Ethiopian University Entrance Examination (EUEE)
Mathematics for Natural Science Stream
GINBOT 2011/JUNE 2019

BOOKLET CODE: 15 **SUBJECT CODE: 02**

Number of Items:65 **Time Allowed: 3 hours**

DIRECTIONS: For each of the following problems, choose the best answer from the given alternatives and carefully **blacken** the letter of your best choice on the separate answer sheet provided.

1. Which one of the following is the set of critical numbers of $f(x)=\frac{3}{8}x^{8/3}-6x^{2/3}$?

   (A) $\{-2, 2\}$ (B) $\{-1, 1\}$ (C) $\{-1, 0, 1\}$ (D) $\{-2, 0, 2\}$

2. What is the greatest lower bound of the sequence $\left\{(-1)^n \frac{1}{n+1}\right\}_{n=0}^{\infty}$?

   (A) -1 (B) 0 (C) $-1/2$ (D) $1/2$

3. Which one of the following is true about the pair of lines: $3x + 9y - 24 = 0$ and $4x+12y+32 = 0$?

   (A) perpendicular lines
   (B) intersecting lines
   (C) parallel and distinct lines
   (D) representing the same lines

4. Which one of the following is equal to $f(x)=\sqrt{(x+4)^2}$, for every $x \in \Re$?

   (A) $g(x)=x+4$
   (B) $g(x)=x+2$
   (C) $g(x)=|x+4|$
   (D) $g(x)=|x|+4$

5. The center of a circle on the line $y = 2x$ and the line $x = 1$ is tangent to the circle at (1,6). How long is the radius of the circle?

(A) 5 (B) 4 (C) 3 (D) 2

6. What is the value of $\lim_{n\to\infty}\left(\frac{3^n + 2^n}{6^n}\right)$?

(A) 1 (B) 0 (C) ∞ (D) $\frac{5}{6}$

7. If $F(x) = \int_0^x e^{-t}dt$, then what is the value of $F'(x)$?

(A) e^{-x} (B) $-e^{-x} - 1$ (C) $\frac{e^{-x+1}}{-x+1}$ (D) $-e^{-x}$

8. If $\neg p \Rightarrow r$ is False and $p \Leftrightarrow q$ is True, then which of the following is True?

(A) $p \vee (\neg q \wedge r)$ (C) $\neg p \Rightarrow (q \vee r)$
(B) $\neg p \wedge (q \Rightarrow r)$ (D) $p \Leftrightarrow (\neg q \vee r)$

9. If $a_1 = 2$, $a_2 = 6$, $a_3 = 10$, $a_4 = 14$, . . . , then $\sum_{n=1}^{100} a_n =$ ____

(A) 20,020 (B) 20,200 (C) 20,000 (D) 22,000

10. What is the value of $\lim_{h\to 0} \frac{8\left(\frac{1}{2} + h\right)^2 - 8\left(\frac{1}{2}\right)^2}{h}$?

(A) 0 (B) 4 (C) 8 (D) The limit does not exist

11. If there are two children in a family, what is the probability that there is at least one girl in the family?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$

12. If $\begin{vmatrix} -1 & 1 & 2 \\ 3 & 2 & x \\ 2 & 4 & 1 \end{vmatrix} = \begin{vmatrix} -x & 3 & 2 \\ 2 & 2 & 3 \\ 1 & -1 & -2 \end{vmatrix}$, then what is the value of x?

(A) $\frac{2}{5}$ (B) $-\frac{2}{5}$ (C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

13. Let $f(x) = 2e^x - k\sin x + 1$. If the equation of the tangent line to the graph of f at $(0,3)$ is $y = 5x + 3$, then what is the value of k?

(A) 3 (B) -3 (C) -5 (D) 2

14. Everyday a person saves 5 cents more than the amount he saved on the previous day. His target is to save the total amount of 3225 cents by the end of 30 days. How much must be the starting saving to meet the target?

(A) 35 cents (B) 25 cents (C) 50 cents (D) 60 cents

15. Which one of the following is the multiplicative inverse of $z = \frac{3+4i}{4-5i}$?

(A) $\frac{8}{25} - \frac{31}{25}i$ (B) $-\frac{8}{25} + \frac{31}{25}i$ (C) $-\frac{8}{25} - \frac{31}{25}i$ (D) $\frac{8}{25} + \frac{31}{25}i$

16. Consider the following system of equations: $\begin{cases} x - 2y + z = 1 \\ -x + y + z = 3 \\ 3x - 5y + z = k \end{cases}$

How much must be the value of k so that the system has a solution?

