1 $\begin{cases} \max\ 5x_1+8x_2 \\ x_1+x_2 \le 6 \\ 5x_1+9x_2 \le 45 \end{cases}$ $\qquad$ $\begin{matrix} -5x_1-8x_2=0 \\ x_1+x_2+s_1=6 \\ 5x_1+9x_2+s_2=45 \end{matrix}$

	x_1	x_2	s_1	s_2	sol.	
z	-5	-8	0	0	0	
s_1	1	1	1	0	6	$\to 6/1=6$
x_2 ~~s_2~~	5	9	0	1	45	$\to 45/9=5$

riga pivot $=\left[\frac{5}{9}, 1, 0, \frac{1}{9}, 5\right]$

$z'=\left[-5-(-8\cdot 5/9), -8-(-8\cdot 1), 0, 0-(-8\cdot 1/9), 0-(-8\cdot 5)\right]=$ coeff. -8

$=\left[-5+\frac{40}{9}, 0, 0, +8/9, 40\right]=\left[-5/9, 0, 0, 8/9, 40\right]$

$s_1'=\left[1-(1\cdot 5/9), 1-(1\cdot 1), 1-(1\cdot 0), 0-(1\cdot 1/9), 6-(1\cdot 5)\right]=$ coeff. 1

$\left[4/9, 0, 1, -1/9, 1\right]$

	x_1	x_2	s_1	s_2	sol	
z	$-5/9$	0	0	$8/9$	40	
x_1 ~~s_1~~	$4/9$	0	1	$-1/9$	1	$\to 1/(4/9)=9/4$
x_2	$5/9$	1	0	$1/9$	5	$\to 5 : 5/9 = \cancel{5}\cdot 9/\cancel{5}=9$

$x_1'=\left[1, 0, 9/4, -1/4, 9/4\right]$

$z'=\left[-5/9-(-5/9\cdot 1), 0, 0-(-5/9\cdot 9/4), 8/9-(+5/9\cdot +1/4), 40+(+5/9\cdot 9/4)\right]=$ coeff $-5/9$

$\left[=0, 0, +5/4, 27/36, 425/4\right]$

$x_2'=\left[5/9-(5/9\cdot 1), 1-(5/9\cdot 0), 0-(5/9\cdot 9/4), 1/9-(5/9\cdot -1/4), 5-(5/9\cdot 9/4)\right]$ coeff $5/9$

$=\left[0, 4/9, -5/4, 1/4, 15/4\right]$

| sol

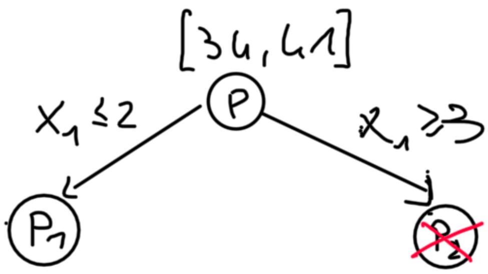
					sol
z	0	0	5/4	27/36	125/4 = 31,25
x_1	1	0	9/4	-1/4	9/4 = 2,25
x_2	0	4/9	-5/4	1/4	15/4 = 3,75

ottima: 31,25
x_1: 2,25
x_2: 3,75

Trovare lower bound arrotondando le soluzioni di x_1 e x_2
upper bound lasciandole così come sono

LB = $x_1 = 2$ max $5x_1 + 8x_2 = 5 \cdot 2 + 8 \cdot 3 = 10 + 24 = 34$
$x_2 = 3$

UB = $x_1 = 2.25$ max $5x_1 + 8x_2 = 5 \cdot 2,25 + 8 \cdot 3,75 =$
$x_2 = 3.75$ $11,25 + 30 = 41,25$



34 = sol. temporanea

P1 impostare $x_1 = 2$, calcolare dai vincoli x_2

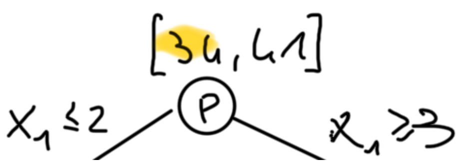
$x_1 + x_2 \leq 6$ $2 + x_2 \leq 6 \rightarrow x_2 \leq 4$
$5x_1 + 9x_2 \leq 45$ $10 + 9x_2 \leq 45 \rightarrow 9x_2 \leq 35 \rightarrow \boxed{x_2 \leq 3.88}$

