Kap 5, p. 37

$$g(t) = e^{P(t)} \qquad P(t) = \int p\,dt$$

$$\frac{dx}{dt} + p(t)\,x = 0 \qquad \Big|\, * g(t) \quad \llcorner e^{P(t)}$$

$$\Rightarrow \; e^{P(t)} \frac{dx}{dt} + e^{P(t)} p(t)\, x = 0 \qquad (*)$$

Ansatz: $h(t, x(t)) = e^{P(t)} \cdot x(t)$

$$\frac{dh}{dt} = \frac{\partial h}{\partial t} + \frac{\partial h}{\partial x}\frac{dx}{dt}$$

$$= \partial_t\left(e^{P(t)}\right) x + e^{P(t)} \frac{dx}{dt}$$

$$= e^{P(t)} \cdot \partial_t(P(t))\, x + e^{P(t)} \frac{dx}{dt}$$

P ist Stammfkt zu p : $P' = \partial_t P = p$

$$= e^{P(t)} p(t)\, x + e^{P(t)} \frac{dx}{dt} \qquad (**)$$

$(**)$ ist die linke Seite von $(*)$

$$\Rightarrow \quad \frac{dh}{dt} = 0 \;\rightarrow\; h(t, x) = C \;\Rightarrow\; \text{b.w.}$$

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$\Rightarrow \quad h(t,x) = e^{P(t)} x = c \qquad | : e^{P(t)}$

$\rightarrow \qquad x(t) = c\, e^{-P(t)}$

ist die allg.

$\rightarrow$ der homogenen Glg.

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p. 40

$\frac{dx}{dt} + \underbrace{t}_{p} x = 0 \qquad \Rightarrow \quad p(t) = t$

$\Rightarrow \quad P(t) = \int p\,dt = \int t\,dt$

$= \frac{1}{2} t^2$

$\Rightarrow \quad x(t) = C e^{-P(t)} = C e^{-\frac{1}{2}t^2}$

Kontrolle: $\frac{dx}{dt} + tx = 0 \Rightarrow \frac{dx}{dt} = -tx \qquad | : x$

$\Rightarrow \quad \frac{1}{x}\frac{dx}{dt} = -t$ separable DGL

$\Rightarrow \int \frac{1}{x} dx = \int -t\,dt \Rightarrow \ln x = -\frac{1}{2}t^2 + c$

$\Rightarrow \quad x = e^{-\frac{1}{2}t^2 + c} = e^{-\frac{1}{2}t^2} \underbrace{e^{c}}_{D} = D e^{-\frac{1}{2}t^2}$ ✓

Kap 5, p. 40

$$\frac{dx}{dt} + \underbrace{\frac{x}{t}}_{p(t)} = \underbrace{t}_{r(t)} \quad \Rightarrow \quad p = \frac{1}{t} \qquad r = t$$

$$\Rightarrow P = \int \frac{1}{t}\,dt$$

$$= \ln t$$

$$\Rightarrow \int e^{P(t)} r(t)\,dt = \int e^{\ln t} \cdot t\,dt$$

$$= \int t \cdot t\,dt = \int t^2\,dt$$

$$= \frac{1}{3}t^3$$

$$\Rightarrow x(t) = e^{-P(t)}\left[\int e^{P(t)} r(t)\,dt + C\right]$$

$$= e^{-\ln t}\left[\frac{1}{3}t^3 + C\right]$$

$$= \frac{1}{e^{\ln t}}\left[\frac{1}{3}t^3 + C\right]$$

$$= \frac{1}{t}\left[\frac{1}{3}t^3 + C\right]$$

erst $\cdot 2$
dann $-\frac{8}{3}$

$$= \frac{1}{3}t^2 + \frac{C}{t}$$

$$\frac{1}{3} = x(2) = \frac{1}{3}2^2 + \frac{C}{2} \Rightarrow \frac{2}{3} - \frac{8}{3} = -2 = C$$

$$\Rightarrow x(t) = \frac{1}{3}t^2 - \frac{2}{t} \quad \text{ist spez. Lsg.}$$

gehackte Hochschule: Kontakt

fra-uas-fb2-ma2-becker@gmx.de

Kap 5, p. 40

$$\underbrace{\frac{dx}{dt} \underbrace{-4}_{p=-4} x}\ = \underbrace{t}_{r=t}$$

„$\int uv' = [uv] - \int u'v$"

$$\Rightarrow \quad P = \int -4\,dt = -4t \qquad (*)$$

$$\int e^{P} r\,dt = \int e^{-4t}\, t\, dt$$

$u \Rightarrow u' = 1$

$v' \Rightarrow v = -\frac{1}{4} e^{-4t}$

$$= \underset{u}{t}\underset{v}{\left(-\frac{1}{4}e^{-4t}\right)} - \int \underset{u'}{1} \cdot \underset{v}{-\frac{1}{4}e^{-4t}}\, dt$$

$$= -\frac{t}{4}e^{-4t} + \frac{1}{4}\left(-\frac{1}{4}e^{-4t}\right)$$

$$= -\frac{t}{4}e^{-4t} - \frac{1}{16}e^{-4t} \qquad (**)$$

b.w.

Kap 5, p.40

$$\Rightarrow x(t) = e^{-P}\Big[\underbrace{\int e^{P} r\,dt}_{(**)} + C\Big]$$

$$= e^{+(+4t)}\Big[-\frac{t}{4}e^{-4t} - \frac{1}{16}e^{-4t} + C\Big]$$

$$= -\frac{t}{4} - \frac{1}{16} + Ce^{4t}$$

$$\frac{dx}{dt} + tx = -2t \qquad p = t \Rightarrow P(t) = \tfrac{1}{2}t^2$$

$$r = -2t$$

$$\int e^{P} r\,dt = \int e^{\frac{1}{2}t^2}\underbrace{(-2t)\,dt}_{dz} \qquad z = t^2$$

$$\Rightarrow \frac{dz}{dt} = 2t$$

$$= -\int e^{\frac{1}{2}z}\,dz \qquad \Rightarrow dz = 2t\,dt$$

$$= -\frac{1}{\frac{1}{2}}e^{\frac{1}{2}z} = -2e^{\frac{1}{2}t^2}$$

$$\Rightarrow x(t) = e^{-P}\Big[\int e^{P} r\,dt + C\Big]$$

$$= e^{-\frac{1}{2}t^2}\Big[-2e^{\frac{1}{2}t^2} + C\Big]$$

$$= -2 + Ce^{-\frac{1}{2}t^2}$$

b.w.

Kap 5, p.40

$$2 = x(0) = -2 + ce^{0} = -2 + c$$

$$\Rightarrow \quad c = 4$$

$$\Rightarrow \quad x(t) = 4e^{-\frac{1}{2}t^2} - 2$$