Calculus(I)
Homework 1, Sep, 11, 2019
Deadline: Sep, 20, 2019

1. $\lim_{x\to 2}(x^2+1) = ?$

   Ans : 5

2. $f(x) = 2x^3 + 7x + 2, Find\ f'(x).$

   Ans : $6x^2 + 7$

3. If $f(x) = \sin x,\ g(x) = \cos x,\ h(x) = \tan x,\ Find\ f'(x),\ g'(x),\ h'(x).$

   Ans : $f'(x) = \cos x,\ g'(x) = -\sin x,\ h'(x) = \sec^2 x.$

4. Let $Sn = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ... + \frac{1}{2^{n-1}} + \frac{1}{2^n}$

   (a) Find Sn.

   Ans : $\because \sum_{k=1}^{n} a_k = a_1 + a_2 + ... + a_n$

   $= a_1 + a_1 r + a_1 r^2 + ... + a_1 r^{n-1}, r\ is\ common\ ratio$

   $= \frac{a_1(1-r^n)}{1-r}, r \neq 1$

   $\therefore a_1 = \frac{1}{2}, r = \frac{1}{2}, S_n = \frac{\frac{1}{2}[1-(\frac{1}{2})^n]}{1-\frac{1}{2}} = 1 - (\frac{1}{2})^n$

   (b) Find $\lim_{n\to\infty} Sn$.

   Ans : $\lim_{n\to\infty} S_n = \lim_{n\to\infty}[1 - (\frac{1}{2})^n] = 1 - 0 = 1$

5. Find the domain of the function

   (a) $f(x) = \frac{x+4}{x^2-9}$

   Ans : $\because x^2 - 9 \neq 0, \therefore x \neq \pm 3,$

   $\therefore f(x)\ domain = \{x | x \in R, x \neq \pm 3\}$

   (b) $g(t) = \sqrt{3-t} - \sqrt{2+t}$

   Ans : $\because 3\text{-}t \geq 0\ and\ 2 + t \geq 0 \Rightarrow t \leq 3\ and\ t \geq -2$

   $\therefore g(t)\ domain = \{t | t \in R, -2 \leq t \leq 3\}$

6. If $f(x) = x + \frac{1}{x}, g(x) = \frac{x+1}{x+2}$
   Find the function (a)$f \circ g$, (b)$g \circ f$, (c)$f \circ f$, (d)$g \circ g$ and their domains.

   (a) $f \circ g = f(g(x)) = f(\frac{x+1}{x+2}) = \frac{x+1}{x+2} + \frac{x+2}{x+1} = \frac{(x+1)^2+(x+2)^2}{(x+2)(x+1)}$

   $x \neq -2, and\ x \neq -1,\ domain = \{x | x \in R, x \neq -2, -1\}$

(b) $g \circ f = g(f(x)) = g(x + \frac{1}{x}) = \frac{x+\frac{1}{x}+1}{x+\frac{1}{x}+2} = \frac{x^2+x+1}{x^2+2x+1} = \frac{x^2+x+1}{(x+1)^2}$

$x \neq 0, and\ x \neq -1,\ domain = \{x | x \in R, x \neq\ 0,\ -1\}$

(c) $f \circ f = f(f(x)) = f(x + \frac{1}{x}) = \frac{x^2+1}{x} + \frac{x}{x^2+1} = \frac{x^4+3x^2+1}{x(x^2+1)}$

$x \neq 0,\ domain = \{x | x \in R, x \neq\ 0\}$

(d) $g \circ g = g(g(x)) = g(\frac{x+1}{x+2}) = \frac{2x+3}{3x+5}$

$x \neq -2, and\ x \neq \frac{-5}{3},\ domain = \{x | x \in R, x \neq\ -2,\ \frac{-5}{3}\}$

7. If $f(x) = \frac{6x}{x^2-9}$ *and* $g(x) = \sqrt{3x}$
Find $f(g(12))$ *value and* $f(g(x))$ *domain*

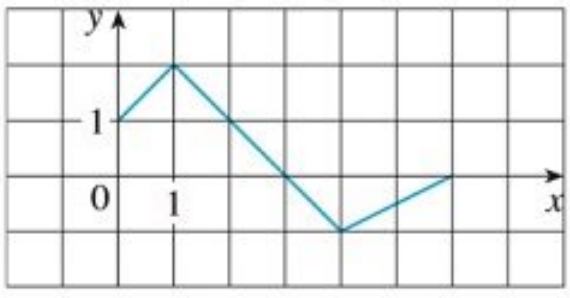
Ans : $f(g(x)) = f(\sqrt{36}) = f(6) = \frac{4}{3}, domain = \{x | x \in R, x \neq \pm 3\}$

8. (Find a formula for the described function and state its domain) A rectangle has area $16m^2$. Express the perimeter of the rectangle as a function of the length of one of its sides.

Ans : if $y = \frac{16}{x}, p = 2x + 2y = \frac{2x^2+32}{x}, domain = \{x | x \in R, x \neq 0\}$

9. The graph of f is given, Use it to graph the following functions.

(a) $y = f(2x)$ (b) $y = f(\frac{1}{2}x)$
(c) $y = f(-x)$ (d) $y = -f(-x)$



10. In multiple expansion of $(2x - y^2)^6$, find the coefficient of x^4y^4.

Ans : $C_2^6(2x)^4(-y^2)^2 = 240x^4y^4$

11. what are your expectation for yourself in Calculus class?