# UNIVERSITY COLLEGE LONDON

## EXAMINATION FOR INTERNAL STUDENTS

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| MODULE CODE | : | **ELEC3005** |
| ASSESSMENT PATTERN | : | **ELEC3005A** |
| MODULE NAME | : | **Digital Signal Processing and Systems** |
| DATE | : | **06-May-09** |
| TIME | : | **10:00** |
| TIME ALLOWED | : | **3 Hours 0 Minutes** |

Answer *FOUR* questions

1. (a) Name and explain the two stages in signal sampling.

[5 marks]

(b) Figure 1.1 shows a RLC circuit.

(i) Show the transfer function $H(s)$ of the system is

$$H(s)=\frac{1}{s^2LC+sRC+1}$$

(ii) Express the un-damped resonant frequency ($\omega_o$) and the damping factor ($\zeta$) in terms of $R$, $L$ and $C$ for the second order system in (i).

(iii) Find the poles of the system.

(iv) Comment on the nature of the system based on the values of the damping factor ($\zeta$).

[10 marks]

(c) Given the following transfer function,

$$H(s)=\frac{s+20}{s+2000}$$

Find

(i) the gain,

(ii) the zero(s), and

(iii) the pole(s) of the system.

Moreover,

(iv) sketch both amplitude and phase of the Bode plot of the system.

[10 marks]

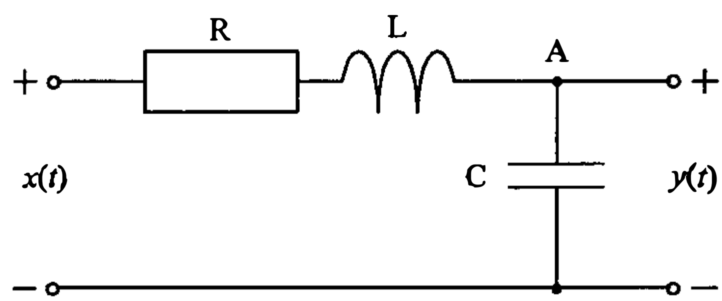


Figure 1.1

2. (a) State, for a complex signal $f_i(t)$, the condition of orthonormality.

[5 marks]

(b) Find the first five responses of the sequence {1, 2, 3}, which is applied to an FIR filter with a transfer function

$$1 + 0.5z^{-1} + 0.25z^{-2},$$

given the discrete-time equation is

$$y(n) = \sum_{k=0}^{n} x(k)h(n-k)$$

[10 marks]

(c) A digital filter is described by the transfer function

$$H(z) = \frac{(z+1)(z^2 - 0.6489z + 1.1025)}{(z^2 + 1.3435z + 0.9025)}$$

(i) Show that the filter is stable.

(ii) Sketch the frequency response.

(iii) Find the maximum gain of the filter in dB.

[10 marks]

Fig. 2.1

3. (a) Explain why spectrum leakage is hard to prevent in practical cases and describe how windowing can minimise the spectrum leakage effect.

[5 marks]

(b) Compare the square window with the Hanning window in the following aspects,

(i) bandwidth,
(ii) peak sidelobe,
(iii) roll-off, and
(iv) processing loss.

[8 marks]

(c) The discrete Fourier transform (DFT) of a sampled signal $x(n)$ is given by:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k \frac{n}{N}}$$

Show that the DFT is

(i) conjugate symmetric about 0 Hz and
(ii) periodic with period $N$.

[8 marks]

(d) Given

$x\{n\} = \{j4, -2\sqrt{2} + j2\sqrt{2}, -4, -2\sqrt{2} - j2\sqrt{2}, -j4, 2\sqrt{2} - j2\sqrt{2}, 4, 2\sqrt{2} + j2\sqrt{2}\}$,

evaluate the first four components of the DFT of $x(n)$

[4 marks]

4. (a) What is meant by the 'canonical form' of a digital filter implementation and why is it used in digital filter design?

[3 marks]

(b) Consider the causal linear time invariant (LTI) system implemented as shown in Figure 4.1. Determine the difference equation relating the output, $y[n]$, to the input, $x[n]$.

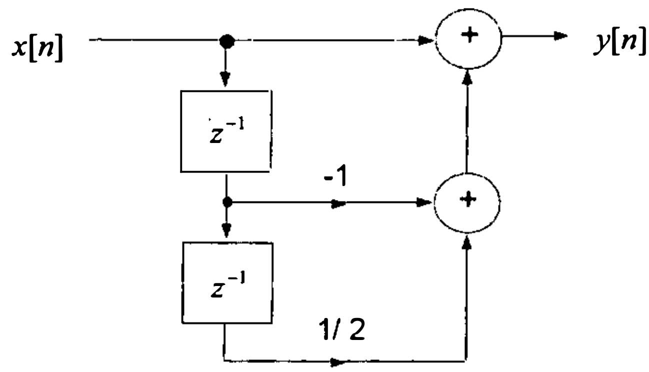


Figure 4.1. Causal LTI system.

