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Theoretical model of the grasp with vacuum gripper

Giacomo Mantriota *

Dipartimento di Ingegneria dell'Ambiente e per lo Sviluppo Sostenibile, Politecnico di Bari, Viale del Turismo, 8, Taranto, Italy

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Abstract

Vacuum grippers are often notably diffused for grasping and moving very large and heavy objects.

The absence of theoretical models able to foresee the behavior of the suction cup notably limits the applications and development of vacuum grippers.

In this work, a study of grasping with a vacuum gripper has been carried out. A mathematical model is proposed for determining the forces of contact between suction cup and object. The principal purpose is to determine the minimum value of the static friction coefficient and the vacuum level in order to guarantee a firm grasp of the object during the movement, or rather without detachment or slipping of the suction cup. Finally, an example of the motion of an object grasped with suction cup is reported.

1. Introduction

Vacuum grippers are largely diffused for moving objects of various natures (glass, marble, sacks, etc.).

We can classify robot-gripping methods in four typologies: impactive, ingressive, astrictive and contigutive [1,2]. The choice of the gripping method is clearly influenced by the object's characteristics such as dimension, weight, superficial conditions, rigidity, the nature of the material, etc. [3]. In numerous applications the astrictive gripping methods are utilized in several applications in which we find magnetic, electroadhesive [4] and vacuum suction retention [5]. Vacuum suction is used extensively throughout the packaging industry, as well as most other fields of robotics [6–12]. In addition to the advantage of producing an attraction force, vacuum grippers have a soft grasp even on large and heavy objects. Monkman [13] has performed the evaluation of response times and relative energy efficiencies of astractive grippers.

The recent applications of vacuum suction are related to wall surface mobile robot with vacuum gripper feet [14–18].

In Mangialardi et al. [19] and Mantriota [20], a criterion has been proposed which determines the most favorable grip points with a view to ensure grasp stability while minimizing the grasp forces or the friction coefficient required to balance any external force acting on the object.

* Fax: +39 080 596 2777.
E-mail address: mantriota@poliba.it

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Nomenclature

a	constant
b	constant
$\mathrm{d}F_{T_x}$	Elementary forces along the edge of the suction cup in tangential direction because of friction, referable to X axis
$\mathrm{d}F_{T_y}$	Elementary forces along the edge of the suction cup in tangential direction because of friction, referable to Y axis
f_{sf}	coefficient of static friction
$F_X^{(e)}$, $F_Y^{(e)}$ and $F_Z^{(e)}$	components of the resultant of the external forces on the grasped object in X, Y and Z direction
$K(x_k, y_k)$	point around which the suction cup has the tendency to rotate
$F^{(e)}$	tangential external force
F_V	Force due to the vacuum level generated inside the suction cup
G	baricenter
L	leg of the square suction cup
p	pressure along the edge of the suction cup in normal direction
p_0	constant
Γ	edge of the suction cup
$T_Z^{(e)}$	torsional torque is present
$T_X^{(e)}$, $T_Y^{(e)}$ and $T_Z^{(e)}$	the components of the resultant moment in X, Y and Z direction
T_x, T_y	resultant of the torques in X and Y direction
ρ	density of the object
β	angle of the rotation

Confronting the numerous applications of vacuum suctions, there have been very few studies regarding suction cup models aimed at determining load capacities while the objects are in movement. The safety measure of suction cups is important for the payload capacity. There are two dangerous circumstances that could occur: one is the object slipping and the other is the object falling. A theoretical analysis of the loading capacity has been performed by Zhu et al. [17] to obtain conditions that prevent slipping and falling. The simple model is related to the multi-vacuum system used in a tracked climbing robot.

The model and the kineostatic analysis of the climbing robot were proposed by Bahr et al. [14,21] and Tummala [22]. During the motion of an object held by a suction cup, often a torsional torque also emerges on the suction cup whose equilibrium relies on the friction force between suction cup and object. In most the cases, the slip phenomenon is caused by presence of tangential forces and torsional torque contemporarily.

To date there have been no models of suction cups that also consider the contemporary presence of tangential forces and torsional torque printed in literature.

In this study a model for determining the pressures of contact between suction cup and object is proposed. The model determines the normal and tangential contact pressures produced by a generic mix of forces and torques acting on the suction cups. The main purpose is to determine the minimum value of the static-friction coefficient and the vacuum level in order to guarantee a stable grasp of the object during motion, or rather without the suction cup falling or slipping.

2. Grasping forces for a single suction cup

Vacuum grippers are constituted by one or more cups inside which a vacuum level, generally not greater than 80%, is created. The presence of an attraction force allows a bilateral constraint in the contact gripper-object to be produced.

The safety analysis is the most important consideration in designing ways to lift an object, which should be secured firmly on the suction cups. There are two dangerous circumstances that could occur: one is slipping and the other is falling of the object.

In this work a model for determining the contact forces between suction cup and object is proposed.

The model determines the normal and tangential pressures of contact that are subsequently produced by overall external forces and torques.

It is assumed that a single rectangular suction cup for grasping the object is used (Fig. 1).

The suction cup produces a force on the grasped object due to the vacuum condition created inside and contact pressure along the edge.

The forces created by the suction cup on the object are referable to (Fig. 2):

Fig. 1. Rectangular suction cup.

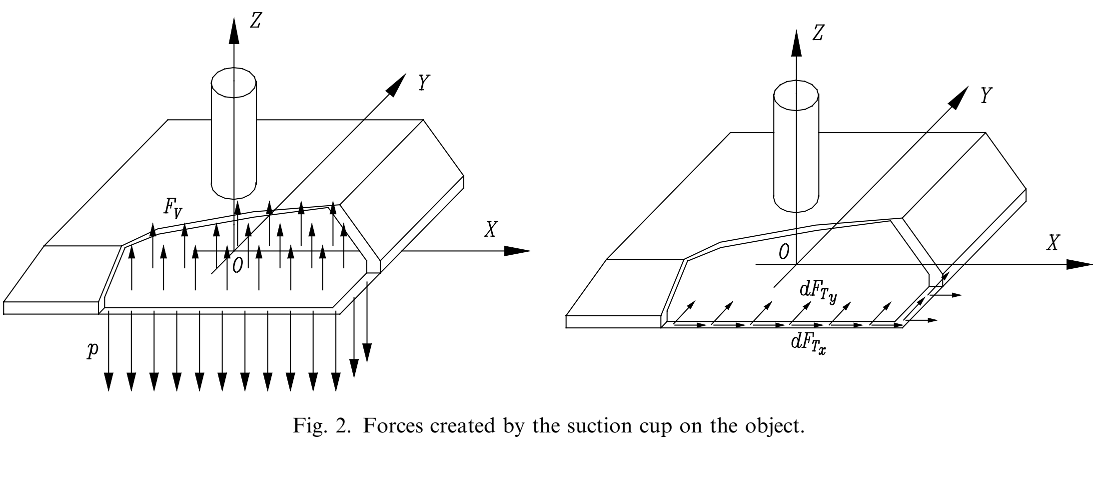

Fig. 2. Forces created by the suction cup on the object.

- Force due to the vacuum level generated inside the suction cup (F_V).
- Pressure "p" along the edge of the suction cup in normal direction.
- Pressure along the edge of the suction cup in tangential direction because of friction, referable to $\mathrm{d}F_{T_x}$ and $\mathrm{d}F_{T_y}$.

The force F_{V}, due to the vacuum level, has the direction of the Z axle of the suction cup (Fig. 2).

The force of the normal pressure "p" along the edge of the suction cup has the direction opposite that of axle Z and the resultant is located inside the perimeter of the suction cup.

If the rigidity of the edge of the suction cup is assumed uniform, the normal contact pressure per unit of length will have a linear law of variation along the edge of the suction cup, or rather

$$p(x,y) = p_0 + ax + by \tag{1}$$

with p_0, a and b constant and x, y the coordinates of the elementary area of contact in relation with the reference system (O, X, Y, Z) of the suction cup (Fig. 2).

