Answer FOUR questions

1. Figure 1.1 shows an air heating system used to regulate the temperature in a house. In this system, air warmed by a furnace is circulated around the house, driven by a fan that has adjustable speed. The furnace is fuelled by gas. It has an on-off characteristic such that if the gas valve is opened the furnace is on; if the gas valve is closed the furnace switches off.

   (a) Identify the following features of the control system:
   (i) Control objective(s)
   (ii) Disturbances
   (iii) Sensors(s) and actuator(s) needed to achieve the control objective. Distinguish between those which are essential and those which might be useful for enhanced performance.
   (iv) Manipulated variable(s) needed to achieve the control objective

   [10 marks]

   (b) Is the system class zero or class one? Explain your answer. Would the controller need integral action?
   Hint: Newton's law of cooling (and common sense) tells us the house will lose heat through its walls and that the rate of heat loss is proportional to temperature.

   [4 marks]

   (c) Draw the block diagram for a basic feedback control system. Explain how your controller would manage the on-off characteristic of the furnace.

   [8 marks]

   (d) Discuss whether a feedforward configuration has anything to offer this control system.

   [3 marks]

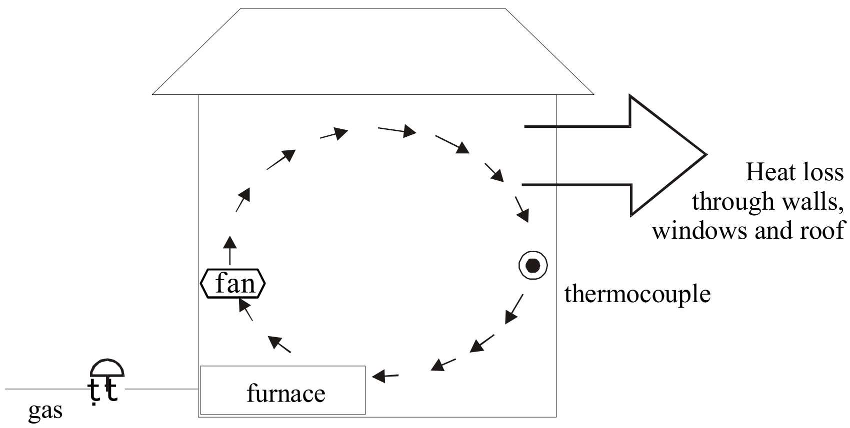

Figure 1.1. Residential air heating system.

2. (a) Show that the transfer function for the system of Figure 2.1 is:

$$\frac{X(s)}{U(s)} = \frac{8\left(s^2 + K_2 s + K_1\right)}{s^3 + 9s^2 + 8K_2 s + 8K_1}$$

How many closed-loop poles does this system have?

[12 marks]

(b) Figure 2.2 shows some root locus diagrams for this system. Explain why two families of curves are required to fully specify the loci of the closed-loop poles.

[3 marks]

(c) From the root locus figures, or otherwise, find values of K_1 and K_2 that will place a pair of closed-loop poles at $s = -1 \pm 2j$.
Show the position of all the closed-loop poles and zeros on the *s*-plane for your choice of K_1 and K_2.

[10 marks]

