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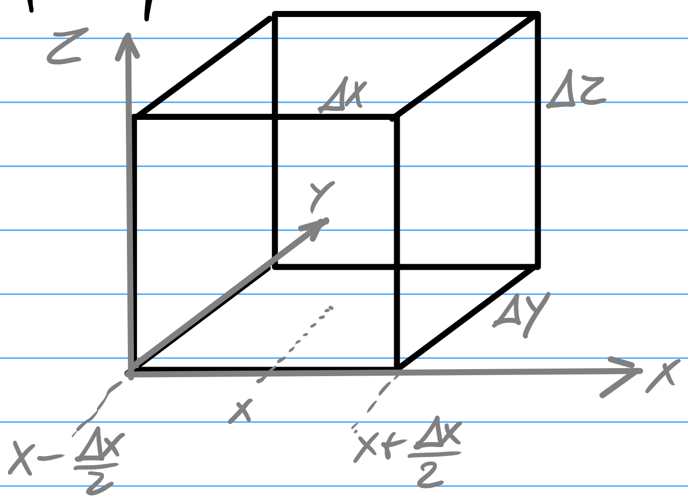

velocity

$$\underline{V} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

density ρ

Massflow in: $\dot{V} = \rho u A$ — but at which location?

$\Rightarrow$ express chang w.r.t. to position x by Taylor-Expansion

$$T(x) = f(x_0) + \frac{\partial f(x_0)}{\partial x}(x - x_0)$$

center of CV $\Rightarrow x$

$\Rightarrow$ boundary left and right: $x - \frac{\Delta x}{2}$, $x + \frac{\Delta x}{2}$

$$f(x) = \dot{V}$$

$$\Rightarrow \quad T(x) = \dot{V}(x) + \frac{\partial \dot{V}}{\partial x}\left(x \mp \frac{\Delta x}{2} - x\right)$$

left / right

$$= \dot{V}(x) \mp \frac{\partial \dot{V}}{\partial x}$$

$$= \rho u A \mp \frac{\partial}{\partial x}(\rho u A) \qquad A = \Delta y \Delta z$$

$$= \left(\rho u \mp \frac{\partial}{\partial x}(\rho u)\right)\Delta y \Delta z$$

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$$\nabla = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \qquad \rho \underline{v} = \begin{pmatrix} \rho u \\ \rho v \\ \rho w \end{pmatrix}$$

$\Rightarrow$ dot product:

$$\nabla \cdot (\rho \underline{v}) = \partial_x(\rho u) + \partial_y(\rho v) + \partial_z(\rho w)$$

$$\nabla \underline{v} = \underbrace{\begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix}}_{3\times 1} \underbrace{\begin{pmatrix} u & v & w \end{pmatrix}}_{1\times 3}$$

without dot!

$$= \begin{pmatrix} \partial_x u & \partial_x v & \partial_x w \\ \partial_y u & \partial_y v & \partial_y w \\ \partial_z u & \partial_z v & \partial_z w \end{pmatrix}$$

$$= (\text{Jacobean Matrix})^T$$

$$\llcorner \begin{pmatrix} \partial_x u & \partial_y u & \partial_z u \\ \partial_x v & \partial_y v & \partial_z v \\ \partial_x w & \partial_y w & \partial_z w \end{pmatrix}$$

$$\underline{\underline{I}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$