Interest Calculations

# Introduction

Interest payments are a vital part of understanding financial instruments. In order to compensate a lender for being without their money for the time they agree to lend it to a borrower, the amount paid back usually exceeds the amount originally lent, thus giving the lender a profit. The extra repayment is known as ‘interest’. In its simplest form, it’s calculated by multiplying the principal (amount lent) by the annual interest rate and then multiplying that by the length of the period the interest is payable for, expressed in years:

**Interest = P x r x t**

**Where:**

**P is the Principal**

**r is the rate of interest**

**t is the amount of time the loan is for, in years.**

So, if you borrowed £100 for two years and were charged 5% interest per year then the total amount you’d have to pay back in interest would be:

100 x 5% (which is the same as saying 0.05) x 2, which equals 10.

Therefore, the interest would be £10.

This is known as ‘simple interest’.

# Time

The timeframe of a loan has a big impact on the amount of interest charged. For example, if a £100 loan had an annual interest rate of 10%, and there was also a £50 loan with a monthly interest rate of 5% then, to find out which offered the best value over the course of a year, we’d need to convert them to the same timeframe and compare them. The first one is straightforward: P x r x t, would work out at 100 x 0.10 x 1 = £10. The second one would need to be converted to a year though. It’s 5% a month and there are 12 months in a year, therefore the ‘r’ part of the equation would be 5% x 12 or 60%. Then P x r x t would be 50 x 0.6 x 1 = £30. Therefore it would be comparably cheaper over the course of a year to borrow £100 for one year with 10% interest, than it would to borrow £50 for one month at 5% interest. This reinforces why it’s so important to ‘annualise’ the interest rate – without a direct time comparison it would be very difficult to compare.

In the event that a time period exceeds a year, then the time period would need to be divided by the number of years. So, for example, if a £100 3-year loan had a total interest of 21%, then the annualised interest would be 21% / 3 or 7%. Similarly, if a £100 18 month loan had a total interest of 15% then the annualised interest would be 15% / 1.5 (18 months is one and a half years) = 10%.

# Compound Interest

If you invest money and are paid interest on that investment then it provides you with extra money. You can then invest that as well, which will increase the amount of money you’re earning interest on overall. For example, let’s say that I invest £1000 with a bank and get 10% interest every 6 months, which is paid every six months (that would be some bank…) In the first 6 months I would get:

£1000 x 10% which is £100. I could then reinvest that in the same account so I’d now have £1,100. Then, in the next 6 months, I’d get 10% of £1100, which would be £110, so that also gets reinvested and I’ve now got £1,210. Then, 6 months later, I’d get 10% of that, which would be £121, taking me to £1,331.

Compound interest is working out the interest for each time period and then adding it to the total. Something to bear in mind is that if you have an annual rate, but are paid every 6 months then you need to halve the annual rate to see how much you’d get every 6 month time period. For example, if you get 10% a year then in the first 6 months you’d get 5%. Therefore if I invested £1000 on a 10% annual interest rate then after 6 months I’d have an extra £50. Then, for the next 6 months, I’d earn 5% of my initial £1000 plus 5% of the new £50 (so 5% of £1050 altogether), which is £52.50. Therefore, after the first year I’d have £1102.50. This is actually more than the 10% stated interest I thought I was going to get (it’s 10.25%), so the compound interest rate is higher. So that’s good news for an investor.

The equation for this is:

**Compound interest rate = (1+(r/m))m - 1**

**Where:**

**r = the annual interest rate (as a decimal)**

**m = how often in a year interest is paid.**

For example, in the above work through the rate of interest was 10% (which is 0.10 in decimal form) and the interest was paid every 6 months, which is 2 times a year. That would make the equation:

Compound interest rate = (1+(0.10/2))2 – 1

Compound interest rate = 0.1025 or 10.25% (to get the percentage, just multiply the decimal by 100)

If the interest was 10% but paid monthly then that would still be 0.1 for ‘r’ but ‘m’ would be 12. So:

Compound interest rate = (1+(0.10/12))12 – 1 = 0.1047 or 10.47%

**Note**: If, like me, you haven’t owned a calculator since dimly remembered school days and instead rely on the Windows calculator, then in order to work out the equation above you’ll need to switch to ‘Scientific mode’. To do this, click on the ‘View’ button at the top of the calculator, then select ‘Scientific’. There are now more buttons, but the key one is xy, which you need to work out ‘to the power of’ equations. In the last example, above, we needed to work out (1+(0.10/12)12. This is essentially 1+0.008333 to the power of 12, or 1.00833312. To calculate this, first type in ‘1.008333’ into the calculator then press the ‘xy’ button, then enter 12.

