Answer FOUR questions

1. (a) The generic form of the transfer function of a second order system is:

$$G(s) = K\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

(i) For the case when $0 < \zeta < 1$, show that the transfer function may also be written as shown below, where $\omega_t = \omega_n\sqrt{1-\zeta^2}$:

$$G(s) = \frac{K\omega_n^2}{(s + \zeta\omega_n - j\omega_t)(s + \zeta\omega_n + j\omega_t)}$$

(ii) Draw a diagram showing the positions of the system poles in the complex plane. Explain the geometrical interpretation of the parameters ω_n and ζ.

[5 marks]

(b) If $0 < \zeta < 1$ show that the partial fraction expansion of the response to a unit step input of the above system is:

$$X(s) = \frac{K}{s} - \frac{K}{2\sqrt{1-\zeta^2}}e^{-j\theta}\frac{1}{(s + \zeta\omega_n - j\omega_t)} - \frac{K}{2\sqrt{1-\zeta^2}}e^{+j\theta}\frac{1}{(s + \zeta\omega_n + j\omega_t)}$$

where θ is an angle such that $\sin(\theta) = \zeta$.

[10 marks]

(c) The generic form of the step response of a second-order system in the time domain is given by:

$$x(t) = K\left(1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\cos(\omega_t t - \theta)\right)$$

where θ is an angle such that $\sin(\theta) = \zeta$.

Examine whether each of the four statements below is true or false, considering the specific second order system obtained by setting: $K = 2$, $\omega_n = 2$, $\zeta = 0.5$. Show your full working in support of your answer for each case. If a statement is false, determine what the correct value is.

(i) The system will reach a steady state output of 8;
(ii) The frequency of the damped oscillations will be 2 $\text{rad} \cdot \text{sec}^{-1}$;
(iii) The system settling time is 4 seconds;
(iv) Doubling the steady state gain will increase the percentage overshoot.

[10 marks]

2. (a) Illustrate what happens to the signs of the poles of $F(s)$ when the zero is present and when it is not present.

$$F(s) = \frac{s + 1.99}{s(s+1)(s+2)}$$

[5 marks]

(b) A lead-lag controller is used to control a third order system as shown in Figure 2.1.

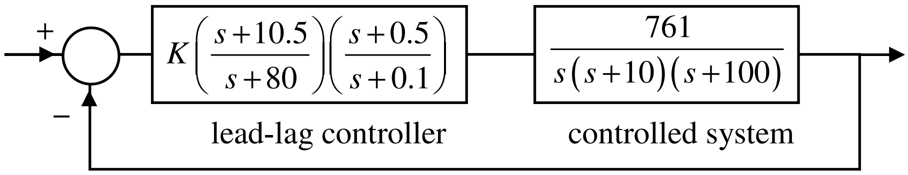

Figure 2.1.

(i) Figure 2.2 shows that the closed loop frequency response of the system of Figure 2.1 was considerably changed by increasing the controller gain from $K = 1$ to $K = 50$. State which gain setting gives better closed loop performance in terms of achieving lower closed loop resonance as well as higher bandwidth (i.e. frequency where the gain is -3dB).

(ii) Answer the same question as for part (i) but now using Figure 2.3, which shows the Nichols plots of the closed loop system of Figure 2.1 for $K = 1$ and $K = 50$.

[10 marks]

(c) Observing that increasing the controller gain K shifts the Nichols plot of the closed loop system of Figure 2.1 upwards, suggest an approximate gain setting for the controller that would decrease the closed loop resonance even further. Explain your suggestion qualitatively based on Figure 2.3. Explain what would happen to the closed loop system response if the gain were increased from $K = 50$ by another $30\,\text{dB}$.

