Algorithms

Computer Science: An Overview
Tenth Edition

by
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Algorithms

Definition of Algorithm

An algorithm is an **ordered** set of **unambiguous**, **executable** steps that defines a **terminating** process.

Algorithm Representation

- Requires well-defined primitives
- A collection of primitives constitutes a programming language.

Figure 5.2 **Folding a bird from a square piece of paper**

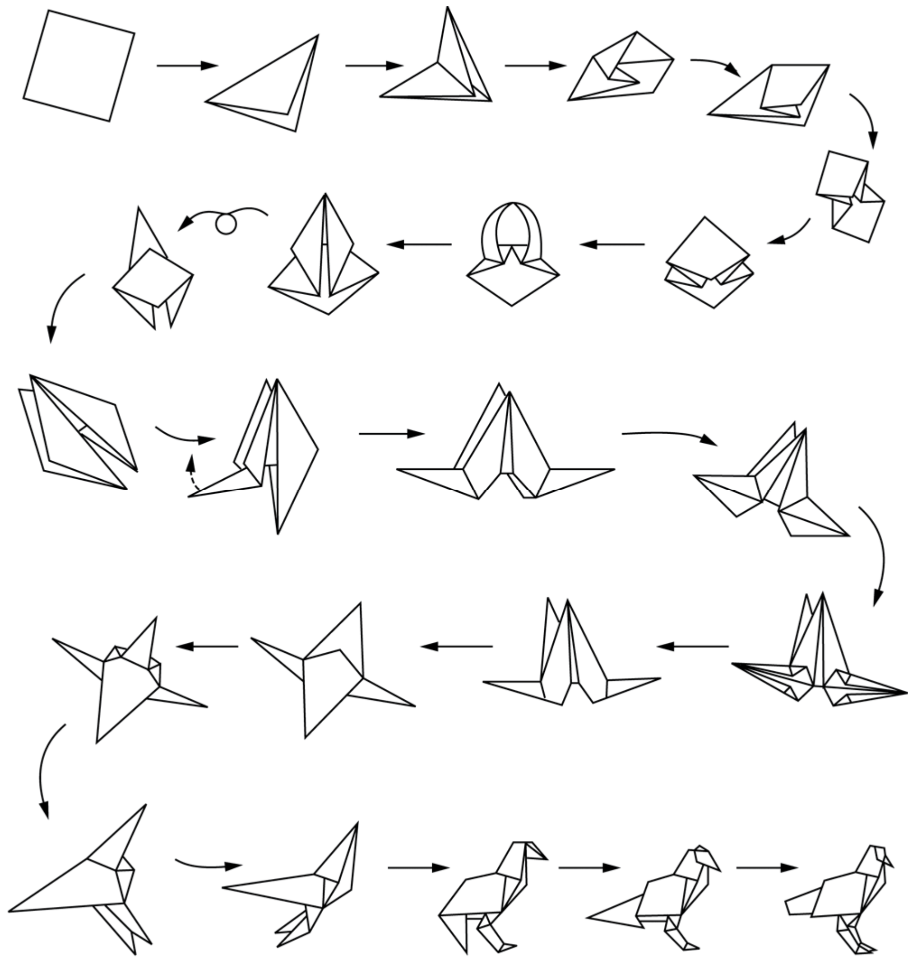
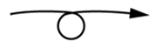
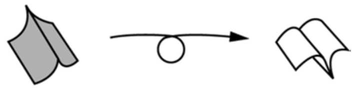
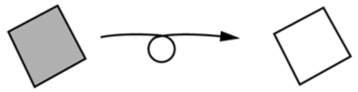
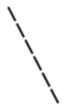
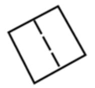
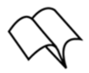
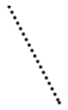
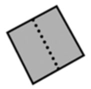
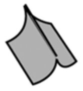
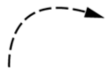
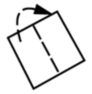
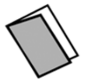
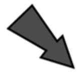
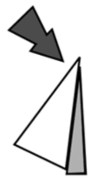
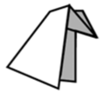


Figure 5.3 **Origami primitives**

Syntax	**Semantics**
	Turn paper over as in 
Shade one side of paper	Distinguishes between different sides of paper as in 
	Represents a valley fold so that  represents 
	Represents a mountain fold so that  represents 
	Fold over so that  produces 
	Push in so that  produces 

Pseudocode Primitives

- Assignment

```
name ← expression
```

- Conditional selection

```
if condition then action
```

Pseudocode Primitives (continued)

- Repeated execution

  while *condition* **do** *activity*

- Procedure

  procedure *name* (*generic names*)

Figure 5.4 **The procedure Greetings in pseudocode**

```
procedure Greetings
Count ← 3;
while (Count > 0) do
    (print the message "Hello" and
        Count ← Count −1)
```

Polya’s Problem Solving Steps

- 1. Understand the problem.
- 2. Devise a plan for solving the problem.
- 3. Carry out the plan.
- 4. Evaluate the solution for accuracy and its potential as a tool for solving other problems.

