Appeared in Proceedings of Genetic and Evolutionary Computation Conference 2006
New York: ACM, (pp. 1629--1636).

Innovization: Innovating Design Principles Through Optimization

Kalyanmoy Deb and Aravind Srinivasan
Kanpur Genetic Algorithms Laboratory (KanGAL)
Indian Institute of Technology Kanpur
Kanpur, PIN 208016, India
{deb,aravinds}@iitk.ac.in

ABSTRACT

This paper introduces a new design methodology (we call it "innovization") in the context of finding new and innovative design principles by means of optimization techniques. Although optimization algorithms are routinely used to find an optimal solution corresponding to an optimization problem, the task of innovization stretches the scope beyond an optimization task and attempts to unveil new, innovative, and important design principles relating to decision variables and objectives, so that a deeper understanding of the problem can be obtained. The variety of problems chosen in the paper and the resulting innovations obtained for each problem amply demonstrate the usefulness of the innovization task. The results should encourage a wide spread applicability of the proposed innovization procedure (which is not simply an optimization procedure) to other problem-solving tasks.

Categories and Subject Descriptors

I.2 [**Artificial Intelligence**]: LearningKnowledge acquisition; J.2 [**Physical Sciences and Engineering**]: Engineering; J.6 [**Computer-aided Engineering**]: Computer-aided design

General Terms

Standardization, design, experimentation

Keywords

Innovative design, multi-objective optimization, Design principles, knowledge discovery.

1. INTRODUCTION

Innovation, defined in Oxford American Dictionary as 'the act of introducing a new process or the way of doing new things' has always fascinated man. In the context of engineering design of a system, a product or a process, researchers and applicationists constantly look for innovative solutions. Unfortunately, there exist very few scientific and systematic procedures for achieving such innovations. Goldberg [8] narrates that a competent genetic algorithm – a search and optimization procedure based on natural evolution and natural genetics – can be an effective mean to arrive at an innovative design for a single objective scenario. Monotonicity analysis [13] is a pre-optimization technique which can be applied to *monotonic* objective and constraint functions to find which variables would get fixed to its lower or upper bounds at the optimal solution, thereby eliminating them from the optimization procedure. However, it cannot be used in a generic problem and to find more interesting properties of variables and their interactions.

In this paper, we extend Goldberg's argument and describe a systematic procedure involving a multi-objective optimization task and a subsequent analysis of optimal solutions to arrive at a deeper understanding of the problem, and not simply to find a single optimal (or innovative) solution. In the process of understanding insights about the problem, the systematic procedure suggested here may often decipher new and innovative design principles which are common to optimal trade-off solutions and were not known earlier. Such commonality principles among multiple solutions should provide a reliable procedure of arriving at a 'blue-print' or a 'recipe' for solving the problem in an optimal manner. Through a number of engineering design problems, we describe the proposed 'innovization' process and present resulting *innovized* design principles which are useful, not obvious from the appearance of the problem, and also not possible to achieve by a single-objective optimization task.

2. SINGLE VERSUS MULTI-OBJECTIVE DESIGN

The main crux of the proposed innovization procedure involves optimization of at least two *conflicting* objectives of a design. When a design is to be achieved for a single goal of minimizing *size* of a product or of maximizing *output* from the product, usually one optimal solution is the target. When optimized, the optimal solution portrays the design, fixes the dimensions, and implies not much more. Although a sensitivity analysis can provide some information about the relative importance of constraints, they only provide local information close to the single optimum solution. Truly speaking, such an optimization task of finding a single optimum design does not often give a designer any deeper understanding than what and how the optimum solution should look like. After all, how much a single (albeit optimal) solution in the entire search space of solutions can offer to anyone?

Let us now think of an optimum design procedure in the context of two or more conflicting goals. In the design of minimizing size of a product, it is intuitive that the obtained optimal design will correspond to having as small a dimension as possible. Visibly, such a minimum-sized solution will look small and importantly will often not be able to deliver too much of an output. If we talk about the design of an electric induction motor involving armature radius, wire diameter and number of wiring turns as design variables and the design goal is to minimize the size of the motor, possibly we shall arrive at a motor which will look small and will deliver only a few horsepower (as shown as solution A in Figure 1), just enough to run a pump for lifting water to a two-storey building. On the other hand, if we design

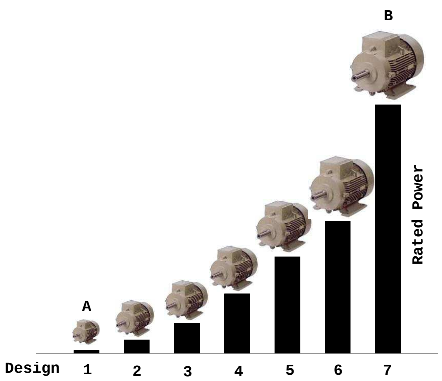

Figure 1: Trade-off designs show a clear conflict between motor size and power delivered in a range of TEFC three-phase squirrel cage induction motors. Despite the differences, are there any similarities in their designs?

the motor for the maximum delivered power using the same technology of motoring, we would arrive at a motor which can deliver, say, a few hundred horsepower, needed to run a compressor in an industrial air-conditioning unit (solution B in Figure 1). However, the size and weight of such a motor will be substantially large. If we let use a bi-objective optimization method of minimizing size and maximizing delivered power simultaneously, we shall arrive at these two extreme solutions and a number of other intermediate solutions (as shown in the figure) with different trade-offs in size and power, including motors which can be used in an overhead crane to hoist and maneuver a load, motors delivering 50 to 70 horsepower which can be used to run a machining center in a factory, and motors delivering about a couple of hundred horsepower which can be used in an industrial exhauster fan.

