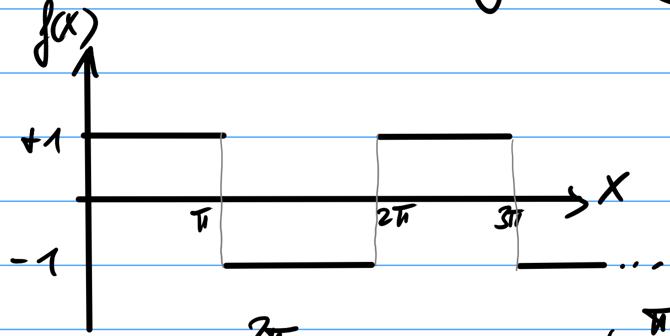
Kap 2, p. 39

$$f(x) = \begin{cases} +1 & 0 < x \leq \pi \\ -1 & \pi < x \leq 2\pi \end{cases}$$



$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\left(\int_0^{\pi} 1\,dx + \int_{\pi}^{2\pi} (-1)\,dx \right)$$

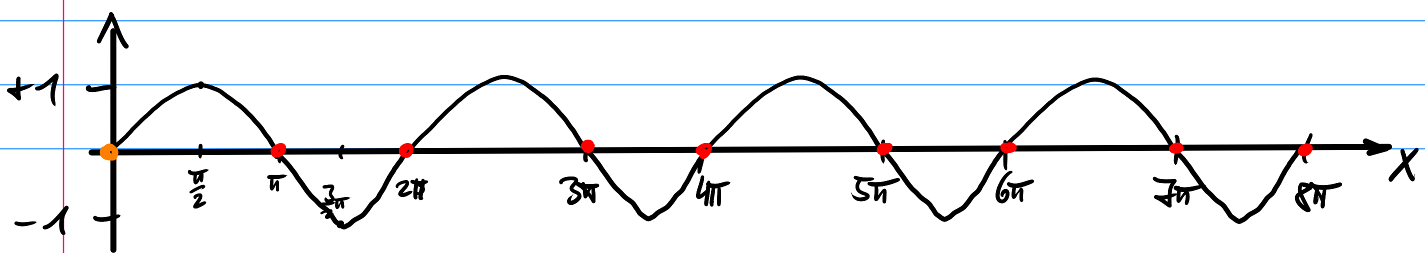
$$= \frac{1}{\pi}\left(\pi + \left((-2\pi) - (-\pi) \right) \right) = 0$$

$$a_m = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos mx\,dx$$

$$= \frac{1}{\pi}\left(\int_0^{\pi} (+1) \cos mx\,dx + \int_{\pi}^{2\pi} (-1) \cos mx\,dx \right)$$

$$= \frac{1}{\pi}\left(\left[\frac{1}{m} \sin mx \right]_0^{\pi} + \left[-\frac{1}{m} \sin mx \right]_{\pi}^{2\pi} \right)$$

$$= \frac{1}{\pi}\left(0 - 0 + 0 - 0 \right) = 0$$

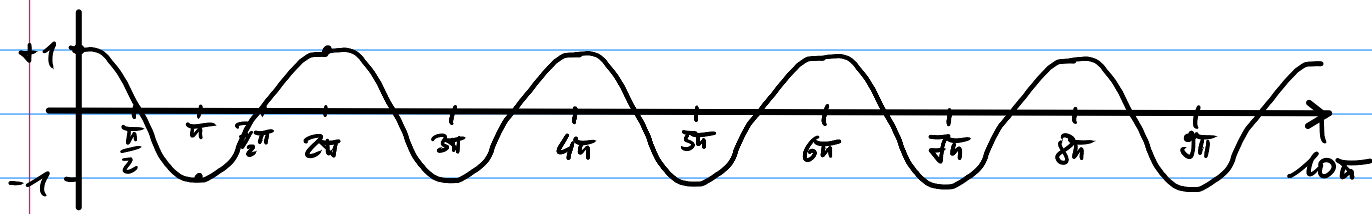


Kap. 2, p. 39 (Fortsetzg.)