## Multiple years

If we invest money for multiple years at a compounded rate of interest then we need to add in the number of years as a variable. That’s done as follows:

**Compound interest rate = (1+(r/m))(m x n) – 1**

**Where:**

**r = the annual interest rate**

**m = how often each year interest is paid**

**n = number of years**

To see how much money we would get, the equation would have to be tweaked very slightly:

**FV = P x (1+(r/m))(m x n)**

**Where:**

**FV = Future value of invested principal**

**P = principal amount invested**

**r = annual interest rate**

**m = how often each year interest is paid**

**n = number of years**

# Simple Interest vs Compound Interest

Compound interest will always provide more money than simple interest because it allows for more money to be invested over time. Consider the following two examples:

Firstly, a simple interest rate of 10% per year without compounding so with interest only accruing on the initial principal and, secondly, a rate of 10% but where the interest amount is then reinvested:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Simple Interest** | | | | | | **Compound Interest** | | | | |
| Year | Starting balance £ | + | Interest £ | = | Ending balance £ | Starting balance £ | + | Interest £ | = | Ending balance £ |
| 1 | 100 | + | 10 | = | 110 | 100 | + | 10 | = | 110 |
| 2 | 110 | + | 10 | = | 120 | 110 | + | 11 | = | 121 |
| 3 | 120 | + | 10 | = | 130 | 121 | + | 12.10 | = | 133.10 |
| 4 | 130 | + | 10 | = | 140 | 133.10 | + | 13.30 | = | 146.40 |

So, at the end of 4 years, the total amount accrued through simple interest is £40 whereas the total amount accrued through compound interest is £46.40.

# Payment Frequency

As the name suggests, payment frequency relates to how often that interest is paid each year. The payment frequency is usually the same as the compounding frequency, which is the number of times each year that interest is earned and reinvested. So, if the payment frequency was monthly then the interest can be reinvested (compounded) on a monthly basis. If you had a bond with a quarterly coupon then quarterly compounding would normally be assumed when it came to calculating the yield.

Whilst it is assumed that compounding frequency per year is equal to the payment frequency per year (unless otherwise stated), the reverse isn’t always true and if you know the compounding frequency you can’t necessarily assume that the payment frequency will be the same. For example, many bank accounts calculate compounding frequency on a daily basis, but only then pay the interest on an annual basis.

If there are two different saving accounts available to you, both with identical interest, but one pays the interest monthly and the other pays it annually, then if you are looking to reinvest the interest you would end up with more money by investing in the one with the higher payment frequency (the monthly one). For example, let’s say you have £1000 and each account offers 5% interest. The annual one would just give you £50 at the end of the year. The monthly one would give you 5/12 as a percentage each month (which is 0.417%). So at the end of the first month you’d have earned £4.17. Then, next month you’ll earn 0.417% of 1004.17. That’s £4.18, which gets added to the £1004.17. Now, in month three, we have £1008.35 and 0.417% of that is £4.20. The annually paying account would give us the equivalent of 5/12 % a month on a flat £1000, which would be 0.417 each month, or £4.17. So we’re already ahead by month 2 with the regular compounding.

To work out what the equivalent rate would be over the course of year by having these monthly payments, we can use the formula **Compound interest rate = (1+(r/m))(m x n) – 1.**

So, that would mean (1+(0.05/12)(12 x 1) – 1, which is (1.0041712) – 1, which is 0.05116, which works out as 5.12%. Therefore the equivalent return is greater than on the savings account that just offers 5% paid annually as that would be (1+(0.05/1)(1 x 1) – 1, which is just 5%.

### Review Question

Suppose that you invest USD 20,000 in a savings deposit that pays interest at 5% per annum and which is compounded weekly. What will the future value of your investment be exactly 6 years from now? Input your answer correct to two decimal places.

