Answer *FOUR* questions

1. (a) Statethe condition(s) if two periodic complex signals *f*(*t*) and *g*(*t*) of period *T* are orthogonal to each other.

[4 marks]

(b) In what case(s), Fourier transform is used instead of Fourier series.

[3 marks]

(c) Find the first five responses of the sequence {1, 2, 1}, which is applied to a FIR filter with a transfer function

$$1 + 0.5z^{-1} + 0.25z^{-2},$$

Given the discrete-time equation is

$$y(n) = \sum_{k=0}^{n} x(k)h(n-k)$$

[8 marks]

(d) A digital filter is described by the transfer function

$$H(z) = \frac{(z+1)(z^2 - 0.6489z + 1.1025)}{(z^2 + 1.3435z + 0.9025)}$$

(i) Evaluate all the zeros and poles
(ii) State and explain if the filter is stable.
(iii) Sketch the frequency response.
(iv) Find the maximum gain of the filter in terms of dB.

[10 marks]

2. (a) Explain why it is useful to find the convolution of two time domain signals in frequency domain.

[2 marks]

(b) Demonstrate, with aid of appropriate equations, the following properties of convolution,

(i) commutative,

(ii) distributive over addition, and

(iii) associative.

[6 marks]

(c) A digital radio system consists of a transmitter on a building and a receiver on another. The impulse response of the transmitter is shown in Fig. 2.1.

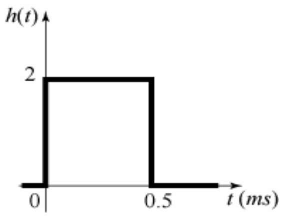

Fig. 2.1: The impulse response of the transmitter

A logic 1 (high) is transmitted by applying a unit impulse to the transmitter. A logic 0 (low) is transmitted by applying an impulse of strength –1 to the transmitter. Assume the time taken for the transmitted pulse to reach the receiver is negligible. The receiver has an impulse response identical to the transmitter.

(i) Draw a block diagram of the system.

(ii) Show the output at the receiver $y(t)$ is

$$\int_{-\infty}^{\infty} h(\tau)h(t-\tau)d\tau$$

(iii) Using (ii)to find the signals at the output of the receiver at the following duration,

- $t \leq 0$,
- $0 \leq t \leq 0.5\times10^{-3}$s,
- $0.5\times10^{-3} \leq t \leq 1.0\times10^{-3}$s, and
- $t \geq 1.0\times10^{-3}$s,

when a logic 1 is transmit.

(iv) Make a detailed sketch of the waveform at the receiver.

[12 marks]

(d) Another signal consists of a logic 0 is transmitted 0.5 ms after the logic 1, i.e.,

$$x(t) = \delta(t) - \delta(t - 0.5 \times 10^{-3})$$

is applied to the transmitter.

(i) Find the signals at the output.

(ii) Draw a detailed sketch of the complete waveform at the output of the receiver.

[5 marks]

3. (a) Explain why spectrum leakage is hard to prevent in practical cases and describe how windowing can minimise the spectrum leakage effect.

[5 marks]

(b) Compare the square window with the Hanning window in the following aspects,
(i) peak sidelobe, and
(ii) roll-off.

[4 marks]

(c) If a N point sequence, the discrete Fourier transform (DFT) of a sampled signal $x(n)$ is given by:

$$X(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j2\pi k\frac{n}{N}}$$

Show that

$$x(n) = \sum_{k=0}^{N-1} X(k) e^{j2\pi k\frac{n}{N}}$$

[8 marks]

(d) For N = 4, using part (b) to find and sketch the inverse DFT for the spectral sequence

$$X[n] = [1, \quad -0.5, \quad 0, \quad -0.5]$$

[8 marks]

4. Consider the causal linear time invariant (LTI) system with discrete input signal $x[n]$ and discrete output signal $y[n]$, sampled at time n, as shown in Figure 4.1. K_1, K_2, K_3, and K_4 are multiplication coefficients which scale the signals passing through the links.

