A Constructive Formalisation of Hoare Logic using the Interactive Theorem Prover Agda

Project Report
Word Count XXXX

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Submitted in partial fulfillment
of the requirements for the degree of
Master of Science
(Computer Science)

at the
University of Birmingham
School of Computer Science
July 2021

Abstract

Problem, Approach, What you Produced, Evaluation, What it all means. Nullam enim nisi, elementum eu pellentesque nec, facilisis id tellus. Sed erat sem, maximus vel fermentum et, fringilla quis est. Aliquam tempus nunc ac velit sollicitudin condimentum. Duis sed rutrum tellus. Curabitur rutrum finibus justo ut malesuada. Nullam tincidunt scelerisque iaculis. Quisque tempor massa id urna elementum, sit amet condimentum tellus euismod. Integer est eros, posuere et lacus finibus, pretium aliquam libero.

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Contents

1 Introduction

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2 Preliminaries & Literature Review

2.1 Weakest Precondition

How is one to give a semantics to computation? One answer is to provide a model of computation that denotes how a mechansim, a computation, or program[1] is to be computed; such as a Turing machine, or a FSA. Another way however is to describe what the mechanism, computation, or program can *do* for us; that is, specifiying what input states it can accept, and what states it will produce.

OR: given a desired output (of states) specifying, how (read, in which input states), if at all, it can produce the desired output.

This approach gives us a way of specifying a ... without caring about the eventual form of the mechanism if indeed it ever takes form at all!

Computation as traversing the state space. Descartes - Cartesian product - [Dijkstra,p12]

Weakest precondition (according to Dijskstra) is unique when considered as a state space, but multiple predicates could denote the same space. (i.e. $x == y$ and $y == x$)

Strongest postcondition? [Gries, exercise 4 section 9.1]

'If for a given P, S, and R, $P \Rightarrow wp(S, R)$ holds, this can often be proved without explicit formulation — or, if you prefer, "computation", or "derivation" — of the predicate $wp(S, R)$'

Note that in the text [dijkstra], $wp(S, R)$, is used interchangeably as a predicate and as the state space that said predicate captures. With our constructive formalisation however, this lack of precision is not possible nor

[1] the three words here being used synonymously

desired, so we end up with the, perhaps superfluous, distinction between predicates and the state space that they describe. Meaning that under our formalisation, $wp(S, F)$ is empty when considered as a state space, but inhabited when considered as a predicate (inhabited uniquely by F itself). This exposition also explains why $<< F >> S << Q >>$ is an inhabited type, as F *is* a valid precondition of any computation for any postcondition (think absurd function, or bluff function). << Actually explained by the fact that what we have formalised is the weakest *liberal* precodnition!

Weakest Liberal Precondition is what has actually been formalised! (Need to work out the translation)

7 regions of the statesapce. As such, we can — if we wish — give a semantics to the notion of a derterministic mechanism as one in which the last four regions of the state space are empty.

2.2 Hoare Logic

2.3 Agda

2.4 Constructive Mathematics

'Agda is a constructive mathematical system by default, which amounts to saying that it can also be considered as a programming language for manipulating mathematical objects.' - MHE

2.5 Formal Proof

2.6 Applications

3 Specification

3.1 Obfuscating Interfaces

Might have also wanted to abstract away expression language (page 42 surface properties (Ligler))

3.2 Exppresion Language

Carving up state space. Every predicate denotes a subset of the statespace (which in our case is infinite).

(day = 23) Dijkstra's example

T/F, x == 2

Relationship between logical operators and set theoretic operators i.e. $\wedge \Leftrightarrow \cap$

Ought to have differentiated between non stuck-ness and termination. I.e. D(E) as domain of expression E, to eliminate divide by zero and non-defined variables in an expression, as that is a problem that can be handled distinctly from termination (i.e. (I think anway) that given a state S, and an expression E, one can deterministically/decidably determine whether or not it is a WFF).

3.3 Language

A SΔ is one of the following:

```
-- Commands/Programs/Mechanisms/Statements
-- Defined as 'SΔ' (read 'State transformer')
-- to emphasise all these different meanings
data Block : Set
data SΔ : Set where
  SKIP : SΔ
  WHILE_DO_ : Exp → Block → SΔ
  IF_THEN_ELSE_ : Exp → Block → Block → SΔ
  _:=_ : Id → Exp → SΔ

data Block where
  -- Terminator:
  _; : SΔ → Block
  -- Separator:
  _;_ : SΔ → Block → Block
```

3.4 Axioms & Rules

4 Implementation

4.1 Constructive Termination

4.2 Small Step Evaluation with Fuel

4.3 Termination Splitting

4.4 Axiom & Rules in Agda

5 Reflections

5.1 Missteps

5.2 Future Work

Gries page 164 'a fine balance between the two' ... but! automation, Infer, parse a C program and create formal proof in background. Complain if fail

Hoare's surprise at test case success (see retrospective)

5.3 Conclusion

6 Appendix

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