Calculus(I) Quiz 1, Oct. 02, 2019

Please show <u>all work</u> and <u>simplify solutions</u>.

1. (16 points) Determine whether the statement is true or false.
   (a) If $f(s) = f(t)$, then $s = t$. False
   (b) If f is a function, then $f(3x) = 3f(x)$. False
   (c) If f and g are functions, then $f \circ g = g \circ f$. False
   (d) If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, then the domain of $g \circ f$ is the close internal $[0,4]$.True
   (e) If $\lim_{x\to c} f(x) = L_1$ and $\lim_{x\to c} g(x) = L_2$, then $\lim_{x\to c}\{f(x)+g(x)\} = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$. ($c$ is a constant and the limit exist) True
   (f) (Following the previous question) $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{L_1}{L_2}$. True or False
   (g) $\lim_{x\to 0}[\sin x] = 0$. ([] is floor function means $n \le x < n+1 \Rightarrow [x] = n,\ n \in \mathbb{Z}$, ex $[1.3] = 1, [2.4] = 2$)False
   (h) $\lim_{x\to 0}[4x - x^2] = 3$. False

2. (20 points) Evaluate the limits.
   (a) (10 points) $\lim_{x\to 1}\frac{x^4-1}{x-1} = \lim_{x\to 1}\frac{(x^2+1)(X^2-1)}{x-1} = \lim_{x\to 1}\frac{(x^2+1)(x+1)(x-1)}{x-1} = 4$
   (b) (10 points) $\lim_{x\to -1}\frac{x^3+1}{x^2-2x-3} = \lim_{x\to -1}\frac{(x+1)(x^2-x+1)}{(x+1)(x-3)} = \frac{3}{-4}$

3. (10 points) Find $\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{\sqrt{1+x}-\sqrt{1-x}}$.

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{\sqrt{1+x}-\sqrt{1-x}}$$
$$= \lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{\sqrt{1+x}-\sqrt{1-x}}\cdot\frac{\sqrt{1+x+x^2}+1}{\sqrt{1+x+x^2}+1}\cdot\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$$
$$= \lim_{x\to 0}\frac{x(1+x)\cdot(\sqrt{1+x}+\sqrt{1-x})}{2x\cdot(\sqrt{1+x+x^2}+1)}$$
$$= \lim_{x\to 0}\frac{(1+x)\cdot(\sqrt{1+x}+\sqrt{1-x})}{2\cdot(\sqrt{1+x+x^2}+1)} = \frac{2}{2\cdot 2} = \frac{1}{2}$$

4. (10 points) Find $\lim_{x\to 0} x sin\frac{1}{x}$. (using squeeze theorem)

$\because$ -1$\le \sin\frac{1}{x} \le 1$ $\therefore$ -x$\le x\sin\frac{1}{x} \le x$

$\because \lim_{x\to 0}(-x) = \lim_{x\to 0} x = 0$ $\therefore$ by the squeeze theorem $\lim_{x\to 0} x\sin\frac{1}{x} = 0$

5. (10 points) Find $\lim_{x\to 10^-}\frac{[x^3]-x^3}{[x]-x}$.

(Def: $n \le x < n+1$⇒[x]=n, n∈ $\mathbb{Z}$)

$\because \lim_{x\to 10^-}[x^3] = 999$

$\lim_{x \to 10^-} [x] = 9$

$\lim_{x \to 10^-} \frac{999 - x^3}{9 - x} = 1$

6. (18 points) If $f(x) = \begin{cases} x^2 & , \text{if } x < 0 \\ x & , \text{if } 0 \leq x < 2 \\ 2 - x & , \text{if } x \geq 2 \end{cases}$, Evaluate the limits

   (a) $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x = 1$

   (b) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x = 1$

   (c) $\lim_{x \to 1} f(x) = 1 \because \lim_{x \to 1^+} x = \lim_{x \to 1^-} x$

   (d) $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (2 - x) = 0$

   (e) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x = 2$

   (f) $\lim_{x \to 2} f(x) =$ do not exist $\because \lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x)$

7. (16 points) If $f(x) = [\![2x]\!] + [\![-x - 4]\!]$, show that $\lim_{x \to 4} f(x)$ exists, but is not equal to $f(4)$.

   Ans:

   $\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} [\![2x]\!] + [\![-x - 4]\!] = \lim_{x \to 4^+} 8 + \lim_{x \to 4^+} -9 = -1$

   $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} [\![2x]\!] + [\![-x - 4]\!] = \lim_{x \to 4^-} 7 + \lim_{x \to 4^-} -8 = -1$

   $\because \lim_{x \to 4^+} = \lim_{x \to 4^-}$, are equal

   $\therefore \lim_{x \to 4} f(x)$ is exist that $\lim_{x \to 4} f(x) = -1$

   but $f(4) = [\![2 * 4]\!] + [\![-4 - 4]\!] = 8 - 8 = 0$

   $f(4)$ is not equal $\lim_{x \to 4} f(x)$