UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : **ELEC3005**

ASSESSMENT PATTERN : **ELEC3005A**

MODULE NAME : **Digital Signal Processing and Systems**

DATE : **13-May-10**

TIME : **14:30**

TIME ALLOWED : **3 Hours 0 Minutes**

2009/10-ELEC3005A-001-EXAM-14

TURN OVER

Answer *FOUR* questions

1. (a) State and explain the four different types of factors that can be represented by a Bode plot of the transfer function *H(s)*.

[8 marks]

(b) Derive the three components of the Bode plot of the following transfer function:

$$H(s) = \frac{s+10}{s+1000}$$

Use them to sketch the plot.

[6 marks]

(c) What are the damping factor and the undamped natural frequency of a system that has the following transfer function?

$$H(s) = \frac{5s}{8s^2 + 48s + 1152}$$

[5 marks]

(d) A low pass second order system has a peak time of 1 second and an overshoot of 10 %. Estimate its bandwidth stating any assumptions made.

[6 marks]

2. (a) Explain, with the aid of diagrams, the term aliasing. Use these to show a minimum sampling frequency is required to avoid aliasing.

[7 marks]

(b) A linear system is represented by the following difference equation.

$$y[n] = 1.6y[n-1] - 0.8y[n-2] + x[n]$$

(i) Determine its transfer function.

(ii) Find the pole(s) and zero(s) of the system.

(ii) Sketch the realization of such a digital filter.

(iv) Evaluate the first 5 outputs for an input sequence of

$$\{0, 0.25, 0.5, 0, 0, 0, \ldots, 0\}.$$

[12 marks]

(c) A digital filter is described by the following transfer function

$$H(z) = \frac{(z+1)(z^2 - 0.6489z + 1.1025)}{(z^2 + 1.3435z + 0.9025)}$$

(i) Determine if the filter is stable or not.

(ii) Find the amplitude response of filter.

(iii) Calculate the gain of the filter at angular frequency $\omega = \dfrac{3\pi}{8\Delta t}$.

[6 marks]

3. (a) Explain why spectrum leakage is hard to prevent in practical cases and describe how windowing can minimise the spectrum leakage effect.

[5 marks]

(b) Compare the square window with the Hanning window in the following aspects,
(i) bandwidth,
(ii) peak sidelobe,
(iii) roll-off, and
(iv) processing loss.

[8 marks]

(c) The discrete Fourier transform (DFT) of signal $x(n)$ comprising N samples is given by:

$$X(k)=\frac{1}{N}\sum_{n=0}^{N-1}x(n)e^{-j2\pi k\frac{n}{N}}$$

Show that
(i)

$$x(n)=\sum_{k=0}^{N-1}X(k)e^{j2\pi k\frac{n}{N}}$$

(ii) $X(k)$ is conjugate symmetric about 0 Hz and
(iii) $X(k)$ is periodic with period N.

[12 marks]

4. (a) Consider the causal linear time invariant (LTI) system implemented as shown in Figure 4.1. Determine the difference equation relating the output, $y[n]$, to the input, $x[n]$.

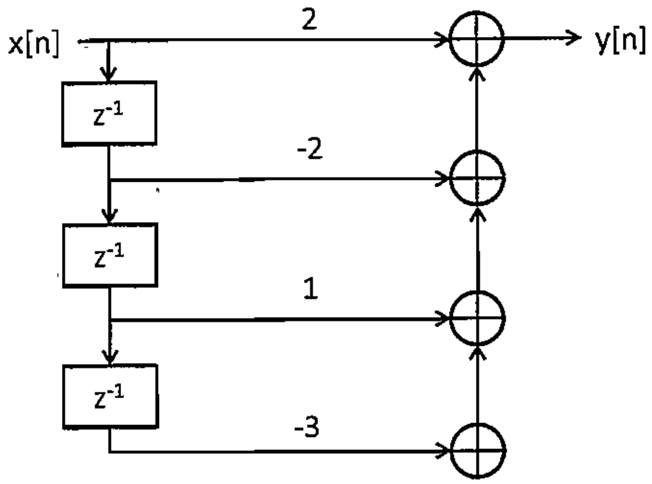


Figure 4.1: A causal LTI system.

[8 marks]

(b) A similar causal LTI system implemented via a lattice structure is shown in Figure 4.2. Determine the difference equation relating the output, $y[n]$, to the input, $x[n]$.

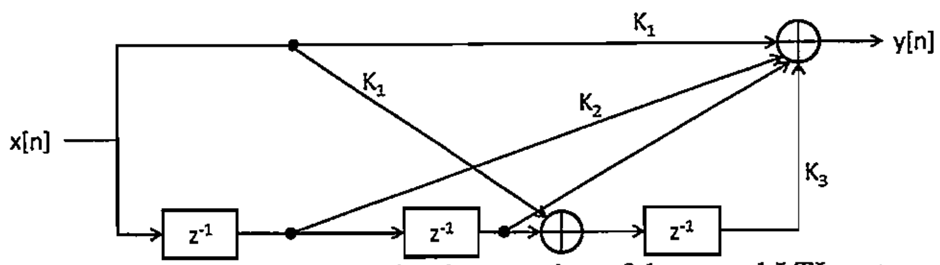


Figure 4.2: A lattice structure implementation of the causal LTI system.

