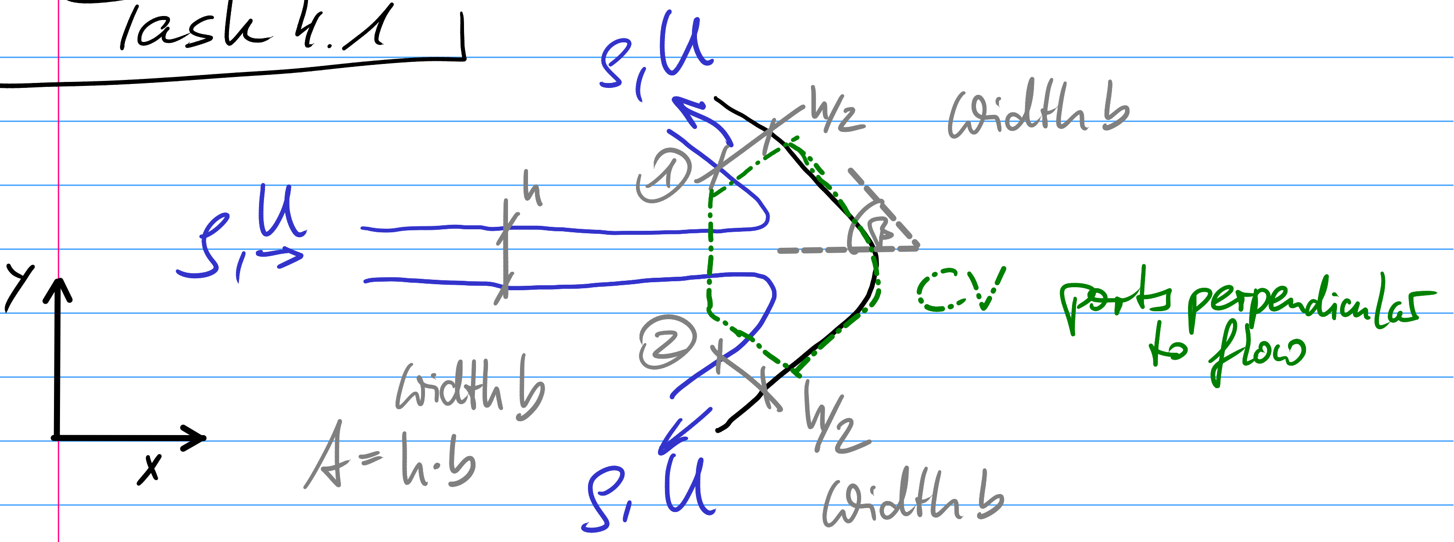
Task 4.1



check continuity eq.

$$\dot{m}_{in} = \dot{m}_{out,1} + \dot{m}_{out,2}$$

$$\cancel{\rho}\,\dot{V}_{in} = \cancel{\rho}\,\dot{V}_{out,1} + \cancel{\rho}\,\dot{V}_{out,2}$$

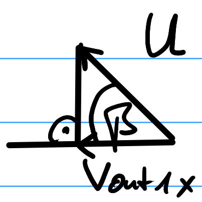
$$A_{in}\cdot \cancel{U} = A_{out,1}\cancel{U} + A_{out,2}\cancel{U}$$

$$h\cdot b = \frac{h}{2} b + \frac{h}{2}\cdot b \quad \checkmark$$

a) Force from momentum eq:

$$\sum_{out} \dot{m}\underline{V} - \sum_{in} \dot{m}\underline{V} = \sum \underline{F} \qquad \underline{V} = \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

$$\dot{m}_{out,1} V_{out,1,x} + \dot{m}_{out,2} V_{out,2,x} - \dot{m}_{in} V_{in,x} = F_{w,x}$$



$$V_{out1,x} = -U\cos\beta$$

↑ negative x-direction

Similarly: $$V_{out2,x} = -U\cos\beta$$

Task 4.1 continued

$$V_{in\,x} = U$$

$$\dot{m}_{in} = \rho_w \dot{V}_{in} = \rho_w A U = \rho h b U$$

$$\dot{m}_{out1} = \rho_w \dot{V}_{out,1} = \rho_w A_{out,1} U = \rho \tfrac{h}{2} b U$$

$$\dot{m}_{out2} = \text{"} = \text{"} = \rho \tfrac{h}{2} b U$$

$$\Rightarrow \underbrace{\rho \tfrac{h}{2} b U(-U\cos\beta)}_{out1} + \underbrace{\rho \tfrac{h}{2} b U(-U\cos\beta)}_{out2}$$

$$- \rho h b U U = F_{\omega,x}$$

$$\Rightarrow F_{\omega x} = -\rho h b U^2 (1 + \cos\beta)$$

b) Max of $F_{\omega,x}$?

