Manipulating and Grasping Forces in Manipulation by Multifingered Robot Hands

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Abstract—**Much research has been done recently on the analysis and design of multifingered robot hands. Several algorithms for determining the fingertip force for a given task have also been proposed. But, the grasping and manipulating forces involved in the manipulation of objects by robotic and human hands have not been defined in a physically reasonable way. The purpose of this paper is to propose a new definition of grasping and manipulating forces for multifingered robot hands. First, a short discussion of the grasping and manipulating forces for two-fingered hands with linear motion is given to explain the motivation more clearly and to give the basic idea of the new definition. Then, for three-fingered hands, based on a representation of the internal force, the grasping force is defined as an internal force that satisfies the static friction constraint. The concept of grasp mode is also introduced. The manipulating force is then defined as a fingertip force that satisfies the following three conditions: 1) It produces the specified resultant force. 2) It is not in the inverse direction of the grasping force. 3) It is orthogonal to the grasping force component. An algorithm for decomposing a given fingertip force into manipulating and grasping forces is presented. Extensions of the result to cases of two-fingered hands with planar motion and four-fingered hands are discussed. Finally, a simple example of synthesizing fingertip force for a given manipulation task is given to illustrate the usefulness of the proposed definition.**

I. Introduction

This paper discusses the question of whether we can define grasping and manipulating forces for multifingered hands in a reasonable way.

Various multifingered hands for manipulating objects skillfully have been developed so far [1]–[6]. Analytical studies of grasping and manipulation by robot hands have also been done by many researchers [6]–[17]. It has been well recognized that internal force is an important factor in the analysis and synthesis of fingertip forces for grasping and manipulation. Mason and Salisbury [5] gave conditions for complete restraint of an object by a grasp in terms of internal force. Kerr and Roth [10] proposed to determine the optimal internal force as that with the minimum norm under an approximated frictional constraint. Kumar and Waldron [13] have a computationally efficient suboptimal solution to this problem. Ji and Roth [16] proposed a method of determining fingertip force of three-fingered hands by first finding the internal force that minimizes the maximum of the angles between the contact normals and the internal force components. Demmel and Lafferriere [18] proposed a different scheme for the same problem. Schwartz and Sharir [19] defined the optimality as minimizing the coefficient of friction needed to attain a grasp in the presence of a given external force and torque, and gave a complexity analysis of the solution algorithm.

Most of these research efforts involve the decomposition of the fingertip force into two components by using the pseudoinverse of a coefficient matrix that relates the resultant force exerted on the object to the fingertip force. One component is given by premultiplying the resultant force by the pseudoinverse of the coefficient matrix. This component is characterized as the one with the minimum norm among those producing the specified resultant force. The other component, which is given by a vector belonging to the null space of the coefficient matrix, is an internal force. Because of their mathematical simplicity and clear meaning in terms of vector norm, we are tempted to define these two components as manipulating and grasping forces (see, e.g., [6]). However, this is not adequate from a physical point of view as will be shown in the next section.

In this paper,a new physically reasonable definition of manipulating force and grasping force is given for two-, three-, and four-fingered hands. First, for a simple case of two-fingered hands with linear motion, the irrelevance of the pseudoinverse decomposition of fingertip force as the basis of defining grasping and manipulating forces is shown, and a new physically reasonable definition is proposed. Then for three-fingered hands a representation of the internal force is given. Based on this representation, the grasping force and the manipulating force are defined. An algorithm for decomposing a given fingertip force into manipulating and grasping forces is presented. It is shown that this definition can be extended to two-fingered hands with planar motion and four-fingered hands. Finally, the usefulness of the obtained analytical result is shown by an example of synthesizing the fingertip force for a given manipulation task.

Note that only fingertip grasps with point contact are treated, and the Coulomb model is used for friction in this paper.

II. Two-Fingered Hand with Linear Motion

To explain the motivation and the main idea of our study, we first consider the problem of decomposing a given fingertip force into grasping and manipulating force components for a simple two-fingered hand as shown in Fig. 1.

Assume that the two fingers have one degree of freedom and can move in the x direction. Let $f_i \in R^1$ (R^n is the set of all n-dimensional real vectors) denote the force applied by the ith finger ($i = 1, 2$) on the object along the x axis. We assume that both fingers can only push, that is, $f_1 \geq 0$ and $f_2 \leq 0$. The relation between the fingertip force $F = [f_1, f_2]^T$ and the resul-

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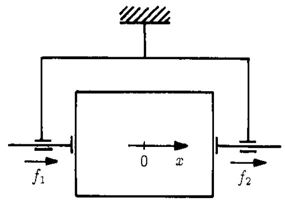


Fig. 1. Two-fingered hand with linear motion.

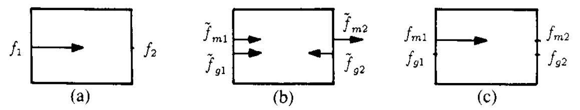


Fig. 2. Decomposition of fingertip force. (a) Fingertip force. (b) Decomposition by (4). (c) Decomposition by (5).

tant force $t \in R^1$ exerted on the object is given by

$$t = f_1 + f_2 = AF \tag{1}$$

where $A = [1, 1]$.

When a resultant force t is given, fingertip F that produces this t is given by the general solution of (1):

$$F = A^+ t + (E_2 - A^+ A) y \tag{2}$$

where A^+ is the pseudoinverse [20] of A, $y \in R^2$ is an arbitrary constant vector, and $E_n \in R^{n \times n}$ ($R^{n \times n}$ is the set of all $n \times n$ real matrices) denotes the unity matrix. The first term of (2) represents the fingertip force with the minimum norm among those that produce the resultant force t. The second term represents an internal force. Now assume that we are given a fingertip force F first instead of t. Substituting (1) into (2) yields

$$F = A^+ AF + (E_2 - A^+ A) F \tag{3}$$

This equation represents a decomposition of the fingertip force into two components. We are tempted to define the manipulating force and the grasping force by the first term $A^+ AF \triangleq [\tilde f_{m1}, \tilde f_{m2}]^T$, and the second term $(E_2 - A^+ A) F \triangleq [\tilde f_{g1}, \tilde f_{g2}]^T$, respectively, on the right-hand side of (3). However, this is not adequate from a physical point of view as is shown in the following.

Since $A^+ = A^T (AA^T)^{-1} = [1/2, 1/2]^T$, we have

$$\begin{bmatrix} \tilde f_{m1} \\ \tilde f_{m2} \end{bmatrix} = \begin{bmatrix} (f_1 + f_2)/2 \\ (f_1 + f_2)/2 \end{bmatrix} \tag{4a}$$

$$\begin{bmatrix} \tilde f_{g1} \\ \tilde f_{g2} \end{bmatrix} = \begin{bmatrix} (f_1 - f_2)/2 \\ -(f_1 - f_2)/2 \end{bmatrix}. \tag{4b}$$

If we consider the case of $f_1 > 0$ and $f_2 = 0$, which is schematically shown in Fig. 2(a), we have $[\tilde f_{g1}, \tilde f_{g2}]^T = [f_1/2, -f_1/2]^T$, implying that the force component for grasping is not zero [see Fig. 2(b)]. However, since $f_2 = 0$, the object will slip out of the two fingers by an application of any external force perpendicular to the x axis however small it may be. Therefore, according to our institution, the grasping force for this case should be zero. Note also that $\tilde f_{m2}$ given by (4) is negative, implying that the manipulating force for the second finger is a pulling force. Hence, $[\tilde f_{m1}, \tilde f_{m2}]^T$ and $[\tilde f_{g1}, \tilde f_{g2}]^T$ are not suitable for the definition of manipulating and grasping forces.

Based on this consideration, we now propose to define the manipulating force $[f_{m1}, f_{m2}]^T$ and the grasping force $[f_{g1}, f_{g2}]^T$ by the following equations:

$$\begin{bmatrix} f_{m1} \\ f_{m2} \end{bmatrix} = \begin{bmatrix} k \\ -(1-k) \end{bmatrix} |f_1 + f_2| \tag{5a}$$

$$\begin{bmatrix} f_{g1} \\ f_{g2} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \min(|f_1|, |f_2|) \tag{5b}$$

where k is an auxiliary parameter that takes a value of 1 when $f_1 + f_2 \geq 0$ and a value of 0 when $f_1 + f_2 < 0$. According to this definition, we always have $f_{m1} \geq 0$, $f_{g1} \geq 0$, $f_{m2} \leq 0$, and $f_{g2} \leq 0$. Also, when $f_1 > 0$ and $f_2 = 0$, we have $[f_{m1}, f_{m2}]^T = [f_1, 0]^T$ and $[f_{g1}, f_{g2}]^T = [0, 0]^T$ [see Fig. 2(c)]. This result agrees with our intuitive picture of the grasping force.

