Elastic Net with Stochastic Noise Strategy and Time-Dependent Parameters for TSP

Gang Yang
Key Laboratory of Data Engineering
and Knowledge Engineering
(Renmin University of China),
MOE, Beijing 100872, P.R. China.
yanggang@ruc.edu.cn

Junyan Yi
Department of
Computer Science and Technology,
Zhejiang University of Technology

Nan Yang
School of Information,
Renmin University of China,
yangnan@ruc.edu.cn

***Abstract*—In this paper, an improved elastic net algorithm is proposed to solve the traveling salesman problem by assembling a stochastic noise strategy and some time-dependent parameters. Based on the observation and analysis of solution status of TSP solving by elastic net, the stochastic noise strategy is introduced into elastic net to overcome the shortcoming of easily trapping in local minima. Being different with other stochastic algorithms, the stochastic noise strategy mainly modifies problem states by using city oscillation randomly and further affects elastic net performance. The time-dependent parameters controlling convergent process increase the ability of matching cities precisely and getting convergence quickly. It is verified by large numbers of simulations that the stochastic noise strategy and time-dependent parameters can enhance the performance of elastic net greatly. And especially the stochastic noise strategy to problem states probably reveals a novel way to optimize some deterministic algorithms.**

I. Introduction

Traveling Salesman Problem (TSP) is a classical combinatorial optimization problem, which is simple to state but very difficult to solve[1][2][7]. The problem is to find the shortest possible tour through a set of N vertices so that each vertex is visited exactly once. This problem is known to be NP-hard, and cannot be solved exactly in polynomial time. Since Hopfield and Tanks proposed Hopfield neural network and applied it to solve TSP, neural networks have been considered as powerful tools for solving combinatorial optimization problems[5][6]. Along with their works, many novel neural networks are presented for TSP[8]. In 1995, Chen and Aihara proposed a transiently chaotic neural network(TCNN) by introducing chaotic dynamics into Hopfield neural network, and TCNN could solve TSP with high successful probability to get optimal solution[15]. Elastic net proposed by Durbin and Willshaw was also an efficient algorithm for TSP, which attracted wide attention for its obvious advantages[3]. Recently, some improved algorithms based on Kohonen map and elastic net were presented and revealed desirable capability in solving combinatorial optimization problems[13][14][9]. Moreover, Gilson and Damper compared several well-known approximate algorithms including elastic net[3], Kohonen map[11] and Burr's modified elastic net[17] et. al for TSP. The results showed that elastic net was more suitable to solve TSP for its geometry corresponding property and unsupervised learning[10]. However, the elastic net is still suffered from some shortcomings such as local minima and parameter tuning problem. Researchers have proposed some improved methods to overcome the disadvantages of elastic net, but analysis shows those modified methods are not effective for their deterministic properties in convergent process.

As is well known, stochastic characteristic embedded in neural network is an useful property for optimization problems, due to random fluctuations could help neural networks escape from local minima. There are a large number of neural network algorithms improved by introducing stochastic characteristic to increase their solving ability[18][19]. They are built by introducing random variations into the network, either by giving the network's neurons stochastic transfer functions, or by giving them stochastic weights. Nevertheless, in our literature there are not appropriated works to introduce stochastic characteristic into elastic net. In order to maintain elastic net geometry property, those normal ways of introducing random variations are not feasible. It is necessary to present a special way to introduce stochastic noise that not only can help elastic net jump out local minima, but also can increase network executive effectiveness.

