

1a)

$$\begin{aligned} P(Y|X) &= \frac{P(YX)}{P(X)}; \text{definition of conditional probability} \\ &= \frac{P(X|Y)P(Y)}{P(X)}; \text{chain rule} \end{aligned}$$

1c)

$$\begin{aligned} P(Y|x_1, x_2, \dots, x_{|w|}) &= \frac{P(Y, x_1, x_2, \dots, x_{|w|})}{P(x_1, x_2, \dots, x_{|w|})}; \text{definition of multivariate conditional probability} \\ &= \frac{P(x_1, x_2, \dots, x_{|w|}|Y)P(Y)}{P(x_1, x_2, \dots, x_{|w|})}; \text{chain rule} \end{aligned}$$

1d)

$$\begin{aligned} P(Y) &= \frac{|\{m|m \text{ is a movie on IMDB that was released in year } Y\}|}{|\{m|m \text{ is a movie on IMDB}\}|} \\ P(Y|x_i) &= \frac{|\{r|r \text{ is a review where word } i \text{ appears } x_i \text{ times and } r\text{'s movie was released in year } Y\}|}{|\{r|r \text{ is an IMDB review where word } i \text{ appears } x_i \text{ times}\}|} \\ P(x_i|Y) &= \frac{|\{r|r \text{ is a review where word } i \text{ appears } x_i \text{ times and } r\text{'s movie was released in year } Y\}|}{P(Y)} \end{aligned}$$

1e)

$$\begin{aligned} P(Y|x_1, x_2, \dots, x_{|w|}) &= \frac{P(x_1, x_2, \dots, x_{|w|}|Y)P(Y)}{P(x_1, x_2, \dots, x_{|w|})}; \text{Bayes Rule} \\ &= \frac{P(x_1|Y)(P(x_2, \dots, x_{|w|}|Y, x_1)P(Y))}{P(x_1, x_2, \dots, x_{|w|})}; \text{chain rule} \\ &\dots \\ &= \frac{P(Y)}{P(x_1, x_2, \dots, x_{|w|})} \prod_{i=1}^{|w|} P(x_1, x_2, \dots, x_i|Y, x_1, x_2, \dots, x_{i-1}) \end{aligned}$$

1i)

$$\begin{aligned} P(Y|x_1, x_2, \dots, x_{|w|}) &= \frac{P(Y)}{P(x_1, x_2, \dots, x_{|w|})} \prod_{i=1}^{|w|} P(x_1, x_2, \dots, x_i|Y, x_1, x_2, \dots, x_{i-1}) \\ &= \frac{P(Y)}{P(x_1, x_2, \dots, x_{|w|})} \prod_{i=1}^{|w|} P(x_i|Y) \end{aligned}$$