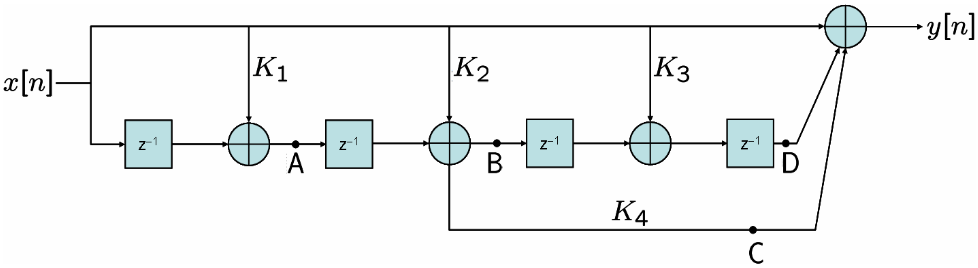

Figure 4.1: A lattice structure implementation of a causal LTI system.

(a) Derive the signal expression at point A.

[2 marks]

(b) Derive the signal expression at point B.

[5 marks]

(c) Derive the signal expression at point C.

[2 marks]

(d) Derive the signal expression at point D.

[8 marks]

(e) Express $y[n]$ as a function of $x[n]$.

[4 marks]

(f) Determine the numerical values of K_1, K_2, K_3, and K_4 so that the difference equations of the systems of Figure 4.1 and Figure 4.2 match.

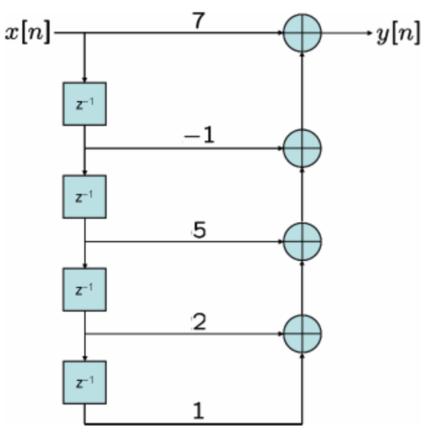

Figure 4.2: A causal LTI system.

[4 marks]

5. You need to design a digital low-pass filter that filters the ambient noise corrupting a sinusoidal signal. The signal is sampled at 200 samples per second and its maximum frequency is 50Hz. The low-pass filter is required to have no ripple and a roll-off rate –12 dB/octave.

   (a) Which type of filter would you choose? Butterworth or Chebyshev and why? Also, give the order of the filter for the design.

   [5 marks]

   (b) It is given that the normalised analogue filter to implement such digital low-pass filter is

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}.$$

   Now, you are asked to use bilinear transformation to realise the digital filter design. To do so, find the analogue filter cut-off frequency prior to warping.

   [4 marks]

   (c) Write down the transfer function of the analogue filter for the design.

   [2 marks]

   (d) Using bilinear transformation, obtain the digital low-pass filter in the z-domain.

   [8 marks]

   (e) Obtain the difference equation showing the input-output relationship of the filtering system.

   [6 marks]

6. Design a linear-phase low pass FIR filter for a system with an 8 kHz sample rate, using an ideal brick wall frequency response with a pass band power gain of 8 dB and a cut-off frequency of 500 Hz. The filter should achieve a stop-band attenuation of at least 50 dB at all frequencies above 3.14 kHz. The design is obtained by the use of Hamming windowing coefficients

$$w_n = 0.54 + 0.46\cos\left(\frac{n\pi}{M}\right),$$

which, with the order of $2M+1$ and sampling interval Δt, has the following properties:

Transition band (Hz)	Stop-band rejection (dB)
$\frac{3.3}{M\Delta t}$	53

(a) Evaluate the transition band.

[2 marks]

(b) Determine the required minimum number of Hamming window weighting coefficients.

[3 marks]

(c) Obtain the resultant FIR filter coefficients.

[20 marks]

END OF PAPER