In this work, we intend to enhance elastic net ability of escaping from local minima by introducing a stochastic noise strategy and decreasing the burden of parameter tuning by using some time-dependent parameters. Being different with other stochastic methods, the stochastic noise strategy brings about the elastic net obtaining random properties from the angle of problem states. To those special algorithms with strict constraints, the stochastic noise strategy is an appropriate way to introduce stochastic characteristic. During the period of elastic net algorithm processing, cities vibrate in their oscillation domains and therefore modify the neurons movement and matching relationship. With gradually decreasing of cities oscillation, the problem is reduced to be the targeted TSP. Thus our proposed algorithm is able to jump out local minima and gets convergence at a superior solution for the targeted TSP. Moreover, some time-dependent parameters are embedded in our algorithm to primarily improve the control granularity and flexibility in determining the relative strength of two forces on the same neuron, which influences the solution quality and the convergent speed. Therefore our proposed algorithm can be expected to have higher ability of escaping from local minima and globally searching optimal and near-optimal solutions in less convergent time. In order to verify our algorithm, it was tested on some randomly generated TSPs from 30 cities to 400 cities. The simulation results show that the stochastic noise strategy and the time-dependent parameters greatly improve the solving ability of elastic net, and our proposed algorithm is superior to the original elastic net in light of the solution quality and CPU times.

II. The elastic net for TSP

The elastic net was proposed as an efficient method for traveling salesman problem in 1987 [3]. Later Durbin et. al [4] gave detailed description, analysis and experimental evaluation on the elastic net in 1989. The elastic net can be described briefly to explain its main properties. We have a set of *n* nodes in the plane within a unit square $X = x_1, x_2, \ldots, x_n$ and a set of *m* dynamic points $Y = y_1, y_2, \ldots, y_m$ defining the rubber band. At the beginning of the algorithm, the rubber band is a small circle around the center of gravity of the nodes. Then, it is stretched toward the nodes by tracking the minima of the energy function:

$$E = -\alpha K \sum_{i=1}^{n} ln \sum_{j=1}^{m} e^{-\|x_i - y_j\|/2k^2} + \beta \sum_{j=1}^{m} \|y_j - y_{j+1}\| \quad (1)$$

where variable K is gradually lowered, corresponds to the temperature in the usual annealing process. The energy equation (1) consists of two items: the first one is responsible for the attraction of the points on the rubber band to the cities; the second one favors the shortest tour. And the constants α and β determine the relative strength of the forces from the cities and the forces from its neighbors on the rubber band. The positions of the points defining the rubber band are updated according to the formula

$$\Delta y_j = \alpha \sum_{i=1}^{n} w_{ij}(x_i - y_j) + \beta K(y_{j+1} - 2y_j + y_{j-1}) \quad (2)$$

where coefficient w_{ij} specifies the influence of city x_i on path node y_j and is defined as below:

$$w_{ij} = e^{-|x_i - y_j|/2k^2} / \sum_{i=1}^{m} e^{-|x_i - y_j|/2K^2} \quad (3)$$

It was proved that when $K \to 0$, the local minima of the energy function is such that city x_i is matched by at least one y_j. And each x_i is matched by exactly one y_j if certain conditions are satisfied equations (2) and (3).

III. The improved elastic net algorithm for TSP

As mentioned above, the elastic net algorithm is geometric in nature and is suitable for solving TSP. However, based on observation and analysis in our previous work[12], it is illustrated there are a few of shortcomings the elastic net suffered. The most obvious one is the local minima problem. Due to elastic net algorithm is a deterministic algorithm, it is easy to trap into the same solution of local minima when the parameters are determined previously for some TSP instances. Moreover for some complex traveling salesman problems elastic net will lost cities or get crossing pathes with slightly high probability[12]. In relative researches of artificial neural networks, it is proved that embedding stochastic noise is an effective method to improve performance of original algorithms. But most of these algorithms mainly focus on the mechanism modification on processing function, few work is achieved on making the problem states stochastic yet.