We will extend the definition of grasping and manipulating forces given by (5) for two-fingered hands to the case of three-fingered hands in Sections III through V.

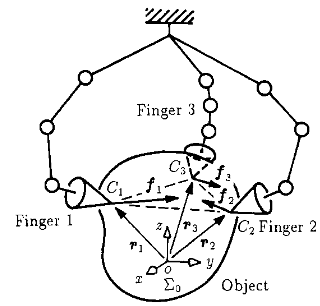


Fig. 3. Three-fingered hand and object.

III. A Representing of Internal Force for Three-Fingered Hands

The basic relations of forces in manipulation of a rigid object by a three-fingered hand and a representation of the internal force will be given in this section under the following assumptions.

A1) Each fingertip makes a frictional point contact with the object.

A2) The three contact points are not located on a straight line.

A3) The mechanism of each finger is such that each fingertip can exert a force to the object in any direction.

Assumption A1 means that the interaction between each finger and the object can be modeled as a pure force through some point, which we call a contact point. Assumptions A2 and A3 are necessary for arbitrary manipulation of the object.

Fig. 3 shows a robot hand and an object under consideration. In the figure, Σ_0 $(0 - xyz)$ is the object coordinate frame fixed to the object, C_i is the contact point of the ith finger $(i = 1, 2, 3)$, $r_i = [r_{ix}, r_{iy}, r_{iz}]^T \in R^3$ is the position vector of C_i from the origin of Σ_0, and $f_i \in R^3$ is the fingertip force applied to the object by the ith finger.

The resultant force $f \in R^3$ and resultant moment $n \in R^3$ due to $(f_i,\ i = 1, 2, 3)$ are given by

$$f = \sum_{i=1}^{3} f_i \tag{6a}$$

$$n = \sum_{i=1}^{3} (r_i \times f_i). \tag{6b}$$

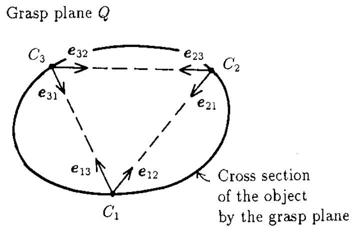


Fig. 4. Vectors e_{ij}.

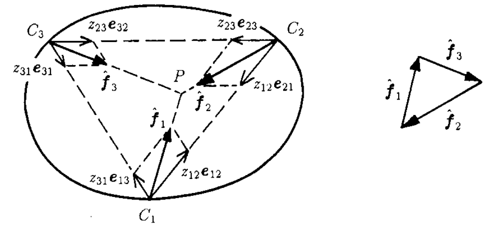


Fig. 5. Internal force and force triangle. (a) Internal force. (b) Force triangle.

From (6) we obtain the basic relation between the total fingertip force $F \triangleq [f_1^T, f_2^T, f_3^T]^T$ and the total resultant force $T \triangleq [f^T, n^T]^T$:

$$T = AF \tag{7}$$

where

$$A \triangleq \begin{bmatrix} E_3 & E_3 & E_3 \\ R_1 & R_2 & R_3 \end{bmatrix} \in R^{6\times 9} \tag{8}$$

$$R_i \triangleq \begin{bmatrix} 0 & -r_{iz} & r_{iy} \\ r_{iz} & 0 & -r_{ix} \\ -r_{iy} & r_{ix} & 0 \end{bmatrix} \in R^{3\times 3}. \tag{9}$$

The general solution F of (7) for a given T is written as

$$F = A^+T + (E_9 - A^+A)y \tag{10}$$

where $y \in R^9$ is an arbitrary constant vector. Although the second term, $(E_9 - A^+A)y$, in (10) represents the internal force among the fingers, this expression is not convenient due to the following fact. From assumption A2 the rank of A is 6 and $\operatorname{rank}(E_9 - A^+A) = 3$. Hence, the number of independent unknown parameters of the internal force is three. Determining the internal force by vector y means determining a three-dimensional quantity by specifying a nine-dimensional quantity. In order to avoid this inconvenience, another expression of the internal force will now be given. Let

$$e_{ij} \triangleq (r_j - r_i)/\|r_j - r_i\|, \qquad i, j = 1, 2, 3,\ i \neq j \tag{11}$$

where $\|r\|$ denotes the Euclidean norm of vector r. The vector e_{ij} is the unit vector directing from C_i to C_j on the grasp plane Q including the three contact points, and $e_{ij} = -e_{ji}$ (see Fig. 4). We have:

Proposition 1: A total fingertip force F is an internal force if and only if there exist a vector $z = [z_{23}, z_{31}, z_{12}]^T \in R^3$ such that

$$F = Gz \tag{12}$$

where

$$G \triangleq \begin{bmatrix} 0 & e_{13} & e_{12} \\ e_{23} & 0 & e_{21} \\ e_{32} & e_{31} & 0 \end{bmatrix} \in R^{9\times 3} \tag{13}$$

Proof: From (8), (9), (11), and (13), we obtain $AG = 0$. Hence, $\operatorname{Range}(G) \subset \operatorname{Range}(E_9 - A^+A)$ where $\operatorname{Range}(\cdot)$ is the range space of a matrix. On the other hand, from assumption A2 and the fact that $\operatorname{rank}(E_9 - A^+A) = 3$, we have $\operatorname{rank}(G) = \operatorname{rank}(E_9 - A^+A)$. Hence, we obtain $\operatorname{Range}(G) = \operatorname{Range}(E_9 - A^+A)$, completing the proof. ■

The property of the internal force expressed by (12) and (13) has been recognized by many researchers (e.g., [4], [10], [13], [16], and [21]). For example, Salisbury and Craig [4] introduced the concept of grasp matrix, which includes G^T as a submatrix. Expression (12) is presented here and proved rigorously because it plays a key role in the following development.

It can be easily shown that any internal force $\hat{F} = [\hat{f}_1^T, \hat{f}_2^T, \hat{f}_3^T]^T$ given by (12) has the following well known geometric characteristics (see, e.g., [21], [16]). As shown in Fig. 5, the load lines of the three fingertip forces $\hat{f}_i$ generally intersect at a point P on plane Q, and the three forces form a closed triangle (force triangle) due to the balance of force and moment. The three degrees of freedom of the internal force can be interpreted as the sum of the two degrees of freedom in the position of the point P on the plane Q, and one degree of freedom in the scale of the force triangle. Therefore, there is one-to-one relation between choosing an internal force and determining both the position of P and the scale of the force triangle. The point P is called the focus of the internal force. Note that when the three load lines are parallel to each other, the point P does not exist. However, the argument developed in this paper is valid by regarding this situation as a limiting case where point P goes to infinity.

IV. Grasping Force for Three-Fingered Hands

Based on the result in the previous section, the grasping force is defined and the concept of grasp mode is introduced for three-fingered hands in this section.

Definition 1: A fingertip force F is called a grasping force if the following two conditions are satisfied.

Condition 1:

$$AF = 0 \tag{14}$$

Condition 2:

$$\frac{f_i^T a_i}{\|f_i\|} > \frac{1}{\sqrt{1+\mu_i^2}}, \qquad i = 1, 2, 3 \tag{15}$$

where a_i is the inward unit normal vector of the object surface and μ_i is the static friction coefficient at the contact point C_i.

Condition 1 implies that F is an internal force. Condition 2 means that each fingertip force should satisfy the friction constraint. Due to this constraint, the location of point P on plane Q in Fig. 5 for grasping force F is also restricted. This restriction will now be analyzed and a condition for the existence of a grasping force will be derived for a given set of three contact points on an object.

