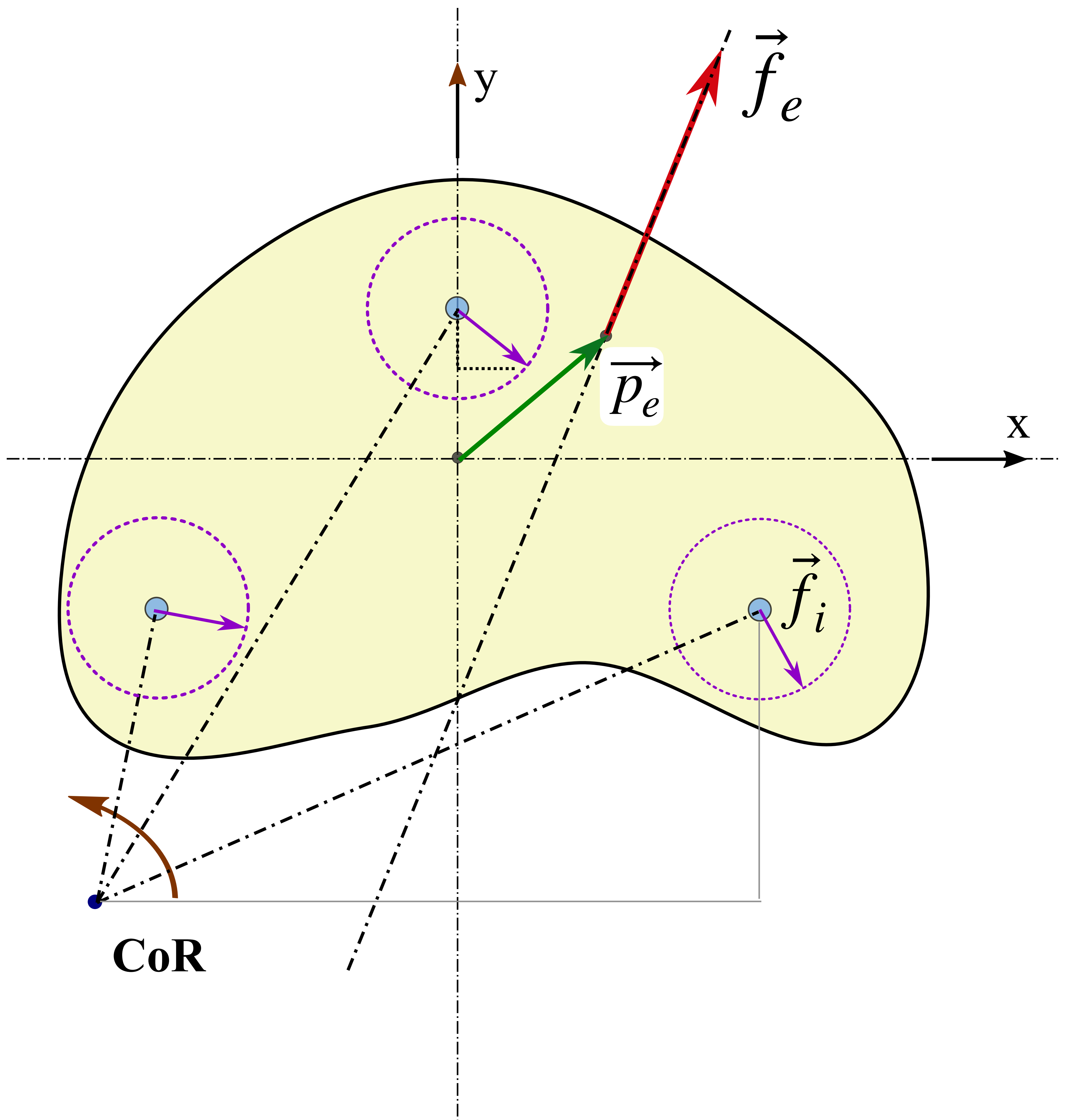
Consider a Sliding object with *n* points of contact