In order to have contact between the edge of the suction cup and the object it is necessary that the normal contact pressure is always positive along the whole edge of the suction cup.

The resultant of the normal contact forces will be equal to:

$$F_Z = \int_\Gamma \mathrm{d}F_N = \int_\Gamma -(p_0 + ax + by)\mathrm{d}l \tag{2}$$

having pointed out with Γ the edge of the suction cup and "dl" the elementary length of the edge.

The resultant moment in X and Y direction (Fig. 2) is produced only by normal pressure and results equal to:

$$T_X = -\int_\Gamma (p_0 + ax + by)y\mathrm{d}l \tag{3}$$

$$T_Y = \int_\Gamma (p_0 + ax + by)x\mathrm{d}l \tag{4}$$

Determining the distribution law of the tangential pressure generated by the suction cup on the object is more complex. The elementary tangential forces ($\mathrm{d}F_T$) are due to the static friction and therefore in every point of contact have a direction located in the contact plane and an intensity that verifies the following relationship:

$$\mathrm{d}F_T \leqslant f_{\mathrm{sf}}\mathrm{d}F_N = f_{\mathrm{sf}}p\mathrm{d}l \tag{5}$$

with f_{sf} the coefficient of static friction.

The two limit conditions for the tangential forces (Fig. 3) are verified when on the suction cup there is a tangential external force ($F^{(\mathrm{e})}$) on the point "O" or when an only torsional torque is present ($T_Z^{(\mathrm{e})}$).

If we hypothesize a constant normal pressure of contact ($a = b = 0$) when there is only a tangential force ($F^{(\mathrm{e})}$) applied in the center of the suction cup to guarantee the equilibrium of the grasped object the static-friction forces ($\mathrm{d}F_T$) will presumably be parallel to the direction of the $F^{(\mathrm{e})}$ (Fig. 3(a)).

In case of incipient slipping of the suction cup (limit condition), the relationship (5) is verified with the sign of equality

$$\mathrm{d}F_T = f_{\mathrm{sf}}p\mathrm{d}l \tag{6}$$

Under conditions different than those of the incipient slip, it is not a given that the ratio between the tangential and the normal pressure is constant along the contact edge. Such a hypothesis will be truer as the operational conditions get closer to the slip effect. The conditions of incipient slipping are precisely those researched in this paper, considering that our main goal is to determine the minimum value of the static-friction coefficient. Therefore, in this paper the hypothesis is to consider constant the ratio between tangential and normal pressure along the contact edge

$$\frac{\mathrm{d}F_T}{\mathrm{d}F_N} = f \leqslant f_{\mathrm{sf}} \tag{7}$$

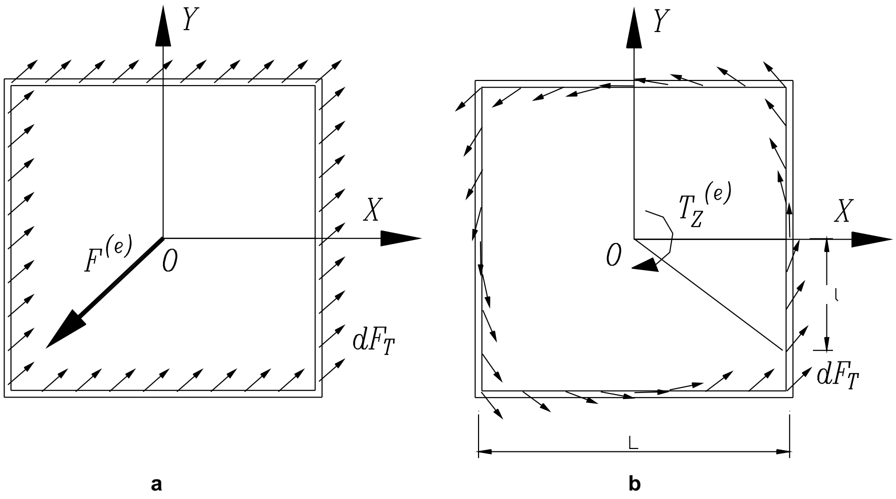

Fig. 3. Static-friction forces: (a) the external force $F^{(e)}$ is only a tangential force; (b) the external force is only torsional torque $T^{(e)}$.

In such hypothesis, when only a tangential force is present on the suction cup and the contact pressure is constant ($a = b = 0$) the resultant of the static-friction forces will be equal to:

$$F_T = \int_\Gamma \mathrm{d}F_T = \int_\Gamma fp\mathrm{d}l = fF_V \tag{8}$$

When only a torsional torque ($T_Z^{(e)}$) is present on the suction cup, the suction cup has the tendency to rotate around its center and the forces of static-friction can be hypothesized perpendicular to the line that connects the contact point with the center of the suction cup (Fig. 3(b)). In which case the equilibrium equation for a square suction cup with leg L ($a = b = 0$) (Fig. 3(b)) with respect to the rotation becomes:

$$T_Z = \int_\Gamma \sqrt{l^2 + \frac{L^2}{4}}\mathrm{d}F_T = 4\int_{-L/2}^{L/2} fp\sqrt{l^2 + \frac{L^2}{4}}\mathrm{d}l = f\frac{F_V}{L}\int_{-L/2}^{L/2}\sqrt{l^2 + \frac{L^2}{4}}\mathrm{d}l = 0.574LfF_V \tag{9}$$

A generic load condition of the suction cup is now considered. Only the edge of the suction cup is regarded as deformable. The relative motion of the body of the suction cup respect to the object is plane motion. If there is slipping between the suction cup and the object, the direction of the elementary tangential forces would be a function of the position of the instantaneous axis of rotation of the suction cup respect to the object. In particular (Fig. 4) such direction would be perpendicular to the line that connects the elementary area with the instantaneous axis of rotation (point K). In case of adhesion, the existence of a point around which there is the "rotation tendency" of the suction cup could be imagined with a consequent direction of the elementary tangential force similar to the case of slipping. This consideration is as true as it is near to the conditions of incipient slipping and, as previously mentioned, the search of such a condition is the objective of this paper.

Subsequently, indicating with $K(x_k, y_k)$ the point around which the suction cup has the tendency to rotate, a distribution of the tangential forces is generated as shown in Fig. 4. Indicating the tangential force with $\mathrm{d}F_T$, we obtain:

$$\mathrm{d}F_T = f\mathrm{d}F_N = fp\mathrm{d}A = f(p_0 + ax + by)\mathrm{d}A \tag{10}$$

The ratio between the components of the elementary tangential forces results equal to:

$$\frac{\mathrm{d}F_{T_y}}{\mathrm{d}F_{T_x}} = -\frac{(x - x_k)}{(y - y_k)} \quad \text{con } \mathrm{d}F_T = \sqrt{\mathrm{d}F_{T_x}^2 + \mathrm{d}F_{T_y}^2} \tag{11}$$

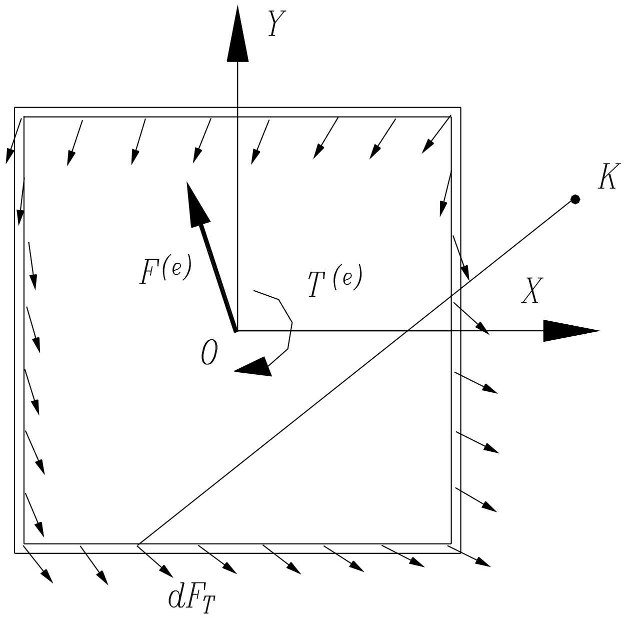

Fig. 4. Direction of the elementary tangential forces.