In order to improve elastic net solution quality and its processing speed, an improved elastic net algorithm is proposed by introducing some time-dependent parameters into the original elastic net and embedding a stochastic noise strategy into TSP states, defined as follows.

$$x_i^{'} = x_i + \frac{\eta(t)}{2} random(-d_i, d_i) \quad (4)$$

$$E = -\alpha(t) K \sum_{i=1}^{n} ln \sum_{j=1}^{m} e^{-\|x_i^{'} - y_j\|/2k^2} + \beta(t) \sum_{j=1}^{m} \|y_j - y_{j+1}\| \quad (5)$$

$$\Delta y_j = \alpha(t) \sum_{i=1}^{n} w_{ij}(x_i^{'} - y_j) + \beta(t) K(y_{j+1} - 2y_j + y_{j-1}) \quad (6)$$

$$\alpha(t+1) = (1-\sigma)\alpha(t) \;\; if \;\; \alpha(t) > 0.1, \;\; else \;\; \alpha(t) = 0.1 \quad (7)$$

$$\beta(t+1) = (1+\varphi)\beta(t) \;\; if \;\; \beta(t) < 12, \;\; else \;\; \beta(t) = 12 \quad (8)$$

$$\eta(t+1) = (1-\lambda)\eta(t) \quad (9)$$

where d_i = distance between city i and its nearest neighbour city, σ = damping factor of the time-dependent $\alpha(t)(0 \le \sigma < 1)$, φ = promoting factor of the time-dependent $\beta(t)(0 \le \varphi < 1)$,$\lambda$ = damping factor of the relative stochastic noise ($0 < \lambda \le 1$).Constants 12 and 0.1 are the upper limit and lower limit correlative with $\beta(t)$ and $\alpha(t)$ respectively, which are confirmed by a large number of simulations. In our proposed algorithm, Eq.(4) and Eq.(9) are responding to the stochastic noise strategy, which contains the decaying control to reduce an oscillation TSP to be targeted problem. Eq.(7) and Eq.(8) are responding to the time-dependent parameters, which provides a flexible tuning parameters to avoid elastic net saturating at non-feasible solutions.

In the improved algorithm, the stochastic noise strategy reveals that annealing stochastic noise on problem states is also an effective way to improve performance of deterministic algorithm. The stochastic noises embedded on confirming city positions of TSP cause the cities oscillation, and further influence the force that moves a neuron on the rubber band towards those cities. With the decrease of the controlling parameter $\eta(t)$, stochastic noises gradually disappear and cities terminate oscillation to locate on their original positions. Due to the limitation of distances d_i the position oscillations are controlled in local domain of every city, our improved elastic net algorithm not only obtains local stochastic searching ability but also is able to avoid getting non-feasible solutions with cross path. If without the limitation d_i, neurons will

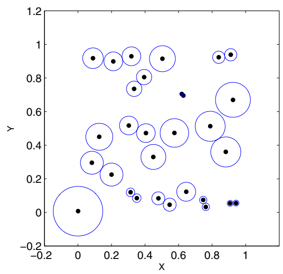

Fig. 1. A random TSP instance with 30 cities and the cities oscillation domain determined by the stochastic noise strategy. The circle around city represents the initial range of city oscillation.

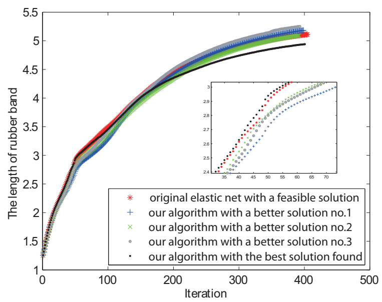

Fig. 2. The comparisons on the different rubber band evolutions for the TSP instance with 30 cities.

be confused utterly to break away the restraint of searching shortest tour length, then the solutions are bad predictably. In final the stochastic noise strategy will disappear and lose control of the problem states, the improved elastic net will get convergence at a superior solution of the relative targeted problems.

In order to show the affection of stochastic noise strategy, we selected a typical TSP instance with 30 cities to demonstrate our proposed strategy, and analyzed the rubber band evolutions in different times running. Fig. 1 and Fig. 2 represent the cities oscillation domain and the length evolution of rubber band respectively. From Fig. 1 we can see clearly that every city vibrates in its own local domain and their oscillation domains are not overlapping each other, so that the attraction force in the first term of Eq. 5 will not drive the rubber band crossing partly. In some TSP instances the distance between two cities sometimes is very small, then the original elastic net is easy to lose competition energy as the distance can not produce large enough discriminating attractive power to distinguish their relative neurons. By using the stochastic noise strategy, the city distances are diversified markedly to be distinguished easily for the elastic net. Fig. 2 shows that in the beginning 50 iterations the evolutions of rubber bands are similar to each other, which means the globally searching of our proposed method are similar to the original elastic net primarily. When the rubber band constructed with neurons expends to the neighborhood of some cities, the stochastic noise increases or decreases the attraction between cities and