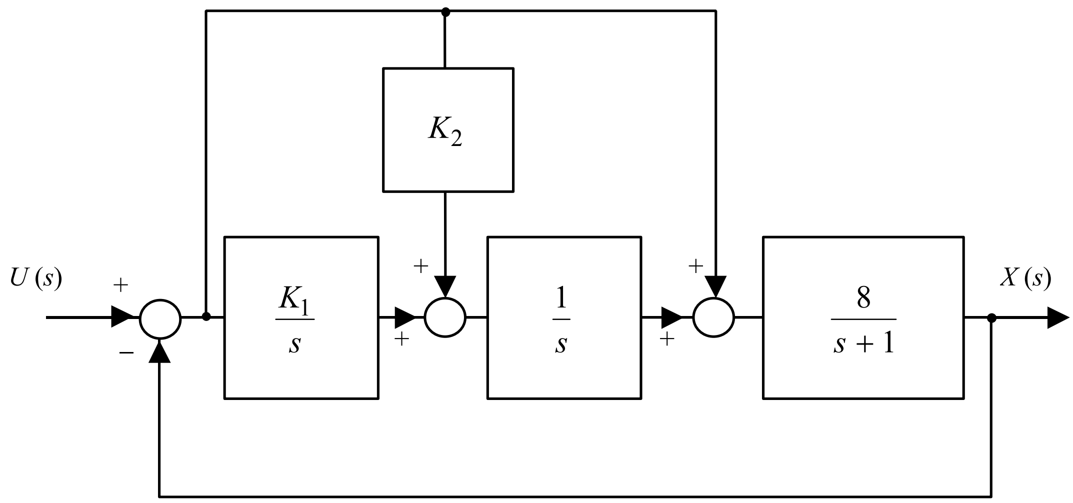

Figure 2.1. Closed-loop system.

QUESTION 2 CONTINUES

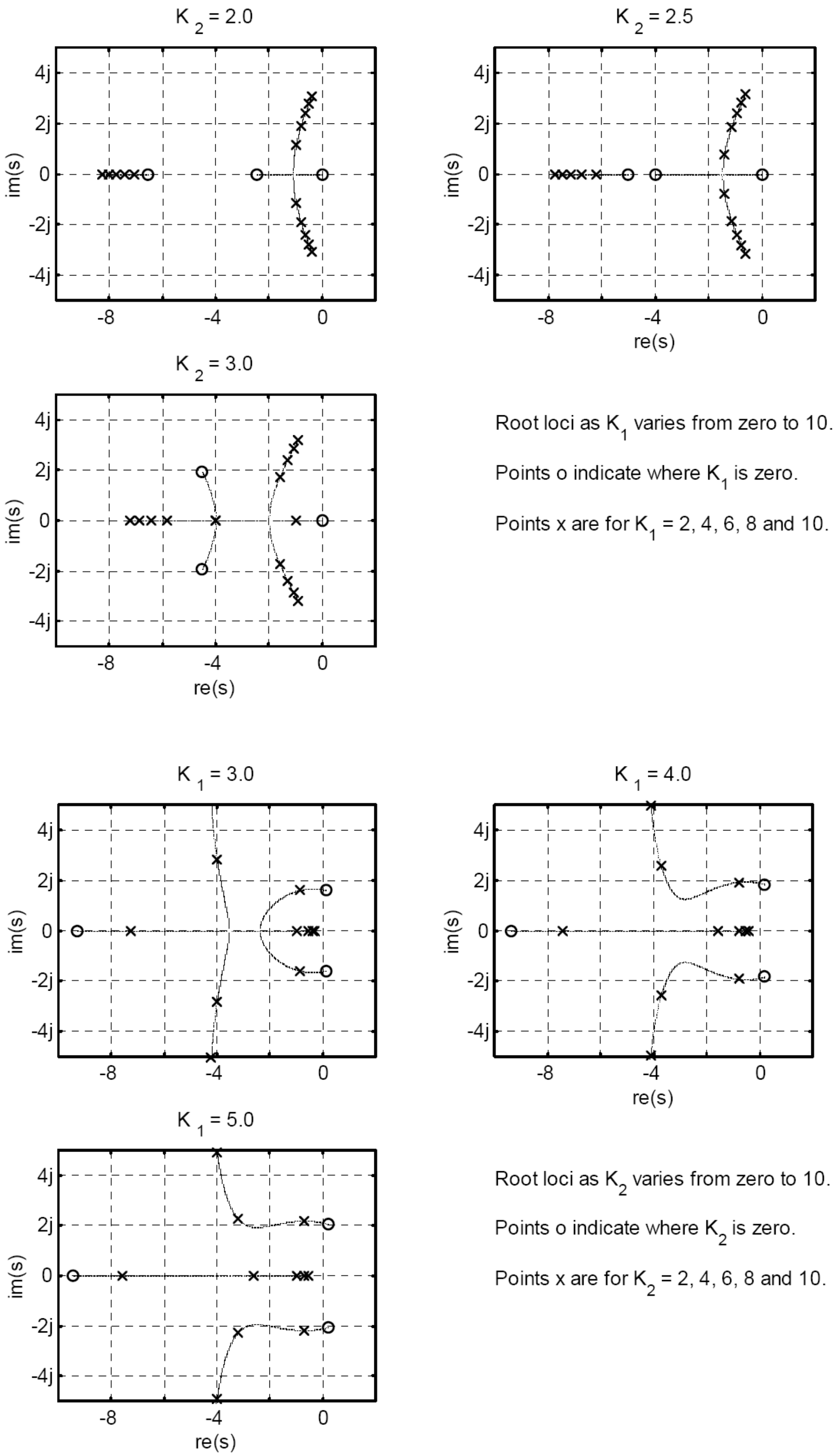

Figure 2.2. Root loci of the system of Figure 2.1.

