THE QUADRATIC EQUATION

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

HOMOGENEOUS LINEAR EQUATIONS:
$ay'' + by' + cy = 0, \quad a \neq 0$

TWO REAL ROOTS:
$y_h = c_1 \cdot e^{r_1 t} + c_2 \cdot e^{r_2 t}$
$y_h' = c_1 r_1 e^{r_1 t} + c_2 r_2 e^{r_2 t}$

REPEATED ROOTS
$y_h = c_1 \cdot e^{rt} + c_2 \cdot t \cdot e^{rt}$
$y_h' = (c_1 r + c_2)e^{rt} + (c_2 r)te^{rt}$

COMPLEX ROOTS:
$y_h = c_1 \cdot e^{\alpha t} \cdot \sin(\beta t) + c_2 \cdot e^{\alpha t} \cdot \cos(\beta t)$
$y_h' = (c_1\alpha - c_2\beta)ES + (c_1\beta + c_2\alpha)EC$

COMPLEX ROOTS (init value):
$c_1 = \frac{y_0' - \alpha y_0}{\beta}, \quad c_2 = y_0$

UNDETERMINED COEFFICIENTS

CASE: $f == C\sin(\beta t) \;||\; C\cos(\beta t)$
SOLU: $y_p = A_1 \sin(\beta t) + A_2 \cos(\beta t)$

CASE: $f == Ct^m e^{rt}$
SOLU: $y_p = t^s P^m e^{rt}$
$s = 0$ if r is not a root of the equation.
$s = 1$ if r is a simple root.
$s = 2$ if r is a double root.

CASE: $f = Ct^m e^{\alpha t} \left\{ \begin{matrix} \sin \beta t \\ \cos \beta t \end{matrix} \right\}$
SOLU: $y_p = t^s P_1^m e^{\alpha t} \cos(\beta t) + t^s P_2^m e^{\alpha t} \sin(\beta t)$
$s = 0$ if $\alpha + i\beta$ is not a root.
$s = 1$ if $\alpha + i\beta$ is a root.

NON-HOM GENERAL SOLUTION
$y = y_h + y_p$

SUPERPOSITION
LET: $F = ay'' + by' + cy$
IF: y_1 is a solution to $F = f_1$
AND: y_2 is a solution to $F = f_2$
THEN: $k_1 y_1 + k_2 y_2$ is a solution to $F = k_1 f_1 + k_2 f_2$