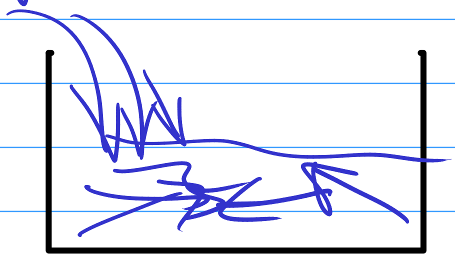
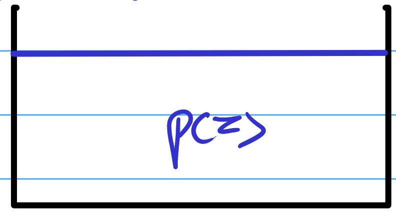
How does pressure spread in fluids?

fill a basin

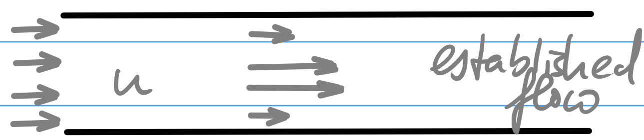


pressure varies wildly

after filling has stopped
after some time



Static

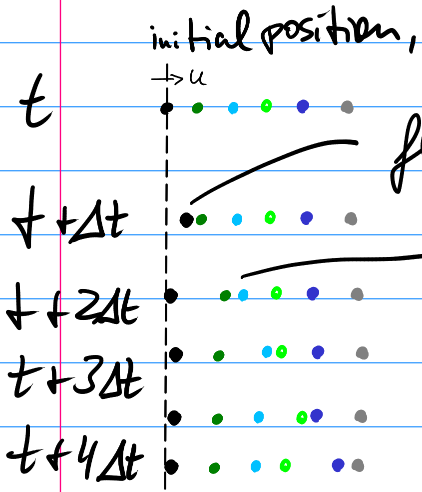


how does pressure help in getting
the fluid in the pipe up to speed?

Look at fluid elements.

$\rho \neq$ constant

fluid element can be squeezed.



fluid elements are squeezed ⇒ higher pressure

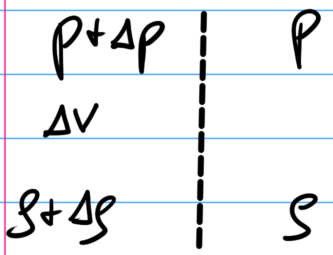
squeezing → higher pressure

squeezing propagates through the fluid

squeezing = higher density, higher pressure

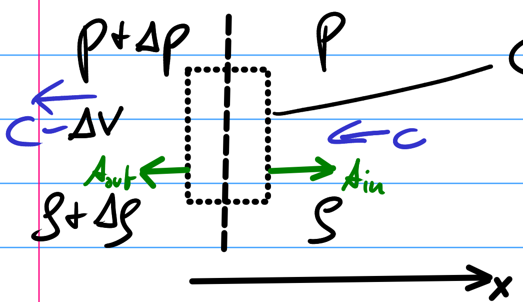
⇒ What is the velocity of spreading pressure?

⇒ The velocity of sound!



fluid at rest

↑ front of pressure expansion with velocity c ⇒ what is c?



CV with speed c

$\underline{A_{in}} \cdot \underline{c} < 0$

$A_{out} \cdot (c-\Delta V) > 0$

Do balances on CV $\qquad |A_{in}| = |A_{out}| = A$

continuity eq: $\quad -\rho A_{in} c + (\rho+\Delta\rho) A_{out} (c-\Delta V) = 0$

$$\Rightarrow -\cancel{\rho A c} + \cancel{\rho A c} + \Delta\rho A c - \rho A \Delta V - \underbrace{\cancel{\Delta\rho A \Delta V}}_{\text{negligible}} = 0$$

product of two small quantities („2nd order")

$$\Rightarrow \quad \Delta\rho \cancel{A} c - \rho \cancel{A} \Delta V = 0$$

$$\Rightarrow \quad \boxed{\Delta V = \frac{\Delta\rho}{\rho} c} \quad \text{(for later)}$$

Momentum balance across CV (x-direction)