The resultant force and the resultant moment of the tangential forces are equal to:

$$F_X = \int_\Gamma \mathrm{d}F_{T_x} = \int_\Gamma f(p_0 + ax + by)\frac{(y - y_K)}{\sqrt{(x-x_K)^2 + (y-y_K)^2}}\mathrm{d}l \tag{12}$$

$$F_Y = \int_\Gamma \mathrm{d}F_{T_y} = -\int_\Gamma f(p_0 + ax + by)\frac{(x - x_K)}{\sqrt{(x-x_K)^2 + (y-y_K)^2}}\mathrm{d}l \tag{13}$$

$$\begin{aligned} T_Z &= \int_\Gamma -y\mathrm{d}F_{T_x} + x\mathrm{d}F_{T_y} \\ &= \int_\Gamma \left[-f(p_0 + ax + by)\frac{y(x - x_K)}{\sqrt{(x-x_K)^2 + (y-y_K)^2}} - f(p_0 + ax + by)\frac{x(y - y_K)}{\sqrt{(x-x_K)^2 + (y-y_K)^2}}\right]\mathrm{d}l \end{aligned} \tag{14}$$

Synthetically the equations of equilibrium result:

$$F_X^{(\mathrm{e})} + F_X = 0 \tag{15}$$

$$F_Y^{(\mathrm{e})} + F_Y = 0 \tag{16}$$

$$F_Z^{(\mathrm{e})} + F_Z + F_V = 0 \tag{17}$$

$$T_X^{(\mathrm{e})} + T_X = 0 \tag{18}$$

$$T_Y^{(\mathrm{e})} + T_Y = 0 \tag{19}$$

$$T_Z^{(\mathrm{e})} + T_Z = 0 \tag{20}$$

with $F_X^{(\mathrm{e})}$, $F_Y^{(\mathrm{e})}$ and $F_Z^{(\mathrm{e})}$ the components of the resultant of the external forces on the grasped object and $T_X^{(\mathrm{e})}$, $T_Y^{(\mathrm{e})}$ and $T_Z^{(e)}$ the components of the resultant moment with respect to the reference system (O, X, Y, Z) (Fig. 2).

Once the external forces on the suction cup are known, the system of 6 Eqs. (15)–(20) contains the 7 unknowns x_k, y_k, p_0, a, b, f, F_V.

In reality, two systems of equations can be distinguished: the first consisting of the Eqs. (17)–(19) in which the unknowns are p_0, a, b, F_V and the second (15), (16), (20) containing the unknowns x_K, y_K, f.

Consequently, for each value of the vacuum level found in the suction cup, and therefore also for the F_V value, it is possible to determine with the first system of equations the values of p_0, a, b and with the second system the values of x_k and y_k and above all for f, which is the minimum value of the static-friction coefficient able to guarantee the grasp. It is necessary however, to verify that the vacuum level is always able to guarantee positive pressure on the edge of the suction cup ($p \geqslant 0$).

For a square suction cup with leg "L" it is possible to develop the integrals that are present inside the Eqs. (2)–(4), (12)–(14). Particularly the resultant of the normal forces and torques T_x and T_y result:

$$F_Z = -\int_\Gamma \mathrm{d}F_N = -\int_\Gamma (p_0 + ax + by)\mathrm{d}l = -4p_0L \tag{21}$$

$$T_X = -\int_\Gamma (p_0 + ax + by)y\mathrm{d}l = -\frac{2}{3}bL^3 \tag{22}$$

$$T_Y = \int_\Gamma (p_0 + ax + by)x\mathrm{d}l = \frac{2}{3}aL^3 \tag{23}$$

The solutions of the integral related to the determination of F_X, F_Y and T_Z (Eqs. (12)–(14)) are reported in the Appendix A.

The minimum pressure value in the edges of the suction cup is worth

$$p_{\min} = p_0 - \frac{L}{2}[|a| + |b|] \tag{24}$$

Such value must be bigger than zero to guarantee the absence of detachment. Subsequently, the minimum value of p_0 results, using the Eqs. (18), (19), and (21)–(23)

$$p_0 \geqslant \frac{L}{2}[|a| + |b|] = \frac{3}{4L^2}\left[|T_X^{(e)}| + |T_Y^{(e)}|\right] \tag{25}$$

Hence the minimum value of the vacuum level ('vl') to guarantee the absence of detachment is equal to (Eqs. (17), (21), (25)):

$$\mathrm{vl}_{\mathrm{Min}} = \frac{\frac{3}{L^3}\left[|T_X^{(e)}| + |T_Y^{(e)}|\right] + \frac{F_Z^{(e)}}{L^2}}{p_{\mathrm{atm}}} 100 \quad [\%] \tag{26}$$

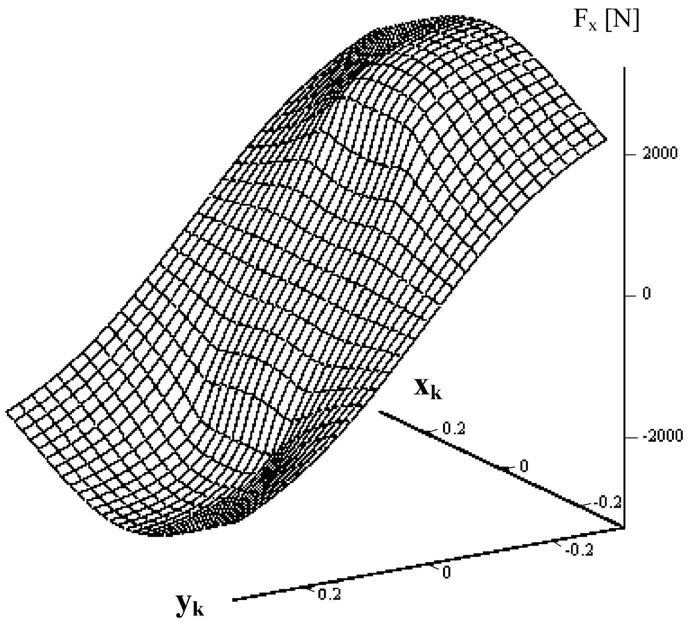

Fig. 5. F_x in function of the position of point K ($p_0 = 4000$ N/m; $f = 1$; $a = b = 0$).

Once the value of the vacuum level has been defined as more than the minimum value obtained by Eq. (26), the non-linear equations system constituted by Eqs. (15), (16), (20) allows us to determine the coordinates x_K, y_K of point K and, above all, the minimum value of the static-friction coefficient.

Considering the grasp with a square suction cup of leg "L", in the hypothesis that T_X and T_Y are zero, the contact pressure will be constant or rather $a = b = 0$. Giving an example, in Figs. 5 and 6 the behavior of forces F_X and F_Y are reported in relation to the range of the position of point K, around which the suction

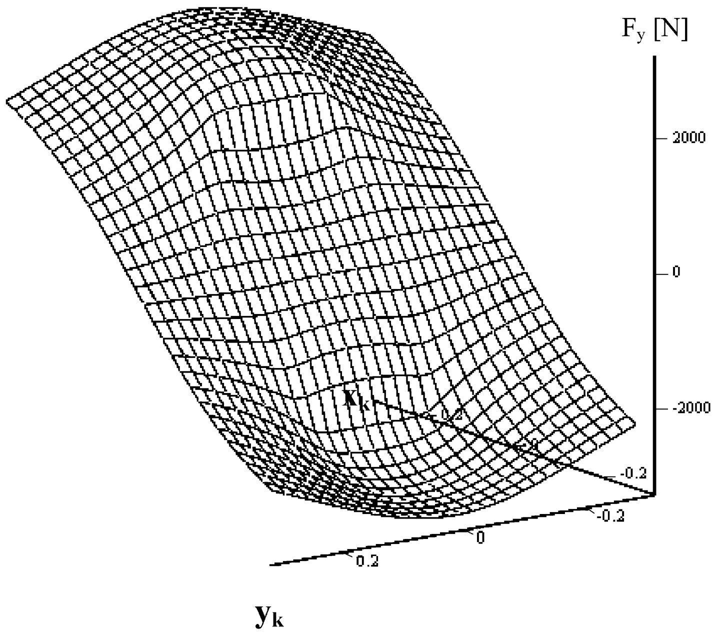

Fig. 6. F_y in function of the position of point K ($p_0 = 4000$ N/m; $f = 1$; $a = b = 0$).

