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Robust Attitude Estimation Method For Underwater Vehicles with External and Internal Magnetic Noise Rejection using Adaptive Indirect Kalman Filter

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Abstract— **This paper considers the question of measuring attitude using magnetometer-equipped Inertial Measurement Units (IMUs) when in the presence of magnetic disturbance. Specifically, for use in micro-observational class underwater vehicles. Such vehicles' small size and mobility suit activities in coral reefs but often face significant magnetic interference due to high metal debris content, especially in artificial reefs. Their small size also means motor thrusters are situated closer to the magnetometers, producing additional interference. An Adaptive Indirect Kalman Filter is proposed that uses IMU and motor speed measurements to deal with two main sources of disturbances: magnetic fields from foreign objects, and from the vehicle's own motors rotating in slow speed. The method has been evaluated using a custom underwater vehicle made in HKUST, and results show that the proposed method is able to obtain accurate orientation representations even when approaching objects with strong magnetic fields or when the motors are producing interference by rotating in slow speeds.**

I. INTRODUCTION

This paper addresses the problem of estimating attitude for an Autonomous Underwater Vehicle equipped with an Inertial Measurement Unit (IMU) with a magnetometer, when in the presence of magnetic disturbances.

Marine biologists need tools to constantly monitor coastal areas where most of the marine habitats are protected, and micro-observational class underwater vehicles provide a suitable solution with their compact size and good mobility. They can help monitor coral reef areas, especially since marine environments experience drastically increased damage in recent years [1]. However, most such vehicles today are manually controlled as the small size limits the use of heavier equipment with stronger processing power. Ideally, for observation purposes we would like these vehicles to perform autonomously. This limitation is fortunately being overcome as manufacturing technology continues to improve, and with it the ease of creating autonomous control systems for such vehicles.

One of the computational requirements in creating autonomous control systems for underwater vehicles is measuring the vehicle's attitude. Classical methods that make use of vehicle kinematics and dynamics have been around for several years[2], but advances in micro-fabrication technology increasingly favor newer methods that are guided by additional sensors such as IMUs. Kalman Filtering is one such method, a popular choice due to its effectiveness in fusing data obtained from IMUs and its low computational cost[3]. Extended Kalman Filtering (EKF) was later introduced to apply the method to non-linear system models. There are different kinds of attitude estimation methods based on EKF or its unscented transform (UKF) [4] with their own advantages and disadvantages.

Other methods exist but also have their disadvantages. Some attitude estimation techniques used by quad-copters can be applied to underwater vehicles, e.g. [5]. Yet there are environmental differences that might affect its effectiveness, such as hydrodynamic and buoyancy forces limiting an underwater vehicle's overall dynamics compared to a quad-copter. Underwater vehicles also face a more limited selection of sensors as electromagnetic wave communications cannot work well underwater, preventing use of methods that use Global Positioning System [6] for example. Other methods attempt navigation with sensors tailored for underwater environments, such as ultrasonic sensors[7] and Doppler Velocity Logs (DVLs) [8]. However, such types of navigation systems only work in deep water environments where the acoustic systems can have stable propagation with less multi-path problems. Applications in shallow water will cause the acoustic signals to bounce multiple times thus impairing the readings. This makes them unsuitable for use in coastal activities such as coral reef area observation.

The challenge with using IMU-based methods such as Kalman Filtering, meanwhile, is that IMUs often use a magnetometer, leaving it vulnerable to magnetic field disturbances. In a small underwater vehicle, the IMU can be close enough to the vehicle's own motors for them to interfere with the magnetometer readings. In addition, newly made coral reef areas are artificially reinforced with metal debris to help the coral's growth. Those and any other metallic foreign objects will cause strong magnetic disturbances to the magnetometer readings.

Therefore, this paper proposes an attitude estimation algorithm that uses quaternion representations and has an adaptive variance in the Indirect Kalman Filter method[9] in order to obtain a stable attitude estimate even in the presence of strong magnetic interference from external objects and motors rotating internally. Most existing attitude estimation techniques for underwater vehicles do not address internal interference from the vehicle's own motors as most underwater vehicles used in research are relatively large. Below we describe the proposed method as well as the results of testing the algorithm on a customized micro-observational class underwater vehicle 1, designed in HKUST laboratories with a dimension less than 40cm x 40 cm x 20 cm to approximate the size of a micro-observational class underwater vehicle.

