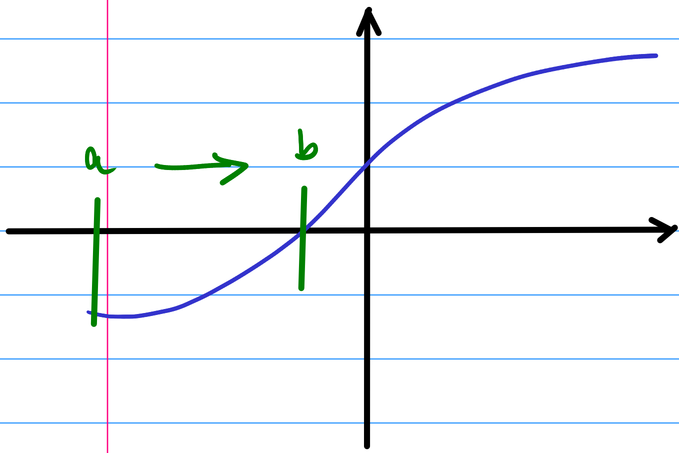
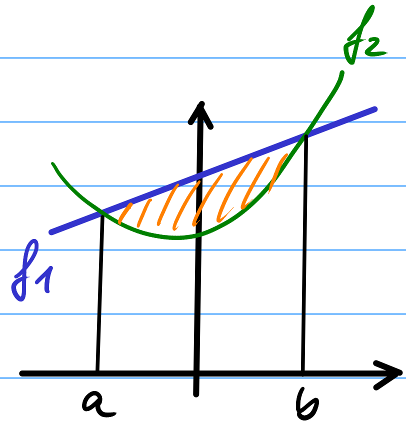
Kap 1, p. 14



$$\int_a^b f(x)\,dx < 0$$



Bsp.: $f_1(x) = \frac{x}{2} + 4$

$f_2(x) = \frac{1}{2}x^2 + 1$

$$\int_a^b f_1(x) - f_2(x)\,dx$$

$\Rightarrow$ a, b aus $f_1(x) = f_2(x)$

$\frac{x}{2} + 4 = \frac{1}{2}x^2 + 1 \quad | \cdot 2$

$x + 8 = x^2 + 2 \quad |$ auf eine Seite

$0 = x^2 - x - 6$

$= (x-p)(x-q)$

$= x^2 - (p+q)x + pq$

$\Rightarrow p+q = 1 \quad p \cdot q = -6 \Rightarrow p = 3 \quad q = -2$

$= (x-3)(x+2)$

$\Rightarrow b = x_2 = 3 \quad \vee \quad a = x_1 = -2$

Kap. 1, p. 14 (Fortsetzg.)

$$\Rightarrow \int_{-2}^{3} \frac{x}{2}+4 - \left(\frac{1}{2}x^2+1\right) dx$$

$$= \int_{-2}^{3} -\frac{1}{2}x^2 + \frac{x}{2} + 3 \; dx$$

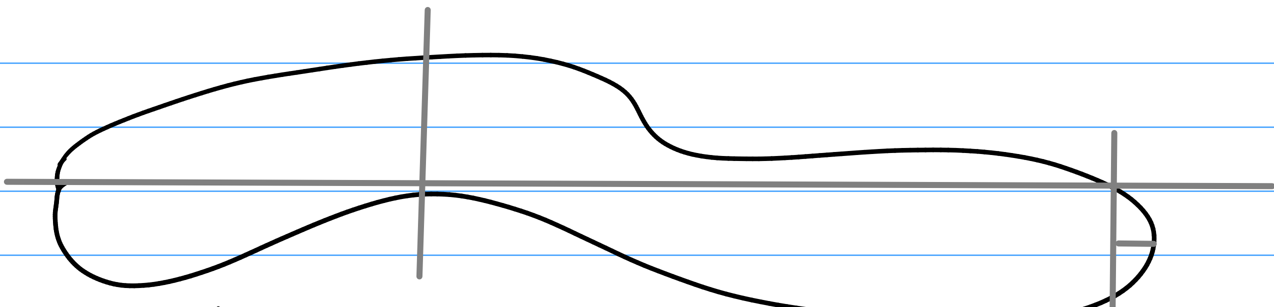
$$= \left[-\frac{1}{2}\frac{1}{3}x^3 + \frac{1}{2}\frac{1}{2}x^2 + 3x \right]_{-2}^{3}$$

$$= -\frac{1}{2}\frac{1}{3}3^3 + \frac{1}{2}\frac{1}{2}3^2 + 3\cdot 3$$
$$- \left(-\frac{1}{2}\frac{1}{3}(-2)^3 + \frac{1}{2}\frac{1}{2}(-2)^2 + (-2)\cdot 3 \right)$$

$$= -\frac{9}{2} + \frac{9}{4} + 9 \underbrace{- \left(\frac{4}{3} + \underbrace{1-6}_{-5} \right)}_{+5}$$