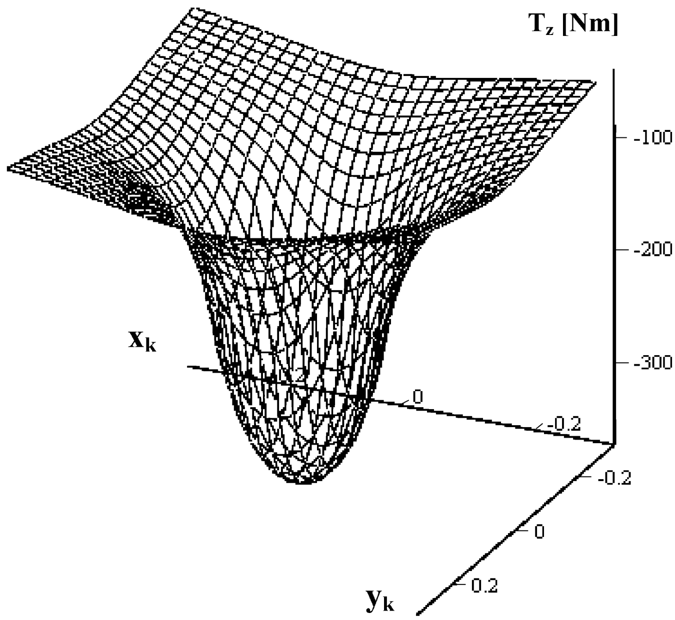

Fig. 7. T_z in function of the position of point K ($p_0 = 4000$ N/m; $f = 1$; $a = b = 0$).

cup has the tendency to rotate ($L = 0.2$ m; $F_V = 3000$ N; $f = 1$). It is clearly seen that when point K is on the axis x ($y_k = 0$), F_X is equal to zero, similarly when $x_k = 0$, F_Y is zero. When point K is sufficiently distant from the suction cup, the tangential resultant tends to the limit value of 3000 N. In Fig. 7 the torque T_Z is obtained in function of the coordinates of point K. The maximum value of the torque is obtained when point K coincides with the center of the suction cup (Fig. 7) and decreases when point K is distant from the center of the suction cup.

Obviously these observations are no longer simple when the contact pressure is not constant, or rather in the presence of torques T_X and T_Y.

3. Example

Supposing it is desired to lift and rotate to the object shown in Fig. 8 using a square suction cup of leg $L = 0.2$ m. The reference system (O, X, Y, Z) originates in the center of the suction cup and therefore, hypothesizing the object being homogeneous, the baricenter has the coordinates $G(0.162, 0, -0.424\text{ m})$. The density of the object is $\rho = 90$ kg/m^3. In the initial position, the axle Z coincides with the vertical axle.

It is assumed that we would like to rotate the object around axle X (Fig. 9) 90° and that the inertial terms are negligible.

During the motion of the object, in comparison to the reference system (O, X, Y, Z), the object applies the following forces and torques on the suction cup:

$$F_X^{(e)} = 0 \tag{27}$$

$$F_Y^{(e)} = -Mg\sin\beta \tag{28}$$

$$F_Z^{(e)} = -Mg\cos\beta \tag{29}$$

$$T_X^{(e)} = -Mgz_G\sin\beta \tag{30}$$

$$T_Y^{(e)} = Mgx_G\cos\beta \tag{31}$$

$$T_Z^{(e)} = -Mgx_G\sin\beta \tag{32}$$

As is seen in Fig. 10 with the growth of angle β, the normal force $Fz^{(e)}$ decreases while the tangential force $F_Y^{(e)}$ increases. Besides the torque T_Y decreases while the torques $T_X^{(e)}$ and $T_Z^{(e)}$ ($T_Z^{(e)}$ is a torsional torque) increase (Fig. 11).

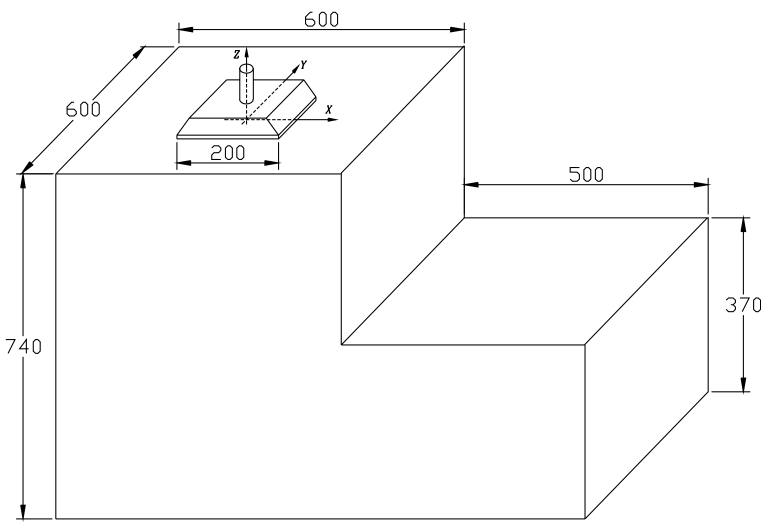

Fig. 8. Object and square suction cup.

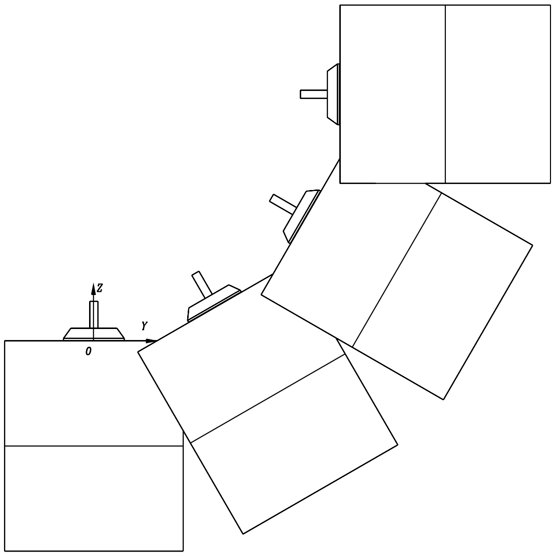

Fig. 9. Motion of the object.