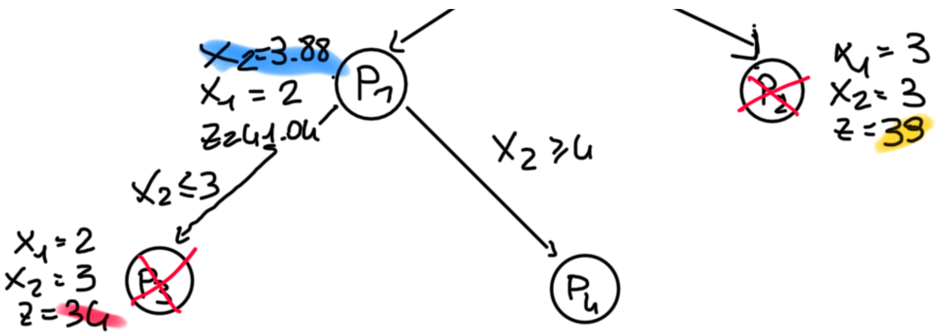
perché è il più basso

f2: $5x_1 + 8x_2 = 5 \cdot 2 + 8 \cdot 3,88 = 41,04$

P2 impostare $x_1 = 3$, calcola x_2 dai vincoli
$3 + x_2 \leq 6 \rightarrow \boxed{x_2 \leq 3}$
$15 + 9x_2 \leq 45 \rightarrow 9x_2 \leq 30 \rightarrow x_2 \leq 3.33$

f2: $5 \cdot 3 + 8 \cdot 3 = 15 + 24 = 39$ i valori di x_1 e x_2 sono interi, e 39 ≥ 34, quindi si aggiorna





P3 $x_1 \le 2,\ x_2 \le 3$

$x_2 = 3 \to x_1 + 3 \le 6 \to \boxed{x_1 \le 3}$

$5x_1 + 27 \le 45 \to x_1 \le 3.6$

$x_1 = 2,\ x_2 = 3 \to 5\cdot 2 + 8\cdot 3 = 34 < 39$ quindi lo si chiude

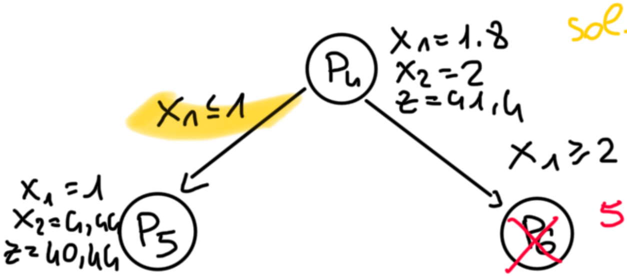
P4 $x_1 \le 2,\ x_2 \ge 4$

$x_2 = 4 \to x_1 + 4 \le 6 \to x_1 \le 2$

$5x_1 + 36 \le 45 \to 5x_1 \le 9 \to \boxed{x_1 \le 1.8}$

$x_1 = 1{,}88,\ x_2 = 4 \to 5\cdot 1{,}88 + 8\cdot 4 = 41{,}4$

sol. = 39



51 > 41 quindi lo chiudiamo

P5 $x_1 = 1$, calcolo x_2

$x_1 = 1 \to 1 + x_2 \le 6 \to x_2 \le 5$

$5 + 9x_2 \le 45 \to 9x_2 \le 40 \Rightarrow 4{,}44$

branch su x_2

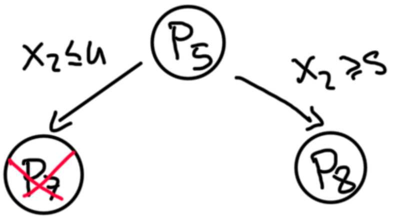
$z = 5\cdot 1 + 8\cdot 4{,}44 = 40{,}52$

P6

P6 $x_1 = 2$, calcolo x_2

$$x_1 = 2 \rightarrow \quad 2 + x_2 \leq 6 \rightarrow x_2 \leq 4$$
$$10 + 9x_2 \leq 45 \rightarrow 9x_2 \leq 35 = 3{,}88$$

$$z = 5 \cdot 4 + 8 \cdot 3{,}88 = 51{,}04$$ → UB deve decrescere, quindi non è valido