Let the position vector of focus P be r_p, and

$$e_{pi} \triangleq \frac{r_p - r_i}{\|r_p - r_i\|}, \qquad i = 1, 2, 3. \tag{16}$$

The vector e_{pi} is the unit vector directing from contact point C_i to focus P. In the case of Fig. 5, the fingertip force $\hat{f}_i$ and the

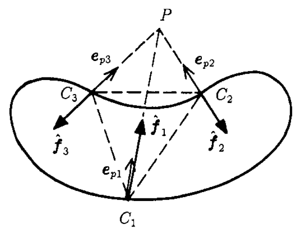


Fig. 6. Relation between $\hat{f}_i$ and e_{pi}.

e_{pi} are in the same direction. However, this does not necessarily hold for all cases. For example, in the case of Fig. 6, $\hat{f}_2$ and e_{p2}, and $\hat{f}_3$ and e_{p3} are in the opposite direction. Taking this point into consideration, the constraint of (15) can be shown to be equivalent to the following two relations:

$$f_i = \operatorname{sgn}\left(e_{pi}^T a_i\right) \| f_i \| e_{pi} \tag{17}$$

$$| e_{pi}^T a_i | > \frac{1}{\sqrt{1 + \mu_i^2}}, \qquad i = 1, 2, 3. \tag{18}$$

In order to express the state of the internal force described by (12), let

$$\begin{aligned} \alpha &\triangleq [\alpha_1, \alpha_2, \alpha_3] \\ \alpha_i &\triangleq \operatorname{sgn}\left(z_{(i+1)(i+2)}\right), \qquad i = 1, 2, 3 \end{aligned} \tag{19}$$

where the subscript j $(= i + 1, i + 2)$ is interpreted as $(j - 3)$ when $j \geq 4$ for notational convenience, and $\operatorname{sgn}(a)$ denotes the sign of a, i.e.,

$$\operatorname{sgn}(a) = \begin{cases} +1, & \text{if } a > 0 \\ \pm 1, & \text{if } a = 0 \\ -1, & \text{if } a < 0. \end{cases} \tag{20}$$

Since $\operatorname{sgn}(z_{i(i+1)})$ tells whether the grasping force between fingers i and $i + 1$ is compression or tension, we can categorize the internal forces by α. A fingertip force that satisfies (12) will be called the internal force of mode α. For example, the internal force in Fig. 5 is of mode $[+1, +1, +1]$ and that in Fig. 6 is of mode $[-1, +1, +1]$. When F is a grasping force, since this mode is useful for classifying it, α will be called the grasp mode. The variable α, and the variables β and γ, which will be introduced later in this section, will be denoted by the symbols $+$ and $-$ hereafter. For example, $[+1, +1, -1]$ will be expressed as $[+, +, -]$. Fig. 7 shows the four essentially different grasp modes. For example, the grasp mode in Fig. 7(d) is the mode that appears when picking up a plate with a hole by inserting three fingers into the hole and opening them out.

Collecting $\operatorname{sgn}(e_{pi}^T a_i)$, $i = 1, 2, 3$, which appeared in (17), we define the parameter β by

$$\beta \triangleq \left[\operatorname{sgn}\left(e_{p1}^T a_1\right), \operatorname{sgn}\left(e_{p2}^T a_2\right), \operatorname{sgn}\left(e_{p3}^T a_3\right)\right]. \tag{21}$$

This parameter β represents the relation between the location of focus P and the shape of the object. We also define regions I through VII on plane Q and their code $\gamma = [\gamma_1, \gamma_2, \gamma_3]$ ($\gamma_i = +1$ or -1) as shown in Fig. 8. Then for an internal force to be a grasping force, it is necessary for the parameters α, β, and γ to satisfy one of the relationships listed in Table I. This can be easily shown by considering the requirements that any internal force should have a focus P (including one at infinity) and that each fingertip force should be in the object's pushing direction.

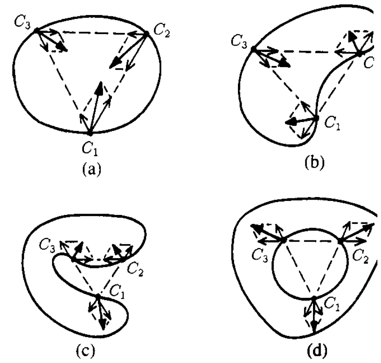


Fig. 7. Four grasp modes. (a) $\alpha = [+ + +]$. (b) $\alpha = [+ + -]$. (c) $\alpha = [+ - -]$. (d) $\alpha = [- - -]$.

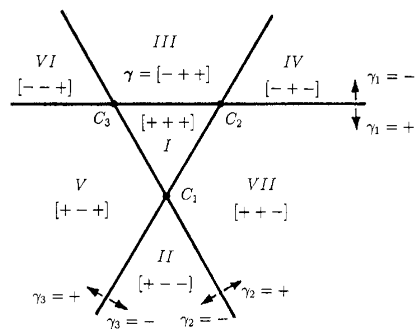


Fig. 8. Seven regions on the plane Q and their codes.

TABLE I
RELATION AMONG α, β, AND γ

Region	γ (Code of Region)	β (Shape of Object)	α (Grasp Mode)
I	$[+ + +]$	$[+ + +]$	$[+ + +]$
		$[- - -]$	$[- - -]$
II	$[+ - -]$	$[+ - -]$	$[+ - -]$
		$[- + +]$	$[- + +]$
III	$[- + +]$	$[- + +]$	$[+ - -]$
		$[+ - -]$	$[- + +]$
IV	$[- + -]$	$[- + -]$	$[- + -]$
		$[+ - +]$	$[+ - +]$
V	$[+ - +]$	$[+ - +]$	$[- + -]$
		$[- + -]$	$[+ - +]$
VI	$[- - +]$	$[- - +]$	$[- - +]$
		$[+ + -]$	$[+ + -]$
VII	$[+ + -]$	$[+ + -]$	$[- - +]$
		$[- - +]$	$[+ + -]$

These relationships can further be condensed to the following conditions:

$$\beta = \gamma \quad \text{or} \quad -\gamma \tag{22}$$

$$\alpha = (\gamma_1 \cdot \gamma_2 \cdot \gamma_3)\beta. \tag{23}$$

Note that (22) is a condition on the relation between the location of P and the shape of the object with respect to P, and (23) tells the grasp mode realizable under the given location of P and the

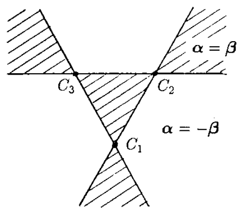


Fig. 9. Relation (23).

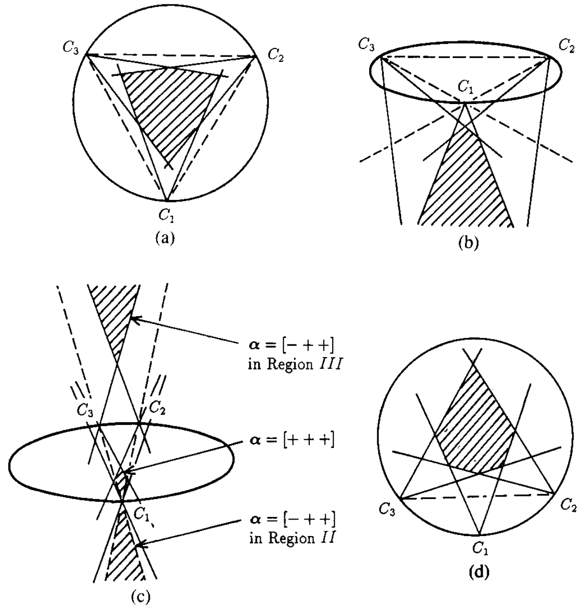


Fig. 10. Examples of realizable mode of grasping force. —— Object. ——— Boundary of Regions. —— Friction cone. ////// Location of P for realizable grasping force. (a) Only $\alpha = [+++]$ is realizable. (b) Only $\alpha = [-++]$ in Region II is realizable. (c) $\alpha = [+++]$ and $\alpha = [-++]$ in Regions II and III are realizable. (d) There is no realizable mode.

shape of the object. Relation (23) is schematically shown in Fig. 9. In the figure, the relation $\alpha = \beta$ should hold if P is in one of the shaded regions and $\alpha = -\beta$ in other regions. Note also that, due to the introduction of the code of region, γ, the relationships listed in Table I have been compactly expressed by (22) and (23).

Summarizing the above argument, we have:

Proposition 2: Suppose that the grasping positions r_i, the inward normal directions a_i, and the static friction coefficients μ_i are given. Then a grasping force can exist if there is a focus P satisfying the following two conditions:

Condition 1: a_i, e_{pi}, and μ_i satisfy the constraint of (18).