$$b_m = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin mx\,dx$$

$$= \frac{1}{\pi}\left(\int_0^{\pi}(+1)\sin mx + \int_\pi^{2\pi}(-1)\sin mx\,dx\right)$$

$$= \frac{1}{\pi}\left(\left[-\frac{1}{m}\cos mx\right]_0^{\pi} + \left[\frac{1}{m}\cos mx\right]_\pi^{2\pi}\right)$$



$$\cos m\pi = \begin{cases} -1 & m \text{ ungerade} \\ +1 & m \text{ gerade} \end{cases}$$

m ungerade:

$$\left[-\frac{1}{m}\cos mx\right]_0^{\pi} = -\frac{1}{m}\left(\cos m\pi - \cos 0\right)$$

$$= -\frac{1}{m}\left(-1 - 1\right) = \frac{2}{m}$$

$$\left[\frac{1}{m}\cos mx\right]_\pi^{2\pi} = \frac{1}{m}\left(\cos(m\,2\pi) - \cos(m\pi)\right)$$

$$= \frac{1}{m}\left(1 - (-1)\right) = \frac{2}{m}$$

$$\Rightarrow \boxed{b_m = \frac{1}{\pi}\left(\frac{2}{m} + \frac{2}{m}\right) = \frac{4}{\pi m}}$$

für m ungerade

m gerade

$$\left[-\frac{1}{m}\cos mx\right]_0^{\pi} = -\frac{1}{m}\left(\cos m\pi - \cos 0\right)$$
$$= -\frac{1}{m}\left(1 - 1\right) = 0$$

$$\left[\frac{1}{m}\cos mx\right]_{\pi}^{2\pi} = \frac{1}{m}\left(\cos m2\pi - \cos m\pi\right)$$
$$= \frac{1}{m}\left(1 - 1\right) = 0$$

$\boxed{b_m = 0}$ für m gerade

$\Rightarrow$ nur Koeffizienten für m ungerade

$m = 2l+1 \qquad l = 0, 1, 2, \ldots$

$m = 2k-1 \qquad k = 1, 2, 3, \ldots$

$$\Rightarrow f(x) = \sum_{l=0} \frac{4}{\pi(2l+1)} \sin\left((2l+1)x\right)$$
$$= \frac{4}{\pi} \sum_{l=0} \frac{\sin\left((2l+1)x\right)}{2l+1}$$
$$= \frac{4}{\pi} \sum_{k=1} \frac{\sin\left((2k-1)x\right)}{2k-1}$$

Kap 2, p. 40

Bisher

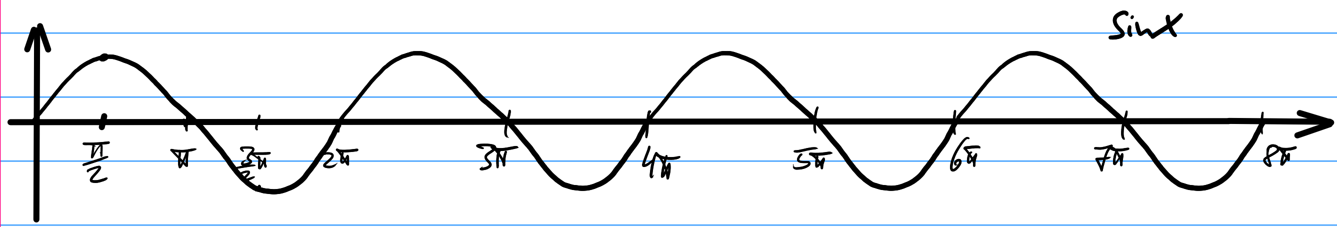
$$f(x) = \frac{a_0}{2} + \sum_{n=1} a_n \cos nx + b_n \sin nx$$

$$\text{für } P = 2\pi$$

Jetzt

$$g(t) = f(\omega_0 t) = \frac{a_0}{2} + \sum a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$\text{mit } \omega_0 = \frac{2\pi}{T} \qquad P = T$$

