Kap 3, p. 79

$$f: \mathbb{R}^3 \to \mathbb{R}: \quad f(\underline{x}) \in \mathbb{R}$$

Nabla Operator

$$\nabla f = \begin{pmatrix} \partial_x f \\ \partial_y f \\ \partial_z f \end{pmatrix}$$

Skalarprodukt

$$\nabla \cdot (\nabla f) = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \cdot \begin{pmatrix} \partial_x f \\ \partial_y f \\ \partial_z f \end{pmatrix}$$

$$= \partial_x(\partial_x f) + \partial_y(\partial_y f) + \partial_z(\partial_z f)$$

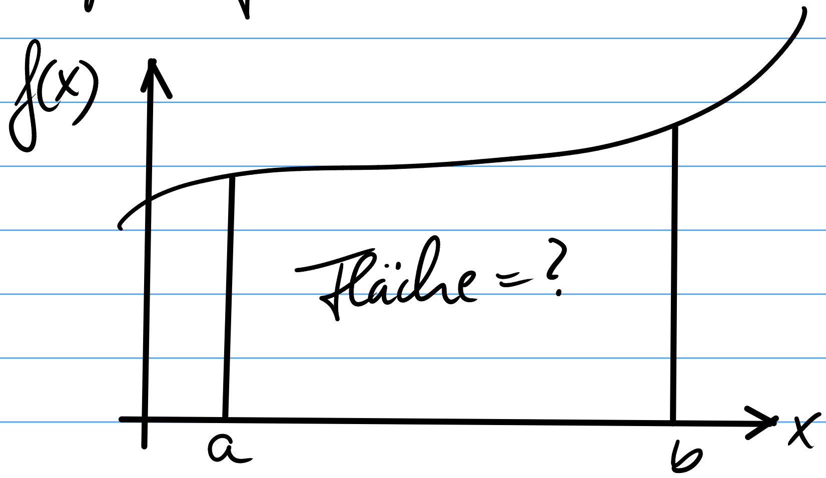
$$= \partial_x^2 f + \partial_y^2 f + \partial_z^2 f = \Delta f$$

$$\left(\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} \cdots\right) = \underbrace{(\partial_x)^2}_{\text{unüblich}} = \partial_x^2\right)$$