Condition 2: β and γ satisfy the condition of (22).

By these two conditions, we can obtain the locations of realizable focus P and the realizable grasp modes from the shape of the object, the grasping position, and the friction coefficient of the surface. Note that there are cases where only one grasp mode is possible,where two or more modes are possible, and where no grasping force exists.

Some examples are given in Fig. 10. In the figure it is assumed that the object is a cylinder with its axis perpendicular to plane Q, so that vectors a_i are in plane Q. It is also assumed that $\mu_i = 0.4$. Fig. 10(a) and (b) shows the cases where there is only one mode $\alpha = [+++]$ and $\alpha = [-++]$, respectively. These modes are obtained by checking the conditions for Proposition 2 and using (23). In the case of Fig. 10(b), for example, by condition 1 of Proposition 2 the focus P can only exist in the shaded area in Region II ($\gamma = [+--]$), and from (21) we have $\beta = [-++]$. Hence, condition 2 of Proposition 2 is satisfied and the grasp mode is $\alpha = [-++]$ from (23). Fig. 10(c) shows the case where two modes $\alpha = [+++]$ and $\alpha = [-++]$ are realizable. Fig. 10(d) shows the case where no grasping force exists. Note that, in the case of Fig. 10(d), by condition 1 of Proposition 2 the focus P can only exist in the shaded area in Region III ($\gamma = [-++]$) and from (21) we have $\beta = [+++]$ for any P in the shaded area, implying that condition 2 does not hold. When there is more than one realizable mode, some additional consideration for selecting one mode will be necessary. This would be a topic for further study.

Once mode α (i.e., the signs of z_{ij}) is determined using (12) for the internal force, an expression of the grasping force $F_g \triangleq [f_{g1}^T, f_{g2}^T, f_{g3}^T]^T \in R^9$ is given by

$$F_g = B_g h_g \tag{24}$$

where

$$B_g \triangleq \begin{bmatrix} 0 & \hat{e}_{13} & \hat{e}_{12} \\ \hat{e}_{23} & 0 & \hat{e}_{21} \\ \hat{e}_{32} & \hat{e}_{31} & 0 \end{bmatrix} \tag{25}$$

$$\hat{e}_{i(i+1)} \triangleq \alpha_{i+2} \cdot e_{i(i+1)}, \qquad i = 1,2,3 \tag{26}$$

$$\hat{e}_{i(i+2)} \triangleq -\hat{e}_{(i+2)i}, \qquad i = 1,2,3 \tag{27}$$

and h_g is an unknown vector satisfying

$$h_g \triangleq \left[h_{g1}, h_{g2}, h_{g3}\right]^T, \qquad h_{gi} \geq 0. \tag{28}$$

The vector $\hat{e}_{ij}$ given by (26) and (27) is the unit vector for the grasping force component at C_i working between fingers i and j. Note that taking mode α into consideration in defining $\hat{e}_{ij}$ makes it possible to assume $h_{gi} \geq 0$. Also note that the condition of (18) must still be satisfied for the existence of the grasping force.

V. Manipulating Force for Three-Fingered Hands

The manipulating force is defined for three-fingered hands, and its relation to the grasping force introduced in the previous section is discussed in this section.

Definition 2: For a given resultant force T, the fingertip force $F = [f_1^T, f_2^T, f_3^T]^T$ is called the manipulating force of mode α if F satisfies the following three conditions:

Condition 1:

$$T = AF \tag{29}$$

Condition 2:

$$f_i^T \hat{e}_{ij} \geq 0, \qquad i = 1,2,3, \; j = (i+1),(i+2) \tag{30}$$

Condition 3:

$$\left(f_i^T \hat{e}_{i(i+1)}\right)\left(f_{i+1}^T \hat{e}_{(i+1)i}\right) = 0, \qquad i = 1,2,3. \tag{31}$$

Note that the $\hat{e}_{ij}$'s are defined by (26) and (27), and so are dependent on mode α. Condition 1 means that F produces the resultant force T. Condition 2 implies that the manipulating force is not in the inverse direction of the grasping force.

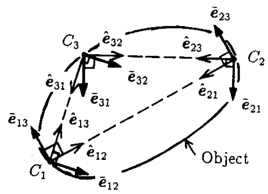


Fig. 11. Relation between $\hat{e}_{ij}$ and $\bar{e}_{ij}$ ($\bar{e}_{i0}$: upward and normal to the plane Q).

Condition 3 means that F contains no force component that results in compression or tension along the line joining C_1 and C_2, C_2 and C_3, or C_3 and C_1. In other words, F is orthogonal to the grasping force component.

In order to give an explicit expression for the manipulating force, we first introduce the following matrix:

$$B_m \triangleq \begin{bmatrix} 0 & (1-k_2)\bar{e}_{13} & k_3\bar{e}_{12} & \bar{e}_{10} & 0 & 0 \\ k_1\bar{e}_{23} & 0 & (1-k_3)\bar{e}_{21} & 0 & \bar{e}_{20} & 0 \\ (1-k_1)\bar{e}_{32} & k_2\bar{e}_{31} & 0 & 0 & 0 & \bar{e}_{30} \end{bmatrix} \quad (32)$$

where

$$\bar{e}_{i0} = \frac{\hat{e}_{i(i+1)} \times \hat{e}_{i(i+2)}}{\|\hat{e}_{i(i+1)} \times \hat{e}_{i(i+2)}\|} \quad (33)$$

$$\bar{e}_{i(i+1)} = \hat{e}_{i(i+2)} \times \bar{e}_{i0} \quad (34)$$

$$\bar{e}_{i(i+2)} = \bar{e}_{i0} \times \hat{e}_{i(i+1)} \quad (35)$$

and k_1, k_2, and k_3 are parameters that can take 1 or 0. The vectors $\bar{e}_{ij}$ given by (33)–(35) play the role of basis vectors for the manipulating force.

Fig. 11 shows the vectors $\hat{e}_{ij}$ and $\bar{e}_{ij}$ schematically for a typical case. Note that $\bar{e}_{i0}$ is normal to the grasp plane Q and

$$\bar{e}_{ij}^T\hat{e}_{ij} \geq 0, \qquad i = 1,2,3,\ j = (i+1),(i+2). \quad (36)$$

The matrix B_m is a function of α and $k = [k_1, k_2, k_3]^T \in R^3$. The parameter $k_1(k_2, k_3)$ determines which of the two vectors $\{\bar{e}_{23}, \bar{e}_{32}\}$, $(\{\bar{e}_{31}, \bar{e}_{13}\}, \{\bar{e}_{12}, \bar{e}_{21}\})$ is included in the matrix B_m. With these preparations, we can give the following proposition.

Proposition 3: A fingertip force F, which produces the resultant force T, is a manipulating force of mode α if it satisfies

$$F = B_m h_m \quad (37)$$

for some α, k, and $h_m \triangleq [h_{m1}, h_{m2}, \cdots, h_{m6}]^T$ with $h_{mi} \geq 0$, $i = 1,2,3$.

Proof: Suppose a fingertip force $\bar{F} \triangleq [\bar{f}_1^T, \bar{f}_2^T, \bar{f}_3^T]^T$ satisfies $T = A\bar{F}$ and (37), i.e.,

$$\begin{aligned} \bar{f}_1 &\triangleq k_3 h_{m3}\bar{e}_{12} + (1-k_2)h_{m2}\bar{e}_{13} + h_{m4}\bar{e}_{10} \\ \bar{f}_2 &\triangleq k_1 h_{m1}\bar{e}_{23} + (1-k_3)h_{m3}\bar{e}_{21} + h_{m5}\bar{e}_{20} \\ \bar{f}_3 &\triangleq k_2 h_{m2}\bar{e}_{31} + (1-k_1)h_{m1}\bar{e}_{32} + h_{m6}\bar{e}_{30}. \end{aligned} \quad (38)$$

It is then enough to show that these $\bar{f}_i$ satisfy (30) and (31). From (33)–(35) we have

$$\hat{e}_{i(i+1)}^T\bar{e}_{i(i+2)} = \hat{e}_{i(i+1)}^T\bar{e}_{i0} = 0 \quad (39)$$

$$\hat{e}_{i(i+2)}^T\bar{e}_{i(i+1)} = \hat{e}_{i(i+2)}^T\bar{e}_{i0} = 0. \quad (40)$$