If we now line up all such motors according to the worse order of one of the objectives, say their increased size, in the presence of two conflicting objectives, they would also get sorted in the other objective in an opposite sense (in their increased output). Obtaining such a wide variety of solutions in a single computational effort is itself a significant matter, discussed and demonstrated in various evolutionary multi-objective optimization (EMO) studies in the recent past [4, 2]. After the multi-objective optimization task, we have a set of optimal solutions specifying the design variables and their objective trade-offs. We can now analyze these solutions to investigate if there exist some *common* principles among all or many of these optimal solutions. In the context of the motor design task, it would be interesting to see if all the optimal solutions have an identical wire diameter or have an armature diameter proportional or in some relation to the delivered power! If such a relationship among design variables and objective values exist, it is needless to say that they would be of great importance to a designer. Such information will provide a plethora of knowledge (or recipe) of how to design the motor in an optimal manner. With such a recipe, the designer can later design a new motor for a new application without resorting to solving a completely new optimization problem again. Moreover, the crucial relationship among design variables and objectives will also provide vital information about the theory of design of a motor which can bring out limitations and scopes of the existing procedure and spur new and innovative ideas of designing an electric motor.

It is argued elsewhere [5] that since the Pareto-optimal solutions are not any arbitrary solutions, rather solutions which mathematically must satisfy the so-called Fritz-John necessary conditions (involving gradients of objective and constraint functions) [9], in engineering and scientific systems and problems, we may be reasonably confident in claiming that there would exist some commonalities (or *similarities*) among the Pareto-optimal solutions which will ensure their optimality.

Such a task has a third dimension in the context of practices in industries. Successful industries standardize their products for reuse, easier maintenance and also for cost reduction. For industries interested in producing a range of products (such as electric motor manufacturing companies produce motors of a particular type ranging from a few horsepower to a few hundred horsepower), if some commonality principles of their designs can be found, this may help save inventory costs by keeping only a few common types of ingredients and raw materials (such as wires, armatures etc.) and also may help simplify the manufacturing process, in addition to cutting down the need for specialized man-powers. Since these innovative principles are derived through the outcome of a carefully performed optimization task, we call this procedure an act of 'innovization' – a process of obtaining innovative solutions and design principles through optimization.

3. INNOVIZATION PROCEDURE

In all case studies performed here, we have used the well-known elitist non-dominated sorting genetic algorithm or NSGA-II [6] as the multi-objective optimization tool. The NSGA-II solutions are then clustered to identify a few well-distributed solutions. The clustered NSGA-II solutions are then modified by using a local search procedure (we have used Benson's method [1, 4] here). The obtained NSGA-II-cum-local-search solutions are then verified by two independent procedures. First, the extreme Pareto-optimal solutions are verified by running a single-objective optimization procedure (a genetic algorithm is used here) independently on each objective function subjected to satisfying given constraints. Second, some intermediate Pareto-optimal solutions are verified by using the normal constraint method (NCM) [11] starting at different locations on the hyperplane constructed using the individual best solutions obtained from the previous step.

When the attainment of optimized solutions and their verifications are made, ideally a data-mining strategy must be used to automatically evolve design principles from the combined data of optimized design variables and corresponding objective values. By no means this is an easy task and is far from being a simple regression task of fitting a model over a set of multi-dimensional data. We mentioned some such difficulties earlier: (i) there may exist multiple relationships which are all needed to be found by the automated programming, thereby requiring to find multiple solutions to the problem simultaneously, (ii) a relationship may exist partially to the data set, thereby requiring a clustering procedure to identify which design principles are valid on which clusters, and (iii) since optimized data may not exactly be the optimum data, exact relationships may not be possible to achieve, thereby requiring to use fuzzy rule or rough set based approaches. While we are currently pursuing various data-mining and machine learning techniques for an automated learning and deciphering of such important design principles from optimized data set, in this paper we mainly use visual and statistical comparisons and graph plotting softwares for the task. We present the proposed innovization procedure here:

Step 1: Find individual optimum solution for each of the objectives by using a single-objective GA (or sometimes using NSGA-II by specifying only one objective) or by a classical method. Thereafter, note down the *ideal* point.

Step 2: Find the optimized multi-objective front by NSGA-II. Also, obtain and note the *nadir* point from the front.

Step 3: Normalize all objectives using ideal and nadir points and cluster a few solutions $Z^{(k)}$ $(k = 1, 2, \ldots, 10)$, preferably in the area of interest to the designer or uniformly along the obtained front.

Step 4: Apply a local search (Benson's method [1] is used here) and obtain the modified optimized front.

Step 5: Perform the normal constraint method (NCM) [11] starting at a few locations to verify the obtained optimized front. These solutions constitute a reasonably confident optimized front.

Step 6: Analyze the solutions for any commonality principles as plausible innovized relationships.

Since the above innovization procedure is expected to be applied to a problem once and for all, designers may not be quite interested in the computational time needed to complete the task. However, if needed, the above procedure can be made faster by parallelizing Steps 1, 2, 4 and 5 on a distributed computing machine.

We now illustrate the working of the above innovization procedure on a number of engineering applications. In all problems solved in this paper, we use a large population and run an evolutionary multi-objective optimization algorithm (NSGA-II) for a large number of generations so as to have confidence on the obtained trade-off frontier.