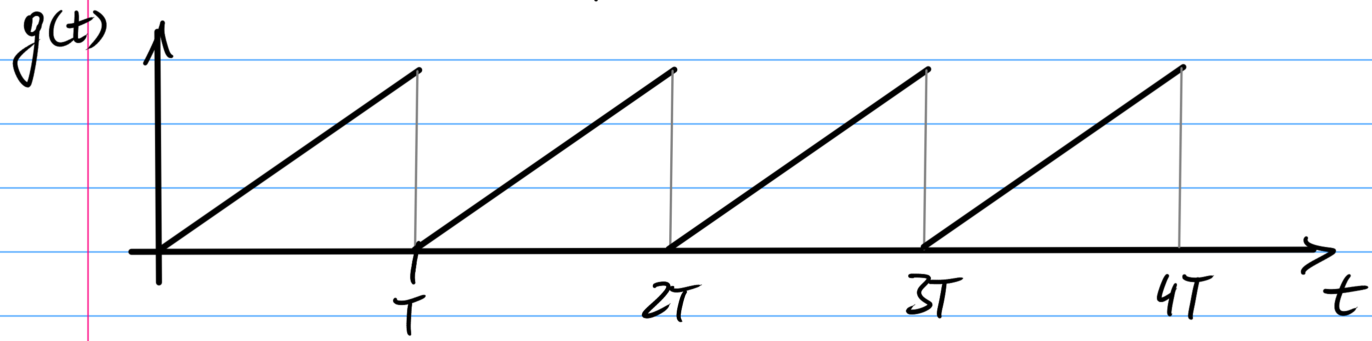


$x \in [0, 2\pi] \Rightarrow$ $\sin x$ durchläuft 1 Periode
$\sin nx$ durchläuft n Perioden
$\cos x$, $\cos nx$ entsprechend

$t \in [0, T]$
$\Rightarrow \omega_0 t \in [0, 2\pi] \Rightarrow$ $\sin \omega_0 t$ durchläuft 1 Periode
$\sin n\omega_0 t$ durchläuft n Perioden
$\cos \omega_0 t$, $\cos n\omega_0 t$ entsprechend

Kap 2, p. 41

$$g(t) = \frac{G}{T} t \qquad 0 \leq t < T$$



$$\omega_0 = \frac{2\pi}{T}$$

$$a_0 = \frac{2}{T} \int_0^T g(t)\, dt = \frac{2}{T} \int_0^T \frac{G}{T} t \, dt$$

$$= \frac{2G}{T^2} \left[\frac{1}{2} t^2 \right]_0^T = \frac{2G}{T^2} \cdot \left[\frac{T^2}{2} - 0 \right] = G$$

$$a_m = \frac{2}{T} \int_0^T g(t) \cos m\omega_0 t \, dt$$

$$= \frac{2}{T} \int_0^T \frac{G}{T} t \cos m\omega_0 t \, dt = \frac{2G}{T^2} \int_0^T t \cos(m\omega_0 t)\, dt$$

$u \Rightarrow u' = 1$

v'

$\Rightarrow v = \frac{1}{m\omega_0} \sin m\omega_0 t$

$m \neq 0$

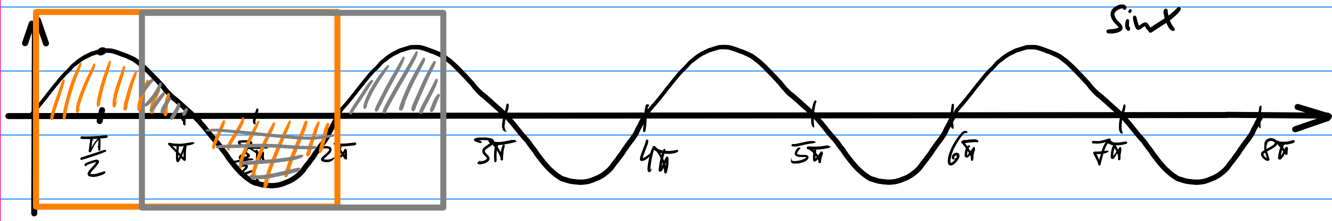
$$= \frac{2G}{T^2} \left(\left[t \frac{1}{m\omega_0} \sin m\omega_0 t \right]_0^T - \int_0^T 1 \frac{1}{m\omega_0} \sin m\omega_0 t \, dt \right)$$