„flow"-direction

$$\sum \dot{m}_o v_x - \sum \dot{m}_{in} v_x = \sum F_x$$

$$\dot{m}_o(-(c-\Delta v)) - \dot{m}_{in}(-c) = (p+\Delta p)A - pA$$

in negative x-direction

left side of CV — right side of CV

pressure force

$$\dot{m}_{in} = \rho A c$$

$$\dot{m}_{out} = (\rho+\Delta\rho)A(c-\Delta v)$$

$$= \rho A c - \rho A \Delta v + \Delta\rho A c - \cancel{\Delta\rho A \Delta v}$$

2nd order of small quantities

$$\Rightarrow (\rho A c - \rho A \Delta v + \Delta\rho A c)(-c+\Delta v) + \rho A c \cdot c = \Delta p A$$

$$\Rightarrow \cancel{-\rho A c^2} + \rho A c \Delta v + \rho A \Delta v c + \cancel{\rho A \Delta v^2}$$

2nd order

$$-\Delta\rho A c^2 - \cancel{\Delta\rho A c \Delta v} + \cancel{\rho A c^2} = \Delta p A$$

2nd order

$$\Rightarrow 2\rho \cancel{A} c \Delta v - \Delta\rho \cancel{A} c^2 = \Delta p \cancel{A}$$

$$\Rightarrow 2\rho c \underbrace{\Delta v}_{\text{from before}} - \Delta\rho c^2 = \Delta p$$

$$\Rightarrow 2\cancel{\rho} c \frac{\Delta\rho}{\cancel{\rho}} c - \Delta\rho c^2 = \Delta p$$

$$\Delta\rho c^2 = \Delta p$$

p.t.o

$$\Rightarrow \quad \boxed{c^2 = \frac{\Delta p}{\Delta \rho}}$$

$\Rightarrow$ limit to go to infinitesimal quantities

$$\Rightarrow \left.\frac{\partial p}{\partial \rho}\right|_S$$

at constant entropy (constraint from thermodynamics, out of scope here)

What does $\left.\frac{\partial p}{\partial \rho}\right|_S$ mean?

Equation of state describes the thermodynamical state of the fluid. $\Rightarrow$ That gives a relation between pressure p and density ρ.

The derivative describes, how density changes affect pressure.

$\Rightarrow$ change of state of our fluid

$\Rightarrow$ <u>fast</u> changes of state of interest here

(Why "fast"? $\Rightarrow$ Heat does not spread.)

$$\Rightarrow \quad \frac{p}{\rho^k} = \text{konst} = C$$

$k = \frac{c_p}{c_V}$

c_p heat capacity at constant pressure

c_V = heat capacity at constant volume

$$\Rightarrow \quad p = C\rho^k$$

$$\Rightarrow \quad \frac{\partial p}{\partial \rho} = kC\rho^{k-1} = kC\frac{\rho^k}{\rho} = k\frac{p}{\rho}$$

Now what is $\frac{P}{\rho}$?

Depends on the fluid.

For ideal gases: $\frac{P}{\rho} = RT$

$$\Rightarrow c^2 = \left.\frac{\partial P}{\partial \rho}\right|_s = k\frac{P}{\rho} = kRT$$

