

# Data Structures Midterm

2020/11/18

# Multiple Choices

拿到考卷後 請先確認選擇題是否批改正確、分數是否計算正確

1	B
2	B
3	E
4	B, C
5	B, D
6	A, B, C, E
7	C
8	A
9	A, D

# 1. Padovan Sequence

$$P_n(x) = \begin{cases} 1, & \text{if } n = 0 \\ x, & \text{if } n = 1 \\ x^2, & \text{if } n = 2 \\ xP_{n-2}(x) + P_{n-3}(x), & \text{if } n \geq 3 \end{cases}$$

$$n = 0 \quad 1$$

$$n = 1 \quad 4$$

$$n = 2 \quad 16$$

$$n = 3 \quad 4 \times 4 + 1 = 17$$

$$n = 4 \quad 4 \times 16 + 4 = 68$$

$$n = 5 \quad 4 \times 17 + 16 = 84$$

$$n = 6 \quad 4 \times 68 + 17 = 289$$

$$n = 7 \quad 4 \times 84 + 68 = 404 \text{ (B)}$$

## 2. B-Tree

A B-tree of order 4 and of height 3 will have a maximum of k keys. What is the value of k? (root = height 1)

height 1(root): 3

height 2:  $3 \times 4 = 12$

height3:  $3 \times 4 \times 4 = 48$

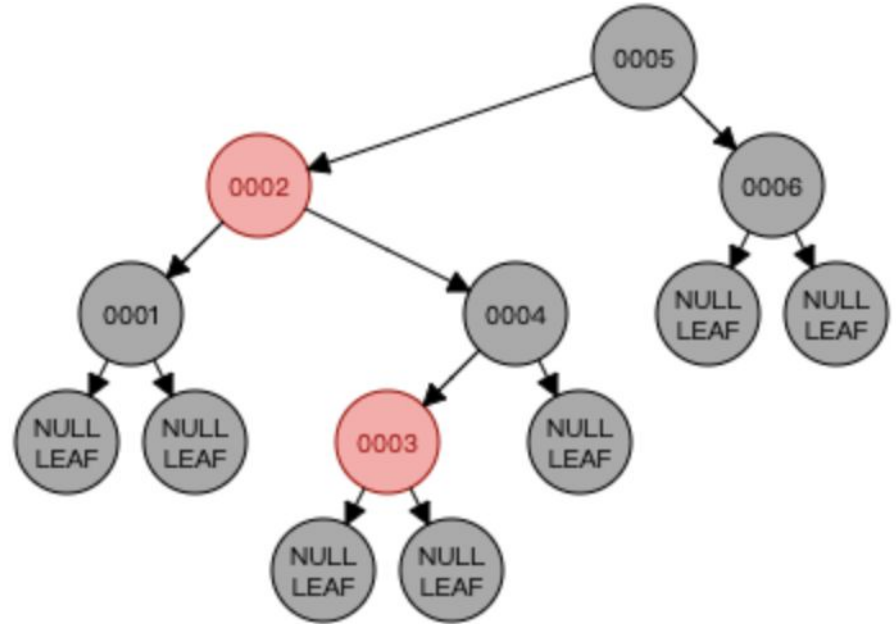
$\Rightarrow 3 + 12 + 48 = 63$  (B)

## \*3, 4: Red-Black Tree

Insert 6, 5, 4, 1, 2, 3 in an empty red black tree in order.

3. height: 5 (E)