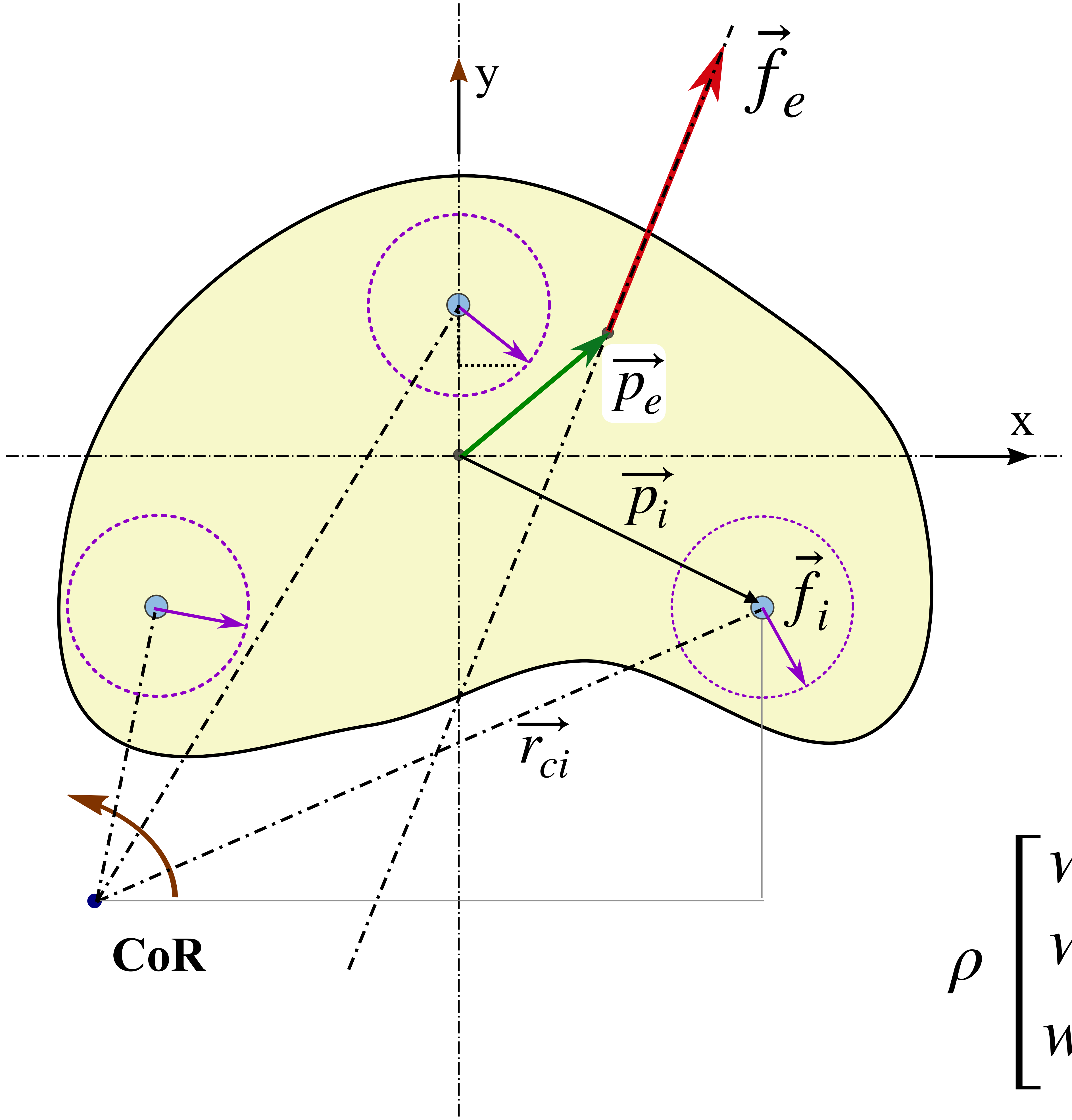


An external force acts upon an object, at location $\boldsymbol{p_e} = [p_{ex}, p_{ey}]'$, resulting in a force and moment at origin. We can write this as a unit external *wrench, w,* with scaling factor ρ:

$$[f_{ex}, f_{ey}, p_e \times f_e] = \rho[w_{ex}, w_{ey}, w_{em}]$$

As the external force magnitude increases, the object will eventually start to slide. It will have some (as yet unknown) instantaneous *twist* $[v_x, v_y, \omega_z]'$ which is equivalent to having an instantaneous CoR (center of rotation) at some location in the plane.