Getting a Foot in the Door

- Try working the problem backwards
- Solve an easier related problem
  - Relax some of the problem constraints
  - Solve pieces of the problem first (bottom up methodology)
- Stepwise refinement: Divide the problem into smaller problems (top-down methodology)

Ages of Children Problem

- Person A is charged with the task of determining the ages of B’s three children.
  - B tells A that the product of the children’s ages is 36.
  - A replies that another clue is required.
  - B tells A the sum of the children’s ages.
  - A replies that another clue is needed.
  - B tells A that the oldest child plays the piano.
  - A tells B the ages of the three children.
- How old are the three children?

Figure 5.5

a. Triples whose product is 36

(1,1,36)	(1,6,6)
(1,2,18)	(2,2,9)
(1,3,12)	(2,3,6)
(1,4,9)	(3,3,4)

b. Sums of triples from part (a)

1 + 1 + 36 = 38	1 + 6 + 6 = 13
1 + 2 + 18 = 21	2 + 2 + 9 = 13
1 + 3 + 12 = 16	2 + 3 + 6 = 11
1 + 4 + 9 = 14	3 + 3 + 4 = 10

Iterative Structures

- Pretest loop:

```
while (condition) do
    (loop body)
```

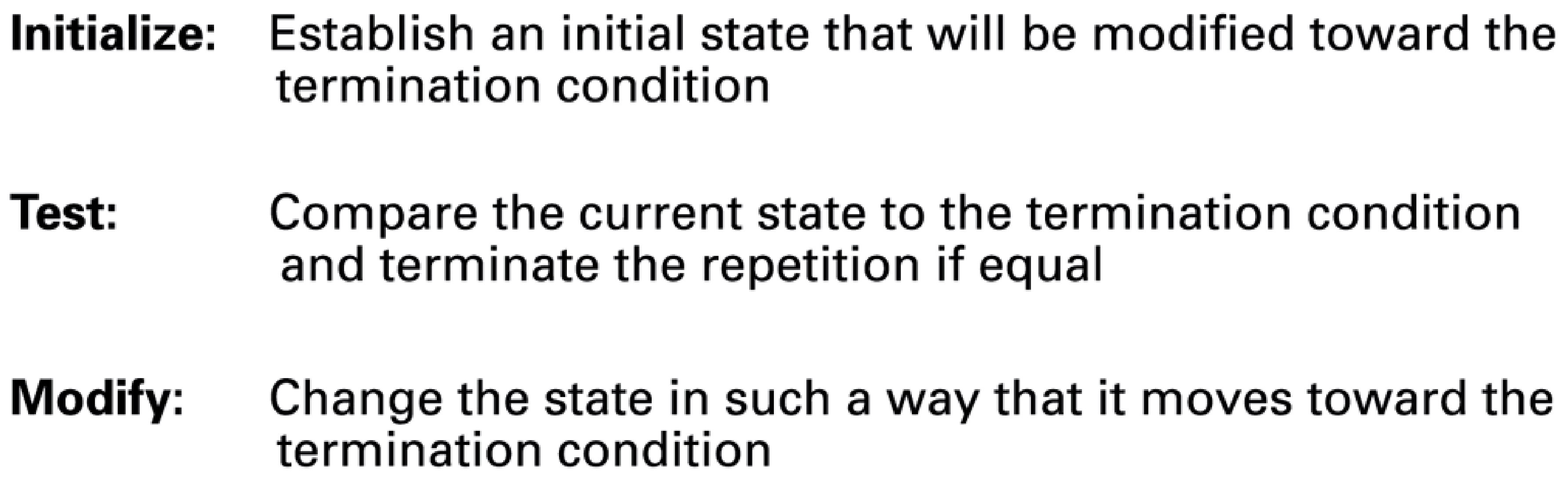
- Posttest loop:

```
repeat (loop body)
    until(condition)
```

Figure 5.6 **The sequential search algorithm in pseudocode**

```
procedure Search (List, TargetValue)
if (List empty)
      then
         (Declare search a failure)
      else
         (Select the first entry in List to be TestEntry;
          while (TargetValue > TestEntry and
                          there remain entries to be considered)
                do (Select the next entry in List as TestEntry.);
          if (TargetValue = TestEntry)
                then (Declare search a success.)
                else (Declare search a failure.)
         ) end if
```

