Definitions

Vector Space

A vector space is a set V with the following properties:

Commutativity:
$u + v = v + u$ for all $u, v \in V$

Associativity:
$(u + v) + w = u + (v + w)$ and $(ab)v = a(bv)$ for all $u, v, w \in V$

Additive Identity:
there exists $0 \in V$ such that $v + 0 = v$ for all $v \in V$

Multiplicative Identity:
for all $v \in V$, there exists $w \in V$ such that $v + w = 0$

Distributive Properties:
$a(u + v) = au + av$ and $(a + b)v = av + bv$ for all $a, b \in \mathbb{R}$ and $u, v \in V$

Linear Combination

A linear combination of a list of vectors $v_1, \ldots, v_n$ is itself a vector, taking the form:

$$a_1 v_1 + \ldots + a_m v_m$$

where each $a_1, \ldots a_n \in \mathbb{R}$

Span

The set of all linear combinations of a list of vectors $v_1, \ldots, v_n$ is called the **span** of $v_1, \ldots, v_n$, or $\mathrm{Span}(v_1, \ldots, v_n)$. Defined as:

$$\mathrm{span}(v_1, \ldots, v_n) = \{a_1 v_1 + \cdots + a_n v_n \ : \ a_1, \ldots, a_m \in \mathbb{R}\}$$

If the span is equal to some space $\mathrm{span}(v_1, \ldots, v_n) = V$, then you could say that $v_1, \ldots, v_n$ **spans** V.

Linearly Independent

For $v_1, \ldots, v_n \in V$ and $a_1, \ldots, a_n \in \mathbb{R}$ such that:

$$a_1 v_1 + \cdots + a_n v_n = 0$$

The list of vectors $v_1, \ldots, v_n$ is called **linearly independent** when

$$a_1 = \cdots = a_n = 0$$

for all possible values of $v_1, \ldots, v_n$.

Inner Product

TODO

Proofs

Law of Cosines

TODO

Triangle Inequality

TODO

Cauchy-Schwartz Inequality

TODO