*Answer FOUR questions*

1. (a) Explain how a root locus diagram is used to predict the behaviour of a system with known open loop dynamics when negative feedback is applied.

[5 marks]

(b) Consider the unity-feedback closed-loop system with forward path

$$G_o(s) = \frac{10(\tau s + 1)}{20\tau s^3 + (10\tau + 2)s^2 + (1 - 10\tau)s - 1}$$

Is the open-loop system stable? Find the condition for $\tau$ such that the closed-loop system is stable.

[10 marks]

(c) Consider the unity-feedback closed-loop system with forward path

$$G_o(s) = \frac{K}{(s + 5)(s + 10)}$$

Derive the gain such that the closed-loop system has 10% steady-state error due to unit step input. Can the system track a ramp input? Justify your answer.

[10 marks]

2. Figure 2.1 shows a level control system for a water tank, with: LI being the level indicator (sensor), LC being the level controller and a valve controlling the incoming flow of water. The transfer functions of the system's components are given below, with: $K_\mathrm{p}$ and $\tau_\mathrm{i}$ the settings of a PI controller, $\tau_\mathrm{v}$ the valve time constant, $F_\mathrm{in}$ the manipulated variable and $F_\mathrm{out}$ the disturbance of the system (in this case the outgoing flow is in fact the disturbance).

$$E(s) = H_\mathrm{sp}(s) - H(s)$$

$$\frac{F_\mathrm{sp}(s)}{E(s)} = K_\mathrm{p}\frac{1 + s\tau_\mathrm{i}}{s\tau_\mathrm{i}}$$

$$\frac{F_\mathrm{in}(s)}{F_\mathrm{sp}(s)} = \frac{1}{1 + s\tau_\mathrm{v}}$$

$$H(s) = \frac{1}{s}[F_\mathrm{in}(s) - F_\mathrm{out}(s)]$$

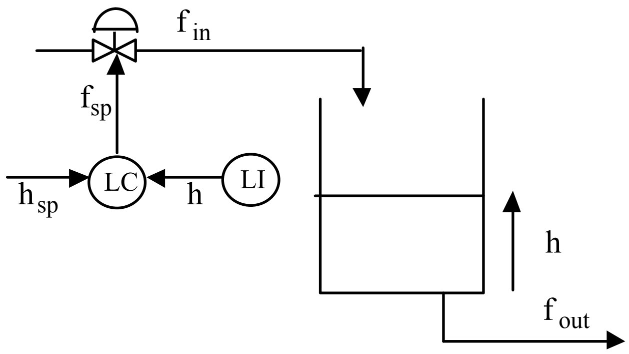


Figure 2.1. Level control system diagram. The time-domain system variables are shown.

(a) Draw a block diagram for the system.

[5 marks]

(b) Figure 2.2 shows two cases for the positions of the open loop poles and zeros of the system.

(i) What value was chosen for the controller integration time, $\tau_\mathrm{i}$ for each case?

(ii) One of the plots of Figure 2.2 corresponds to the case when the demands on the movement of the valve are below its velocity limit and the time constant of the valve is: $\tau_\mathrm{v} \cong 0.10$ seconds. The other is for the case of a large step leading to a rate-limited movement and the time constant of the valve is: $\tau_\mathrm{v} \cong 0.27$ seconds. Which is which? Explain your answer.

[10 marks]

(c) This controller becomes unstable for large valve movements. Suggest a simple alternative choice of controller that would ensure the closed loop system would not become unstable for large valve movements. Explain your choice.

[10 marks]

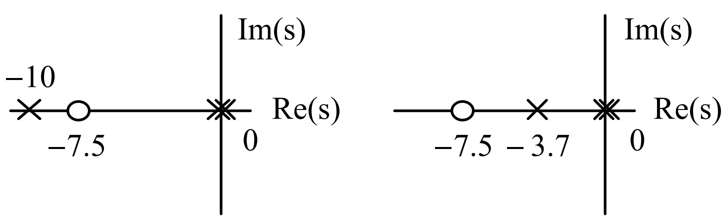


Figure 2.2. Two cases for the open-loop poles and zeros of the system of Figure 2.1.