(A) 7 (B) 1 (C) 0 (D) -1

17. What is the value of $\int_1^2 \frac{\ln x}{x^2} dx$?

(A) $-\frac{\ln 2}{2} - \frac{1}{2}$ (B) $\frac{\ln 2}{2} - \frac{1}{2}$ (C) $-\frac{\ln 2}{2} + \frac{1}{2}$ (D) $\frac{\ln 2}{2} + \frac{1}{2}$

18. Among 2000 students who took a regional exam, the percentile of certain student's score is 90. Which of the following is correct about the student's score?
(A) The student have answered 90% of the questions correctly.
(B) The student's score is greater than or equal to that of 1800 students.
(C) The student's score is the same as the top10% of the scores.
(D) The score of the student is as good as that of 90% of the students.

19. If A is a 3×3 matrix and $\det(A) = 5$, then how much is $\det(2A^TA)$?
(A) 200 (B) 100 (C) 50 (D) 20

20. What is the simplified form of $\dfrac{a^{-1}b^{-1}}{a^{-3}-b^{-3}}$?
(A) $\dfrac{a^2b^2}{b^2-a^2}$ (B) $\dfrac{a^2b^2}{b^3-a^3}$ (C) $\dfrac{a^3-b^3}{ab}$ (D) $\dfrac{a^3-b^3}{a-b}$

21. If the circle passing through the point $(-1,0)$ touches the $y-$axis at $(0,2)$, then what is the equation of the circle?
(A) $x^2+y^2+5x+4y+4=0$ (C) $x^2+y^2-5x-4y+4=0$
(B) $x^2+y^2-5x+4y+4=0$ (D) $x^2+y^2+5x-4y+4=0$

22. Let z be a complex number and $w=3+4i$. If $\dfrac{z^2+1}{z+i}=|w|z-\dfrac{1}{i}\overline{w}$, then what is the value of z ?
(A) $-4-2i$ (B) $4-2i$ (C) $-1+i$ (D) $-1-i$

23. What is the value of the left-hand side limit, $\lim\limits_{x\to0^-}\dfrac{\sin x}{x^2}\sqrt{4x^3+9x^2}$?
(A) -3 (B) 3 (C) 2 (D) Does not exist

24. What is the value of $\lim_{x\to 2}\frac{x^3-8}{x^2-4}$?

(A) 2
(B) 3
(C) –2
(D) The limit does not exist.

25. If $f(x)=ax^3+\frac{b}{x}+5$ has local minimum at $(2,-3)$, what are the values of a and b?

(A) $a=-\frac{1}{4}$, b = 12
(B) $a=-\frac{1}{4}$, b = -12
(C) $a=\frac{1}{4}$, b = 12
(D) $a=\frac{1}{4}$, b = -12

26. What is the value of $\int\frac{x^{e-1}+e^{x-1}}{x^e+e^x}dx$?

(A) $\frac{1}{e}\ln\left(x^e+e^{x+1}\right)+c$
(B) $\frac{1}{e}\ln\left(x^e+e^x\right)+c$
(C) $\frac{1}{e}\ln\left((x+1)^e+e^{x+1}\right)+c$
(D) $\frac{1}{e}\ln\left(x^{e+1}+e^{x+1}\right)+c$

27. What is the area of the region bounded by the lines $x=0$, $x=2$, and $y=1$ and the curve $y=e^{2x}$?

(A) $\frac{e^4}{2}-\frac{5}{2}$
(B) $\frac{e^2}{2}-\frac{1}{2}$
(C) $\frac{e^3}{3}-\frac{1}{3}$
(D) $-\frac{e^4}{2}+\frac{5}{2}$

28. If the region enclosed by the graphs of $f(x)=x^2$ and $f(x)=x^3$ from $x=0$ to $x=1$ rotates about the $x-axis$, what is the volume of the solid of revolution?

(A) $\frac{2\pi}{27}$ cubic units
(B) $\frac{2\pi}{25}$ cubic units
(C) $\frac{2\pi}{5}$ cubic units
(D) $\frac{2\pi}{35}$ cubic units

29. Which one of the following is a valid argument?

(A) If I don't change my oil regularly, my engine will die. My engine died, Thus, I didn't change my oil regularly.

(B) If I am literate, then I can read and write. I can read but I can't write. Thus I am not literate.

(C) If you do every problem in the book, then you will learn the subject. You learned the subject. Thus, you did every problem in the book.

(D) If it rains or snows, then my roof leaks. My roof is leaking. Thus, it is raining and snowing.

