Kap 3, p. 29

$$f(x,y) = \frac{x-y}{x+y}$$

$$\partial_x f = \frac{1\cdot(x+y) - (x-y)\cdot 1}{(x+y)^2} = \frac{2y}{(x+y)^2}$$

$$\partial_y f = \frac{(-1)(x+y) - (x-y)\cdot 1}{(x+y)^2} = -\frac{2x}{(x+y)^2}$$

$$\partial_y(\partial_x f) = \frac{2(x+y)^{\cancel{2}} - 2y\, 2\cancel{(x+y)}\cdot 1}{(x+y)^{\cancel{4}\,3}}$$

$$= \frac{2x+2y-4y}{(x+y)^3} = \frac{2x-2y}{(x+y)^3}$$

$$\partial_x(\partial_y f) = -\frac{2(x+y)^{\cancel{2}} - 2x\, 2\cancel{(x+y)}\cdot 1}{(x+y)^{\cancel{4}\,3}}$$

$$= -\frac{2x+2y-4x}{(x+y)^3} = -\frac{-2x+2y}{(x+y)^3}$$

$$= \frac{2x-2y}{(x+y)^3}$$

Kap 3, p. 23

$$f(x,y) = \ln(x^2+y)$$

$$\partial_x f = \frac{1}{x^2+y} \cdot 2x = 2x(x^2+y)^{-1}$$

$$\partial_y f = \frac{1}{x^2+y} \cdot 1 = (x^2+y)^{-1}$$

$$\partial_y(\partial_x f) = \partial_{yx} f = 2x(-1)(x^2+y)^{-2} \cdot 1$$

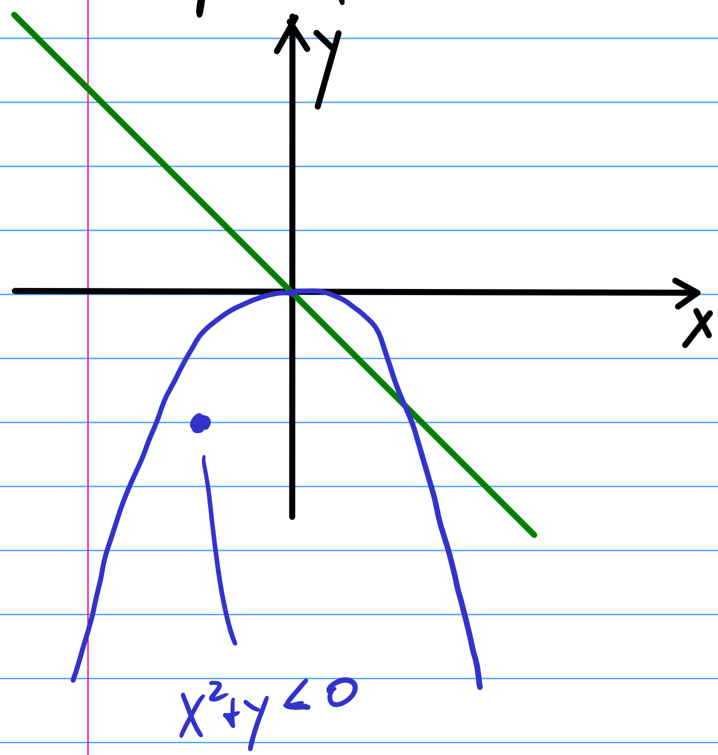
$$= -\frac{2x}{(x^2+y)^2}$$

$$\partial_x(\partial_y f) = \partial_{xy} f = (-1)(x^2+y)^{-2} \cdot 2x$$

$$= -\frac{2x}{(x^2+y)^2}$$

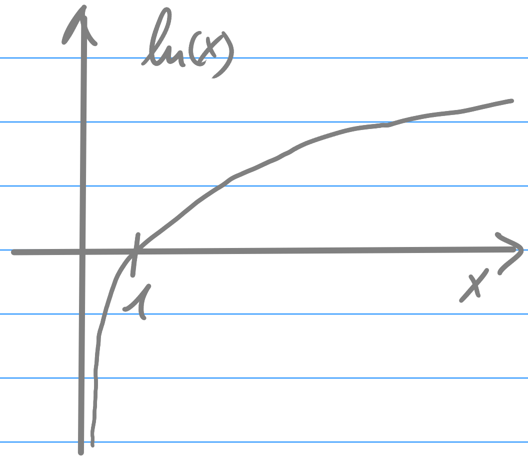
$= \checkmark$

Kap 3, p. 29



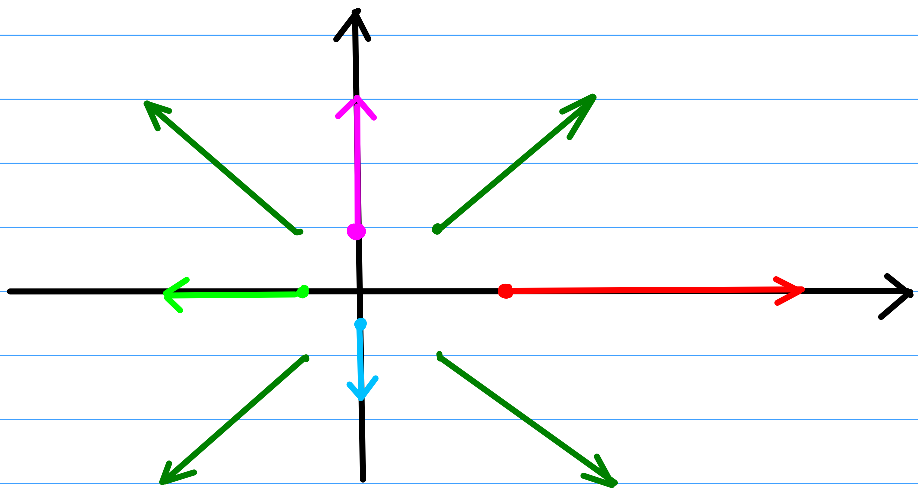
$x+y=0 \Rightarrow y=-x$

$x^2+y=0 \Rightarrow y=-x^2$



p. 30

$f(x,y)=x^2+y^2 \Rightarrow \text{grad} f=(2x,2y)$



Kap3, p.32

