Week 5

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Q 1

```python
# We leave the derivation of the [T] and [M] matrices as an exercise
# for the reader :-) You will similarly use eq (8) and (9)
from sympy import diag
xuu = simplify(diff(xu, u))
xuv = simplify(diff(xu, v))

Tb = Matrix([list(xuu/normpu), list(xuv/normpvpos)]).transpose()

T = yu.transpose() * Tb
T = simplify(T)
print("\nT = ")
pprint(T)

M = diag(normpu, normpvpos)
print("\nM = ")
pprint(M)
```

which results in

```
T =
⎡   -tan(u) ⎤
⎢0  ────────⎥
⎣      R    ⎦

M =
⎡R     0    ⎤
⎢           ⎥
⎣0  R⋅cos(u)⎦
```

same as in Montana's paper.

Q 2

Q 2.1

I would expect the contact point to move with $v_y = -\omega_x \times r$ (in the plane reference frame), similarly to a cylindrical wheel. SO the point should move in the negative y-axis direction.

The result from the script matches the expectation.

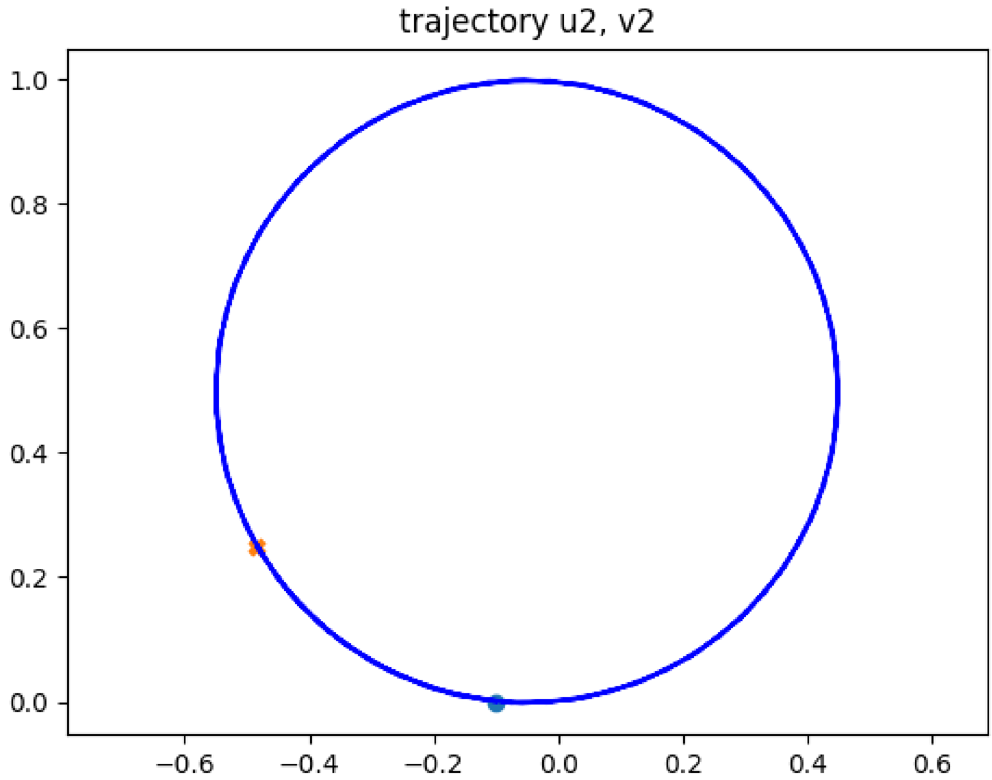
Q 2.2

Considering

$$\omega = \begin{pmatrix} 0 \\ \omega_y \\ \omega_z \end{pmatrix} \qquad \forall \quad \omega_y, \omega_z \in \mathbb{R}$$

The ball describe a circular trajectory of radius $r = \frac{\omega_y}{\omega_z}$.

Example for $\omega_y = 1.0,\ \omega_z = 2.0$ ($r = 0.5$)



Q 2.3

It is interesting to note that we do not obtain a circle if we set $\omega = (\omega_x, 0, \omega_z)$. This can be also be seen by inspecting $\dot{u}_2$ and $\dot{\psi}$, for which we have

$$\dot{\mathbf{u}}_2 = \begin{pmatrix} -\omega_x \sin(\psi) - \omega_y \cos(\psi) \\ -\omega_x \cos(\psi) + \omega_y \sin(\psi) \end{pmatrix}$$

$$\dot{\psi} = \omega_x \tan(u_2) + \omega_z$$

To describe a circle on the plane we would like to have $\dot{\mathbf{u}}_2$ in a form similar to $(\cos(\theta),\ sin(\theta))$, but the $\tan(u_2)$ term introduce an extra dependency on u_2. This term can be attributed to how the sphere is parametrized in the example being considered, and can be traced back to how the torsion matrix T is defined for the sphere.