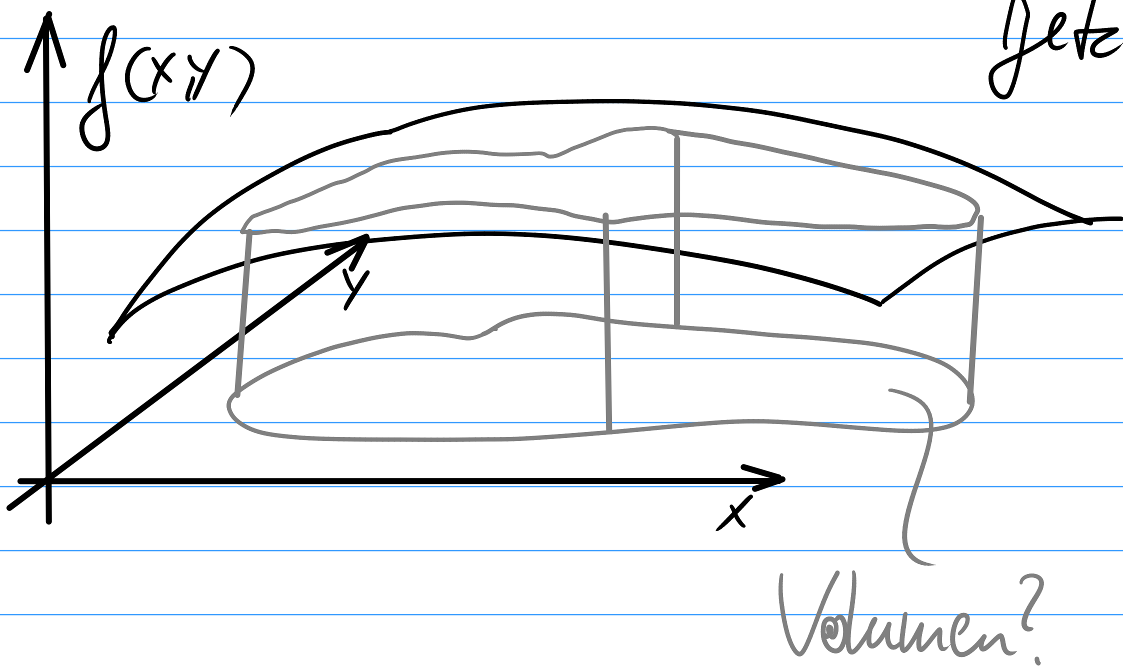
Laplace Operator

Kap 4, p. 3/4

bisher

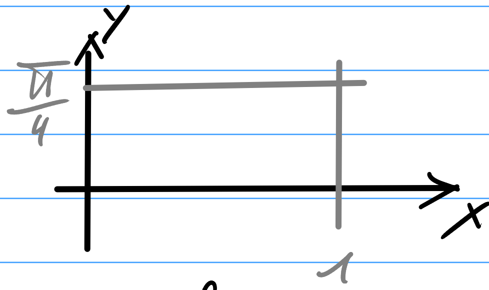


Jetzt



Kap 4, p. 10

$$f(x,y) = x\cos 2y$$



Draufsicht

$$\int_0^1\int_0^{\frac{\pi}{4}} f(x,y)\,dy\,dx$$

$$= \int_0^1\int_0^{\pi/4} x\cos 2y\,dy\,dx$$

$$= \int_0^1 \left[x\tfrac{1}{2}\sin 2y\right]_0^{\pi/4} dx$$

$$= \int_0^1 \left(x\tfrac{1}{2}\underbrace{\sin 2\tfrac{\pi}{4}}_{=1} - \cancel{x\tfrac{1}{2}\underbrace{\sin 2\cdot 0}_{=0}}\right) dx$$

$$= \int_0^1 \tfrac{1}{2}x\,dx = \left[\tfrac{1}{2}\tfrac{1}{2}x^2\right]_0^1$$

$$= \tfrac{1}{4}\cdot 1^2 - \tfrac{1}{4}\cdot 0^2 = \tfrac{1}{4}$$

Kap 4, p. 10

andere Integrationsreihenfolge

$$\int_0^{\pi/4}\int_0^1 f(x,y)\,dx\,dy$$

$$= \int_0^{\frac{\pi}{4}}\int_0^1 x\cos 2y\,dx\,dy$$

$$= \int_0^{\pi/4}\left[\frac{1}{2}x^2\cos 2y\right]_0^1 dy$$

$$= \int_0^{\pi/4} \frac{1}{2}1^2\cos 2y - \cancel{\underbrace{\frac{1}{2}0^2}_{0}\cdot\cos 2y}^{\,0}\,dy$$

$$= \int_0^{\pi/4}\frac{1}{2}\cos 2y\,dy = \frac{1}{2}\left[\frac{1}{2}\sin 2y\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left(\frac{1}{2}\underbrace{\sin 2\tfrac{\pi}{4}}_{=1} - \cancel{\frac{1}{2}\underbrace{\sin 2\cdot 0}_{0}}^{\,0}\right) = \frac{1}{4}$$

wie zuvor

Kap 4, p. 10 (aus einem Lehrbuch)

$$\int_0^{3/2}\int_{x=1}^{5y} y e^x \,dx\,dy$$

Produktregel

$$= \int_0^{3/2} \left[y e^x\right]_{x=1}^{5y} dy = \int_0^{3/2} \underbrace{y}_{u}\,\underbrace{e^{5y}}_{v'} - y e^1 \; dy$$

$u' = 1$ $v = \frac{1}{5}e^{5y}$

$$= \left[\underbrace{y}_{u}\,\underbrace{\tfrac{1}{5}e^{5y}}_{v}\right]_0^{3/2} - \int_0^{3/2} 1\cdot\frac{1}{5}e^{5y}dy - \left[e\frac{1}{2}y^2\right]_0^{3/2}$$