$$0 = \frac{\partial F}{\partial\beta} = -\rho h b U^2 (0 + (-\sin\beta))$$

$$= \rho h b U^2 \sin\beta$$

$\Rightarrow \beta = 0° \lor \beta = 180° \Rightarrow \beta = 180°$ ruled out

$\Rightarrow$ „no wall"

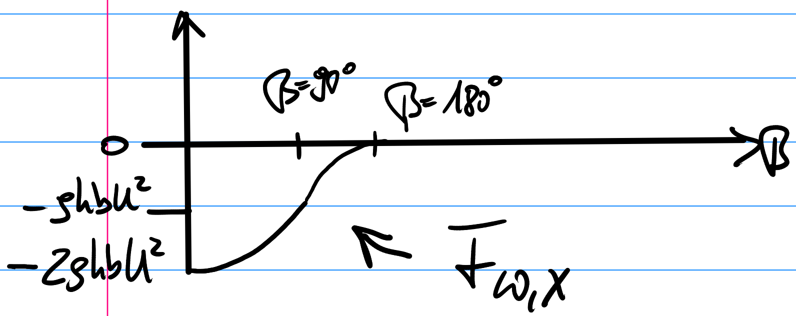
Max? $\quad \frac{\partial^2 F}{\partial\beta^2} = \rho h b U^2 \cos\beta$

$\Rightarrow$ for $\beta = 0$

$\Rightarrow$ $$\frac{\partial^2 F}{\partial \beta^2} = \rho h b U^2 \cdot 1 \gg 1$$

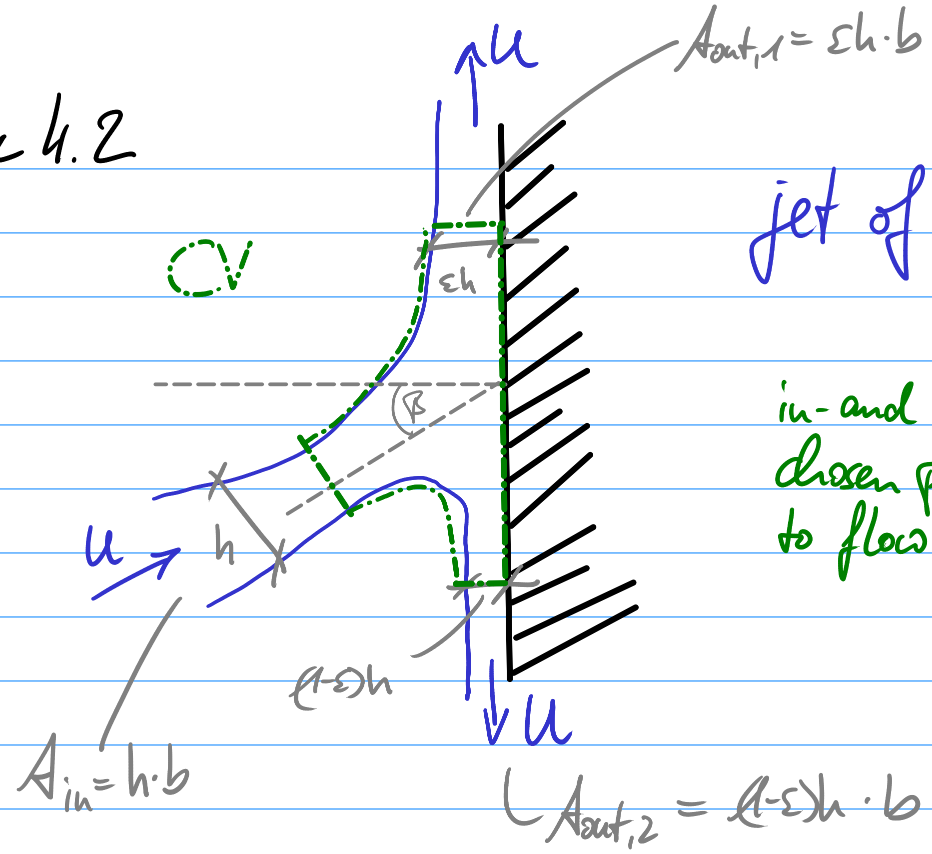
$\Rightarrow$ But this means minimum!

