UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE	:	**ELEC3005**
ASSESSMENT PATTERN	:	**ELEC3005A**
MODULE NAME	:	**Digital Signal Processing and Systems**
DATE	:	**16-May-12**
TIME	:	**10:00**
TIME ALLOWED	:	**3 Hours 0 Minutes**

2011/12-ELEC3005A-001-EXAM-21

TURN OVER

Answer FOUR questions

1 (a) State the following three properties of convolution;
(i) commutativity,
(ii) distributivity over addition, and
(iii) associativity.

[6 marks]

(b) A digital TV system consists of a transmitter and a receiver. The impulse response of the transmitter is shown in Fig. 1.1. A logic 1 (high) is transmitted by applying a unit impulse to the transmitter. A logic 0 (low) is transmitted by applying an impulse of strength −1 to the transmitter. The time taken for the transmitted pulses to arrive at the receiver is negligible. The receiver has an impulse response identical to that of the transmitter.

(i) Draw a block diagram of the system.
(ii) Using convolution and with the aid of diagrams, find the output mathematical expressions of the receiver when a logic 0 is transmitted. Make a detailed sketch of the final waveform.

[10 marks]

(c) In bad weather there are two paths between the transmitter and the receiver. The first has a negligible time delay and a gain of 1. The second has a gain of -1 and a delay of 0.25 ms. The contributions from the two paths add together at the input to the receiver.

(i) Draw a diagram of the complete system.
(ii) Make a detailed sketch of the waveform at the output of the receiver when a logic 1 is transmitted.

[9 marks]

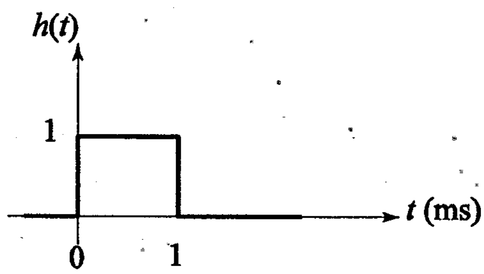

Fig. 1.1

2 (a) Given the impulse response of a zero-order-hold (ZOH) is:

$$h_{ZOH}(t) = \begin{cases} 1, & 0 \le t < \Delta t \\ 0, & \text{otherwise} \end{cases}$$

Find;

(i) the transfer function $H(s)$, and

(ii) the frequency response.

Sketch;

(iii) the magnitude of the response and

(iv) the phase of the response.

[12 marks]

(b) Figure 2.1 shows a response sequence,

(i) Write down the impulse response of the sequence,

(ii) Find the transfer function $H(z)$ of the sequence

(iii) Sketch the pole/zero pattern in the z-plane

(iv) State the stability condition of digital filters.

(v) Derive the inverse z-transform and draw the filter block diagram.

[13 marks]

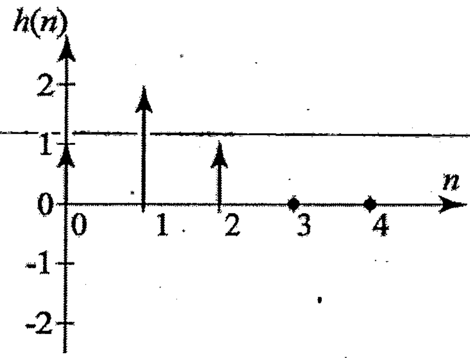

Figure 2.1

CONTINUED

3 (a) The discrete Fourier transform (DFT) of a sampled signal $x(n)$ is given by:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi k \frac{n}{N}}$$

Show that DFT is

(i) conjugate symmetric, and

(ii) $X(N-k) = X^*(k)$, and

(iii) $X(N+k) = X(k)$.

[5 marks]

(b) Show that an N-point DFT can be partitioned into two DFTs each of size N/2. Assume N is an even number.

[5 marks]

(c) Explain what a radix-2 in-place, decimation-in-time FFT algorithm is.

[5 marks]

(d) For N = 4, devise a fast Fourier transform (FFT) and draw the complete signal flow diagram.

[10 marks]

4 A signal sampled at 18kHz is to be filtered using three IIR filters in parallel. The first filter is low pass with a cut-off at 3kHz. The second is a high pass with a cut-off at 6kHz. The third is a band pass with cut-offs at 3 and 6 kHz. The digital filters should be designed using a bilinear transformation and based on a first order Butterworth filter with the following transfer function:

$$H(s)=(1+s)^{-1}$$

(a) Find the analogue filter cut-off frequency prior to warping for the low pass filter.

[5 marks]

(b) Apply the bilinear transformation to obtain the digital low pass filter.