Therefore, from (38), (39), (40), and (36)

$$\bar{f}_1^T\hat{e}_{12} = k_3 h_{m3}\bar{e}_{12}^T\hat{e}_{12} \geq 0 \quad (41)$$

$$\bar{f}_1^T\hat{e}_{13} = (1-k_2)h_{m3}\bar{e}_{13}^T\hat{e}_{13} \geq 0 \quad (42)$$

and

$$(\bar{f}_1^T\hat{e}_{12})(\bar{f}_2^T\hat{e}_{21}) = k_3(1-k_3)h_{m3}^2(\bar{e}_{12}^T\hat{e}_{12})(\bar{e}_{21}\hat{e}_{21}) = 0. \quad (43)$$

Hence, (30) and (31) for $i = 1$ are satisfied.It can also be shown by similar arguments that (30) and (31) for $i = 2$ and 3 are satisfied by $\bar{F}$. ■

The manipulating force will be expressed by $F_m \triangleq [f_{m1}^T, f_{m2}^T, f_{m3}^T]^T$ hereafter. From Proposition 3 a manipulating force F_m of mode α, which produces the resultant force T, satisfies

$$T = AF_m = AB_m h_m. \quad (44)$$

Hence, to obtain F_m for a given T, we first calculate $\hat{h}_m \triangleq [\hat{h}_{m1}, \hat{h}_{m2}, \cdots, \hat{h}_{m6}]^T \in R^6$ from

$$\hat{h}_m = (AB_m)^{-1}T \quad (45)$$

for eight different combinations of k_1, k_2, and $k_3(k_1, k_2, k_3 = 1$ or $0)$. We then select the one for which $\hat{h}_{mi} \geq 0$ $(i = 1,2,3)$ as the vector h_m. Then $F_m = B_m h_m$ with the above selected values h_m and $k = [k_1, k_2, k_3]^T$ is the desired manipulating force. The value k which satisfies $\hat{h}_{mi} \geq 0$ $(i = 1,2,3)$ is uniquely determined except for degenerate cases where at least one of the three $\hat{h}_{mi}$ is zero.

A relation between the manipulating and grasping forces is given by the following proposition.

Proposition 4: For any given manipulating force F_m of mode α,

$$\|F_m\| \leq \|F_m + F_g\| \quad (46)$$

for any grasping force F_g of the same mode α.

Proof: From (24), (25), (28), (30), and (37)

$$F_m^T F_g = \sum_{i=1}^{3}(f_{mi}^T f_{gi}) \geq 0. \quad (47)$$

Therefore,

$$\|F_m + F_g\|^2 = \|F_m\|^2 + 2F_m^T F_g + \|F_g\|^2 \geq \|F_m\|^2. \quad (48)$$ ■

VI. Decomposition of Fingertip Force into Manipulating and Grasping Forces

A method for decomposing a given fingertip force into a manipulating force and a grasping force for three-fingered hands will be given.[1]

Suppose that $\boldsymbol{r}_i$, $\boldsymbol{a}_i$, μ_i $(i = 1,2,3)$, and a fingertip force $\boldsymbol{F}$ are given, and that this $\boldsymbol{F}$ can be expressed as a sum of a manipulating force $\boldsymbol{F}_m$ and a grasping force $\boldsymbol{F}_g$ for some mode α. Then

$$\boldsymbol{F} = \boldsymbol{F}_g + \boldsymbol{F}_m = \boldsymbol{BH} \tag{49}$$

where

$$\boldsymbol{B} \triangleq [\boldsymbol{B}_g, \boldsymbol{B}_m] \tag{50}$$

$$\boldsymbol{H} \triangleq [\boldsymbol{h}_g^T, \boldsymbol{h}_m^T]^T \tag{51}$$

and $\boldsymbol{B}_g$ and $\boldsymbol{B}_m$ are given by (25) and (32). An algorithm for decomposition of $\boldsymbol{F}$ is as follows.

1) Find the set of all realizable modes $\boldsymbol{\alpha}$ from $\boldsymbol{r}_i$, $\boldsymbol{a}_i$, μ_i $(i = 1,2,3)$.
2) Pick up one realizable mode $\boldsymbol{\alpha}$ from the set obtained in 1). Calculate $\hat{\boldsymbol{H}} \triangleq [\boldsymbol{h}_g^T \ \hat{\boldsymbol{h}}_m^T]^T$ by
$$\hat{\boldsymbol{H}} = \boldsymbol{B}^{-1}\boldsymbol{F} \tag{52}$$
for the eight different values of $\boldsymbol{k}$.
3) Select $\boldsymbol{k}$ for which $\hat{h}_{gi} \ge 0$ $(i = 1,2,3)$ and $\hat{h}_{mi} \ge 0$ $(i = 1,2,3)$. Let $\hat{\boldsymbol{H}}$ for this $\boldsymbol{k}$ be $\boldsymbol{H}$ and calculate $\boldsymbol{F}_g = \boldsymbol{B}_g \boldsymbol{h}_g$. Then, if this $\boldsymbol{F}_g$ satisfies the friction constraint, the pair $\{\boldsymbol{F}_m = \boldsymbol{B}_m \boldsymbol{h}_m, \boldsymbol{F}_g = \boldsymbol{B}_g \boldsymbol{h}_g\}$ is a decomposition of $\boldsymbol{F}$.
4) Repeat steps 2 and 3 until all the realizable modes are checked.

Note that the decomposition may not be unique because of the existence of multiple grasp modes. Also, there are cases where such decomposition does not exist. Note also that whenever a fingertip force $\boldsymbol{F}$ can be decomposed into $\boldsymbol{F}_m$ and $\boldsymbol{F}_g$, $\|\boldsymbol{F}_m\| \le \|\boldsymbol{F}\|$ from Proposition 4.

A simple example of decomposition will be given in the following. The object is a cylinder of radius 5 with its axis coinciding with the z axis of the object coordinate frame Σ_0. Suppose that the three contact points and the fingertip forces are given by

$$\begin{aligned} \boldsymbol{r}_1 &= [0, -5, 0]^T & \boldsymbol{f}_1 &= [1, 8, 0]^T \\ \boldsymbol{r}_2 &= [5\sqrt{3}/2, 5/2, 0]^T & \boldsymbol{f}_2 &= [-3, -2, 0]^T \\ \boldsymbol{r}_3 &= [-5\sqrt{3}/2, 5/2, 0]^T & \boldsymbol{f}_3 &= [6, -2, 0]^T. \end{aligned} \tag{53}$$

Therefore, the plane Q is given by the x-y plane of Σ_0, and

[1] The decomposition of a given fingertip force into grasping and manipulating force has both theoretical and practical meanings. Theoretical meaning: If a definition of grasping and manipulating forces is sound, we should be able to decompose a given fingertip force into these two forces. Section VI gives an answer to this theoretical question. Practical meaning: When we choose the grasping and manipulating forces as controlled variables in a controller for a robotic hand, we usually need to know the current values of the grasping and manipulating forces from the measured fingertip force in order to implement the force servo loops in terms of these variables.

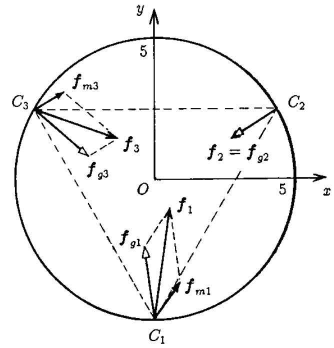


Fig. 12. Example of decomposition of fingertip force into manipulating and grasping forces (the scale of forces is $\times 0.5$).