[10 marks]

QUESTION 2 CONTINUES

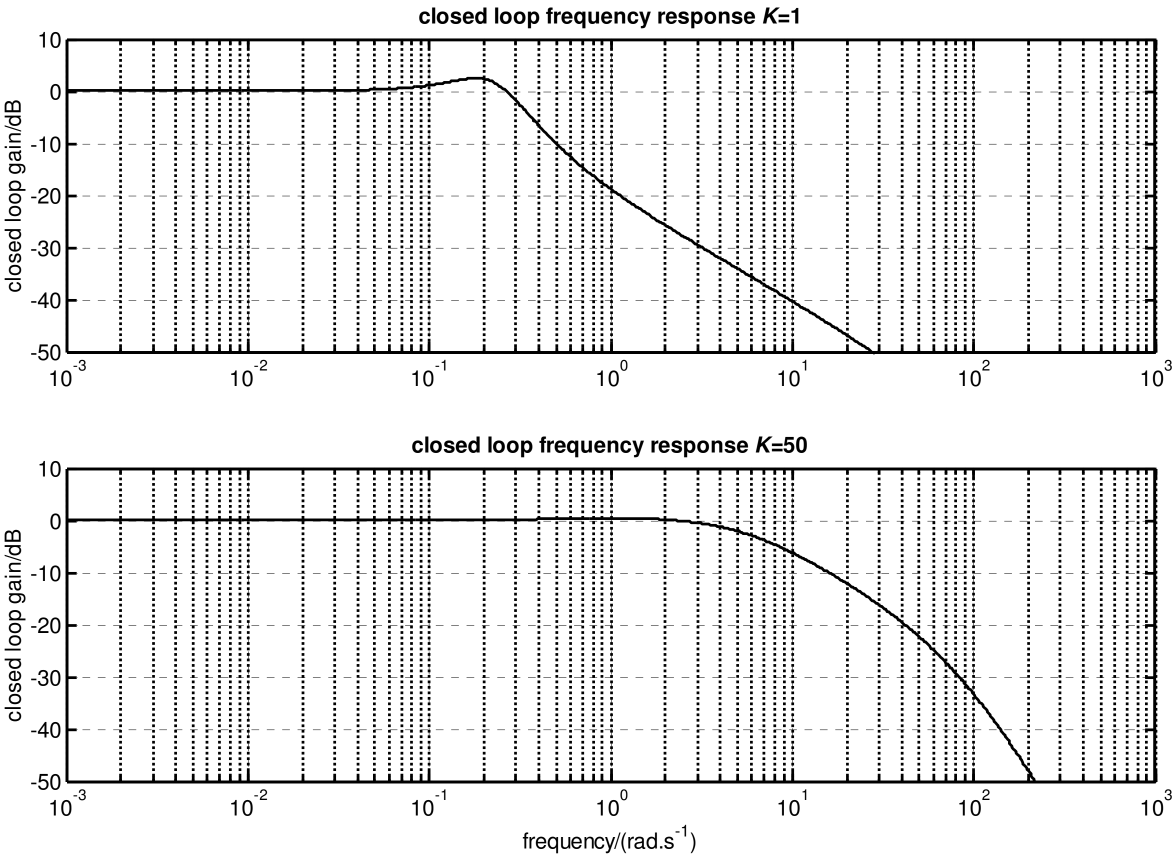

Figure 2.2.

