Calculus(II) Quiz 1, Mar. 19, 2020

Please show <u>all work</u> and <u>simplify solutions</u>.

1. (30 points) Evaluate the following integrals:

(a) $\int \frac{1-\tan^2 x}{\sec^2 x}\, dx$.

$\Rightarrow \int (\frac{1}{\sec^2 x} - \frac{\tan^2 x}{\sec^2 x})\, dx = \int \cos^2 x - \frac{\sin^2 x}{\cos^2 x}\cos^2 x\, dx$

$\int \cos^2 x - \sin^2 x\, dx$, know $\cos 2x = \cos^2 x - \sin^2 x$

so $\int \cos^2 x - \sin^2 x\, dx = \int \cos 2x\, dx = \frac{1}{2}\sin 2x + c$.

(b) $\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\, dy$.

$\overset{kt.}{\Rightarrow} \frac{A}{y} + \frac{B}{y+2} + \frac{C}{y-3} = \frac{4y^2-7y-12}{y(y+2)(y-3)}$

calculate... $\Rightarrow\ A = 2,\ B = \frac{9}{5},\ C = \frac{1}{5}$

$\int_1^2 \frac{2}{y}\, dy + \int_1^2 \frac{9}{5} \times \frac{1}{y+2}\, dy + \int_1^2 \frac{1}{5} \times \frac{1}{y-3}\, dy$

$\Rightarrow \Big[2\ln|y| + \frac{9}{5}\ln|y+2| + \frac{1}{5}\ln|y-3|\Big]_1^2$

$= \frac{27}{5}\ln 2 - \frac{9}{5}\ln 3$ or $(\frac{9}{5}\ln|\frac{8}{3}|)$.

(c) $\int \frac{e^{2x}}{e^{2x}+3e^x+2}\, dx$.

kt. $u = e^x,\ du = e^x\, dx,\ so \Rightarrow \int \frac{u\, du}{u^2+3u+2} = \int \frac{u\, du}{(u+1)(u+2)}$

$\frac{u}{(u+1)(u+2)} = \frac{A}{u+1} + \frac{B}{u+2} \xrightarrow[\text{calculate}]{} A = -1,\ B = 2$

so $\frac{u}{(u+1)(u+2)} = \frac{-1}{u+1} + \frac{2}{u+2}$

$\int \frac{u\, du}{(u+1)(u+2)} = \int \frac{-1}{u+1} + \frac{2}{u+2}\, du = -\ln|u+1| + 2\ln|u+2| + C$

$= \int \frac{(u+2)^2}{|u+1|} + C \Rightarrow \ln\frac{(e^x+2)^2}{e^x+1} + C.$

2. (10 points) $\int t^3 e^{-t^2}\ dt$.

Let $u = -t^2,\ du = -2t\ dt \Rightarrow dt = \frac{du}{-2t}$

$\int t^3 e^{-t^2}\ dt = \int t^3 e^u \frac{du}{-2t} = \int -\frac{1}{2} t^2 e^u\ du = \int \frac{1}{2} u e^u\ du$

$\int \frac{1}{2} u e^u\ du = \frac{1}{2} u e^u - \int \frac{1}{2} e^u\ du = \frac{1}{2} u e^u - \frac{1}{2} e^u + C$

$\Rightarrow \int t^3 e^{-t^2}\ dt = \frac{1}{2}(-t^2 e^{-t^2} - e^{-t^2}) + C = -\frac{e^{-t^2}(t^2+1)}{2} + C.$

3. (10 points) $\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^5\theta\ d\theta$

$= \int_0^{\frac{\pi}{2}} \sin^7\theta \cos^4\theta \cos\theta\ d\theta$

$\cos^2\theta = 1 - \sin^2\theta$, so $\int_0^{\frac{\pi}{2}} \sin^7\theta (1-\sin^2\theta)^2 \cos\theta\ d\theta$

Let $u = \sin\theta,\ du = \cos\theta\ d\theta$

$\int_0^{\frac{\pi}{2}} \sin^7\theta \cos^5\theta\ d\theta = \int_{\sin(0)}^{\sin(\frac{\pi}{2})} \sin^7\theta \cos^5\theta\ d\theta = \int_0^1\ u^7(1-u^2)^2\ du$

