Università di Pisa

Computer Engineering

Performance Evaluation of Computer Systems and Networks

Slotted random-access wireless network

Group Project Report

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1 Introduction

1.1 Problem Description

From the group project assignment:

*In a **slotted random-access network**, **N couples transmitter-receiver** share the same communication medium, which consists of **C separate channels**. Multiple attempts to use the same channel in the same slot by different transmissions will lead to collision, hence no receiver listening on that channel will be able to decode the message. Assume that each of the N transmitters generate packets according to an **exponential inter-arrival distribution**, and picks its channel at random on every new transmission. Before sending a packet, it keeps extracting a value from a **Bernoullian RV with success probability p** on every slot, until it achieves success. Then it transmits the packet and starts over. If a collision occurs, then the transmitter backs off for a random number of slots (see later), and then starts over the whole Bernoullian experiment. The number of back-off slots is extracted as* $U(1, 2^{x+1})$*, where x is the number of collisions experienced by the packet being transmitted.*

1.2 Objectives

The aim of the project report is the *Assessment of the Effectiveness of the Slotted Random-Access Network Protocol* described in the latter paragraph.

1.3 Performance Indexes

In order to define a metric of performance of the objective, the following Performance Indexes are defined:

- **Throughput**: let Tp be the Throughput to be measured, N_p the number of packets successfully sent to the corresponding receiver, N_t the number of time-slot considered in the count of N_p, T_{slot} the period (in seconds) of a time slot, the Throughput(per slot) can be measured as:

$$Tp(slot) = \frac{N_p}{N_t} \quad [packets/slot]$$

  We can also convert this performance metric in a more standard form, dealing with packets per second:

$$Tp = Tp(slot) * T_{slot} \quad [packets/s]$$

- **Response Time**: defined as the time that occurs from the first appearance of one packet at the Transmitter up to the reception of the packet at the Receiver.

- **Mean Number of Packets in the queues**: due to the fact that the this metric is time dependent, will be computed as:

$$E[N] = \frac{\sum_{i=0}^{N} N_i * (t_{i+1} - t_i)}{t_N - t_0}$$

2 Modeling

2.1 Introduction

We model the system as N couples of transmitter and receiver which communicate through C Channels. A collision can occur on a channel if more than one transmitters want to transmit a packet in that channel. The transmitter stores in a queue the packets that it wants to transmit and, then, it sends them; the channels "knows" if a collision occurs and handles it; the receiver only receive packets.

2.2 General Assumptions

The following general assumptions have been made:

- **Slotted**: packets are attempted to be transmitter by the Transmitter only at the **beginning of the time-slot**
- **Constant Packet Size and Transmission Rate**: each packet has a **constant packet size** and each transmitter has a constant and equal transmission rate for which to transmit a packet (without collision) from the receiver to the transmitter will last **one time-slot**.
- **No Propagation Error in the channel**: the only cause of a failed transmission has to be considered as the packet collision. Other causes, (i.e. path-loss, shadowing, small-scale fading), are neglected.
- **FIFO Queues of unlimited Capacity at the Transmitter**
- **Transmitters and Receivers always synchronized with the time-slot period**: in addition the receiver knows in which channel the transmitter will try to send his packet in each time-slot and the receiver will be ready to listen in the correct channel.
- **After an eventual collision the packet will change his channel choice**

2.3 Preliminar Validation TODO Forse cancella

Before the implementation a preliminar validation phase is necessary to ensure that the model is correct. Let analyze if the assumptions made in the previous section are reasonable:

- The pure slotted assumption is reasonable due to the fact that exist some network protocols which work under this assumption, see for instance Slotted Aloha that is the most famous one.
- We consider the packet size constant because, in a network, there are small packet and huge packet, but if we want to consider the performance, then we have to take a mean length, otherwise it's possible that we consider too small or too large packets. Furthermore if the packet length is so large that more than one slots are needed, we can consider, from the viewpoint of the model, that this unique packet send in two different slots is like two packets of fixed-length send each one in a slot.
- The critical issue of every slotted network is the one related to the collisions: they have an huge impact on the general performance of the network, so it is reasonable to neglect the other propagation errors that are not network-specific or that depend from the environment (as path-loss).
- It's reasonable that the transmitter and the receiver are synchronized with the time-slot period because otherwise it would be very difficult every type of communications between the two entities. Moreover also slotted ALOHA requires a synchronization of this type.

- When a packet collide it's reasonable to think that the transmitter will change the transmission channel in order to avoid another collision. Indeed this along with the back-off time are the techniques that should avoid another collision.

2.4 Factors

The following factors have been defined which may affect the performance of the system:

- **N**: Transmitter-Receiver Couples.
- **C**: numbers of Channels.
- **p**: probability of success for sending a packet in the current time-slot for a Transmitter.
- λ: exponential distribution rate of packets arrival at the Transmitter.
- T_{slot}: time-slot duration.

3 Implementation

3.1 Modules

The following modules have been defined:

- **Transmitter**: duty of sending a packet to a specific channel. Parameters: *lambda*, p, packetsInQueue Signal (for the queue)
- **Channel**: duty of checking every channel in each timeslot for any collision. Parameters: $T_{timeslot}$, Throughput Signal
- **Receiver**: duty of receiving a packet from the channel. Parameters: Response Time Signal

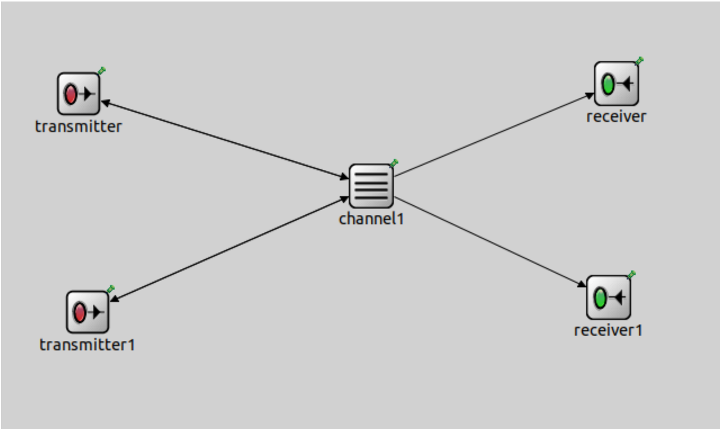


Figure 1: Network example

3.2 Messages

A new format of message has been defined, in order to store all packet information,with the following fields:

- *simtime_t creationTime*: time in which the packet arrives for the first time at the Transmitter
- *int idChannel*: Channel chosen for the current transmission, may change in case of collision
- *int idTransmitter*: Index of the module Transmitter related to the message
- *int indexTx*: Index of the gate in which the Transmitter is linked to the Channel

3.3 Modules Behaviour

3.3.1 *Transmitter* Module Behaviour in the implementation

1. **Message Arrival** at the *Transmitter*:
   - **IF** an *ACK* has been received the packet at the top of the queue can be removed. **GOTO (2)**
   - **ELSE IF** a *NACK* has been received, then the *Transmitter* starts his backoff time and **waits for another message**.
   - **ELSE IF** an *Synchronization message* has been received **AND** the *Transmitter* is not in backoff-time **GOTO (2)**

- **ELSE IF** a packet arrives at the *Transmitter*, the Transmitter stores the packet in the queue, make a reschedule of the arrival of the next packet and **waits for another message**

2. The *Transmitter* tries to send the packet

    - **IF** success on the Bernoullian Experiment, then the packet will be forwarded to the *Channel.*
    - **ELSE waits for another message**

3.3.2 *Channel* Module Behaviour in the implementation

1. The *Channel* wakes up at the beginning of each time.slot and checks his channels status.

    - **IF** two or more packets have arrived in the same channel, the *Channel* will send a NACK to the *Transmitters* that have forwarded the packets in that specific channel.
    - **ELSE IF** one and only one packet has arrived in a channel, the *Channel* will send an ACK to the relative *Transmitter* and will forward the packet to the relative *Receiver.*
    - **ELSE** the *Transmitters* that did not send a packet will receive from the Channel a *Synchronization Message*

2. The *Channel* will gather of the packets for the current time.slot, to be processed in the next one.

    So this means that if the channel receives packets in time-slot j, then the information about collisions are provided to transmitters in time-slot $j+1$. Hence, from the point of view of the transmitter, the information received at $j+1$ are referred to events took place in j

3.3.3 *Receiver* Module Behaviour in the implementation:

1. The *Receiver* wakes up when a packet arrives. Then are computed some statistics (concerning Response Time).

4 Verification

In this section we present tests performed in order to verify that our implementation reflects correctly our model.

4.1 Degeneracy Tests

In the degeneracy tests we verify the behaviour of our simulator with parameters set to 0 values. In all tests the simulator works properly. In particular the following observations can be inferred:

- If the number of channel is 0 then the simulation stops immediately because has no sense running a simulation with 0 channels.
- If the time slot size is 0 then the simulation doesn't stop and goes to infinite because on instant 0 time slots are continuously triggered.
- If the exponential mean is 0 then the simulation doesn't stop and continues to infinite because packets arrive at time 0 continuously.

4.2 Consistency Test

The consistency test verifies that the system react consistently with the output. In order to test this we perform two tests with the following parameters.