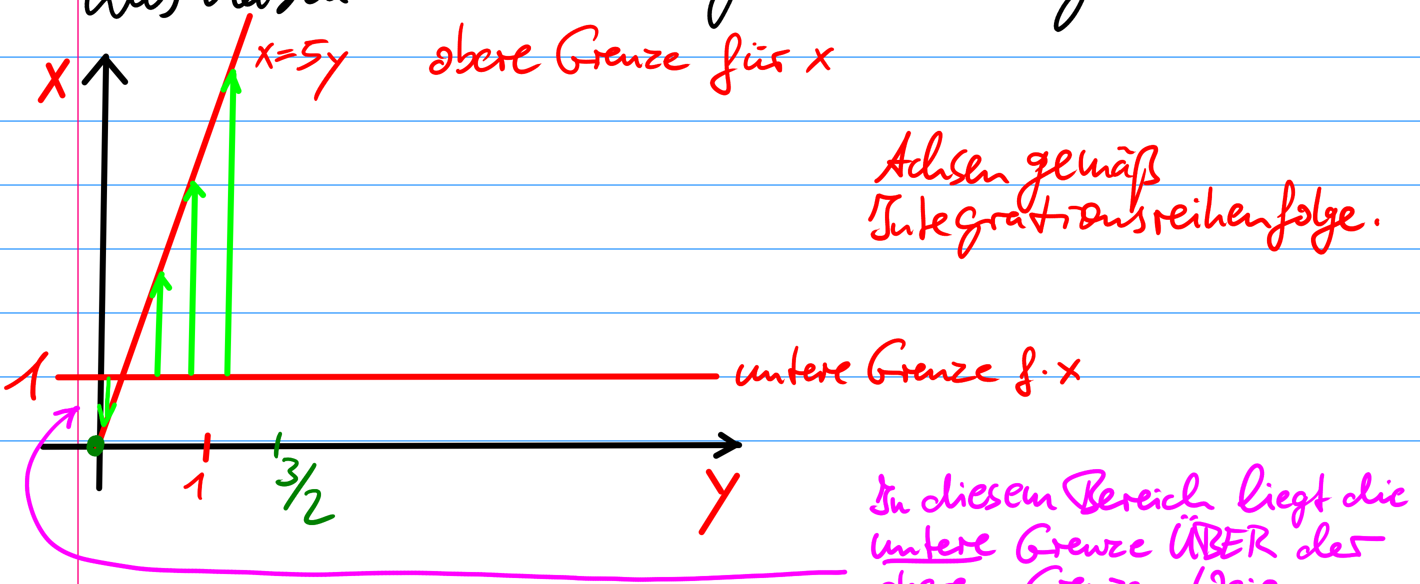
$$= \frac{3}{2}\,\frac{1}{5}e^{5\cdot\frac{3}{2}} - \cancel{0\cdot\frac{1}{5}e^{5\cdot 0}} - \left[\frac{1}{5^2}e^{5y}\right]_0^{3/2} - \left(\frac{e}{2}\left(\frac{3}{2}\right)^2 - \cancel{0^2}\right)$$

$$= \frac{3}{10}e^{15/2} - \left(\frac{1}{25}e^{\frac{15}{2}} - \frac{1}{25}e^0\right) - \frac{9e}{8}$$

$$= \left(\frac{3}{10} - \frac{1}{25}\right)e^{15/2} + \frac{1}{25} - \frac{9}{8}e$$

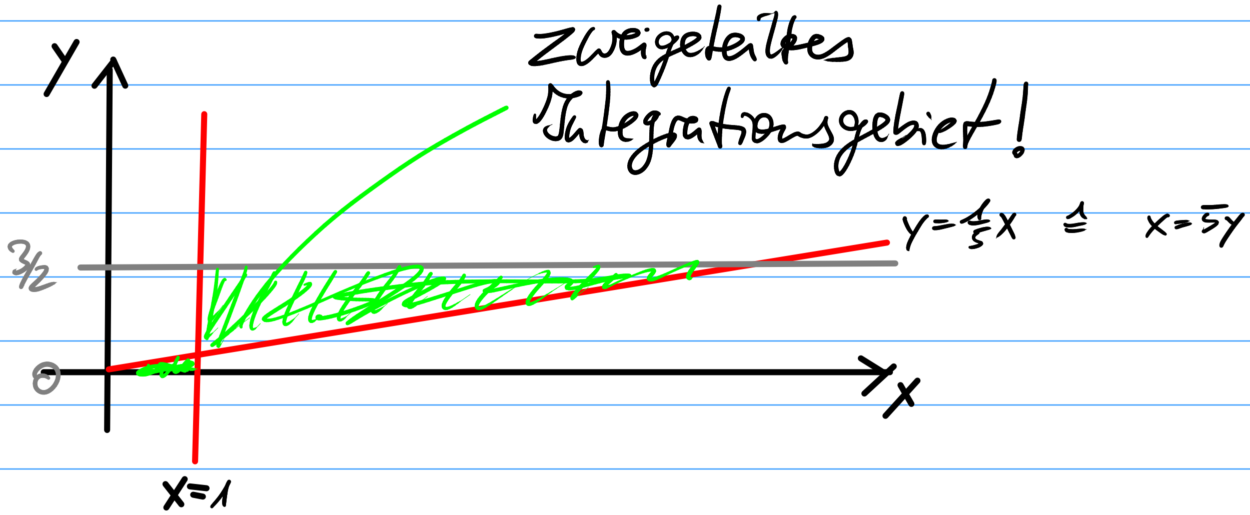
Super Ergebnis

Was haben wir hier eigentlich integriert?