Sliding object with *n* points of contact



An external force acts upon an object, at location $\boldsymbol{p_e} = [p_{ex}, p_{ey}]'$, resulting in a force and moment at origin. We can write this as a unit external wrench with scaling factor:

$$[f_{ex}, f_{ey}, p_e \times f_e] = \rho[w_{ex}, w_{ey}, w_{em}]$$

Each sliding point creates a friction force, $\boldsymbol{f_i} = [f_{xi}, f_{yi}]$ where the magnitude of the force is known: $|\boldsymbol{f_i}| = \mu f_{ni}$

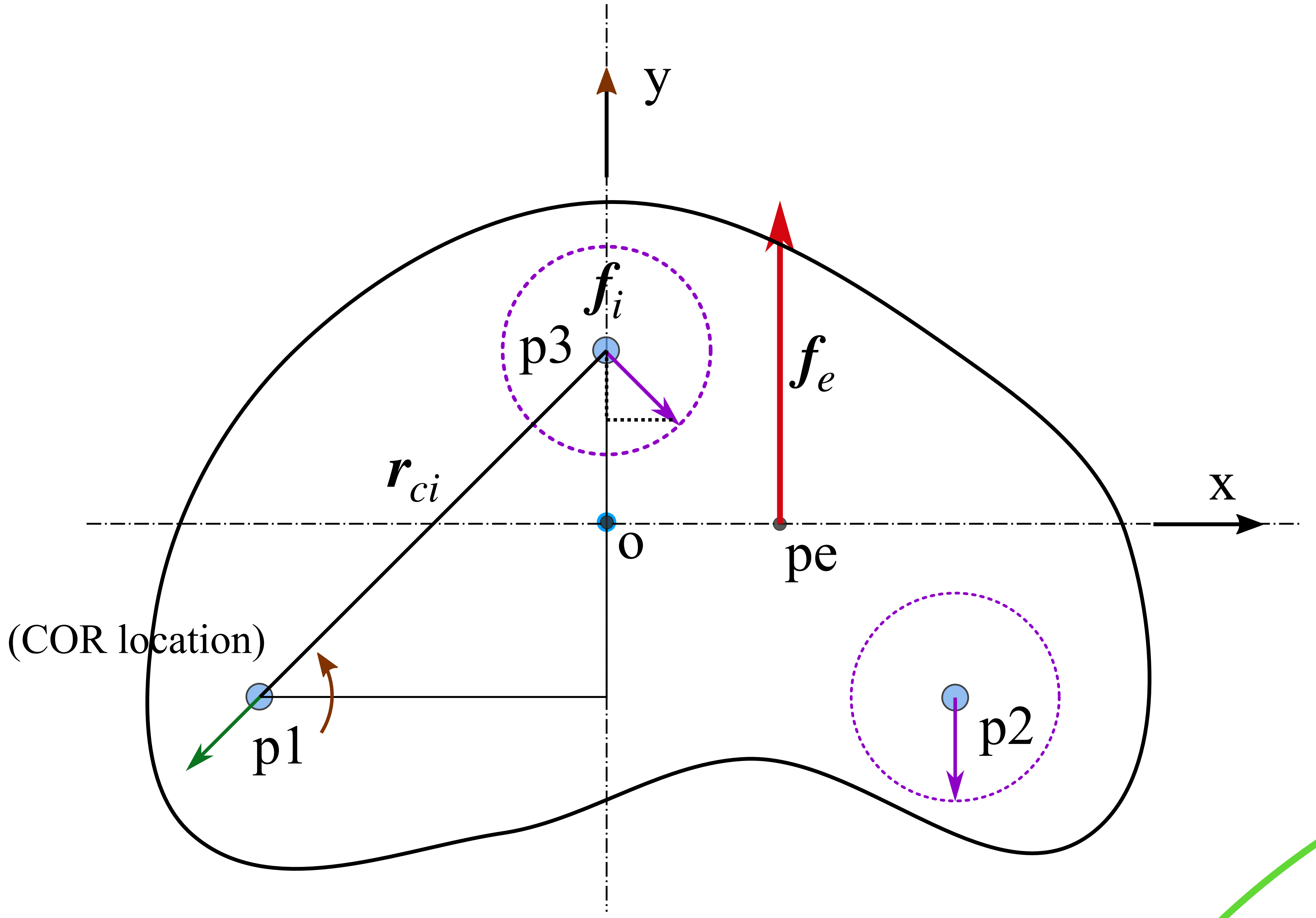
Thus the quasi-static equilibrium equations are:

