UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE	:	**ELEC3005**
ASSESSMENT PATTERN	:	**ELEC3005A**
MODULE NAME	:	**Digital Signal Processing and Systems**
DATE	:	**01-May-14**
TIME	:	**14:30**
TIME ALLOWED	:	**3 Hours 0 Minutes**

Answer FOUR questions

1 (a) Write the mathematical expressions of the following three properties of convolution:
(i) commutativity,
(ii) distributivity over addition, and
(iii) associativity.

[6 marks]

(b) A communication link has a transmitter with an impulse response as shown in Fig. 1.1. A logic 1 (high) is transmitted by applying a unit impulse to the transmitter. A logic 0 (low) is transmitted by applying an impulse of strength -1 to the transmitter. Assume that the time taken for the transmitted pulses to arrive at the receiver is negligible. The receiver has an impulse response identical to that of the transmitter.

(i) Draw the block diagram of the communication link, according to the above description.
(ii) Using convolution find the mathematical expressions of the output of the receiver when a logic 1 is transmitted. Use diagrams to illustrate the convolution. Make a detailed sketch of the final waveform.

[10 marks]

(c) In the presence of clutter, the communication link has two paths between the transmitter and the receiver. The first has a gain of 1 and a negligible time delay. The second has a gain of 1 and a delay of 1 ms. The contributions from the two paths add together at the input to the receiver.

(i) Draw a diagram of the communication link in the presence of clutter.
(ii) Make a detailed sketch of the waveform at the output of the receiver when a logic 1 is transmitted in this case.

[9 marks]

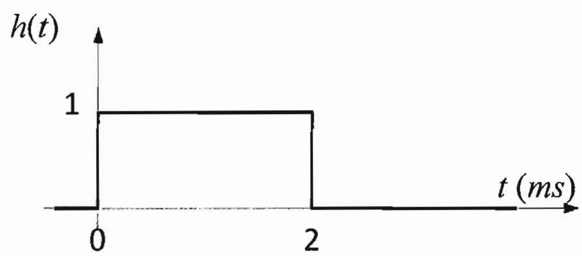

Fig. 1.1

2. (a) Explain, with the aid of diagrams, the term aliasing. Use these to show that a minimum sampling frequency is required to avoid aliasing.

[7 marks]

(b) A linear system is represented by the following differential equation.

$$y[n] = 0.8y[n-1] - 0.2y[n-2] + x[n] - 0.3x[n-1]$$

(i) Determine its transfer function.

(ii) Find the pole(s) and zero(s) of the system.

(iii) Sketch the realization of such a digital filter.

(iv) Evaluate the first 5 outputs for an input sequence of

$$\{0, 0.25, 0.5, 0, 0, 0, \ldots, 0\}.$$

[10 marks]

(c) A digital filter is described by the transfer function

$$H(z) = \frac{z^2 + 1.0404}{z^2 - 1.3718z + 0.9409}$$

(i) Sketch the zero-pole diagram
(ii) State and explain if the filter is stable.
(iii) Sketch the frequency response.
(iv) Find the maximum gain of the filter in terms of dB.

[8 marks]

3 (a) The discrete Fourier transform (DFT) of a sampled signal $x(n)$ is given by:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k \frac{n}{N}}$$

Show that

(i) the DFT is conjugate symmetric, and

(ii) $X(N-k) = X^*(k)$, and

(iii) $X(N+k) = X(k)$.

[5 marks]

(b) Show that an N-point DFT can be partitioned into two DFTs each of size N/2. Assume N is an even number.

[5 marks]

(c) Explain what a radix-2 in-place, decimation-in-time fast Fourier transform (FFT) algorithm is.

[5 marks]

(d) For N = 4, devise a FFT and draw the complete signal flow diagram.

[10 marks]

4. Given the amplitude response of an n-th order normalised low-pass Butterworth filter

$$|H(\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}$$

Work out the followings:

(a) Show that this filter is maximally flat.

[5 marks]

(b) Explain the meaning of having a filter roll off factor –6 dB/octave.

[3 marks]

(c) Show that this filter's roll off factor is –6 dB/octave.

[5 marks]

(d) Find out the poles of the Butterworth filter.