Fig. 1. Photos of Hermit AUV

II. INDIRECT KALMAN FILTER

This paper is based on Indirect Kalman Filtering from Trawny[9]. Indirect Kalman Filtering is a special version of Extended Kalman Filtering that is used for attitude estimation. Instead of Euler Angle representations it uses quaternions instead. Just like in every Kalman Filter equation, there will be a propagation step and an update step. Define a seven-element state vector consisting of the quaternion and the gyro bias :

$$x(t) = \begin{bmatrix} \bar{q}(t) \\ b(t) \end{bmatrix}$$

where $\bar{q}(t)$ is a 4-element quaternion representation at time t and $\bar{b}(t)$ is the 3-element gyro bias at time t. Usually the error vector is expressed in terms of arithmetic differences between the state vector and its estimate, however, since quaternion representation is used and it needs to maintain unit length,the quaternion difference is used.

$$\bar{q}(t) = \delta\bar{q} \otimes q_m$$

δq is the difference between the estimated quaternion and the measured quaternion q_m from the accelerometer and magnetometer. In case of small rotations, the error quaternion can be represented as :

$$\delta\bar{q} = \begin{bmatrix} \delta q \\ \delta q_w \end{bmatrix} = \begin{bmatrix} \hat{k} sin(\theta/2) \\ cos(\theta/2) \end{bmatrix} \approx \begin{bmatrix} \frac{1}{2}\delta\theta \\ 1 \end{bmatrix}$$

The $\delta\theta$ will be used as a 3-dimensional error vector that will be combined with the bias difference to construct a 6-dimensional error state and 6x6 error covariance matrix $P(t)$.

$$\delta\bar{x}(t) = \begin{bmatrix} \delta\theta \\ \Delta b \end{bmatrix}$$

where $\delta\theta$ is the error angle vector obtained from small rotations between the state quaternion and the estimated quaternion. Δb is the difference between state bias and estimated bias. This algorithm then involves 2 routines, propagation and update.

A. Propagation Equation

In this step, data obtained from a 3 axis gyroscope will be used to propagate the quaternion estimate into a new estimate. Measured angular velocity will be modeled as follows:

$$\omega_m = \omega + b + \mu_g \in R^3$$

where the measured angular velocity ω_m is the sum of the real angular velocity ω acting on the body frame, a time-varying bias b and measurement noise μ_g. Besides ω_m the current state $x_{k|k}$ will be used to estimate the new state $x_{k+1|k}$ based on gyroscope integration in one time step. Initially,

$$\hat{x}(0) = \begin{bmatrix} \hat{q}_{0|0} \\ \hat{b}_{0|0} \end{bmatrix} = \begin{bmatrix} q_m \\ 0_{3\times1} \end{bmatrix}$$

$$\hat{P}_{0|0} = I_{6\times6}$$

where q_m is the initial quaternion obtained from combining the first magnetometer and accelerometer using same method on update step. $0_{3\times1}$ is the 3-dimensional vector consists of all zeros. Lastly, $I_{6\times6}$ is 6 by 6 identity matrix.

After initialization, the propagation step proceeds as follows:

1) Propagate the previous bias and correct the measured angular velocity

$$\hat{b}_{k+1|k} = \hat{b}_{k|k}$$

$$\hat{\omega}_{k+1|k} = \omega_{m_{k+1}} - \hat{b}_{k+1|k}$$

2) Propagate the quaternion state $\hat{\bar{q}}_{k+1|k}$ using a first order approximation integrator with $\hat{\omega}_{k+1|k}, \hat{\omega}_{k|k}$, and $\hat{\bar{q}}_{k|k}$