$f(x,y)\ ,\ g(x,y)\ :\ \mathbb{R}^2 \to \mathbb{R}$

$$\operatorname{grad}(f\cdot g) = \left(\partial_x(f\cdot g)\ ,\ \partial_y(f\cdot g)\right)$$

$$= \left((\partial_x f)\cdot g + f(\partial_x g),\ (\partial_y f)\cdot g + f(\partial_y g)\right)$$

$$= \left((\partial_x f)g\ ,\ (\partial_y f)g\right) + \left(f(\partial_x g),\ f(\partial_y g)\right)$$

$$= g\,\underbrace{(\partial_x f, \partial_y f)} + f\,\underbrace{(\partial_x g, \partial_y g)}$$

$$= g\ \operatorname{grad} f + f\ \operatorname{grad} g$$

Kap 3, p. 33

$$f(x,y) = e^x - e^{-y}$$
$$g(x,y) = x^2 + y^{-2}$$

N.R.:
$$\partial_y(-e^{-y}) = -\partial_y(e^{-y}) = -(-e^{-y}) = e^{-y}$$

$$f \cdot g = (e^x + e^{-y})(x^2 + y^{-2})$$

$\Rightarrow$ Ableitungen häßlich / länglich / fehleranfällig!

$$\operatorname{grad} f = (e^x \,,\, e^{-y})$$
$$\operatorname{grad} g = (2x \,,\, -2y^{-3})$$

$$\operatorname{grad}(f \cdot g) = g \operatorname{grad} f + f \operatorname{grad} g$$

im Prinzip fertig

$$\operatorname{grad}(c \cdot f) = c \operatorname{grad} f = \begin{pmatrix} ce^x \\ ce^{-y} \end{pmatrix}^T$$
$$\operatorname{grad}(f+g) = \operatorname{grad} f + \operatorname{grad} g$$
$$\operatorname{grad}\left(\frac{f}{g}\right) = \frac{1}{g^2}\left(g \operatorname{grad} f - f \operatorname{grad} g\right)$$