Test 1

- Number of couple tx-rx: 1
- Number of channels: 500
- Send probability: 1
- Mean inter-arrival time: 10s (deterministic)
- Time slot size: 5s

Test 2: Two couple TX-RX with half packets arrival rate

- Number of couple tx-rx: 2
- Number of channels: 500
- Send probability: 1
- Mean inter-arrival time: 20s (deterministic)
- Time slot size: 5s

We expect that the result of the two tests are more or less equal because the behaviour of one source transmitting every 10 seconds must be similar to the behaviour of two sources transmitting every 20 seconds. We set the number of channels at 500 in order to neglect the effect of collisions (in any case it is possible that a collision occur, but in the following tests no collisions have been detected).
The graph in figure 2 and in figure 3 show the results of the tests previously explained. We can see that the behaviour is very similar in both cases and so that the systems works as we expect. In fact the Mean Throughput per slot tend in both cases to 0.50 packets per slot.

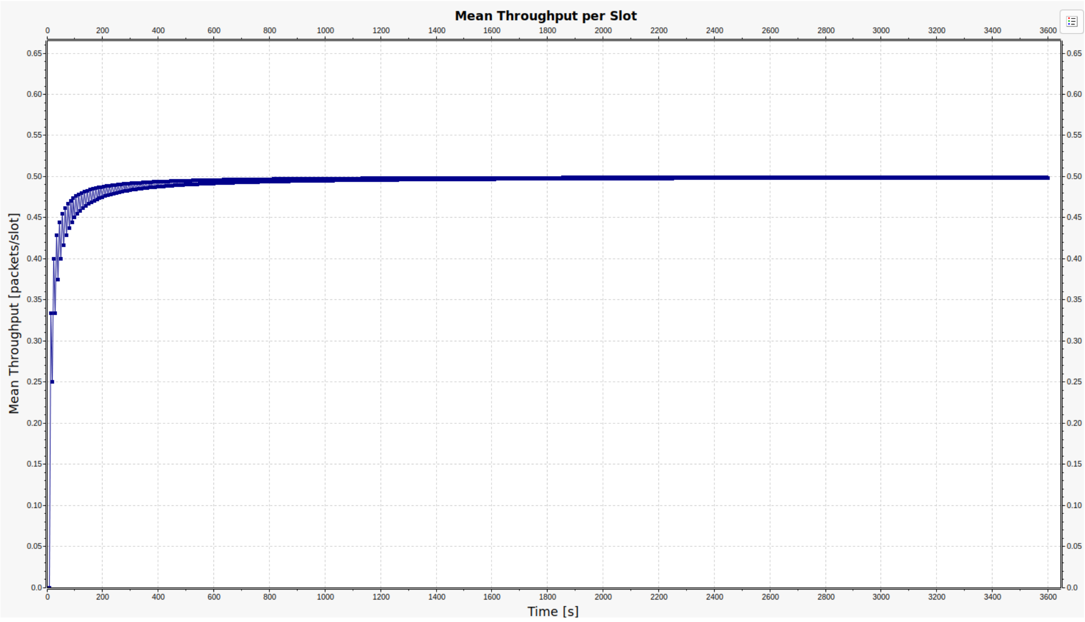


Figure 2: Consistency Test 1

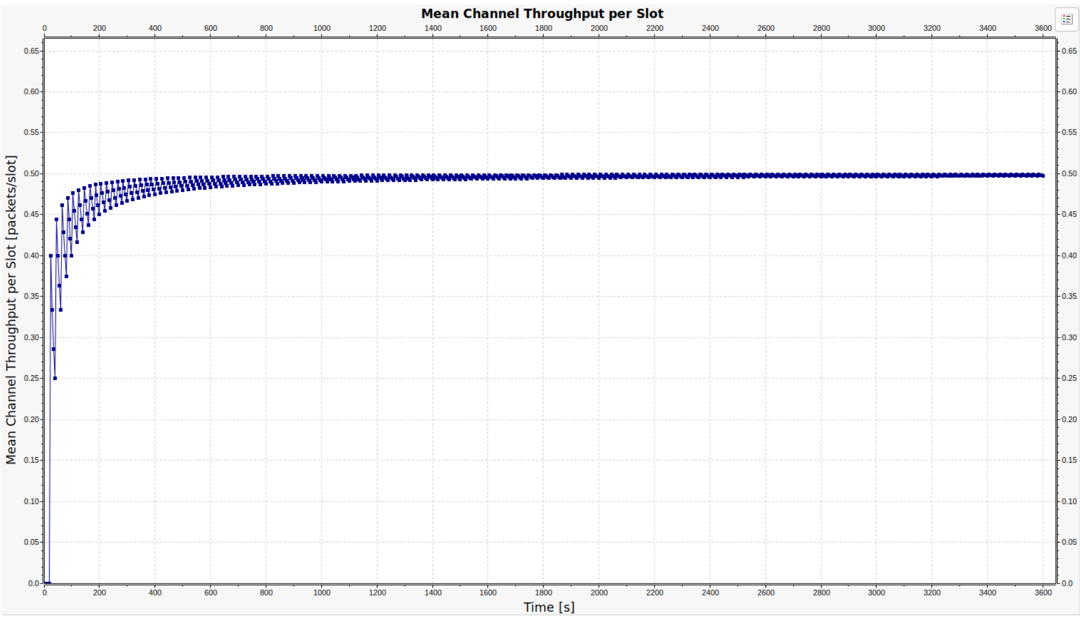


Figure 3: Consistency Test 2

The oscillations at the beginning are due to the fact that the mean inter-arrival time is bigger with respect to the time slot size. In fact we can observe that in the second test there are larger oscillations because the difference between the mean inter-arrival time and the time slot size is bigger than the one of the first test.
Now let analyse the response time, we can see its measure in both tests in figure 4 and 5.

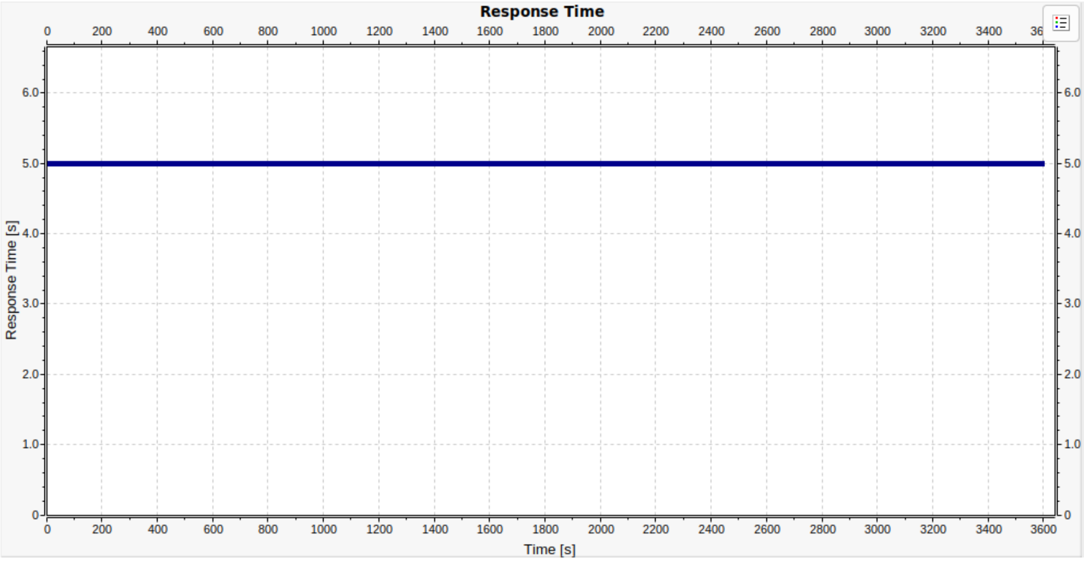


Figure 4: Consistency Test 2 - Response Time

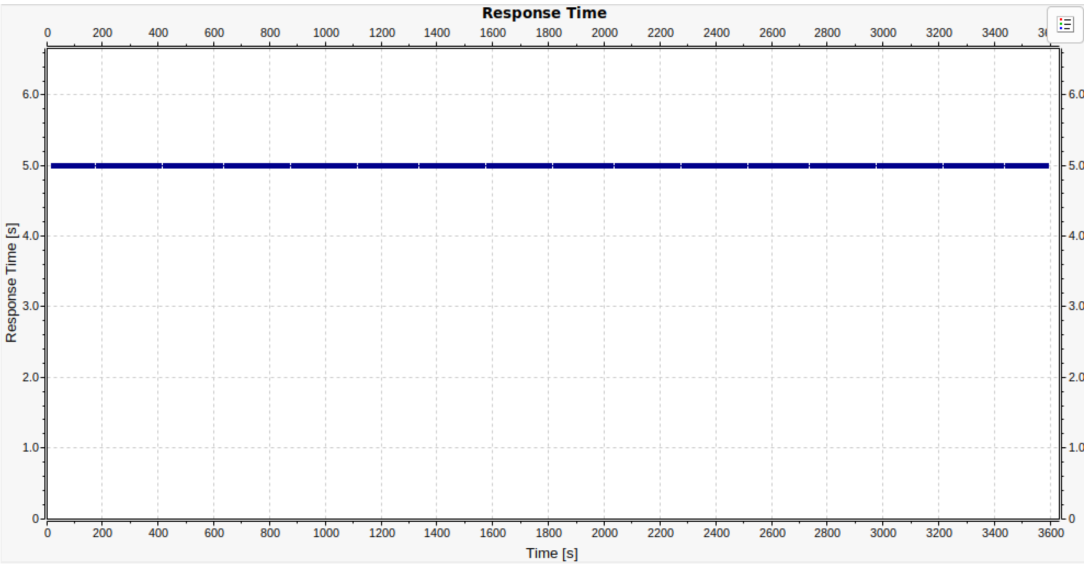


Figure 5: Consistency Test 2 - Response Time

The mean response time in both cases is 5 seconds, this is a lower bound for the response time because in both tests 5 seconds is the size of the slot time so it's impossible to have a response time lower than the slot size.
We can see that in the second test we have a less continue plot, this is due to the fact that the mean inter-arrival time is bigger in the second case and so the packet are more distant in time. We can conclude that also for what concerns the response time the consistency is ensured because it is the same in the case in which we have a source transmitting each 10 seconds and in the case in which we have two sources transmitting each 20 seconds.