[12 marks]

(c) Determine the numerical values of K_1, K_2 and K_3 so that the difference equations of the systems of Figure 4.1 and Figure 4.2 match.

[5 marks]

5. (a) What are the advantages and disadvantages of digital FIR filters in comparison to digital IIR filters?

[5 marks]

(b) Find the minimum order Butterworth filter to meet the following specifications and obtain the frequency response of the required Butterworth filter:

(i) Passband (0-ωc): 0-100 k rad/s
(ii) Minimum power gain at ωc: 0.5 (-3 dB)

[10 marks]

(c) Find the minimum order normalised Chebychev filter to meet the following specifications and obtain the frequency response of the required Butterworth filter:

(i) Maximum passband ripple: -1 dB
(ii) For $|\omega| > 4$ rad/s, stopband attenuation must be < -40 dB

[10 marks]

6. Consider the two-output discrete-time system shown in Figure 5.1 (a) where H and G are two digital filters (LTI discrete-time systems) with frequency responses H(ω) and G(ω) as shown in Figure 5.2.

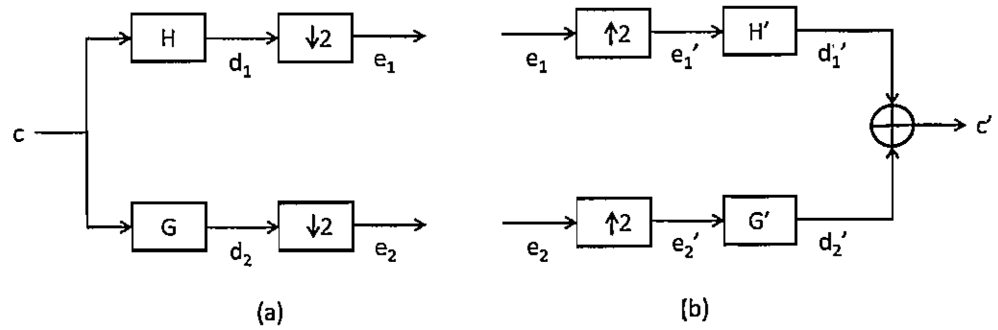


Figure 5.1: The two-output discrete-time system.

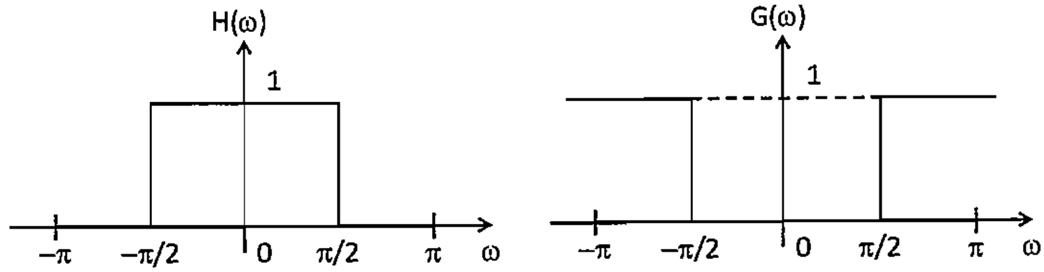
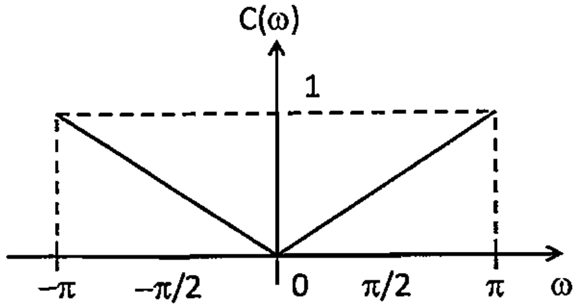


Figure 5.2: The frequency responses for H(ω) and G(ω).

The goal is to reconstruct c from e_1 and e_2. We propose to study the system shown in Figure 5.1(b) where H' and G' are two digital filters.

(a) Suppose that the Fourier transform C(ω) of c is as follows:



Plot successively the Fourier transforms $D_1(\omega)$, $D_2(\omega)$, $E_1(\omega)$ and $E_2(\omega)$ of d_1, d_2, e_1 and e_2 for $\omega\in[-\pi,\pi]$. In these plots, make sure to specify the coordinates of the points of discontinuity and the points where the slop changes.

[17 marks]

(b) Plot the frequency responses H'(ω) and G'(ω) of H' and G' so that $d_1=d_1'$ and $d_2=d_2'$. In this case, what is the output c'?

[8 marks]

END OF PAPER