END OF QUESTION 2

3. (a) Explain what is meant by the gain margin and the associated cross-over angular frequency of an open-loop system. Illustrate your answer using sketches of Nyquist and open-loop Bode plots.

[7 marks]

For the remainder of this question, consider the system shown in Figure 3.1. The system is controlled by a proportional-only controller (parameter K_p). Note that it is a class zero system.

(b) Write down the transfer functions $E(s)/U(s)$ and $X(s)/U(s)$.

[5 marks]

(c) Show using a method of your choice that the control loop becomes unstable if the controller gain, K_p, exceeds a critical value. What is the critical value of the gain that causes instability?

[8 marks]

(d) Determine the steady-state values of the error and of the output, $x(t)$, when the input is a unit step.

[5 marks]

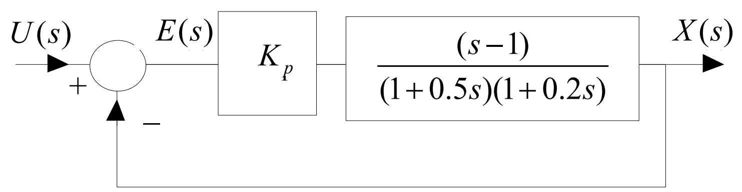

Figure 3.1. System with proportional-only control.

4. (a) A plant $G(s)$ is to be controlled by a PI controller $C(s)$ in a unity feedback configuration where:

$$G(s) = \frac{1}{s^2 + 2s + 1}$$

and
$$C(s) = K_p\left(1 + \frac{1}{\tau_i s}\right)$$

Using the Routh Array, or otherwise, determine whether the closed loop will be stable when $K_p = 10$ and $\tau_i = 5$.

[7 marks]

(b) Explain what is wrong with the following reasoning:

"The Laplace transform of $\cos(t)$ is $\frac{s}{s^2+1}$. Since $\frac{d}{dt}\cos(t) = -\sin(t)$ it follows that the Laplace transform of $-\sin(t)$ is $s \times \frac{s}{s^2+1} = \frac{s^2}{s^2+1}$"

[5 marks]

(c) Consider the (open-loop) system:

$$Y(s) = \frac{3(s+1)}{s(s-1)(s+2)^2}$$

Use a partial-fraction expansion to determine the time-domain signal $y(t)$.

What is the value of $y(t)$ as $t \to \infty$?

[13 marks]

5. (a) Explain how the feedforward control structure of Figure 5.1 enables a control system to reject disturbances. Your answer should compare the time-domain responses of the controlled variable and the manipulated variable when feedforward control is present to the same responses when the feedforward control is not present. Why might disturbance rejection with a feedforward system be less than perfect?

[7 marks]

(b) Using Figure 5.1 as a basis, draw an enhanced block diagram that shows a feedforward plus feedback control structure. Figure 5.2 suggests additional blocks and signals that might be useful for this task.

[4 marks]

(c) Explain why the disturbance rejection of a combined feedforward and feedback control system is generally better than that of a feedforward-only system or a feedback-only system.

[7 marks]

(d) By considering the response of the controlled variable to a step change in the set point, explain why integral action is needed in the forward path of a feedback system if the steady state error is to be zero.