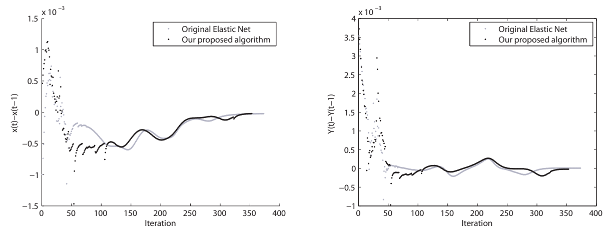

Fig. 3. Evolutions of the position changes of a typical neuron that converged to the same city for a 30 city TSP in the original elastic net and the proposed algorithm: (a)evolutions of the changes δx of the neuron and (b)evolution of the changes δy of the neuron.

neurons, which helps the elastic net escape from local minima. In Fig. 2 the effect of stochastic noise is revealed distinctly in the period from 50 iteration to final convergent iteration. Moreover, from the relationship between length transforming of rubber band and solution quality in Fig.2, we can see that rubber band expansion in earlier stage demonstrably influences the final solution of elastic net. Larger expansion means elastic net can yield more effective searching in global field. The rubber band of our proposed algorithm obviously holds enough expansion in the beginning period, shown in the enlarged view of Fig. 2, and finally our algorithm escapes from local minima and gets convergence at a better solution with the help of stochastic noise strategy. In elastic net, α and β are important parameters to balance two forces which separately affect the rubber band expanding to match cities and restrict the rubber band length to getting shortest path. It is difficult to set α β appropriate values to drive elastic net converge to optimal solution. Here the time-dependent parameters $\alpha(t)$ and $\beta(t)$ are introduced and flexibly tunes the balance of two force. In the process of our proposed algorithm, initially $\alpha(t)$ is set as a large value, $\beta(t)$ is initialized as a small value. Then the first item $-\alpha(t)K\sum_{i=1}^{n} ln \sum_{j=1}^{m} e^{-\|x_i^{'}-y_j\|/2k^2}$ of the energy function in Eq.(5) would play a key role in the network at the early stage. That means the force that affects the rubber band expanding to match cities is very strong, so network gains faster speed to drive the neurons move nearby those relative cities. As previous presentation, large expending of rubber band in earlier stage can improve the solution quality partly, so parameter $\alpha(t)$ not only accelerates the executive speed but increases the probability of finding optima solution in some degree. As time goes on, $\alpha(t)$ is decreased and $\beta(t)$ is increased gradually according to Eqs.(7) and (8). When $\beta(t)$ becomes sufficiently large, and $\alpha(t)$ reaches a relatively small value, the second item $\beta(t)K(y_{j+1}-2y_j+y_{j-1})$ of the energy function in Eq.(5) would play an important role to strengthen the force that pulls neurons towards their neighbor neurons on the rubber band. Strong attraction by their path neighbors restricts the rubber band length to form the shortest tour and accelerates the convergence process of the network generally. In this way, the time-dependent parameters could promote our proposed algorithm find better quality solution in fewer steps. Based on the analysis of various parameters involved in the elastic net and our experiments, the values of $\alpha(t)$ and $\beta(t)$

are found that they must be kept in bounds with $\alpha(t) \geq 0.1$ and $\beta(t) \leq 12$, otherwise the network may be uncontrolled gradually.