Ok, so for this we are being asked to determine the ‘Future Value’ of our deposit. The equation for that can be found in the ‘Multiple Years’ section above and it is:

**FV = P x (1+(r/m))(m x n)**

**Where: FV = Future value of invested principal**

**P = principal amount invested**

**r = annual interest rate (as a decimal)**

**m = how often each year interest is paid**

**n = number of years**

So, let’s input the information we have:

FV = 20,000 x (1+(0.05/52)(52 x 6)

FV = 20,000 x 1.0009615312

FV = 20,000 x 1.34966

FV = 26993.28510

FV = USD 26,993.29

# Continuous Compounding

The example that was just worked through is an illustration of ‘discrete compounding’ (which sounds like the kind of compounding that might have an affair, but wouldn’t tell everyone about it). This covers where interest is compounded at a discrete (specific) number of points during the year. As seen above, the more often interest is compounded, the higher the rate of compound interest. This makes sense because it’s basically saying that the more frequently we receive interest payments, the more frequently we can then reinvest them and add to the total amount we’re earning money on.

If compounding was to be done constantly (not once a month, week, day or even second), then interest is said to be continuously compounded. There needs to be some sort of rule to calculate this so that everyone uses the same methodology and then rates can be compared (what if interest was calculated and paid every millisecond or 100 times a millisecond?) so, there is a separate formula for this, which takes into account continuous compounding. It uses ‘e’, which is a well-known mathematical principle and irrational number. It’s often called ‘Euler’s Number’ (pronounced ‘Oilers Number’) and amongst other things it’s the base of the natural logarithms (invented by John Napier). However, what’s important is the change to the formulae:

**Continuously Compounded interest = P x (e(r x n) – 1)**

**And Future Value determination:**

**FV = P x e(r x n)**

**Where:**

**P = principal amount invested**

**e = the base of the exponential function (approx. 2.718281828)**

**r = annual interest rate (as a decimal)**

**n = number of years**

Don’t worry about all these letters and what is meant by ‘the base of the exponential function’ – the important thing is that whenever you have a scenario where you need to calculate interest you can put the correct information into the equation, and then get the correct information out afterwards.

For example, let’s say we had a situation where a £10,000 investment will earn a continuously compounded interest rate of 8% per annum for two years. To work out how much it will be worth at that time we just need to plug in the numbers into the right places in the equation:

**FV = P x e(r x n)**

So, in our example, the principal is £10,000, the rate of interest is 8% (or 0.08 as a decimal) and the number of years is 2. Therefore:

FV = 10,000 x e(0.08 x 2)

Multiplying 0.08 by 2 is 0.16 so we need to calculate ‘e’ to the power of 0.16

Note: To get ‘e’ on a windows calculator, you need to use the ‘View’ button, switch to ‘Scientific’ and then – in this example – enter ‘0.16’ then click on the ‘Inv’ button and next to it an ex button will then magically appear (where it previously said ‘In’). Just click on that ex button and it’ll work it out.

That gives 1.17351087.

FV = 10,000 x 1.17351087

FV = £11,735.11

Remember, the more often interest is compounded, the higher the future value amount. So, continuous compounding should yield a higher FV than if it were calculated on a daily basis. We can check though. For the daily basis, using the same example of £10,000 at 8% over two years, the maths would be:

**FV = P x (1+(r/m))(m x n)**

FV = 10,000 x (1+ 0.08/365)(365 x 2)

FV = 10,000 x 1.000219730

FV = £11,734.90

Remember, with continuous compounding we got £11,735.11. So there it is: continuous compounding has yielded a higher future value than daily compounding, just as we thought.

