B10815057 Algorithms homework2

(1). lg(n!) = lg(n\*(n-1)\*(n-2)…\*2\*1)=lg(n)+lg(n-1)+lg(n-2)…+lg(2)+lg(1)

而此式必小於 nlogn =lg(n)+lg(n)+….lg(n)+lg(n) (n個lg(n))

故lg(n!) = O(nlogn)

(2).設S = lg(n!) , T = lg(1)+lg(2)…+lg(n/2) ,

U = lg((n/2)+1)+ lg((n/2)+2)…+ lg(n)

S = T + U

T的下界(lower bound) = lg(1)+lg(1)…+lg(1) = 0+0…+0 = 0

U的下界(lower bound) = lg(n/2)+lg(n/2)…+lg(n/2) = n/2\*lg(n/2)

所以S的下界(lower bound) = T的下界 + U的下界 = 0 + n/2\*lg(n/2)

n/2\*lg(n/2) >= nlogn ---> n/2\*lg(n/2) = Ω(nlogn)

故lg(n!) = Ω(nlogn)

因為lg(n!) = Ω(nlogn)且lg(n!) = O(nlogn)

所以lg(n!) = θ(nlogn)

(a). 𝑇(𝑛) = 32𝑇(𝑛/4) +

f(n) = = = = θ()

因此屬於case 2

故𝑇(𝑛) =θ() = θ()

(b). 𝑇(𝑛) = 3𝑇(𝑛/9) +

f(n) = = ( 時可成立)

且滿足𝑎𝑓 ≤ 𝑐𝑓(𝑛) 🡪 (c取 )

因此屬於 case 3

故𝑇(𝑛) =θ() =θ()

(c). 𝑇(𝑛) = 8𝑇(𝑛/4) +

f(n) = =θ()

因此屬於case 2

故𝑇(𝑛) =θ() = θ()

(d). 𝑇(𝑛) = 𝑇(𝑛 - 1) +

𝑇(𝑛) = 𝑇(𝑛 - 1) +

= (𝑇(𝑛 - 2) + ) +

= [(𝑇(𝑛 - 3) + ) + ] +

…

…

…

= 𝑇(1) +

= 𝑇(1) +

=

由1-2的結論可得知lg(n!) = θ(nlogn)

故𝑇(𝑛) = θ(nlogn)

(e). 𝑇(𝑛) = 2𝑇(𝑛/2 - 1)

𝑇(𝑛) = 2𝑇(𝑛/2 - 1)

= 2𝑇((𝑛-2)/2)

設u = n-2

𝑇(𝑛) = 𝑇 (u+2) = 2𝑇(u/2)

f(n) = 0 = O( 時可成立)

因此屬於case 1

𝑇(𝑛) = θ() = θ() = θ(n)