The time-dependent parameters modify the neuron movement states and matching corresponding relation. Fig. 3 plots the evolutions of the changes δx and δy of one selected neuron in original elastic net and the proposed algorithm. We can see the neuron position differences in our algorithm change greatly compared with that of original elastic net, especially in the field of iteration (50-120). The oscillations of neurons are dominated in the beginning period, it means our proposed algorithm has more powerful expanding force again. In the last stage of Fig. 3, the neurons do the local searching as the influence of the decaying parameters. In the middle period from 150 to 250 iterations, the neurons movement states are similar in both algorithms because of the controlling balance of parameter $\alpha(t)$ and $\beta(t)$. In Fig. 3 our proposed algorithm could use less iterations, about 25 iterations for the 30 cities problem, to get near 4% shorter tour length than the original elastic net. Obviously, with the help of time-dependent parameters and stochastic noise strategy our proposed algorithm displays a superior performance in solving TSP.

IV. SIMULATION

In order to evaluate the performance of our proposed method, extensive and systematic experiments on TSP were conducted in regard to solution quality and convergent speed. The results were promising and encouraging. The transiently chaotic neural network(TCNN), the original elastic net and our proposed algorithm had been implemented for the comparison. In our experiments the TSPs involved from 30 to 400 cities are considered, which uniformly distributed over a unit square[16]. To examine the credibility of the proposed algorithm, several kinds of traveling salesman problems with different cities density and distribution are selected as the tested instances. For each graph instance, 20 simulation runs are repeated from different initial values of neurons inputs to avoid the initial state dependency. Parameters in our proposed algorithm are set as follows: $\alpha(0) = 0.6; \beta(0) = 1.0; \sigma = 0.001; \varphi = 0.0025; \lambda = 0.01; \eta = 1.0; K = 0.5$ Similarly, the initial parameter values in original elastic net are set as the standard values taken from [3]: $\alpha = 0.2, \beta = 2.0, K = 0.2$. The parameters in TCNN are set as the standard values presented in [15]. Table I shows the summarized computational results, in which the "non-F*" means algorithm can't get a feasible solution. The column "BS" represents the shortest path length found by simulated algorithms within 20 times running. The columns "Original EN" and "Our IEN" respectively represent the original elastic net and our improved elastic net. From Table I, we can get some observations listed in follows:

- With the helping improvement of stochastic noise strategy and time-dependent parameters, generally our proposed algorithm could find out superior solution not only in the aspect of solution quality but in the executive steps. Although in some times running, our proposed algorithm sometimes find out a near-optimal solution being slightly worse than the relative one found by original elastic net, yet the average solution quality of our algorithm is superior than that of the original elastic net.
- The original elastic net is easy to lose cities and get convergence at some infeasible solutions when the size of TSP is larger than 100 cities. If the convergence speed of elastic net is accelerated by increasing the decaying temperature K, this shortcoming is more obvious. On the contrary, our proposed algorithm can avoid the disadvantage and can possess faster convergence speed.
- In some special instances, our proposed algorithm gets the same tour length as original elastic net and TCNN. The solution found by three algorithms with same tour length maybe is the optimal solution, which will be verified by other heuristic algorithms in the near future.
- Original elastic net and our improved algorithm manifest a steady characteristic in the aspect of computing steps. Their steps running are basically maintained in the domain from 300 steps to 450 steps, and our proposed algorithm uses less steps than original elastic net slightly. On the contrary, the steps running of TCNN show a linearly increasing characteristic primarily meaning large computational cost, which is the uniform shortcoming of Hopfield-type neural networks possibly.

TABLE I
COMPARISON AMONG ORIGINAL ELASTIC NET, TCNN AND OUR IMPROVED ELASTIC NET ON SOME RANDOM INSTANCES OF TSP.

