Week 2

Francesco Iori

Q1

$w_1 = [0, -1, 0, 0, 0, 1]$

Q2

$$W_n = \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix}$$

Q3

- rank$(W_n) = 2$
- The unconstrained degrees of freedom (DOFs) are
  - translation on the x and z axis
  - rotation around the x and y axis.
- It does not satisfy *force closure*, as the object still has unconstrained DOFs (W_n is not full rank)

Q4

$$W_f \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & -0 & 0 & -1 & -0 & 0 & 1 & -0 \\ 0 & -1 & -0 & 0 & -1 & -0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & -0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & -0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Q5

- rank$(W_f) = 6$ (full rank, null-space is null)
- This matrix is a good candidate to satisfy *force closure*.

Q6

$$G = \left[\begin{array}{ccc|ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ \hline 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 0 & 1 & 1 & 0 & 0 \\ \hline -2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ -1 & -2 & 0 & 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 & 1 & -2 & 0 \end{array}\right] \quad \text{for } k = \begin{pmatrix} f_{x1} \\ f_{y1} \\ f_{z1} \\ f_{x2} \\ f_{y2} \\ f_{z2} \\ f_{x3} \\ f_{y3} \\ f_{z3} \end{pmatrix}$$

- Simple example of a force that does not satisfy frictional constraint: an external force acting only on the x-axis of the object.

Q7

- Solving the problem for $f_{zi} \geq 1.0$ and for null external forces brings as a result:

$$k_0 = (0, 0, 1, 0, 0, 1, 0, 0, 2)^T$$

- Repeating, considering an external force of $mg = (0, 0, -5)^T$, and considering that the grasp should generate an opposite force, we get:

$$k_{mg} = (0, -1.3, 1, 0, -1.3, 1, 0, 2.5, 2)^T$$

  - Frictional forces (f_{xi}, f_{yi}) appear, as they are needed to hold the object against an external z-axis force).
  - However, the frictional constraints are not respected (considering $\mu < 1$).

Q8

$$G \cdot k_{mg} = (0, 0, 5, 0, 0, 0 | 0, 7.5, -7.5)^T$$

- Internal forces act on the object. It is reasonable, as the object is squeezed by the f_{zi}.