Answer FOUR questions

1. (a) Prove that the transfer function, $H_d(s)$, of a system with a time delay, T_d, is related to the transfer function, $H(s)$, of the original (delay-free) system by:

$$H_d(s) = e^{-sT_d} H(s)$$

[5 marks]

(b) Consider the vertical mechanical system shown in Figure 1.1.

(i) Find the two differential equations at the points where displacements $x_1(t)$ and $x_2(t)$ are indicated in the figure. The system is excited only by initial conditions.

[7 marks]

(ii) A force $f(t)$ is applied downwards to the mass M. Find the transfer function from the applied force to the displacement, $x_1(t)$, of the mass; that is, find $X_1(s)/F(s)$.

[8 marks]

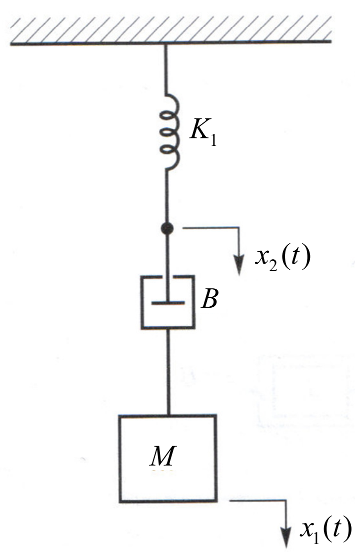

Figure 1.1. Vertical mechanical system.

(c) Draw the Bode plot for the amplitude of the following transfer function:

$$G(s) = \frac{1}{s(s+1)(s+10)}$$

explaining the changes in the each part of your drawing incurred by each factor of the transfer function.

[5 marks]

2. (a) You are given the following measurements of the response, $c(t)$, of the plant system of Figure 2.1 under a unit step input, $u(t)$:
   - At $t = 0.5$: $c(t) = 1.633$
   - At $t = 1.0$: $c(t) = 2.433$

   Find the unknown parameters, K and τ, of the transfer function of the system.

   [10 marks]

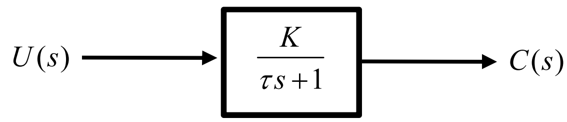

Figure 2.1. First order plant system.

(b) Find the mathematical expression of the final value of the closed-loop system of Figure 2.2 when the input is a unit step function.

[5 marks]

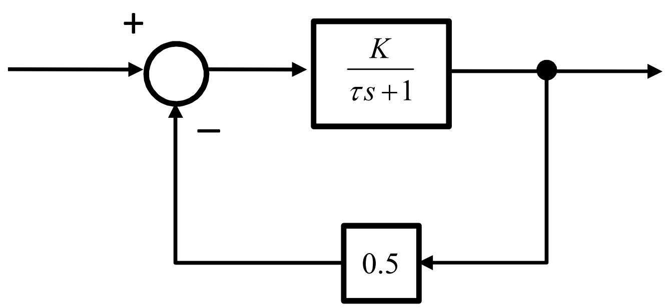

Figure 2.2. Closed-loop system.

(c) Draw a block diagram of the ideal PI-D structure for an industrial controller. Explain why PI-D should generally be used in preference to the PID structure.

[10 marks]

3. Consider a field controlled d.c. motor whose transfer function is:

$$G(s) = \frac{50}{s(s+1)(s+10)}$$

It is to be driven by a harmonic input $u(t) = \cos\omega t$.

(a) By writing $u(t)$ as a sum of two complex exponential signals, show:

$$U(s) = \frac{s}{(s - j\omega)(s + j\omega)}$$

[5 marks]

(b) Plot the pole-zero pattern for the output $X(s) = G(s)U(s)$ on the complex plane.

[5 marks]

(c) Find the partial fraction expansion for the output signal $X(s)$ and the time-domain expression of the output signal, $x(t)$, showing all intermediate derivations leading to the final results.

[15 marks]

4. Consider the system shown in Figure 4.1. For each case given, find the steady-state error for a unit step input and a unit ramp input. Assume in each case that the closed-loop system is stable.