[7 marks]

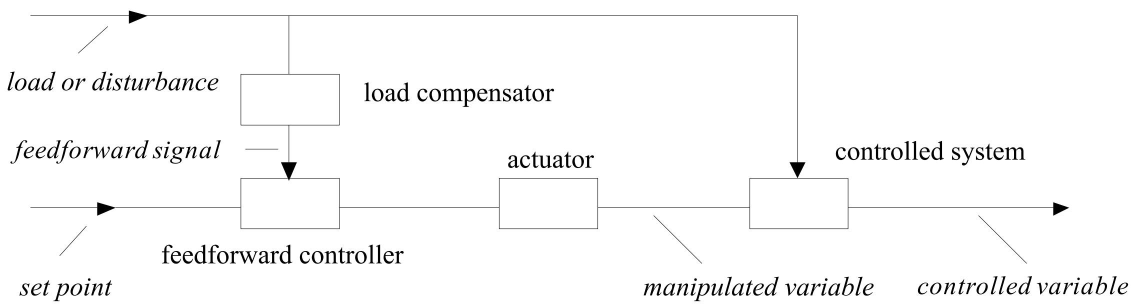

Figure 5.1. Feedforward control structure.

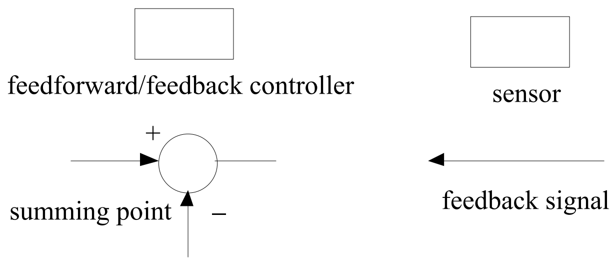

Figure 5.2. Suggested additional elements.

6. The volume of liquid in the vessel shown in Figure 6.1 may be controlled to make maximum use of the available vessel capacity in order to smooth out disturbances in the inflow. Figure 6.2 shows the block diagram of the vessel under PI control. For simplicity in the exposition, all the variables shown in both Figure 6.1 and Figure 6.2 are the deviation variables (that is: the actual value of a variable minus its average value).

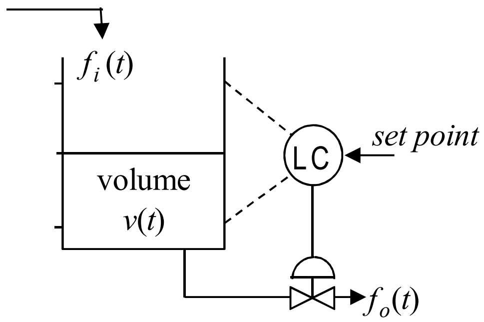

Figure 6.1. Control system.

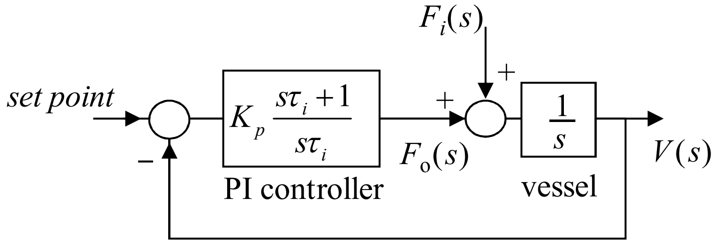

Figure 6.2. Block diagram of control system.

(a) Write down the closed-loop transfer from the disturbance $F_i(s)$ to the tank volume $V(s)$ and express it in the form below, stating clearly the expressions for ζ and ω_n in terms of K_p and τ_i.

$$V(s) = \frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2} F_i(s)$$

[4 marks]

(b) Show for the critically damped case (i.e. when $\zeta = 1$) that the proportional gain K_p and integration time constant τ_i of the controller satisfy the relationships:

$$K_p \tau_i = 4 \qquad \text{and} \qquad \omega_n = \frac{K_p}{2}$$

[4 marks]

(c) For the critically damped case determine the time-domain response of the tank volume when the disturbance $f_i(t)$ has a unit step change. Show also that the maximum deviation in the volume is $2 \times e^{-1} / K_p$.

Hint: The following Laplace transform may be of use: $L\left(t\, e^{s_i t}\right) \leftrightarrow \dfrac{1}{(s - s_i)^2}$.

[10 marks]

(d) Discuss the response of the volume of liquid in the tank to disturbances in the inflow when the system is made critically damped using low gain K_p and large integration time τ_i compared to a critically damped system using high gain and a small integration time.

[7 marks]

END OF PAPER