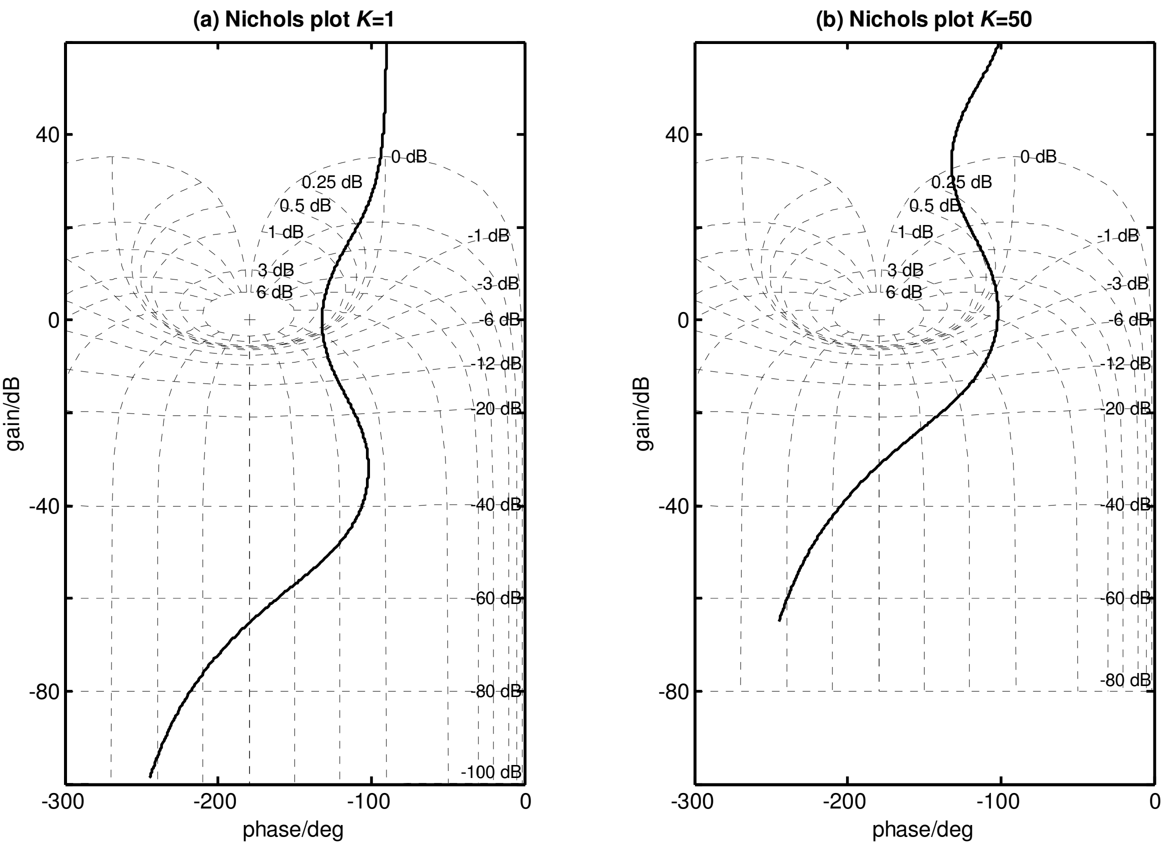

Figure 2.3.

END OF QUESTION 2

3. (a) A plant $G(s)$ is to be controlled by a PI controller $C(s)$ in a unity feedback configuration (shown in Figure 3.1), where:

$$G(s) = \frac{1}{s^2 + 2s + 1}$$

and

$$C(s) = K_p\left(1 + \frac{1}{\tau_i s}\right)$$

Using the Routh Array, or otherwise, determine whether the closed loop will be stable when $K_p = 10$ and $\tau_i = 5$.

[5 marks]

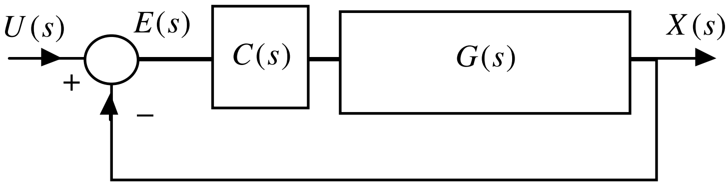

Figure. 3.1.

(b) Devise a numerical algorithm for the digital implementation of the controller output signal $C_{\text{out}}(s) = C(s)(U(s) - X(s))$ when $K_p = 10$ and $\tau_i = 5$s. The algorithm should compute a control increment $\Delta c_{\text{out}}(t)$ based upon samples of the set point $u(t)$ and system output $x(t)$ using a sampling interval of 0.1s.

[10 marks]

(c) Explain the practical issue of integral windup when the controller error is large.

[10 marks]

CONTINUED

4. (a) For a system having non-linear dynamics of the form

$$\frac{dx}{dt} + f(x) = u(t)$$

show that if the input u is constant then the linearised dynamics can be written as:

$$\frac{d\Delta x}{dt} + g \times \Delta x = 0$$

where

$$g = \left.\frac{\partial f}{\partial x}\right|_{x=x_0}$$

where x_0 is the operating point (i.e. the steady state value of the output) and Δx is the deviation from the operating point. State and explain the important condition that must be fulfilled by the quantity g as defined in the expression above.

[5 marks]

(b) In the case where the input is not constant but instead deviates either side of the input operating point value then the input may be expressed as $= u_0 + \Delta u$. In that case, show the linearised dynamics are

$$\frac{d\Delta x}{dt} + g \times \Delta x = \Delta u$$

[10 marks]

(c) A nonlinear system has the following dynamics:

$$\frac{dx}{dt} - 2e^{-4x} = u$$

(i) Find the steady state x_0 when $u = -1$;

(ii) Show that the time constant of the linearised system is $0.25s$.