P7 $x_2 = 4$, $x_1 = ?$

$$x_1 + 4 \leq 6 \rightarrow x_1 \leq 2$$
$$5x_1 + 36 \leq 45 \rightarrow x_1 \leq 1.8$$ → non vale $x_1 \leq 1$ e quindi deve essere $x_1 \leq 1$

$$z = 5 \cdot 1 + 8 \cdot 4 = 37 < 39$$ quindi:
→ soluzione intera $(1, 4)$
→ ma minore quindi no

P8 $x_2 = 5$, $x_1 = ?$

$$x_1 + 5 \leq 6 \rightarrow x_1 \leq 1$$
$$5x_1 + 45 \leq 45 \rightarrow \boxed{x_1 \leq 0}$$

$$x_1 = 0, x_2 = 5 \rightarrow 5 \cdot 0 + 8 \cdot 5 = 40 > 39$$ quindi:
→ soluzione intera $(0, 5)$
→ ma maggiore quindi ok

---

2 max $x_1 + 3x_2$ $\qquad -x_1 - 3x_2$

$x_1 + 5x_2 \leq 21 \qquad x_1 + 5x_2 + s_1 = 21$

$8x_1 + 2x_2 \leq 35 \qquad 8x_1 + 2x_2 + s_2 = 35$

		x_1	x_2	s_1	s_2	sol	
	z	-1	-3	0	0	0	
x_2	~~s_1~~	1	5	1	0	21	→ $21/5 = 4.2$
	s_2	8	2	0	1	35	→ $35/2 = 17.5$

$x_2 = [1/5, 1, 1/5, 0, 4.2]$

$z' = [-1-(-3 \cdot 1/5), -3-(-3 \cdot 1), 0-(-3 \cdot 1/5), 0, 0-(-3 \cdot 4.2)]$ = coeff -3

$= [-2/5, 0, 3/5, 0, 12.6]$

$S_2' = [8-(2 \cdot 1/5), 2-(2 \cdot 1), 0-(2 \cdot 1/5), 1-(2 \cdot 0), 35-(2 \cdot 4.2)]$ = coeff 2

$= [7.6, 0, -2/5, 1, 26.6]$

	x_1	x_2	S_1	S_2	sol
z	$-2/5$	0	$3/5$	0	12.6
x_2	$1/5$	1	$1/5$	0	4.2
~~S_2~~ x_1	7.6	0	$-2/5$	1	26.6

→ $4.2 : 1/5 = 4.2 \cdot 5 = 21$

→ $26.6 / 7.6 = 3.5$

$x_1' = [1, 0, 0.05, 0.13, 3.5]$

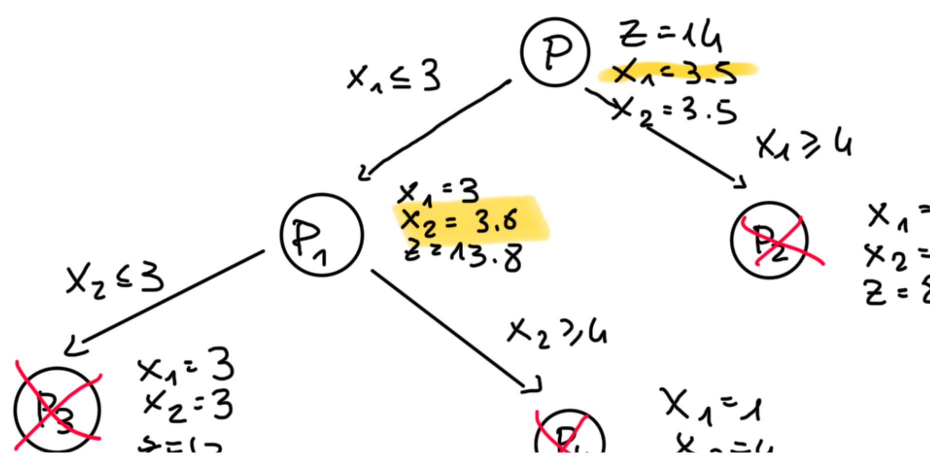
$x_2' = [0, 1-(0.2 \cdot 0), 0.2-(0.2 \cdot 0.05), 0-(0.2 \cdot 0.13), 4.2-(0.2 \cdot 3.5)] = [0, 1, 0.19, -0.026, 3.5]$ coeff. $1/5 = 0.2$

