Q1 2-1 3, 5

3. Using (1) with $f(x) = 4x - 3x^2$ and $P(2, -4)$ [we could also use (2)],

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to 2} \frac{(4x - 3x^2) - (-4)}{x - 2} = \lim_{x \to 2} \frac{-3x^2 + 4x + 4}{x - 2}$$

$$= \lim_{x \to 2} \frac{(-3x - 2)(x - 2)}{x - 2} = \lim_{x \to 2} (-3x - 2) = -3(2) - 2 = -8$$

5. Using (1), $m = \lim_{x \to 1} \frac{\sqrt{x} - \sqrt{1}}{x - 1} = \lim_{x \to 1} \frac{(\sqrt{x} - 1)(\sqrt{x} + 1)}{(x - 1)(\sqrt{x} + 1)} = \lim_{x \to 1} \frac{x - 1}{(x - 1)(\sqrt{x} + 1)} = \lim_{x \to 1} \frac{1}{\sqrt{x} + 1} = \frac{1}{2}$.

Tangent line: $y - 1 = \frac{1}{2}(x - 1) \quad \Leftrightarrow \quad y = \frac{1}{2}x + \frac{1}{2}$

Q2 2-1 27,28

27. Use (4) with $f(t) = (2t+1)/(t+3)$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\dfrac{2(a+h)+1}{(a+h)+3} - \dfrac{2a+1}{a+3}}{h} = \lim_{h\to 0} \frac{(2a+2h+1)(a+3)-(2a+1)(a+h+3)}{h(a+h+3)(a+3)}$$

$$= \lim_{h\to 0} \frac{(2a^2+6a+2ah+6h+a+3)-(2a^2+2ah+6a+a+h+3)}{h(a+h+3)(a+3)}$$

$$= \lim_{h\to 0} \frac{5h}{h(a+h+3)(a+3)} = \lim_{h\to 0} \frac{5}{(a+h+3)(a+3)} = \frac{5}{(a+3)^2}$$

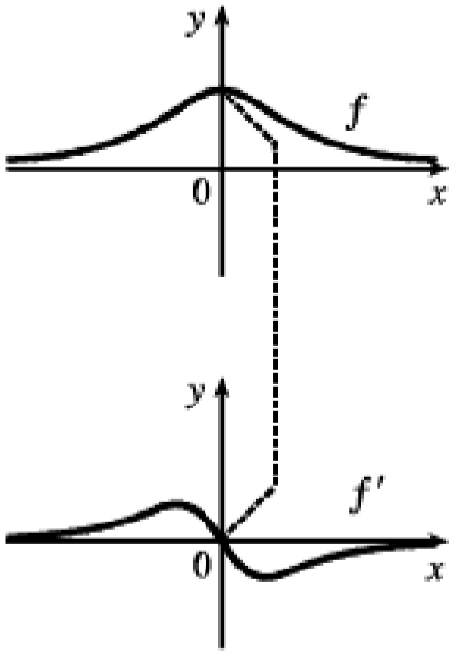
28. Use (4) with $f(x) = x^{-2} = 1/x^2$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\dfrac{1}{(a+h)^2} - \dfrac{1}{a^2}}{h} = \lim_{h\to 0} \frac{\dfrac{a^2-(a+h)^2}{a^2(a+h)^2}}{h} = \lim_{h\to 0} \frac{a^2-(a^2+2ah+h^2)}{ha^2(a+h)^2}$$

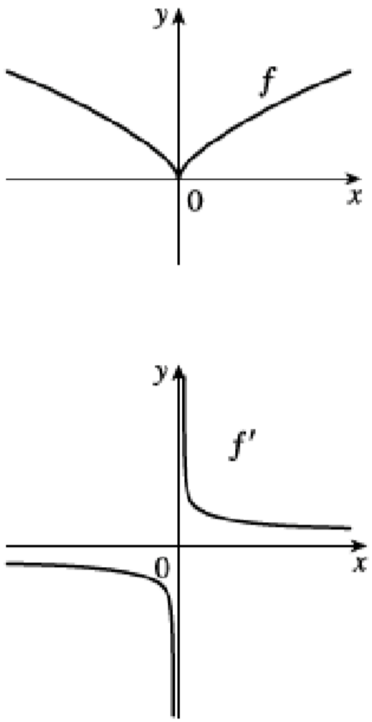
$$= \lim_{h\to 0} \frac{-2ah-h^2}{ha^2(a+h)^2} = \lim_{h\to 0} \frac{h(-2a-h)}{ha^2(a+h)^2} = \lim_{h\to 0} \frac{-2a-h}{a^2(a+h)^2} = \frac{-2a}{a^2(a^2)} = \frac{-2}{a^3}$$