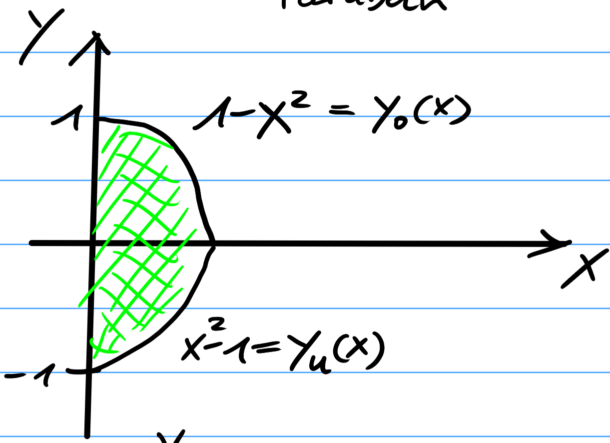
Kap 4, p. 10

übliche Achsen:



Begrenzung durch Parabeln

Kap 4, p. 10



$$f(x,y) = x^2+y^2$$

$$\int_0^1 \int_{y_u}^{y_o} f(x,y)\,dy\,dx = \underbrace{\int_0^1 \underbrace{\int_{y_u}^{y_o} x^2+y^2\,dy}_{A}\,dx}_{B}$$

A
$$\int_{y_u}^{y_o} x^2+y^2\,dy = \left[x^2 y + \frac{1}{3}y^3\right]_{y_u}^{y_o}$$

$$= x^2(1-x^2) - x^2(x^2-1) + \frac{1}{3}(1-x^2)^3 - \frac{1}{3}(x^2-1)^3$$

$$= x^2 - x^4 - x^4 + x^2 + \frac{1}{3}\left(1^3 - 3\cdot 1^2\cdot x^2 + 3\cdot 1(x^2)^2 - (x^2)^3\right)$$

$$- \frac{1}{3}\left((x^2)^3 - 3\cdot(x^2)^2\cdot 1 + 3x^2 1^2 - 1^3\right)$$

$$= \cancel{-2x^4} + \cancel{2x^2} + \frac{1}{3} - \cancel{x^2} \cancel{+x^4} - \frac{1}{3}x^6$$

$$- \frac{1}{3}x^6 \cancel{+x^4} \cancel{-x^2} + \frac{1}{3}$$

$$= \frac{2}{3} - \frac{2}{3}x^6$$

Kap. 4, P. 10 (Fortsetzg.)

(B) $$\int_0^1 \frac{2}{3} - \frac{2}{3}x^6 \, dx$$

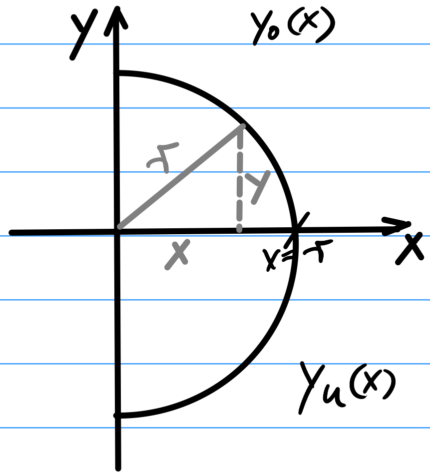
$$= \left[\frac{2}{3}x - \frac{2}{3}\cdot\frac{1}{7}x^7\right]_0^1$$

$$= \frac{2}{3} - \frac{2}{21} = \frac{12}{21} = \frac{4}{7}$$

I.d.R. nicht die Grenzen als Funktionen vorgegeben, sondern durch Geometrie.

$\Rightarrow$ Man muss sich die Funktionen basteln.

Bsp.



Grundgebiet soll Halbkreis mit Radius r sein. $\Rightarrow$?

$$\left.\begin{aligned} y_o &= -x^2 + r \\ y_u &= x^2 - r \end{aligned}\right] \text{so nicht}$$