30. The earth's orbit has a semi-major axis $a \approx 149.6$ Gm (gigameters) and an eccentricity of $e \approx 0.017$. What is the approximate value of the semi-minor axis?

(A)152.14 Gm (B) 145.32 Gm (C)149.06 Gm (D) 149.58 Gm

31. The marks of 50 students is given below:

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	5	8	f_1	10	f_2

If the median of the data is 26, what are the values of f_1 and f_2?

(A) $f_1 = 7, f_2 = 20$ (C) $f_1 = 15, f_2 = 12$

(B) $f_1 = 12, f_2 = 15$ (D) $f_1 = 20, f_2 = 7$

32. Suppose 2,500 items are produced by a machine and 2 % of the product are randomly selected and tested. If 5 of the tested items have a defect, then what is the probability that an item produced by the machine has NO defect?

(A) 0.80 (B) 0.85 (C) 0.90 (D) 0.95

33. If $2\begin{pmatrix} 2x & x \\ -5 & -3 \end{pmatrix}^{-1} = \begin{pmatrix} 3 & 2 \\ -5 & -4 \end{pmatrix}$, then what is the value of x?

(A) -2 (B) 2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

34. If the second and fifth terms of a geometric progression are $-\frac{1}{2}$ and $\frac{1}{16}$, respectively, what is the sum of the first eight terms of the sequence?

(A) $\frac{85}{128}$ (B) $\frac{255}{256}$ (C) $\frac{128}{85}$ (D) $\frac{256}{255}$

35. What is the limit of the sequence $1, \frac{2}{2^2-1^2}, \frac{3}{3^2-2^2}, \frac{4}{4^2-3^2}, \ldots$?

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{3}{2}$

36. At what value of x does the function $f(x)=\frac{4x^3}{3}-x^4$ attains its maximum value?

(A) -1 (B) 0 (C) 1 (D) $\frac{4}{3}$

37. What is the solution of $\int 4x\left(\ell n\, x+\frac{1}{x^2}\right)dx$?

(A) $4x^2(\ell n\, x+1)-2x^2+c$ (C) $x^2(2\ell n\, x+1)+4\ell n\, x+c$

(B) $x^2(4\ell n\, x-1)+2\ell n\, x+c$ (D) $x^2(2\ell n\, x-1)+4\ell n\, x+c$

38. If $f(x)=ax-b$ and $f^{-1}(x+1)=\frac{1}{2}x+2$, for each $x\in\Re$, then what must be the values of a and b?

(A) $a=1/2,\ b=-2$ (C) $a=1,\ b=1$

(B) $a=2,\ b=2$ (D) $a=2,\ b=3$

39. Which one of the following is true about the graph of $f(x)=\frac{x^2+5x+6}{x^2-4}+3$?

(A) The graph has a hole at $x=2$.

(B) The vertical asymptotes of the graph are $x=-2$ and $x=2$.

(C) A horizontal asymptote of the graph is $y=4$.

(D) The graph has $y-$intercept at $(0,-\frac{3}{2})$

40. Fatuma can solve 90% of the problems given in a book and Mesfin can solve 70%. What is the probability that at least one of them will solve the problem, selected at random from the book?

(A) 0.77 (B) 0.87 (C) 0.97 (D) 0.67

41. What is the polar form of $\frac{7-i}{3-4i}$?

(A) $\sqrt{2}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

(B) $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(C) $2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

(D) $2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

42. What is the sum of the infinite series $\sum_{k=0}^{\infty} 5\left(\frac{2^k + 5^k}{10^k}\right)$?

(A) 15 (B) $\frac{25}{4}$ (C) 5 (D) $\frac{65}{4}$

43. If $f(x) = \ln\left(2^{\tan x}\right)$, then what is the value of $f'(0)$?

(A) $\ln 2$ (B) $-2\ln 2$ (C) $\frac{\ln 2}{2}$ (D) 1

44. If $h(x) = \sqrt{1+\sqrt{x}}$, then which of the following is equal to $h'(x)$?

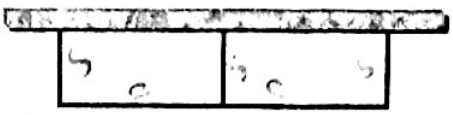
(A) $\frac{1}{2\sqrt{1+\sqrt{x}}}$

(B) $\frac{1}{4\sqrt{x+x\sqrt{x}}}$

(C) $\frac{x}{2\sqrt{1+\sqrt{x}}}$

(D) $\frac{x}{4\sqrt{x+x\sqrt{x}}}$

45. A man wants to fence two identical rectangular enclosures in a field alongside a straight river as shown in the following figure.