Q3 2-2 6, 8, 9

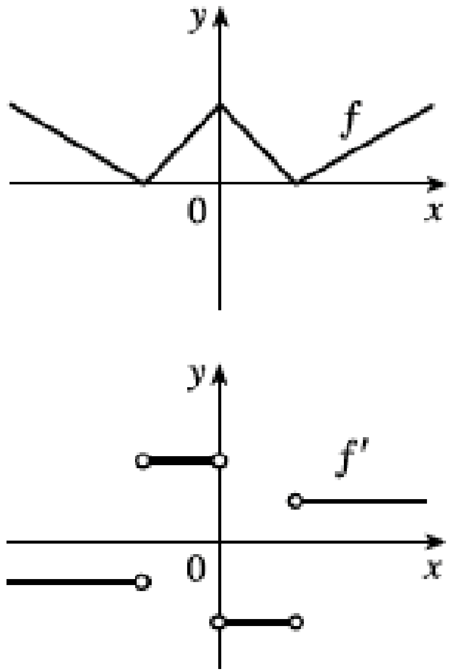
6.



8.

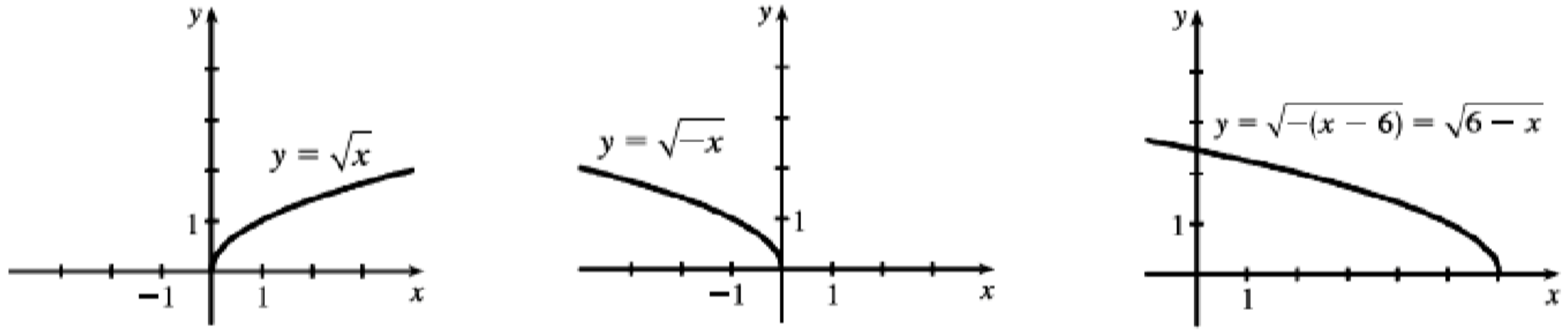


9.



Q4 2-2 28

28. (a)



Q5 2-3 8, 10, 22

8. $y = \sin t + \pi \cos t \quad \Rightarrow \quad y' = \cos t + \pi(-\sin t) = \cos t - \pi \sin t$

10. $h(x) = (x-2)(2x+3) = 2x^2 - x - 6 \quad \Rightarrow \quad h'(x) = 2(2x) - 1 - 0 = 4x - 1$

22. $y = \dfrac{x^2 - 2\sqrt{x}}{x} = x - 2x^{-1/2} \quad \Rightarrow \quad y' = 1 - 2\left(-\tfrac{1}{2}\right)x^{-3/2} = 1 + 1/(x\sqrt{x})$

Q6 2-4 2

2. Quotient Rule: $F(x) = \dfrac{x - 3x\sqrt{x}}{\sqrt{x}} = \dfrac{x - 3x^{3/2}}{x^{1/2}} \quad \Rightarrow$

$$F'(x) = \frac{x^{1/2}\left(1 - \frac{9}{2}x^{1/2}\right) - \left(x - 3x^{3/2}\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}\right)^2} = \frac{x^{1/2} - \frac{9}{2}x - \frac{1}{2}x^{1/2} + \frac{3}{2}x}{x} = \frac{\frac{1}{2}x^{1/2} - 3x}{x} = \frac{1}{2}x^{-1/2} - 3$$

Simplifying first: $F(x) = \dfrac{x - 3x\sqrt{x}}{\sqrt{x}} = \sqrt{x} - 3x = x^{1/2} - 3x \quad \Rightarrow \quad F'(x) = \frac{1}{2}x^{-1/2} - 3$ (equivalent).