$$\begin{aligned} \hat{\bar{q}}_{k+1|k} = (exp(\frac{1}{2}\Omega(\frac{\hat{\omega}_{k+1|k} + \hat{\omega}_{k|k}}{2})\Delta t) \\ + \frac{1}{48}(\Omega(\hat{\omega}_{k+1|k})\Omega(\hat{\omega}_{k|k}) \\ - \Omega(\hat{\omega}_{k|k})\Omega(\hat{\omega}_{k+1|k})))\Delta t^2)\hat{\bar{q}}_{k|k} \end{aligned}$$

where $\Omega(w)$ is a 4x4 skew-symmetric matrix with 0 in its diagonal configured as:

$$\bar{x}(t) = \begin{bmatrix} 0 & w_z & -w_y & w_x \\ -w_z & 0 & w_x & w_y \\ w_y & -w_x & 0 & w_z \\ -w_x & -w_y & -w_z & 0 \end{bmatrix}$$

3) Compute state transition matrix ϕ and discrete time noise covariance matrix Q_d to propagate the state covariance Matrix as follows:

$$P_{k+1|k} = \phi P_{k|k} \phi^T + Q_d$$

Derivation and values of the transition matrix ϕ as well as the state covariance matrix can be found in [9]

B. Update Equation

In the update step, data obtained from the accelerometer and magnetometer will be used to construct a quaternion representation of the vehicle in global frame. Accelerometer data will be modeled as :

$$a_m = C_G^L(a + g) + \mu_a \in R^3$$

where the measured acceleration a_m is the sum of device acceleration a and the gravitational acceleration g acting on the global frame added with measurement noise μ_a. Rotation matrix C_G^L maps the total acceleration in global frame to the local frame. Magnetometer data will be modeled as :

$$m_m = WC_G^L m + m_d + \mu_m \in R^3$$

where the measured magnetic field value m_m is the sum of the Earth's magnetic field m affected by soft-iron interference W, hard iron interference m_d and measurement noise μ_m These two 3-dimensional vectors will be used to construct a 3x3 rotation matrix that represents the underwater vehicle's orientation. Using NED representation when:

$$C_m = I_{3\times3} \in SO(3)$$

This identity matrix represents the orientation in the global frame which orients the vehicle x-axis to the north of the geomagnetic field, y-axis to the east and z-axis parallel to the direction of the gravity vector. Using these measurements, the data will be processed as follows:

1) Gram-Schmidt orthonormalization algorithm will be used to combine both magnetometer and accelerometer data to obtain a 3x3 rotation matrix that represents vehicle attitude corresponding to the aforementioned global frame:

$$C_m = \begin{bmatrix} \frac{a_m \times (a_m \times m_m)}{|a_m \times m_m|} & \frac{(a_m \times m_m)}{|a_m \times m_m|} & a_m \end{bmatrix}$$

This C_m will be the measured rotation matrix representation which corresponds orthonormally to the gravity vector. Gravity vector obtained in a_m is used as the basis since the gravitational vector is always pointing downward to the Earth's core wherever the vehicle is located on Earth. In contrast, geomagnetic fields have a wide variety not only in its magnitude but also declination angle which means the geomagnetic north will not necessarily be parallel to the gravity vector. Therefore, using this orthonormalization vector ensures that the measured orientation provides stable data independent of the vehicle's location on earth.

2) Construct quaternion representation q_m from the 3x3 matrix constructed on the previous step.
3) Compute the residual r and represent it as a quaternion difference:

$$\begin{aligned} q_m &= \delta\bar{q} \otimes \hat{\bar{q}} \\ C(\bar{q}) &= C(\delta\bar{q})C(\hat{\bar{q}}) \\ where, C(\delta\bar{q}) &\approx I - \lfloor \delta\theta \times \rfloor \\ z = C(\hat{\bar{q}}) \begin{bmatrix} \delta\theta \\ \bar{b} \end{bmatrix} &= H \begin{bmatrix} \delta\theta \\ \bar{b} \end{bmatrix} + \mu_z \\ r &= z - \hat{z} \end{aligned}$$

4) Compute the covariance of the residual, S:

$$S = HPH^T + R$$

where H is the measurement matrix obtained from the previous step and R is the 6x6 covariance matrix that corresponds to the accelerometer and magnetometer data noise.

