1. You are required to design a digital high pass filter without ripple and a roll-off rate –12 dB/octave. The signal is sampled at 800 samples/s and its cut-off frequency is 5 kHz. The following table is provided to aid your design.

n	Butterworth Polynomials in factored form
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	$(s^2+s+1)(s+1)$
4	$(s^2+0.76536s+1)(s^2+1.84776s+1)$
5	$(s+1)(s^2+0.168s+1)(s^2+0.168s+1)$
6	$(s^2+0.518s+1)(s^2+1.414s+1)(s^2+1.932s+1)$

Also, it is known that a digital high pass filter can be obtained from a low pass filter using the following relationship, where w_s is the angular sampling frequency, w_{CHP} is the high pass cut-off frequency and w_{CLP} is the cut-off frequency for the low pass filter. Also, $H_{\mathrm{HP}}(z)$ is the z-transfer function of the high pass filter while $H_{\mathrm{LP}}(z)$ is the z-transfer function of the low pass filter:

$$\omega_{\mathrm{CHP}} = \frac{\omega_s}{2} - \omega_{\mathrm{CLP}} \qquad H_{\mathrm{HP}}(z) = H_{\mathrm{LP}}(z)|_{z\to -z}$$

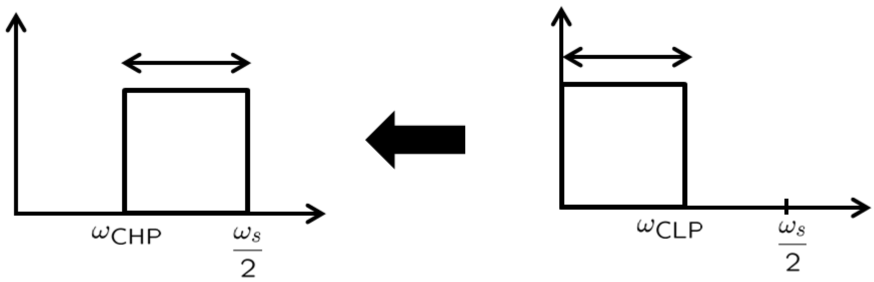

Figure 4.1

(a) Would you use Butterworth or Chebyshev filter for the design? Why?

[2 marks]

(b) Obtain the transfer function of the required normalised analogue filter.

[3 marks]

(c) Find the analogue filter cut-off frequency prior to wrapping.

[5 marks]

(d) Use bilinear transformation to obtain the digital high pass filter.

[15 marks]

2. Design a linear-phase low pass FIR filter for a system with an 8 kHz sample rate, using an ideal low pass frequency response (i.e., brick wall) with a pass band power gain of 8 dB and a cut-off frequency of 500 Hz. The filter should achieve a stop-band attenuation of at least 70 dB at all frequencies above 3.14 kHz. The following table shows the properties for different windowing functions with order N=2M+1 where N is the number of taps (odd) and sampling interval Δt.

Window	Transition band (Hz)	Stopband rejection (dB)
Rectangular	$\frac{1}{N\Delta t}$	**21**
Hanning	$\frac{3.1}{N\Delta t}$	**44**
Hamming	$\frac{3.3}{N\Delta t}$	**53**
Kaiser, β =6	$\frac{4}{N\Delta t}$	**63**
Blackman	$\frac{5.5}{N\Delta t}$	**74**
Kaiser, β =9	$\frac{5.7}{N\Delta t}$	**90**

Some of the windowing functions are also given below to aid your design.

Hanning: $\omega_n = \frac{1}{2}[1+\cos(\frac{n\pi}{M})]$

Hamming: $\omega_n = 0.54 + 0.46\cos(\frac{n\pi}{M})$

Blackman $\omega_n = 0.42 + 0.5\cos(\frac{n\pi}{M}) + 0.08\cos(\frac{2n\pi}{M})$

(a) Which windowing function should be used for the design and why?

[4 marks]

(b) Calculate the bandwidth of the transition band and find the minimum number of window weighting coefficients.

[6 marks]

(c) Obtain the resultant FIR filter coefficients.

[15 marks]

3. A communications channel has a transfer function $0.3+2z^{-1}$. The signal, $x(n)$, is zero mean and white with variance (or power) of 1.5. The additive noise, $N(n)$, is white with zero mean and variance of 0.5 and is uncorrelated with the signal, where n is the time index. The objective is to design a two-tap Weiner filter equaliser with a lag of 1 to minimise the mean square error (MSE) of the signal. The design is illustrated in the following diagram:

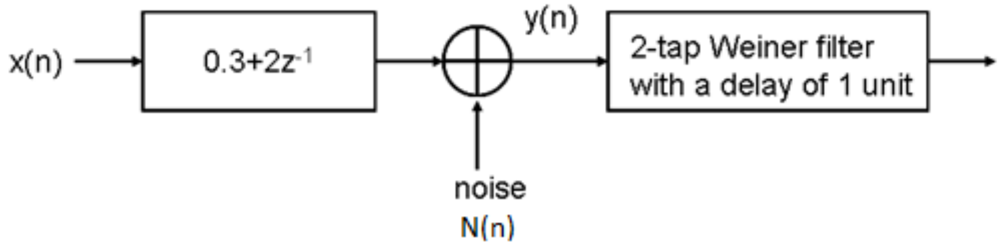

Figure 6.1

(a) Find the power spectral density of the signal y and the autocorrelation matrix.

[10 marks]

(b) Find the cross-correlation vector of the signal y and the reference signal.

[5 marks]

(c) Obtain the Weiner filter coefficients.

[10 marks]

END OF PAPER