Answer FOUR questions

1. (a) Show how the second-order linear differential equation:

$$\frac{d^2x(t)}{dt^2} + K_1\frac{dx(t)}{dt} + K_2x(t) - K_3u(t) = 0$$

with K_1, K_2, K_3 being constants, can be converted into the state-space form:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\,\mathbf{x}(t) + \mathbf{b}\,u(t)$$

with $\mathbf{x}(t)$ a 2×1 vector derived from $x(t)$, $\mathbf{A}$ a 2×2 matrix of constants, and $\mathbf{b}$ a 2×1 vector of constants multiplied by the system input, $u(t)$.

[5 marks]

(b) Consider the mechanical system of Figure 1.1. Initially, there are no external forces and the system is in equilibrium (zero-valued initial conditions, we ignore gravity from the problem). We now apply a force $f(t)$ downwards to the mass M. Find the transfer function from the applied force to the displacement, $x_1(t)$, of the mass; that is, find $X_1(s)/F(s)$.

[10 marks]

(c) Consider the closed-loop, negative unit feedback system with forward path:

$$G(s) = \frac{K}{s(s+5)}$$

Find the range of K for which the system is under-damped, critically damped, over-damped. For the system to have minimum settling time, is the value of K relevant? Justify your answer.

[10 marks]

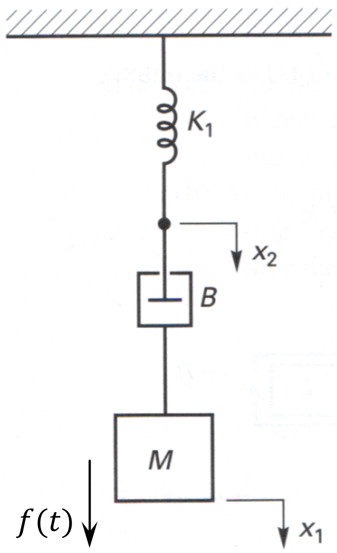

Figure 1.1. Mechanical system.

2. (a) Explain the difference between delay and lag in the response of a control system. Illustrate your answer by a simple example.

[5 marks]

(b) Consider the closed-loop system with negative unit-feedback and forward path:

$$G(s) = \frac{K}{s(s+1)(s+3)}$$

Find the steady-state error for unit-step input, assuming the system is stable. Find the range of K for which the system is stable. Find the location of all roots of the characteristic equation of the system for the value of K allowing for marginal stability.

[10 marks]

(c) Figure 2.1 shows the block diagram of the servo-control system for one of the axes of a digital plotter. The input θ_r is the output of a digital computer and the output θ_p is the position of the servomotor shaft. It is assumed that the pen-positioning system connected to the motor shaft is rigid (no dynamics) within the system bandwidth. When $K_d = 1$, find the range of K_v values for which the system is stable. When $K_v = 0.3$, find the range of K_d values for which the system is stable.

[10 marks]

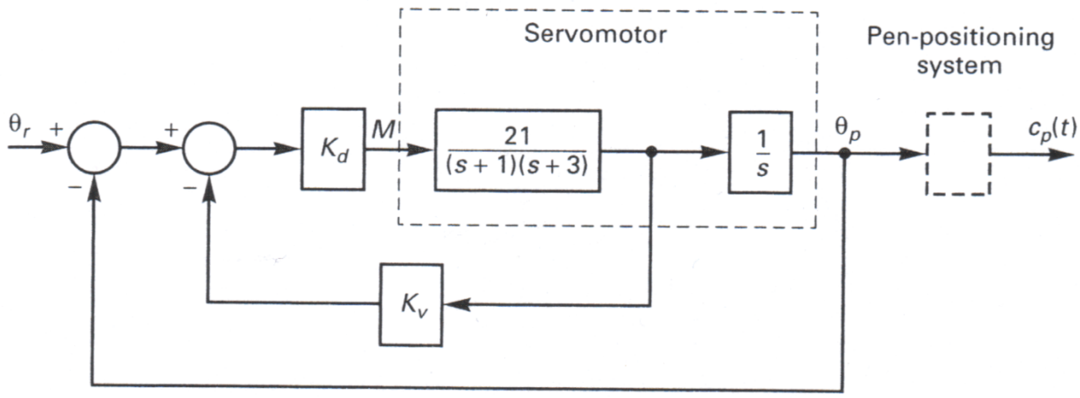

Figure 2.1. Block diagram of the servo-control system for one of the axes of a digital plotter.