5) Compute the Kalman gain K:

$$K = PH^T S^{-1}$$

6) Compute the correction:

$$\delta\hat{x}(+) = \begin{bmatrix} \delta\hat{\theta}(+) \\ \Delta\hat{b}(+) \end{bmatrix} = \begin{bmatrix} 2 \cdot \delta\hat{q}(+) \\ \Delta\hat{b}(+) \end{bmatrix} = Kr$$

7) Update the quaternion from the correction:

$$\hat{\bar{q}}_{k+1|k+1} = \delta\hat{\bar{q}} \otimes \hat{\bar{q}}_{k+1|k}$$

where $\delta\hat{\bar{q}}$ can be obtained from

$$\delta\hat{\bar{q}} = \begin{bmatrix} \delta\hat{q}(+) \\ \sqrt{1 - \delta\hat{q}(+)^T \delta\hat{q}(+)} \end{bmatrix}$$

or if $\delta\hat{q}(+)^T \delta\hat{q}(+) > 1$, instead using

$$\delta\hat{\bar{q}} = 1\sqrt{1 + \delta\hat{q}(+)^T \delta\hat{q}(+)} \begin{bmatrix} \delta\hat{q}(+) \\ 1 \end{bmatrix}$$

8) Update the bias and use the new bias to update the estimated turn rate:

$$\hat{b}_{k+1|k+1} = \hat{b}_{k+1|k} + \Delta\hat{b}(+)$$

$$\hat{\omega}_{k+1|k+1} = \omega_{m_{k+1}} - \hat{b}_{k+1|k+1}$$

9) Lastly, update the covariance matrix:

$$P_{k+1|k+1} = NP_{k+1|k}N^T + KRK^T$$

where $N = I_{6\times6} - KH$

III. DESIGN CHANGES

Even with calibrated magnetometer and accelerometer data, Indirect Kalman Filtering is still prone to errors. These errors arise from inherent properties of the accelerometer that measure not only pure gravity, but also external acceleration. As mentioned before, magnetometers also measure external magnetic fields when the underwater vehicle approaches magnets or ferromagnetic objects, not just the earth's geomagnetic field. In addition, in micro-observation class vehicles, thrusters are placed close to the sensor which affects the measurements as well. Therefore, several design changes are introduced in this paper:

1) Accelerometer Norm: In ideal conditions when the IMU is kept stable on a horizontal plane, the accelerometer will return $a_m = [0 \ 0 \ 1 \ ]^T$ with the norm being equal to $1g = 9.80665 m/s^2$. No matter the direction, as long as it is stable, the norm will always have a value of 1. When an external disturbance is present or the underwater vehicles' motors engage in motion, however, the accelerator's norm value will suddenly deviate from 1. There are several approaches we can take to mitigate this problem, such as increasing the variance of the update, R, that is related to the accelerometer data. The downside is that this approach will still affect the data in small portions of the reading, such that for short durations of several milliseconds

there will be false readings affecting the estimation. Therefore, in a dynamic situation, taking into account gyroscope data as a whole allows us to produce more accurate results. The proposed solution will extract estimated gravity from the attitude estimation.

$$\hat{C}(\hat{\bar{q}}) = [\hat{c1}\ \hat{c2}\ \hat{c3}\ ]$$

The values in the third column will be used to substitute the accelerometer reading when the vehicle is in dynamic motion $a_m^{'} = \hat{c3}$. It represents the best estimate of the vehicle's z axis relative to the gravity vector. This will be fed into the Gram-Schmidt orthonormalization.

$$c_m1 = (a_m^{'} \times (a_m^{'} \times m_m))/|a_m^{'} \times m_m|$$
$$c_m2 = (a_m^{'} \times m_m)/|a_m^{'} \times m_m|$$
$$c_m3 = a_m^{'}$$

Afterwards the update step proceeds normally using the new rotation matrix. This procedure will be invoked whenever the norm of (acceleration - 1 g) is less than the threshold α. Our experiments suggest 0.2 g to be a suitable threshold.