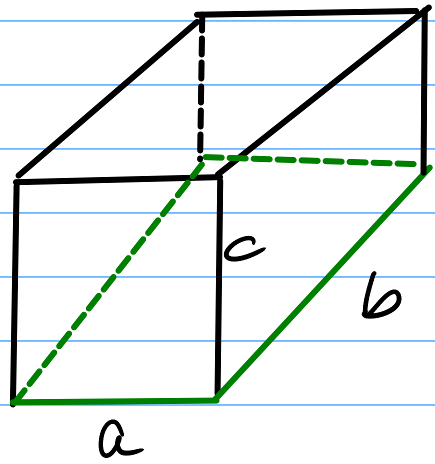
$$= -\frac{9}{4}\cdot\frac{3}{3} - \frac{4}{3}\cdot\frac{4}{4} + 14$$

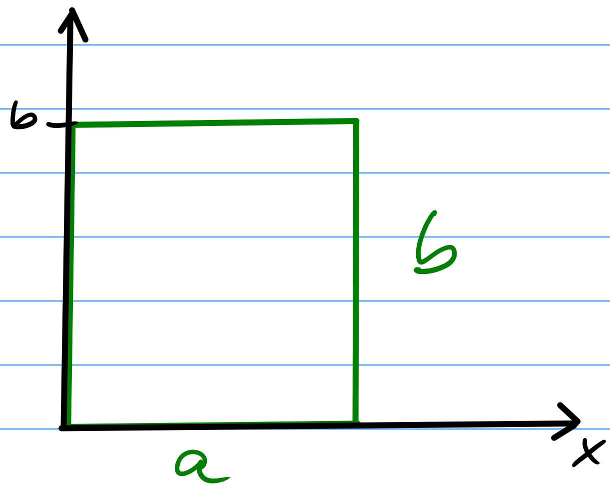
$$= -\frac{43}{12} + 14\,\frac{12}{12} = \frac{168-43}{12} = \frac{125}{12}$$



komplizierte Gebiete aufteilen.

Kap 1, p. 15



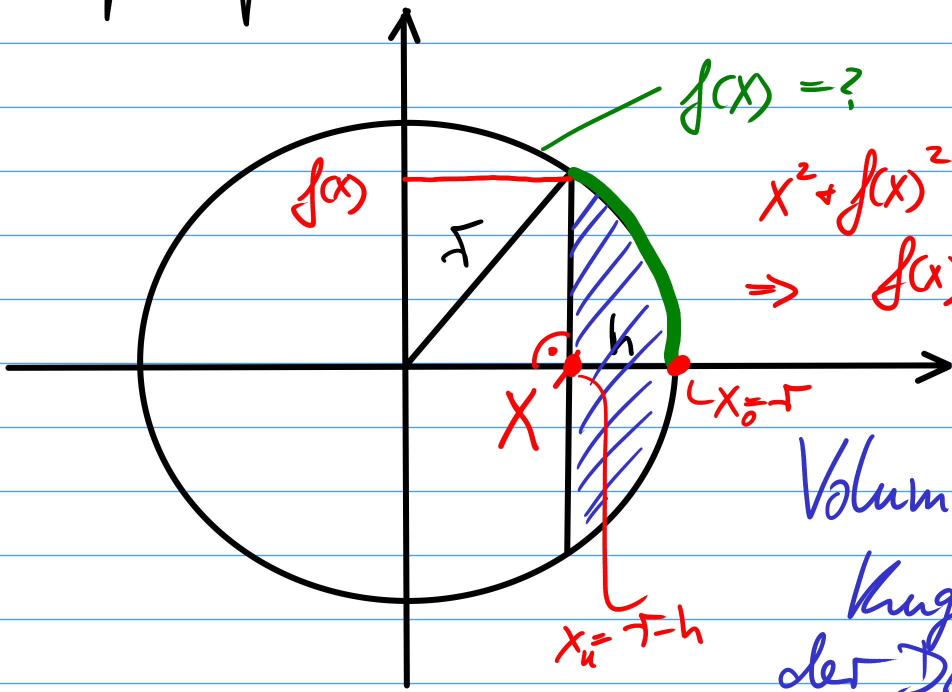


Grundfläche: $\int_0^a b\,dx = a \cdot b = A$

Volumen: $V = A \cdot h$

$$\int_0^c A\,dx = (a \cdot b) \cdot c$$

Kap1, p.19



$$x^2 + f(x)^2 = r^2$$

$$\Rightarrow \quad f(x) = \sqrt{r^2 - x^2}$$

Volumen des Kugelabschnitts der Dicke h ⇒ ?