Figure 5.7 **Components of repetitive control**



Initialize:	Establish an initial state that will be modified toward the termination condition
Test:	Compare the current state to the termination condition and terminate the repetition if equal
Modify:	Change the state in such a way that it moves toward the termination condition

Figure 5.8 **The while loop structure**

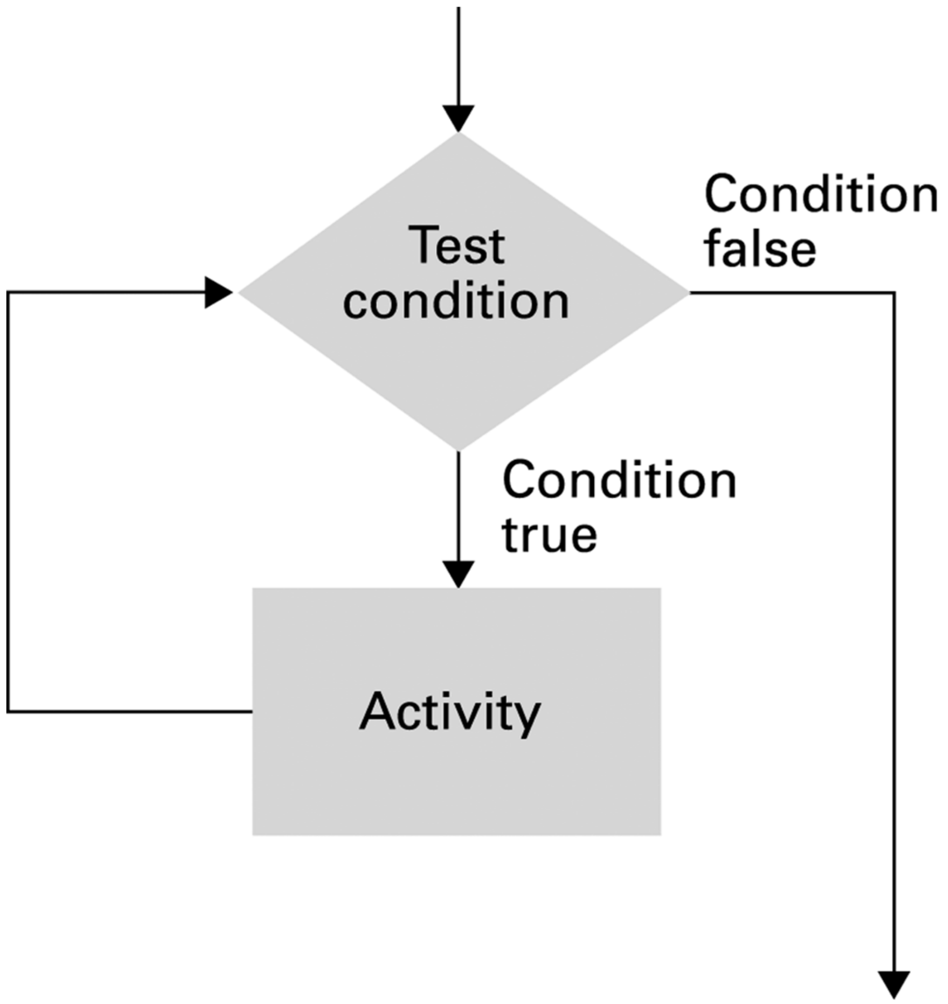


Figure 5.9 **The repeat loop structure**

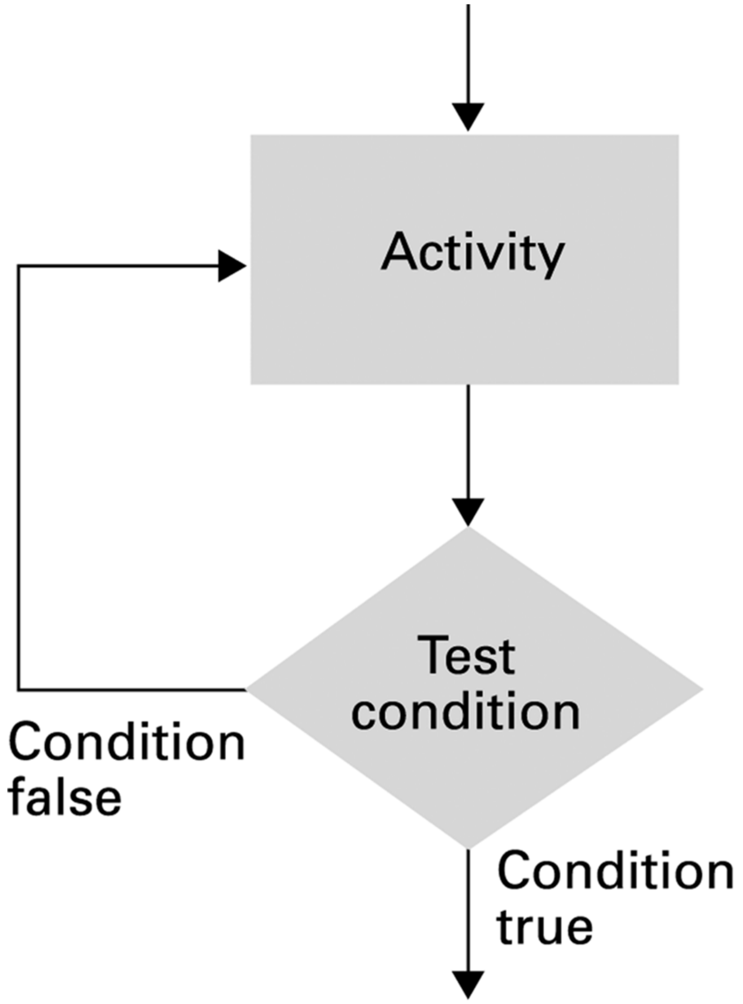


Figure 5.10 **Sorting the list Fred, Alex, Diana, Byron, and Carol alphabetically**

