ hydrostatic pressure for this column:



$$p_a = p_0 + \Delta p + g \rho_2 h_2$$

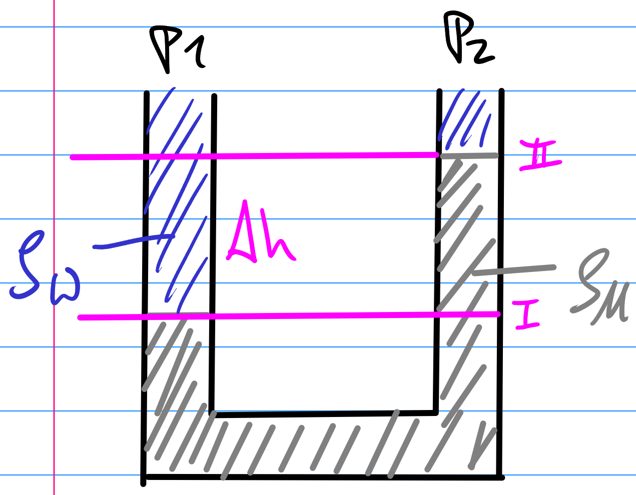
$$\Rightarrow \cancel{p_0} + \Delta p + g \rho_2 h_2 + g \rho_1 h_1 = \cancel{p_0} + \frac{\rho_1}{2} u^2$$

$$\Rightarrow \quad u^2 = \left(\Delta p + g \rho_2 h_2 + g \rho_1 h_1\right) \frac{2}{\rho_1}$$

$$\Rightarrow \quad u = \sqrt{\frac{2}{\rho_1}\left(\Delta p + g \rho_2 h_2 + g \rho_1 h_1\right)}$$

$$= \ldots$$ (Taschenrechner)

Task 3.3



$$p_{l,I} = p_{r,I}$$

$$p_1 + g\rho_w\Delta = p_2 + g\rho_M\Delta h$$

$$\Delta p = p_1 - p_2$$
$$= g\rho_M\Delta h - g\rho_w\Delta h$$
$$= g\Delta h(\rho_M - \rho_w)$$
$$= g\Delta h\,\Delta\rho$$
$$= \dots$$ (Taschenrechner)

Attention:

$$\frac{A_2}{A_2} = \frac{\pi\frac{d_2^2}{4}}{\pi\frac{d_1^2}{4}} = \frac{d_2^2}{d_1^2} \neq \frac{d_2}{d_1}$$