4. MULTIPLE-DISK CLUTCH BRAKE DESIGN

In this problem, a multiple clutch brake [12], as shown in Figure 2, needs to be designed. Two conflicting objectives

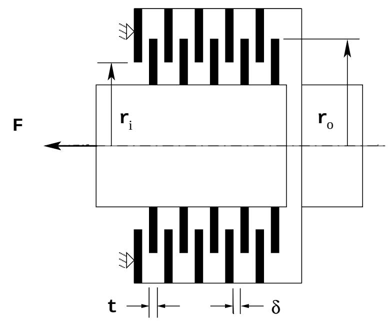

Figure 2: A multiple-disk clutch brake.

are considered: (i) minimization of mass (f_1 in kg) of the brake system and (ii) minimization of stopping time (T in s). There are five decision variables: $\vec{x} = (r_i, r_o, t, F, Z)$, where $r_i \in [60, 80]$ (in steps of one) is the inner radius in mm, $r_o \in [91, 110]$ (in steps of one) is the outer radius in mm, $t \in [1, 3]$ (in steps of 0.5) is the thickness of discs in mm, $F \in [600, 1000]$ (in steps of 10) is the actuating force in N and $Z \in [2, 10]$ (in steps of one) is the number of friction surfaces (or discs). The optimization problem is formulated below:

$$\begin{array}{rl}
\text{Minimize} & f_1(\vec{x}) = \pi({x_2}^2 - {x_1}^2)x_3(x_5+1)\rho, \\
\text{Minimize} & f_2(\vec{x}) = T = \frac{I_Z\omega}{M_h+M_f}, \\
\text{Subject to} & g_1(\vec{x}) = x_2 - x_1 - \Delta R \geq 0, \\
& g_2(\vec{x}) = L_{\max} - (x_5+1)(x_3+\delta) \geq 0, \\
& g_3(\vec{x}) = p_{\max} - p_{rz} \geq 0, \\
& g_4(\vec{x}) = p_{\max}V_{sr,\max} - p_{rz}V_{sr} \geq 0, \\
& g_5(\vec{x}) = V_{sr,\max} - V_{sr} \geq 0, \quad g_6(\vec{x}) = M_h - sM_s \geq 0, \\
& g_7(\vec{x}) = T \geq 0, \quad g_8(\vec{x}) = T_{\max} - T \geq 0, \\
& r_{i,\min} \leq x_1 \leq r_{i,\max}, \quad r_{o,\min} \leq x_2 \leq r_{o,\max}, \\
& t_{\min} \leq x_3 \leq t_{\max}, \quad 0 \leq x_4 \leq F_{\max}, \quad 2 \leq x_5 \leq Z_{\max}.
\end{array} \tag{1}$$

The parameters are given below:

$$\begin{array}{l}
M_h = \frac{2}{3}\mu x_4 x_5 \frac{{x_2}^3 - {x_1}^3}{{x_2}^2 - {x_1}^2} \text{ N·mm}, \omega = \pi n/30 \text{ rad/s}, \\
A = \pi({x_2}^2 - {x_1}^2) \text{ mm}^2, p_{rz} = \frac{x_4}{A} \text{ N/mm}^2, V_{sr} = \frac{\pi R_{sr} n}{30} \text{ mm/s}, \\
R_{sr} = \frac{2}{3}\frac{{x_2}^3 - {x_1}^3}{{x_2}^2 - {x_1}^2} \text{ mm}, \Delta R = 20 \text{ mm}, L_{max} = 30 \text{ mm}, \mu = 0.5, \\
p_{\max} = 1 \text{ MPa}, \rho = 0.0000078 \text{ kg/mm}^3, V_{sr,\max} = 10 \text{ m/s}, \\
s = 1.5, T_{max} = 15 \text{ s}, n = 250 \text{ rpm}, M_s = 40 \text{ Nm}, \\
M_f = 3 \text{ Nm}, I_z = 55 \text{ kg·m}^2, \delta = 0.5 \text{ mm}, r_{i,\min} = 60 \text{ mm}, \\
r_{i,\max} = 80 \text{ mm}, r_{o,\min} = 90 \text{ mm}, r_{o,\min} = 110 \text{ mm}, \\
t_{\min} = 1.5 \text{ mm}, t_{\max} = 3 \text{ mm}, F_{\max} = 1,000 \text{ N}, Z_{\max} = 9.
\end{array}$$

Individual minimum solutions are found by a single-objective NSGA-II and are shown in Table 1. The trade-off between two objectives is clear from the table. The Pareto-optimal

Table 1: The extreme solutions for the multiple-disk clutch brake design.

Solution	x_1	x_2	x_3	x_4	x_5	f_1	f_2
Min. f_1	70	90	1.5	1000	3	0.4704	11.7617
Min. f_2	80	110	1.5	1000	9	2.0948	3.3505

front obtained using NSGA-II is shown in Figure 3. The ex-

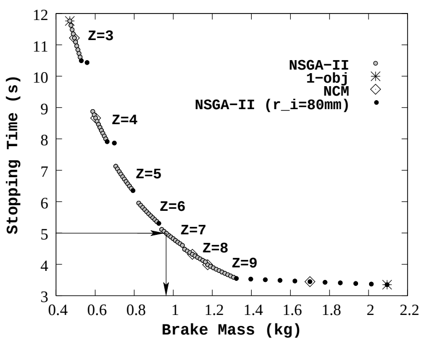

Figure 3: NSGA-II solutions for the clutch brake design problem.

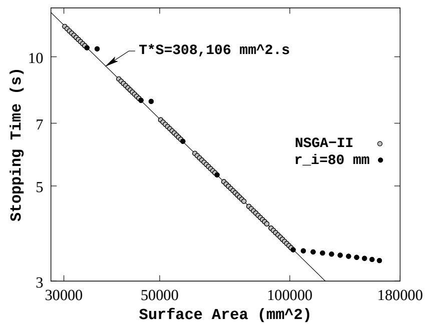

Figure 4: Stopping time (s) versus braking area (mm^2) for the optimal solutions of the clutch brake design problem.