$= \int_0^1\ u^7 - 2u^9 + u^{11}\ du = (\frac{u^8}{8} - \frac{2u^{10}}{10} + \frac{u^{12}}{12})_0^1 = (\frac{1}{8} - \frac{2}{10} + \frac{1}{12}) - 0 = \frac{1}{120}.$

4. (10 points) $\int \frac{dx}{x^2\sqrt{4-x^2}},\ x = 2\sin\theta.$

$\Rightarrow\ dx = 2\cos\theta\ d\theta$

$\sqrt{4-x^2} = \sqrt{4-(2\sin\theta)^2} = \sqrt{4-4\sin^2\theta} = \sqrt{4-cos^2\theta} = 2\cos\theta$

$\int \frac{dx}{x^2\sqrt{4-x^2}} = \int \frac{2\cos\theta\ d\theta}{(2\sin\theta)^2(2\cos\theta)} = \frac{1}{4}\int \frac{d\theta}{\sin^2\theta} = -\frac{1}{4}\cot\theta + C$

$\because x = 2\sin\theta,\ \cot\theta = \frac{\sqrt{4-x^2}}{x},\ \therefore\ \int \frac{dx}{x^2\sqrt{4-x^2}} = -\frac{\sqrt{4-x^2}}{4x} + C.$

5. (10 points) $\int \sin^3 x \cos^2 x \ dx$.

$= \int \sin x(1-\cos^2 x)\cos^2 \ dx = \int \sin x(\cos^2 x - \cos^4 x) \ dx$

$= -\int \cos^2 x - \cos^4 x \ d\cos x$

Let $u = \cos x$

$= -\int u^2 - u^4 \ du = -(\frac{1}{3}u^3 - \frac{1}{5}u^5) + C = -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C.$

6. (20 points) Determine if the following integral converges or diverges?

If the integral converges determine its value.

(a) $\int_{-5}^{1} \frac{1}{10+2x} \ dx$.

$$= \lim_{t\to\ -5^+} \int_t^1 \frac{1}{10+2x} \ dx = \lim_{t\to\ -5^+} \left(\frac{1}{2}\ln|10+2x|\Big|_t^1\right)$$

$$= \lim_{t\to\ -5^+} \left(\frac{1}{2}\ln|12| - \frac{1}{2}\ln|10+2t|\right) = \frac{1}{2}\ln|12| + \infty = \infty$$

Ans:diverges.

(b) $\int_2^{\infty} \frac{9}{(1-3x)^4} \ dx$.

$$= \lim_{t\to\infty} \int_2^t \frac{9}{(1-3x)^4} \ dx = \lim_{t\to\infty} \frac{1}{(1-3x)^3}\Big|_2^t$$

$$= \lim_{t\to\infty} \left[\frac{1}{(1-3t)^3} - \left(\frac{1}{-125}\right)\right] = \frac{1}{125}.$$

7. (10 points) $\int_0^2 \frac{1}{1+x^6}\, dx,\ n = 8$. Use the Trapezoidal Rule to approximate the given integral with the specified value of n.(Round answers to six decimal places.)

$f(x) = \frac{1}{1+x^6}$

$f(0) = 1$

$f(0.25) = 0.999756$

$f(0.5) = 0.984615$

$f(0.75) = 0.848912$

$f(1) = 0.5$

$f(1.25) = 0.207697$

$f(1.5) = 0.0807062$

$f(1.75) = 0.0336441$

$f(2) = 0.0153846$

$$\Delta x = \frac{2-0}{8} = 0.25$$

$$\int_0^2 \frac{1}{1+x^6} dx \approx \frac{1/4}{2}[1 + 2(0.999756) + 2(0.984615) + 2(0.848912) + 2(0.5) + 2(0.207697) + 2(0.0807062) + 2(0.0336441) + (0.0153846)] \approx \frac{1}{8}(8.3260452) \approx 1.040756.$$