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Determine the form of a particular solution to the equation $y^{(4)} - 5y'' + 4y = 10\cos(t) - 20\sin(t)$

The corresponding homogeneous equation,

$$y^{(4)} - 5y'' + 4y = 0$$

has a characteristic equation of

$$r^4 - 5r^2 + 4r = 0$$

which can be factored into

$$r(r-1)(r^2 + r - 4) = 0$$

In the third term, we need to know if has real or complex roots, so we can determine that checking if $b^2 - 4ac < 0$,

$$(1)^2 - 4(1)(-4) \;=\; 9 \quad \not< 0$$

Thus, the third term has real roots.

Ok, now lets look at all the roots together,

$$\begin{aligned} r_1 &= 0 \\ r_2 &= 1 \\ r_3 &\in \mathbb{R} \\ r_4 &\in \mathbb{R} \end{aligned}$$

Since we don't have any complex roots, we don't need to worry about adding extra t terms to our particular solution(s).

At least, I don't *think* we need to worry about it...

Ok, back to the original differential equation. First thing to notice is that we can use the superposition principle here,

$$\begin{aligned} k_1 f_1 + k_2 f_2 = 10\cos(t) - 20\sin(t) \\ k_1 &= 10 \\ k_2 &= -20 \\ f_1 &= \cos(t) \\ f_2 &= \sin(t) \end{aligned}$$

So we can get particular solutions for each of them separately,

$$\begin{aligned} y_{p1} &= A\sin(t) + B\cos(t) \\ y_{p2} &= C\sin(t) + D\sin(t) \end{aligned}$$

And then do a linear combination of them to get the form of our particular solution,

$$\boxed{y_p = (A + C)\sin t + (B + D)\cos t}$$