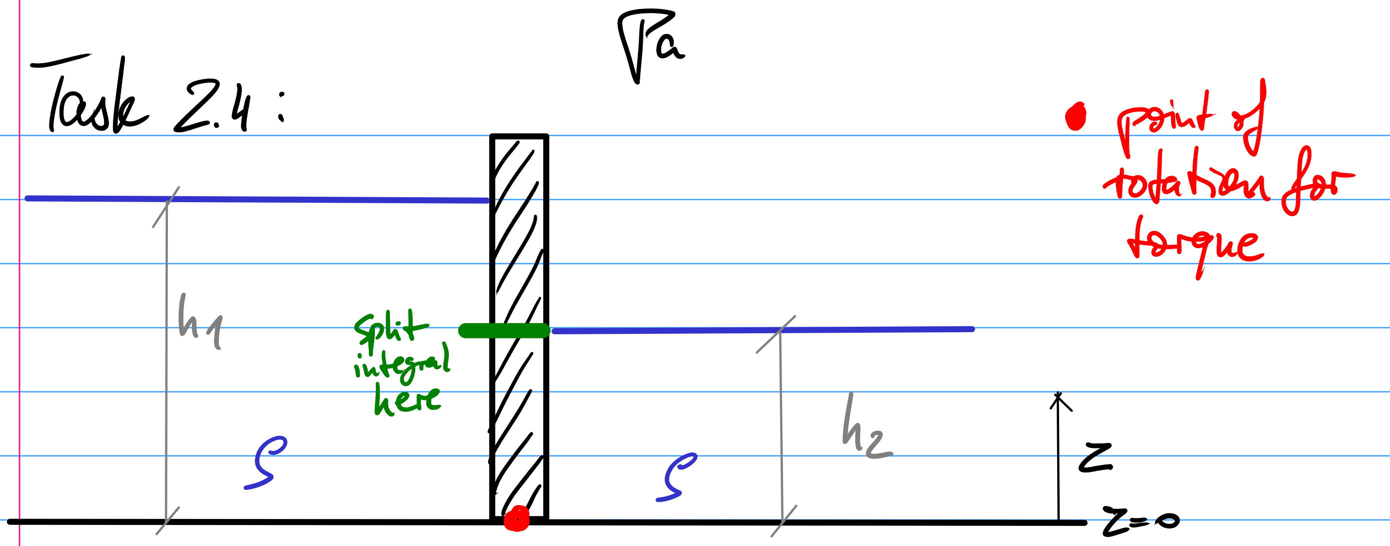
Task 2.4:



$$dF = dF_\ell - dF_r \qquad \text{cf. lecture}$$

$$F_s = \int dF \qquad \text{Correct, but formulae change along the way} \Rightarrow \text{split!} \qquad dA = B\,dz$$

$$= \int_{z=0}^{h_2} dF + \int_{h_2}^{h_1} dF \qquad \hookleftarrow \text{Now formulae in each integral fixed.}$$

$0 \le z \le h_2$: $\quad p_\ell = p_a + g\rho(h_1 - z) \qquad dF_\ell = p_\ell\, dA$

$\qquad\qquad\quad\;\; p_r = p_a + g\rho(h_2 - z) \qquad dF_r = p_r\, dA$

$h_2 \le z \le h_1$: $\quad p_\ell = p_a + g\rho(h_1 - z) \qquad dF_\ell = p_\ell\, dA$

$\qquad\qquad\quad\;\; p_r = p_a \qquad dF_r = p_r\, dA$

$$\ldots = \int_0^{h_2} (p_\ell - p_r)\,dA + \int_{h_2}^{h_1} (p_\ell - p_r)\,dA$$

$$= \int_0^{h_2} \left(\cancel{p_a} + g\rho(h_1 - \cancel{z})\right) - \left(\cancel{p_a} + g\rho(h_2 - \cancel{z})\right) B\,dz$$