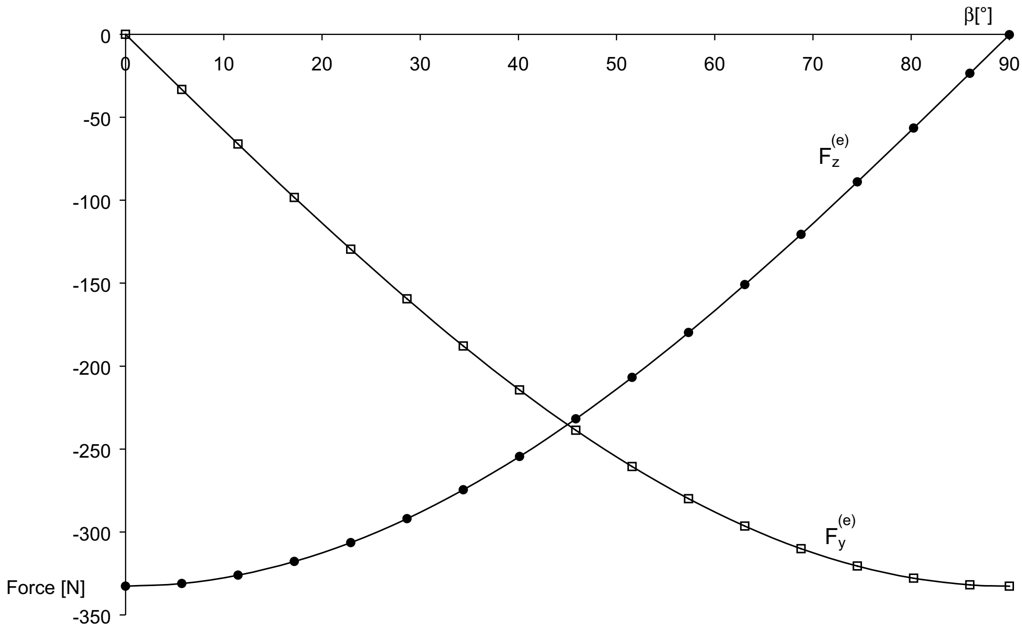

Fig. 10. External forces $F_y^{(e)}$ and $F_z^{(e)}$ in function of angle β.

To guarantee the contact between suction cup and object it is necessary that the normal pressure "p" is positive in every point of contact.

Using Eqs. (26), (29)–(31) it is possible to calculate the minimum value of the vacuum level in relation to angle β (Fig. 11) through the following equation:

$$\mathrm{vl}_{\mathrm{Min}} = \frac{Mg}{L^2}\left[3\frac{|z_G \sin\beta| + |x_G \cos\beta|}{L} + |\cos\beta|\right] \tag{33}$$

It is possible to note (Fig. 12) how the vacuum level needed in order to avoid the separation of the suction cup raises to the growth of β, reaching the maximum value ($\mathrm{vl} = 61\%$) corresponding to $\beta = atg\frac{z_G}{x_G} = 69°$.

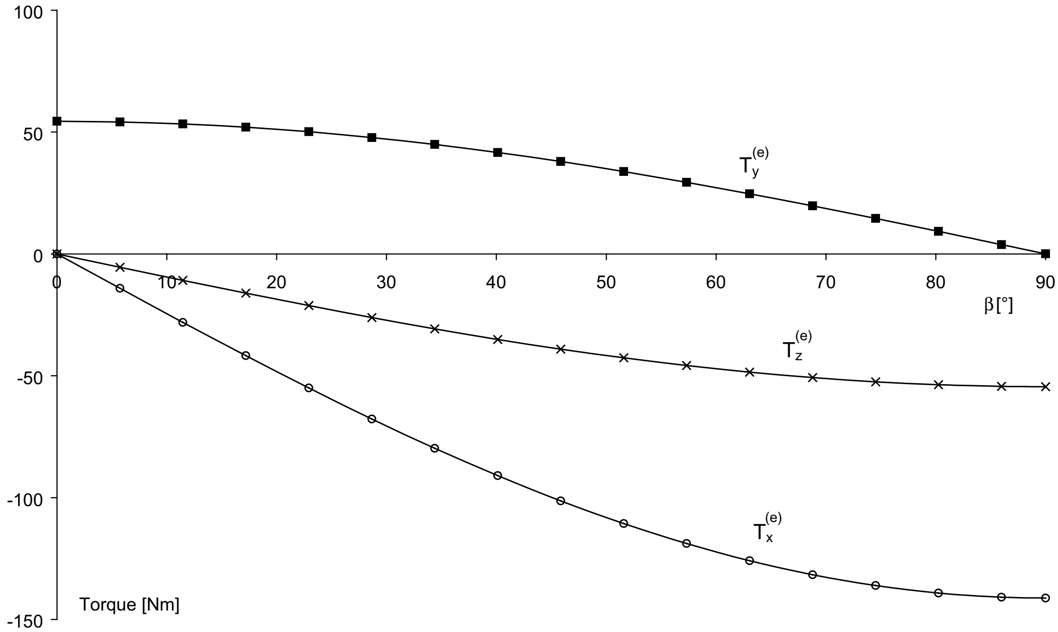

Fig. 11. External torques $T_x^{(e)}$, $T_y^{(e)}$ and $T_z^{(e)}$ in function of angle β.

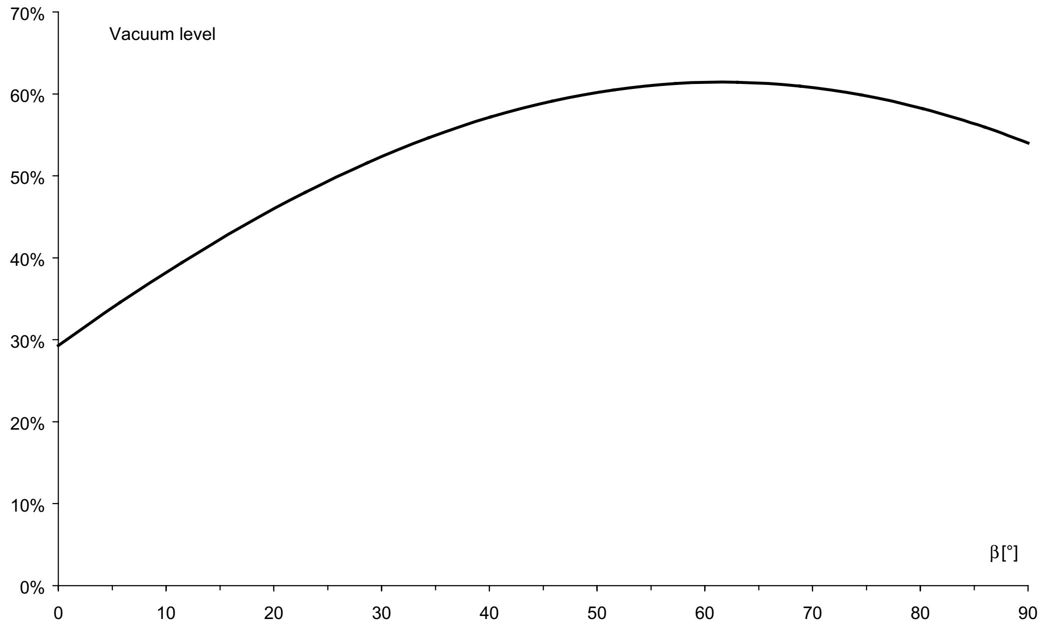

Fig. 12. Minimum vacuum level in function of angle β.

Considering a vacuum level of 64% (more than the minimum value), during the motion the average contact pressure p_0 varies between 2710 and 3120 N/m (Fig. 13), increasing slightly because of the reduction of the force F_z. The contact pressures on the corners of the suction cup have their behaviour reported in Fig. 13, therefore, the contact pressure becomes positive in every point of the suction cup edge; more accurately it does not cause separation of the suction cup from the object.

