RESEARCH STATEMENT

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The primary objective of my graduate research at the University of Oregon was to better understand the bulk transport properties of metals. During this time, my advisor, Dietrich Belitz, and I, together with our collaborator Theodore R. Kirkpatrick, obtained both new and improved expressions for the thermoelectric coefficients of metallic magnets; in order to calculate an expected value for the electric current and the heat current, which both emerge in response to either an applied electric field or an applied temperature gradient, we studied the linearized Boltzmann equation [1]. I then used our method to determine the leading beyond semi-classical corrections to the bulk resistivity of a metallic ferromagnet, which arise due to interference between magnetic fluctuations and weak disorder.

Qualitative Background

Methods for solving the many-body problem associated with normal metals stem from Landau's Fermi liquid theory [2], which is analytically tractable on account of Pauli's principle. Indeed, for temperatures T much less than the Fermi energy ϵ_F, there arises a Fermi surface, which delineates the degenerately packed region of electronic phase space from its complement, wherein the occupation numbers are suppressed; this feature all but restricts the spectrum to include only those elementary excitations that are continuously connected to a state of the free electron gas. Moreover, when $T \ll \epsilon_F$, these modes are slow to relax for two reasons: (1) the volume of phase space within the Fermi sphere into which they may decay is limited and (2) there are few charge inhomogeneities available to facilitate a downward transition [3]. Hence, low-lying fluctuations of the interacting system may be represented as long-lived quasi-particles with an electron-like energy-momentum relation

Now then, the physics of a clean metal open to its low temperature environment may be captured by coupling the mobile quasi-electrons of Landau's Fermi liquid theory to lattice originating fields governed by distribution functions thermalized at temperature T. To determine which couplings provide the dominant mechanism by which electronic fluctuations are drawn towards equilibrium, we leverage the hierarchy of scales which characterize the dynamical degrees of freedom of a metallic ferromagnet. It follows that the only propagating modes that need to be considered as interacting with the electrons are acoustic phonons and ferromagnons, which emerge from the broken continuous translational symmetry brought about by the lattice and the spontaneous symmetry breaking associated with the magnetization field, respectively.

Towards addressing the fact that real materials are marred by structural defects, we include collisions of electrons with impurities; such distortions of the lattice are introduced by way of a random potential field [4].

Thus, while electron Bloch waves travel freely in an ideal crystal at zero temperature, and therefore accelerate indefinitely under external forces, physical materials have inherent channels for the dissipation of electronic energy and momentum; on modeling the scattering of conduction electrons by lattice vibrations, variations in the local electromagnetic field, and defects in the crystal, we obtain expressions for the transport coefficients.

Quantitative Aspects

Here, we aim to illuminate the mathematical structure underlying our method by sketching a determination of the electrical conductivity σ, which obeys Ohm's law

$$J = \sigma E, \tag{1}$$

where J is the electric current and E is the applied electric field. Following Kubo [5], one finds that the DC conductivity is given by an inner product

$$\sigma_{ij} = \left(k_i, C^{-1} \circ k_j\right) \tag{2}$$

that sandwiches the inverse collision operator C^{-1} (where C defines the relevant Boltzmann equation) between two bare momentum modes k; the subscripts i, j are spatial indices. To realize the action of C^{-1}, we note that C is self-adjoint under $(\cdot,\cdot)$ and seek its spectral representation. An asymptotically exact inversion of the collision operator is then obtained by leveraging a singularity that is inherent to the Boltzmann equation when global electronic momentum conservation is weakly violated; in this case, C admits a lowest nonzero eigenvalue λ that is isolated, nondegenerate, and corresponds to an eigenvector that is perturbatively connected to k. It follows that the leading contributions to the resistivity are entirely contained in the nonconserving corrections to the momentum mode, i.e. one need only solve the eigenvalue problem

$$C \circ e_i = \lambda e_i, \tag{3}$$

where

$$e_i \approx k_i + \mathcal{O}(T, \gamma), \quad \lambda_p \approx \mathcal{O}(T, \gamma), \tag{4}$$

with γ controlling the impurity strength. Thus, we arrive at a Drude formula

$$\sigma = \frac{ne^2}{2m\lambda}\left[1 + \mathcal{O}(T, \gamma)\right], \tag{5}$$

where all details of the complicated many-body physics are buried in the eigenvalue λ; here n is the electronic density, e is the electronic charge, and m is the electronic mass.

Primary Result

Using our spectral method, I calculated the dominant ballistic corrections to the electrical resistivity of a weakly disordered metallic ferromagnet; the response is derived, via an effective action that contains vertices for both magnon exchange and impurity scattering. In this way, one can algorithmically generate the beyond semi-classical contributions to the collision operator and then construct the ensuing corrections to the transport rate in a second von Neumann series about either the ideal solution or the zero temperature solution; we find a Matthiessen's rule-like interpolation formula between the regime of weaker magnons and the regime of weaker disorder.

As a result, we corroborate the arguments of Belitz and Kirkpatrick [6, 7], which indicated that the observed scaling of the resistivity $\rho \propto T^{3/2}$, when $T \ll T_1$, in metallic ferromagnets can be attributed to interference between two scattering mechanisms: ferromagnons and static impurities.

Moving Forward

Our method may prove useful in the context of active systems which weakly violate energy conservation. In particular, a future project might investigate whether meta-materials can be designed which are more fatigue resistant than their traditional counterparts.

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