Fractulum Mathathon	Marks obtained ↓
March 21, 2022 Total questions: **12** Total points: **12**	
Team Name: Time: 90+ Minutes	

Instructions: Read carefully before preceding.

1. The Contest limit is 90+ Minutes.
2. Non Programmable calculators are allowed.
3. No books or other aids are permitted.
4. Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.
5. If you have any questions, please raise your hand.
6. When you are told that time is up, stop working.

Question	Points	Score
1	1	
2	1	
3	1	
4	1	
5	1	
6	1	
7	1	
8	1	
9	2	
10	1	
11	1	
12	0	
Total:	12	

1. (1 point) Let $y = x^2 + bx + c$ be a quadratic function. it has only one real root. if b is postive, find $\dfrac{b+2}{\sqrt{c}+1}$.

2. (1 point) Prove that the product of four consecutive natrual numbers cannot be the square of an integer.

3. (1 point) Define $d(n)$ to be the number of divisors of a postivie integer n (including 1, n), show that if

$$n = p_1^{e^2} p_2^{e^2} \dots p_t^{e^t}$$

is the prime factoriztion of n then

$$d(n) = (e_1 + 1)(e_2 + 1) \dots (e_t + 1).$$

4. (1 point) Find all pairs of integers (x, y) that satisfy the equation

$$xy + 10x - 10y = 131$$

5. (1 point) What's the product of the real roots of the equation

$$x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$$

6. (1 point) What's the largest integer n for which $n^3 + 100$ is divisible by $n + 10$?

7. (1 point) When n standard six-sided dice are rolled, the probability of obtaining a sum of 2022 is greater than zero, and the same as the probability of obtaining a sum of S. what's the smallest possible value of S?

8. (1 point) Prove that for all possible integers n

$$1 \times \binom{n}{1} + 2 \times \binom{n}{2} + 3 \times \binom{n}{3} + \cdots + n \times \binom{n}{n} = n \times 2^{n-1}.$$

9. (2 points) Prove that

$$\frac{\sin x}{x} = \prod_{n=1}^{\infty} \cos(\frac{x}{2^n}).$$

10. (1 point) Let $f_0(x) = (\sqrt{e})^x$ and recursively define $f_{n+1}(x) = f'_n(x)$ for all integers $n \geq 0$. Compute $\sum_{i=0}^{\infty} f_i(1)$

11. (1 point) Prove that consecutive Fibonaaci numbers are relativley prime.

12. (0 points) Written on a blackboard is the polynomial: $x^2 + x + 2022$. Mona and Ibrahim take turns alternatively (starting with Mona) in the following game:

    - During her turn, Mona either increases or decreases the coefficient of x by 1.
    - During his turn, Ibrahim either increases or decreases the constant coefficient by 1.

    If at any time the polynomial on the blackboard has an integer root, Mona wins. Does Mona have a winning strategy? If yes, describe such strategy. If not, show how Ibrahim can play in such a way to prevent Mona from winning.