Calculus(II)
Homework 1, Mar, 05, 2020
Deadline: Mar, 20, 2020

1. Evaluate the integral.
   (a) $\int x \cos 5x \; dx$.
   Let $u = x, \; du = dx$
   then $dv = \cos 5x \; dx$
   $\Rightarrow \; v = \int \cos 5x \; dx = \frac{1}{5} \int \cos 5x \cdot 5 \; dx \Rightarrow \; v = \frac{1}{5} \sin 5x$
   $\therefore \int u \; dv = uv - \int v \; du$
   $\Rightarrow \int x \cos 5x \; dx = x(\frac{1}{5} \sin 5x) - \int (\frac{1}{5} \sin 5x) \; dx$
   $= \frac{1}{5} x \sin 5x - \frac{1}{5}(-\frac{1}{5} \cos 5x) + c = \frac{1}{5} x \sin 5x + \frac{1}{25} \cos 5x + c.$
   (b) $\int \ln(2x+1) \; dx$.
   Let $u = 2x + 1, \; du = 2dx$
   $\Rightarrow \int \ln u \frac{du}{2} = \frac{1}{2} \int \ln u \; du = \frac{1}{2}(u \ln u - u)$
   $= \frac{1}{2}[(2x+1) \ln(2x+1) - (2x+1)] + c = \frac{1}{2}(2x+1) \ln(2x+1) - x + c.$

2. First make a substitution and then use integration by parts to evaluate the integral.
   (a) $\int \cos \sqrt{x} \; dx$.
   Let $u = \sqrt{x}, \; u^2 = x, \; dx = 2u \; du, \; du = \frac{dx}{2u}$
   $\int \cos \sqrt{x} \; dx = \int \cos u \cdot 2u \; du = 2 \int u \cos u \; du$
   $= 2u(\sin u) - 2 \int \sin(u) \; du = 2u \sin(u) + 2 \cos(u) + c$
   $= 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + c.$
   (b) $\int t^3 e^{-t^2} \; dt$.
   Let $u = -t^2, \; du = -2t \; dt \Rightarrow dt = \frac{du}{-2t}$
   $\int t^3 e^{-t^2} \; dt = \int t^3 e^u \frac{du}{-2t} = \int -\frac{1}{2} t^2 e^u \; du = \int \frac{1}{2} u e^u \; du$
   $\int \frac{1}{2} u e^u \; du = \frac{1}{2} u e^u - \int \frac{1}{2} e^u \; du = \frac{1}{2} u e^u - \frac{1}{2} e^u + c$
   $\Rightarrow \int t^3 e^{-t^2} \; dt = \frac{1}{2}(-t^2 e^{-t^2} - e^{-t^2}) + c = -\frac{e^{-t^2}(t^2+1)}{2} + c.$

3. Evaluate the integral.

   (a) $\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^5\theta \; d\theta.$

   $= \int_0^{\frac{\pi}{2}} \sin^7\theta \cos^4\theta \cos\theta \; d\theta$

   $\cos^2\theta = 1 - \sin^2\theta,$ so $\int_0^{\frac{\pi}{2}} \sin^7\theta(1-\sin^2\theta)^2 \cos\theta \; d\theta$

   Let $u = \sin\theta,\; du = \cos\theta \; d\theta$

   $\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^5\theta \; d\theta = \int_{\sin(0)}^{\sin(\frac{\pi}{2})} \sin^7\theta \cos^5\theta \; d\theta = \int_0^1 \; u^7(1-u^2)^2 \; du$

   $= \int_0^1 \; u^7 - 2u^9 + u^{11} \; du = (\frac{u^8}{8} - \frac{2u^{10}}{10} + \frac{u^{12}}{12})_0^1 = (\frac{1}{8} - \frac{2}{10} + \frac{1}{12}) - 0 = \frac{1}{120}.$