4.3 Continuity Test

In this test the aim is to prove that the output changes slightly if the input changes a bit. In order to do prove this, 2 simulations have been performed with the parameters shown in table 1. With this parameters we have changed slightly the input, and so we expect that the outputs don't show particular differences. Obviously some differences will be present (in particular due to collisions and the increasing in the number of transmitters) but they shouldn't affect a lot the results.

Test	Number of Transmitters	Number of Channels
1	8	20
2	10	20

Table 1: Continuity test parameters

The remaining parameters are the same of all of four tests:

- Send probability: 1
- Mean Inter-arrival time: 10 sec (deterministic)
- Time slot size: 5 sec
- Threshold: 20 sec

The results of the simulation as shown in the figure 6 and 7 (we measure the mean throughput).

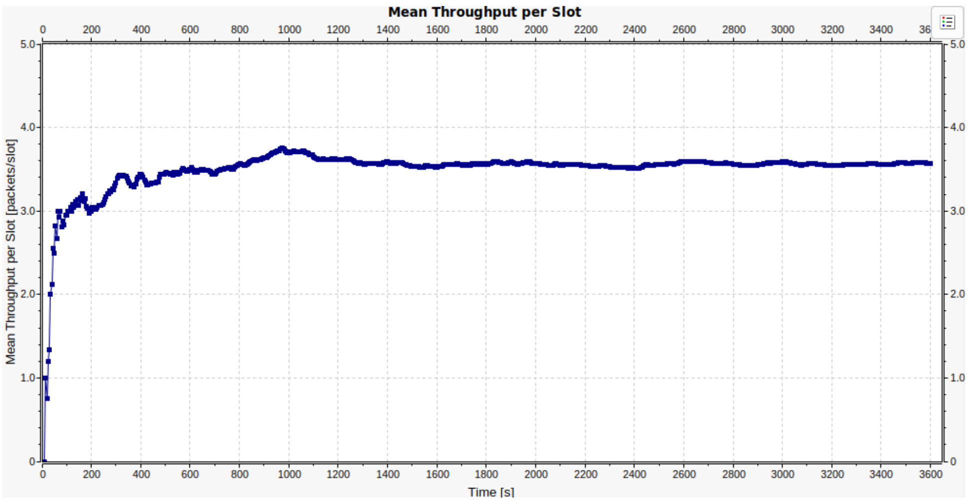


Figure 6: Continuity Test 1 - Mean Throughput per Slot - Collisions detected: 272

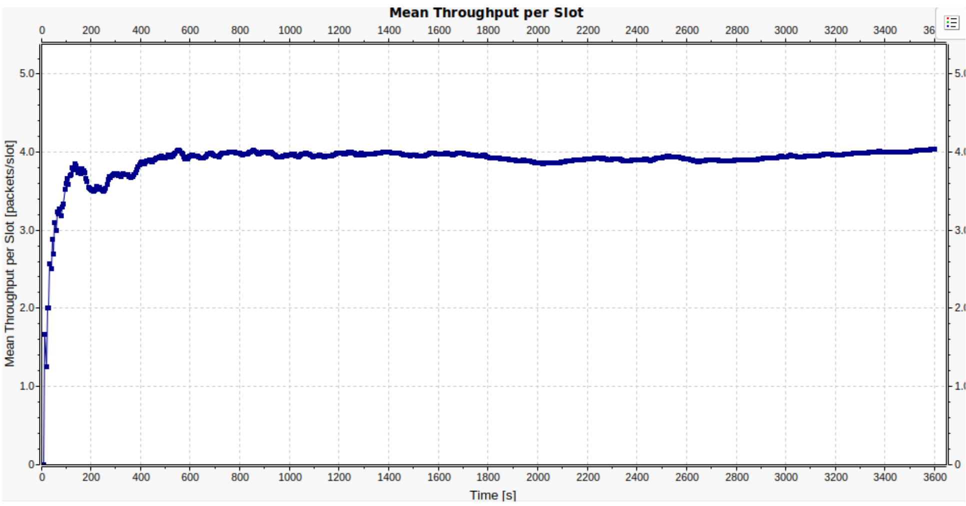


Figure 7: Continuity Test 2 - Mean Throughput per Slot - Collisions detected: 390

We can observe that the output changes slightly between the two cases. In particular it's possible to infer that if the number of transmitter increases then the throughput increases too, but this increasing is reduced by the collisions, in fact if the number of channels is fixed, the more the transmitter the more the collisions.
At the end of the day we can see that in the first case the mean throughput is settled to a value about 3.6 and in the second case about 4. So changing slightly the input changes slightly the output.
If we analyze the response time we can see that the difference is bigger between the two cases because the response time is sensible to the variation of transmitters and channels, and this is something that must be taken into account. In fact, as we have seen previously, there are a lot of collisions in the second case and this is the main reason for the increasing of response time in the second test w.r.t the first one.

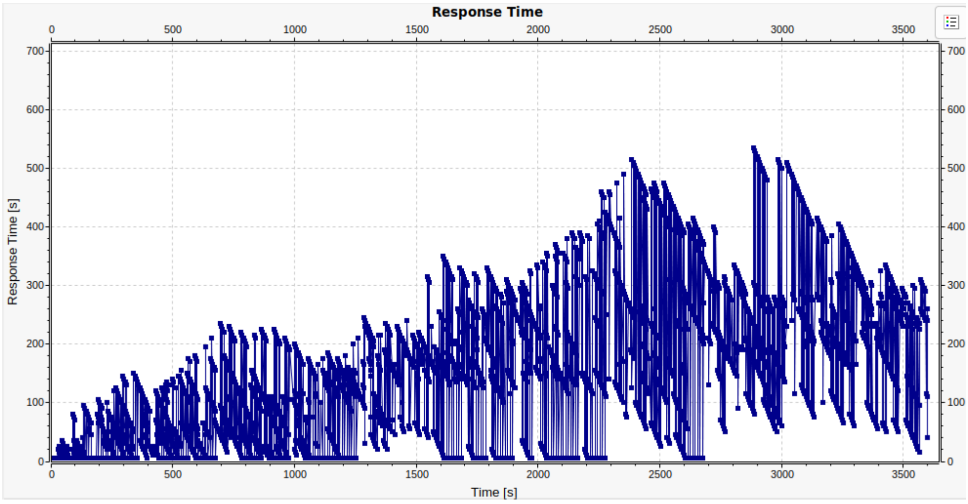


Figure 8: Continuity Test 1 - Response Time - Collisions detected: 272

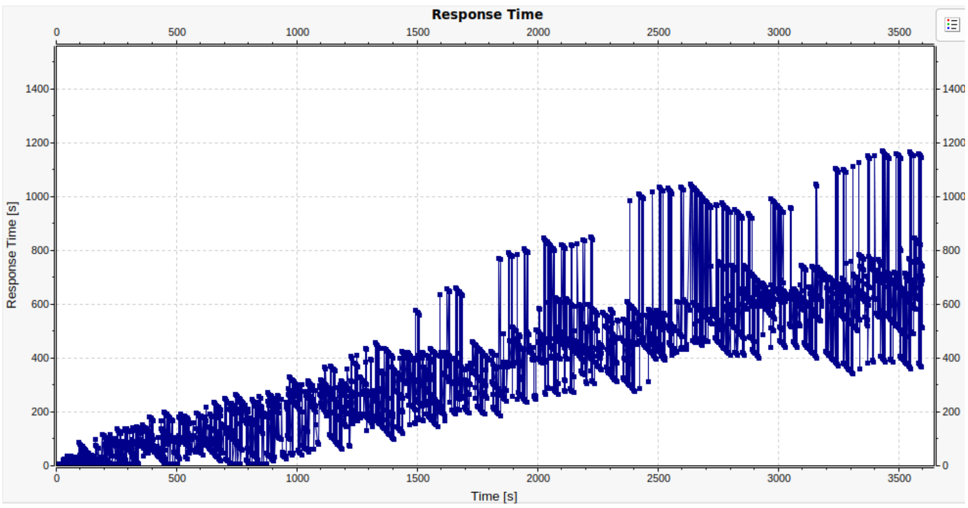


Figure 9: Continuity Test 2 - Response Time - Collisions detected: 390

It's possible to see the response time measured in the two tests in the figure 8 and 9. In any case we can see that the continuity test is correct because the system works as we expect: channels fixed, the more the transmitters, the more the collisions, the more the response time.