$\omega_0 = \frac{2\pi}{T}$

Kap2, p. 41 (Fortsetzg)

$$= \frac{2G}{T^2}\left(\underbrace{T\frac{1}{m\omega_0}\sin(m\underbrace{\omega_0 T}_{2\pi})}_{=0} - \underbrace{0\cdot\frac{1}{m\omega_0}\sin 0}_{=0} - \frac{1}{m\omega_0}\underbrace{\int_0^T \sin m\omega_0 t\, dt}_{\text{Integral über volle Periode} = 0} \right)$$

$= 0 = a_m$ für $m \neq 0$



$$b_m = \frac{2}{T}\int_0^T g(t)\sin m\omega_0 t\, dt$$

$$= \frac{2}{T}\int_0^T \frac{G}{T} t \sin m\omega_0 t\, dt = \frac{2G}{T^2}\int t \sin m\omega_0 t\, dt$$

$u \Rightarrow u' = 1$ $\quad v'$

$\Rightarrow v = -\frac{1}{m\omega_0}\cos m\omega_0 t$

$$= \frac{2G}{T^2}\left(\left[t\left(-\frac{1}{m\omega_0}\cos m\omega_0 t\right)\right]_0^T - \underbrace{\int_0^T 1\cdot\left(-\frac{1}{m\omega_0}\cos m\omega_0 t\right) dt}_{\text{Integration über volle Periode} \Rightarrow = 0} \right)$$

Kap 2, p. 41 (Fortsetzg.)

$$\Rightarrow \dots \quad b_m = \frac{2G}{T^2}\left(\left(-\frac{T}{m\omega_0}\cos m\omega_0 T\right) - \underbrace{\left(-\frac{0}{m\omega_0}\cos m\omega_0 0\right)}_{=0}\right)$$

$$= -\frac{2GT}{T^2 m\omega_0}\cos(m\omega_0 T) \quad \Big| \quad \omega_0 = \frac{2\pi}{T}$$

$$= -\frac{\cancel{2}G\cancel{T}}{\cancel{T^2}\, m\,\frac{\cancel{2}\pi}{\cancel{T}}}\cos\left(m\frac{2\pi}{\cancel{T}}\cancel{T}\right)$$

$$= -\frac{G}{m\pi}\underbrace{\cos(m2\pi)}_{=1} = -\frac{G}{m\pi}$$

$m = 1, 2, 3, \dots$

Zusammen

$$g(t) = \frac{a_0}{2} + \sum_{m=1} b_m \sin m\omega_0 t$$

$$= \frac{G}{2} + \sum_{m=1}\left(-\frac{G}{m\pi}\right)\sin(m\omega_0 t)$$

$$= \frac{G}{2} - \frac{G}{\pi}\sum_{m=1}\frac{\sin(m\omega_0 t)}{m}$$

$$= \frac{G}{2} - \frac{G}{\pi}\left(\frac{\sin\omega_0 t}{1} + \frac{\sin 2\omega_0 t}{2} + \frac{\sin 3\omega_0 t}{3} + \dots\right)$$