City Num	Set	Original EN		TCNN		Our IEN	
		BS	Steps	BS	Steps	BS	Steps
30	1	4.962	342	4.962	529	4.910	320
	2	4.741	339	4.741	503	4.741	311
	3	4.513	369	4.522	514	4.496	335
	4	**4.739**	**397**	**4.739**	**538**	**4.739**	**347**
	5	4.808	414	4.808	586	4.791	362
60	1	6.418	382	6.402	723	6.362	358
	2	6.400	358	6.412	739	6.388	393
	3	6.222	339	6.302	746	6.222	362
	4	6.518	362	6.611	773	6.518	312
	5	6.527	411	6.571	782	6.500	342
100	1	8.163	361	8.413	862	8.012	332
	2	**8.647**	**397**	**8.632**	**871**	**8.274**	**343**
	3	**8.324**	**418**	**8.835**	**728**	**8.324**	**383**
	4	8.103	338	8.712	832	8.732	332
	5	8.821	387	9.103	842	8.801	366
200	1	11.929	401	12.310	1022	11.512	382
	2	**non-F***	**381**	**12.832**	**1093**	**11.129**	**336**
	3	10.968	442	11.382	1103	10.732	358
	4	**non-F***	427	11.981	1181	10.931	363
	5	11.525	395	12.341	1121	11.183	374
400	1	**non-F***	431	18.391	2293	17.812	407
	2	**16.555**	**453**	**16.512**	**2231**	**16.508**	**425**
	3	**non-F***	434	17.162	2018	16.874	439
	4	**non-F***	383	**non-F***	2192	16.723	421
	5	**non-F***	419	**non-F***	2321	17.383	427

To demonstrate the comparison of solution quality between original elastic net and our proposed algorithm, a typical TSP instance with 40 cities is selected to show the final solution

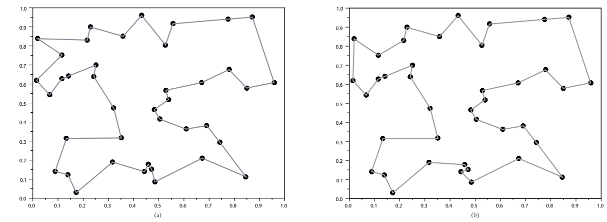

Fig. 4. The tour paths gotten by original elastic net(a) and our proposed elastic net(b).

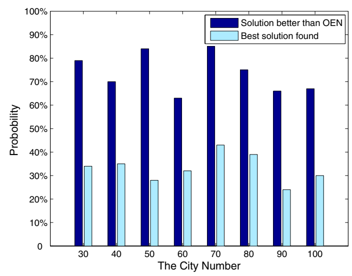

Fig. 5. The probability of finding near-optimal solutions in several instances of TSP. The upper blue bars represent the probability of finding better solutions than original elastic net, and the lower red bars represent the probability of getting the best solutions ever found.

in Fig. 4, solved by both algorithms. As shown in Fig.4 a superior solution with 5.191 tour length, which is equal to the solution gotten by genetic algorithm, is procured in 367 steps by using our proposed algorithm; however the original elastic net should cost 434 steps to obtain a worse solution with 5.214 tour length. Our algorithm could execute local adjustment on some problem region and escape from local minima to get a better solution, just as illustrated in Fig.4. Moreover aiming to show the efficiency of our proposed algorithm, we give out the successful probability of finding near-optimal solution in some TSP instances with 30 cities to 100 cities. Fig. 5 shows the test results. From Fig. 5, we can see that the probability of finding shorter tour length than that of original elastic net(OEN) is about 70% averagely. And the probability of finding the best solution ever found by our proposed algorithm is over 24% in 100 times running. In general the improvement of our algorithm by using stochastic noise strategy and time-dependent parameters is clear in Fig. 5.

V. Conclusion

In this article, we have proposed an improved elastic net algorithm to solve traveling salesman problem by introducing a stochastic noise strategy and some time-dependent parameters. The stochastic noise strategy is greatly useful for increasing stochastic dynamics of elastic net to avoid bad solutions in local minima. The time-dependent parameters enhance the globally searching ability of elastic net and accelerate the convergence speed. Simulations show that the novel elastic net embedded with the stochastic noise strategy and time-dependent parameters performs better than the classical elastic net for TSP in light of solution quality and convergence speed. Moreover the stochastic noise strategy provides a novel way to bring stochastic characteristic to some deterministic algorithms. In the near future we will focus on constructing a self-adaptive tuning method to control the stochastic noise in our algorithm, and attempt to affect the oscillation orientation of stochastic dynamics.

