Fluid Dynamics

Task 3.6

a) Bernoulli-eq. from 0 to 6

$$\cancel{p_0} + \cancel{\frac{\rho}{2} v_0^2}^{\to 0} + g\rho(H+s) = \cancel{p_0} + \underbrace{g\rho s}_{\text{like chap 2}} + \frac{\rho}{2} v_6^2 + g\rho \cdot 0$$

$A_0 \gg A_1, A_N$ like before

$z=0$

$$\Rightarrow \quad g\cancel{\rho} H + \cancel{g\rho s} = \cancel{g\rho s} + \frac{\cancel{\rho}}{2} v_6^2 \quad \Big| \text{ solve for } v_6$$

$$\Rightarrow \quad v_6 = \sqrt{2gH}$$

b) Bernoulli: $0 \to 1$

$$p_0 + \cancel{\frac{\rho}{2} v_0^2} + \cancel{g\rho z_0} = p_1 + \overbrace{\frac{\rho}{2} v_1^2}^{p_{dyn,1}} + \cancel{g\rho z_1}$$

like above

$z_0 = z_1$

Need v_1: $\quad \dot{V}_1 = \dot{V}_6$ from a)

$$v_1 A = \dot{V}_6 \Rightarrow v_1 = \frac{\dot{V}_6}{A} = 4\,\mathrm{m/s}$$

$$p_{dyn,1} = \frac{\rho}{2} v_1^2 = 8000\,\mathrm{Pa}$$

$$\Rightarrow \quad p_1 = p_0 - p_{dyn,1} = 92000\,\mathrm{Pa}$$

Bernoulli $0 \to 2$

$$p_0 + \cancel{\frac{\rho}{2} v_0^2} + g\rho z_0 = p_2 + \overbrace{\underbrace{\frac{\rho}{2} v_2^2}}^{p_{dyn,2}} + g\rho z_2$$

like before

$$\dot{V}_1 = \dot{V}_2 \qquad A = const$$

$$\Rightarrow v_1 = v_2$$

$$\Rightarrow p_{dyn,2} = p_{dyn,1}$$

$$p_2 = p_0 - p_{dyn,2} + g\rho\underbrace{(z_0 - z_2)}_{-h}$$

$$= 42000 \text{ Pa}$$

Bernoulli $0 \to 3$

like for Point 2, because $z_2 = z_3$ and $v_2 = v_3$ because the diameter does not chang

$$\Rightarrow p_3 = p_2$$

Bernoulli $0 \to 4$

$$p_0 + \cancel{\frac{\rho}{2} v_0^2} + g\rho z_0 = p_4 + \overbrace{\frac{\rho}{2} v_4^2}^{p_{dyn,4} = p_{dyn,1}} + g\rho z_4$$

like before

$$p_4 = p_0 - p_{dyn,4} + g\rho\overbrace{(z_0 - z_4)}^{+H}$$
$$= 892\,000 \text{ Pa}$$

Bernoulli $0 \to 5$

$$p_0 + \underbrace{\cancel{\frac{\rho}{2} v_0^2}}_{\text{like before}} + g\rho z_0 = p_5 + \overbrace{\frac{\rho}{2} v_5^2}^{p_{dyn,5} = p_{dyn,1}} + g\rho z_5$$

$$\Rightarrow \quad p_5 = p_0 - p_{dyn,5} + g\rho \underbrace{(z_0 - z_5)}_{H+S}$$

$$= 1052000 \text{ Pa}$$

Bernoulli $0 \to 6$

$$p_0 + \underbrace{\cancel{\frac{\rho}{2} v_0^2}}_{\text{like before}} + g\rho z_0 = p_6 + \underbrace{\frac{\rho}{2} v_6^2}_{p_{dyn,6} \neq p_{dyn1}} + g\rho z_6$$

$$v_6 \neq v_1$$

$$p_6 = p_0 - p_{dyn,6} + g\rho \underbrace{(z_0 - z_6)}_{H+S}$$

$$= 260000 \text{ Pa}$$

hydrostatic pressure at depth S

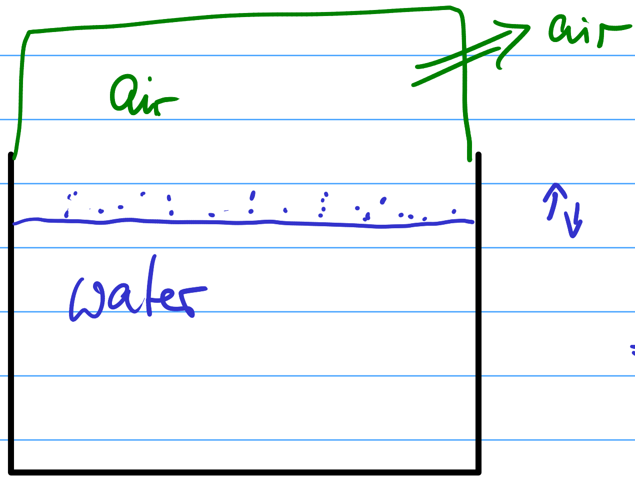
$$p(z) = p_0 + g\rho z$$

$$p(S) = p_0 + g\rho S = 260000 \text{ Pa}$$

$=!$

c)

idea of vapor pressure



Vapor pressure

= pressure at which liquid vaporizes

⇒ explosion like expansion in volume

⇒ to be protected against, because this is damaging.

point of lowest pressure:

Somewhere between 2 ⇔ 3

velocity at point 6 given by Bernoulli eq.

→ Bernoulli-Eq. between point 3 → 6

$$p_3 + \frac{\rho}{2} v_3^2 + g\rho z_3 = p_6 + \frac{\rho}{2} v_6^2 + g\rho z_6$$

c) (Continued)

$$\underset{\uparrow \\ V_3 \text{ can change}}{p_3 + \frac{\rho}{2}V_3^2} = \underbrace{p_6 + \frac{\rho}{2}V_6^2 + g\rho(z_6 - z_3)}_{\text{given by geometry of reservoirs}}$$

$$A \cdot V_3 = \underbrace{A_N^*}_{\text{any Nozzle diameter}} V_6 \quad \Rightarrow$$

$\Rightarrow$ changing the Nozzle diameter changes the velocity <u>in the pipe</u>!

$$p_3 = \underbrace{p_6 + \frac{\rho}{2}V_6^2 + g\rho(\overbrace{z_6 - z_3}^{-(H+h+s)})}\ \underbrace{- \frac{\rho}{2}V_3^2}_{\text{lowers } p_3 \text{ for higher values of } V_3}$$

$$V_3 = V_6 \frac{A_N^*}{A}$$

$p_3 = p_{\text{vapor pressure}}$ is the lowest possible pressure

$\Rightarrow$ solve for A_N^*:

$$p_{vap} = \underbrace{p_6 + \frac{\rho}{2}V_6^2}_{p_{dyn,6}} - g\rho(h+H+s) - \frac{\rho}{2}V_6^2\left(\frac{A_N^*}{A}\right)^2$$

$$\frac{\rho}{2} V_6^2 \frac{A_N^{*2}}{A^2} = p_6 - p_{vap} + p_{dyn,6} - g\rho(h+H+s)$$

$$A_N^* = \sqrt{\frac{2A^2}{\rho V_6^2}\left(p_6 - p_{vap} + p_{dyn,6} - g\rho(h+H+s)\right)}$$

$$= 0{,}244 \ m^2$$

limiting $\dot{V}^*$

$$\dot{V}^* = A_N^* \cdot V_6 = 9{,}75 \ m^3/s$$

The pipe cannot deliver more without cavitating and destroying itself!