Citations	From References: 0	From Reviews: 0

MR4166627 31E05 05C50 31C20 37F10 47A75 58J50
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Spectral decimation of the magnetic Laplacian on the Sierpinski gasket: solving the Hofstadter-Sierpinski butterfly. **(English summary)**

Comm. Math. Phys. **380** (2020), *no. 1,* 187–243.

On an electrical network $G=(V,E)$ with conductance c, given a unitary connection ω, which can be interpreted as a complex number $\omega_{xy}=e^{2\pi i\theta}$ on each edge (θ is named the magnetic flux) such that $\omega_{xy}=\overline{\omega_{yx}}$, the magnetic Laplacian is defined as

$$\mathcal{L}^{\omega}_{(G,c)}u(x)=\sum_{y\sim x}c_{x,y}(u(x)-w_{xy}u(y)),\quad \forall\, u\in l(V).$$

In this paper, the authors establish the spectrum of the magnetic Laplacian, as a set of real numbers with multiplicities, on the Sierpiński gasket graph where the magnetic fluxes (product of the unitary connection along the simple cycle) equal α through the upright triangles, and β through the downright triangles. This provides a quantitative answer to a question of J. V. Bellissard [in *Ideas and methods in quantum and statistical physics (Oslo, 1988)*, 118–148, Cambridge Univ. Press, Cambridge, 1992; MR1190523] on the relationship between the dynamical spectrum and the actual magnetic spectrum.

The computation is based on the spectral decimation method [M. Fukushima and T. Shima, Potential Anal. **1** (1992), no. 1, 1–35; MR1245223], but the authors also make important improvements in that the model considered in the present paper involves irrational decimation functions. *Shiping Cao*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.