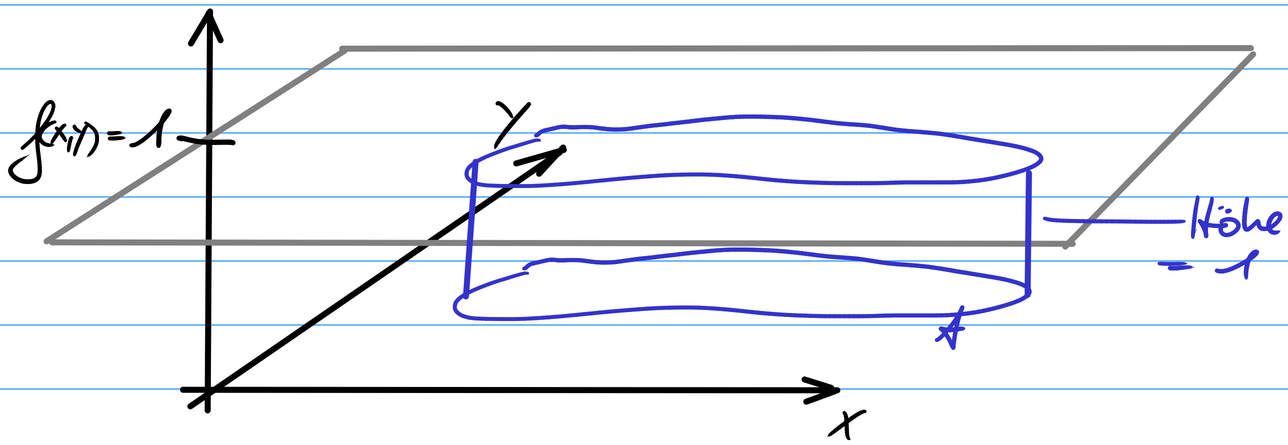
$\Rightarrow r^2 = y^2 + x^2 \Rightarrow y^2 = r^2 - x^2 \Rightarrow y = \pm\sqrt{r^2 - x^2}$

$\Rightarrow y_o = +\sqrt{r^2-x^2}$

$y_u = -\sqrt{r^2-x^2}$

Kap. 4, p. 10

Spezialfall: $f(x,y) = 1$



$$\iint\limits_A f \, dy dx = \iint\limits_A 1 \, dy dx = \text{Volumen}$$

= Grundfläch mal Höhe

⇒ Zahlenwert des Volumes =
= Zahlenwert der Fläche

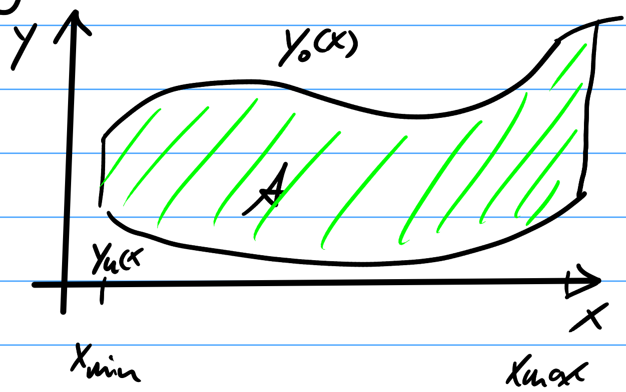
$$\iint\limits_A 1 \, dA = \text{Fläche}$$

⇒ Flächenberechnung für unregelmäßige Flächen!

Kap 4, p. 10

Flächenberechnung jetzt

z.B.: $dA = dy\,dx$



$$F_A = \int_{x_{min}}^{x_{max}} \int_{y_u}^{y_o} 1 \, dy\,dx$$

$$= \int_{x_{min}}^{x_{max}} [y]_{y_u}^{y_o} \, dx = \int_{x_{min}}^{x_{max}} y_o(x) - y_u(x) \, dx$$

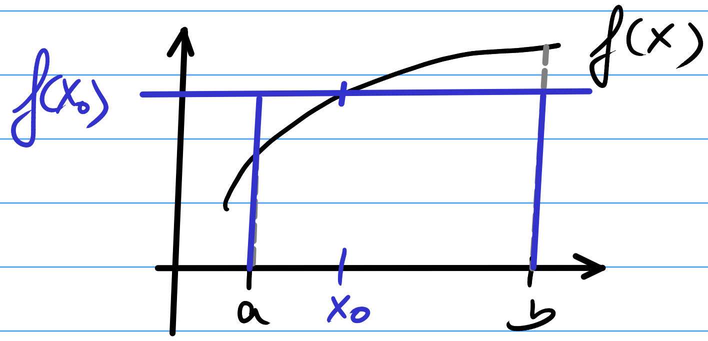
Wie in Mathematik 1
Kap. 7, p. 47

Hinweis f. Übungszettel 4-1:

$$x_s = \frac{1}{A} \iint_A x \, dA$$

Wie kann man das verstehen?

⇒ mit MA1:
Mittelwertsatz



$$\int_a^b f(x)dx = f(x_0)(b-a)$$

└ Fläche des Rechtecks

$$\Rightarrow \quad f(x_0) = \frac{1}{(b-a)} \int_a^b f \, dx$$

└ Mittelwert von f auf dem Intervall $[a,b]$

⇒ x_s ist der „Mittelwert der x-Koordinate"
über diese Fläche ⇒ Schwerpunkt.