[5 marks]

(e) Express the transfer function of the filter in terms of the poles in (d).

[2 marks]

(f) Determine the Butterworth polynomial for n=4.

[5 marks]

5. Design a linear-phase low pass FIR for a system with a 2.5 kHz sampling rate using an ideal (i.e., brick wall) low pass frequency response with a pass band power gain of 5 dB and a cut-off frequency of 100 Hz. The filter should achieve a stop-band attenuation of at least 48 dB at all frequencies above 2 kHz. The following table shows the properties for different windowing functions with order N=2M+1 where N is the number of taps (odd) and sampling interval Δt.

Window	Transition band (Hz)	Stopband rejection (dB)
Rectangular	$\frac{1}{N\Delta t}$	21
Hanning	$\frac{3.1}{N\Delta t}$	44
Hamming	$\frac{3.3}{N\Delta t}$	53
Kaiser, β =6	$\frac{4}{N\Delta t}$	63
Blackman	$\frac{5.5}{N\Delta t}$	74
Kaiser, β =9	$\frac{5.7}{N\Delta t}$	90

Some of the windowing functions are also given below to aid your design.

Hanning: $\omega_n = \frac{1}{2}[1+\cos(\frac{n\pi}{M})]$

Hamming: $\omega_n = 0.54 + 0.46\cos(\frac{n\pi}{M})$

Blackman $\omega_n = 0.42 + 0.5\cos(\frac{n\pi}{M}) + 0.08\cos(\frac{2n\pi}{M})$

(a) Which windowing function should be used for the design and why?

[2 marks]

(b) Calculate the bandwidth of the transition band and find the minimum number of window weighting coefficients.

[3 marks]

(c) Obtain the resultant FIR filter coefficients.

[12 marks]

(d) If a rectangular window was used while keeping the same number of taps in the original design, what would be the attenuation (in dB) at 5.5 kHz?

[8 marks]

6. A communications channel has a transfer function $1.5+0.8z^{-1}$. The signal, $x(n)$, is zero mean and white with variance (or power) of 1.5. The additive noise, $N(n)$, is white with zero mean and variance of 0.5 and is uncorrelated with the signal, where, n, is the time index. The objective is to design a two-tap Weiner filter equaliser with a lag of 1 to minimise the mean square error (MSE) of the signal. The design is illustrated in the following diagram:

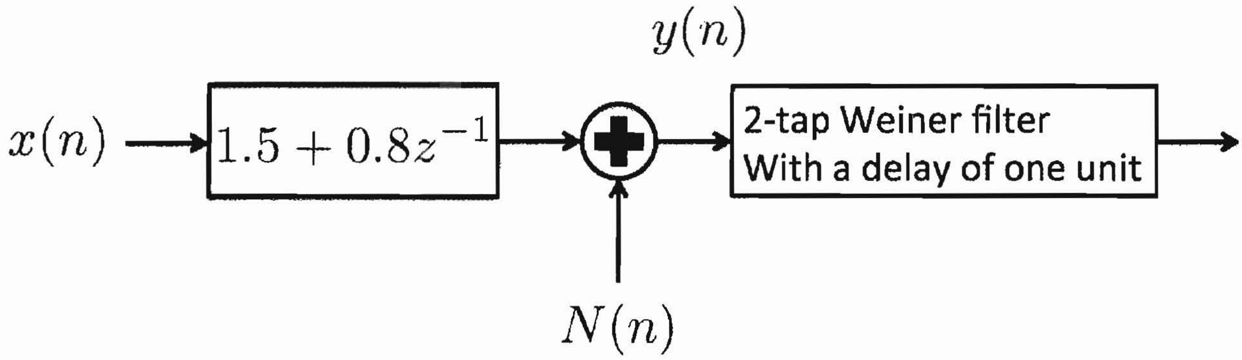

Figure 6.1

(a) Find the power spectral density of the signal $y(n)$ and the autocorrelation matrix.

[10 marks]

(b) Find the cross-correlation vector of the signal $y(n)$ and the reference signal.

[5 marks]

(c) Obtain the Weiner filter coefficients.

[5 marks]

(d) Find the MSE of the resultant filter.

[5 marks]

END OF PAPER