Answer FOUR questions

1. (a) Derive the state-space representation for the following electrical circuit in matrix form.

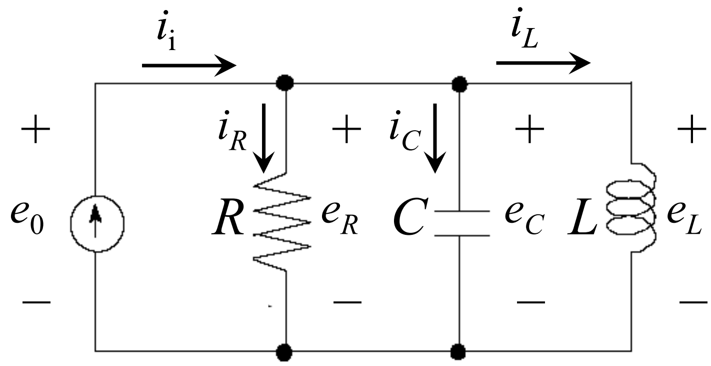

Figure 1.1. Electrical circuit.

[15 marks]

(b) Find the conditions for the time constant τ to guarantee that the closed-loop (with unit feedback) system is stable, when the open-loop transfer function is:

$$G(s) = \frac{10(\tau s + 1)}{20\tau s^3 + (10\tau + 2)s^2 + (1 - 10\tau)s - 1}$$

[10 marks]

2. (a) Explain how the use of partial fractions enables rapid inversion of a Laplace transform. Illustrate your answer by showing how the time domain output of the system $G(s)=\dfrac{1}{s+1}$ is calculated when the input to the system is a unit step.

[6 marks]

(b) Show that the criteria for a point in the s-plane to be a closed loop pole for the system shown in Figure 2.1 are given by the following expressions:

$$\angle(s+2)-\angle s-\angle(s+1)-\angle(s+10)=-\pi \text{ rad} \quad (180°)$$

$$\frac{K|s+2|}{|s||s+1||s+10|}=1$$

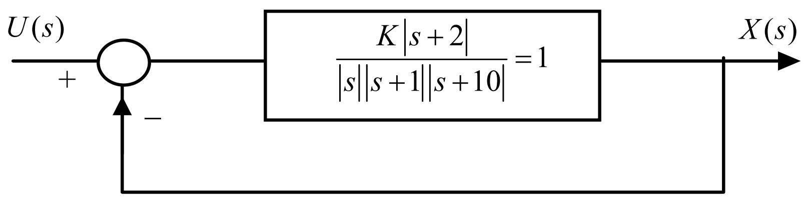

Figure 2.1.

[5 marks]

(c) When $K=25$ the three closed loop poles are at $s=-1.982\pm j1.783$ and $s=-7.035$. Plot the pole zero diagram for the closed loop transfer function. Write down an expression for the closed loop transfer function in rational function form (i.e. with the gain and any zeros in the numerator and poles in the denominator).

[4 marks]

(d) Draw the pole zero diagram for the signal $X(s)$ when the set point is a step input ($U(s)=\dfrac{1}{s}$). What is the steady-state value of x?

[5 marks]

(e) Show that the closed loop step response is given by a time domain expression of the form shown below where A is equal to 1. Do *not* attempt to evaluate the coefficients B, $|C|$ and $\angle C$.

$$x(t)=A+Be^{-7.035t}+2|C|e^{-1.982t}\cos(1.783t+\angle C)$$

[5 marks]

3. (a) State the final value theorem and explain its derivation using an appropriate example.

[8 marks]

(b) The feedback control systems shown in Figure 3.1 have a controlled system $G_o(s)$ together with a lead compensator and, in the case of Figure 3.1(b), a lag compensator. Show that the poles of the closed loop systems are the values of s that satisfy the following polynomial equations. Do not attempt to find the numerical values of the poles (they are given in section (c) of this question).