Kap 2. p. 41

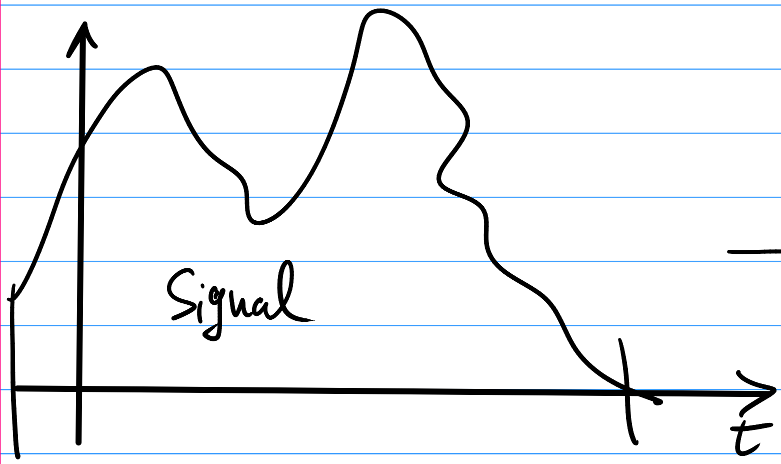
Bsp Matlab

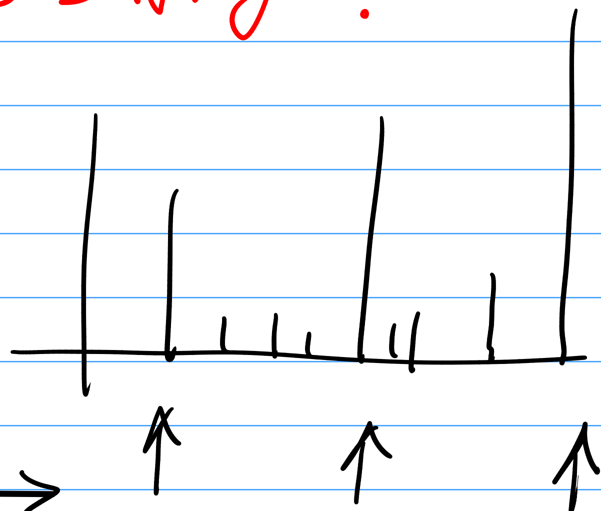
Fourier-Reihe

$$f(x) = \frac{a_0}{2} + \sum_{m=1} a_m \cos mx + b_m \sin mx$$

Amplituden

⇒ Wichtigkeit des Beitrags!





Fourier-Koeffizienten

⇒ einige dominant!

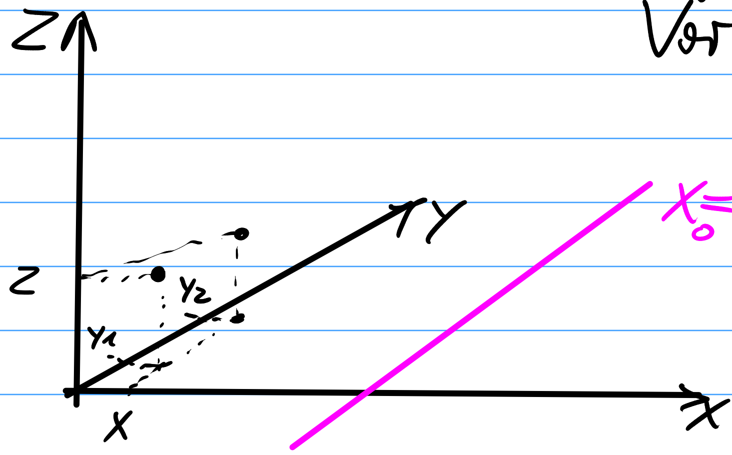
Je nach Anwendung Anregung mit diesen Frequenzen vermeiden!

„And now for something completely different .... “

Fragen nach der Vorlesung.

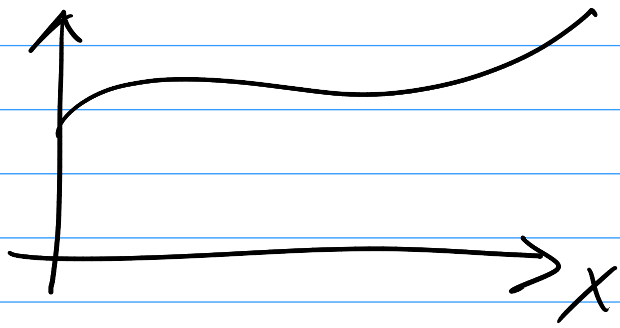
$\rho(x,y,z)$

$\underline{v}(x,y,z)$



$x_0 = \text{const}$

$f(x)$



$\rho(x_0,y,z)$