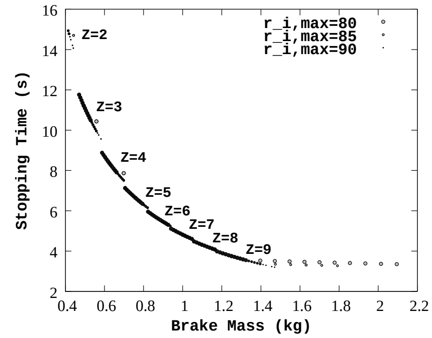

Figure 5: Effect of $r_{i,\max}$ on the trade-off solutions of the clutch brake design problem.

treme solutions shown in the table are also members of the Pareto-optimal front. The front is also verified by finding a number of optimal solutions using the NC method.

4.1 Innovized Principles

Following observations are made by analyzing the NSGA-II results.

1. The Pareto-optimal front is fragmented into a number of contiguous regions of identical Z values, as shown in Figure 3. This means that fixing the number of discs is the highest-level decision-making process by which the location of specific stopping time and brake mass value get more or less set. But for solutions to be optimal, the range of brakes from the least-weight design to quickest-acting design must be achieved with a monotonically increasing number of discs. For the smaller-weight solutions, fewer number of discs are needed, whereas for a quicker-acting design, the number of discs required are more.

2. Interestingly, there are two distinct relationships observed among the solutions of the Pareto-optimal front. For every fixed-Z portion of the front, there is a trade-off which starts with a small value of brake-mass having smallest values of r_i (70 mm) and r_o (90 mm), but in order to remain optimal both radii increase linearly by maintaining a difference of exactly $\Delta R = 20$ mm, making the constraint g_1 active. When r_i reaches its maximum limit (80 mm), r_i remains constant at this upper limit, but r_o keeps on increasing to produce faster stopping time solutions. These fixed-r_i solutions are marked in filled circles in Figure 3.

3. For all optimal solutions, the following decision variables take identical values: $t = 1.5\text{mm}, \quad F = 1,000\text{N}$. The disc thickness ($t$) of all solutions are identical to the lower allowable value ($t_{\min} = 1.5$ mm) and the applied force must be set to the largest allowable value ($F_{\max} = 1,000$ N). These innovative relationships for an optimal solution is far from being intuitive and can only be inferred from the obtained optimized data.

4. It is also interesting to note that the stopping time (T) is inversely proportional to the total braking area (S) of the system, as shown in Figure 4. Although it may be intuitive to a designer that a quicker stopping time solution is expected to be achieved for a braking system having a larger braking area, NSGA-II solutions bring out an exact relationship ($T \cdot S = 308,106$ mm^2·s) between the two quantities in this problem.

As indicated above, it is clear from the results that fixing the number of discs is the highest-level decision-making in this design process. Say for example, if we need to design a brake system capable of stopping in a maximum of 5 seconds, Figure 3 immediately indicates that a minimum of $Z = 7$ discs are needed with a smallest weight of 0.964 kg, requiring $r_i = 72$ mm and $r_0 = 92$ mm. Any optimal design with a particular stopping time T must have an overall surface area of contact equal to $S = 308,106/T$ mm^2. Moreover, if a brake with a stopping time in the range 4.6 seconds to about 5.1 seconds is required, the optimal design should have seven discs with its weight ranging between 0.941 kg to 1.047 kg. Thus, Figure 3 and the corresponding decision variables values can be used as a 'recipe' of arriving at an optimal design for a particular desired performance of the braking system.

4.2 Higher-Level Innovizations

It seems from the above simulation runs that the parameter $r_{i,\max}$ is an important one. In order to investigate the effect of this parameter on the obtained Pareto-optimal solutions, we rerun NSGA-II for two other $r_{i,\max}$ values (85 and 90 mm) and plot the frontiers in Figure 5. In both these cases, $\Delta R = 20$ mm, $t = 1.5$ mm, $F = 1,000$ N remain as innovizations. Three higher-level innovized principles are obtained from this study:

1. With larger upper limit of r_i, the gap between trade-off fragments of two consecutive Z values reduce. Thus, the deviation of optimal solutions (shown with filled circles) from fixed $T \cdot S$ relationship observed in Figure 4 is purely due to the fixation of the upper limit of r_i.

2. Solutions obtained with fixed $r_i = r_{i,\max}$ are better for a larger $r_{i,\max}$ value. However, solutions having $r_i < r_{i,\max}$ and obtained with different $r_{i,\max}$ values all follow the $T \cdot S = 308,106$ relationship. Thus, the T-S relationship is independent of the choice of $r_{i,\max}$.

3. With larger upper limit of r_i, more light-weight brakes with only $Z = 2$ discs having a stopping time less than or equal to 15 seconds are possible. Recall that with $r_{i,\max} = 80$ mm,

$Z = 2$ solutions were infeasible. These light-weight brakes have larger r_i and r_o values than the earlier case, thereby allowing to have a larger surface area per disc.

With the limit of $r_{i,\max} = 85$ or 90 mm, the lightest weight brake weighs 0.4145 kg having $r_i = 84$ mm and $r_o = 104$ mm, as opposed to 0.4704 kg obtained with $r_{i,\max} = 80$ mm. Similarly, a quicker-acting brake can be designed with an increase in $r_{i,\max}$ ($T = 3.20$ seconds with $r_{i,\max} = 90$ mm compared to $T = 3.35$ seconds with $r_{i,\max} = 80$ mm).

