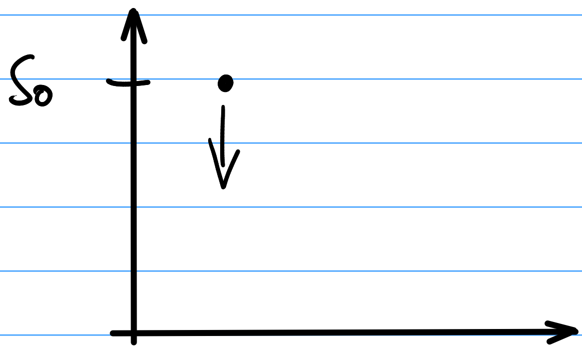
Kap. 5, p. 7

$$F_G = -mg = ma = m \cdot \ddot{s}$$

$$\Rightarrow \quad \ddot{s} = -g \qquad\qquad g \text{ const.}$$

$$\dot{s} = -gt + c$$

$$s(t) = -\tfrac{1}{2} g t^2 + ct + d$$



$$s(0) = s_0 = d$$

$$\dot{s}(0) = v_0 = c$$

p. 21

$$f(t) = A e^{-4t} \quad \Rightarrow \quad \dot{f} = -4 \underbrace{A e^{-4t}}_{f} = -4f$$

vgl. p. 19: $\quad \dot{x} + 2x = 0$

$$\Rightarrow \dot{x} = -2x$$

$$\ddot{f} + \lambda^2 f = 0 \Rightarrow \ddot{f} = -\lambda^2 f$$

$$f = \sin(\lambda t)$$

p. 22

$$f(t) = A \sin \lambda t + B \cos \lambda t$$

$$\dot{f}(t) = \lambda A \cos \lambda t - \lambda B \sin \lambda t$$

$$\ddot{f}(t) = -\lambda^2 A \sin \lambda t - \lambda^2 B \cos \lambda t$$

$$= -\lambda^2 (A \sin \lambda t + B \cos \lambda t)$$

$$= -\lambda^2 f$$

p. 23

$$\dot{x} = 4t^2 \quad \Rightarrow \quad x(t) = 4 \cdot \frac{1}{3} t^3 + c$$

$$\ddot{x} = t^3 - 2t \quad \Rightarrow \quad \dot{x}(t) = \frac{1}{4} t^4 - t^2 + c$$

$$x(t) = \frac{1}{4}\frac{1}{5} t^5 - \frac{1}{3} t^3 + ct + d$$

$$\dot{x} = -6x \quad \Rightarrow \quad x = A e^{-6t}$$

$$\ddot{x} = -\underset{\uparrow}{6}x \quad \Rightarrow \quad x = A \sin \sqrt{6}t + B \cos \sqrt{6} t$$

$$\lambda^2 \Rightarrow \lambda = \sqrt{6}$$

Was ist mit additiver Konstante?

$$x = e^{-6t} + A \quad \Rightarrow \quad \dot{x} = -6e^{-6t} + 0$$

$$\neq -6x$$

$$= -6(e^{-6t} + A)$$

$\Rightarrow$ Ist dann keine Lösung. $\Rightarrow$ Wir lernen noch, wie man das ausrechnet.

Kap 5, p.25

$$f' = -4f \Rightarrow f(t) = Ae^{-4t}$$

$$f(0) = 2 \Rightarrow 2 = f(0) = Ae^{-4\cdot 0} = A$$

$t=0$ → einsetzen

$$\ddot{f} + \lambda^2 f = 0 \Rightarrow f(t) = A\sin\lambda t + B\cos\lambda t$$

$f(0) = 0$

$\dot{f}(0) = 1$

$$\Rightarrow \underline{0} = f(0) = A\sin\lambda\cdot 0 + B\cos\lambda\cdot 0$$

$$= 0 + \underline{B}\cdot 1$$

s.o.

$$\Rightarrow 1 = \dot{f}(0) = A\lambda\cos\lambda 0 - B\lambda\sin\lambda 0$$

$$= A\lambda \quad 1 \qquad 0$$

$$\Rightarrow \underline{A = \frac{1}{\lambda}}$$

Kap 5, p. 27

$$\int g(x)\frac{dx}{dt}\,dt = \int h(t)\,dt \quad \Big|\ \text{Subst.}$$

$$\underbrace{\frac{dx}{dt}\,dt}_{\text{Substitution}}$$

$$\Rightarrow \int g(x)\,dx = \int h(t)\,dt$$

Handwerklich

$$g(x)\frac{dx}{dt} = h(t) \quad \Big|\cdot dt$$

$$g(x)\,dx = h(t)\,dt \quad \Big|\ \text{„Integrieren"}$$

$$\int g(x)\,dx = \int h(t)\,dt$$

$$G(x) = H(t)$$

p. 28 $\frac{dx}{dt} = \dot{x} = 4xt \Rightarrow \underbrace{\frac{1}{x}}_{g(x)}\frac{dx}{dt} = \underbrace{4t}_{h(t)}$

$$\Rightarrow \int g(x)\,dx = \int h(t)\,dt$$

$$\int \frac{1}{x}\,dx = \int 4t\,dt$$

eine Konstante reicht.

$$\Rightarrow \ln x = 2t^2 + c \quad \Big|\ e^{(\dots)}$$

$$\Rightarrow x = e^{2t^2+c} = e^{2t^2}\underbrace{e^{c}}_{D} = De^{2t^2}$$

p.28

$$\frac{dx}{dt} = kx \qquad \Big| :x \quad \Big| \cdot dt$$

$$\Rightarrow \frac{1}{x}\, dx = k\, dt \qquad \Big| \int$$

$$\Rightarrow \int \frac{1}{x}\, dx = \int k\, dt$$

$$\Rightarrow \ln x = kt + c \qquad \Big| e^{(\dots)}$$

$$\Rightarrow x(t) = e^{kt+c} = e^{kt}\underbrace{e^{c}}_{D} = De^{kt}$$

$$\frac{dx}{dt} = \frac{bx}{t} \qquad \Big| :x \quad \Big| \cdot dt \quad \Big| \int$$

$$\int \frac{1}{x}\, dx = \int \frac{b}{t}\, dt$$

$$\Rightarrow \ln x = b \ln t + c \qquad \Big| e^{(\quad)}$$

noch nicht

$$= \ln t^{b} + c \qquad \Big| c = \ln D$$

$$= \ln t^{b} + \ln D$$

$$= \ln(t^{b} \cdot D) \qquad \Big| e^{(\quad)}$$

$$\Rightarrow x = Dt^{b}$$

Kap 5, p. 28 $\qquad$ $x(4) = 9$

$$t^2 \frac{dx}{dt} = \frac{1}{x} \quad \Big| \cdot x \Big| : t^2 \Big| \cdot dt \quad \Big| \int$$

$$\int x\,dx = \int \frac{1}{t^2}\,dt$$

$$\frac{1}{2}x^2 = -\frac{1}{t} + C \qquad \Big| \sqrt{}$$

$$x = \overset{+}{\ominus}\sqrt{2C - \frac{2}{t}}$$

nicht erlaubt wg. $x > 0$

$$9 = x(4) = \sqrt{2C - \frac{2}{4}} \qquad \Big|\,{}^2$$

$$81 = 2C - \frac{1}{2} \qquad \Big| + \frac{1}{2}$$

$$81 + \frac{1}{2} = \frac{162}{2} + \frac{1}{2} = \boxed{\frac{163}{2} = 2C}$$

$$\Rightarrow \quad x(t) = \sqrt{\frac{163}{2} - \frac{2}{t}}$$