Calculus(I) Quiz 3, Dec. 18, 2019

Please show all work and simplify solutions.

1. (10 points) Find f if $f''(x) = (3x-1)^2$, $f(0) = 3$, $f(1) = 5$.

   $f''(x) = (3x-1)^2 = 9x^2 - 6x + 1$

   $\Rightarrow f''(x) = 9 \times \frac{1}{2+1}x^{2+1} - 6 \times \frac{1}{1+1}x^{1+1} + x + c = 3x^3 - 3x^2 + x + c$

   $\Rightarrow f(x) = 3 \times \frac{1}{3+1}x^{3+1} - 3 \times \frac{1}{2+1}x^{2+1} + \frac{1}{1+1}x^{1+1} + (x+1) = \frac{3}{4}x^4 - x^3 + \frac{1}{2}x^2 + cx + D$

   $f(0) = D = 3$

   $f(1) = \frac{3}{4} - 1 + \frac{1}{2} + c + D = \frac{1}{4} + c + 3 = 5 \Rightarrow c = \frac{7}{4}$

   $\therefore f(x) = \frac{3}{4}x^4 - x^3 + \frac{1}{2}x^2 + \frac{7}{4}x + 3$

2. (20 points) Evaluate the following integrals:

   (a) $\int x^2(x^3 - 2x + 5)\ dx$

   $= \int x^5 - 2x^3 + 5x^2\ dx$

   $= \frac{1}{6}x^6 - 2 \times \frac{1}{4}x^4 + 5 \times \frac{1}{3}x^3 + c$

   $= \frac{1}{6}x^6 - \frac{1}{2}x^4 + \frac{5}{3}x^3 + c$

   (b) $\int \sqrt{x}\ dx$

   $= \int x^{\frac{1}{2}}\ dx$

   $= \frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} + c$

   $= \frac{2}{3}x^{\frac{3}{2}+c}$

   (c) $\int \sin x + \cos x\ dx$

   $= -\cos x + \sin x + c$

   (d) $\int \frac{x^2+2}{\sqrt{x}}\ dx$

   $= \int x^{2-\frac{1}{2}} + 2x^{\frac{-1}{2}}\ dx$

   $= \int x^{\frac{3}{2}} + 2x^{\frac{-1}{2}}\ dx$

   $= \frac{1}{\frac{3}{2}+1}x^{\frac{3}{2}+1} + 2 \times \frac{1}{\frac{-1}{2}+1}x^{\frac{-1}{2}+1} + c$

   $= \frac{2}{5}x^{\frac{5}{2}} + 2 \times 2x^{\frac{1}{2}} + c$

   $= \frac{2}{5}x^{\frac{5}{2}} + 4x^{\frac{1}{2}} + c$

3. (20 points) Evaluate the following integral by using the Substitution Rule.

   (a) $\int 6x^7\sqrt{3x^8+7}\ dx$

   If $u = 3x^8 + 7$, then $du = 24x^7\ dx$

   $\therefore \int 6x^7\sqrt{3x^8+7}\ dx$

   $= \int \frac{1}{4}\sqrt{u}\ du$

   $= \frac{1}{4} \times \frac{1}{\frac{1}{2}+1} \times u^{\frac{1}{2}+1} + c$

   $= \frac{1}{4} \times \frac{1}{\frac{3}{2}}u^{\frac{3}{2}} + c = \frac{1}{6}u^{\frac{3}{2}} + c = \frac{1}{6}(3x^8+7)^{\frac{3}{2}} + c$

(b) $\int \sec^2 x e^{\tan x}\ dx$

Let $u = \tan x,$ then $du = \sec^2 x\ dx$

$\therefore \int \sec^2 x e^{\tan x}\ dx$

$= \int e^u\ du = e^u + c = e^{\tan x} + c$

4. (10 points) Prove $\int \tan x\ dx = ln|\sec x| + c$ by using Substitution Rule.

$\int \tan x\ dx = \int \frac{\sin x}{\cos x}\ dx$

Let $u = \cos x$, then $du = -\sin x\ dx$

$\int \frac{\sin x}{\cos x} dx = \int -\frac{1}{u} du = -ln|u| + c = -ln|\cos x| + c = ln|\sec x| + c$

5. (20 points) Evaluate the following define integrals.

(a) $\int_1^4 \frac{dx}{\sqrt{x}}$

$= \int_1^4 x^{\frac{-1}{2}}\ dx = \frac{1}{\frac{-1}{2}+1} x^{\frac{-1}{2}+1}|_1^4 = 2x^{\frac{1}{2}}|_1^4 = 2\sqrt{4} - 2\sqrt{1} = 4 - 2 = 2$

(b) $\int_0^5\ -4x\ dx$

$= -4 \times \frac{1}{2}x^2|_0^5 = -2 \times 5^2 = -50$

(c) $\int_{-3}^4 \frac{8}{x^3}\ dx$

$= \int_0^4 \frac{8}{x^3}\ dx + \int_{-3}^0 \frac{8}{x^3} dx$

$= \frac{8}{-2} \times \frac{1}{x^2}|_0^4 + \frac{8}{-2} \times \frac{1}{x^2}|_{-3}^0$

$= -4 \times \frac{1}{x^2}|_0^4 + (-4)\frac{1}{x^2}|_{-3}^0$

$= -4 \times \frac{1}{16} - (-\infty) + (-\infty) - (-4) \times \frac{1}{(-3)^2}$

$\Rightarrow$ Divergence

$\therefore \int_{-3}^4 \frac{8}{x^3} dx$ does not exist.

(d) $\int_0^{2\pi}\ |\sin x|\ dx$

$\Rightarrow \int_0^{2\pi} |\sin x|\ dx = 4\int_0^{\frac{\pi}{2}} \sin x\ dx$

$= 4 \times (-\cos x)|_0^{\frac{\pi}{2}} = -4 \times (\cos\frac{\pi}{2} - \cos 0) = -4 \times (-1) = 4$

6. (10 points) The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these three seconds.

t(s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v(m/s)	0	1.9	3.3	4.5	5.5	5.9	6.2

Lower Estimates: L6 $= 0.5 \times (0 + 1.9 + 3.3 + 4.5 + 5.5 + 5.9) = 10.55m$

Upper Estimates: U6 $= 0.5 \times (1.9 + 3.3 + 4.5 + 5.5 + 5.9 + 6.2) = 13.65m$

7. (10 points) We know $\int_1^3 f(x)dx = 2$, $\int_1^2 g(x)dx = 3$, $\int_3^5 f(x)dx = 7$, and $\int_2^5 g(x)dx = 5$. Try to find the value of $\int_1^5 (2f(x) - 3g(x))dx$.

$\because \int_1^5 f(x)dx = \int_3^5 f(x)dx + \int_1^3 f(x)dx = 7 + 2 = 9$

$\int_1^5 g(x)dx = \int_2^5 g(x)dx + \int_1^2 g(x)dx = 5 + 3 = 8$

$\therefore \int_1^5 2f(x) - 3g(x)\ dx = 2\int_1^5 f(x) - 3\int g(x)dx = 2 \times 9 - 3 \times 8 = 18 - 24 = -6$