[10 marks]

5. (a) State what features of a system are represented by a curve on a Nichols plot and explain clearly what the Nichols iso-contours are used for.

[5 marks]

(b) Consider the *closed loop* system with negative unity feedback and forward path:

$$G(s) = \frac{K}{s(s+1)(s+3)}$$

Find the range of K for which the system is stable. Assuming K is selected from that range, determine the steady state error for a unit step input.

[10 marks]

(c) Find the final value for the *closed loop* response (with negative unity feedback) to a unit step input for:

(i) the system with forward path: $\frac{10}{s(1+s)}$;

(ii) the system with forward path: $\frac{1}{(1+s)}$;

(iii) the system with forward path: $\frac{5(2-s)}{(1+s)(s^2+s+1)}$; determine if this system is stable.

[10 marks]

6. (a) Describe the concept of the root locus method for predicting how a system will behave when negative feedback is applied.

[5 marks]

(b) Figure 6.1 shows the root locus for the unity negative feedback system whose forward path is

$$G_o(s) = \frac{10K}{s(s+1)(s+10)}$$

(i) By measurement from the root locus diagram, or otherwise, determine the value of K at which the closed loop system becomes unstable.

(ii) Suppose the system represents the dynamics of the position control motor and body of an astronomical telescope, which rotates as it tracks a feature in the night sky. The telescope has to make a total of 2160 steps in 1440 minutes (each step is one $1/6$th of a degree and the telescope rotates 360° per 24 hours, or per 1440 minutes). Therefore it stays at each position for 0.667 minutes or 40 seconds.
If the dominant *closed loop poles* of the closed loop system with forward path given by $G_o(s)$ are placed at $s = -1 \pm j$, discuss whether this is a suitable choice in terms of convergence time to steady state for the telescope positioning system.

[10 marks]

(c) Use a pole placement method to design a phase lead controller to be inserted into the forward path in order to obtain dominant closed loop poles at $s = -1 \pm j$.
Hints: Don't forget to calculate a value for the gain K as well as selecting the location of the zero and pole of the controller. Also note that there are many possible positions for the pole/zero placement of the controller (i.e. the solution is not unique).

[10 marks]

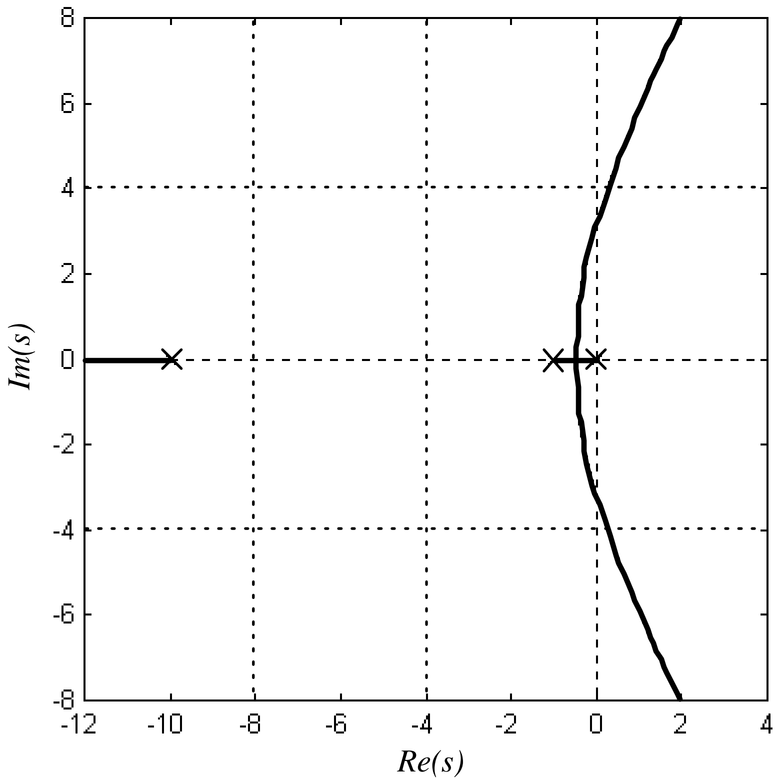

Figure 6.1.

END OF PAPER