4 A signal sampled at 18kHz is to be filtered using three IIR filters in parallel. The first filter is low pass with a cut-off at 3kHz. The second is a high pass with a cut-off at 6kHz. The third is a band pass with cut-offs at 3 and 6 kHz. The digital filters should be designed using a bilinear transformation and based on a first order Butterworth filter with the following transfer function:

$$H(s)=(1+s)^{-1}$$

(a) Find the analogue filter cut-off frequency prior to warping for the low pass filter.

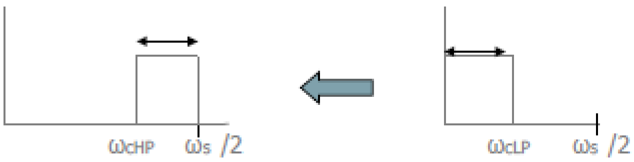
[5 marks]

(b) Apply the bilinear transformation to obtain the digital low pass filter.

[5 marks]

(c) It is known that a high pass filter can be obtained from a low pass filter using the following relationship:

$$\omega_{CHP} = \omega_s/2 - \omega_{CLP}$$



$$H_{HP}(z) = H_{LP}(z)\Big|_{z \to -z}$$

Then, find the transfer function of the high pass filter.

[5 marks]

(d) Now, it is given that a band pass filter can be obtained from a low pass filter using the following relationship:

$$z_{LP}^{-1} = \frac{-z^{-1}(z^{-1}-\alpha)}{(1-\alpha z^{-1})}$$

$$\omega_{clp} = \omega_{cu} - \omega_{cl}$$

$$\alpha = \frac{\cos[\frac{\pi(\omega_{cu}+\omega_{cl})}{\omega_s}]}{\cos[\frac{\pi(\omega_{cu}-\omega_{cl})}{\omega_s}]}$$

Then, find the transfer function of the band pass filter.

[10 marks]

5 Design a linear-phase low pass FIR filter for a system with a 4 kHz sample rate, using an ideal brick wall frequency response with a pass band power gain of 3 dB and a cut-off frequency of 400 Hz. The filter should achieve a stop-band attenuation of at least 40 dB at all frequencies above 2 kHz. The design is obtained by the use of Hanning windowing coefficients

$$0.5\left(1 - \cos\frac{2n\pi}{N}\right)$$

which, with the order of N=2M+1 and sampling interval Δt, has the properties:

Window	Transition band (Hz)	Stopband rejection (dB)
Rectangular	$\frac{1}{N\Delta t}$	**21**
Hanning	$\frac{3.1}{N\Delta t}$	**44**

(a) Evaluate the transition band.

[2 marks]

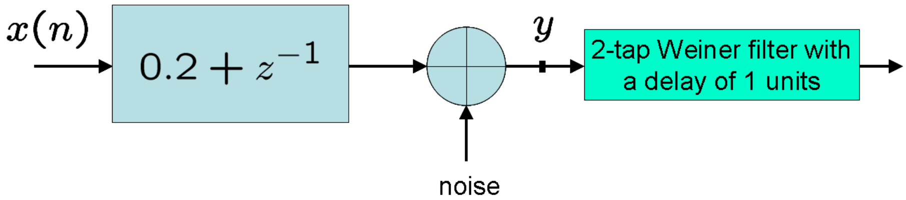
(b) Determine the required minimum number of window weighting coefficients.

[3 marks]

(c) Obtain the resultant FIR filter coefficients.

[20 marks]

6 A communications channel has a transfer function $0.2+z^{-1}$. The signal x(n) is zero mean and white with variance (or power) of 1.5. The additive noise is white with zero mean and variance of 0.5 and is uncorrelated with the signal. The objective is to design a two-tap Weiner filter equaliser with a lag of 1 to minimise the mean square error (MSE) of the signal. The design is illustrated in the following diagram:



(a) Find the power spectral density of the signal y and the autocorrelation matrix.

[10 marks]

(b) Find the cross-correlation vector of the signal y and the reference signal.

[5 marks]

(c) Obtain the Weiner filter coefficients.

[10 marks]