For this problem, simplifying first seems to be the better method.

Q7 2-4 4, 17

4. $f(x) = \sqrt{x}\sin x \quad \Rightarrow \quad f'(x) = \sqrt{x}\cos x + \sin x\left(\frac{1}{2}x^{-1/2}\right) = \sqrt{x}\cos x + \dfrac{\sin x}{2\sqrt{x}}$

17. $f(t) = \dfrac{2t}{2+\sqrt{t}} \quad \overset{\text{QR}}{\Rightarrow} \quad f'(t) = \dfrac{(2+t^{1/2})(2) - 2t\left(\frac{1}{2}t^{-1/2}\right)}{(2+\sqrt{t})^2} = \dfrac{4+2t^{1/2}-t^{1/2}}{(2+\sqrt{t})^2} = \dfrac{4+t^{1/2}}{(2+\sqrt{t})^2}$ or $\dfrac{4+\sqrt{t}}{(2+\sqrt{t})^2}$

Q8 2-4 28

28. $y = \dfrac{\sqrt{x}}{x+1} \;\Rightarrow\; y' = \dfrac{(x+1)\left(\dfrac{1}{2\sqrt{x}}\right) - \sqrt{x}\,(1)}{(x+1)^2} = \dfrac{(x+1)-(2x)}{2\sqrt{x}\,(x+1)^2} = \dfrac{1-x}{2\sqrt{x}\,(x+1)^2}$. At $(4, 0.4)$, $y' = \frac{-3}{100} = -0.03$, and an equation of the tangent line is $y - 0.4 = -0.03(x-4)$, or $y = -0.03x + 0.52$.

Q9 2-5 2, 6

2. Let $u = g(x) = 2x^3 + 5$ and $y = f(u) = u^4$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (4u^3)(6x^2) = 24x^2(2x^3 + 5)^3$.

6. Let $u = g(x) = \sqrt{x}$ and $y = f(u) = \sin u$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (\cos u)\left(\frac{1}{2}x^{-1/2}\right) = \dfrac{\cos u}{2\sqrt{x}} = \dfrac{\cos\sqrt{x}}{2\sqrt{x}}$.

Q10 2-5 17, 24, 36

17. $f(x) = (2x-3)^4(x^2+x+1)^5 \quad \Rightarrow$

$$f'(x) = (2x-3)^4 \cdot 5(x^2+x+1)^4(2x+1) + (x^2+x+1)^5 \cdot 4(2x-3)^3 \cdot 2$$

$$= (2x-3)^3(x^2+x+1)^4[(2x-3)\cdot 5(2x+1) + (x^2+x+1)\cdot 8]$$

$$= (2x-3)^3(x^2+x+1)^4(20x^2-20x-15+8x^2+8x+8) = (2x-3)^3(x^2+x+1)^4(28x^2-12x-7)$$

24. $f(x) = \dfrac{x}{\sqrt{7-3x}} \quad \Rightarrow$

$$f'(x) = \frac{\sqrt{7-3x}(1) - x\cdot\frac{1}{2}(7-3x)^{-1/2}\cdot(-3)}{(\sqrt{7-3x})^2} = \frac{\sqrt{7-3x} + \dfrac{3x}{2\sqrt{7-3x}}}{(7-3x)^1}$$

$$= \frac{2(7-3x)+3x}{2(7-3x)^{3/2}} = \frac{14-3x}{2(7-3x)^{3/2}}$$

36. $y = \sin(\sin(\sin x)) \quad \Rightarrow \quad y' = \cos(\sin(\sin x))\,\dfrac{d}{dx}(\sin(\sin x)) = \cos(\sin(\sin x))\,\cos(\sin x)\,\cos x$

Q11 2-5 58

58. (a) $h(x) = f(f(x)) \quad \Rightarrow \quad h'(x) = f'(f(x))f'(x)$. So $h'(2) = f'(f(2))f'(2) = f'(1)f'(2) \approx (-1)(-1) = 1$.

(b) $g(x) = f(x^2) \quad \Rightarrow \quad g'(x) = f'(x^2) \cdot \dfrac{d}{dx}(x^2) = f'(x^2)(2x)$. So $g'(2) = f'(2^2)(2 \cdot 2) = 4f'(4) \approx 4(2) = 8$.