5. SPRING DESIGN

A helical compression spring needs to be designed for minimum volume and for minimum developed stress. Three variables are used for this purpose: the wire diameter d which is a discrete variable taking a few values mentioned below, the mean coil diameter D which is a real-valued parameter varied in the range [1,30] in, and the number of turns N, which is an integer value varied in the range [1,32]. The wire diameter d takes one of 42 non-equi-spaced values (as given in [10]). Denoting the variable vector $\vec{x} = (x_1, x_2, x_3) = (N, d, D)$, we write the two-objective, eight-constraint optimization problem as follows:

$$
\begin{array}{ll}
\text{Min.} & f_1(\vec{x}) = 0.25\pi^2 {x_2}^2 x_3 (x_1 + 2),\\
\text{Min.} & f_2(\vec{x}) = \frac{8KP_{max}x_3}{\pi {x_2}^3},\\
\text{s.t.} & g_1(\vec{x}) = l_{max} - \frac{P_{max}}{k} - 1.05(x_1+2)x_2 \geq 0,\\
& g_2(\vec{x}) = x_2 - d_{min} \geq 0, \quad g_3(\vec{x}) = D_{max} - (x_2 + x_3) \geq 0,\\
& g_4(\vec{x}) = C - 3 \geq 0, \quad g_5(\vec{x}) = \delta_{pm} - \delta_p \geq 0,\\
& g_6(\vec{x}) = \frac{P_{max}-P}{k} - \delta_w \geq \quad g_7(\vec{x}) = S - \frac{8KP_{max}x_3}{\pi {x_2}^3} \geq 0,\\
& g_8(\vec{x}) = V_{max} - 0.25\pi^2 {x_2}^2 x_3 (x_1+2) \geq 0,\\
& x_1 \text{ is integer, } x_2 \text{ is discrete, } x_3 \text{ is continuous.}
\end{array} \quad (2)
$$

The parameters used are as follows:

$$
\begin{array}{l}
K = \frac{4C-1}{4C-4} + \frac{0.615x_2}{x_3}, P = 300 \text{ lb}, D_{max} = 3 \text{ in}, k = \frac{G{x_2}^4}{8x_1{x_3}^3},\\
P_{max} = 1,000 \text{ lb}, \delta_w = 1.25 \text{ in}, \delta_p = \frac{P}{k}, l_{max} = 14 \text{ in},\\
\delta_{pm} = 6 \text{ in}, S = 189 \text{ ksi}, d_{min} = 0.2 \text{ in}, C = x_3/x_2,\\
G = 11,500,000 \text{ lb/in}^2, V_{max} = 30 \text{ in}^3.
\end{array}
$$

First, to obtain the individual minimum solutions, we use NSGA-II for solving each objective alone and obtain the solutions shown in Table 2. The non-dominated front found by NSGA-II also contains the same extreme solutions. Figure 6 shows the non-dominated front obtained by NSGA-II. The solutions obtained by several starting solutions by the NC method are also shown in the figure. A good agreement between single-objective results, NSGA-II results and NCM results give us confidence in the optimality of the obtained front.

Table 2: The extreme solutions for the spring design problem.

Solution	x_1	x_2 (in)	x_3 (in)	f_1 (in^3)	f_2 (psi)
Min. Vol.	9	0.283	1.223	2.659	187,997.203
Min. Str.	21	0.5	1.969	27.943	56,626.148

5.1 Innovized Principles

Let us now analyze the optimal solutions to find if there are any innovized principles which can be gathered about the spring design problem.

1. The Pareto-optimal front is fragmented and every fragment corresponds to a fixed value of wire diameter d, as shown in Figure 6. Of 42 different allowed d values, only five values make their places on the Pareto-optimal frontier. Here, fixing the d value fixes the range of optimal objective values on the Pareto-optimal frontier, thereby making the selection of this parameter the most important decision-making task in the design process.

2. Moreover, not every combination of D and N turns out to be optimal for these five values of d. Figure 6 shows (with a solid line) the complete non-dominated front obtained by keeping d constant and using only D and N as decision variables. As evident from the figure, some part of each front does not qualify (gets dominated by members of other fragments) to remain as Pareto-optimal when all d values are allowed.

3. For an optimal solution having a small volume, a small d must be chosen. However, the smallest available wire diameter ($d = 0.009$ in) is not an optimal choice. In fact, the smallest optimal wire diameter is $d = 0.283$ in.

4. When the non-dominated solutions are plotted in a logarithmic scale, optimal objective values (volume (V) and stress (S)) are found to have an interesting relationship: $SV^{0.517} =$ constant.

5. An investigation reveals that $d \propto D^{3/4}$ (Figure 8) for all and $N \propto 1/D^3$ (Figure 7) is a constant for all non-dominated solutions. Combining the two relationships, we conclude that ND^3/d^4 is a constant. Interestingly, this quantity is proportional to the inverse of spring constant $k = Gd^4/(8ND^3)$. By substituting the constant derived from the NSGA-II solutions, we obtain $k = 560$ lb/in for all optimal solutions. This reveals an innovation for this spring design problem. In order to create an optimal solution, we simply need to have a spring with a fixed spring constant of 560 lb/in for the chosen parameters of the design problem. Obviously, one can have different combinations of d, D and N to achieve this magical spring constant value. Figure 6 shows all such solutions which will make a non-dominated optimal combination of two objectives.

6. Another interesting aspect of the obtained NSGA-II solutions is that the constraint g_6 is active for all solutions. By substituting the fixed parameters in the mathematical constraint function (g_6), we obtain $k = (1000-300)/1.25 = 560$ lb/in, thereby explaining the specific value of the spring constant observed in the obtained data above.