4. Evaluate the integral using the indicated trigonometric substitution. Sketch and label the associated right triangle.

   (a) $\int \frac{dx}{x^2\sqrt{4-x^2}},\; x = 2\sin\theta.$

   $\Rightarrow \; dx = 2\cos\theta \; d\theta$

   $\sqrt{4-x^2} = \sqrt{4-(2\sin\theta)^2} = \sqrt{4-4\sin^2\theta}$

   $= \sqrt{4 - cos^2\theta} = 2\cos\theta$

   $\int \frac{dx}{x^2\sqrt{4-x^2}} = \int \frac{2\cos\theta \; d\theta}{(2\sin\theta)^2(2\cos\theta)} = \frac{1}{4}\int \frac{d\theta}{\sin^2\theta} = -\frac{1}{4}\cot\theta + c$

   $\because x = 2\sin\theta,\; \cot\theta = \frac{\sqrt{4-x^2}}{x},\; \therefore \; \int \frac{dx}{x^2\sqrt{4-x^2}} = -\frac{\sqrt{4-x^2}}{4x} + c.$

5. Evaluate the integral.

   (a) $\int_0^1 \frac{2}{2x^2+3x+1} \; dx.$

   $(2x^2+3x+1) = (x+1)(2x+1)$

   Let $\frac{2}{2x^2+3x+1} = \frac{A}{x+1} + \frac{B}{2x+1} \Rightarrow A(2x+1) + B(x+1) = 2$

   $\Rightarrow A = -2,\; B = 4,\; \therefore \int_0^1 \frac{2}{2x^2+3x+1} \; dx = \int_0^1 \frac{-2}{x+1} + \frac{4}{2x+1} \; dx$

   $\Rightarrow \int_0^1 \frac{-2}{x+1} + \frac{4}{2x+1} \; dx = \Big[-2\ln(x+1) + 2\ln(2x+1) \Big]_0^1$

   $= -2\ln 2 + 2\ln 3 = 2\ln\frac{3}{2}.$

6. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a) $\int_3^\infty \frac{1}{(x-2)^{\frac{3}{2}}}\,dx.$ $\Rightarrow$ Let $t \to \infty$, $\therefore \lim_{t\to\infty} \int_3^t (x-2)^{-\frac{3}{2}}\,dx$

Let $u = x-2,\ du = dx,\ \lim_{t\to\infty} \int_3^t u^{-\frac{3}{2}}\,du = \lim_{t\to\infty} (-2u^{-\frac{1}{2}})_3^b$

$= \lim_{t\to\infty} \left[\frac{-2}{\sqrt{x-3}}\right]_3^t = \lim_{t\to\infty} \frac{-2}{\sqrt{t-2}} + 2 = 2.$

(b) $\int_0^\infty \frac{1}{\sqrt[4]{1+x}}\,dx.$ $= \lim_{t\to\infty} \int_0^t \frac{1}{\sqrt[4]{1+x}}\,dx = \lim_{t\to\infty} \left[\frac{4}{3}(1+x)^{\frac{3}{4}}\right]_0^t$

$= \lim_{t\to\infty} \frac{4}{3}\left[(1+t)^{\frac{3}{4}} - (1+0)^{\frac{3}{4}}\right] = \lim_{t\to\infty} \frac{4}{3}\left[(1+t)^{\frac{3}{4}} + 1\right]$

$= \frac{4}{3}\left[\infty - 1\right] = \infty.$ Answer:divergent.

(c) $\int_{-\infty}^0 \frac{1}{3-4x}\,dx.$ $= \lim_{t\to-\infty} \left[-\frac{1}{4}\ln(3-4x)\right]_t^0$

$= \lim_{t\to-\infty} \left[-\frac{1}{4}\ln 3 - \left(-\frac{1}{4}\ln(3-4x)\right)\right]$

$= \lim_{t\to-\infty} \left[-\frac{1}{4}\ln 3 + \frac{1}{4}\ln(3-4t)\right]$

$= -\frac{1}{4}\ln 3 + \infty = \infty.$ Answer:divergent.