2) Magnetometer Norm: In the magnetometer's case, the norm of an undistorted magnetometer reading will take the range 25 - 65 μT, depending on the geographical location. After the magnetometer is calibrated, the norm will not deviate from its original value as long as the vehicle is relatively static compared to the earth's location. Small deviations will occur from the sensor's white noise or if the calibration was performed poorly. In case of significant amounts of deviation, it means that the underwater vehicle encountered some metal debris such as a shipwreck which impaired the magnetometer measurements. Let:

$$\delta M = |m_m| - |m_o|$$

$$\delta M > \beta$$

β will be the threshold used to determine the presence of external disturbance. If it is exceeded disturbance is considered present and the reading will be discarded. The current estimated attitude will instead be used as a replacement.

$$m_m{'} = \hat{c1}$$

The best estimate for the current magnetic field vector will be the first column of $\hat{C}(\hat{\bar{q}})$. It will be used to substitute the magnetometer reading when δM ¿β. After that the Gram-Schmidt process will occur and the update process continues using the new magnetometer data. If both thresholds α and β are exceeded, the update phase will be skipped and the next propagation phase will fully rely on gyroscope readings.

3) Internal Disturbance from Thrusters : When the thrusters are arranged physically close to the IMU, interference from the motor will affect magnetometer readings. The figure(1) shows that low speed rotation

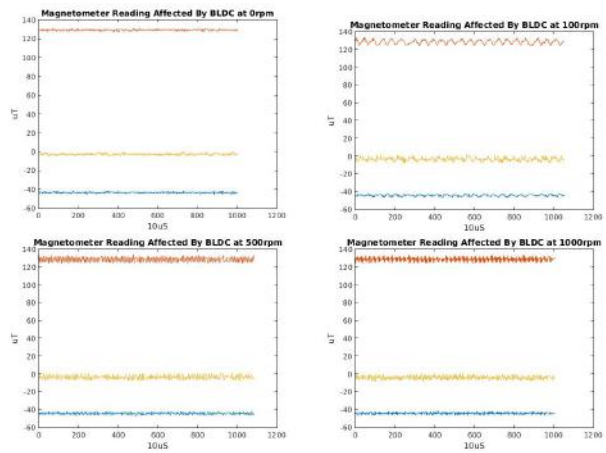

Fig. 2. Magnetometer data collected with different rpm speeds of Blue Robotics T200 thruster at $\pm$15cm from the IMU. The top left is static / no rotation, top right runs at 100rpm, bottom left at 500 rpm and bottom right at 1000 rpm

at 100-500 rpm will create significant sinusoidal interference with peak to peak values of more than 10 μT. This phenomenon can impair the estimation algorithm since it cannot differentiate noise from the motor and actual periodic rotations of the robot. However, at relatively high-speeds at ¿500 rpm , it interferes at relatively higher frequencies compared to the sensor dynamics and can be filtered out easily. In this case, there will be another parameter γ so that

$$\gamma = \begin{cases} 1 & -100 \geq \omega_{thr} \leq 100 \\ (500 - \omega_{thr})/400 & 100 \leq \omega_t hr \leq 500 \\ (500 + \omega_{thr})/400 & -500 \leq \omega_t hr \leq -100 \\ 0 & \omega_{thr} \geq 500, \omega_{thr} \leq -500 \end{cases}$$

In this case γ will have a value between 0 and 1 and will be used to adaptively increase the variance in the measurement phase scaled by c1.

$$R(+) = R + \begin{bmatrix} I_{3\times3}(\gamma c1) & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix}$$

4) Slow Varying Magnetic Field Interference : some magnetic field interference does not change the norm of the magnetometer reading but it significantly interferes with reading the direction of true north. This phenomenon usually occurs several milliseconds before approaching strong magnetic field interference. This type of interference can be detected by comparing the quaternion difference between the estimated quaternion and the measured quaternion which is indicated by the 3-dimensional vector $\delta\theta$. The value of $\delta\theta$ is ideally 0. However,the value will be non-zero in practice due to either gyroscope drift, timing differences, and measurement noise. Significant changes in the values of $\delta\theta$, however, indicates that the gyroscope integration

in consecutive time steps is significantly different from the one obtained by the magnetometer and accelerometer. Since gyroscope integration is more reliable while the vehicle undergoes a certain motion, it can be deduced that the magnetometer or accelerometer is affected by external disturbances that do not change the norm.