The above innovized principles provide us with a *recipe* of designing a spring optimally. For example, if a spring has to be designed with a material having yield strength of 100,000 psi, Figure 6 clearly shows that an optimal design must be made from a wire of diameter $d = 0.394$ in. Other design variables must take $D = 1.779$ in and $N = 11$ turns and the spring will have a volume of 8.857 in^3. The results can also be interpreted as follows. If the designer is looking for designing a range of springs with materials having yield strength ranging from 130,000 psi to 165,000 psi, the optimal spring must be made from a wire of diameter 0.331 in, thereby requiring to maintain a small inventory for storing only one-sized wires for making optimal springs. What is also important here to note that all such springs

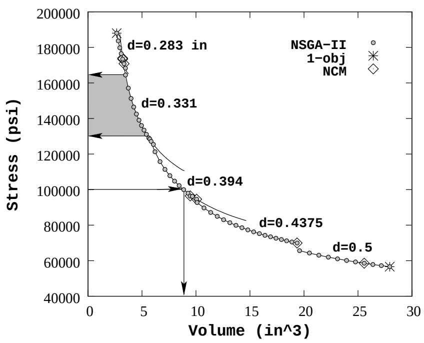

Figure 6: Pareto-optimal front obtained using NSGA-II for the spring design problem.

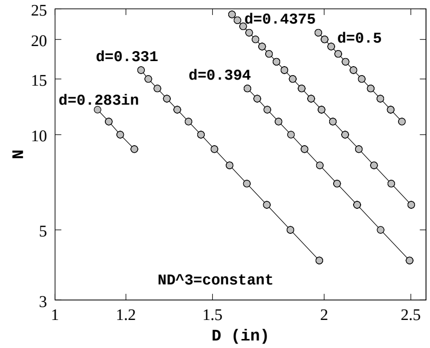

Figure 7: Optimal solutions follow ND^3 = constant **relationship for the spring design problem.**

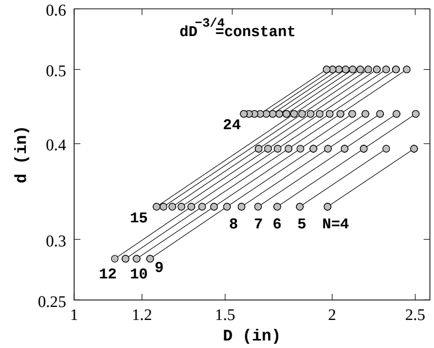

Figure 8: Optimal solutions follow $dD^{-3/4}$ = constant **relationship for the spring design problem.**

will be *optimal* from a dual consideration of volume (size) and strength (performance). The information about specific wire diameters (only five out of chosen 42 different values) for optimality and a common property of having a fixed spring constant (of 560 lb/in) are all innovations, which will be difficult to arrive at, otherwise.

5.2 Higher-Level Innovizations

Next, we increase δ_w to twice to its previous value, that is, we set $\delta_w = 2.5$ in. When we redo the proposed innovization procedure, we once again observe that the constraint g_6 is active for all solutions. Substituting other parameters, we then obtain $k = (1000 - 300)/2.5 = 280$ lb/in, half of what was achieved previously. Substituting the new values of the design variables in the stiffness term k, we observe that all solutions possess more or less an identical $k = 280$ lb/in, as can also be seen from Figure 9. We repeat the study for

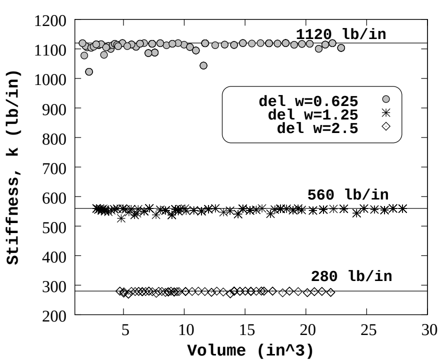

Figure 9: Different δ_w causes different Pareto-optimal frontiers, each causing an identical spring stiffness in all its solutions.

$\delta_w = 0.625$ in and observe that the corresponding stiffness of solutions come close to $k = 1,120$ lb/in. This clearly brings out an important innovization: All Pareto-optimal solutions must have an identical spring stiffness and the stiffness value depends on the chosen values of fixed parameters.

6. WELDED BEAM DESIGN

The welded beam design problem is well studied in the context of single-objective optimization [14]. A beam needs to be welded on another beam and must carry a certain load. It is desired to find four design parameters (thickness of the beam, b, width of the beam t, length of weld ℓ, and weld thickness h) for which the cost of the beam is minimum and simultaneously the vertical deflection at the end of the beam is minimum. The overhang portion of the beam has a length of 14 in and $F = 6,000$ lb force is applied at the end of the beam. The mathematical formulation of the two-objective optimization problem of minimizing cost and the end deflection can be found elsewhere [7, 3].

Table 3 presents the two extreme solutions obtained by the single-objective GA and also by NSGA-II. An intermediate solution, T (which will be explained latter), obtained by NSGA-II, is also shown. Figure 10 shows these two extreme solutions and a set of Pareto-optimal solutions obtained using NSGA-II. The obtained front is verified by finding a number of Pareto-optimal solutions using the NC method.

Table 3: The extreme solutions for the welded-beam design problem.

Solution	x_1 (h) (in)	x_2 (ℓ) (in)	x_3 (t) (in)	x_4 (b) (in)	f_1	f_2 (in)
Cost	0.244	6.215	8.299	0.244	2.3815	0.0157
Defl.	1.557	0.543	10.000	5.000	36.4403	0.0004
Soln.(T)	0.233	5.330	10.000	0.236	2.5094	0.0093

6.1 Innovized Principles

Let us now analyze the NSGA-II solutions to decipher innovized design principles:

1. Although Figure 10 shows an apparent inverse relationship between the two objectives, the logarithmic plot (inset) shows that there are two distinct behaviors between the objectives. From an intermediate transition solution T (shown in Table 3 and in Figure 10) near the smallest-cost (having comparatively larger deflection) solutions, objectives behave differently than in the rest of the trade-off region. For small-deflection solutions, the relationship is almost polynomial ($f_1 \approx O(f_2^{-0.890})$).