$$\Rightarrow \boxed{c = \sqrt{kRT}}$$

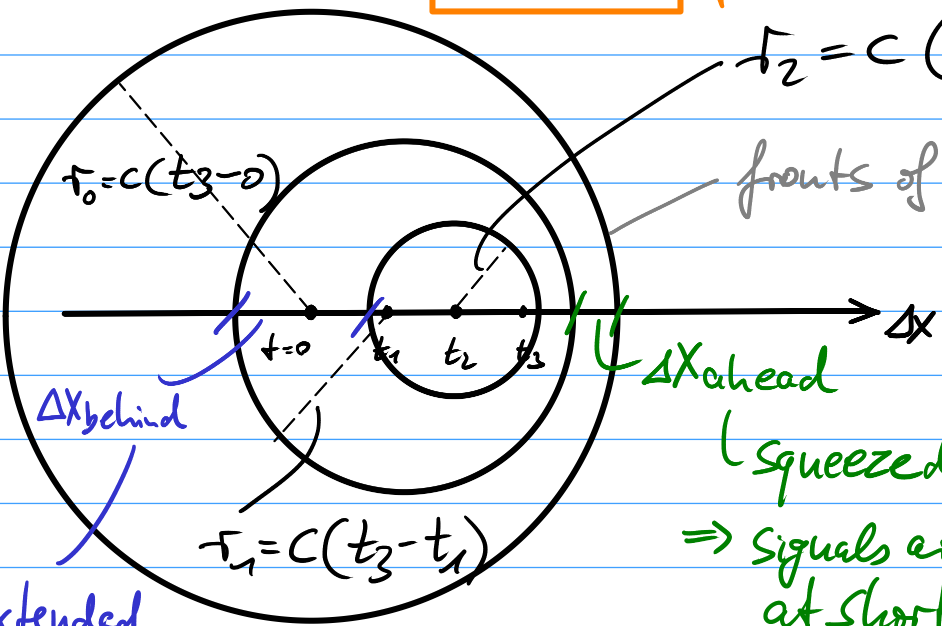
Speed of sound

= " of spreading

of pressure.

See how sound spreads from a moving point.
Start at time $t=0$. Now at time t_3.

$\boxed{V < C}$ point moves slower than sound



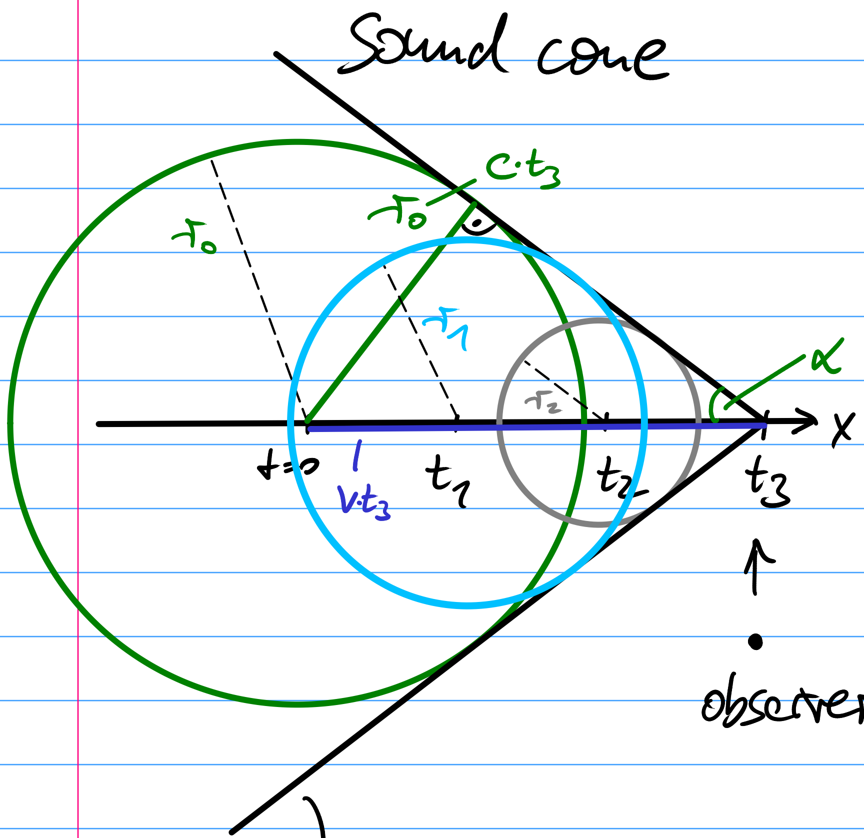
squeezed
$\Rightarrow$ signals arrive at shorter intervalls
$\Rightarrow$ higher frequency
$\Rightarrow$ higher tone

extended
drawn out
$\Rightarrow$ signals arrive at longer intervalls
$\Rightarrow$ lower frequency
$\Rightarrow$ low tone

Doppler-Effect

What if $v > c$?
If the point moves faster than sound?
We are again at time t_3. What happened so far?

$$\boxed{v > c}$$



$r_0 = c(t_3 - 0)$
$r_1 = c(t_3 - t_1)$
$r_2 = c(t_3 - t_2)$

observer: sees point at position at time t_3 but does not hear a sound, because that has not yet arrived.

„Schallmauer"
<u>Sound barrier</u>

$$\sin\alpha = \frac{c \cdot t_3}{v \cdot t_3} = \frac{c}{v} \quad \text{for } v > c$$

angle of sound cone.