$$\delta\theta = \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \\ \delta\theta_3 \end{bmatrix}$$

In order to mitigate the problem, the measurement Variance R will have a linearly dependant function with respect to $\delta\theta$ and scaled by c2.

$$R(+) = R + \begin{bmatrix} I_{3x3} \begin{bmatrix} \delta\theta_1 c2 \\ \delta\theta_2 c2 \\ \delta\theta_3 c2 \end{bmatrix} + & 0_{3\times 3} \\ 0_{3\times 3} & o_{3\times 3} \end{bmatrix}$$

IV. RESULTS

The results of the proposed algorithm is tested on the Hermit AUV as shown in 1. It is developed and built in HKUST's Robotics Institute. The vehicle is equipped with a MPU9250 IMU that has a 16-bit 3-axis accelerometer, gyroscope and magnetometer accurate to $\pm$2G, $\pm$250 degree/s, and $\pm$4800 μT respectively. The Hermit AUV has 6 brushless 3-phase motors (150W each) mounted according to figure3. Each

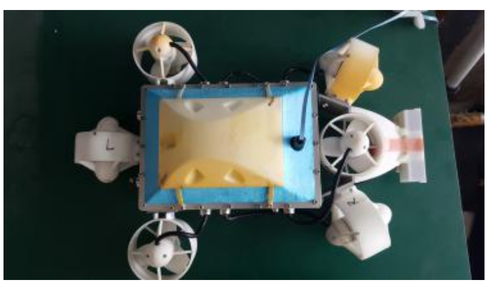

Fig. 3. Thruster configuration of Hermit AUV

thruster is equipped with an electronic speed control (ESC) based on the Texas Instrument InstaSpin FOC algorithm to get accurate motor control and rotation speed feedback. Throughout this experiment, the propagation equation is updated at 500Hz frequency according to the gyroscope data and the update equation runs at 100Hz, limited by the rate of magnetometer readings.

Figure 5 shows the results of attitude estimation on land despite magnetic interference. The Hermit AUV is first held in a static position on land such that its actual attitude remains constant throughout the observation. A periodic thruster rpm increase of 100 rpm every 1s is then introduced to see how it affects the attitude estimate. In addition, near the end of the figure, an external magnet is brought close to the vehicle several times. Figure 4 shows there is indeed strong disturbance in the magnetometer data, however there are no significant changes shown in the attitude estimation result in figure 5. This shows that in this type of setup, the

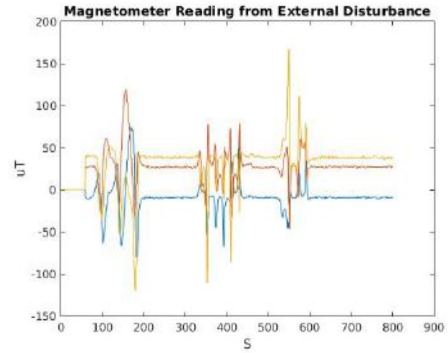

Fig. 4. Magnetometer data in the presence of external and internal disturbance

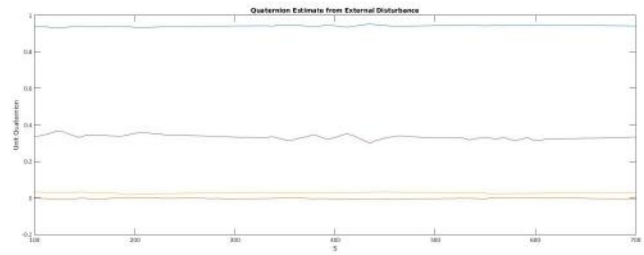

Fig. 5. Quaternion in the presence of external and internal disturbance

proposed algorithm can minimize external disturbances and estimate the correct attitude.