In addition to this, some simulation were taken in order to **assess the monotonicity** of some KPI by changing some factors:

- **Mean Throughput**: By **increasing N** (the numbers of couples tx-rx), with a high number of channels to avoid collisions, an **increase on the mean throughput is expected**. On the contrary, by **increasing** $\frac{1}{\lambda}$ (the mean inter-arrival time) an **opposite result is expected**. The following results were obtained:

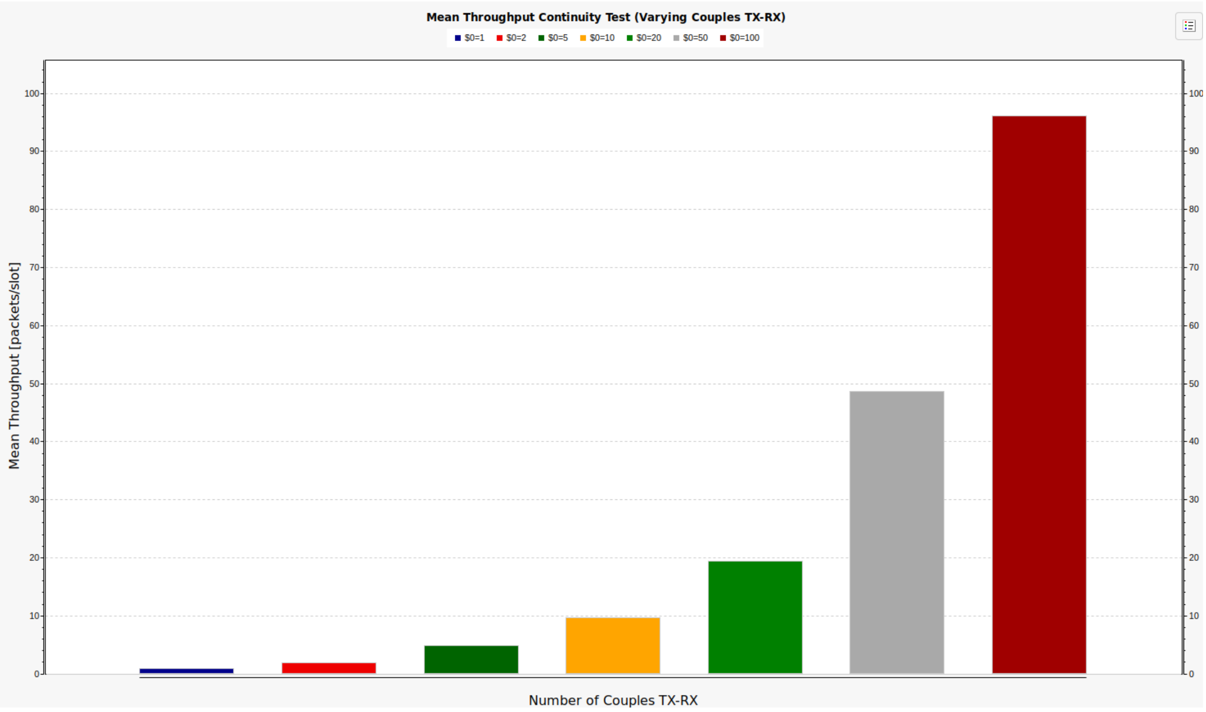


Figure 10: Continuity Test - Increasing Number of TX-RX (Main factors: **N** = 1, 2, 5, 10, 20, 100; **C** = 20000; $\frac{1}{\lambda}$ = 20 ms; T_{slot} = 5ms; p = 1)

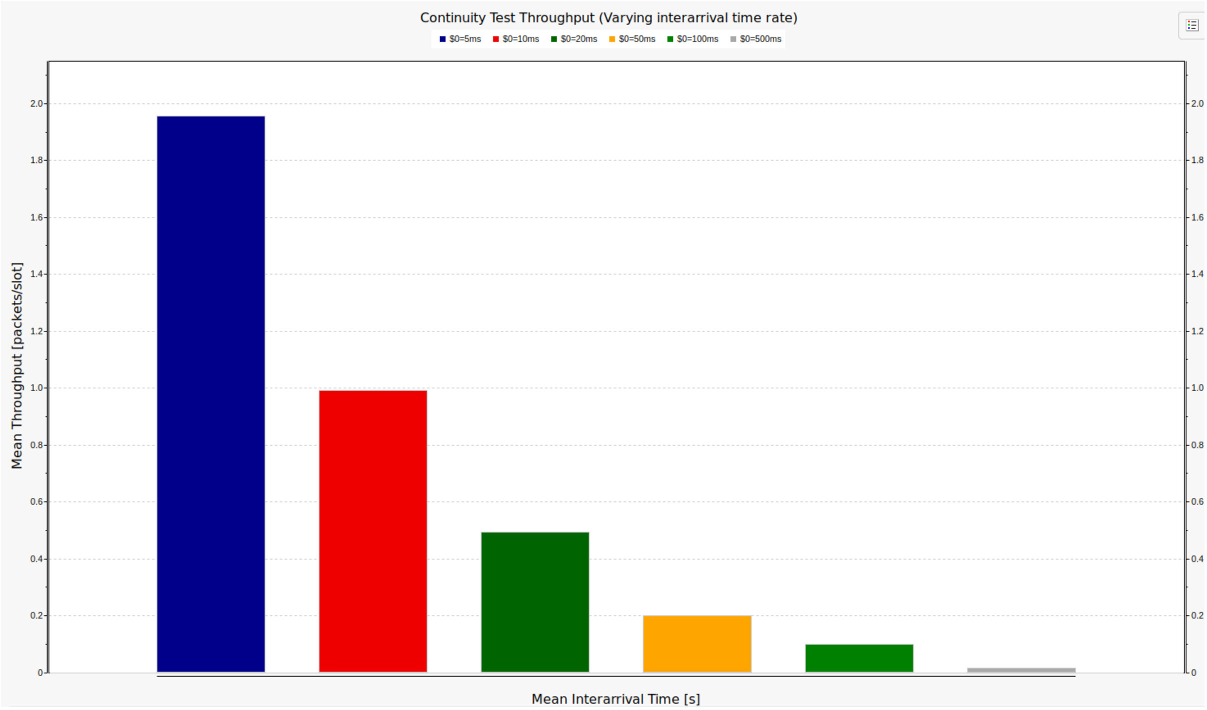


Figure 11: Continuity Test - Increasing Mean Inter-arrival Time (Main factors: **N** = 2; **C** = 20000; $\dfrac{1}{\lambda}$ = 5ms, 10ms, 20ms, 50ms, 100ms, 500ms; T_{slot} = 5ms; $p = 1$)

- **Mean Response Time**: By **Increasing N** ()the number of couples tx-rx), with low numbers of channels, a **increase on the mean response time is expected**. By increasing the transmission probability p (with a low number of couples tx-rx) a decrease on the mean response time is expected due to more transmissions. The following results were obtained:

