LINEAR EQUATION

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$y = \frac{\int \mu Q\, dx + C}{\mu}, \quad \mu = e^{\int P\, dx}$$

EXACT EQUATIONS

$$M\, dx + N\, dy = 0$$

Exactness: $\frac{\partial}{\partial y} M = \frac{\partial}{\partial x} N$
My Trick: $F = \int M\, dx + C(y) = \int N\, dy + C(x)$
Solution: $F(x, y) = C$

SPECIAL INTEGRATING FACTOR

$$A = \left(\frac{\frac{\partial}{\partial y} M - \frac{\partial}{\partial x} N}{N} \right), \quad B = \left(\frac{\frac{\partial}{\partial x} N - \frac{\partial}{\partial y} M}{M} \right)$$

Use A if it only depends on x.
Use B if it only depends on y.

HOMOGENEOUS EQUATIONS

$$\frac{dy}{dx} = G\left(\frac{y}{x}\right)$$

$$\text{the test:} \quad f(tx, ty) = f(x, y)$$

Let $v = y/x$, then $dy/dx = v + x(dv/dx)$, and the transformed equations in the variables v and x is separable.

ANOTHER SUBSTITUTION

$$\frac{dy}{dx} = G(ax + by)$$

Let $z = ax + by$. Then $dz/dx = a + b(dy/dx)$, and the transformed equation in the variables z and x is separable.

BERNOULLI EQUATIONS

$$\frac{dy}{dx} = P(x)y = Q(x)y^n$$

For $n \neq 0, 1$, Let $v = y^{(1-n)}$. Then $\frac{dv}{dx} = (1-n)y^n \left(\frac{dy}{dx}\right)$, and the transformed equation in the variables v and x is linear.

LINEAR COEFFICIENTS

$$(a_1 x + b_1 y + c_1)dx + (a_2 x + b_2 y + c_2)dy = 0$$

For $a_1 b_2 \neq a_2 b_1$, let $x = u + h$ and $y = v + k$ where h and k satisfy

$$a_1 h + b_1 k + c_1 = 0$$
$$a_2 h + b_2 k + c_2 = 0$$

Then the transformed equation in the variables u and v is homogeneous.

EULERS METHOD

$x_{n+1} = x_n + h$
$y_{n+1} = y_n + hf(x_n, y_n)$

IMPROVED_EULER(x_0, y_0, c, N) :

$x = x_0$
$y = y_0$
$h = (c - x_0)/N$
FOR i FROM 0 TO N :
  $F = f(x, y)$
  $G = f((x + h),\ (y + h * F))$
  $x = x + h$
  $y = y + h(F + G)/2$

NEWTONS MOTION

$$x(t) = \frac{mg}{b} t + \frac{m}{b} \left(v_0 - \frac{mg}{b} \right) (1 - e^{-bt/m})$$

SPRING-MASS OSCILLATOR

$$my'' + by' + ky = F_{\text{ext}}(t)$$

$$y(t) = A\cos(\Omega t) + B\sin(\Omega t)$$

HOMOGENEOUS LINEAR EQUATIONS

$$ay'' + by' + cy = 0, \qquad a \neq 0$$

Two Roots $\rightarrow y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$
Same Root $\rightarrow y(t) = c_1 e^{rt} + c_2\, t\, e^{rt}$

THE QUADRATIC EQUATION

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$