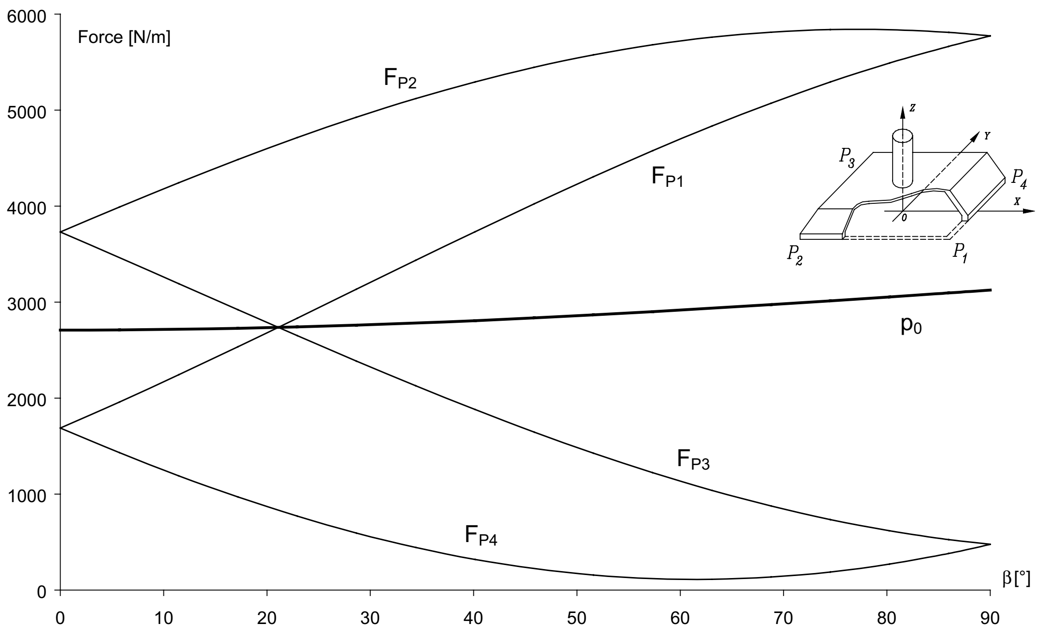

Fig. 13. Average contact pressure p_0 and contact pressures on the corners of the suction cup in function of angle β.

In consideration of the slip check, solving the non-linear system constituted by Eqs. (15), (16), (20), (27), (28), (32) for every value of angle β, the coordinates of point K around which the suction cup has the tendency to rotate when submitted to the external actions have been calculated (Fig. 14). As can be seen, despite F_x always being zero, y_k is not zero because the contact pressure is not constant due to the torques T_x and T_y.

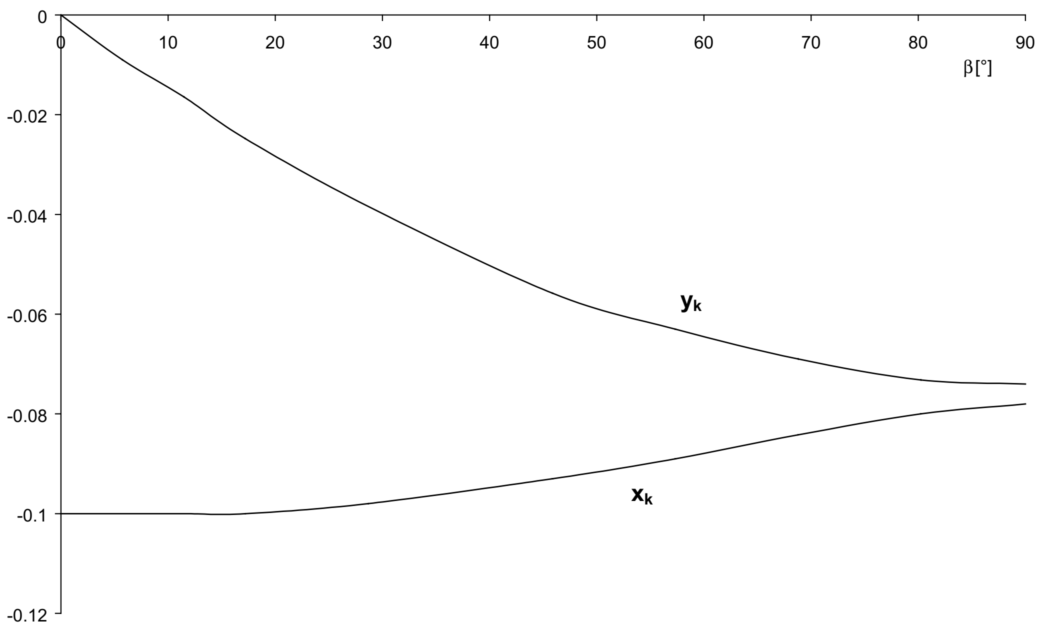

Fig. 14. Coordinates of the point K in function of angle β.

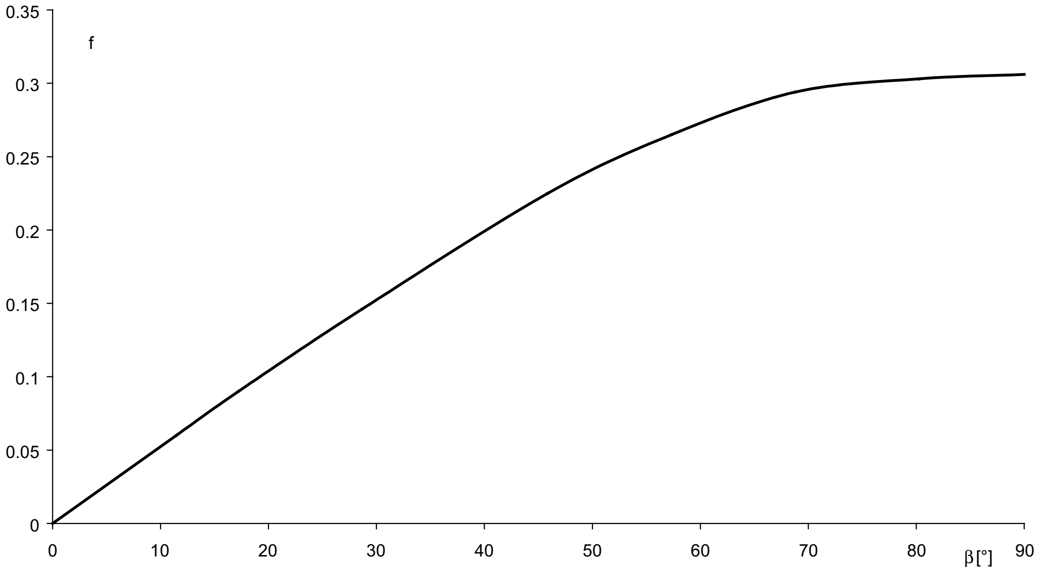

Fig. 15. Minimum value of the static friction coefficient in function of angle β.

Table 1
Values of the parameters when $\beta = 90°$

vl = 64%
$\theta = 90°$
$p_0 = 3120$ N/m
$a = 0$
$b = -26480$ m
$F_{P1} = F_{P2} = 5773$ N/m
$F_{P3} = F_{P4} = 475$ N/m
$x_k = -0.078$ m
$y_k = -0.074$ m
$f = 0.306$

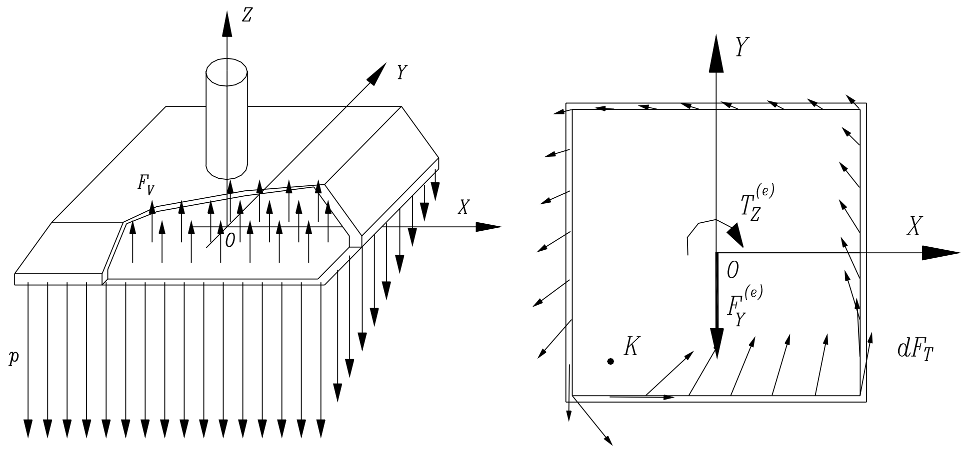

Fig. 16. Normal and tangential elementary forces when $\beta = 90°$.

