# Prime Factor Space

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = 2^0 \cdot 3^1 \cdot 5^2 \cdot 7^3 \cdot 11^4$$

# Definitions

Let $A$ represent an ordered set $(a_1, \ldots, a_n)$, and let:

$$P_A = 2^{a_1} \cdot 3^{a_2} \cdot 5^{a_3} \cdots (p_n)^{a_n}$$

Where each of the factors is in the order set of Prime Numbers, and $p_n$ denotes the $n^{th}$ prime number.

To define the sets that these things are in:

$$A \in \mathbb{R}^n \text{ and } P_A \in \mathbb{R}$$

For all $A \in \mathbb{R}^n$ and $c \in \mathbb{R}$:

Let Addition be defined:

$$P_A + P_B = P_{A+B}$$

Let Multiplicative be defined:

$$P_A \cdot P_B = P_{AB}$$

Let scalar addition be defined:

$$c + A = P_{c+A} = 2^{c+a_1} \cdots {p_n}^{c+a_n}$$

Let scalar multiplication be defined:

$$c \cdot A = P_{cA} = 2^{c \cdot a_1} \cdots {p_n}^{c \cdot a_n}$$

Let there exist a $\vec{0}$ such that:

$$\vec{0} = P_0 = 2^0 \cdots {p_n}^0 = 1$$

# Proof of Vector Space

The goal is to show that our special $P$ is a vector space $V$

## Commutative

$A + B = B + A$ for all $A, B \in V$

*Proof*:

Let $A = (a_1, \dots a_n)$, and $B = (b_1, \dots b_n)$. Then by MYDEF,

$$A = 2^{a_1} \cdots {p_n}^{a_n}$$
$$B = 2^{b_1} \cdots {p_n}^{b_n}$$

Since addition of A and B is written like:

$$A + B = \left(2^{a_1} \cdots (P_n)^{a_n}\right) \cdot \left(2^{b_1} \cdots (P_n)^{b_n}\right)$$

Since both sides are esssentially real numbers. Then you can rewrite the expression like this:

$$\left(2^{b_1} \cdots (P_n)^{b_n}\right) \cdot \left(2^{a_1} \cdots (P_n)^{a_n}\right) = B + A$$

Thus,

$$A + B = B + A$$

□

It might be easier to show that they become real numbers.

$$\begin{aligned} A + B &= P_A \cdot P_B \\ &= P_B \cdot P_A \\ &= B + A \end{aligned}$$

## Associative

For all $A, B, C \in V$ and $a, b \in F$:

- $(A + B) + C = A + (B + C)$
- $(ab)A = a(bA)$

*Proof*

$$\begin{aligned} (A + B) + C &= (P_A \cdot P_B) \cdot P_C \\ &= P_A \cdot (P_B \cdot P_C) \\ &= A + (B + C) \end{aligned}$$

□

## Additive Identity

There exists an element $0 \in V$ such that $A + 0 = A$ for all $A \in V$

*Proof*

This makes use of the definition of exponents. All real numbers taken to the $0^{\text{th}}$ power equal to 1.

$$\begin{aligned} A + 0 &= P_A \cdot P_0 \\ &= P_A \cdot (2^0 \cdots {p_n}^0) \\ &= P_A \cdot (1 \cdots 1) \\ &= P_A \cdot 1 \\ &= P_A \\ &= A \end{aligned}$$

□

## Additive Inverse

For every $A \in V$ there exists a $B \in V$ such that $A + B = \vec{0}$

*Proof*

Suppose $A = P_A$ and $B = P_{-A}$, Then:

$$\begin{aligned} A + B &= P_A \cdot P_{-A} \\ &= \left(2^{a_1} \cdots {p_n}^{a_n}\right) \cdot \left(2^{-a_1} \cdots {p_n}^{-a_n}\right) \\ &= \left(2^{a_1} \cdot 2^{-a_1} \cdots (p_n)^{a_n} \cdot (p_n)^{-a_n}\right) \\ &= \left(2^{a_1 - a_1} \cdots {p_n}^{a_n - a_n}\right) \\ &= \left(2^0 \cdots {p_n}^0\right) \\ &= P_0 \\ &= \vec{0} \end{aligned}$$

□

It's important to note at this point that $\vec{0}$, which is an ordered list of $0$s, becomes $P_0$, which is equal to $1$.

Just because it's always fun to write things like this:

$$\vec{0} = 1$$

which is true only in our special Factor Space.

## Multiplicative Identity

$1A = A$ for all $A \in V$.

*Proof*

$$
\begin{aligned}
1A &= P_{1A} \\
&= \left(2^{1a_1} \cdots {p_n}^{1a_n}\right) \\
&= \left(2^{a_1} \cdots {p_n}^{a_n}\right) \\
&= P_A \\
&= A
\end{aligned}
$$

□

## Distributive

For all $c, d \in F$ and $A, B \in V$,

- $c(A + B) = cA + cB$
- $(c + d)A = cA + dA$

*Proof Part 1*

$$
\begin{aligned}
cA + cB &= P_{cA} + P_{cB} \\
&= (2^{ca} \cdots {p_n}^{ca_n}) \cdot (2^{cb} \cdots {p_n}^{cb_n}) \\
&= 2^{ca+cb} \cdots {p_n}^{ca+cb} \\
&= 2^{c(a+b)} \cdots {p_n}^{c(a+b)} \\
&= P_{c(A+B)} \\
&= c(A + B)
\end{aligned}
$$

□

*Proof Part 2*

$$
\begin{aligned}
(c + d)A &= P_{(c+d)A} \\
&= 2^{(c+d)A} \cdots {p_n}^{(c+d)A} \\
&= 2^{cA+dA} \cdots {p_n}^{cA+dA} \\
&= 2^{cA} \cdot 2^{dA} \cdots {p_n}^{cA} \cdot {p_n}^{dA} \\
&= (2^{cA} \cdots {p_n}^{cA}) \cdot (2^{dA} \cdots {p_n}^{dA}) \\
&= P_{cA} \cdot P_{dA} \\
&= cA + dA
\end{aligned}
$$

□

# Extras

If $A$ is a list of positive integers, $A \in \mathbb{N}^n$, then

$$P_A \in \mathbb{Z}$$

because of the Fundamental Theorem of Arithmetic.

```
def primes(x):
    print(2)
    primes = [2]
    n = 3
    while len(primes) < x:
        notPrime = False
        for p in primes:
            if n % p == 0:
                notPrime = True
                break
        if not notPrime:
            primes += [n]
            print(n)
        n += 1
    return primes
```