$$\rho \begin{bmatrix} w_{ex} \\ w_{ey} \\ w_{em} \end{bmatrix} = \begin{bmatrix} \sum \mu f_{ni} \frac{r_{ciy}}{|r_{ci}|} \\ -\sum \mu f_{ni} \frac{r_{cix}}{|r_{ci}|} \\ \sum \mu f_{ni} (\frac{-r_{cix}}{|r_{ci}|} p_{iy} + \frac{r_{ciy}}{|r_{ci}|} p_{ix}) \end{bmatrix}$$

friction forces and moment at origin for all sliding points, for a given (unkown) COR location

What are the unknown quantities in these equations?

Sakurai (MIT 1990) example - used for assignment



An external force $\vec{f}_e$ acts upon object, pulling, as with a string attached at p_e, resulting in a force and moment at origin.

We know the direction of $\vec{f}_e$ and its line of action, given by p_e, but not the magnitude. Thus we know the ratios f_{ey}/f_{ex} and $|f_e|/m_e$ where m_e is the moment. We write this as a unit external wrench with scaling factor ρ:

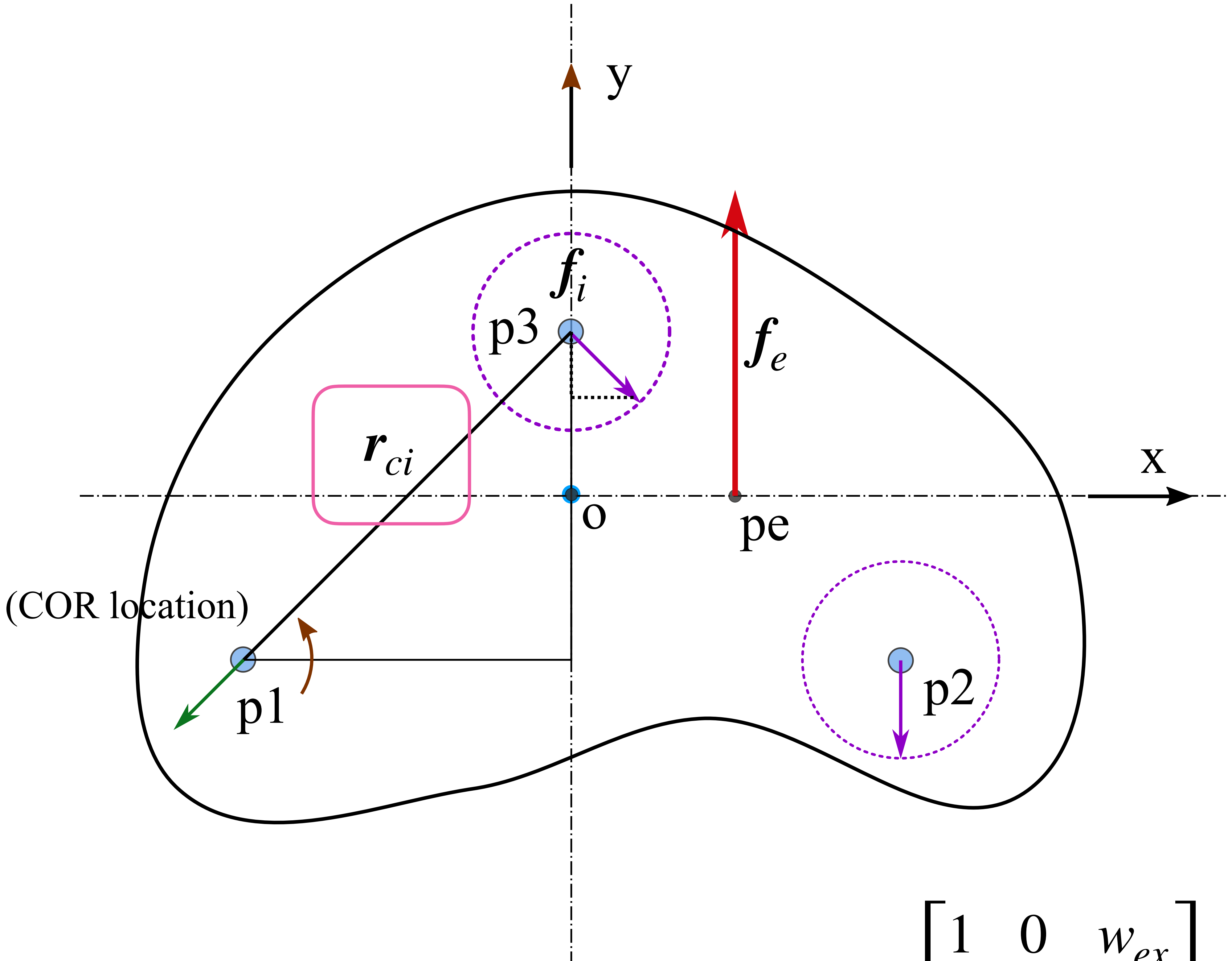
$$[f_{ex}, f_{ey}, p_e \times f_e] = \rho[w_{ex}, w_{ey}, w_{em}]$$

