Homework 5

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Problems from 2.6

7.

p	q	$\neg q$	$p \wedge \neg q$	$q \wedge \neg q$	$(p \wedge \neg q) \rightarrow (q \wedge \neg q)$	$p \rightarrow q$
1	1	0	0	0	1	1
1	0	1	1	0	0	0
0	1	0	0	0	1	1
0	0	1	0	0	1	1

Therefore, $(p \wedge \neg q) \rightarrow (q \wedge \neg q) \;=\; p \rightarrow q$.

8.

p	q	$\neg p$	$\neg q$	$p \rightarrow \neg q$	$\neg q \rightarrow p$	$(p \rightarrow \neg q) \wedge (\neg q \rightarrow p)$	$\neg p \leftrightarrow q$
1	1	0	0	0	1	0	0
1	0	0	1	1	1	1	1
0	1	1	0	1	1	1	1
0	0	1	1	1	0	0	0

Therefore, $\neg p \leftrightarrow q \;=\; (p \rightarrow \neg q) \wedge (\neg q \rightarrow p)$.

10.

p	q	r	$\neg q$	$\neg r$	$p \to q$	$p \wedge \neg q$	$(p \wedge \neg q) \wedge \neg r$	$\neg((p \wedge \neg q) \wedge \neg r)$	$(p \to q) \vee r$
1	1	1	0	0	1	0	0	1	1
1	1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	1	0	1	1
0	0	0	1	1	1	0	0	1	1
0	1	1	0	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1	1

Therefore, $\neg((p \wedge \neg q) \wedge \neg r) \;=\; (p \to q) \vee r$. Yes, they are logically equivalent.

11.

p	q	$\neg p$	$p \to q$	$q \to p$	$(\neg p) \wedge (p \to q)$	$\neg(q \to p)$
1	1	0	1	1	0	0
1	0	0	0	1	0	0
0	1	1	1	0	1	1
0	0	1	1	1	1	0

Therefore, $(\neg p) \wedge (p \to q) \;\neq\; \neg(q \to p)$. No, they are not logically equivalent. In the case where both p is false and q is false, the two statements are not the same.

12.

p	q	$\neg q$	$p \to q$	$\neg(p \to q)$	$p \wedge \neg q$
1	1	0	1	0	0
1	0	1	0	1	1
0	1	0	1	0	0
0	0	1	1	0	0

Therefore, $\neg(p \to q) \;=\; p \wedge \neg q$. Yes, they are logically equivalent.

Problems from 2.7

1. "The square of all real numbers is greater than zero." $\leftarrow$ False.

3. "There exists a real number, a, such that for all real numbers, x, the statement $ax = x$ is true." $\leftarrow$ True.

4. "All subsets of the Natural Numbers are also subsets of the Real Numbers." $\leftarrow$ True.

5. "For all natural numbers, n, there exists a subset of the Natural Numbers, such that the cardinality of that subset is less than n." $\leftarrow$ True.

7. "For all subsets of the Natural Numbers, there exists an integer, n, such that the cardinality of that subset is equal to n." $\leftarrow$ True.