TRIGONOMETRY
$\tan x = \sin x / \cos x$
$\cot x = 1/\tan x$
$\sec x = 1/\cos x$
$\csc x = 1/\sin x$
$\sin^2 x + \cos^2 x = 1$
$\sin 2x = 2\sin x \cos x$

INTEGRALS
$\int u\,dv = uv - \int v\,du$
$\int(1/x) = \ln|x|$
$\int(1/(ax+b)) = \ln|ax+b|/a$
$\int \tan ax = -\ln|\cos ax|/a = \ln|\sec ax|/a$
$\int \cot ax = \ln|\sin ax|/a$
$\int \sec^2 x = \tan x$
$\int(1/(1+x^2)) = \arctan x$
$\int(1/\sqrt{1-x^2}) = \arcsin x$
$\int(x/e^{ax}) = e^{-ax}(ax-1)/a^2$

THE QUADRATIC EQUATION
$x = \left(-b \pm \sqrt{b^2-4ac}\right)/2a$

HOMOGENEOUS LINEAR EQUATIONS:
$ay'' + by' + cy = 0, \quad a \neq 0$

TWO REAL ROOTS:
$y_h = c_1 \cdot e^{r_1 t} + c_2 \cdot e^{r_2 t}$
$y_h' = c_1 r_1 e^{r_1 t} + c_2 r_2 e^{r_2 t}$

REPEATED ROOTS
$y_h = c_1 \cdot e^{rt} + c_2 \cdot t \cdot e^{rt}$
$y_h' = (c_1 r + c_2)e^{rt} + (c_2 r)te^{rt}$

COMPLEX ROOTS:
$y_h = c_1 \cdot e^{\alpha t} \cdot \sin(\beta t) + c_2 \cdot e^{\alpha t} \cdot \cos(\beta t)$
$y_h' = (c_1\alpha - c_2\beta)ES + (c_1\beta + c_2\alpha)EC$

COMPLEX ROOTS (init value):
$c_1 = \frac{y_0' - \alpha y_0}{\beta}, \quad c_2 = y_0$

UNDETERMINED COEFFICIENTS

CASE: $f == C\sin(\beta t) \;||\; C\cos(\beta t)$
SOLU: $y_p = A_1 \sin(\beta t) + A_2 \cos(\beta t)$

CASE: $f == Ct^m e^{rt}$
SOLU: $y_p = t^s P^m e^{rt}$
$s = 0$ if r is not a root of the equation.
$s = 1$ if r is a simple root.
$s = 2$ if r is a double root.

CASE: $f = Ct^m e^{\alpha t}\left\{ \begin{smallmatrix} \sin\beta t \\ \cos\beta t \end{smallmatrix} \right\}$
SOLU: $y_p = t^s P_1^m e^{\alpha t}\cos(\beta t) + t^s P_2^m e^{\alpha t}\sin(\beta t)$
$s = 0$ if $\alpha + i\beta$ is not a root.
$s = 1$ if $\alpha + i\beta$ is a root.

NON-HOM GENERAL SOLUTION
$y = y_h + y_p$

SUPERPOSITION
LET: $F = ay'' + by' + cy$
IF: y_1 is a solution to $F = f_1$
AND: y_2 is a solution to $F = f_2$
THEN: $k_1 y_1 + k_2 y_2$ is a solution to $F = k_1 f_1 + k_2 f_2$

VARIATION OF PARAMETERS
$y_p = v_1 y_1 + v_2 y_2$
$W[y_1, y_2] = y_1 y_2' - y_1' y_2$

$$v_1 = \int \frac{-f y_2}{aW[y_1,y_2]}, \quad v_2 = \int \frac{+f y_1}{aW[y_1,y_2]}$$

$y_1 v_1' + y_2 v_2' = 0,$
$y_1' v_1' + y_2' v_2' = f/a$

CAUCHY-EULER (Oily Couch)
$at^2 y'' + bty' + cy = f$
CHARACTERISTIC: $ar^2 + (b-a)r + c = 0$
REAL: $y_h = t^{r_1} + t^{r_2}$
DOUBLE: $y_h = t^r + t^r \ln(t)$
COMPLEX: $y_h = t^\alpha \sin(\beta \ln t) + t^\alpha \cos(\beta \ln t)$

REDUCTION OF ORDER
$y'' + Py' + Qy = 0$
Given: y_1

$$y_2 = y_1 \int \frac{e^{-\int P}}{(y_1)^2}$$

$y_h = y_1 + y_2$

LINEAR INDEPENDENCE TEST
For whole interval: $W[y_1, y_2] = 0$