Sakura (eq 4.2.9) writes the equilibrium equation as:
$[W] \cdot \boldsymbol{f} = -\rho \boldsymbol{w}$ where $\boldsymbol{f}$ is a vector with all the friction forces (x and y components) and $[W]$ is essentially a wrench matrix of the sort we have seen before.

$$\begin{bmatrix} 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 0 & 1 \\ -y_1 & x_1 & \cdots & -y_n & x_n \end{bmatrix} \begin{bmatrix} f_{1x} \\ f_{1y} \\ \vdots \\ f_{nx} \\ f_{ny} \end{bmatrix} = -\begin{bmatrix} w_{cx} \\ w_{cy} \\ w_{cm} \end{bmatrix}$$

Note: Sakurai uses ‘I’ for ρ, which can be a bit confusing…

Possibly it rotates about one of the contact points?



In this case, the COR is assumed known, so we know immediately the friction forces for all the other sliding contacts ($j \neq i$).

Again,
$[f_{ex}, f_{ey}, p_e \times f_e] = \rho[w_{ex}, w_{ey}, w_{em}]$

So we can rewrite things as:

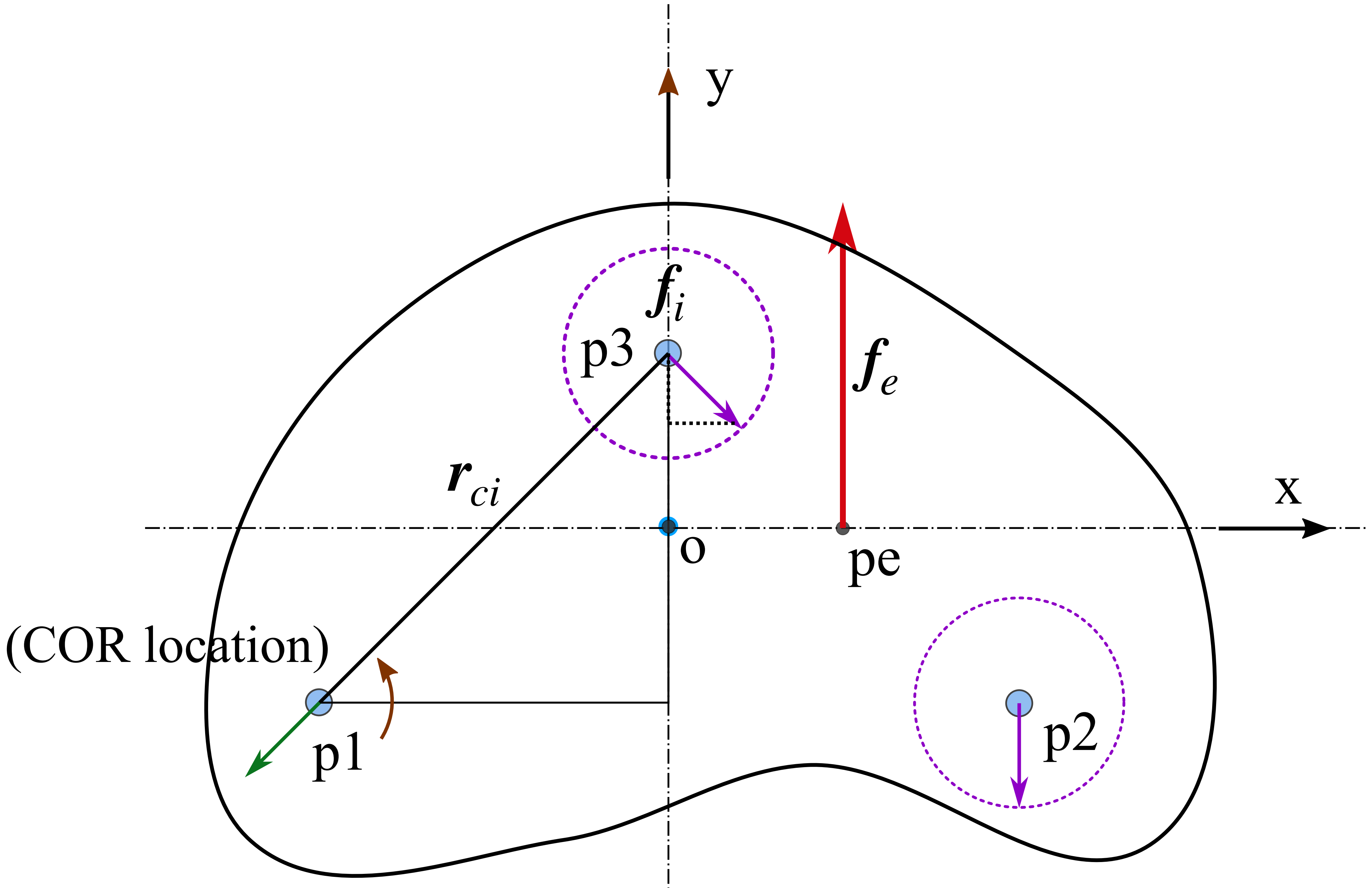
Possibly COR is at one of the contact points (shown as p1 here)
$\boldsymbol{r}_{ci}$ = vector from COR to a sliding point

$$\begin{bmatrix} 1 & 0 & w_{ex} \\ 0 & 1 & w_{ey} \\ 0 & 0 & w_{em} \end{bmatrix} \begin{matrix} f_{cx} \\ f_{cy} \\ \rho \end{matrix} = \begin{bmatrix} \sum \mu f_{ni} \frac{r_{ciy}}{|r_{ci}|} \\ -\sum \mu f_{ni} \frac{r_{cix}}{|r_{ci}|} \\ \sum \mu f_{ni} (\frac{-r_{cix}}{|r_{ci}|} p_{iy} + \frac{r_{ciy}}{|r_{ci}|} p_{ix}) \end{bmatrix}$$