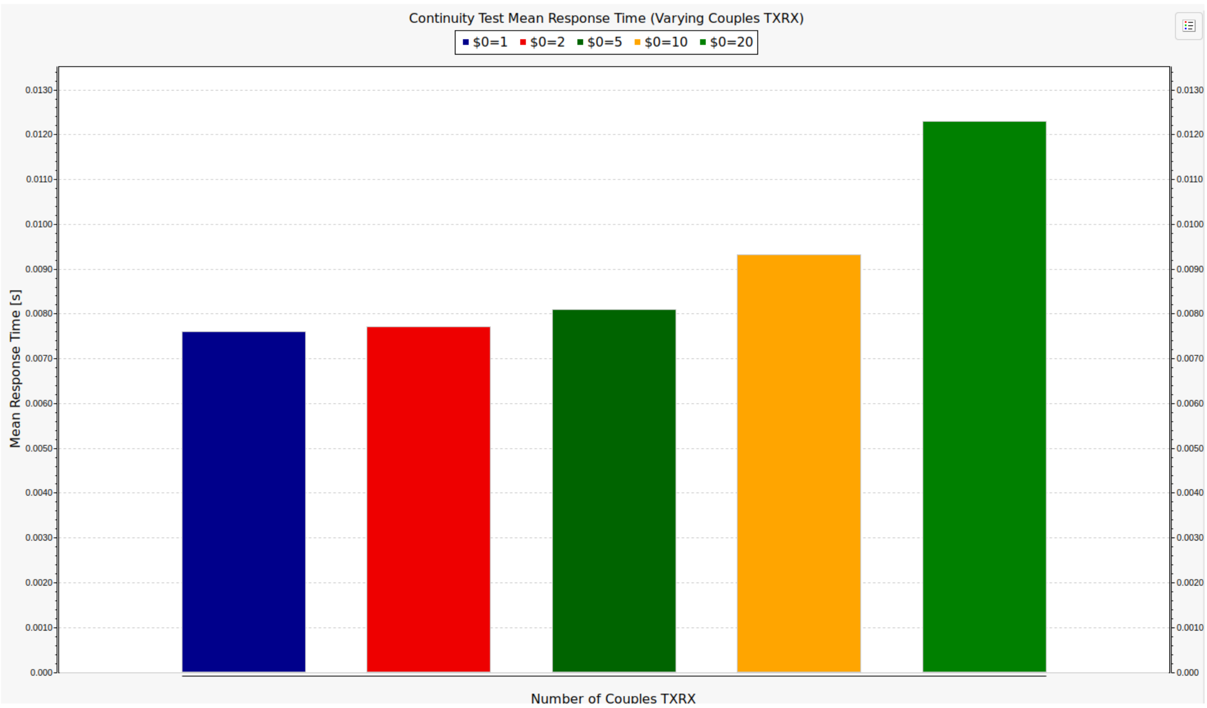


Figure 12: Continuity Test Mean Response Time- Increasing Number of TX-RX

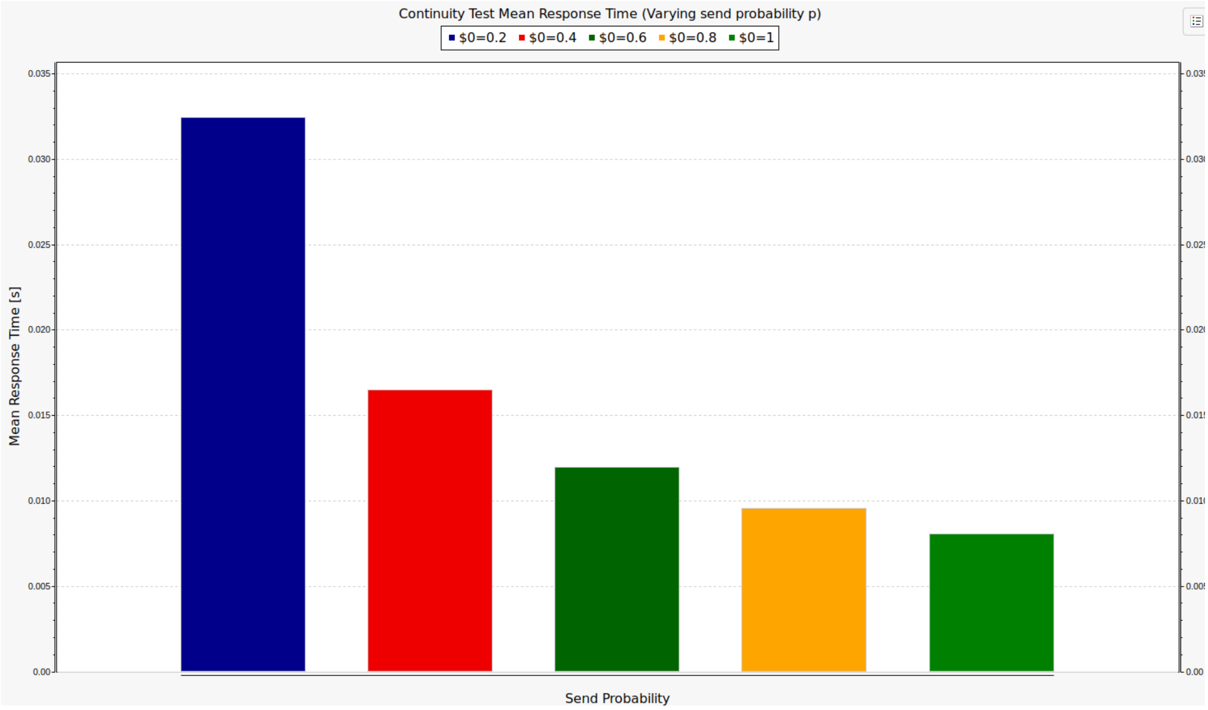
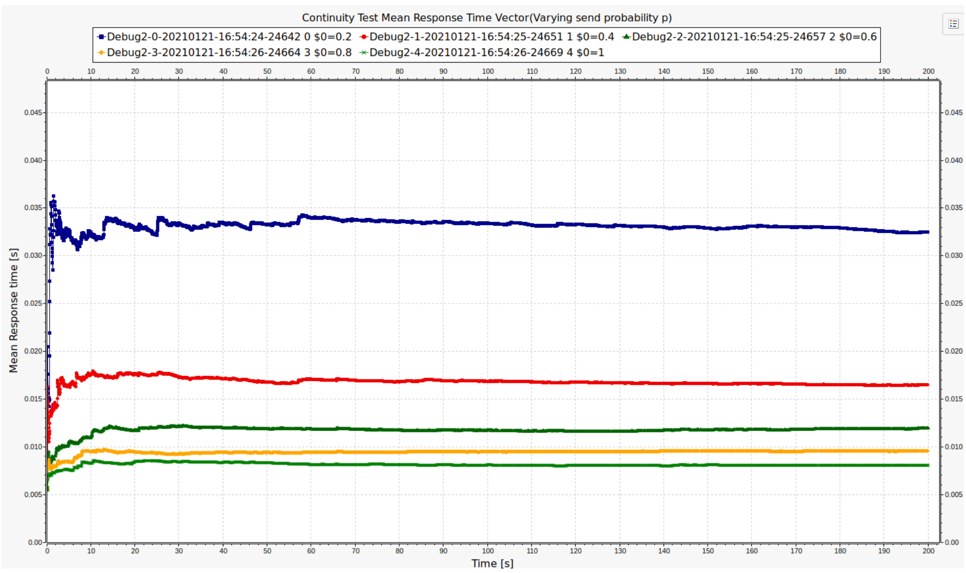


Figure 13: Continuity Test Mean Response Time- Increasing Sending Probability P(Main factors: $\mathbf{N} = 5$; $\mathbf{C} = 4$; $\frac{1}{\lambda} = 200\text{ms}$; $T_{slot} = 5\text{ms}$; $p = 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1$)

For the Mean Response Time was also checked the steady state reach by just plotting that the relative vector stabilizes at some point (this was done in general to make conclusion with this KPI). Here an example for the varying of the Transmission Probability p:



4.4 Test Simulations with Binomial Model (1)

This test simulation has been performed with the following parameters:

Parameters

- Number of couple tx-rx: 1

- Number of channels: 1
- Send probability: ${0.05, 0.1, 0.15, 0.2, 0.4, 0.5, 0.6, 0.8} (p)
- Mean inter-arrival time: 1s (deterministic) ($lambda$)
- Time slot size: 2s (T_{slot})
- simulation-duration: 3600s (T_{sim})
- repeat: 100
- seed-set: ${repetition}

In this simplified context there are no collisions (only one couple) and the transmitter will have, for every slot, at least one packet to sent ($lambda < T_{slot}$). We can model this particular case as a repeated Bernoullian Experiment, in which a success event correspond to a successful packet sent. In this simplified model we can define X as the number of success in n repeated trials (in independent condition), so $X \sim Bin(n, p)$. For this reason the PMF is the following:

$$p(i) = P\{X = i\} = \binom{n}{i} p^i (1-p)^{n-i} \tag{1}$$

Where n represents the number of repeated trials and i the number of successes in those trials. With this distribution the mean and the variance are:

$$E[X] = np \qquad Var(X) = np(1-p)$$

In our context we can state the following:

$$n = \left\lfloor \frac{T_{slot}}{T_{sim}} \right\rfloor = 1800 \tag{2}$$

We would expect in the case of $p = 0.5$:

$$E[X] = np = 900 \tag{3}$$

And the results after the run of 100 test simulation with different seeds, the following results are returned (with 95% CI):

$$\overline{X} \in [893.08, 901.94] \tag{4}$$

Which is in line with our expectations. The latter computations have been repeated for different values of p and the following plot can sum up the results:

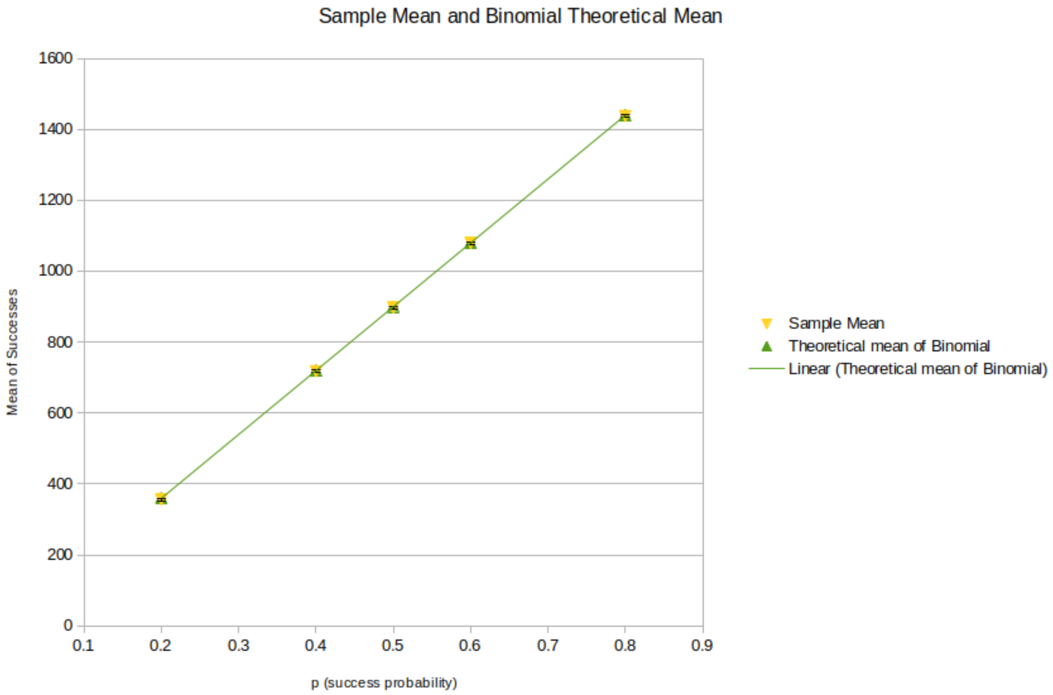


Figure 14: Test Binomial Model

As we can see it is difficult to recognize the theoretical results with the results obtained with the test simulations and the Confidence Interval can be barely seen.

