Calculus(I)
Homework 3, Dec. 04, 2019
Deadline: Dec. 20, 2019

1. Evaluate each of the following indefinite integrals.
   (a) $\int 5x^5\ dx = 5 \times \frac{1}{6} + c = \frac{5}{6}x^6 + c$
   (b) $\int \cos x - \sin x\ dx = \sin x + \cos x + c$
   (c) $\int e^{3x}\ dx = \int e^{3x}\ dx = \int \frac{1}{3}e^{3x}\ d(3x) = \frac{1}{3}e^{3x} + c$
   (d) $\int 8x^5 + \sqrt[3]{x^2}\ dx = \int 8x^5 + x^{\frac{2}{3}}\ dx = 8 \times \frac{1}{6}x^6 + \frac{3}{5}x^{\frac{5}{3}} + c = \frac{4}{3}x^6 + \frac{3}{5}x^{\frac{5}{3}} + c$

2. Evaluate each of the following definite integrals.
   (a) $\int_{-3}^{1} 6x^2 - 5x + 2\ dx$
   $= (6 \times \frac{1}{3}x^3 - 5 \times \frac{1}{2}x^2 + 2x)|_{-3}^{1} = (2x^3 - \frac{5}{2}x^2 + 2x)|_{-3}^{1} = (2 - \frac{5}{2} + 2) - (-54 - \frac{45}{2} - 6) = 84$
   (b) $\int_0^{\frac{\pi}{3}} 2\sin\theta - 5\cos\theta d\theta$
   $= (2 \times (-\cos\theta) - 5\sin\theta)|_0^{\frac{\pi}{3}} = (-2\cos\frac{\pi}{3} - 5\sin\frac{\pi}{3}) - (-2\cos 0 - 5\sin 0)$
   $= -2 \times \frac{1}{2} - 5 \times \frac{\sqrt{3}}{2} - (-2) = -1 - \frac{5\sqrt{3}}{2} + 2 = 1 - \frac{5\sqrt{3}}{2}$

3. A stone was dropped off a cliff and hit the ground with a speed of 120 ft/s. What is the height of the cliff? (Hint: Acceleration of gravity= 32 ft/s, downward.)
   Ans:

   1
   When working with feet (ft), the acceleration due to gravity on earth at any time t is

   $a(t) = -32$ ft/s^2

   We use the convention that down is the negative direction.

   2
   The antiderivative of that is the velocity

   $v(t) = -32t + C$

   Since it was dropped, the initial velocity was 0, and if we let $t = 0$ seconds be the time it was dropped then $v(0) = 0$ and we solve for C

   $$0 = -32(0) + C$$
   $$C = 0$$

   So the velocity function is just $v(t) = -32t$ ft/s

   If the stone hits the ground at 120 ft/s then we can find the time of impact

   $-32t = -120$ (negative because moving downwards)

   $$t = \frac{-120}{-32} = 3.75 \text{ s}$$

   3
   The antiderivative of velocity is the position, $s(t)$, with another unknown constant C_2

   $$s(t) = -32 \cdot \frac{1}{2}t^2 + C_2$$
   $$= -16t^2 + C_2$$

   If we choose to make ground level be 0, then the position at time 3.75 s is $s(3.75) = 0$ ft. Use this to solve for C_2

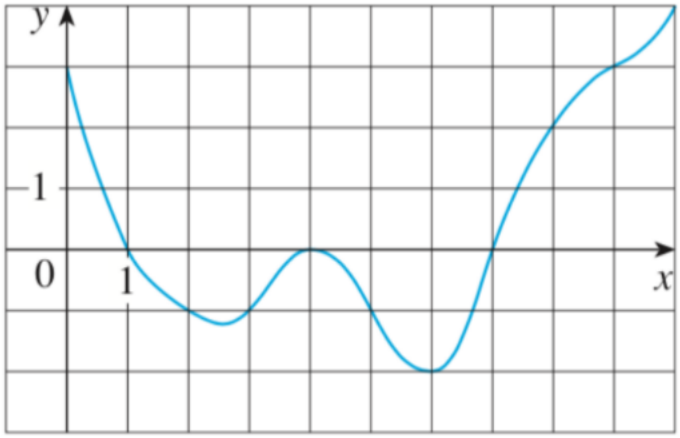
   $$0 = -16(3.75)^2 + C_2$$
   $$225 = C_2$$

   So the position function is $s(t) = -32t^2 + 225$

   The position at time $t = 0$ is the height of the cliff

   $$s(0) = -32(0)^2 + 225 = 225 \text{ ft}$$

4. The graph of a function f is given. Estimate $\int_0^{10} f(x)dx$ using five subintervals with (a) right endpoints, (b) left endpoints, and (c) midpoints.



Ans:

(a) $\displaystyle\int_0^{10} f(x)\,dx \approx R_5 = [f(2)+f(4)+f(6)+f(8)+f(10)]\,\Delta x$

$= [-1+0+(-2)+2+4](2) = 3(2) = 6$

(b) $\displaystyle\int_0^{10} f(x)\,dx \approx L_5 = [f(0)+f(2)+f(4)+f(6)+f(8)]\,\Delta x$

$= [3+(-1)+0+(-2)+2](2) = 2(2) = 4$

(c) $\displaystyle\int_0^{10} f(x)\,dx \approx M_5 = [f(1)+f(3)+f(5)+f(7)+f(9)]\,\Delta x$

$= [0+(-1)+(-1)+0+3](2) = 1(2) = 2$

5. Evaluate the integral by interpreting it in terms of area.

$$\int_0^{10} |x-5|dx$$

Ans:

$\int_0^{10} |x-5|\ dx$ can be interpreted as the sum of the areas of the two shaded triangles; that is, $2\left(\frac{1}{2}\right)(5)(5) = 25$.