net friction forces and moment at origin for all sliding points, given COR location

What are the unknown quantities in these equations?

If it rotates about one of the contact points



In this case, the COR is assumed known so we know immediately the friction forces for all the other sliding contacts ($j \neq i$).

We set up equilibrium equations for the unknown forces f_{cx}, f_{cy} at the COR location and ρ

Possibly COR is at one of the contact points (shown as p1 here)
r_{ci} = vector from COR to a sliding point

$$\begin{bmatrix} 1 & 0 & w_{ex} \\ 0 & 1 & w_{ey} \\ 0 & 0 & w_{em} \end{bmatrix} \begin{matrix} f_{cx} \\ f_{cy} \\ \rho \end{matrix} = \begin{bmatrix} \sum \mu f_{ni} \frac{r_{ciy}}{|r_{ci}|} \\ -\sum \mu f_{ni} \frac{r_{cix}}{|r_{ci}|} \\ \sum \mu f_{ni} \left(\frac{-r_{cix}}{|r_{ci}|} p_{iy} + \frac{r_{ciy}}{|r_{ci}|} p_{ix} \right) \end{bmatrix}$$

net friction forces and moment at origin for all sliding points, given COR location

If solution results in values of f_{cx}, f_{cy} that are below the friction limit, <u>it is the solution.</u>