4.5 Test Simulations with Collisions(2)

This test simulation has been performed with the following parameters:

Parameters

- Number of couple tx-rx (N): ${2, 5, 10, 30}
- Number of channels (C): 1
- Send probability: ${0.2, 0.4, 0.6, 0.8} ($p$)
- Mean inter-arrival time: 1s (deterministic) ($lambda$)
- Time slot size: 2s (T_{slot})
- simulation-duration: 3600s (T_{sim})
- repeat: 40
- seed-set: ${repetition}
- No Backoff in case of collision

The aim of this verification is to assess if the mean throughput is comparable with some equations that will be found even in the case of presence of collisions.

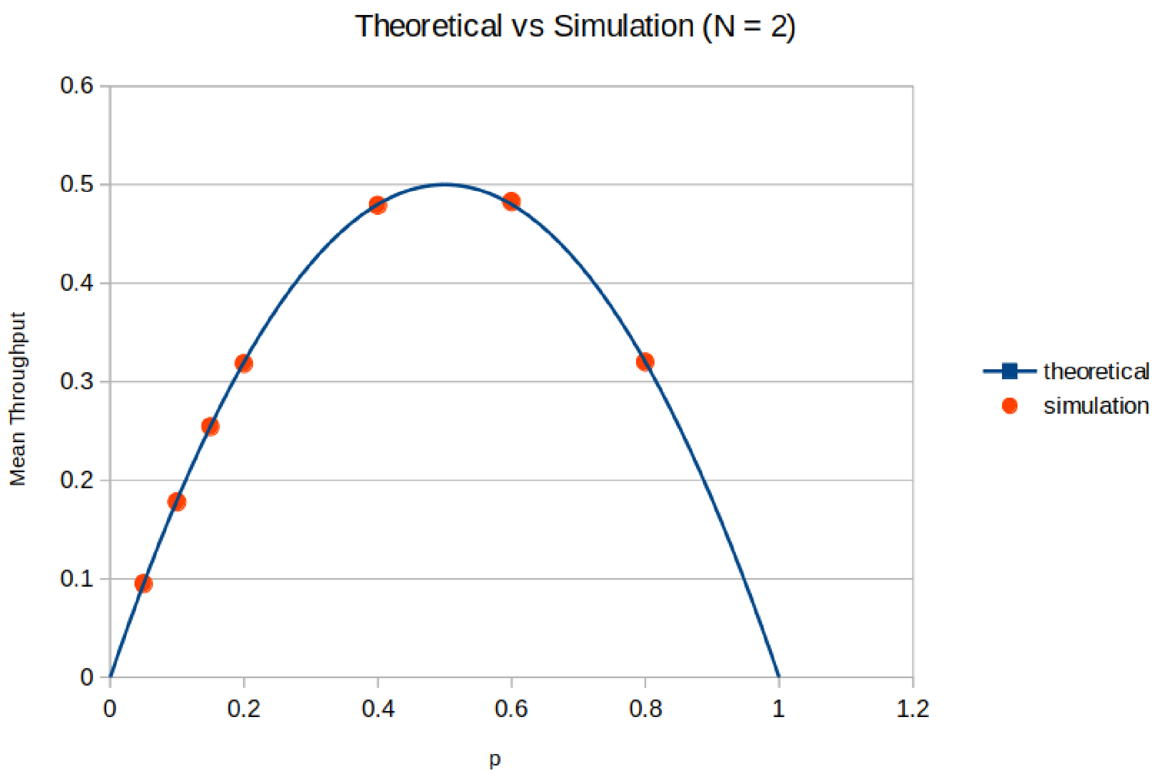
The probability of a successful sent in a particular timeslot, in this case, is the following:

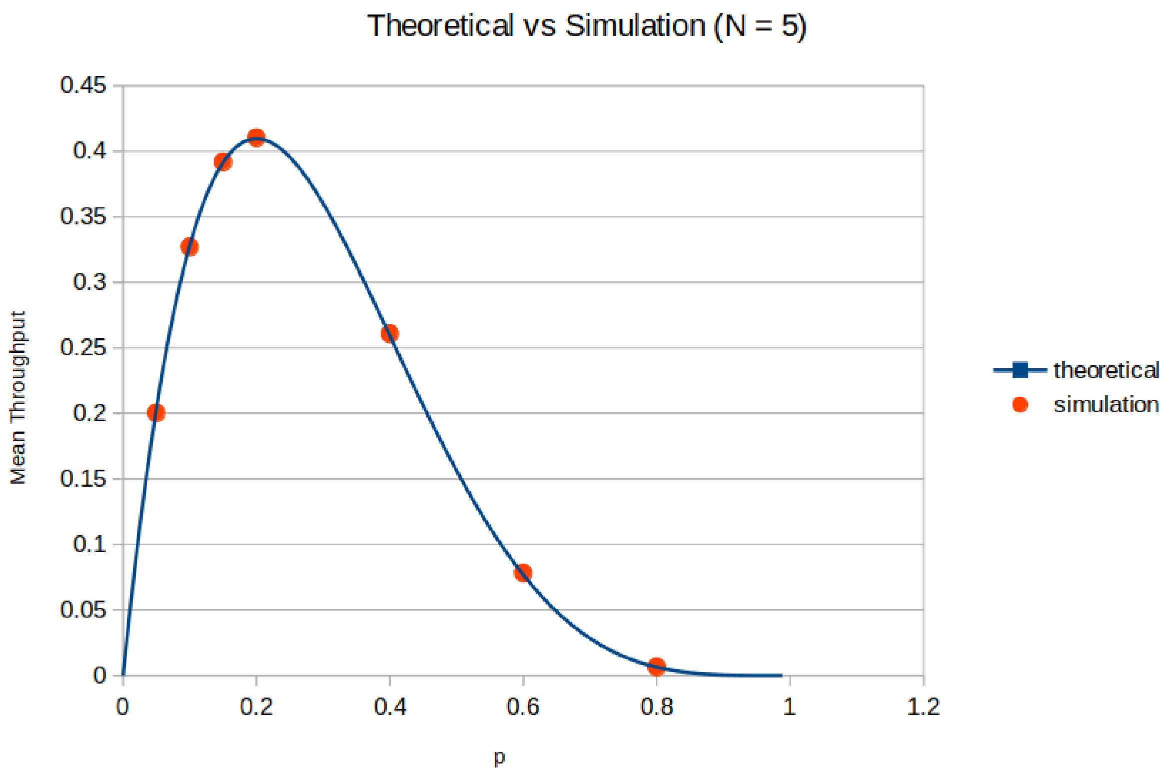
$$P\{"successful\ transmission"\} = P\{"only\ one\ tx\ transmit"\} = N \cdot p \cdot (1-p)^{N-1} \tag{5}$$