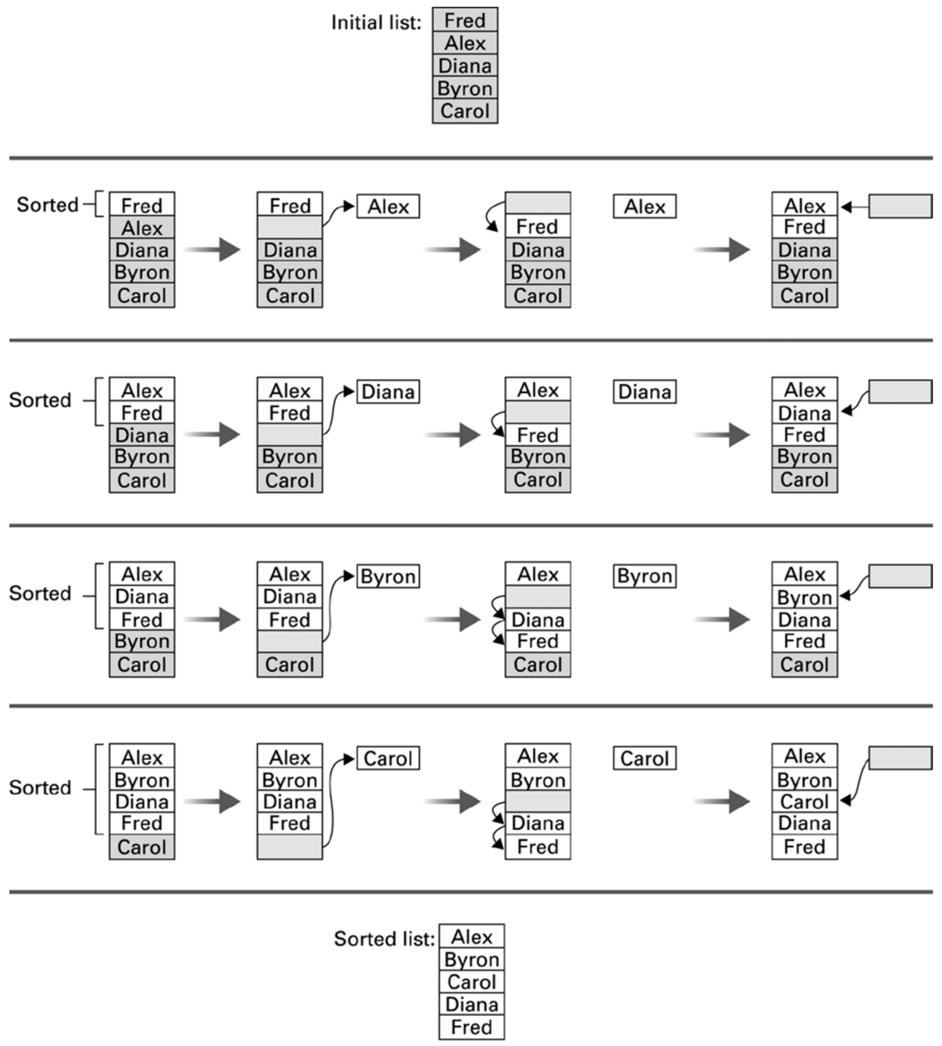


Figure 5.11 **The insertion sort algorithm expressed in pseudocode**

```
procedure Sort (List)
N ← 2;
while (the value of N does not exceed the length of List) do
      (Select the Nth entry in List as the pivot entry;
       Move the pivot entry to a temporary location leaving a hole in List;
        while (there is a name above the hole and that name is greater than the pivot) do
              (move the name above the hole down into the hole leaving a hole above the name)
        Move the pivot entry into the hole in List;
        N ← N + 1
      )
```