3. (a) Explain what is meant by gain margin and phase margin in Bode diagrams, illustrating your answer by an example.

[5 marks]

(b) Show that the transfer function for the car suspension system shown in Figure 3.1 is:

$$\frac{X_1(s)}{F(s)} = \frac{Bs + K_1}{M_1M_2s^4 + B(M_1 + M_2)s^3 + [K_1(M_1 + M_2) + K_2M_1]s^2 + K_2Bs + K_1K_2}$$

where x_1 is the deviation of mass M_1 from its equilibrium and f is the force on mass M_2. *Hints*: useful intermediate results are:

$$X_1(s) = \frac{Bs+K_1}{M_1s^2+Bs+K_1}X_2(s) \quad \text{and} \quad X_2(s) = \frac{Bs+K_1}{M_2s^2+Bs+K_1+K_2}X_1(s) + \frac{1}{M_2s^2+Bs+K_1+K_2}F(s).$$

It is not necessary to include the force due to gravity in the analysis. The force due to gravity determines the equilibrium position but does not influence deviations from the equilibrium.

[10 marks]

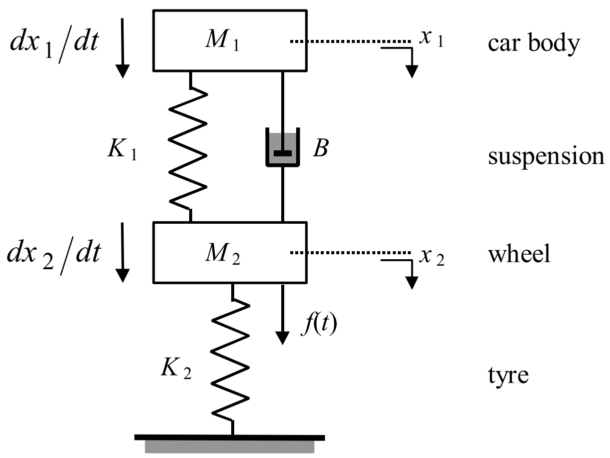

Figure 3.1 Simplified mechanics of car suspension system.

(c) Figure 3.2 shows the frequency responses for a particular car suspension system as B and K_2 vary. The viscosity factor of the fluid in the damper increases as the system ages. Under-inflated tyres result in decreased K_2. Using the features of the plots, describe the effect of:

(i) ageing,

(ii) an under-inflated tyre,

on the quality of the ride in the car.

[10 marks]

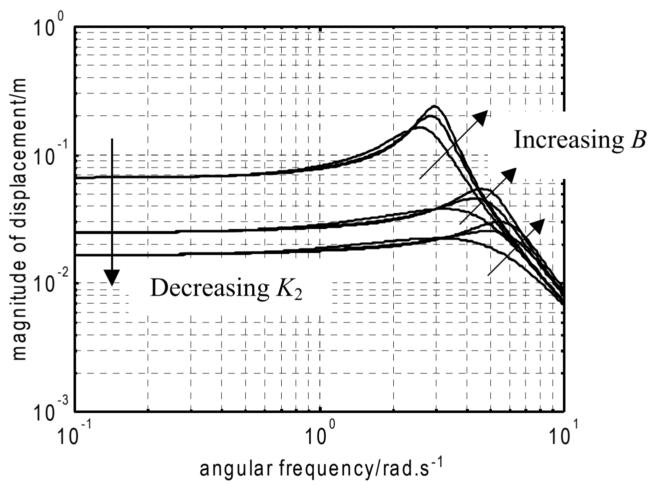

Figure 3.2. Frequency response of car suspension system.

