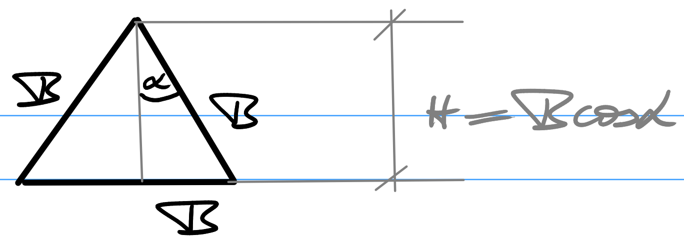
Fluid Dynamics

Task 2.5 c



$$dF_S = \Delta p(z)\, b(z)\, dz$$

$$\Delta p(z) = p_a + g\rho(H - z) - p_a$$

$$b(z) = \frac{H - z}{\cos\alpha} \qquad \text{(vgl. p. 16)}$$

$$\Rightarrow F_S = \int_0^H \Delta p(z)\, b(z)\, dz$$

$$= \int_0^H g\rho (H - z) \frac{(H - z)}{\cos\alpha}\, dz$$

$$= \frac{g\rho}{\cos\alpha} \int_0^H (H - z)^2\, dz \qquad \text{NR: s.u.}$$

$$= \frac{g\rho}{\cos\alpha} \left[-\frac{1}{3}(H - z)^3 \right]_0^H$$

$$= \frac{g\rho}{\cos\alpha} \left(0 - \left(-\frac{1}{3} H^3\right)\right) = \frac{1}{3} \frac{g\rho H^3}{\cos\alpha}$$

$$= 2500\,N$$

p.t.o.

NR: $f(x) \Rightarrow F(x) = \int f(x)\,dx$ bekannt

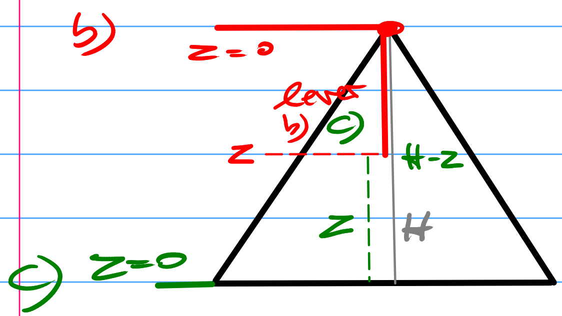
$$\Rightarrow \int f(\alpha x + \beta)\, dx = \frac{1}{\alpha} F(\alpha x + \beta) + c$$

Hier: $f(x) = x^2 \Rightarrow F(x) = \frac{1}{3}x^3$

mit $\alpha = -1$ u. $\beta = H$

$$dM_s = \Delta p(z)\, b(z) \overbrace{\underbrace{(H-z)}_{\text{why?}}}^{\text{lever}} dz$$

$\Rightarrow$ Because of this:



In b) we used z as the lever. So the application point of the torque is the origin at $z=0$ in b)!

Therefore, to have a ~~correct~~ comparison, we must use the triangle tip as the application point for the torque as well!

This means, in the green coordinate system of part c), the lever is given by $\underline{H-z}$!

$$\Rightarrow M_s = \int_0^H \Delta p(z)\, b(z)\,(H-z)\,dz$$

$$= \int_0^H g\rho (H-z) \frac{H-z}{\cos\alpha} (H-z)\, dz$$

$$= \frac{g\rho}{\cos\alpha} \int_0^H (H-z)^3 dz$$

$$= \frac{g\rho}{\cos\alpha} \left[-\frac{1}{4}(H-z)^4 \right]_0^H$$

$$= \frac{g\rho}{\cos\alpha} \left[0 - \left(-\frac{1}{4}H^4\right) \right] = \frac{1}{4}\frac{g\rho}{\cos\alpha} H^4$$

$$= 1623{,}8 \text{ Nm}$$

Fluid Dynamics
Task 2.5.c continued

In the coordinate system of this task, the lever for a torque applying to the tip of the triangle, is $H-z$. Hence:

$$M_S = (H - z_S)\, F_S$$

$$\Rightarrow \quad z_S = H - \frac{M_S}{F_S}$$

$$= 0{,}2165 \text{ m}$$