4. Red nodes: 2 (B) & 3 (C)



## 5. Notations

Little-Oh是嚴格邊界

(A)  $n = o(8n)$  X

(B)  $2^n = o(4^n)$  O

(C)  $2^n = o(n^3)$  X

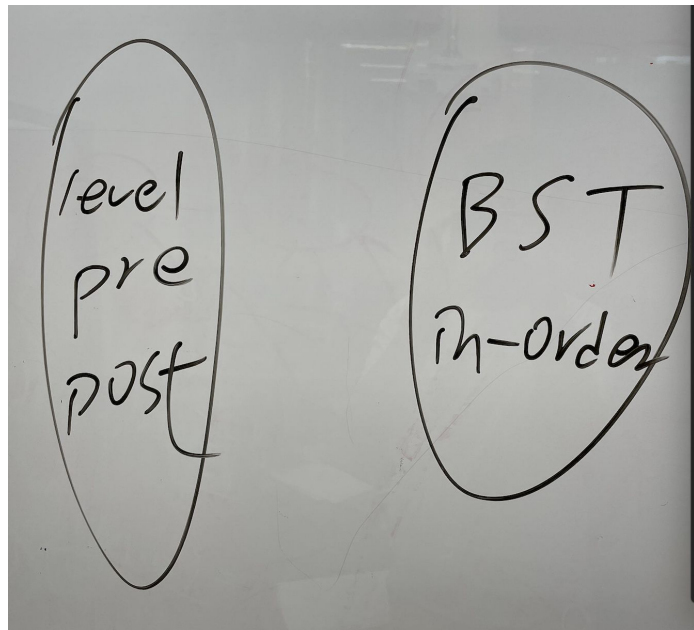
(D)  $1 = o(n^2)$  O

(E)  $n = o(\log n)$  X

## \*6. Binary Tree

Which of the following selections offer you enough information to build an unique Binary Tree?

- (A) pre-order + in-order
  - (B) in-order + level-order
  - (C) The tree is a completed binary tree + post-order
  - (D) The tree is a Binary Search Tree + in-order
- 錯誤, in-order在BST一定是由小到大, 沒用
- (E) The tree is a Binary Search Tree + post-order



## 7. BST

2-3樹的結構並非BST (C)

AVL Tree, RB Tree, Splay Tree, 都是balanced BST



## 8. B-Tree

(A) All leaves are at the exact same depth. V

(B) All nodes contain the exact same number of entries. X, 可能有不同數量

(C) All non-leaf nodes have the exact same number of children. X, 2-3樹很多例外

(D) All entries of a node are greater than or equal to the entries in the node's children. X, 例如若node的其中一個entry是7, 他的children可能包含6

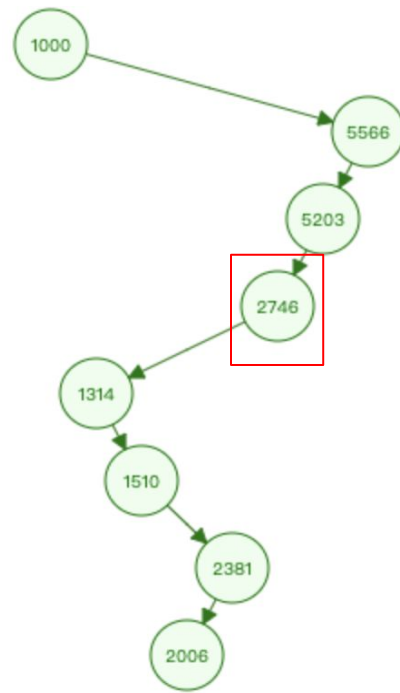
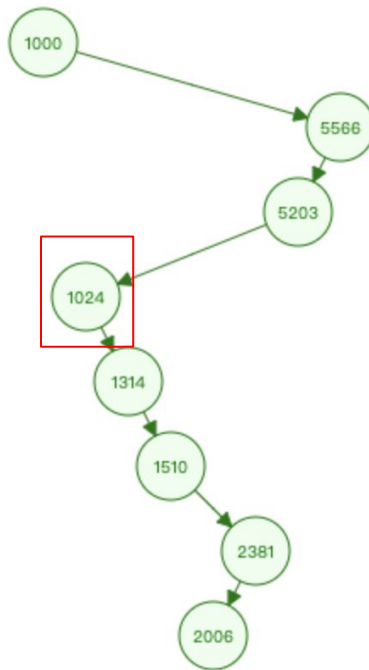
## 9. BST

1000, 5566, 5203,  $k$ , 1314, 1510, 2381, 2006.

$1000 < k < 1314$

$2381 < k < 5203$

1024(A) & 2746(D)



## 10. Postfix Transformation

批改方式：因有加多餘括號造成可能有多組答案，例如 $(+/($ ， $((+/($ 都算對

若是寫當前輸出的內容也算對

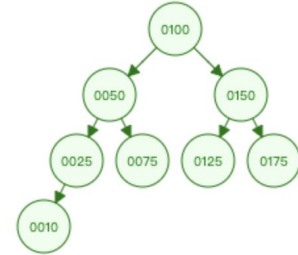
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## 11. Insertion Function for Queue

