

Quiz 20201211**Name:****School ID:**

1. (5%) A smartphone company makes a marketing report to predict whether its new product will be a success. So far, 60% of the company's new products have been successes. Furthermore, 70% of their successful products were predicted to be successes, while 40% of their failed products were predicted to be successes. Find the probability that the company's new smartphone will be successful if its success has been predicted.

Solution:

S : a product is successful. P : a product is predicted to be success. So, $p(S) = 0.6, p(P|S) = 0.7, p(\neg P|S) = 0.3, p(P|\neg S) = 0.4, p(\neg P|\neg S) = 0.4$.

$$p(S|P) = \frac{p(P|S)p(S)}{p(P|S)p(S) + p(P|\neg S)p(\neg S)} = \frac{0.7 \times 0.6}{0.7 \times 0.6 + 0.4 \times 0.4} \approx 0.72$$

2. (5%) Suppose that we flip a fair coin until either a tail comes up or we have flipped it six times. What is the expected number of times we flip the coin?

Solution:

$$1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{8} + 4 \cdot \frac{1}{16} + 5 \cdot \frac{1}{32} + 6 \cdot \frac{2}{64} = \frac{63}{32}$$

3. (5%) What is the variance of the number of heads that come up when a fair coin is flipped 10 times?

Solution:

Let X be the number of heads that come up in 10 flips, then $V[X] = 10 \cdot \frac{1}{2} \cdot (1 - \frac{1}{2}) = 2.5$.