$$+ \int_{h_2}^{h_1} \cancel{p_a} + g\rho(h_1 - z) - \cancel{p_a} \qquad B\,dz$$

$$= \int_0^{h_2} B g \rho (h_1 - h_2)\, dz + \int_{h_2}^{h_1} B g \rho (h_1 - z)\, dz$$

$$= B g \rho (h_1 - h_2) \left[z \right]_0^{h_2}$$
$$+ \quad B g \rho \left[h_1 z - \tfrac{1}{2} z^2 \right]_{h_2}^{h_1}$$

$$= B g \rho (h_1 - h_2) h_2$$
$$+ B g \rho \left(\underbrace{(h_1^2 - \tfrac{1}{2} h_1^2)}_{\frac{1}{2} h_1^2} - (h_1 h_2 - \tfrac{1}{2} h_2^2) \right)$$

$$= \cancel{B g \rho h_1 h_2} - B g \rho\, h_2^2$$
$$+ B g \rho \tfrac{1}{2} h_1^2 - \cancel{B g \rho h_1 h_2} + B g \rho \tfrac{1}{2} h_2^2$$

$$= B g \rho \tfrac{1}{2} h_1^2 - B g \rho \tfrac{1}{2} h_2^2 = F_S$$

Torque for Rotation point at $z = 0$ (lower end of the wall)

$$M_S = \int_0^{h_1} \underbrace{z}_{\text{lever}}\, dF = \int_0^{h_2} z\, dF + \int_{h_2}^{h_1} z\, dF$$

use pressure expressions from above:

$$= \int_0^{h_2} (p_e - p_r) z \underbrace{dA}_{B dz} + \int_{h_2}^{h_1} (p_e - p_r) z \underbrace{dA}_{B dz}$$

p.t.o.

$$= \int_0^{h_2} \Big(\big(\cancel{p_a} + g\rho(h_1 - \cancel{z})\big) - \big(\cancel{p_a} + g\rho(h_2 - \cancel{z})\big) \Big) z B \, dz$$

$$+ \int_{h_2}^{h_1} \Big(\big(\cancel{p_a} + g\rho(h_1 - z)\big) - \cancel{p_a} \Big) z B \, dz$$

$$= \int_0^{h_2} B g \rho (h_1 - h_2) \, z \, dz$$

$$+ \int_{h_2}^{h_1} B g \rho \underbrace{(h_1 - z) z}_{h_1 z - z^2} \, dz$$

$$= B g \rho (h_1 - h_2) \left[\frac{1}{2} z^2 \right]_0^{h_2}$$

$$B g \rho \left[h_1 \frac{1}{2} z^2 - \frac{1}{3} z^3 \right]_{h_2}^{h_1}$$

$$= B g \rho (h_1 - h_2) \frac{1}{2} h_2^2 - 0$$

$$+ B g \rho \left(\underbrace{\left(\frac{1}{2} h_1^3 - \frac{1}{3} h_1^3 \right)}_{\frac{1}{6} h_1^3} - \left(h_1 \frac{1}{2} h_2^2 - \frac{1}{3} h_2^3 \right) \right)$$

$$= \cancel{B g \rho h_1 \frac{1}{2} h_2^2} - B g \rho \frac{1}{2} h_2^3$$

$$+ B g \rho \frac{1}{6} h_1^3 - \cancel{B g \rho h_1 \frac{1}{2} h_2^2} + B g \rho \frac{1}{3} h_2^3$$

$$= B g \rho \frac{1}{6} \left(h_1^3 - h_2^3 \right) = M_s$$

p.t.o.

Substitution point from:

$$M_S = z_S F_S$$

Note: This is in fact the correct lever for the point of interest here.

$$\Rightarrow \quad z_S = \frac{M_S}{F_S} = \frac{\cancel{\rho g s}\,\frac{1}{6}\left(h_1^3 - h_2^3\right)}{\cancel{\rho g s}\,\frac{1}{2}\left(h_1^2 - h_2^2\right)}$$

$$= \frac{1}{3}\,\frac{h_1^3 - h_2^3}{h_1^2 - h_2^2} = \dots$$ Numbers...