In Fig. 15 the minimum value of the needed static-friction coefficient to guarantee the grasp of the object without any slip effect is reported. The value of f is zero when $\beta = 0°$ and increases to reach the maximum value of 0.306, representing therefore the minimum value of the static-friction coefficient to guarantee the absence of slipping.

When $\beta = 90°$, the average pressure is worth $p_0 = 3120$ N/m and point K, around which the suction cup has the tendency to rotate, has coordinates $x_k = -0.078$ m and $y_k = -0.074$ m (Table 1).

The contact pressure is located between 5773 N/m corresponding with the points with $y = -0.1$ m and a minimum of 475 N/m for $y = 0.1$ m. The distribution of the tangential contact pressures is reported in Fig. 16 for $\beta = 90°$.

4. Conclusions

Vacuum grippers are largely diffused for moving objects of varied nature. In this work a mathematical model was proposed to determine the normal and tangential pressures of contact between suction cup and object. The original aspect of this work is, above all, relative to the possibility of considering the torsional torque effect on the suction cup.

Hypothesizing a constant normal rigidity along the edge of the suction cup, equations were obtained to determine the distribution of the normal pressure, the point around which the suction cup tends to rotate and, finally, the distribution of the tangential pressures.

The principal purpose of the model was to determine the minimum vacuum level to guarantee grasping the object without detachment and the minimum value of the necessary static-friction coefficient to avoid the object slipping.

The model was used in an example to verify the possibility of moving an irregular object.

Future experiments will be conducted in order to verify the model presented.

Appendix A

Solutions of the integral (Eqs. (12)–(14)) related to the determination of F_X, F_Y and T_Z (square suction cup with leg "L")

