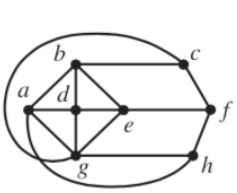
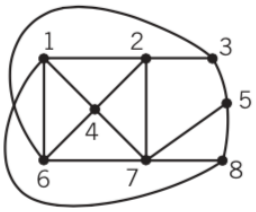
**Quiz 20201218**

**Name:** **School ID:**

1. **(5%)** Prove whether the following graphs are isomorphic.

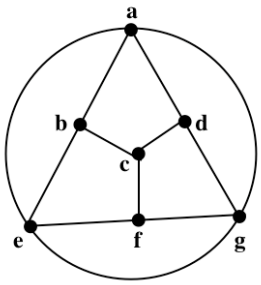



**Solution:**

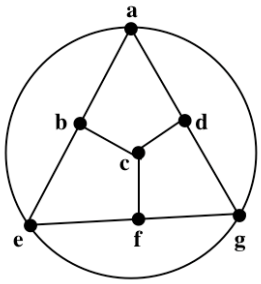
There is a triangle containing only vertices of degree 5 and degree 3 in the right graph, while in the left graph there is no such a triangle. So the two graphs are not isomorphic.

2. **(5%)** Prove whether the following graph is bipartite.




**Solution:** Not bipartite, since there is a triangle in the graph.

3. **(5%)** Use the adjancency matrix to find the number of paths of length 3 from vertex $a$ to vertex $f$ in the following graph.




**Solution:**

The adjacency matrix $A = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$. Then $A^2 = \begin{pmatrix} 4 & 1 & 2 & 1 & 2 & 2 & 2 \\ 1 & 3 & 0 & 2 & 1 & 2 & 2 \\ 2 & 0 & 3 & 0 & 2 & 0 & 2 \\ 1 & 2 & 0 & 3 & 2 & 2 & 1 \\ 2 & 1 & 2 & 2 & 4 & 1 & 2 \\ 2 & 2 & 0 & 2 & 1 & 3 & 1 \\ 2 & 2 & 2 & 1 & 2 & 1 & 4 \end{pmatrix}$ and the required number of paths is $(0, 1, 0, 1, 1, 0, 1) \cdot (2, 2, 0, 2, 1, 3, 1) = 6$.