$$V = \pi \int_{x_u = r-h}^{x_o = r} \left(\sqrt{r^2 - x^2}\right)^2 dx$$

$$= \pi \int_{r-h}^{r} r^2 - x^2 \, dx = \pi \left[r^2 x - \frac{1}{3}x^3 \right]_{r-h}^{r}$$

$$= \pi \left[r^3 - \frac{1}{3}r^3 - \left(r^2(r-h) - \frac{1}{3}(r-h)^3 \right) \right]$$

$$= \pi \left[\frac{2}{3}r^3 - \left(r^3 - r^2 h - \frac{1}{3}\left(r^3 - 3r^2 h + 3rh^2 - h^3\right)\right)\right]$$

$$= \pi \left[\underbrace{\frac{2}{3}r^3 - r^3}_{=0} + \underbrace{r^2 h}_{} + \underbrace{\frac{1}{3}r^3}_{} - \underbrace{\frac{3}{3}r^2 h}_{=0} + \frac{3}{3}rh^2 - \frac{1}{3}h^3 \right]$$

$$= \pi \left[rh^2 - \frac{1}{3}h^3 \right]$$

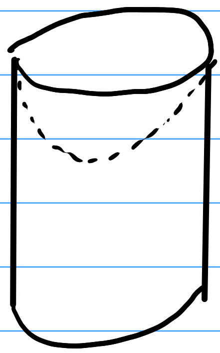
Was ist mit $h = 2r$ (Vollkugel!)

$$V = \pi \left[r(2r)^2 - \frac{1}{3}(2r)^3 \right] = \pi \left[\frac{3}{3} 4r^3 - \frac{8}{3}r^3 \right] = \pi \frac{4}{3} r^3$$

✓

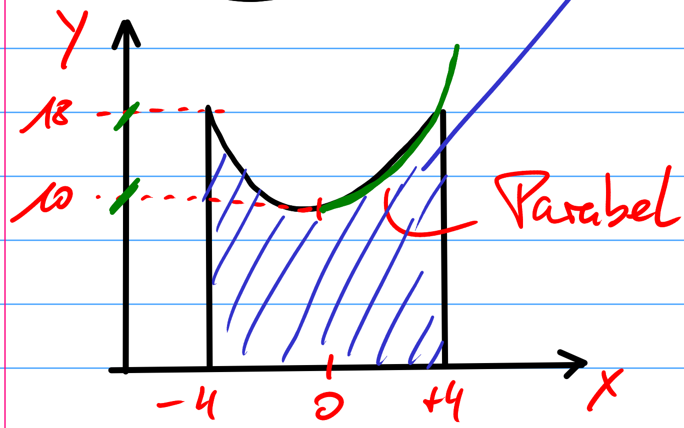
Kap 1, p. 19

Bsp. Hohlkörper



Paraboloid ausgefräst

Volumen des Hohlkörpers

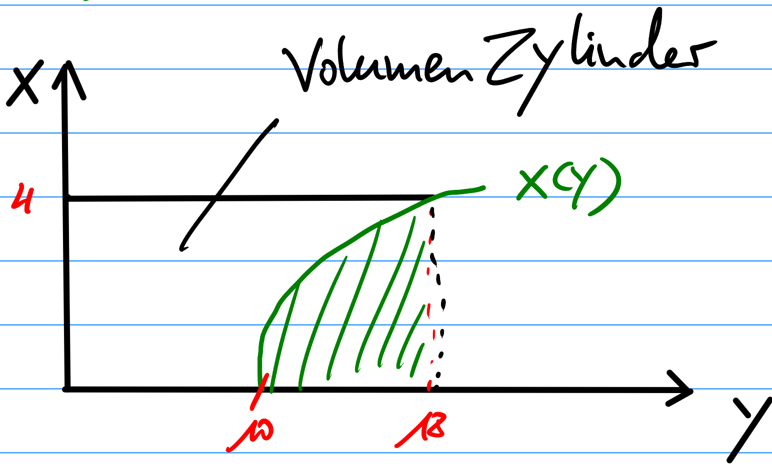


Rotationsachse ist die y-Achse

$y(x) = \frac{1}{2}x^2 + 10$

$\Rightarrow$ $g(y)$ als begrenzende Fkt.

$\Rightarrow x(y) = \sqrt{2(y-10)}$ $\quad y \in [10, 18]$



Volumen Paraboloid

$\Rightarrow$ Volumen Hohlkörper = Volumen Zylinder − Volumen Paraboloid

Kap 1, p. 19 (Fortsetzg.)   cm als Einheit d. Achsen

$V_{Zyl} = \pi (4)^2 \cdot 18 \text{ cm}^3 \approx 904{,}78 \text{ cm}^3$

$$V_{Paraboloid} = \pi \int_{10}^{18} (x(y))^2 \, dy$$

$$= \pi \int_{10}^{18} \left(\sqrt{2(y-10)}\right)^2 dy = \pi \int_{10}^{18} 2y - 20 \, dy$$

$$= \pi \left[y^2 - 20y \right]_{10}^{18} = \pi \left(18^2 - 20 \cdot 18 - (10^2 - 200) \right)$$

$$\approx 201{,}06 \text{ cm}^3$$

$$V_{HK} = V_{Zyl} - V_{Parab.} = \ldots = 703{,}72 \text{ cm}^3$$