2. Figure 11 plots the constraint values for all trade-off solutions. It is apparent that for all optimal solutions the

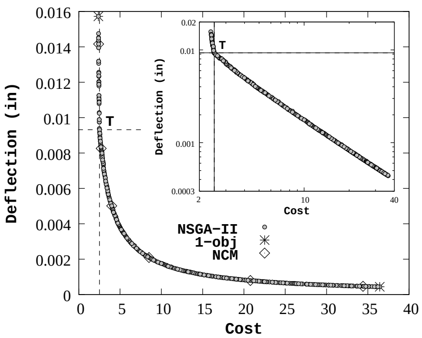

Figure 10: NSGA-II solutions are shown for the welded-beam design problem.

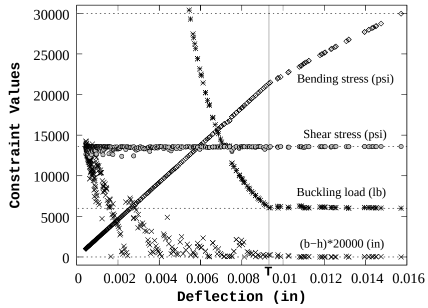

Figure 11: Constraint values of all Pareto-optimal solutions are shown for the welded-beam design problem.

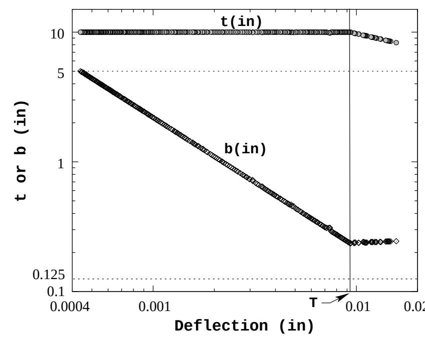

Figure 12: Variations of design variables t and b across the Pareto-optimal front are shown for the welded-beam design problem.

shear stress constraint is most critical and active. For small-deflection (or large-cost) solutions, the chosen bending strength (30,000 psi) and allowable buckling load (6,000 lb) are quite large compared to the developed stress and applied load. Any Pareto-optimal solution must achieve the maximum allowable shear stress value (13,600 psi). Thus, in order to improve the design, selection of a material having a larger shear strength capacity would be wise.

3. The transition point (point T) between two trade-off behaviors (observed in Figure 10) happens mainly from the buckling consideration. Designs having larger deflection values (or smaller cost values) reduce the buckling load capacity, as shown in Figure 11. When the buckling load capacity becomes equal to the allowable limit (6,000 lb), no further reduction is allowed. This happens at a deflection value close to 0.00932 in (having a cost of 2.509).

4. Interestingly, there are further innovizations with the design variables. For small-deflection solutions, the decision variable b must reduce inversely ($b \propto 1/f_2$)) with deflection objective (f_2) to retain optimality.

5. For small-deflection solutions, the decision variable t remains constant, as shown in Figure 12. This indicates that for most Pareto-optimal solutions, the height of the beam must be set to its upper limit. Although t causes an inverse effect to cost and deflection, as apparent from the equations, the active shear stress constraint involves t. Since shear stress value reduces with an increase in t (apparent from the formulation), it can be argued that fixing t to its upper limit would make a design optimal. Thus, if in practice solutions close to the smallest-cost solution are not desired, a beam of identical height ($t = 10$ in) may only be procured, thereby simplifying the inventory.

6. However, an increase of ℓ and a decrease in h with an increase in deflection (or a decrease in cost) are not completely monotonic. These two phenomena are not at all intuitive and are also difficult to explain from the problem formulation. However, the innovized principles for arriving at optimal solutions seem to be as follows: for a reduced cost solution, keep t fixed to its upper limit, increase ℓ and reduce h and b. This 'recipe' of design can be practiced only till the applied load is strictly smaller than the allowable buckling load.

7. Thereafter, any reduction in cost optimally must come from (i) reducing t from its upper limit, (ii) increasing b, and (iii) adjusting other two variables so as to make buckling, shear stress, and constraint g_4 active. In these solutions, with decreasing cost, the dimensions are reduced in such a manner so as to make the bending stress to increase. Finally, the minimum cost solution occurs when the bending stress equals to the allowable strength (30,000 psi, as in Figure 11). At this solution all four constraints become active, so as to optimally utilize the materials for all four purposes.

8. To achieve very small cost solutions, the innovized principles are different: for a reduced cost solution, reduce t and increase ℓ, h and b. Thus, overall a larger ℓ is needed to achieve a small cost solution.

6.2 Higher-Level Innovizations

Here, we redo the innovization procedure for a different set of allowable limits: shear strength in constraint g_1 is increased by 20%, bending strength in constraint g_2 is increased by 20%, and buckling limit load in constraint g_4 is reduced by 50%. We change them one at a time and keep the other parameters identical to their previous values. Figure 13 shows the corresponding Pareto-optimal frontiers for these three cases. Following innovizations are obtained:

1. It is clear that all three cases produce similar dual behavior (different characteristics on either side of a transition point) in the Pareto-optimal frontier, as was also observed in the previous case. All other innovizations (such as t being constant and b being smaller with increasing deflection, etc.) mentioned earlier remains the same in all three cases.