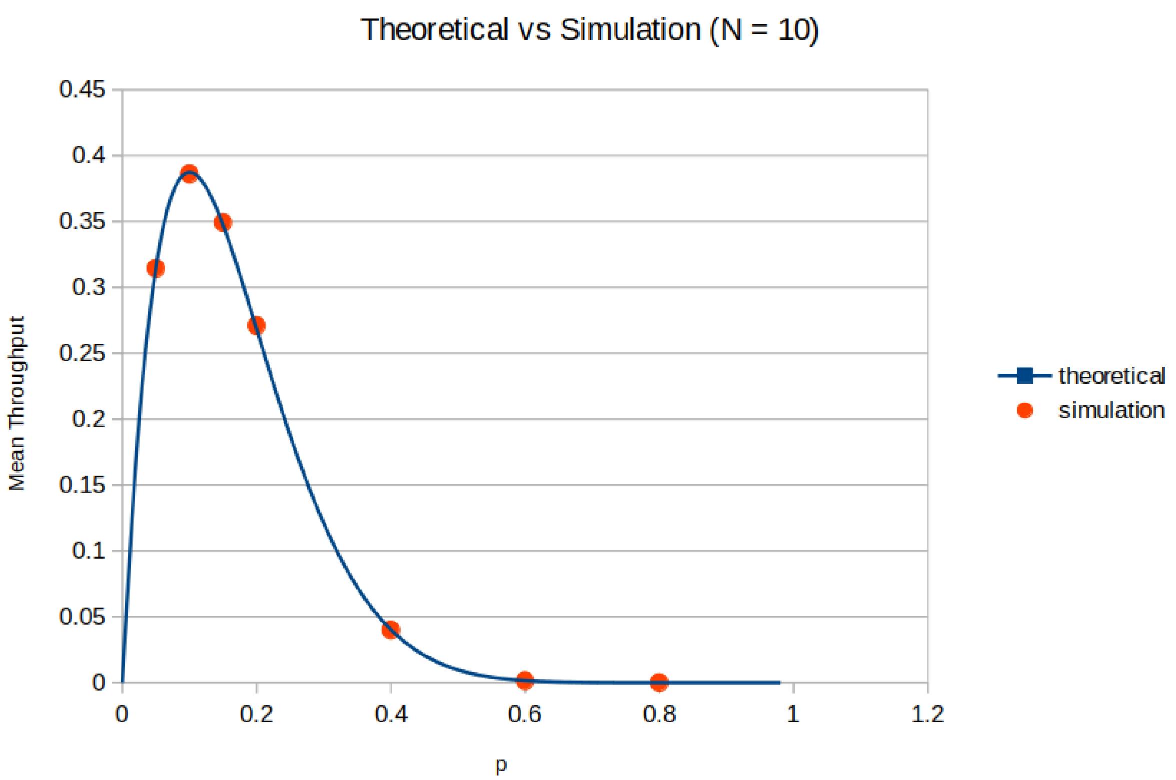
This probability of successful transmission can be seen as the mean throughput of the system in the single channel case. In fact, let N_p the number of packets successfully sent to the corresponding receiver, N_t the number of time-slot considered in the count:

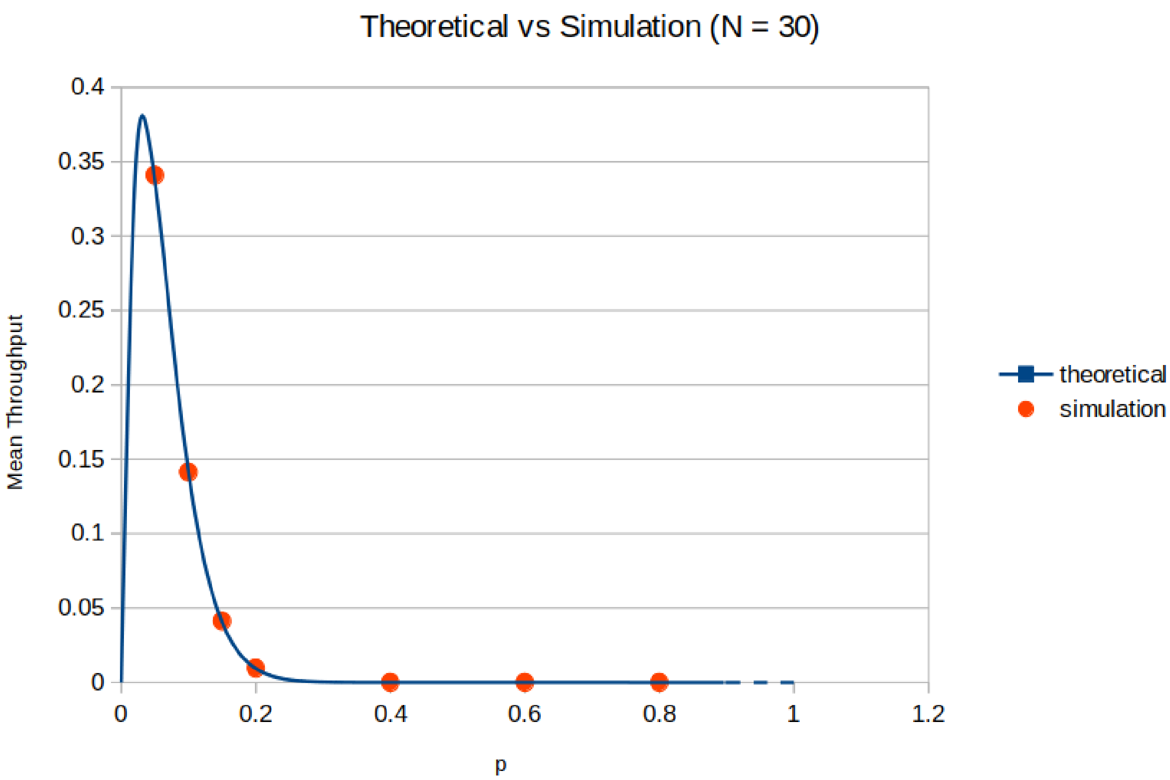
$$Tp\ (slot) = \frac{N_p}{N_t} = \frac{N_t \cdot P\{"successful\ transmission"\}}{N_t} = N \cdot p \cdot (1-p)^{N-1}$$

By comparing the above formula with the results of the simulation the following results are obtained (95% CI too small to be seen)




Theoretical vs Simulation (N = 5)
Mean Throughput
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
1.2
p
theoretical
simulation


Theoretical vs Simulation (N = 10)
Mean Throughput
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
1.2
p
theoretical
simulation



At this point we can state that a proper amount of verification of the implementation of the model has been carried out to make some simulation and gather some insight. Before doing so, an observation of the result can be carried out at this point: with a good number of couple tx-rx a huge sending probability (i.e. greater than 0.5) is pointless to obtain a high throughput. This result will be considered during the scenario calibration in the next chapter.

5 Simulations Experiments

5.1 Scenario Calibration

In order to calibrate the simulator parameters, the following range of values were used:

- **Number of Couples Tx-Rx (N)**: [5, 30]
- **Number of Channels (C)** : [6, 100] (Resource Blocks in LTE for different Frequencies)
- **Mean Inter-arrival Time** ($\frac{1}{\lambda}$): [25ms, 500ms]
- **Time-slot duration** (T_{slot}): 0.5 ms (Timeslot duration in LTE)
- **Send Probability (p)**: [0.1, 0.5]

5.2 Calibration of Warm-Up Period and Simulation duration

For calibrating the warm-up different simulation were made (with the factors range in the latter paragraph) with 10 repetition each. The worst case in terms of convergence time was encountered with the **mean throughput** with N = 5, C = 6, $\dfrac{1}{\lambda}$ = 500ms, p = 0.5:

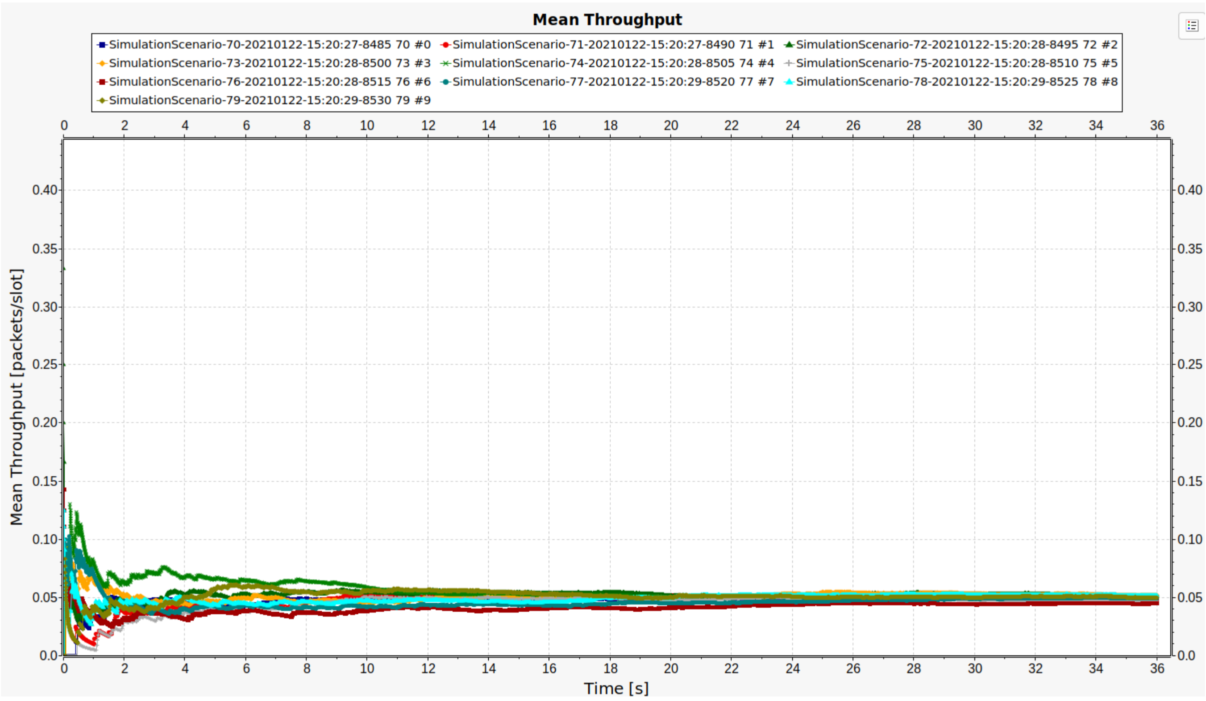


Figure 15: Worst Case Warm-up Throughput

With the **mean response time** the worst case is the following with N = 5, C = 100, $\dfrac{1}{\lambda}$ = 25ms, p = 0.5:

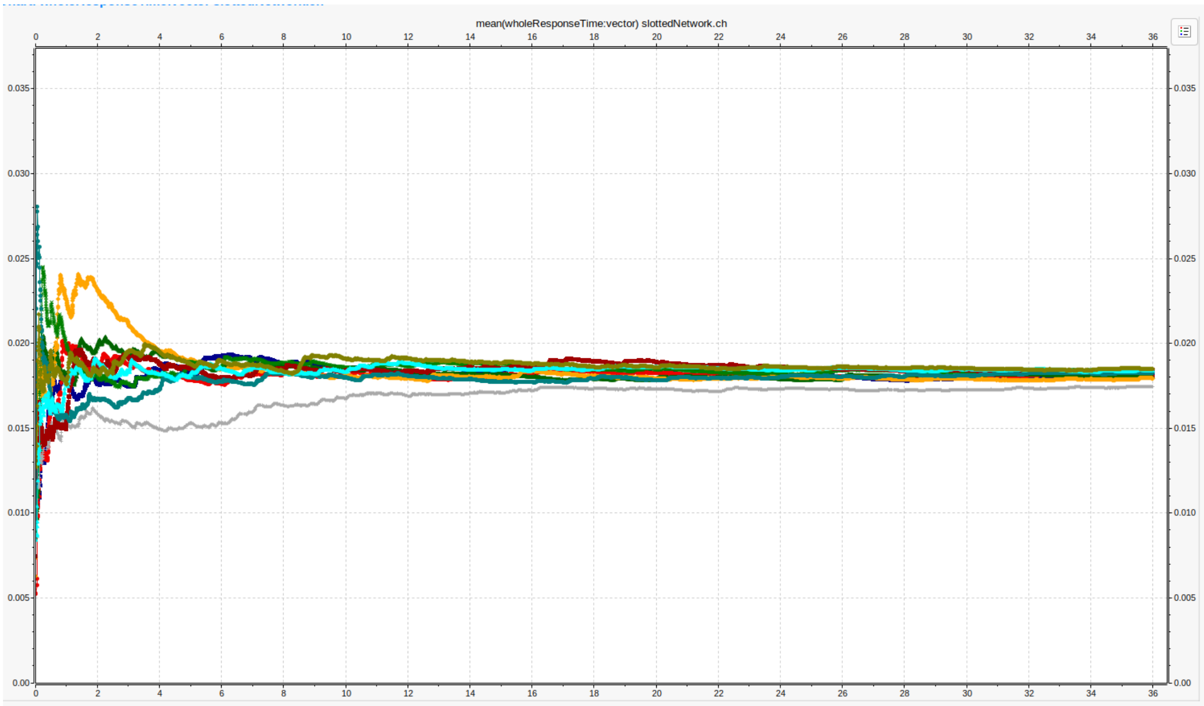


Figure 16: Worst Case Warm-up Response Time

A warm-up period of 10s was chosen.
For what concerns the simulation duration was made a trade-off between the memory consumption and the length of the simulation itself. This was done because there are not stochastic elements in the model (like a particular error probability with a low percentage) that will need a particular amount of time to be shown. Obviously the duration has to be greater than the warm-up duration. All things considered, **a simulation-duration of 100s was chose**

5.3 Design of Experiments

5.3.1 Factorial Analysis $r2^k$ on Throughput

In order to analyse the contribution of the factors on the throughput performance, we perform a $r2^k$ analysis with $r = 5$ and $k = 4$ (so we perform $5 * 2^4 = 80$ experiments). We take into account the following factors:

- Number of Couples Tx-Rx: [5, 30] **(A)**
- Number of Channels C : [6, 100] **(B)**
- Send Probability p: [0.2, 0.5] **(C)**
- Mean Inter-arrival Time: [25ms, 500ms] **(D)**

The first step is to check the hypothesis, in particular we have to control that the residuals are normal and that its standard deviation is constant (a.k.a. homoskedasticity). For what concerns the normal hypothesis it's possible to see (Figure 17) that the QQ plot of residuals vs normal show a linear tendency and so the hypothesis is verified.
For the homoskedasticity we have a QQ plot residuals vs predicted response and we can see (Figure 18) that indeed there is a trend, however the errors (y axis) are two order of magnitude below the predicted response (x axis) and so we can ignore trends and state that the homoskedasticity hypothesis is respected.

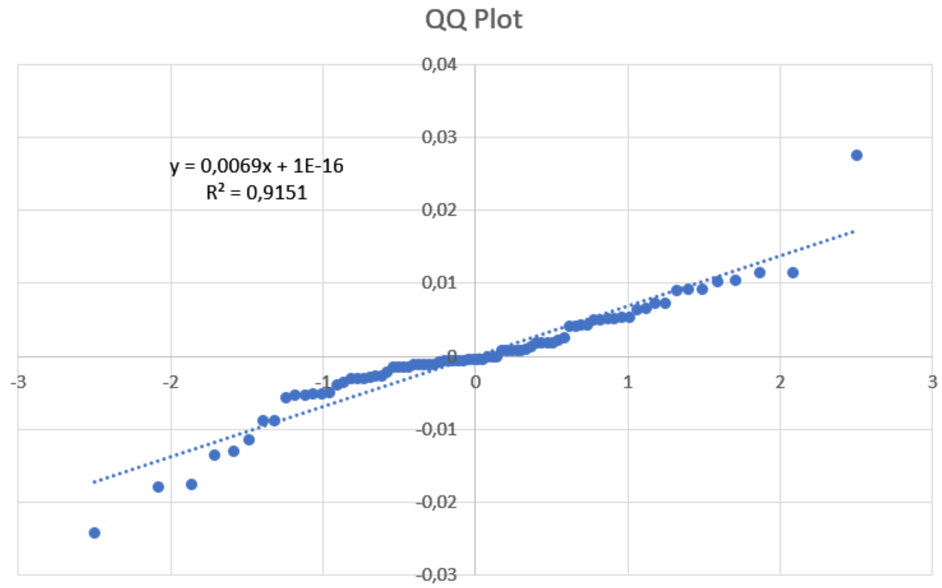


Figure 17: QQ Plot for testing the normal hypothesis

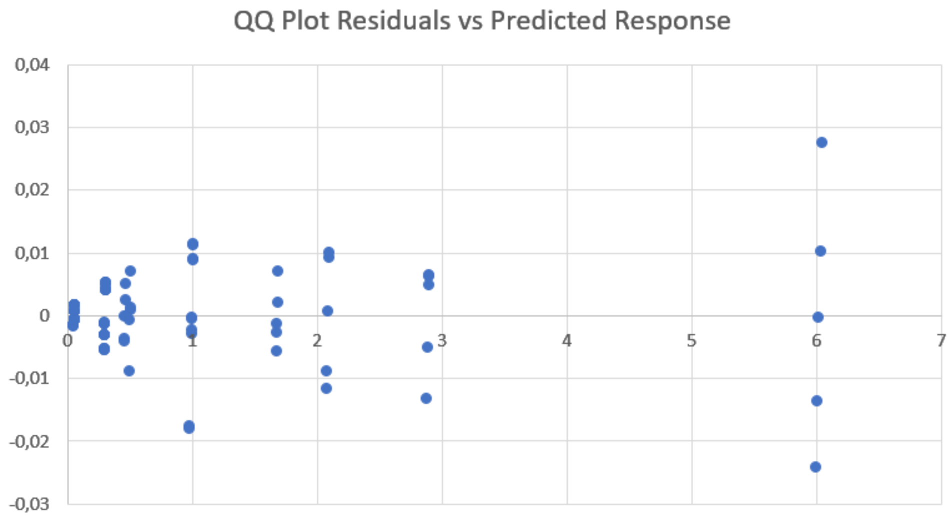


Figure 18: QQ Plot for testing homoskedasticity

Now we can analyse the obtained results. The most relevant ones are the following (the other factors have an impact on the variability that is very low, below 5%, and so they are non relevant):

- **Number of Couples**

  It has a positive impact on throughput, in particular $qi = [0,6689; 0,6701]$[1] and it accounts for the 19,67% of the variability. This means that the higher the number of couples the higher the throughput. In fact with more transmitters we have more packets and so we have an higher throughput.

- **Mean Inter-Arrival Time**

  It has a negative impact: $qi = [-0,8899; -0,8887]$ and it accounts for the 34,71% of the varibility. Thus we can say that the higher the mean inter-arrival time, the lower the

[1]This and the following are 95% confidence interval

throughput. This happens due to the fact that when we increase the mean inter-arrival time it's more likely that a transmitter have an empty buffer and so it has no packets to transmit, then the throughput decreases.

- **Jointly Effect of Number of Couples and Mean Inter-Arrival Time**

  The jointly effect of the above factors accounts for the 13,07% of the variability and it has a negative impact ($qi = [-0,5463; -0,5451]$). This because the effect of the mean inter-arrival time is greater with respect to the one of the number of couples, so if both increase then the throughput decreases. Indeed, if we have an higher number of transmitters, but we have the most of them which have an empty buffer (due to the previously explained phenomenon caused by the increasing of the mean inter-arrival time), then the throughput decreases because there are too few packets to transmit.

5.4 Result Analysis

6 Conclusions