$z' = [0, 0-(-0.4 \cdot 0), 3/5-(-0.4 \cdot 0.05), 0-(-0.4 \cdot 0.13), 12.6-(-0.4 \cdot 3.5)] = [0, 0, 0.602, 0.052, 14]$ coeff -0.4

$LB = 3 + 3 \cdot 3 = 12 \Rightarrow$ SOL. TEMPORANEA

	x_1	x_2	S_1	S_2	sol
z	0	0	0.602	0.052	14
x_2	0	1	0.19	-0.026	3.5
x_1	1	0	0.05	0.13	3.5

→ NO INTERE quindi si prova il branching su X_1 per esempio



MASSIMIZZAZIONE:

UB decresce o = stima migliore

LB qualsiasi soluzione temporanea

$z = 13$

P1 $x_1 \leq 3 \rightarrow x_2 = ?$

$x_1 = 3$

$3 + 5x_2 \leq 21 \rightarrow \boxed{x_2 \leq 3.6}$

$24 + 2x_2 \leq 35 \rightarrow x_2 \leq 5.5$

LOWER BOUND

$\max z = x_1 + 3x_2 = 3 + 3 \cdot 3.6 = 13.8$

P2 $x_1 \geq 4 \rightarrow x_2 = ?$

$x_1 = 4$

$4 + 5x_2 \leq 21 \rightarrow x_2 \leq 4.25$

$32 + 2x_2 \leq 35 \rightarrow \boxed{x_2 \leq 1.5}$

$\max z = x_1 + 3x_2 = 4 + 3 \cdot 1.5 = 8.5 \rightarrow 8.5 < 12$

CHIUDO perché UB deve decrescere o =

P3 $x_1 \leq 3, \underbrace{x_2 \leq 3}_{\text{fisso}} \rightarrow$

$x_2 = 3$

$x_1 + 15 \leq 21 \rightarrow x_1 \leq 6$

$8x_1 + 6 \leq 35 \rightarrow \underbrace{x_1 \leq 3.625}$

NO perché $x_1 \leq 3$

quindi $x_1 \leq 3, x_2 \leq 3$

$\max z = x_1 + 3x_2 = 3 + 3 \cdot 3 = 12 \rightarrow$ CHIUDO

soluzione intera ✓

$12 \geq \underbrace{12}_{\text{temp.}}$ ✓

P4 $x_1 \leq 3, x_2 \geq 4 \rightarrow x_2 = 4$

$x_1 + 20 \leq 21 \rightarrow \boxed{x_1 \leq 1}$

$8x_1 + 8 \leq 35 \rightarrow x_1 \leq 3.375$

NO perché $x_1 \leq 3$

$x_1 = 1, x_2 = 4 \rightarrow \max z = x_1 + 3x_2 = 1 + 3 \cdot 4 = 12 + 1 = 13 \rightarrow$ CHIUDO

soluzioni intere ✓

$13 \geq 12$ ✓

sol. migliore

Esercizio 3

$$\max \; 5x_1 + 4x_2 \qquad\qquad -5x_1 - 4x_2$$
$$3x_1 + 2x_2 \le 16 \qquad\qquad 3x_1 + 2x_2 + S_1 = 16$$
$$2x_1 + 3x_2 \le 16 \qquad\qquad 2x_1 + 3x_2 + S_2 = 16$$

	~~x_1~~	x_2	S_1	S_2	sol	
z	-5	-4	0	0	0	
x_1 ~~S_1~~	3	2	1	0	16	$\to 16/3 = 5{,}33$
S_2	2	3	0	1	16	$\to 16/2 = 8$

$$x_1' = \left[1, \tfrac{2}{3}, \tfrac{1}{3}, 0, \tfrac{16}{3}\right]$$

$$z' = -5-(-5\cdot 1), -4-(-5\cdot \tfrac{2}{3}), 0-(-5\cdot\tfrac{1}{3}), 0-(-5\cdot 0), 0-(-5\cdot \tfrac{16}{3})] \quad \text{coeff } -5$$
$$= \left[0, -\tfrac{2}{3}, \tfrac{5}{3}, 0, \tfrac{80}{3}\right]$$