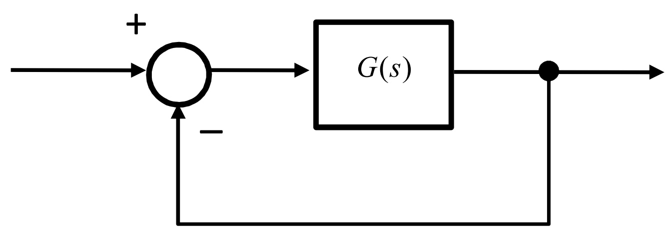

Figure 4.1. Closed-loop system.

(a) $G(s) = \dfrac{10}{(s+1)(s+3)}$

[5 marks]

(b) $G(s) = \dfrac{10}{s(s+1)(s+6)}$

[5 marks]

(c) $G(s) = \dfrac{7(s+2)}{s^2(s+6)}$

[5 marks]

(d) $G(s) = \dfrac{6s^2+2s+10}{s(s^2+4)}$

[5 marks]

(e) If the closed-loop transfer function of the system of Figure 4.1 is $G_c(s) = \dfrac{10}{(s+1)(s-2)}$, find the open-loop transfer function.

[5 marks]

5. (a) Figure 5 shows the block diagram for an aircraft automatic landing system in which the output is the direction in which the plane is heading. Two disturbances are present. One is a disturbance from the wind and the other represents noise from the radar.
Write down the closed-loop transfer function, $G_w(s)$, representing the effect of the wind disturbance on the control loop output, and the closed-loop transfer function, $G_R(s)$, representing the effect of radar noise on the control loop output, where:

$$G_w(s) = \frac{X(s)}{W(s)} \text{ and } G_R(s) = \frac{X(s)}{R(s)}.$$

[8 marks]

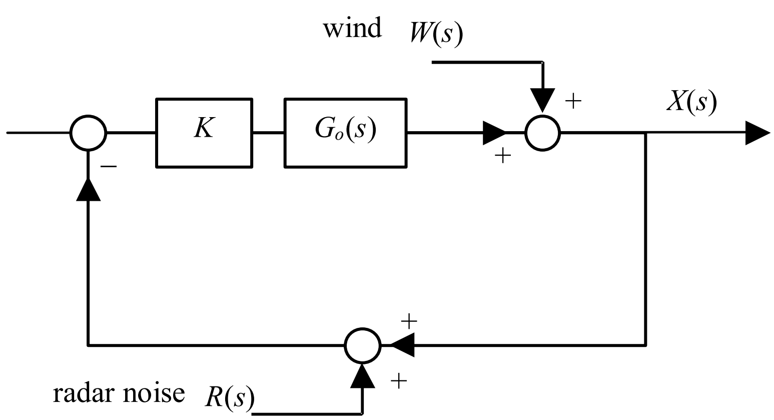

Figure 5.1. Block diagram of an aircraft automatic landing system.

(b) Explain the problem that can arise when a loop is put into AUTO mode after it has been previously running in a mode in which the operator sets the control signal manually. Provide appropriate diagrams showing time trends of the relevant signals. How does an industrial control algorithm handle the problem?

[9 marks]

(c) The Cohen-Coon tuning rules give formulae for the parameters K, τ_1 and τ_2. What experiment needs to be done to generate the data from which these parameters may be calculated? Why might the plant manager raise objections to the loop tuning activities of the control engineer?

[8 marks]

6. (a) The transfer function below is to be used as the forward path of a unity negative feedback system. Determine the value of K at which the closed-loop system will become unstable.

$$KG_o(s) = K \times \frac{10}{s(s+1)(s+5)}$$

[7 marks]

(b) The forward path transfer function below has the same poles as that in part (a), but it also has a zero. By consideration of the form of the root locus plot, or otherwise, determine for what values of K, if any, the unity negative feedback closed-loop system is unstable.

$$KG_o(s) = K \times \frac{10(s+2)}{s(s+1)(s+5)}$$

[5 marks]

(c) One of the points:

$$s = -2.7 \text{ or } s = -1.7$$

is a closed-loop pole for the system of part (b).

(i) Determine which one, and determine the gain, K, needed to locate a closed-loop pole at that point. Show that with the selected value of K there are other closed-loop poles located at, approximately, $s = -1.65 \pm 2.9j$.

(ii) With the determined pole and the determined gain, plot the pole-zero diagram for the closed-loop system. Comment on the performance of the closed-loop system in response to a step input.

[13 marks]

END OF PAPER