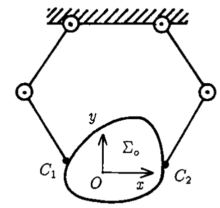


Fig. 13. Two-fingered hand with planar motion.

the $\boldsymbol{a}_i$'s are

$$\begin{aligned} \boldsymbol{a}_1 &= [0 \quad 1 \quad 0]^T \\ \boldsymbol{a}_2 &= [-\sqrt{3}/2 \quad -1/2 \quad 0]^T \\ \boldsymbol{a}_3 &= [\sqrt{3}/2 \quad -1/2 \quad 0]^T. \end{aligned} \tag{54}$$

Also suppose that $\mu_i = 0.4$, $i = 1,2,3$. Then it can be easily shown by using Proposition 2 and (19) that there is only one mode $[+ + +]$, and using the above algorithm we obtain the following manipulating and grasping forces of mode $[+ + +]$:

$$\begin{aligned} \boldsymbol{f}_{m1} &= [7/4, 4 - 3\sqrt{3}/4, 0]^T \\ \boldsymbol{f}_{m2} &= [0, 0, 0]^T \\ \boldsymbol{f}_{m3} &= [9/4, 3\sqrt{3}/4, 0]^T \\ \boldsymbol{f}_{g1} &= [-3/4, 4 + 3\sqrt{3}/4, 0]^T \\ \boldsymbol{f}_{g2} &= [-3, -2, 0]^T \\ \boldsymbol{f}_{g3} &= [15/4, -2 - 3\sqrt{3}/4, 0]^T. \end{aligned} \tag{55}$$

This is shown in Fig. 12. It can be easily seen that these forces satisfy Definitions 1 and 2.

In the following section, it will be shown that similar discussions can also be presented for two other types of hands, i.e., two-fingered hands with planar motion and four-fingered hands.

VII. Other Types of Hands

A. Two-Fingered Hands with Planar Motion

We consider a two-fingered hand grasping an object as shown in Fig. 13. Assume that each finger has two degrees of freedom and can move its fingertip in a plane. Assume also that the two fingertips make point contacts with friction with a grasped

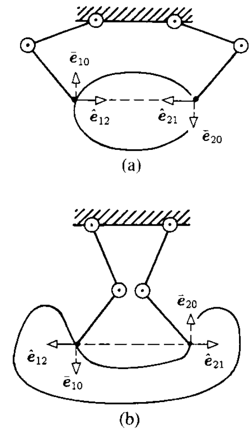


Fig. 14. Grasp mode of two-fingered hands with planar motion. (a) $\alpha = +1$. (b) $\alpha = -1$.

object. Hence, the hand can force the object to make a planar motion. Similar to the case of three-fingered hands, we consider an object coordinate frame $\Sigma_0(0 - xy)$. Let $f_i = [f_{ix}, f_{iy}]^T \in R^2$ denote the force applied by the ith finger ($i = 1, 2$) on the object in the x-y plane, and let $r_i = [r_{ix}, r_{iy}]^T \in R^2$ denote the position of contact point C_i of the ith finger. We assume that no force works in the direction normal to the plane x-y. The resultant force $f \in R^2$ and the resultant moment $n \in R^1$ due to $\{f_1, f_2\}$ are given by

$$f = \sum_{i=1}^{2} f_i \tag{56}$$

$$n = \sum_{i=1}^{2} (r_{ix} f_{iy} - r_{iy} f_{ix}). \tag{57}$$

Then the relation between the total fingertip force $F = [f_1^t, f_2^T]^T \in R^4$ and the total resultant force $T = [f_x, f_y, n]^T \in R^3$ is expressed as

$$T = AF \tag{58}$$

where

$$A = \left[\begin{array}{cc|cc} \multicolumn{2}{c|}{E_2} & \multicolumn{2}{c}{E_2} \\ \hline -r_{1y} & r_{1x} & -r_{2y} & r_{2x} \end{array}\right] \in R^{3\times 4}. \tag{59}$$

By a similar argument to that in the preceding sections, we can show that the grasping force and the manipulating force are given as follows. Let $e_{12} \in R^2$ denote the unit vector directing from C_1 to C_2 and let $e_{21} = -e_{12}$. Then the grasping force $F_g = [f_{g1}^T, f_{g2}^T]^T \in R^2$ is given by

$$F_g = B_g h_g \tag{60}$$

where h_g is an arbitrary scalar satisfying $h_g \geq 0$, and

$$B_g = \begin{bmatrix} \hat{e}_{12} \\ \hat{e}_{21} \end{bmatrix}. \tag{61}$$

Here, $\hat{e}_{12} = \alpha e_{12}$, and α is a scalar taking a value of $+1$ or -1. There are two grasp modes represented by $\alpha = +1$ and $\alpha = -1$, which are shown in Fig. 14. Note that the grasping force must also satisfy the friction constraint similar to (15), imposing a restriction on the shape of object and the location of contact points on the object. Let $\hat{e}_{12} = [\hat{e}_{12x}, \hat{e}_{12y}]^T$, then the

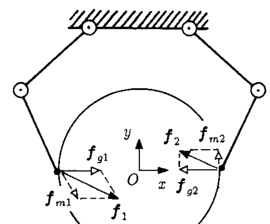


Fig. 15. Example of decomposition of fingertip force into manipulating and grasping forces.

manipulating force $F_m \in R^4$ is given by

$$F_m = B_m h_m \tag{62}$$

where

$$B_m = \begin{bmatrix} k\hat{e}_{12} & \bar{e}_{10} & 0 \\ (1-k)\hat{e}_{21} & 0 & \bar{e}_{20} \end{bmatrix} \in R^{4\times 3} \tag{63}$$

$$\bar{e}_{10} = [-\hat{e}_{12y}, \hat{e}_{12x}]^T, \; \bar{e}_{20} = -\bar{e}_{10} \tag{64}$$

and $h_m = [h_{m1}, h_{m2}, h_{m3}]^T$ with $h_{m1} \geq 0$. Unit vectors $\bar{e}_{10}$ and $\bar{e}_{20}$ are also shown in Fig. 14. Using (60) and (62) we can decompose a total fingertip force into the grasping and manipulating forces as in Section VI. For example, consider the case shown in Fig. 15 where

$$\begin{aligned} r_1 &= [-2, 0]^T & r_2 &= [2, 0]^T \\ a_1 &= [1, 0]^T & a_2 &= [-1, 0]^T \\ \mu_1 &= \mu_2 = 0.6. \end{aligned} \tag{65}$$

Then the fingertip force given by

$$f_1 = [1.5, -0.7]^T \qquad f_2 = [-1, 0.5]^T \tag{66}$$

is decomposed into

$$\begin{aligned} f_{g1} &= [1, 0]^T & f_{g2} &= [-1, 0]^T \\ f_{m1} &= [0.5, -0.7]^T & f_{m2} &= [0, 0.5]^T. \end{aligned} \tag{67}$$

This decomposition is also shown in Fig. 15.

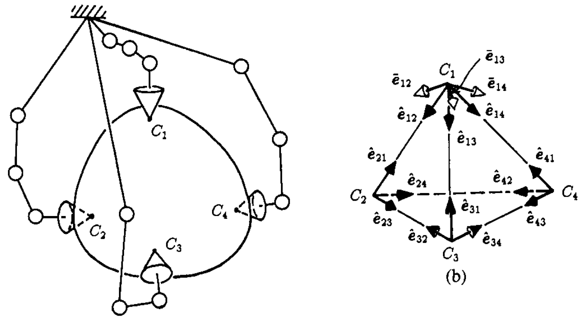
Note that a similar argument can also be developed for two-fingered hands with linear motion treated in Section II, and it can be shown that there are two grasp modes corresponding to those in Fig. 14.

B. Four-Fingered Hand

We can define the grasping and manipulating forces also for four-fingered hands as shown in Fig. 16. Since the arguments are quite similar to those for three-fingered hands, we will just give the expressions of these forces corresponding to (24) and (37).