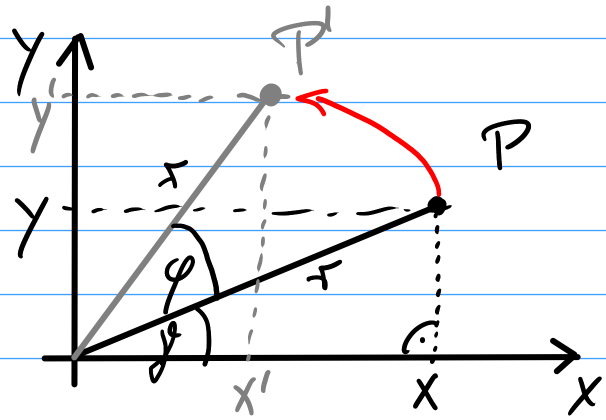
im Prinzip fertig

$$= \frac{1}{g} \operatorname{grad} f - \frac{f}{g^2} \operatorname{grad} g$$

Kap 3, p. 38/39

$$g_\varphi(x,y) = \begin{pmatrix} x\cos\varphi - y\sin\varphi \\ x\sin\varphi + y\cos\varphi \end{pmatrix}$$

$$= \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$



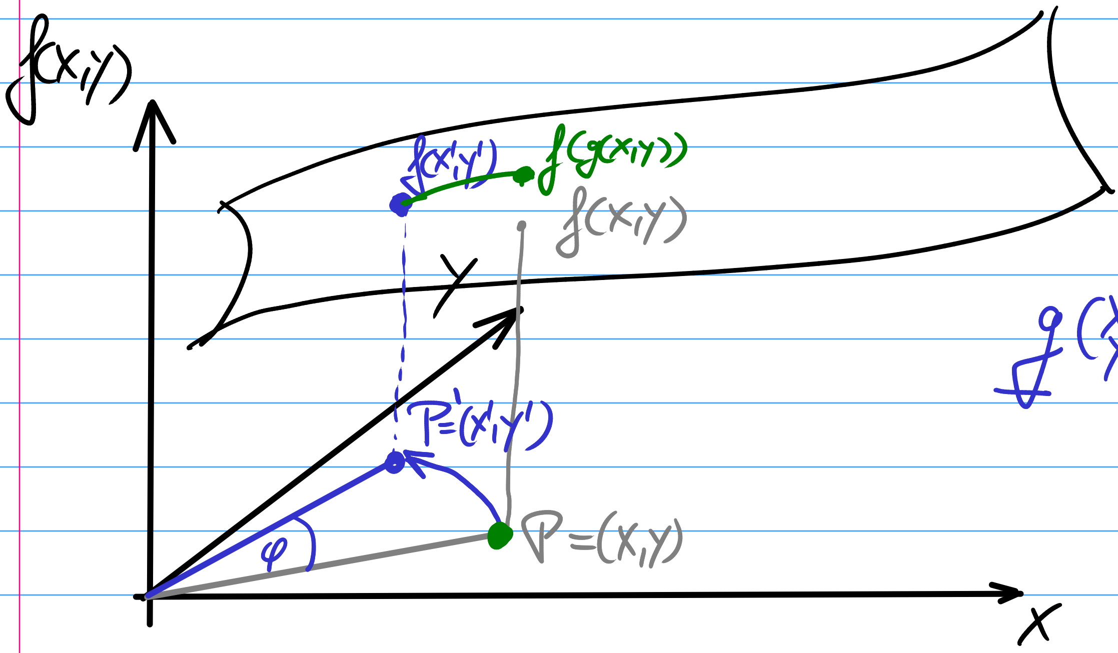
Drehung um Winkel φ um den Ursprung

$$x' = r\cos(\varphi+\psi) = r(\cos\varphi\cos\psi - \sin\varphi\sin\psi)$$
$$= \cos\varphi\,\underbrace{r\cos\psi} - \sin\varphi\,\underbrace{r\sin\psi}$$
$$= \cos\varphi\quad x \quad - \sin\varphi \quad y$$

$$y' = r\sin(\varphi+\psi) = r(\sin\varphi\cos\psi + \cos\varphi\sin\psi)$$
$$= \sin\varphi\,\underbrace{r\cos\psi} + \cos\varphi\,\underbrace{r\sin\psi}$$
$$= \sin\varphi\quad x \quad + \cos\varphi\quad y$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

Kap 3, p. 38/39



$g\binom{x}{y} \Rightarrow$ Drehung <u>gegen den</u> Uhrzeigersinn

x,y → .... (gerechnet) → Zahlenwert f(g(x,y))

Die Plottroutine macht, das was sie soll:
Sie malt den Zahlenwert f(g(x,y)) über die Stelle x,y. ⇒ Drehung <u>im</u> Uhrzeigersinn!