$s^3 + 9s^2 + 70s + 162 = 0$ for the system in Figure 3.1(a)

$s^4 + 9.1s^3 + 70.9s^2 + 189s + 76.2 = 0$ for the system in Figure 3.1(b)

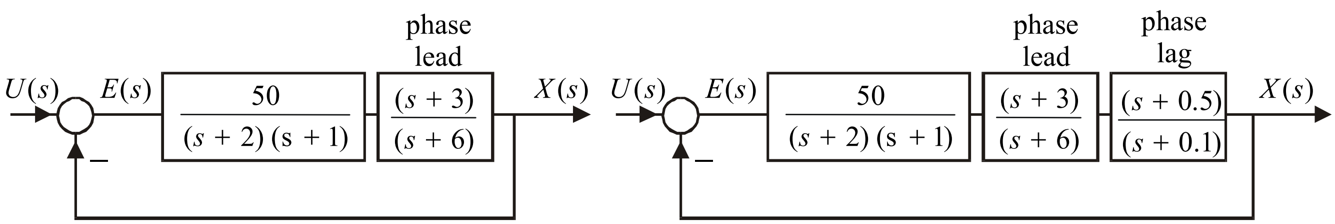

Figure 3.1(a) Figure 3.1(b)

[5 marks]

(c) The closed loop poles of the system are:

$s = -2.93 \pm j6.56$, $s = -3.14$ for the system in Figure 3.1(a)

$s = -2.72 \pm j6.48$, $s = -3.16$, and $s = -0.48$ for the system in Figure 3.1(b)

Find the numerical values of the zeros of the transfer functions $E(s)/U(s)$ for the two systems. Plot pole-zero diagrams for the transfer functions $E(s)/U(s)$ for the two systems. Highlight any closed loop pole and zero that originates from the phase lag compensation.

[5 marks]

(d) If the input is a unit step show that the use of the phase lag compensator gives the benefit of reducing the steady state error by a factor of 4.7 compared to the case when there is no phase lag compensation. What is the disadvantage of the use of the phase lag compensator?

[7 marks]

4. You are given a closed-loop system with unit feedback and open-loop transfer function: $G(s) = \dfrac{K}{s(s+1)(s+2)}$ for $K > 0$.

    (a) For $K_c = 6$, find out the points of the root locus diagram of the closed-loop system that touch the imaginary axis.

    [5 marks]

    (b) Design the root locus diagram for the closed-loop system detailing the steps you take to determine the number of sections of the diagram, the number of the asymptotes and their angles with the real axis, and the breaking points of the diagram.

    [15 marks]

    (c) Find out the value of K such that the poles of the closed-loop system are in the positions: $s_{1,2} = -0.3337 \pm j0.5780$. What is the third pole of the system in this case?

    [5 marks]

5. Suppose that the input to a linear time-invariant system was:

$$u(t) = e^{t} \sin(2t), \quad t \geq 0$$

and this resulted in the output:

$$y(t) = e^{\alpha t} \cos(2t) + e^{t} \sin(2t) + e^{-3t}, \quad t \geq 0$$

All initial conditions are zero.

(a) Let $\alpha = 1$. Show by algebraic manipulation that the transfer function of the system has only one pole at $s = -3$.

[15 marks]

(b) Let $\alpha = 0$. Show by algebraic manipulation that the transfer function of the system has two additional poles on the imaginary axis.

[10 marks]

Note: Useful Laplace transforms:

$$L[e^{\alpha t} \cos(\beta t)] = \frac{s-\alpha}{(s-\alpha)^2 + \beta^2}, \quad L[e^{\alpha t} \sin(\beta t)] = \frac{\beta}{(s-\alpha)^2 + \beta^2}.$$

6. (a) Derive the state-space representation for the following mechanical system in matrix form. State clearly any assumptions you make.
In the figure below, x_1 and x_2 are measured from unstreched spring position.

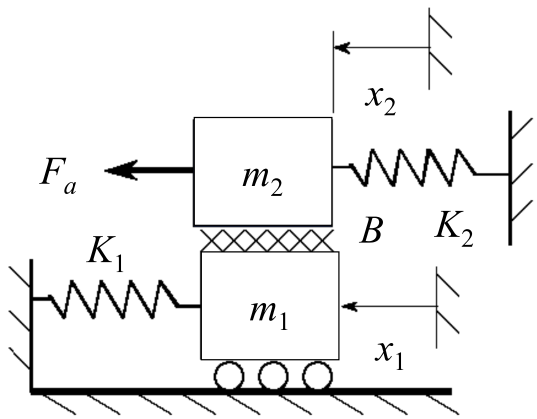

Figure 6.1. Mechanical system. Friction is assumed only between the two masses, with coefficient B.

[15 marks]

(b) Consider the system with transfer function:

$$G(s) = \frac{30s + 30}{s^2 + 7s + 10}$$

Derive the impulse response of the system in the time domain.

[7 marks]

(c) Explain briefly why the locations of the poles and zeros in a system's transfer function affect the response of the system to any input.

[3 marks]

END OF PAPER