3. (a) Explain how the use of partial fractions enables rapid inversion of a Laplace transform. Illustrate your answer by showing how the time domain output of the system $G(s) = \frac{1}{s+1}$ is calculated when the input to the system is a unit step.

[5 marks]

(b) The following open loop transfer function has a zero in the right half of the $s$-plane. Two equivalent forms of the transfer function are given; in the second the poles have been converted to the $(1 + s\tau)$ form.

$$G_o(s) = \frac{10(s-1)}{(s+2)(s+5)} = \frac{s-1}{(1+0.5s)(1+0.2s)}$$

(i) Show that the step response output of this open loop system is given by the expression for $x(t)$ below. It is more convenient to use the first of the two equivalent forms of $G_o(s)$ for this task.

$$x(t) = 5e^{-2t} - 4e^{-5t} - 1$$

(ii) Sketch an approximate graph of $x(t)$ on a time axis by evaluating the expression at a series of instants such as $t = 0$, $t = 0.2$, $t = 0.7$ and $t = 2.5$ and also finding the steady-state value of $x$.

[10 marks]

(c) The system is placed in a unity negative feedback loop. The system is controlled by a P-only controller, as shown in Figure 3.1. Note that it is a class zero system.

(i) For the closed loop configuration, write down the transfer functions $\frac{E(s)}{U(s)}$ and $\frac{X(s)}{U(s)}$. Determine the steady state values of the error and of the output, $x$, when the input is a unit step. Comment on any unusual features of these steady state values.

(ii) Using a method of your choice, show that the control loop becomes unstable if the controller gain, $K_\text{p}$, exceeds a critical value. What is the critical value of the gain that causes instability?

[10 marks]

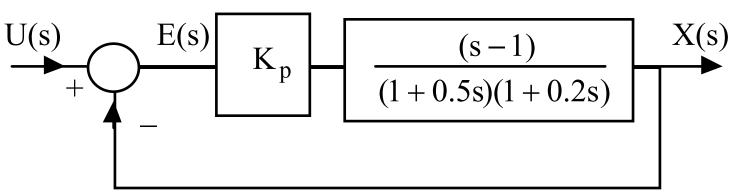


Figure 3.1. Block diagram of the system controlled by a P-only controller.

4. Consider the circuit of Figure 4.1.

(a) Find the voltage transfer function $\frac{V_2(s)}{V_1(s)}$.

[5 marks]

(b) Suppose that an inductor $L_2$ is connected across the output terminals in parallel with $R_3$. Find the new transfer function $\frac{V_2(s)}{V_1(s)}$ for this case.

[10 marks]

(c) For the circuit of Figure 4.1, a constant input voltage $v_1(t) = 10$ V is applied to the circuit. Using the final value theorem, find the steady-state value of the output voltage $v_2$.

[10 marks]

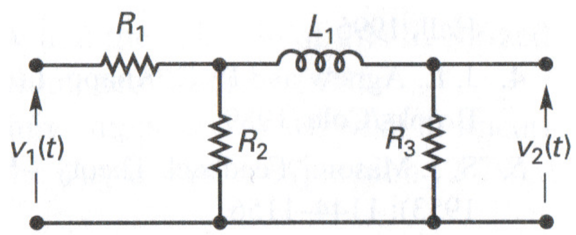


Figure 4.1. RL circuit diagram.

5. (a) Explain the basic assumption of the describing function technique for examining the behaviour of a non-linear system via a linear approximation.

[5 marks]

(b) Consider the system shown in Figure 5.1.

(i) Make an approximate sketch of the output from the non-linearity when its input is $e(t) = e_0 \sin(\omega t)$ using the describing function technique.