$\Rightarrow$ We look for a maximum!



$\Rightarrow$ Yes, the minimum is the maximum in the negative direction!

Task 4.2



check continuity eq.:

$$\dot V_{in} = \dot V_{out,1} + \dot V_{out,2}$$

$$\cancel{u} \cdot A_{in} = \cancel{u} A_{out1} + \cancel{u} A_{out2}$$

$$A_{in} = hb = \varepsilon hb + (1-\varepsilon) hb \quad \checkmark$$

a) $F_{W,x}$ force on the wall in x-direction

momentum eq. for x-direction from

$$\sum_{out} \dot m \underline{v} - \sum_{in} \dot m \underline{v} = \sum \underline{F} \qquad \underline{V} = \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

at outlets $\underline{V}_{out1} = \begin{pmatrix} 0 \\ u \end{pmatrix}$, $\underline{V}_{out2} = \begin{pmatrix} 0 \\ -u \end{pmatrix}$

$\Rightarrow$ no contributions to x-component from outlets!

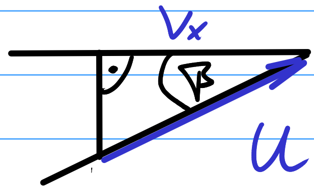
only one inlet: $\quad -\dot m V_x = \sum F_x$

Task 4.2 a (continued)

$$\dot{m} = \rho \dot{V} = \rho A U \qquad \frac{kg}{s}$$

v_x from triangle:

$= U \cos\beta$



forces: ambient pressure from all sides
$\Rightarrow$ cancel.

$$\Rightarrow \quad \sum F_x = F_{w,x}$$

force of wall on the fluid in x-direction

$$\Rightarrow \quad -\dot{m} v_x = F_{w,x}$$

$$-\rho A U \, U \cos\beta = F_{w,x} \qquad A = h \cdot b$$

$$\Rightarrow \quad F_{w,x} = -\rho \, h b U^2 \cos\beta$$

(force on the wall is minus force on the fluid

$$F_{wall,mech} = \rho \, h b U^2 \cos\beta \,)$$

Task 4.2 b)

Relation between ε and β from momentum eq. in y-direction.

$$\dot{m}_{out,1} V_{out1,y} + \dot{m}_{out2} V_{out2,y} - \dot{m}_{in} V_{in,y} = 0$$

no wall forces

$$\underline{V}_{out,1} = \begin{pmatrix} 0 \\ U \end{pmatrix} \qquad \underline{V}_{in} = \begin{pmatrix} U\cos\beta \\ U\sin\beta \end{pmatrix}$$

$$\underline{V}_{out,2} = \begin{pmatrix} 0 \\ -U \end{pmatrix}$$

$$\dot{m}_{in} = \rho \dot{V}_{in} = \rho A U = \rho h b U$$

$$\dot{m}_{out,1} = \rho \dot{V}_{out,1} = \rho A_{out,1} = \rho \varepsilon h b U$$

$$\dot{m}_{out,2} = \qquad = \rho (1-\varepsilon) h b U$$

$$\rho\varepsilon h b U U + \rho(1-\varepsilon) h b U(-U) - \rho h b U U \sin\beta = 0$$

$$\cancel{\rho}\varepsilon \cancel{h b U^2} - \cancel{\rho}\cancel{h b U^2} + \cancel{\rho}\varepsilon\cancel{h b U^2} - \cancel{\rho}\cancel{h b U^2}\sin\beta = 0$$

$$\varepsilon \quad - 1 \quad \varepsilon \quad - \sin\beta = 0$$

$$2\varepsilon = 1 + \sin\beta$$

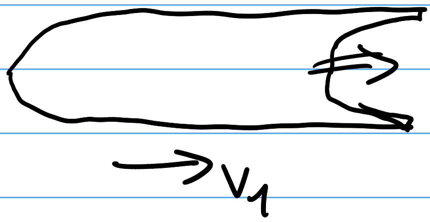
$$\varepsilon = \frac{1}{2}(1 + \sin\beta)$$

Task 4.4b) Hints:

$P = F \cdot v \Rightarrow$ which velocity?

$\Rightarrow$ mechanical power to move rocket.

F force on wall from a)



Work/Energy $\quad W = F \cdot \Delta s \quad$ Δs distance traveled by piece

Power = Work/Time $\quad P = \frac{W}{\Delta t}$

$$= F \underbrace{\frac{\Delta s}{\Delta t}}_{v_1} \neq v_E$$

Scratch

$$\underline{v}_{out1} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$