What is the maximum area of each enclosure that he can make with 192 meter fencing material if the side along the river does not need a fence?

(A) 1530 m^2 (B) 1564 m^2 (C) 1664 m^2 (D) 1536 m^2

46. For real numbers x and y, which one of the following is **NOT** true?

(A) $(\forall x)(\forall y)\ (y^2 + x^2 \geq -1)$

(B) $(\forall x)(\exists y)\ (y \geq x^2 + 1)$

(C) $(\exists x)(\forall y)\ (y \geq x^2 + 1)$

(D) $(\exists x)(\exists y)\ (y \geq x^2 + 1)$

47. If $f(x) = x^3 - 2x + 1$, then which one of the following is NOT true?

(A) $f(c) = 0$ for some $c \in [-2, 0]$.

(B) $f(c) = \frac{1}{2}$ for some $c \in [0, 1]$.

(C) $f(c) = \frac{1}{4}$ for some $c \in [-1, 0]$.

(D) $f(c) = 3$ for some $c \in [1, 2]$.

48. A water tank is a rectangular parallelepiped with base length 3 m, width 2 m and height 2.5 m. If water is flowing into the tank at the rate of 0.12 m^3/sec, then how fast does the level of water rise up in the tank?

(A) 0.02 m/sec (C) 0.04 m/sec

(B) 0.03 m/sec (D) 0.06 m/sec

49. If f is the greatest integer function and g is the absolute value function, then what is the value of $(f \circ g)(\frac{3}{2}) + (g \circ f)(-\frac{4}{3})$?

(A) 1 (B) 3 (C) -1/2 (D) 2

50. There are three children in a room with, ages four, five, and six. If a five-year-old child enters the room, then which of the following statement is correct ?
(A) mean age will stay the same but the standard deviation will decrease.
(B) mean age will stay the same but the standard deviation will increase.
(C) mean age and standard deviation will increase.
(D) mean age and standard deviation will stay the same.

51. If $f(x) = \frac{1}{x}$, then what is the value of $f^{(n)}(x)$?

(A) $f^{(n)}(x) = \frac{(-1)^n n!}{x^n}$
(B) $f^{(n)}(x) = \frac{(-1)^{n+1}(n+1)!}{x^{n+1}}$
(C) $f^{(n)}(x) = \frac{(-1)^n n!}{x^{n-1}}$
(D) $f^{(n)}(x) = \frac{(-1)^n n!}{x^{n+1}}$

52. What is the value of $\cot 270° + 2\cos 90° + 4\sec^2 180°$?
(A) -2 (B) 8 (C) 4 (D) 7

53. $\forall n \in N$, $3^n - 2$ is prime that can be proved or disproved by which one of the following mathematical proofs?
(A) Direct proof
(B) Proof by exhaustion
(C) Disprove by counter example
(D) Proof by contradiction

54. If the point $(\alpha,0,3)$ is on the sphere centered at $(1,2,3)$ with radius 2, what is the value of α ?
(A) 0 (B) 1 (C) 2 (D) -3

55. Consider a rectangle ABCD with base vertices A=(0,3) and B=(4,0) and the other vertices, C and D, in the first-quadrant of the coordinate plane. If its height $\overline{BC}$ is half of the length of its base, then which of the following indicates the coordinates of vertex C?
(A) (4, 5/2) (B) (6, 3/2) (C) (5/2, –2) (D) (11/2, 2)

56. Which one of the following is a valid assertion that can be proved by the principle of mathematical induction?

(A) $3n+25<3^n$, for every integer $n\geq 3$.

(B) $2^n > n+20$, for every integer $n\geq 4$.

(C) $n^2 \leq 2^n$, for every integer $n\geq 1$.

(D) n^3-n is divisible by 6, for every integer $n\geq 1$.

57. Ship A and B depart from the same point at the same time on the course N60°E and N40°E, respectively. If the speed of ship A is 20 km per hour and the speed of ship B is 30 km per hour, what is the distance between the two ships just after 30 minutes of their departure?

[*You may take*: $\cos(40°)=0.77$, $\cos(20°)=0.94$, $\sin(20°)=0.34$]

(A) $\sqrt{40}$ km (B) $\sqrt{43}$ km (C) $\sqrt{50}$ km (D) $\sqrt{53}$ km

58. What is the image of the circle $x^2+y^2-4x-6y+11=0$, when the origin is shifted to the point (1,1) after translation of axes?