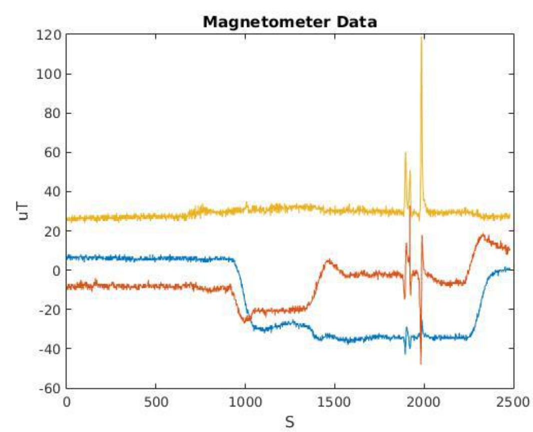

Fig. 6. Magnetometer data

Previous test results are done on land to ensure the stability of the estimation algorithm. Figure 7 shows the data of experiments when the Hermit AUV is tested in a small swimming pool with depth 1m. In figure 7, the Hermit AUV has a PID controller to automatically maintain its attitude with the estimated attitude as the input. Since this underwater vehicle has 6 DOF, it can maintain its own roll, pitch and yaw angle as well as perform heave, surge and sway motions. In this data set, the AUV was tasked with covering a rectangular area so that the motion covers 90 degree angle turns and forward movement alternately for a period of time. At the 3rd point of the rectangular pattern,

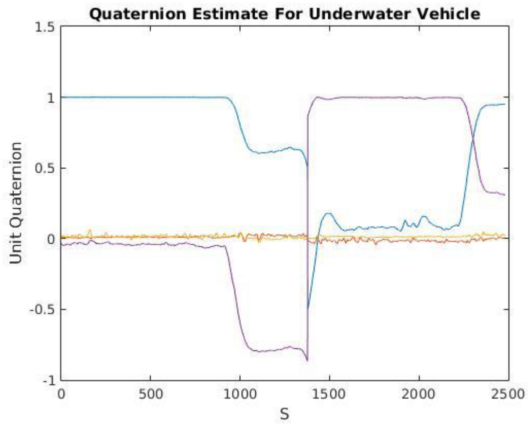

Fig. 7. Quaternion estimate

external disturbance in the form of a varying magnetic field is applied on top of the AUV. This disturbance can be seen in figure 6 towards the end. The estimated quaternion shows adequate stability performance both when the robot moves forward and when the vehicle needs to do 90 degree turns. This can be seen by the low variance of the estimated quaternion throughout the whole data. The quaternion estimate also shows insignificant change despite the application of external disturbances that strongly affect the magnetometer reading.

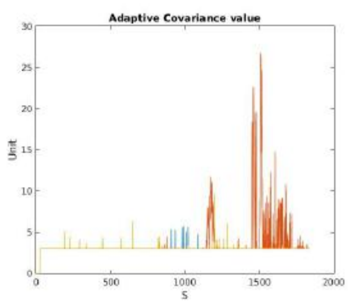

Fig. 8. Covariance Values

Figure 8 shows the modified R matrix after being scaled with c2. The figure shows multiple spikes, which means the algorithm has picked up notable slow-varying external disturbances and increased R values accordingly so that only small portions of the reading will be used to update the estimate.

V. CONCLUSION

This paper is focused on the attitude estimation problem for underwater vehicles. The problem addressed in this paper specifically targets micro-observational class underwater vehicles that are relatively small in shape such that magnetic fields from tight thruster arrangements can interfere with sensor measurements. In addition to aforementioned internal disturbances, micro-observational class vehicles are often used to do coral observations where magnetic disturbance from artificial reefs made of metal objects are also commonly found nowadays. The proposed solution is based on the Indirect Kalman Filter, which provides an Extended Kalman Filtering solution using a quaternion representation. Several design changes to the Indirect Kalman Filter are then proposed to minimize the effect of internal and external disturbances. Proposed design changes are tested and confirmed to be able to reject external and internal magnetic disturbances to provide reliable attitude estimation. This proposed design will be beneficial in the application of coral surveys, by having a small underwater vehicle that can maintain its orientation and navigate autonomously to keep track of coral growth across long coastal lines without being affected by metal debris. In the future this algorithm can be combined with vision navigation and periodic resurfacing to do GPS correction providing autonomous position in addition to orientation.

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