

**Quiz - 20201127****Name:****School ID:**

1. (5%) Flip a fair coin ten times. Find the probability that there are more heads than tails.

**Solution:**

The probability that there are equal number of heads and tails is  $\frac{1}{2^{10}} \binom{10}{5}$ . Since the remaining cases are either more heads than tails or more tails than heads, the required probability is  $\frac{1 - \frac{1}{2^{10}} \binom{10}{5}}{2}$ .

2. (5%) Suppose that we flip a fair coin until either it comes up tails twice or we have flipped it six times. What is the probability that the experiment needed 6 flips?

**Solution:**

$$Prob(\text{need 6 flips}) = Prob(\text{no more than 1 tail in the first 5 flips}) = \left(\frac{1}{2}\right)^5 + \binom{5}{1} \left(\frac{1}{2}\right)^5$$

3. (5%) What is the probability that a randomly generated 10-digit binary string contains the substring "111"?

**Solution:**

Consider the position where the first "111" starts:

- Digit 1:  $2^7$  cases.
- Digit 2:  $1 \times 2^6$  cases.
- Digit 3:  $2^1 \times 2^5$  cases.
- Digit 4:  $2^2 \times 2^4$  cases.
- Digit 5:  $(2^3 - 1) \times 2^3$  cases.
- Digit 6:  $(2^4 - 3) \times 2^2$  cases.
- Digit 7:  $(2^5 - 8) \times 2$  cases.
- Digit 8:  $(2^6 - 20) \times 1$  cases.

Sum up the above, then divide by  $2^{10}$ .