[5 marks]

(c) It is known that a high pass filter can be obtained from a low pass filter using the following relationship:

$$\omega_{\mathrm{CHP}} = \frac{\omega_s}{2} - \omega_{\mathrm{CLP}} \qquad H_{\mathrm{HP}}(z) = H_{\mathrm{LP}}(z)|_{z \to -z}$$

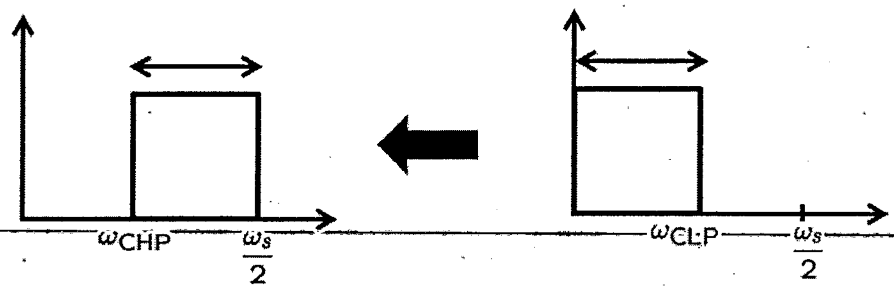

Figure 4.1

Then, find the transfer function of the high pass filter.

[5 marks]

(d) Now, it is given that a band pass filter can be obtained from a low pass filter using the following relationship:

$$\omega_{\mathrm{CLP}} = \omega_{\mathrm{cu}} - \omega_{\mathrm{cl}} \qquad z_{\mathrm{LP}}^{-1} = \frac{-z^{-1}(z^{-1}-\alpha)}{1-\alpha z^{-1}}$$

$$\alpha = \frac{\cos\left(\frac{\pi(\omega_{\mathrm{cu}}+\omega_{\mathrm{cl}})}{\omega_s}\right)}{\cos\left(\frac{\pi(\omega_{\mathrm{cu}}-\omega_{\mathrm{cl}})}{\omega_s}\right)}$$

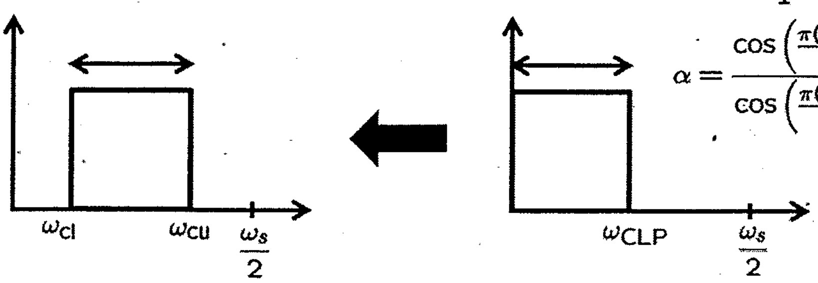

Figure 4.2

Then, find the transfer function of the band pass filter.

[10 marks]

5 Design a linear-phase low pass FIR filter for a system with a 4 kHz sample rate, using an ideal brick wall frequency response with a pass band power gain of 3 dB and a cut-off frequency of 400 Hz. The filter should achieve a stop-band attenuation of at least 40 dB at all frequencies above 2 kHz. The design is obtained by the use of Hanning windowing coefficients

$$0.5\left(1-\cos\frac{2n\pi}{N}\right)$$

which, with the order of N=2M+1 and sampling interval Δt, has the properties:

Window	Transition band (Hz)	Stopband rejection (dB)
Rectangular	$\frac{1}{N\Delta t}$	21
Hanning	$\frac{3.1}{N\Delta t}$	44

(a) Evaluate the transition band.

[2 marks]

(b) Determine the required minimum number of window weighting coefficients.

[3 marks]

(c) Obtain the resultant FIR filter coefficients.

[20 marks]

6 A communications channel has a transfer function $0.2+z^{-1}$. The signal x(n) is zero mean and white with variance (or power) of 1.5. The additive noise is white with zero mean and variance of 0.5 and is uncorrelated with the signal. The objective is to design a two-tap Weiner filter equaliser with a lag of 1 to minimise the mean square error (MSE) of the signal. The design is illustrated in the following diagram:

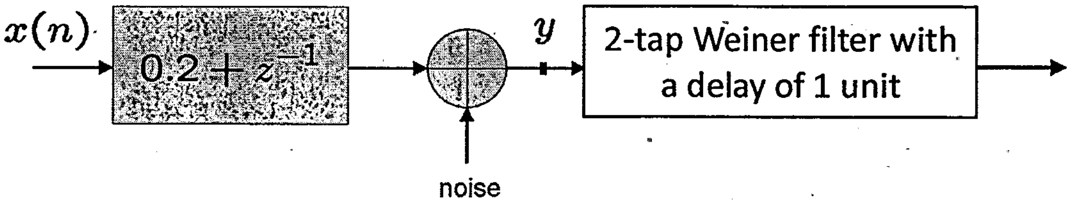

Figure 6.1

(a) Find the power spectral density of the signal y and the autocorrelation matrix.

[10 marks]

(b) Find the cross-correlation vector of the signal y and the reference signal.

[5 marks]

(c) Obtain the Weiner filter coefficients.

[10 marks]

END OF PAPER