## Comparisons between different compounding frequencies

If we were to put together a table to show the impacts of different compounding frequencies on an investment of £1 with an interest rate of 8% over 1 year (so ‘n’ would be 1), it would look like this:

|  |  |  |  |
| --- | --- | --- | --- |
| **Compounding Frequency** | **r/m** | **m x n** | **Future Value of £1** |
| Annual | 0.08/1 = 0.08 | 1 | 1.08 |
| Semi-annual | 0.08/2 = 0.04 | 2 | 1.0816 |
| Quarterly | 0.08/4 = 0.02 | 4 | 1.08243216 |
| Monthly | 0.08/12 = 0.006667 | 12 | 1.082999507 |
| Daily | 0.08/365 = 0.000219 | 365 | 1.083277572 |
| Continuous | 0.08 | 1 | 1.083287068 |

This shows that a £1 investment earning 8% compounded semi-annually would effectively earn 8.16% interest over the course of a year. Therefore there can be a difference between the stated interest rate and the ‘effective annual rate’ (EAR). So an 8% stated interest rate has an EAR of 8.16% with semi-annual compounding.

In the US, the EAR is known as the ‘annual percentage yield’ (APY) and the stated interest rate is known as the ‘annual percentage rate’ (APR).

### Example

Suppose you deposit USD 10,000 in a bank account that pays an APR of 7%, compounded quarterly. What is the EAR of this investment?

So the EAR is just the compound interest rate, which is:

**Compound interest rate = (1+(r/m))(m x n) – 1**

So, let’s put in the numbers:

EAR = (1+(0.07/4))(4 x 1) – 1

EAR = (1+0.0175)4 – 1

EAR = 1.01754 – 1

EAR = 0.07185903

EAR = 7.19%

# Rearranging the Formulas

If you start off knowing the compound interest rate and instead want to work backwards to find out what the stated annual interest rate will be, then you can just rearrange the formula. This works for when you have a discrete case of compounding as well as a continuously compounding scenario, although the formulae are different:

## Discrete Case

CI = (1 + r/m)(m x n) – 1

Can be rearranged as:

**r = m x [(1 + CI)(1/(m x n)) – 1]**

## Continuous Case

CI=er – 1

Can be rearranged as:

**r = In(1+CI)**

**Where:**

**CI = compound interest rate**

**r = stated interest rate**

**In = the natural logarithm** (don’t worry about exactly what’s meant by this – just press the ‘In’ button on a calculator when required)

### Example

Your bank tells you that the discretely compounded interest on your USD 10,000 deposit is 10.5% per annum. This is based on semi-annual compounding. What is the stated interest rate published by the bank?

First off, this would be discrete case so we need the formula:

**r = m x [(1 + CI)(1/(m x n)) – 1]**

Now we put in the details we have:

r = 2 x [(1 + 0.105)(1/2 x 1) – 1]

r = 2 x [(1.1050.5)– 1]

r = 2 x 0.0511898

r = 0.1023796

r =10.24%

### Example

What would the stated bank interest rate be if you were told that the continuously compounded interest rate was 10.5% per annum?

So, now we need the continuous case equation where we know CI:

**r = In(1+CI)**

r = In(1+0.105)

r = In(1.105)

r = 0.099845

r = 9.98%

Note: to get the ‘In’ part of the equation, change to the ‘Scientific’ windows calculator, then enter ‘1.105’ then click the ‘In’ button. Nice and easy.

# Comparing Different Compounding Bases

There can be a need to compare different compounding bases. For example, you might wish to compare two different investments where the interest is compounded quarterly on one and monthly on the other. We may might want to know what the stated interest of the quarterly one would be if the interest was compounded monthly. Then we could compare the two against each other.

There are more formulas (yay!) to draw upon here and, as before, there’s one for the discrete case (which covers when interested is compounded at specific points throughout a year) and the continuous case (when it’s just compounded constantly):

## Discrete Case

**r2 = [(1 + r1 / m1)m1/m2 – 1] x m2**

**Where:**

**r1 = the stated interest rate with a compounded frequency of m1**(basically which ever stated interest rate you already know for r1 and how many times a year that interest is currently paid for m1)

**r2 = the stated interest rate with a compounded frequency of m2**(basically, r2 is the interest rate you are looking to work outand m2 is how many times a year that interest rate would be paid)

## Continuous Case

**R = n(eR/n – 1)**

**Where:**

**R = the interest rate on a continuous compounding basis**

**r = the stated interest rate with a compounding frequency of n**

**e = the base of the exponential function**

### Example

Let’s say that there’s an investment compounded semi-annually with a stated interest rate of 9% p.a. and you want to know what the stated interest rate would be if it was compounded quarterly. So, as the compounding is done a specified number of times a year, we need the discrete case:

**r2 = [(1 + r1 / m1)m1/m2 – 1] x m2**

Now we need to input the numbers:

Remember: we’re trying to work out the rate of interest for the compounding happening at a different frequency (r2) so we don’t have a value for that. r1 is the rate of interest currently against the compounding, expressed as a decimal. In this instance, it’s 9% so that would be 0.09 as a decimal. m1 is how many times a year the compounding is currently being done. It’s semi-annual so that means it’s being done 2 times (or twice, if you will) so that would be 2. m2 is how often the compounding would be done in our new world order; how frequently do we want it to occur? In this example that’s quarterly so 4 times a year. Therefore, we’ve got the numbers and now we just need to put them into the right places:

r1 = 0.09

m1 = 2

m2 = 4

Therefore:

r2 = [(1 + 0.09/2)2/4 – 1] x 4

r2 = [(1 + 0.045)0.5 – 1] x 4

r2 = [1.022252 – 1] x 4

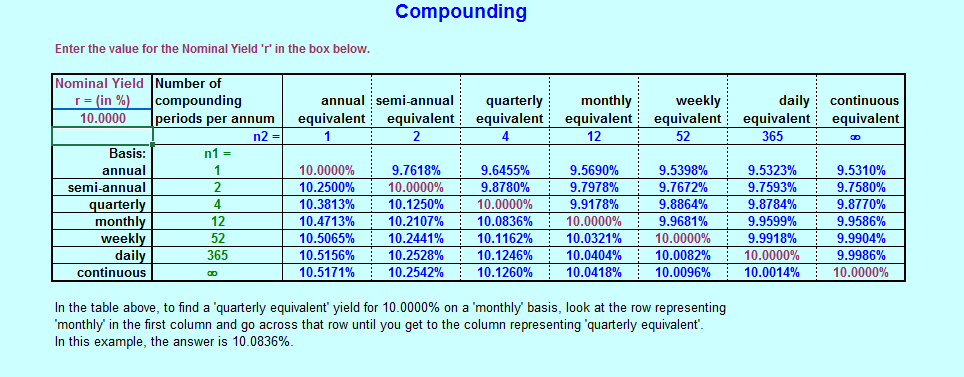
r2 = 0.022252 x 4

r2 = 0.089008

r2 = 8.90%

This means that the stated rate on an investment with compounding on a semi-annual basis is higher than the equivalent rate if the compounding were done more frequently. This makes sense because we saw earlier on that if the stated rate was the same then the compounding rate increases the more frequently compounding occurs. Therefore, for the compounding rate to stay the same, the standard rate must reduce the more often compounding occurs.

As a demonstration of how the rates change, see the below table:



As you can see, if you have a rate of 10% and wish to convert it to see what the equivalent rate would be if compounding took place less regularly (from weekly to annually, for example) then the rate has to go up to compensate for the fact that you’re adding less money to the investment.

# Day Count

When calculating interest on a daily basis over the course of a year, most right-minded people would probably assume that the year contained 365 days. Already though there could be conflict as what if it was a leap year? Different parties might input different numbers; a bank might advertise a savings account with compounding occurring daily and assume 365 days but an investor might work on the principle that it contained 366 and demand returns accordingly. To confuse things further, some people calculate interest based upon a year containing just 360 days, and different months can contain different days than you might expect.

Depending on the ‘day count basis’ on which interest is calculated, the amount of interest due is different for a given principal amount, interest period and interest rate.

There are two types of day count basis: bond basis and money market basis.

## Bond Basis

Bond basis is usually used for longer term fixed income securities. It’s important to be able to accurately calculate the number of days between two dates that interest is being paid (coupon dates) because if the bond is traded in between two coupon dates then the number of days left until the next coupon will affect its value (as interest will be accrued over this time period). There are various ways of calculating the day count:

### Actual / 365 basis

This uses a day count fraction equal to the actual (think of it as ‘real’) number of days in an investment period divided by 365 (regardless of whether it might actually be a leap year). For example, if we invest £1,000,000 in a bond carrying an interest rate of 10% p.a. where the principal and interest are returned to us after one year then we would receive:

1,000,000 x 0.1 x 365/365 = 100,000 (remember, the very first calculation we came across was **Interest = P x r x t** where we multiply the principal by the rate of interest (as a decimal) by the time period)

However, it was a leap year then the equation would be:

1,000,000 x 0.1 x 366/365 = 100,273.97

### Actual / Actual

This uses a day count fraction equal to the actual number of days in an investment period divided by the actual number of days in that year (so 365 in a standard year but 366 if it’s a leap year).

