$$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \frac{q_{\text{enc}}}{\epsilon_0}$$

1. spatial symmetry
2. surface, $\vec{\mathbf{E}} \cdot \hat{\mathbf{n}} = E$
3. Φ through surface (left-side)
4. q_{enc} (right-side)
5. Evaluate for $\vec{\mathbf{E}}$

Spherical Symmetry

$$\vec{\mathbf{E}}_p = E_p(r)\ \hat{\mathbf{r}}$$

$$\Phi = E_p 4\pi r^2$$

$$\|E(r)\| = \frac{1}{4\pi\epsilon_0}\frac{q_{\text{enc}}}{r^2}$$

$$4\pi r^2 E = \frac{q_{\text{enc}}}{\epsilon_0}$$

Cylindrical Symmetry

$$\vec{\mathbf{E}} = E_p(r)\ \vec{\mathbf{r}}$$

$$\Phi = 2\pi r L E$$

$$q_{\text{enc}} = \lambda_{\text{enc}} L$$

$$\|E(r)\| = \frac{\lambda_{\text{enc}}}{2\pi\epsilon_0}\frac{1}{r}$$

$$2\pi r L E = \frac{q_{\text{enc}}}{\epsilon_0}$$

Planar Symmetry

$$\vec{\mathbf{E}} = E(z)\ \hat{\mathbf{z}}$$

$$\Phi = E_p A$$

$$q_{\text{enc}} = \sigma_0 A$$

$$\vec{\mathbf{E}}_p = \frac{\sigma_0}{2\epsilon_0}\ \hat{\mathbf{n}}$$

$$2AE = \frac{q_{\text{enc}}}{\epsilon_0}$$

Conductors

Inside a conductor: $\vec{\mathbf{E}}_{\text{net}} = \vec{\mathbf{0}}$

just above surface: $E = \dfrac{\sigma}{\epsilon_0}$

Other

Field of infinite wire:

$$\vec{\mathbf{E}}(z) = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{z}\ \hat{\mathbf{k}}$$

Field of infinite plane

$$\vec{\mathbf{E}} = \frac{\sigma}{2\epsilon_0}\ \hat{\mathbf{k}}$$

1-dim (wire), Line charge density λ

2-dim (plate), Surface charge density σ

3-dim (sphere), volume charge density ρ