Week 3

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Q1

Considering a jacobian of the right finger as

$$J_{\theta_2} \begin{bmatrix} 0 & -link & -link \\ link & 0 & 0 \\ 0 & -link & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{1}$$

and

$$^P_BJ_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & w \\ 0 & 0 & 1 & 0 & -w & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{2}$$

we get

```
--- Kb2 ---
Matrix([
[kb + kc,   -kc,      0, 0,        0,    -kc*w],
[    -kc,    kc,      0, 0,        0,     kc*w],
[      0,     0,     ka, 0,    -ka*w,        0],
[      0,     0,      0, 0,        0,        0],
[      0,     0, -ka*w, 0, ka*w**2,        0],
[  -kc*w, kc*w,      0, 0,        0, kc*w**2]])

--- Kbtotal ---
Matrix([
[2*kb + 2*kc,     0,     0, 0,          0,    -2*kc*w],
[          0, 2*kc,     0, 0,          0,          0],
[          0,     0, 2*ka, 0,          0,          0],
[          0,     0,     0, 0,          0,          0],
[          0,     0,     0, 0, 2*ka*w**2,          0],
[    -2*kc*w,     0,     0, 0,          0, 2*kc*w**2]])
```

which matches the result from the paper

$$K_b = \begin{bmatrix} 2k_b + 2k_c & 0 & 0 & 0 & 0 & -2k_c w \\ 0 & 2k_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 2k_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2k_a w^2 & 0 \\ -2k_c w & 0 & 0 & 0 & 0 & 2k_c w^2 \end{bmatrix} \tag{3}$$

Q2

To consider a soft finger contact model, we modify H to include torque along the (local) z-axis

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Adding also the C_{tip} matrix to the total compliance matrix, we obtain

$$K_b = \begin{bmatrix} 4k_a & 0 & 0 & 0 & 0 & -2k_a w \\ 0 & 2k_a & 0 & 0 & 0 & 0 \\ 0 & 0 & 2k_a + 2k_q & -2k_q & 0 & 0 \\ 0 & 0 & -2k_q & 2k_q & 0 & 0 \\ 0 & 0 & 0 & 0 & 2w^2\left(k_a + k_q\right) & 0 \\ -2k_a w & 0 & 0 & 0 & 0 & 2k_a w^2 \end{bmatrix} \tag{5}$$

Q3

$$K_j = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2f_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2f_n w & 0 \\ 0 & 0 & 0 & 0 & 0 & -2f_n w\left(w+1\right) \end{bmatrix} \tag{6}$$

In particular, we obtain a tilting moment, due to geometry, that would tend to unstabilize the object (but the final effect depends also on the value of K_b).

Q4

By adding the translation (in the local $\{l_i, m_i, n_i\}$ reference frame) produced by the infinitesimal rotation in `diff_i` (considered also to be in the local reference frame), we obtain a new ΔJ .

This result in a new geometric stiffness

$$K_j = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2f_n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2f_n\left(R-w\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2f_n\left(R-w\right)\left(w+1\right) \end{bmatrix} \tag{7}$$

If we consider the torque produced by this geometric effect around the body's z-axis,

$$\tau_{b,z}^{j} = 2f_n\left(R-w\right)\left(w+1\right)\delta\theta_z^2 \tag{8}$$

we can see that it is positive (i.e. restoring torque) for $R > w$. So, for an object that is comparatively thin compared to the radius of the fingertips, the geometric effect is also stabilizing.