[8 marks]

(c) A similar causal LTI system implemented via a lattice structure is shown in Figure 4.2. Determine the difference equation relating the output, $y[n]$, to the input, $x[n]$.

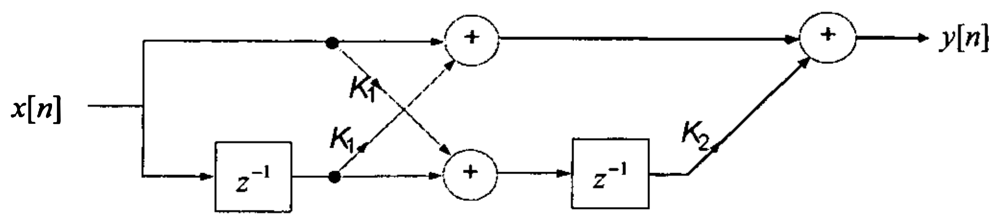


Figure 4.2. Lattice structure implementation of the causal LTI system.

[8 marks]

(d) Determine the numerical values of $K_1$ and $K_2$ so that the difference equations of the systems of Figure 4.1 and Figure 4.2 match.

[6 marks]

5. Consider a lossless image codec system processing the input image line-by-line. Each input image line is represented as input signal $x[n]$ and the result of the codec processing is output signal $y[n]$. The signal analysis and synthesis stages used within the codec are shown in Figure 5.1 and they are indicated by the dashed areas.

Consider $H_0(z) = 1 + z^{-1}$ and $H_1(z) = 1 - z^{-1}$ (both are half-band pass filters) when answering the following questions.

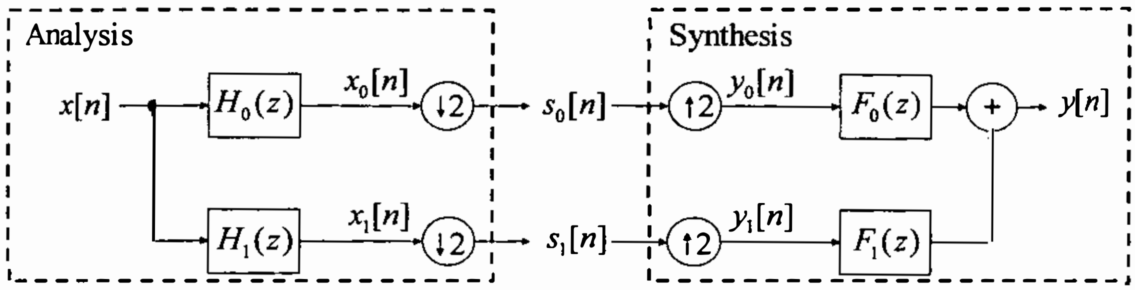


Figure 5.1. Signal analysis and signal synthesis within the lossless image codec.

(a) What is 'multi-rate processing' and why is it used? Is the analysis part in Figure 5.1 a multi-rate processing system, and if so, why?

[3 marks]

(b) In Figure 5.1:

$$s_i[n] = x_i[2n],\ i = 0,1.$$

Determine the $Z$-domain expression of $s_0[n]$ and $s_1[n]$.

[8 marks]

(c) In Figure 5.1:

$$y_i[n] = \begin{cases} s_i[\frac{n}{2}], & n \text{ even} \\ 0, & n \text{ odd} \end{cases},\ i = 0,1.$$

Determine the $Z$-domain expression of $y[n]$ in function of the $Z$ transform of the input $x[n]$.

[6 marks]

(d) Determine the $Z$ transform of the synthesis filters, $F_0(z)$ and $F_1(z)$, so that $y[n] = x[n-1]$, i.e. the output of the overall system is equal to the input delayed by one time unit for all possible signals.

[8 marks]

6. (a) What are the advantages and disadvantages of digital FIR filters in comparison to digital IIR filters?

[5 marks]

(b) For the following three parts of the question, consider the causal linear time invariant (LTI) system described by the difference equation:

$$y[n] = j \cdot y[n-1] + x[n] - x[n-4], \text{ where } j = \sqrt{-1}.$$

(i) Determine the $Z$-domain transfer function of the system.

[7 marks]

(ii) Find the output $y[n]$ for the input $x[n] = 4 + \sin(\pi n)$, $-\infty < n < \infty$.

[5 marks]

(iii) Determine the impulse response of the system. Is the system FIR or IIR?

[8 marks]



END OF PAPER