### 30/360

In the US corporate bond market, the 30/360 basis is used, which assumes that every month is composed of exactly 30 days and a year is therefore exactly 360 days long. Not accurate, but easier mathematically.

## Money Market Basis

The money market basis uses a day count fraction equal to the number of days of the investment period divided by 360 (actual / 360), except in the UK and other Commonwealth countries where it’s the number of days of the investment period divided by either 365 or 366 depending on whether it’s a leap year or not (actual / actual). For example, in the £1,000,000 investment example detailed under the ‘Actual / 365’ basis in the Bond Basis, above, using the money market basis would earn more:

1,000,000 x 0.1 x 365/360 = 101,388.89

However, using the UK ‘actual / actual’:

1,000,000 x 0.1 x 365/365 = 100,000

So the (non-UK) method of calculating the time period is clearly more attractive for investors.

## Link between Bond Basis and Money Market Basis

There’s a better yield using the money market basis (non-UK) than using the bond basis, as the money market equation multiplies ‘x/360’ while the bond equation multiplies ‘x/365’. The difference between the two is 1.38889%. Therefore a bond basis investment has to be more than 1.389% p.a. greater than a money market basis investment to make it worthwhile investing.

The equation to convert one rate to another would be as follows:

Converting from bond basis to money market basis

**rb = rm x 365/360**

Converting from bond basis to money market basis:

**rm = rb x 360/365**

**Where:**

**rb = interest rate expressed on a bond basis**

**rm = interest rate expressed on a money market basis**

### Example

Would it be more worthwhile to invest £100,000 for one year with Bank A, which offers you 8% p.a. on a bond basis, or with Bank B, which offers you 7.80% on a money market basis?

So, let’s convert the offering from Bank A from bond basis to money market basis:

rm = rb x 360/365

rm = 0.08 x 360/365

rm = 0.08 x 0.986301

rm = 0.789041

rm = 7.89%

Therefore, Bank A’s offer is better because it’s the equivalent of a money market basis of 7.89%, but Bank B is offering less than that at 7.80% on a money market basis.

#### Shortcut

Another way to work it out is to see if the bond basis rate is greater than 1.389% more than the money market basis. To find this out, simply divide the bond rate basis by the money market basis and minus 1, then see if the resulting number is greater than 0.01389. If so then the bond basis is better. In this example, (8/7.80) – 1 = 0.0256, and 0.0256 is greater than 0.01389 so that confirms that the offering is better than Bank B.

### Example 2

So far we’ve been focusing on investments of a year, but what if the investment period isn’t as long as a year? Let’s say I want to see what the rate of interest would be if I invested £1000 for 30 days at a rate of 10% p.a. In that case all that needs to be done is to insert the number of days corresponding to the investment period (also known as the investment horizon) into the equation:

**Interest = P x r x t**

Here, ‘t’ is the number of days in the investment period divided by the number of days in the year (which is dependent on which day count basis is being used). So the formula could be rewritten as:

**Interest = P x r x d/D**

**Where:**

**d = investment horizon in days**

**D = total number of days in the year**

So, in the example we would input:

Interest = 1000 x 0.1 x 30/365 (that 365 might be 366 if it was a leap year and it might be 360 using a money market basis).

Interest = 1000 x 0.1 x 0.082192

Interest = £8.22

Therefore the interest rate is 8.22/1000, which is 0.82%

### Example 3

You have borrowed GBP 5,000 from a bank for 75 days. You have to pay back GPB 102.74 at the end of this period. Assuming that it is a non-leap year, what is the annual interest correct to two decimal places?

So, we need to find the rate of interest – ‘r’. To solve this we need to input the data we have into this equation:

**Interest = P x r x d/D**

That gives us:

102.74 = 5000 x r x 75/365

102.74 = 5000 x r x 0.20548

102.74 = r x 1027.40

r = 1027.40 / 102.74

r = 10.

