B10815057 Algorithms homework2

1.

(1). lg(n!) = lg(n*(n-1)*(n-2)…*2*1)=lg(n)+lg(n-1)+lg(n-2)…+lg(2)+lg(1)

而此式必小於 nlogn =lg(n)+lg(n)+….lg(n)+lg(n) (n 個 lg(n))

故 lg(n!) = O(nlogn)

(2).設 S = lg(n!) , T = lg(1)+lg(2)…+lg(n/2) ,

U = lg((n/2)+1)+ lg((n/2)+2)…+ lg(n)

S = T + U

T 的下界(lower bound) = lg(1)+lg(1)…+lg(1) = 0+0…+0 = 0

U 的下界(lower bound) = lg(n/2)+lg(n/2)…+lg(n/2) = n/2*lg(n/2)

所以 S 的下界(lower bound) = T 的下界 + U 的下界 = 0 + n/2*lg(n/2)

n/2*lg(n/2) >= nlogn ---> n/2*lg(n/2) = Ω(nlogn)

故 lg(n!) = Ω(nlogn)

因為 lg(n!) = Ω(nlogn)且 lg(n!) = O(nlogn)

所以 lg(n!) = θ(nlogn)

2.

(a). $T(n)$ = 32$T(n/4)$ + $n^2\sqrt{n}$

f(n) = $n^{2+0.5}$ = $n^{2.5}$ = $n^{log_4 32}$ = θ($n^{log_4 32}$)

因此屬於 case 2

故$T(n) = \theta\,(n^{log_b a}\, lg\, n) = \theta\,(n^{2.5}\, lg\, n)$

(b). $T(n) = 3T(n/9) + n^4\, lg\, n$

f(n) = $n^4\, lg\, n = \Omega(n^{log_9 3-\epsilon})$ ($\epsilon = 1$ 時可成立)

且滿足$af\left(\frac{n}{b}\right) \le cf(n)$ → $3\left(\frac{n}{9}\right)^4 lg\left(\frac{n}{9}\right) \le \frac{1}{3}n^4 lgn$ (c 取 $\frac{1}{3}$)

因此屬於 case 3

故$T(n) = \theta\,(f\,(n)) = \theta\,(n^4\, lg\, n)$

(c). $T(n) = 8T(n/4) + n\sqrt{n}$

f(n) = $n^{1+0.5} = n^{1.5} = n^{log_4 8} = \Theta(n^{log_4 8})$

因此屬於 case 2

故$T(n) = \theta\,(n^{log_b a}\, lg\, n) = \theta\,(n^{1.5}\, lg\, n)$

(d). $T(n) = T(n - 1) + lg\, n$

$$
\begin{aligned}
T(n) &= T(n-1) + lg\, n \\
&= (T(n-2) + lg(n-1)) + lg\, n \\
&= [(T(n-3) + lg(n-2)) + lg(n-1)] + lg\, n \\
&\dots \\
&\dots \\
&\dots \\
&= T(1) + lg\, 1 + lg\, 2 + lg\, 3 + \cdots lg(n-1) + lg\, n \\
&= T(1) + lg\, n! \\
&= lg\, n!
\end{aligned}
$$

由 1-2 的結論可得知 lg(n!) = Θ(nlogn)

故$T(n)$ = Θ(nlogn)

(e). $T(n)$ = 2$T(n/2$ - 1)

$T(n)$ = 2$T(n/2$ - 1)

= 2$T((n$-2$)/2)$

設 u = n-2

$T(n)$ = T (u+2) = 2T(u/2)

f(n) = 0 = O$(n^{log_2 2-\epsilon})$($\epsilon = 0.5$ 時可成立)

因此屬於 case 1

$T(n)$ = θ $(n^{log_b a})$ = θ $(n^{log_2 2})$ = θ (n)