(A) $x^2+y^2-6x-8y+23=0$

(B) $x^2+y^2-4x-6y+3=0$

(C) $x^2+y^2+6x+8y-23=0$

(D) $x^2+y^2-4x-2y+3=0$

59. When the plane is rotated 45^0 about the point (1, –2), then what would be the image of the point (2, 4)?

(A) $\left(1-\frac{5\sqrt{2}}{2}, -2+\frac{7\sqrt{2}}{2}\right)$

(B) $\left(1-\frac{\sqrt{2}}{2}, -2+\frac{\sqrt{2}}{2}\right)$

(C) $\left(\frac{5\sqrt{2}}{2}, \frac{7\sqrt{2}}{2}\right)$

(D) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

60. If $\mathbf{u} = 2\vec{j} - \vec{k}$ and $\mathbf{v} = \vec{i} - 8\vec{j} + 3\vec{k}$, then what is the unit vector in the direction of $5\mathbf{u} + \mathbf{v}$?

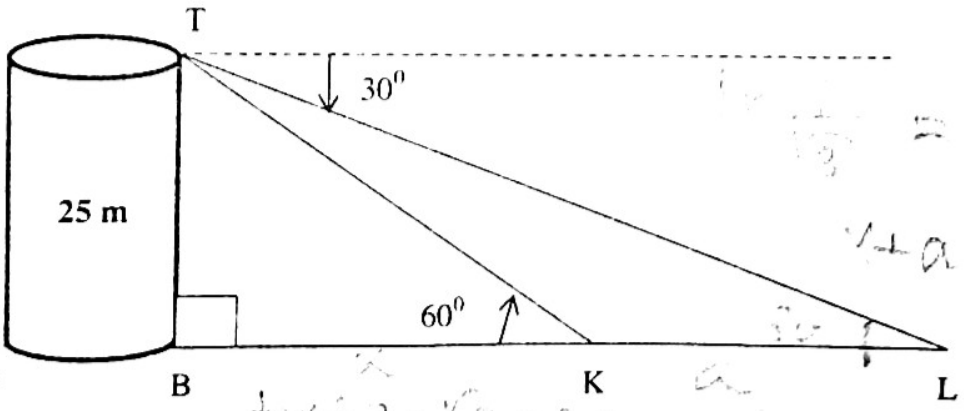
(A) $\vec{i} + 2\vec{j} - 2\vec{k}$

(B) $\frac{2}{3}\vec{i} + \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k}$

(C) $\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{2}{3}\vec{k}$

(D) $\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}$

61. The diagram below is a representation of a 25 m vertical observation tower TB and two cars K and L on a road. The angle of depression from T to car L is 30°. The angle of elevation from car K to the top of the tower is 60°. B, K and L lie in a straight line and lie on the same horizontal plane as the base of the tower.



What is the distance between the two cars?

(A) $\frac{50\sqrt{3}}{3}$ m (B) $50\sqrt{3}$ m (C) $\frac{50\sqrt{3}}{2}$ m (D) $50 + \sqrt{3}$ m

62. Let L be the line given by the vector equation $(x, y) = (1,1) + t(\sqrt{3},1)$, $t \in \mathbb{R}$. What is the equation of the image of L after being rotated 15° about (1,1) and then translated by vector $\mathbf{u} = (-1,1)$?

(A) $-x + y = 2$

(B) $x - y = 2$

(C) $\sqrt{3}x - y = 2$

(D) $-x + \sqrt{3}y = 1$

63. If the angle between the vectors $\vec{A} = (2,-1,1)$ and $\vec{B} = (1,1,\alpha)$ is $\frac{\pi}{3}$, then what is the value of α?

(A) 1 (B) -1 (C) 2 (D) -2

64. What is the solution set of $\sin^2 x - \sin x \cos x = 0$ over $[0,2\pi]$?

(A) $\{0,\pi,\frac{5\pi}{4},2\pi\}$ (C) $\{0,\frac{\pi}{4},\pi,\frac{5\pi}{4},2\pi\}$

(B) $\{0,\frac{\pi}{4},\pi,2\pi\}$ (D) $\{0,\frac{\pi}{4},\pi,\frac{5\pi}{4}\}$

65. Which of the following is a correct assertion that can be proved by the principle of mathematical induction?

(A) $\frac{1}{n+1} \le 1$, for each real number $n \ge 1$.

(B) $m! \le 4^m$, for each positive integer m.

(C) $2^p - 1$ is prime for each prime integer p.

(D) $k! \ge 2^k$, for each integer $k \ge 4$.

THE END