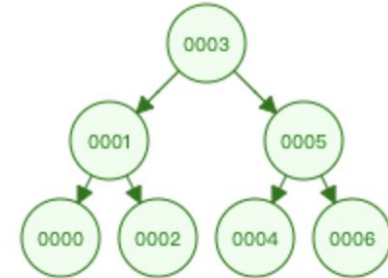
```
if(rear == MAX-1)
    printf("OVERFLOW");
else if(front == -1 && rear == -1)
    front = rear = 0;
else
    rear++;
queue[rear] = num;
```

## 12. Binary Tree

(A) What is the minimum number of nodes for a complete binary tree with level  $N$ ?  $2^n$

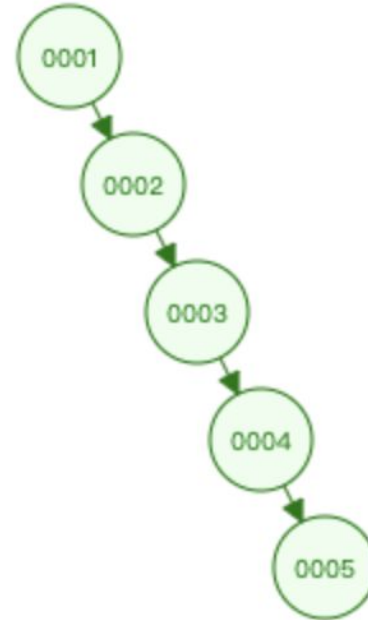


(B) For a binary search tree with  $N$  nodes, what is the minimum level of the tree?  $\text{floor}(\log_2 n)$



## 12. Binary Tree

(C) For a binary tree with  $N$  nodes, what is the maximum level of the tree?  **$N-1$**



## 13. Lower-triangular matrix (9/28投影片P.16)

Given a lower-triangular matrix  $A$ , the size of columns and rows are both 100. If we leverage a memory  $B$  to store the matrix by using row-major order:

(A) How many memory elements do we need?  $(1+100) * 100 / 2 = 5050$

(B) Which memory block will store  $A_{70,50}$ ?  $(1+69) * 69 / 2 + 50 = 2465$

(C) Which element in  $A$  will be stored in  $B_{152}$ ?

$$1 + [(1 + 2 + 3 + \dots + (i - 1)) + (j - 1)] \times 1 = 1 + \left[ \frac{1 + (i - 1)}{2} \times (i - 1) + (j - 1) \right] \times 1 = 152$$

$$\because i \geq j$$

$$\therefore i = 17 \quad j = 16$$

## 14. Recursion (9/23 投影片P.9)

- Take  $T(n) = 3T\left(\frac{n}{4}\right) + n \log_2 n$  for example
  - $a = 3, b = 4$ , and  $f(n) = n \log_2 n \Rightarrow n^{\log_b a} = n^{\log_4 3}$
  - $f(n) = \Omega(n^{\log_4 3 + \epsilon})$
  - $af\left(\frac{n}{b}\right) = 3f\left(\frac{n}{4}\right) = 3 \frac{n}{4} \log_2 \frac{n}{4} = 3 \frac{n}{4} (\log_2 n - \log_2 4)$   
 $= \frac{3}{4} n \log_2 n - \frac{3}{2} n \leq cn \log_2 n = cf(n)$ , when  $c = \frac{3}{4}$
  - $T(n) = \Theta(f(n)) = \Theta(n \log_2 n)$
  - Case3



# 15. Time Complexity

Average cost for inserting x in a BST.	(D) $O(\log n)$
Maximum cost for deleting x in a BST.	(C) $O(n)$
Search for Kth biggest number in an AVL tree.	(C) $O(n)$ or (D) $O(\log n)$
Insert a data in an AVL tree.	(D) $O(\log n)$
Delete a data in an AVL tree.	(D) $O(\log n)$
Output all elements in order from an AVL tree.	(C) $O(n)$
Insert an element in a circular queue.	(E) $O(1)$
Delete an element in a circular queue.	(E) $O(1)$

# Scores

80+: 8 (7%)

70-79: 10 (9%)

60-69: 20 (17%)

50-59: 23 (20%)

40-49: 19 (16%)

Below 40: 37 (31%)

Median: 51 & Average: 50.1

Recall the grading policy...

(認真寫作業很難被當)

Homework 55% 記得交作業!

**Midterm 25%**

Final 30%

# The Deadline of Homework #2 has Extended!

~~11/25 23:55~~ -> 11/29 23:55

第二題小題記得step-by-step insert the seq into the 2-3 tree, 並搭配code說明

接下來作業會較密集!!