Let $e_{ij} \in R^3$ denote the unit vector directing from the ith contact point C_i to the jth contact point C_j, $i, j = 1, 2, 3, 4$; $i \neq j$. Also let $\hat{e}_{ij} = z_{ij} e_{ij}$, where $z_{ij} \in R^1$ satisfies $z_{ij} = z_{ji}$, and $\alpha = [\mathrm{sgn}(z_{23}), \mathrm{sgn}(z_{13}), \mathrm{sgn}(z_{12}), \mathrm{sgn}(z_{14}), \mathrm{sgn}(z_{24}), \mathrm{sgn}(z_{34})] \in R^6$ represents the grasp mode. Then the grasping force $F_g = [f_{g1}^t, f_{g2}^T, f_{g3}^T, f_{g4}^T]^T \in R^{12}$ is given by

$$F_g = B_g h_g \tag{68}$$



(a)

Fig. 16. Four-fingered hands. (a) Hand and an object. (b) Unit vectors $\hat{e}_{ij}$ for grasping force and $\bar{e}_{1j}$ for manipulating force ($\bar{e}_{2j}$, $\bar{e}_{3j}$, $\bar{e}_{4j}$ are not shown).

where

$$h_g = \left[h_{g1}, \cdots, h_{g6}\right]^T \in R^6, \qquad h_{gi} \geq 0,\ i = 1, 2, \cdots, 6 \tag{69}$$

$$B_g = \begin{bmatrix} 0 & \hat{e}_{13} & \hat{e}_{12} & \hat{e}_{14} & 0 & 0 \\ \hat{e}_{23} & 0 & \hat{e}_{21} & 0 & \hat{e}_{24} & 0 \\ \hat{e}_{32} & \hat{e}_{31} & 0 & 0 & 0 & \hat{e}_{34} \\ 0 & 0 & 0 & \hat{e}_{41} & \hat{e}_{42} & \hat{e}_{43} \end{bmatrix}. \tag{70}$$

Note that the essential degree of arbitrariness of the internal force (or grasping force) is six in the present case because we are supposed to adjust the six-dimensional resultant force vector $\boldsymbol{T}$ by using the twelve-dimensional fingertip force vector $\boldsymbol{F}$. In (70), the expression of the grasping force, this arbitrariness is represented by the six pairs $\{\hat{e}_{ij}, \hat{e}_{ji}\}$ of unit force vectors along the six edges of the tetrahedron formed by the four contact points C_i as shown in Fig. 16(b) by black arrows.

On the other hand, the manipulating force is given by

$$F_m = B_m h_m \tag{71}$$

where

$$h_m = \left[h_{m1}, \cdots, h_{m6}\right]^T \in R^6, \qquad h_{mi} \geq 0,\ i = 1, 2, \cdots, 6 \tag{72}$$

$$B_m = \begin{bmatrix} 0 & (1-k_2)\bar{e}_{13} & k_3\bar{e}_{12} & k_4\bar{e}_{14} & 0 & 0 \\ k_1\bar{e}_{23} & 0 & (1-k_3)\bar{e}_{21} & 0 & k_5\bar{e}_{24} & 0 \\ (1-k_1)\bar{e}_{32} & k_2\bar{e}_{31} & 0 & 0 & 0 & k_6\bar{e}_{34} \\ 0 & 0 & 0 & (1-k_4)\bar{e}_{41} & (1-k_5)\bar{e}_{42} & (1-k_6)\bar{e}_{43} \end{bmatrix} \tag{73}$$

and $\bar{e}_{ij}$ is the unit vector normal to the plane including the three contact points other than C_j and satisfies $\bar{e}_{ij}^T \hat{e}_{ij} > 0$. An example of the set $\bar{e}_{ij}$ is shown by white arrows in Fig. 16(b) for $i = 1$. Note that, for example, $\bar{e}_{14}$ is perpendicular to $\hat{e}_{12}$, $\hat{e}_{23}$ and $\hat{e}_{31}$, which are unit vectors for grasping force components between C_1 and C_2, C_2 and C_3, and C_3 and C_1. Hence, when the location of contact points C_i ($i = 1, 2, 3, 4$), friction coefficients μ_i ($i = 1, 2, 3, 4$), and the grasp mode $\boldsymbol{\alpha}$ are given, the decomposition of a given fingertip force $\boldsymbol{F}$ into grasping force $\boldsymbol{F}_g$ and manipulating force $\boldsymbol{F}_m$ can be done by a procedure similar to that in Section VI. Note, however, that the concept of the focus of the internal force introduced for three-fingered hands cannot be directly extended to the case of four-fingered hands. Hence, there is no direct extension of Proposition 2 on the existence of grasping force. Obtaining conditions for the

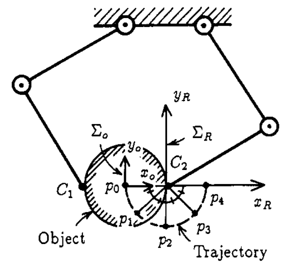


Fig. 17. Example of synthesis of fingertip force.

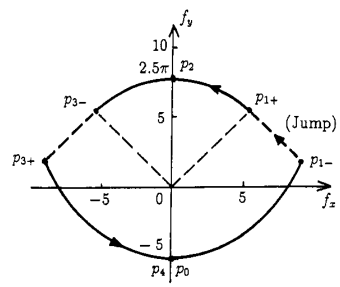


Fig. 18. Trajectory of the total resultant force $\boldsymbol{T} = [f_x, f_y, 0]^T$(N) for the object trajectory $p_0 - p_4$ in Fig. 17.

existence of a grasping force for four-fingered hands will be a topic for future research.

For hands with more than four fingers, we have not succeeded in defining the grasping and manipulating forces. This will also be a topic for future research.

VIII. Application to the Synthesis of Fingertip Force

A simple application of the concepts of grasping and manipulating forces to the synthesis of fingertip force for a given task of manipulation will be presented in this section.

Consider a two-fingered hand with planar motion and an a cylindrical object with a radius of 0.05 m, a mass of 1 kg, and friction coefficient $\mu = 0.5$ as shown in Fig. 17. Suppose that the given task is to carry the object following a circular trajectory. First we define the object frame Σ_0, its origin at the center of the cylinder. We assume that the contact points are given by $\boldsymbol{r}_1 = [-0.05, 0]^T$ and $\boldsymbol{r}_2 = [0.05, 0]^T$. The desired trajectory of the origin of Σ_0 is assumed to be given by a half circle with a radius of 0.05 m starting from p_0 of the reference frame Σ_R shown in Fig. 17, going through p_1, p_2, and p_3, and reaching to p_4. The trajectory accelerates by 5 m/s^2 during the path (p_0, p_1), goes with constant speed during(p_1, p_3), and decelerates by -5 m/s^2 during (p_3, p_4). The orientation of the object is assumed to be fixed. Then for the case of no gravitational effect, the time history $f(t)$ (N) of the desired total resultant force to be applied to the origin of the object is given in Fig. 18.

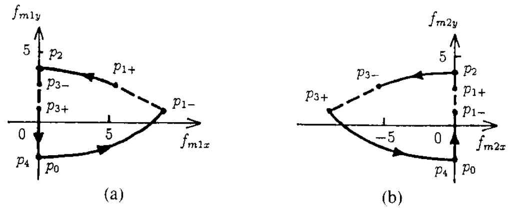


Fig. 19. Trajectory of manipulating force $F_m(t)$ (N). (a) $f_{m1}(t)$. (b) $f_{m2}(t)$.

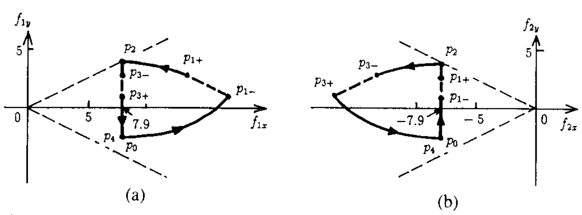


Fig. 20. Trajectory of fingertip force $F(t) = [f_1(t), f_2(t)]^T$ (N) for $h_g = 7.9$ (N). ---- Frictional limit. (a) $f_1(t)$. (b) $f_2(t)$.

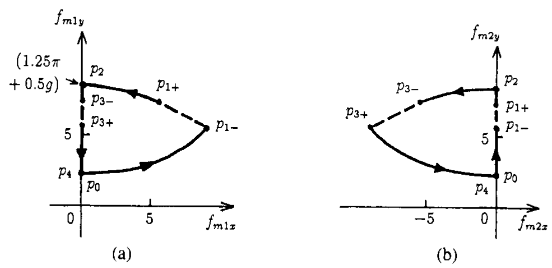


Fig. 21. Trajectory of fingertip force $F_m(t)$ (N) under gravity in minus y direction. (a) $f_{m1}(t)$. (b) $f_{m2}(t)$.

Noticing that the realizable grasp mode is given by $\alpha = +1$ and using (63) with $k = 1$ for the path (p_0, p_2) and $k = -1$ for the path (p_2, p_4), the manipulating force $F_m(t) = [f_{m1}^T(t), f_{m2}^T(t)]^T$ is given in Fig. 19.