(ii) Derive the coefficient of the fundamental term in the Fourier series for this waveform and hence show that the describing function of the non-linearity is given by the expression below. *Hint*: The waveform is a sum of two simple periodic signals.

$$N(e_0) = \frac{4a}{\pi e_0} + k$$

[10 marks]

(c) Copy the following table into your answer book and calculate the values of $N(e_0)$ and $-\frac{1}{N(e_0)}$ for the cases $a = 0.1, k = 0.1$ and $a = 0.1, k = 1$.

[10 marks]

| | $a = 0.1, k = 0.1$ | | $a = 0.1, k = 1$ | |
|---|---|---|---|---|
| $e_0$ | $N(e_0)$ | $-\frac{1}{N(e_0)}$ | $N(e_0)$ | $-\frac{1}{N(e_0)}$ |
| 0 | | | | |
| 0.2 | | c8 | | |
| 2 | | | | |
| 20 | | | | |
| 200 | | | | |

Table 5.1. Controller non-linearity and stability point $-\frac{1}{N(e_0)}$ values for different input amplitudes $e_0$.

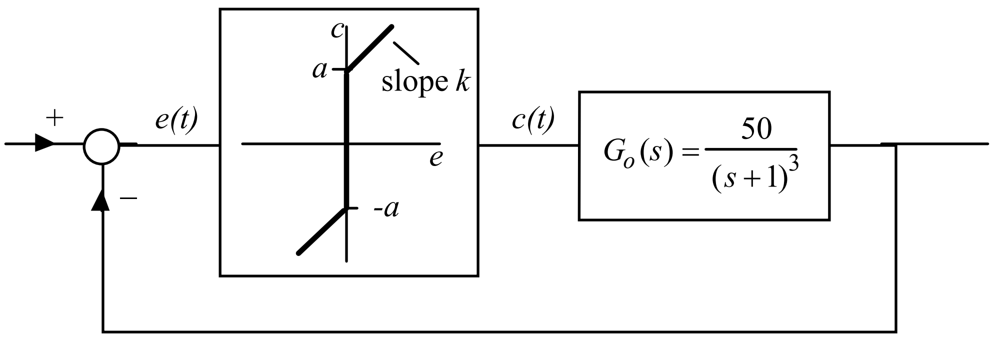


Figure 5.1. System with non-linear controller in the forward path.

6. The system in Figure 6.1 shows a system with a proportional controller in the forward path. The system has a set point input $U(s)$ and is also subject to a disturbance $D(s)$, which is summed with the control signal. Because it is a linear system the overall response can be treated with the principle of superposition of inputs.

(a) Based on the principle of superposition, draw a diagram showing the system response solely due to input $U(s)$ and a diagram showing the system response solely due to input $D(s)$.

[5 marks]

(b) Draw a diagram representing the transfer from $U(s)$ to $E_1(s)$, which is the error stemming solely from input $U(s)$. Also draw a diagram representing the transfer from $D(s)$ to $E_2(s)$, which is the error stemming solely from input $D(s)$.

[10 marks]

(c) By deriving the expression for the transfer function $\frac{E_1(s)}{U(s)}$ show that the final value for the error $e_1(t)$ is zero when the set point is a unit step ($U(s) = \frac{1}{s}$). Show additionally from the transfer function $\frac{E_2(s)}{D(s)}$ that the final value of $e_2(t)$ in response to a step in the disturbance ($D(s) = \frac{1}{s}$) is *not* zero.

[10 marks]

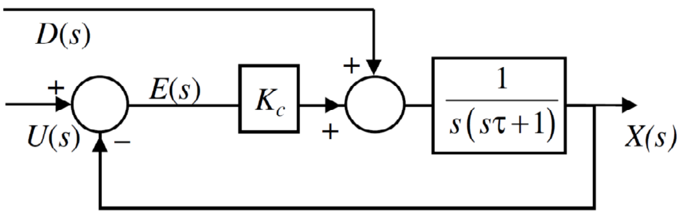


Figure 6.1 System with proportional-only control having both set-point and disturbance inputs.



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