4. (a) Explain what is a class-zero, a class-one and a class-two system. Does the class-two system present problems with respect to stability? Why?

[5 marks]

(b) Consider the closed-loop system with negative unit feedback and forward path:

$$G_1(s) = \frac{4(s+3)}{s^2 - 2s + 10}$$

Is the closed-loop system stable? Is the system stable when opening the loop? Repeat your analysis of open-loop and closed-loop stability when the forward path is:

$$G_2(s) = \frac{4(s-3)}{s^2 + 2s + 10}$$

[10 marks]

(c) Consider the system of Figure 4.1. Let

$$G_1(s) = \frac{10K}{5s+1}, G_2(s) = 1, H(s) = 1.$$

Suppose that $d(t) = 5u(t)$, with $u(t)$ the unit step function. Find the value of K that will limit the steady-state component in $c(t)$ due to $d(t)$ to 1% of the value of $d(t)$. With that value of K, if $r(t) = 10u(t)$, find the steady-state error in $c(t)$ due to $r(t)$.
Hint: Use the principle of superposition when answering this question, i.e. ignore one input to the system when studying the effect of the other input to the output value in steady-state.

[10 marks]

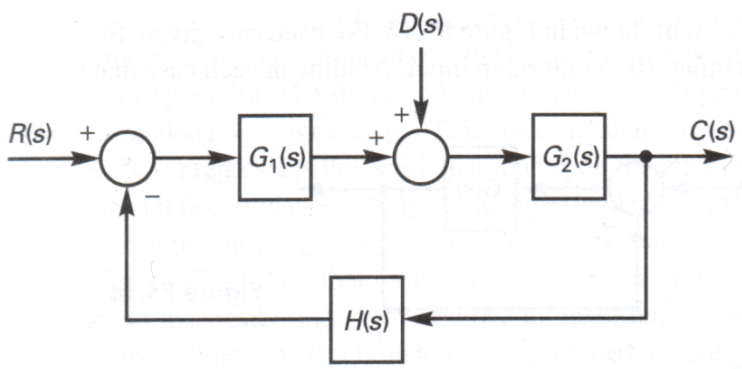

Figure 4.1. System with two inputs.

5. (a) State what quantities are represented by the graph axes of a Nichols' chart (i.e. what is being plotted) and state what additional variable parameterises the curve.

[5 marks]

(b) The generic form of a transfer function for a second order system is:

$$G(s) = K\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For the case when $0 < \zeta < 1$, show the factorised form of the denominator of the transfer function, where the two poles are presented explicitly. Draw a clear diagram showing the positions of the system poles in the complex plane. Explain the geometrical interpretation of the parameters ω_n and ζ using your diagram.

[10 marks]

(c) Describe the feedback control structures known as *ideal-PID*, *PI-D* and *I-PD*. For each one give a block diagram and the controller transfer function. Explain what considerations govern the choice between them.

[10 marks]

6. (a) You are given the controller transfer function:

$$G(s) = \frac{1.45s + 0.35}{s + 0.07}$$

Answer the following:

(i) Find the dc gain of the controller.
(ii) Find the high-frequency gain of the controller.
(iii) Is the controller a phase-lead or a phase-lag compensator? Why?

[5 marks]

(b) Consider the following systems being in the forward path of two separate closed-loop negative unit feedback systems:

$$G_1(s) = \frac{K}{s^2(s+1)(s+3)}$$
$$G_2(s) = \frac{s-1}{s(s-2)(s-5)(s-7)}$$

Find the steady-state error of for each of the closed-loop systems to a unit-ramp input.

[10 marks]

(c) Given the compensator transfer function:

$$G_p(s) = \frac{0.294s + 0.0706}{s + 0.0706}$$

Answer the following:

(i) Find a PI compensator such that both the high-frequency gain and the position of the zero is the same as for $G_p(s)$.
(ii) Give the DC gain of both compensators.
(iii) Is $G_p(s)$ a phase lead or a phase lag compensator? Justify your answer.
(iv) Discuss the differences between the two compensators (if any) at low (close to DC) frequencies and high frequencies.

[10 marks]

END OF PAPER