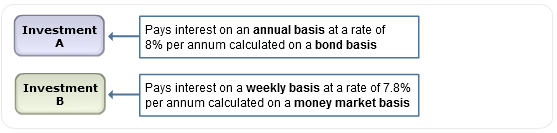
Therefore the rate of interest is 10.00%

# Procedure for Comparing Investments

If you need to compare two different investments that have different compounding bases and different day count bases then you have to convert them so that both the compounding bases and day count bases are the same. First of all you need to convert them to same day count basis, then once that’s happened you can convert them to the same compounding basis. Interestingly (I say ‘interestingly’… I do like to play fast and loose with semantics), when you convert them to the same compounding basis, it doesn’t have to be equal to the original compounding basis of either of the investments. For example, one investment might have a weekly compounding basis and another might have a monthly compounding basis so you could just convert them both to an annual compounding basis.

### Example

Consider the below. How would we compare these two investments:



Let’s say we want to see both of them on a bond basis with semi-annually compounded rates.

So, we need to first of all convert Investment B to bond basis. The relevant conversion calculation given earlier was:

**rb = rm x 365/360**

so:

rb = 0.078 x 365/360

rb = 0.078 x 1.013889

rb = 0.07908

rb = 7.91%

Then we have to convert Investment B from a weekly basis to a semi-annual basis so we need our conversion equation to see what the new rate would be. It was paid weekly so m1 would be 52 (as there are 52 weeks in a year) and we want it to be paid semi-annually so m2 would be 2:

**r2 = [(1 + r1 / m1)m1/m2 – 1] x m2**

r2 = [(1 + 0.0791/ 52)52/2 – 1] x 2

r2 = [1.0015226 – 1] x 2

r2 = [1.04031 – 1] x 2

r2 = 0. 04031 x 2

r2 = 0.08062

r2 = 8.06%

Now we have to convert Investment A from an annual basis to a semi-annual basis:

**r2 = [(1 + r1 / m1)m1/m2 – 1] x m2**

so:

r2 = [(1 + 0.08/ 1)1/2 – 1] x 2

r2 = [1.080.5 – 1] x 2

r2 = 0.03923 x 2

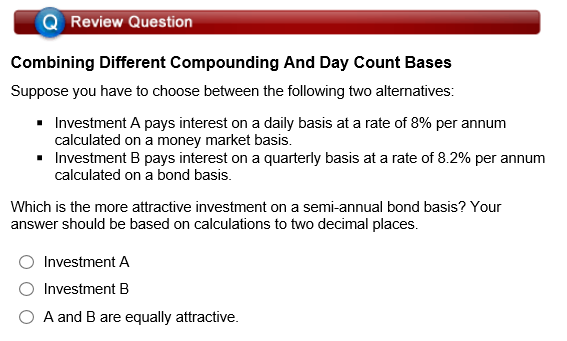
r2 = 0.07846

r2 = 7.85%

Right. Now we can see how the two investments compare. Investment A pays interest on a semi-annual basis at a rate of 7.85% p.a. and Investment B pays interest on a semi-annual basis at a rate of 8.06% p.a. So we made sure both were comparable by both being converted to bond basis (we could have done it on a money market basis had the urge taken us) and on a semi-annual compounding basis (could have been annual if we’d wanted though).

Altogether, now we can see that Investment B gives a better return.

### Example from slide 51 of Intuition



I found this interesting because if you calculate it by converting Investment B to money market basis and then convert both to semi-annual then you get a different answer about how they compare than if you convert Investment A to bond basis and then convert both to semi-annual:

First of all, let’s answer the question. We need to convert Investment A to bond basis, then convert both investments to semi-annual. So first of all to get A from money market to bond basis we need to use the conversion formula:

**rb = rm x 365/360**

So, rb = 0.08 x 365/360

= 0.08 x 1.01389

= 0.0811

Then we convert it from daily to semi-annual:

**r2 = [(1 + r1 / m1)m1/m2 – 1] x m2**

r2 = [(1 + 0.0811 / 365)365/2 – 1] x 2

r2 = [(1 + 0.00022)182.5 – 1] x 2

r2 = [1.04138 – 1] x 2

r2 = 0.08277

r2 = 8.28% (to 2 decimal places)

Now we have to convert investment B from quarterly to semi-annual using the same conversion equation:

r2 = [(1 + 0.082 /4)4/2 – 1] x 2

r2 = [(1 + 0.082 /4)4/2 – 1] x 2

r2 = [(1+ 0.205)2 – 1] x 2

r2 = [1.04142 -1] x 2

r2 = 0.08284

r2 = 8.28% (to 2 decimal places)

So they’re the same, which is the answer that Intuition is looking for.

