Quiz 20201106 Solution

Name: **School ID:**

1. **(5%)** A domino is a tile of size 2×1. Find a recurrence relation for the number of ways to arrange n dominoes to fill a $2 \times n$ checkerboard.

Solution:
$a_n = a_{n-1} + a_{n-2}$

2. **(5%)** Find the generating function for modeling the number of ways to make r cents change using 1 cent, 5 cent, and 10 cent coins.

Solution:
$(1 + x + x^2 + ...)(1 + x^5 + x^{10} + ...)(1 + x^{10} + x^{20} + ...) = \frac{1}{(1-x)(1-x^5)(1-x^{10})}$

3. **(5%)** Find the solution of the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$, where $a_0 = 0$ and $a_1 = 1$.

Solution:
The characteristic equation is $x^2 - 5x + 6 = 0$, which has roots 2 and 3. Therefore the solution is of the form $a_n = s \cdot 2^n + t \cdot 3^n$. With $a_0 = 0$ and $a_1 = 1$, we have $s + t = 0$ and $2s + 3t = 1$ that result in $s = -1$ and $t = 1$. So, $a_n = 3^n - 2^n$.