2. The minimum-cost solution depends on all three constraint (g_1, g_2 and g_4) limits, but the minimum-deflection solution only depends on the limit on shear stress constraint (g_1). However, at this solution, variables t and b take their largest allowable values of 10 in and 5 in, respectively.

3. An increase of bending strength by 20% does not change smaller-deflection solutions and the location of the transition point.

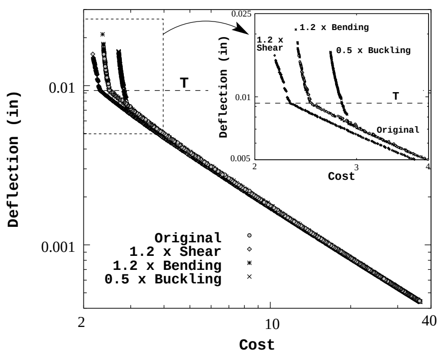

Figure 13: Effect of material strength and buckling load limit on the Pareto-optimal frontier for the welded-beam design problem.

4. Finally, a decrease in the buckling load limit by 50% changes the location of the transition point (which moves towards a larger cost solution), however the rest of the original Pareto-optimal frontier remains identical to the original front.

Thus, we conclude with confidence that (i) shear strength has a major role to play in deciding the optimal variable combinations (the shear stress constraint remains active in all cases), (ii) bending strength has an effect on the smallest-cost solution alone, as only this solution makes the bending constraint active, and (iii) buckling load limit has the sole effect in locating the transition point on the Pareto-optimal front. These information provide adequate knowledge about relative importance of each constraint and variable interactions for optimally designing a welded-beam over an entire gamut of cost-deflection trade-off. It is unclear how such valuable innovative information could have been achieved otherwise merely from a mathematical problem formulation.

7. CONCLUSIONS

In this paper, we have introduced a new design procedure (through a new terminology, we called 'innovization procedure') based on multi-objective optimization and a post-optimality analysis of optimized solutions. We have argued that the task of a single-objective optimization results in a single optimum solution which may not provide enough information about useful relationships among design variables, constraints and objectives for achieving different trade-off solutions. On the other hand, consideration of at least two conflicting objectives of design should result in a number of optimal solutions, trading-off the two objectives. Thereafter, a post-optimality analysis of these optimal solutions should provide useful information and design principles about the problem, such as relationships among variables and objectives which are common among the optimal solutions and the differences which make the optimal solutions different from each other. We have argued that such information should often introduce new principles for optimal designs, thereby allowing designers to learn innovations about solving the problem at hand.

On a number of engineering design problems having mixed discrete and continuous design variables, many useful innovizations (innovative design principles) are deciphered. Interestingly, many such innovizations were not intuitive and not known before. The ease of application of the proposed innovization procedure has also become clear from different applications. It is also clear that the proposed procedure is useful and ready to be used in other more complex design tasks.

However, the innovization procedure suggested here must now be made more automatic and problem-independent as far as possible. In this regard, an efficient data-mining technique is in order to evolve innovative design relationships from the Pareto-optimal solutions. Although some apparent hurdles of this task have been pointed out in this paper, effort is underway at Kanpur Genetic Algorithms Laboratory (KanGAL) in this direction. Higher-level innovization principles can also be extracted for more than two objectives and also for different levels of robustness and reliability considerations.

Acknowledgments

Authors acknowledge the support provided by STMicroelectronics, Agarte and Noida for performing this study.

8. REFERENCES

[1] H. P. Benson. Existence of efficient solutions for vector maximization problems. *Journal of Optimization Theory and Applications*, 26(4):569–580, 1978.

[2] C. A. C. Coello, D. A. VanVeldhuizen, and G. Lamont. *Evolutionary Algorithms for Solving Multi-Objective Problems*. Boston, MA: Kluwer Academic Publishers, 2002.

[3] K. Deb. An efficient constraint handling method for genetic algorithms. *Computer Methods in Applied Mechanics and Engineering*, 186(2–4):311–338, 2000.

[4] K. Deb. *Multi-objective optimization using evolutionary algorithms*. Chichester, UK: Wiley, 2001.

[5] K. Deb. Unveiling innovative design principles by means of multiple conflicting objectives. *Engineering Optimization*, 35(5):445–470, 2003.

[6] K. Deb, S. Agrawal, A. Pratap, and T. Meyarivan. A fast and elitist multi-objective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, 2002.

[7] K. Deb and A. Kumar. Real-coded genetic algorithms with simulated binary crossover: Studies on multi-modal and multi-objective problems. *Complex Systems*, 9(6):431–454, 1995.

[8] D. E. Goldberg. *The design of innovation: Lessons from and for Competent genetic algorithms*. Kluwer Academic Publishers, 2002.

[9] J. Jahn. *Vector optimization*. Berlin, Germany: Springer-Verlag, 2004.

[10] B. K. Kannan and S. N. Kramer. An augmented Lagrange multiplier based method for mixed integer discrete continuous optimization and its applications to mechanical design. *ASME Journal of Mechanical Design*, 116(2):405–411, 1994.

[11] A. Messac and C. A. Mattson. Normal constraint method with guarantee of even representation of complete Pareto frontier. *AIAA Journal*, in press.

[12] A. Osyczka. *Evolutionary algorithms for single and multicriteria design optimization*. Heidelberg: Physica-Verlag, 2002.

[13] P. Y. Papalambros and D. J. Wilde. *Principles of optimal design: Modeling and computation*. Cambridge, UK: Cambridge University Press, 2000.

[14] G. V. Reklaitis, A. Ravindran, and K. M. Ragsdell. *Engineering optimization methods and applications*. New York : Wiley, 1983.