Calculus(I) Quiz 2, Nov. 13, 2019

Please show <u>all work</u> and <u>simplify solutions</u>.

1. (10 points) Find an equation of the tangent line to the curve at the given point.

(a) $f(x) = 2x^2 - 5$; $(1, -3)$

$f'(x) = 4x \to f'(1) = 4$

$y - (-3) = 4(x - 1)$

$y + 3 = 4x - 4$

$\boldsymbol{y - 4x + 7 = 0.}$

(b) $f(x) = \sqrt[3]{x}$; $(8, 2)$

$f'^{(x)} = \frac{1}{3}x^{\frac{-2}{3}} \to f'(8) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

$y - 2 = \frac{1}{12}(x - 8)$

$12y - 24 = x - 8$

$\boldsymbol{12y - x - 16 = 0.}$

2. (10 points) Find $f'(x)$.

(a) $f(x) = -8$

$\boldsymbol{f'(x) = 0.}$

(b) $f(x) = \sqrt{7}$

$\boldsymbol{f'(x) = 0.}$

3. (10 points) Find $\frac{dy}{dx}$.

(a) $y = (\sin x)(x^4 + x)$

$$\frac{dy}{dx} = (\sin x)'(x^4 + x) + (\sin x)(x^4 + x)'$$

$$= \boldsymbol{\cos x\,(x^4 + x) + \sin x\,(4x^3 + 1).}$$

(b) $y = \frac{5x-3}{x^2+1}$

$$\frac{dy}{dx} = \frac{(5x-3)'(x^2+1) - (5x-3)(x^2+1)'}{(x^2+1)^2}$$

$$= \frac{5(x^2+1) - 2x(5x-3)}{(x^2+1)^2} = \frac{5x^2+5-10x^2+6x}{(x^2+1)^2} = \boldsymbol{\frac{-5x^2+6x+5}{(x^2+1)^2}}.$$

4. (10 points) Find $\frac{d^{87}(\sin x+\cos x)}{dx^{87}}$.

$$\frac{d}{dx}(\sin x+\cos x)=\cos x-\sin x$$

$$\frac{d^2}{dx^2}(\sin x+\cos x)=-\sin x-\cos x$$

$$\frac{d^3}{dx^3}(\sin x+\cos x)=-\cos x+\sin x$$

$$\frac{d^4}{dx^4}(\sin x+\cos x)=\sin x+\cos x$$

$\because 87\ \%\ 4=3\quad \therefore \frac{d^{87}(\sin x+\cos x)}{dx^{87}}=\boldsymbol{-\cos x+\sin x}$.

5. (10 points) If $y=(x^3-2x^2+1)^5$, find $\frac{dy}{dx}$.

$$\frac{dy}{dx}=5(x^3-2x^2+1)^4(x^3-2x^2+1)'$$
$$=\boldsymbol{5(x^3-2x^2+1)^4(3x^2-4x).}$$

6. (10 points) If $y=\sin(\tan x)$, find $\frac{dy}{dx}$.

$$\frac{dy}{dx}=\frac{d(\sin(\tan x))}{d\tan x}\cdot\frac{d\tan x}{dx}$$
$$=\boldsymbol{\cos(\tan x)\cdot\sec^2 x.}$$

7. (10 points) If $y=(x^2+1)\frac{3x^5}{x^2-1}$, and $\frac{dy}{dx}=\frac{ax^8-bx^6-cx^4}{(x^2-1)^2}$, find a, b, c.

$$\frac{dy}{dx}=(x^2+1)'\left(\frac{3x^5}{x^2-1}\right)+(x^2+1)\left(\frac{3x^5}{x^2-1}\right)'$$
$$=2x\left(\frac{3x^5}{x^2-1}\right)+(x^2+1)\left[\frac{(3x^5)'(x^2-1)-(3x^5)(x^2-1)'}{(x^2-1)^2}\right]$$
$$=\frac{6x^6}{x^2-1}+(x^2+1)\left[\frac{(15x^4)(x^2-1)-(3x^5)(2x)}{(x^2-1)^2}\right]$$
$$=\frac{6x^6(x^2-1)+(15x^4)(x^4-1)-6x^6(x^2+1)}{(x^2-1)^2}$$
$$=\frac{6x^8-6x^6+15x^8-15x^4-6x^8-6x^6}{(x^2-1)^2}$$
$$=\frac{15x^8-12x^6-15x^4}{(x^2-1)^2}\qquad \therefore \boldsymbol{a=15, b=12, c=15.}$$

8. (10 points) If $y = \sqrt[3]{x+y}$, find $\frac{dy}{dx}$.

$$y^3 = x + y$$

$$\frac{dy^3}{dx} = \frac{d(x+y)}{dx}$$

$$\frac{dy^3}{dx} \times \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$3y^2 \times \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$(3y^2 - 1)\frac{dy}{dx} = 1 \rightarrow \frac{dy}{dx} = \frac{\mathbf{1}}{\mathbf{3y^2 - 1}}.$$

9. (10 points) If $y^2 + 2y = x^2$, $y'' = \frac{b}{(y+a)^3}$, find a, b. (Hint: a, b is constant)

$$\frac{d}{dx}(y^2 + 2y) = \frac{dx^2}{dx}$$

$$\frac{dy^2}{dx} + \frac{d(2y)}{dx} = 2x$$

$$2y \times \frac{dy}{dx} + 2\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{2x}{2y+2} = \frac{x}{y+1}$$

$$y'' = \frac{d^2y}{dx} = \frac{d}{dx}\left(\frac{x}{y+1}\right) = \frac{1(y+1) - x(y+1)\prime}{(y+1)^2} = \frac{y+1-x\frac{dy}{dx}}{(y+1)^2} = \frac{y+1-x\left(\frac{x}{y+1}\right)}{(y+1)^2} = \frac{(y+1)^2 - x^2}{(y+1)^3}$$

$$= \frac{y^2 + 2y + 1 - x^2}{(y+1)^3} = \frac{(y^2 - x^2) + 2y + 1}{(y+1)^3} = \frac{1}{(y+1)^3}$$

$\therefore \boldsymbol{a = 1, b = 1.}$

10. (10 points) Two cars start moving from the same point. One travels south at 30km/h and the other travels west at 72 km/h. At what rate is the distance between the cars increasing two hours later?

If South Travel $= 30$ km/h $\Rightarrow S = 60$ km (two hours later)

If West Travel $= 72$ km/h $\Rightarrow W = 144$ km (two hours later)

Let D = the distance between the 2 cars.

The travel distance triangle is $W^2 + S^2 = D^2$, then $2D\frac{dD}{dt} = 2W\frac{dW}{dt} + 2S\frac{dS}{dt}$

$$D^2 = 144^2 + 60^2 \rightarrow D = 156, W = 144, \frac{dW}{dt} = 72, S = 60, \frac{dS}{dt} = 30$$

$$2D\frac{dD}{dt} = 2W\frac{dW}{dt} + 2S\frac{dS}{dt} \quad \longrightarrow \quad 2 \times 156 \times \frac{dD}{dt} = 2 \times 144 \times 72 + 2 \times 60 \times 30$$

$\therefore \frac{dD}{dt} = \frac{(60\times30)+(144\times72)}{156} = \mathbf{78}$ **km/h.**