Acknowledgment

This research was partially supported by the grants from the Natural Science Foundation of China (No.61003205,No.60773216); Zhejiang Provincial Natural Science Foundation of China(No.Y1101062).

References

[1] M.Padberg and G.Rinaldi, A branch-and-cut algorithm for the resolution of large-scale symmetric traveling salesman problems, *SIAM Review*,vol.33, pp.60-100, 1991.

[2] S.Lin, B.W. Kernighan, An effective heuristic algorithm for the travelling-salesman problem, *Operations Research*, vol. 21, pp.498-516, 1973.

[3] Durbin, R. and D. Willshaw, An analogue approach to the traveling salesman problem using an Elastic Net approach,*Nature*, vol. 326, pp.689-691, 1987.

[4] Durbin, R., R. Szeliski and A. Yuille,An analysis of the Elastic Net approach to the traveling salesman problem, *Neural computation*, vol. 1, pp.348-358, 1989.

[5] J. J. Hopfield, Neural networks and physical systems with emergent collective computational abilities, *Proceedings of the National Academy of Sciences of the USA*, vol.79, pp.2554-2558, 1982.

[6] J.J. Hopfield and D.W. Tank, Computing with neural circles: a model, Science, vol. 233, pp.625-633, 1986.

[7] Lawler, E. L., J. K. Lenstra, A. H. G. Rinnooy Kan, and D. B. Shmoys, The travelling salesman problem: A guided tour of combinatorial optimization, Springer, Berlin, 1985.

[8] Vakhutinsky, A. I. and B. L. Golden, The co-adaptive neural network approach to the Euclidean Traveling Salesman Problem, *Neural Networks*, *16*, pp.1499-1525, 2003.

[9] Vakhutinsky, A. I. and B. L. Golden, A hierarchical strategy for solving traveling salesman problems using elastic nets, *Journal of Heuristics*, vol.1, pp.67-76, 1995.

[10] S.J. Gilson, Deformable contour methods for shape extraction from binary edge-point images, Ph.D. Thesis, University of Southampton, UK, June 1999.

[11] T. Kohonen, Clustering, taxonomy, and topological maps of patterns, in: Proceeding of the Sixth international Conference on Pattern Recognition, pp.114-128, 1982.

[12] Junyan Yi, Gang Yang, Zhiqiang Zhang and Zheng Tang, An Improved Elastic Net Method with Time-dependent Parameters for Traveling Salesman Problem, International Journal of Innovative Computing, Information and Control, vol.5, no.4, pp.1089-1100, 2009.

[13] Boeres M.C.S., de Carvalho L.A.V. and Barbosa V.C. A faster elastic-net algorithm for the traveling salesman problem, Neural Networks, 1992. IJCNN., International Joint Conference on Volume 2, 7-11 June 1992 Page(s):215–220 vol.2.

[14] Jan van den Berg and Jock H. Geselschap, An analysis of various elastic net algorithms, Discussion Paper, Erasmus University Rotterdam, Faculty of Economics, 1995.

[15] Chen, L. and K. Aihara, Chaotic simulated annealing by a neural network model with transient chaos, *Neural Networks*, vol.8, pp. 915–930, 1995.

[16] Reinelt, G., TSPLIB-a travelling salesman problem library, *ORSA Journal on Computing*, vol.3, pp.376-384, 1991.

[17] D.J.Burr, An improved elastic net method for the travelling salesman problem. in: Proceeding of the IEEE international Conference on Neural Network, vol.1, pp. 69-76, 1988.

[18] Pavlovic V, Schonfeld D, Friedman G. Stochastic noise Process enhancement of Hopfield neural networks. IEEE Transactions on Circuits and Systems II: Express Briefs. vol.52(4), pp.213-217, 2005.

[19] Wang J, Cai Y, Yin J. Multi-start stochastic competitive Hopfield neural network for frequency assignment problem in satellite communications. Expert Systems with Applications. vol. 38(1), pp. 131-145, 2011.