Now we consider the selection of the grasping force F_g. Since the general form of F_g in this case is $F_g = [1, 0, -1, 0]^T h_g$ from (60), the fingertip force $F(t) = [f_1^T(t), f_2^T(t)]^T$ for a fixed h_g is given in Fig. 20. Therefore, if we select a grasping force satisfying $h_g \geq 2.5\pi$(N) $\cong$ 7.9(N), then the given task can be achieved without slipping. If the object is under gravitational effect, $F_m(t)$ is given as in Fig. 21, and $h_g \geq (2.5\pi + g)$(N) $\cong$ 17.7(N) (g is the gravity constant) is necessary for achieving the task without slipping. Further, if some external disturbance force $f_e(t)$ such that $\| f_e(t) \| < 2$(N) is expected to work at the origin of Σ_0 and should be canceled out by the fingertip force, then the time history of $f(t)$ may deviate within the shaded region shown in Fig. 22. Obtaining the corresponding region of the manipulating force shown in Fig. 23, we find that it is enough to select a grasping force satisfying $h_g \geq 2.5\pi + 2$(N) $\cong$ 9.9(N) to achieve the task. These simple selections of grasping force can usually be done off-line for each given task, reducing the necessary real-time computation for synthesizing the fingertip forces.If the given task is more complicated and just one selection of the grasping force is not appropriate, we can decompose the task into several sequential subtasks and determine a suitable grasping force for each subtask.

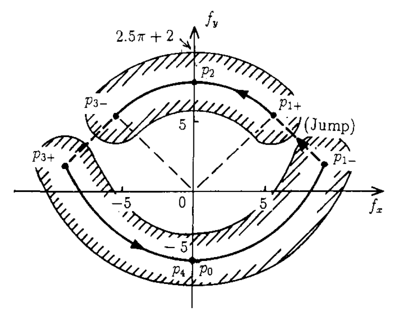


Fig. 22. Region of possible trajectory of the total resultant force $T = [f_x, f_y, 0]^T$ (N) for the object trajectory $p_0 - p_4$ in Fig. 17 and external disturbance $f_e(t)$ such that $\| f_e(t) \| < 2$(N).

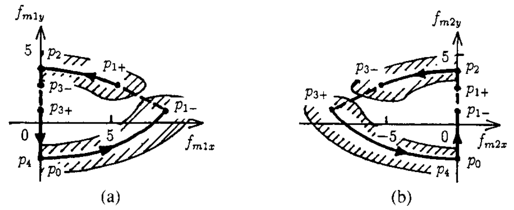


Fig. 23. Trajectory of manipulating force $F_m(t) = [f_{m1}^T(t), f_{m2}^T(t)]^T$ (N) for the resultant force in Fig. 22. (a) Region of $f_{m1}(t)$. (b) Region of $f_{m2}(t)$.

IX. Conclusion

The concepts of the manipulating force and grasping force, which have sometimes been used intuitively for describing some force components involved in manipulation of objects by human hands, have been studied for multifingered robot hands. The main results obtained in this paper are summarized as follows.

1) The grasping and manipulating forces for two-fingered hands with linear motion have been discussed to explain the motivation and the main idea of this paper.

2) Three-fingered hands have been studied in detail. First, the grasping force has been defined as an internal force that satisfies the static friction constraint. The concept of grasp mode has been introduced. Then the manipulating force has been defined as a fingertip force satisfying the following three conditions: 1) It produces the specified resultant force. 2) It is not in the inverse direction of the grasping force. 3) It is orthogonal to the grasping force component. An explicit expression of the manipulating force has been given. An algorithm for decomposing a given fingertip force into manipulating and grasping forces has also been given.

3) It has been shown that similar arguments can be developed for two-fingered hands with planar motion and four-fingered hands.

4) To show the usefulness of the concepts of grasping and manipulating forces, a simple problem of synthesizing the fingertip force of a two-fingered hand with planar motion for a given task of manipulating a cylindrical object has been discussed.It has been shown that the grasping force can be determined off-line, implying that the load of real-time computation is reduced.

These results will be useful for developing control algorithms for multifingered robot hands and cooperated manipulation of objects by multiple robots. In particular, the concepts of grasping force and grasp mode would be helpful for securely grasping objects with various shapes. In applying these results to on-line control, however, we will have to pay attention to the rather large computational amount for calculating grasping and manipulating forces.

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References

[1] T. Okada, "Object-handling system for manual industry," *IEEE Trans. Syst. Man Cybern.*, vol. SMC-9, no. 2, pp. 79-89, 1979.

[2] H. Hanafusa and H. Asada, "Stable prehension by a robot hand with elastic fingers," in *Proc. 7th Int. Symp. Industrial Robots*, Tokyo, 1977, pp. 361-368.

[3] S. C. Jacobsen *et al.*, "The version I UTA/MIT dextrous hand," in *Robotics Res., 2nd Int. Symp.* Cambridge, MA: MIT Press, 1985, pp. 301-308.

[4] J. K. Salisbury and J. J. Craig, "Articulated hands: Force control and kinematic issues," *Int. J. Robotics Res.*, vol. 1, no. 1, pp. 4-17, 1982.

[5] M. T. Mason and J. K. Salisbury, *Robot Hands and the Mechanics of Manipulation.* Cambridge, MA: MIT Press, 1985.

[6] H. Kobayashi, "Control and geometrical consideration for an articulated robot hand," *Int. J. Robotics Res.*, vol. 4, no. 1, pp. 3-12, 1985.

[7] T. Yoshikawa, "Manipulability of robotic mechanisms," *Int. J. Robotics Res.*, vol. 4, no. 2, pp. 3-9, 1985.

[8] H. Kobayashi, "On the articulated hands," in *Robotics Res., 2nd Int. Symp.* Cambridge, MA: MIT Press, 1985, pp. 293-300.

[9] H. Hanafusa, T. Yoshikawa, Y. Nakamura, and K. Nagai, "Structural analysis and robust prehension of robotic hand-arm system," in *Proc. Int. Conf. Advanced Robotics*, Tokyo, 1985, pp. 311-318.

[10] J. Kerr and B. Roth, "Analysis of multifingered hands," *Int. J. Robotics Res.*, vol. 4, no. 4, pp. 3-17, 1986.

[11] J. M. Abel, W. Holzmann, and J. M. McGarthy, "On grasping planar objects with two articulated fingers," in *Proc. IEEE Int. Conf. Robotics Automat.*, 1985, pp. 576-581.

[12] J. C.Trinkle, J. M. Abel, and R. P. Paul, "Enveloping, frictionless, planar grasping," in *Proc. IEEE Int. Conf. Robotics Automat.*, 1987, pp. 246-251.

[13] V. Kumar and K. J. Waldron, "Sub-optimal algorithms for force distribution in multifingered grippers," in *Proc. IEEE Int. Conf. Robotics Automat.*, 1987, pp. 252-257.

[14] Z. Li and S. Sastry, "Task oriented optimal grasping by multifingered robot hands," in *Proc. IEEE Int. Conf. Robotics Automat.*, 1987, pp. 389-394.

[15] Y. Nakamura, K. Nagai, and T. Yoshikawa, "Mechanics of cooperative manipulation by multiple robotic mechanisms," in *Proc. IEEE Int. Conf. Robotics Automat.*, 1987, pp. 991-998.

[16] Z. Ji and B. Roth, "Direct computation of grasping force for three-finger tip-prehension grasps," *ASME J. Mechanisms, Transmissions, Automat. Design*, vol. 110, pp. 405-413, 1988.

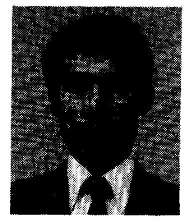
[17] V. D. Nguyen, "Constructing force-closure grasps," *Int. J. Robotics Res.*, vol. 7, no. 3, pp. 3-16, 1988.

[18] J. Demmel and G. Lafferriere, "Optimal three finger grasps," in *Proc. IEEE Int. Conf. Robotics Automat.*, 1989, pp. 936-942.

[19] J. T. Schwartz and M. Sharir, "Finding effective force-targets for two-dimensional multifinger frictional grips," in *Proc. 25th Allerton Conf. Commun., Control, Computing*, 1987, pp. 843-848.

[20] T. L. Boullion and P. L. Odell, *Generalized Inverse Matrices.* New York: Wiley, 1971, pp. 1-11.

[21] J. K. Salisbury and B. Roth, "Kinematic and force analysis of articulated mechanical hands," *ASME J. Mechanisms, Transmissions, Automat. Design*, vol. 105, pp. 35-41, 1983.



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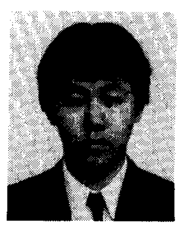

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