However, if I was to convert Investment B to money market basis then change to semi-annual it would be:

0.082 x 360/365 = 0.08087 or 8.09%

Then:

r2 = [(1 + 0.0809/ 4)4/2 – 1] x 2

r2 = 0.08172 = 8.17%

Then convert A from daily to semi-annual:

r2 = [(1 + 0.08/ 365)365/2 – 1] x 2

r2 = 0.08161 = 8.16%

So there is a slight difference between the two investments but it’s not so obvious when converting both to bond basis. I just found that interesting. On my own? Alrighty then.

# Decompounding

We’ve had an absolute whale of a time compounding away merrily, but you’ll be able to amaze your friends and confound your enemies by reversing the process and ‘decompounding’. This is where you work backwards from a compounded rate or amount (the amount being the principal plus interest) in order to work out the original rate or principal. The formula is the old favourite used when switching from one compounding basis (frequency) to another:

**r2 = [(1 + r1 / m1)m1/m2 – 1] x m2**

Suppose that interest is paid semi-annually at the rate of 10% p.a. What is the equivalent rate that is paid quarterly instead?

Using the above formula we can see:

r2 = [(1 + 0.1/2)0.5 – 1] x 4 = 9.88%

Logically, the semi-annual rate (denoted as r6)should be equal to two lots of the quarterly rate (denoted as r3). This is expressed as:

(1 + r6/2) = (1 + r3/4) x (1 + r3/4)

Note: the numbers after the / in each set of brackets are indicative of how often the interest payments occur each year.

And sure enough:

(1 + 0.1/2) = (1 + 0.0988/4) x (1 + 0.0988/4)

If we wanted to see the value of the investment then we would just multiply both sides by the initial principal (P):

P x (1 + r6/2) = P x (1 + r3/4) x (1 + r3/4)

### Example

“Suppose that an investment of USD 15,000 pays interest quarterly at the rate of 9% p.a. What would be the equivalent monthly interest rate such that at the end of the three months, the investor is no better or worse off regardless of the frequency of compounding? Input your answer correct to three decimal places.”

Oakaly doakaly. So, we just need to use the usual change in compounding frequency:

**r2 = [(1 + r1 / m1)m1/m2 – 1] x m2**

r2 = [(1 + 0.09/4)4/12 – 1] x 12

r2 = [(1.0225)1/3 – 1] x 12

r2 = 0.007444442749 etc. x 12

r2 = 0.08933 (to three decimal places)

r2 = 8.933%

Now we can check it to see if it works, taking into account that the quarterly payments are due 4 times a year and the monthly payments are due 12 times a year. Also, it would be logical to expect a quarterly payment (which occurs every 3 months) to be equal to 3 monthly payments):

P x (1 + r3/4) = P x (1 + r1/12) x (1 + r1/12) x (1 + r1/12)

So, our principal (P) is the USD 15,000.

15,000 x (1 + 0.09/4) = 15,000 x (1 + 0.08933/12) x (1 + 0.08933/12) x (1 + 0.08933/12)

15,000 x 1.0225 = 15,000 x 1.00744 x 1.00744 x 1.00744

15.337.50 = 15,337.49

That’s pretty damn good (and had we extended the 8.933 to beyond 3 decimal places then it would be even closer).

# Conclusion

Hopefully you should now be confident in running interest calculations. Remember: the first half of the battle is working out which equation to use, and the next half is then making sure you identify the correct information to input into the equation. The maths part itself is actually straightforward.

Further explanation of interest rate compounding can be found here: <http://www.mathsisfun.com/money/compound-interest-periodic.html>, which is a very easily digestible introduction to it all.