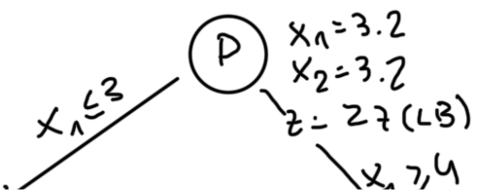
$$S_2' = 2-(2\cdot 1), 3-(2\cdot\tfrac{2}{3}), 0-(2\cdot\tfrac{1}{3}), 1-(2\cdot 0), 16-(2\cdot\tfrac{16}{3})] = \quad \text{coeff } 2$$
$$= \left[0, \tfrac{5}{3}, -\tfrac{2}{3}, 1, \tfrac{16}{3}\right]$$

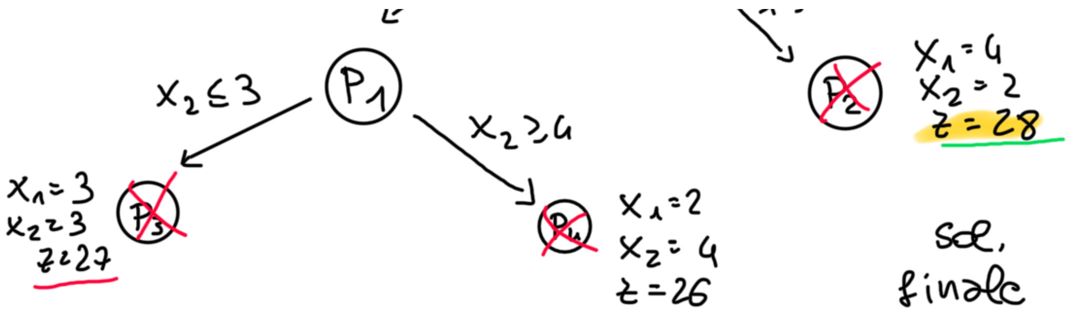
	x_1	x_2	S_1	S_2	sol
z	0	$-2/3$	$5/3$	0	$80/3$
x_1	1	$2/3$	$1/3$	0	$16/3$
S_2	0	$5/3$	$-2/3$	1	$16/3$

$\to$ continua...

$x_1 = 3.2$
$x_2 = 3.2$ $\to$ $5\cdot 3.2 + 4\cdot 3.2 = 28.8$ DEGENERE quindi B&B

(LB) $x_1 = 3$
$x_2 = 3$
$5\cdot 3 + 4\cdot 3 = 27$





sol. finale $x_1 = 4,\ x_2 = 2$
$z = 28$

P1 $x_1 \leq 3,\ x_2 = ?$

$$3 \cdot 3 + 2x_2 \leq 16 \rightarrow x_2 \leq 3.5$$
$$6 + 3x_2 \leq 16 \rightarrow \boxed{x_2 \leq 3.3}$$

$$x_1 = 3, x_2 = 3.3 \rightarrow \max z = 5 \cdot 3 + 4 \cdot 3.3 = 28.32$$

LB deve decrescere o = quindi OK

P2 $x_1 \geq 4,\ x_2 = ?$

$$3 \cdot 4 + 2x_2 \leq 16 \rightarrow \boxed{x_2 \leq 2}$$
$$8 + 3x_2 \leq 16 \rightarrow x_2 \leq 2.667$$

$$x_1 = 4, x_2 = 2 \rightarrow \max z = 5 \cdot 4 + 4 \cdot 2 = 28$$

$28 > 27$, nuova soluzione temp. poiché x_1 e x_2 sono interi

P3 $x_1 \leq 3,\ x_2 \leq 3 \rightarrow x_2 = 3$

$$3x_1 + 6 \leq 16 \rightarrow \boxed{x_1 \leq 3{,}33}$$
$$2x_1 + 9 \leq 16 \rightarrow x_1 \leq 3.5$$

ma $x_1 \leq 3$ quindi teniamo quello

$$x_1 = 3, x_2 = 3 \rightarrow \max z = 5 \cdot 3 + 4 \cdot 3 = 27$$

$27 < 28$ quindi NON VALIDO

P4 $x_1 \leq 3,\ x_2 \geq 4 \rightarrow x_2 = 4$

$3x_1 + 8 \le 16 \rightarrow x_1 \le 2.66$

$2x_1 + 12 \le 16 \rightarrow \boxed{x_1 \le 2}$

$x_1 = 2,\ x_2 = 4 \rightarrow \max z = 5 \cdot 2 + 4 \cdot 4 = 26$

$\downarrow$

$26 < 28$ quindi

non valido