$$
\begin{aligned}
\frac{F_X}{f} = {} & \left(p_0 + \frac{ax_k}{2} - \frac{bL}{2} + \frac{aL}{4}\right)\sqrt{\left(\frac{L}{2}-x_k\right)^2 + \left(\frac{L}{2}+y_k\right)^2} - \left(p_0 + \frac{ax_k}{2} - \frac{bL}{2} - \frac{aL}{4}\right)\sqrt{\left(\frac{L}{2}+y_k\right)^2 + \left(\frac{L}{2}+x_k\right)^2} \\
& + \left(p_0 + \frac{ax_k}{2} + \frac{bL}{2} + \frac{aL}{4}\right)\sqrt{\left(\frac{L}{2}-y_k\right)^2 + \left(\frac{L}{2}-x_k\right)^2} - \left(p_0 + \frac{ax_k}{2} + \frac{bL}{2} - \frac{aL}{4}\right)\sqrt{\left(\frac{L}{2}-y_k\right)^2 + \left(\frac{L}{2}+x_k\right)^2} \\
& - \frac{a}{2}\left\{\left(\frac{L}{2}+y_k\right)^2 \ln\left[\frac{\frac{L}{2}-x_k+\sqrt{\left(\frac{L}{2}+y_k\right)^2+\left(\frac{L}{2}-x_k\right)^2}}{\frac{-L}{2}-x_k+\sqrt{\left(\frac{L}{2}+y_k\right)^2+\left(\frac{L}{2}+x_k\right)^2}}\right] + \left(\frac{L}{2}-y_k\right)^2 \ln\left[\frac{\frac{L}{2}-x_k+\sqrt{\left(\frac{L}{2}-y_k\right)^2+\left(\frac{L}{2}-x_k\right)^2}}{\frac{-L}{2}-x_k+\sqrt{\left(\frac{L}{2}-y_k\right)^2+\left(\frac{L}{2}+x_k\right)^2}}\right]\right\} \\
& + \left\{-\frac{L}{2}\left[p_0 + by_k + a\left(\frac{-L}{2}-x_k\right)\right] + x_k(p_0+by_k)\right\}\ln\left[\frac{\frac{L}{2}-y_k+\sqrt{\left(\frac{L}{2}-y_k\right)^2+\left(\frac{L}{2}+x_k\right)^2}}{\frac{-L}{2}-y_k+\sqrt{\left(\frac{L}{2}+y_k\right)^2+\left(\frac{L}{2}+x_k\right)^2}}\right] \\
& + \left\{\frac{L}{2}\left[p_0 + by_k + a\left(\frac{L}{2}-x_k\right)\right] + x_k(p_0+by_k)\right\}\ln\left[\frac{\frac{L}{2}-y_k+\sqrt{\left(\frac{L}{2}-y_k\right)^2+\left(\frac{L}{2}-x_k\right)^2}}{\frac{-L}{2}-y_k+\sqrt{\left(\frac{L}{2}+y_k\right)^2+\left(\frac{L}{2}-x_k\right)^2}}\right] \\
& - b\left(\frac{L}{2}+x_k\right)\left[\sqrt{\left(\frac{L}{2}-y_k\right)^2+\left(\frac{L}{2}+x_k\right)^2} - \sqrt{\left(\frac{L}{2}+y_k\right)^2+\left(\frac{L}{2}+x_k\right)^2}\right] \\
& + b\left(\frac{L}{2}-x_k\right)\left[\sqrt{\left(\frac{L}{2}-y_k\right)^2+\left(\frac{L}{2}-x_k\right)^2} - \sqrt{\left(\frac{L}{2}+y_k\right)^2+\left(\frac{L}{2}-x_k\right)^2}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{F_Y}{f} = {} & \left(p_0 - \frac{ay_k}{2} - \frac{bL}{2} + \frac{aL}{4}\right)\sqrt{\left(\frac{L}{2}+y_k\right)^2 + \left(\frac{L}{2}+x_k\right)^2} - \left(p_0 - \frac{ay_k}{2} - \frac{bL}{2} - \frac{aL}{4}\right)\sqrt{\left(\frac{L}{2}-y_k\right)^2 + \left(\frac{L}{2}+x_k\right)^2} \\
& + \left(p_0 - \frac{ay_k}{2} + \frac{bL}{2} + \frac{aL}{4}\right)\sqrt{\left(\frac{L}{2}+y_k\right)^2 + \left(\frac{L}{2}-x_k\right)^2} - \left(p_0 - \frac{ay_k}{2} + \frac{bL}{2} - \frac{aL}{4}\right)\sqrt{\left(\frac{L}{2}-y_k\right)^2 + \left(\frac{L}{2}-x_k\right)^2} \\
& - \frac{a}{2}\left\{\left(\frac{L}{2}+x_k\right)^2 \ln\left[\frac{\frac{L}{2}+y_k+\sqrt{\left(\frac{L}{2}+y_k\right)^2+\left(\frac{L}{2}+x_k\right)^2}}{\frac{-L}{2}+y_k+\sqrt{\left(\frac{L}{2}-y_k\right)^2+\left(\frac{L}{2}+x_k\right)^2}}\right] + \left(\frac{L}{2}-x_k\right)^2 \ln\left[\frac{\frac{L}{2}+y_k+\sqrt{\left(\frac{L}{2}+y_k\right)^2+\left(\frac{L}{2}-x_k\right)^2}}{\frac{-L}{2}+y_k+\sqrt{\left(\frac{L}{2}-y_k\right)^2+\left(\frac{L}{2}-x_k\right)^2}}\right]\right\} \\
& + \left\{-\frac{L}{2}\left[p_0+bx_k+a\left(\frac{-L}{2}+y_k\right)\right]+y_k(p_0+bx_k)\right\}\ln\left[\frac{\frac{-L}{2}-x_k+\sqrt{\left(\frac{L}{2}-y_k\right)^2+\left(\frac{L}{2}-x_k\right)^2}}{\frac{L}{2}+x_k+\sqrt{\left(\frac{L}{2}-y_k\right)^2+\left(\frac{L}{2}+x_k\right)^2}}\right] \\
& + \left\{\frac{L}{2}\left[p_0+bx_k+a\left(\frac{L}{2}+y_k\right)\right]+y_k(p_0+bx_k)\right\}\ln\left[\frac{\frac{L}{2}-x_k+\sqrt{\left(\frac{L}{2}+y_k\right)^2+\left(\frac{L}{2}-x_k\right)^2}}{\frac{-L}{2}-x_k+\sqrt{\left(\frac{L}{2}+y_k\right)^2+\left(\frac{L}{2}+x_k\right)^2}}\right] \\
& - b\left(\frac{L}{2}-y_k\right)\left[\sqrt{\left(\frac{L}{2}-y_k\right)^2+\left(\frac{L}{2}-x_k\right)^2}-\sqrt{\left(\frac{L}{2}-y_k\right)^2+\left(\frac{L}{2}+x_k\right)^2}\right] \\
& + b\left(\frac{L}{2}+y_k\right)\left[\sqrt{\left(\frac{L}{2}+y_k\right)^2+\left(\frac{L}{2}-x_k\right)^2}-\sqrt{\left(\frac{L}{2}+y_k\right)^2+\left(\frac{L}{2}+x_k\right)^2}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{T_Z}{f} = {} & \left[\frac{a}{3}\left[\left(\frac{L^2}{4}+\frac{Lx_k}{2}+x_k^2\right)-2\left(\frac{L}{2}+y_k\right)^2\right]+\frac{aL}{2}\left(\frac{L}{2}+y_k\right)+\frac{1}{2}\left(p_0-\frac{bL}{2}\right)\left(\frac{-L}{2}+x_k\right)\right]\sqrt{\left(\frac{L}{2}+x_k\right)^2+\left(\frac{L}{2}+y_k\right)^2} \\
& + \left[\frac{a}{3}\left[\left(\frac{L^2}{4}+\frac{Lx_k}{2}+x_k^2\right)-2\left(\frac{L}{2}+y_k\right)^2\right]+\frac{aL}{2}\left(\frac{L}{2}+y_k\right)+\frac{1}{2}\left(p_0-\frac{bL}{2}\right)\left(\frac{L}{2}+x_k\right)\right]\sqrt{\left(\frac{L}{2}-x_k\right)^2+\left(\frac{L}{2}+y_k\right)^2} \\
& + \left[\frac{a}{3}\left[\left(\frac{L^2}{4}-\frac{Lx_k}{2}+x_k^2\right)-2\left(\frac{L}{2}-y_k\right)^2\right]+\frac{aL}{2}\left(\frac{L}{2}-y_k\right)+\frac{1}{2}\left(p_0+\frac{bL}{2}\right)\left(\frac{-L}{2}+x_k\right)\right]\sqrt{\left(\frac{L}{2}+x_k\right)^2+\left(\frac{L}{2}-y_k\right)^2} \\
& + \left[\frac{a}{3}\left[\left(\frac{L^2}{4}+\frac{Lx_k}{2}+x_k^2\right)-2\left(\frac{L}{2}-y_k\right)^2\right]+\frac{aL}{2}\left(\frac{L}{2}-y_k\right)+\frac{1}{2}\left(p_0+\frac{bL}{2}\right)\left(\frac{L}{2}+x_k\right)\right]\sqrt{\left(\frac{L}{2}-x_k\right)^2+\left(\frac{L}{2}-y_k\right)^2} \\
& - \left(\frac{L}{2}+y_k\right)\left[\frac{1}{2}\left(p_0-\frac{bL}{2}\right)\left(\frac{-L}{2}+y_k\right)+ax_ky_k\right]\ln\left[\frac{\frac{L}{2}-x_k+\sqrt{\left(\frac{L}{2}-x_k\right)^2+\left(\frac{L}{2}+y_k\right)^2}}{\frac{-L}{2}-x_k+\sqrt{\left(\frac{L}{2}+x_k\right)^2+\left(\frac{L}{2}+y_k\right)^2}}\right] \\
& + \left(\frac{L}{2}-y_k\right)\left[\frac{1}{2}\left(p_0+\frac{bL}{2}\right)\left(\frac{L}{2}+y_k\right)+ax_ky_k\right]\ln\left[\frac{\frac{L}{2}-x_k+\sqrt{\left(\frac{L}{2}-x_k\right)^2+\left(\frac{L}{2}-y_k\right)^2}}{\frac{-L}{2}-x_k+\sqrt{\left(\frac{-L}{2}-x_k\right)^2+\left(\frac{L}{2}-y_k\right)^2}}\right] \\
& + \left[\frac{b}{3}\left[\left(\frac{L^2}{4}+\frac{-Ly_k}{2}+y_k^2\right)-2\left(\frac{L}{2}+x_k\right)^2\right]+\frac{bL}{2}\left(\frac{L}{2}+x_k\right)+\frac{1}{2}\left(p_0-\frac{aL}{2}\right)\left(\frac{-L}{2}+y_k\right)\right]\sqrt{\left(\frac{L}{2}+x_k\right)^2+\left(\frac{L}{2}+y_k\right)^2} \\
& + \left[\frac{b}{3}\left[\left(\frac{L^2}{4}+\frac{Ly_k}{2}+y_k^2\right)-2\left(\frac{L}{2}+x_k\right)^2\right]+\frac{bL}{2}\left(\frac{L}{2}+x_k\right)+\frac{1}{2}\left(p_0-\frac{aL}{2}\right)\left(\frac{L}{2}+y_k\right)\right]\sqrt{\left(\frac{L}{2}+x_k\right)^2+\left(\frac{L}{2}-y_k\right)^2} \\
& + \left[\frac{b}{3}\left[\left(\frac{L^2}{4}-\frac{Ly_k}{2}+y_k^2\right)-2\left(\frac{L}{2}-x_k\right)^2\right]+\frac{bL}{2}\left(\frac{L}{2}-x_k\right)+\frac{1}{2}\left(p_0+\frac{aL}{2}\right)\left(\frac{-L}{2}+y_k\right)\right]\sqrt{\left(\frac{L}{2}-x_k\right)^2+\left(\frac{L}{2}+y_k\right)^2}
\end{aligned}
$$

$$+\left[\frac{b}{3}\left[\left(\frac{L^2}{4}+\frac{Ly_k}{2}+y_k^2\right)-2\left(\frac{L}{2}-x_k\right)^2\right]+\frac{bL}{2}\left(\frac{L}{2}-x_k\right)+\frac{1}{2}\left(p_0+\frac{aL}{2}\right)\left(\frac{L}{2}+y_k\right)\right]\sqrt{\left(\frac{L}{2}-x_k\right)^2+\left(\frac{L}{2}-y_k\right)^2}$$

$$-\left(\frac{L}{2}+x_k\right)\left[\frac{1}{2}\left(p_0-\frac{aL}{2}\right)\left(\frac{-L}{2}+x_k\right)+bx_ky_k\right]\ln\left[\frac{\frac{L}{2}-y_k+\sqrt{\left(\frac{L}{2}+x_k\right)^2+\left(\frac{L}{2}-y_k\right)^2}}{\frac{-L}{2}-y_k+\sqrt{\left(\frac{L}{2}+x_k\right)^2+\left(\frac{L}{2}+y_k\right)^2}}\right]$$

$$+\left(\frac{L}{2}-x_k\right)\left[\frac{1}{2}\left(p_0+\frac{aL}{2}\right)\left(\frac{L}{2}+x_k\right)+bx_ky_k\right]\ln\left[\frac{\frac{L}{2}-y_k+\sqrt{\left(\frac{L}{2}-x_k\right)^2+\left(\frac{L}{2}-y_k\right)^2}}{\frac{-L}{2}-y_k+\sqrt{\left(\frac{L}{2}-x_k\right)^2+\left(\frac{L}{2}+y_k\right)^2}}\right]$$

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