Homework 6

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Problems from 2.10

1. $(x \leq 0) \vee (y \geq 0)$
2. $(x \in \mathbb{P}) \wedge (\sqrt{x} \in \mathbb{Q})$
3. $\exists p \in \mathbb{P},\ \forall q \in \mathbb{P},\ q \leq p$
4. $\exists \epsilon \in \mathbb{R}^+,\ \forall \delta \in \mathbb{R}^+,\ \Big(|x-a| < \delta\Big) \wedge \Big(|f(x) - f(a)| \geq \epsilon\Big)$
5. $\exists \epsilon \in \mathbb{R}^+,\ \forall M \in \mathbb{R}^+,\ \Big(x > M\Big) \wedge \Big(|f(x) - b| \leq \epsilon\Big)$
6. $\forall a \in \mathbb{R}, \exists x \in \mathbb{R}, a + x \neq x$
7. I eat a thing with a face.
   $\exists x \in \{\text{Things that I eat}\}, x \in \{\text{Things with faces}\}$.

Problems from 4.1

1. Suppose $\neg((\neg P) \wedge (\neg Q))$ is true.
   Suppose $\neg Q$ is true.

$$\begin{aligned} \neg(\neg P \wedge \neg Q) &= \neg(\neg P \wedge \text{true}) \\ &= P \vee \text{false} \\ &= P \end{aligned}$$

   Since $\neg(\neg P \wedge \neg Q)$ is true, P is also true.
   Therefore P.

2. Suppose the statements R, $P \to R$, and $P \vee Q$ are all true.
   Since $\neg R$ is false, R must be true.

$$\begin{aligned} P \to R &= \neg P \vee R \\ &= \neg P \vee \text{false} \\ &= \neg P \end{aligned}$$

   Since $\neg P$ is true, P must be false.

$$\begin{aligned} P \vee Q &= \text{false} \vee Q \\ &= Q \end{aligned}$$

   Since $P \vee Q$ is true, Q is also true.
   Therefore Q.