Sakurai example where CoR happens to be at p_1

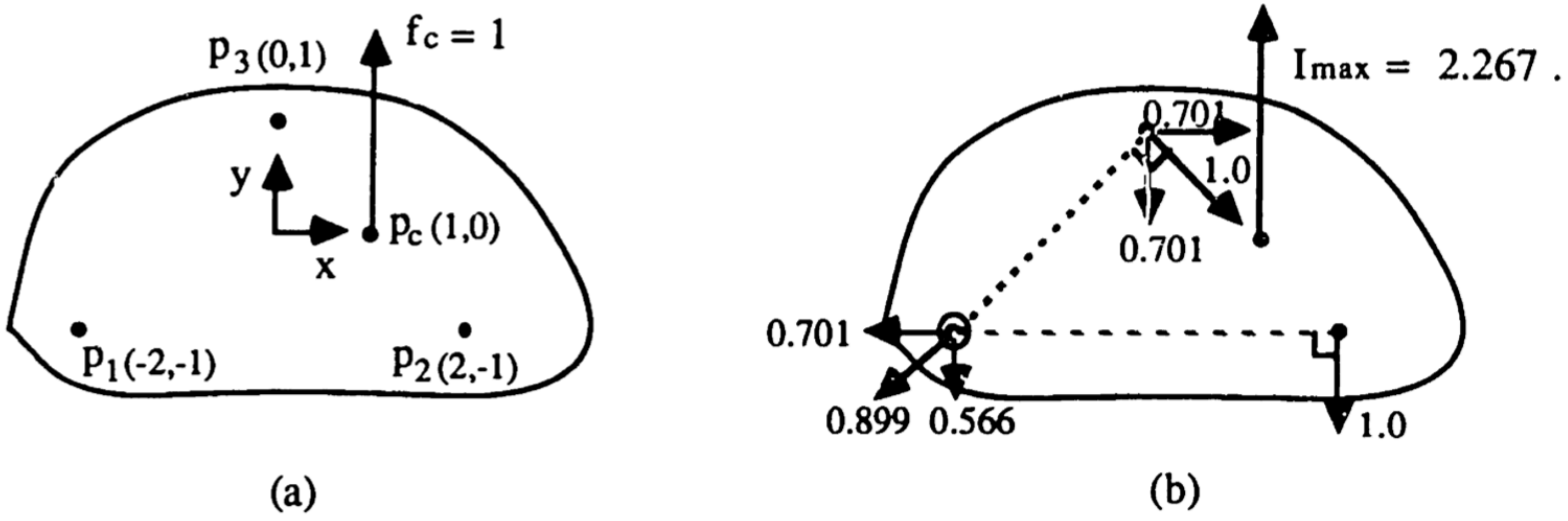
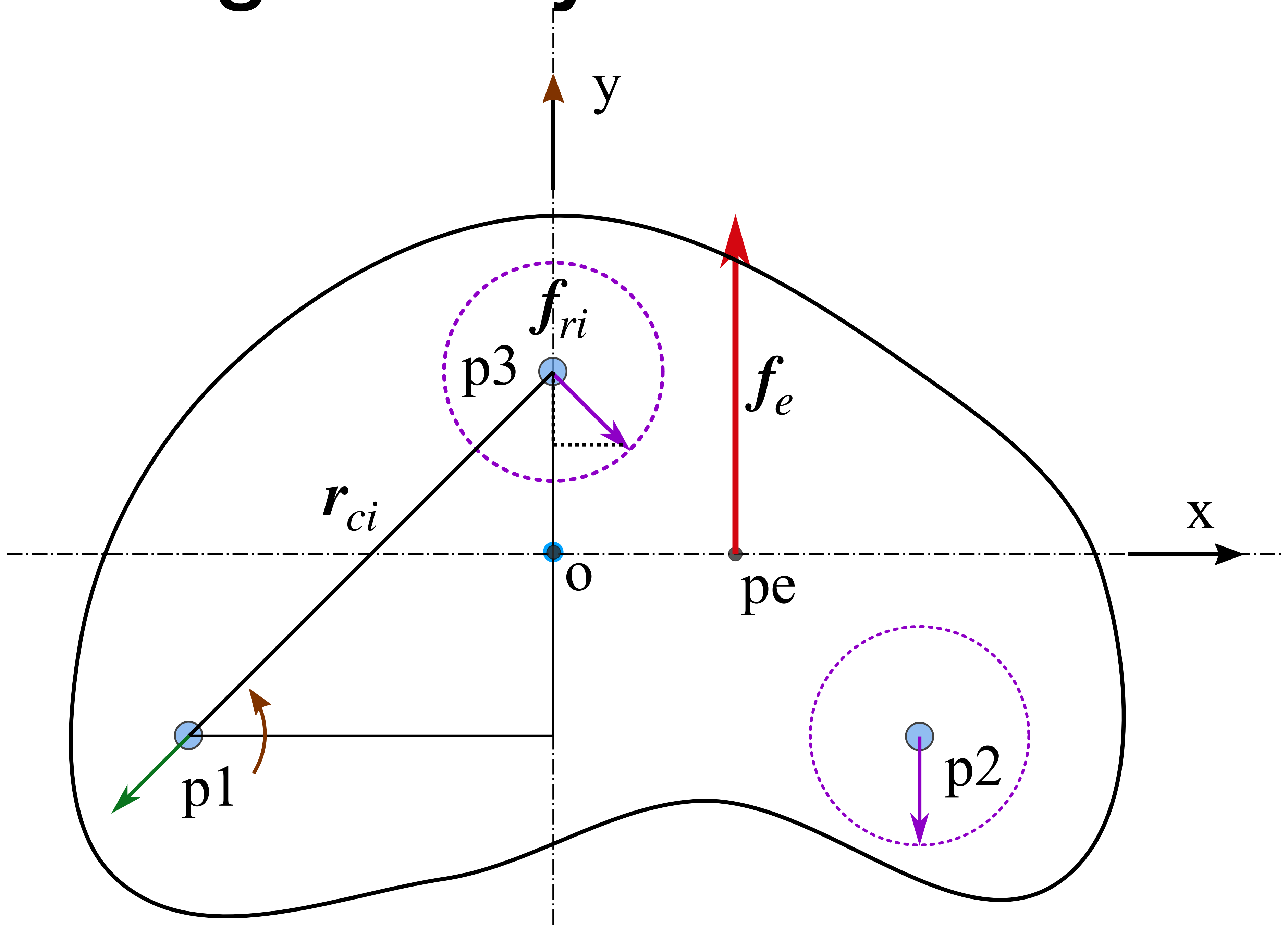


Figure 4.2.7 An example of maximum magnitude method

Note that Sakurai does not actually solve this special case using the method on the last slide. Instead he applies the general Maximum Work optimization method (see next slide), which gives the same result.

More generally…

(This method is implemented in SakuraiFriction.py)



$$\begin{bmatrix} 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 0 & 1 \\ -y_1 & x_1 & \cdots & -y_n & x_n \end{bmatrix} \begin{bmatrix} f_{1x} \\ f_{1y} \\ \vdots \\ f_{nx} \\ f_{ny} \end{bmatrix} = - \begin{bmatrix} w_{cx} \\ w_{cy} \\ w_{cm} \end{bmatrix}$$

eq (4.2.9) from Sakurai expresses the friction wrench that is equal and opposite to the external wrench

Sakurai eq 4.2.18 - 4.2.20:

$[W]\boldsymbol{f}$ produces a net friction wrench on the body. At sliding, it is equal and opposite to the external wrench, $\boldsymbol{w}$, produced by $\boldsymbol{f_e}$.

So we seek $\boldsymbol{f}$ that will maximize the product $([W]\boldsymbol{f}) \cdot \boldsymbol{w}$.

We can rewrite this as

Maximize:

$$\rho = (-w' \cdot [W])\boldsymbol{f}$$

where ρ is a scaling factor and w is a unit wrench in the direction of the external wrench.

Subject to the constraint that the net friction force must be antiparallel to the external force so:

$w \times [W\boldsymbol{f}] = 0$ (Aeq, beq in linprog())

We must also satisfy **bounds** due to friction constraints:

$f_{ix}^2 + f_{iy}^2 \leq (\mu f_{in}^2)$

which we can approximate with linear friction polygons.