Recursion

- The execution of a procedure leads to another execution of the procedure.
- Multiple activations of the procedure are formed, all but one of which are waiting for other activations to complete.

Figure 5.12 **Applying our strategy to search a list for the entry John**

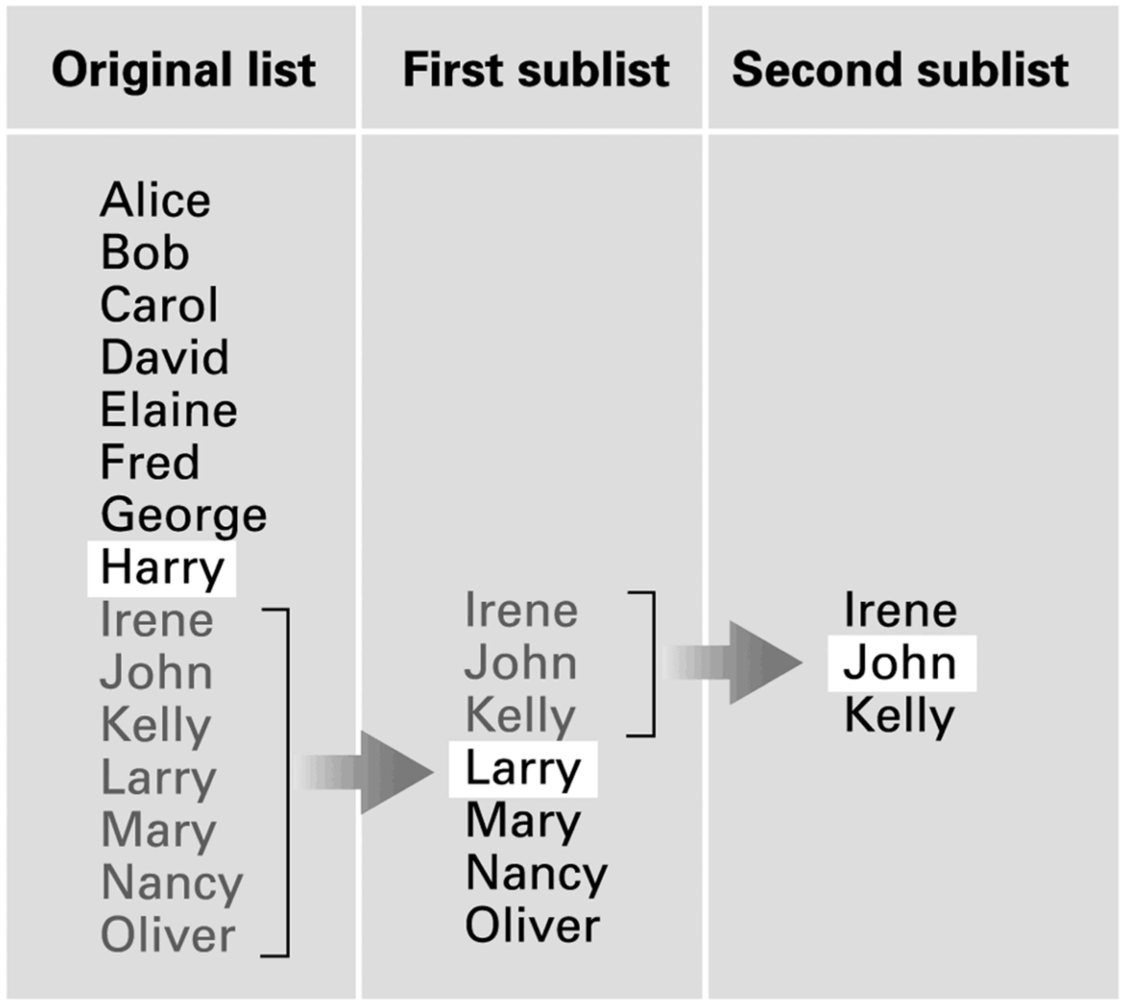


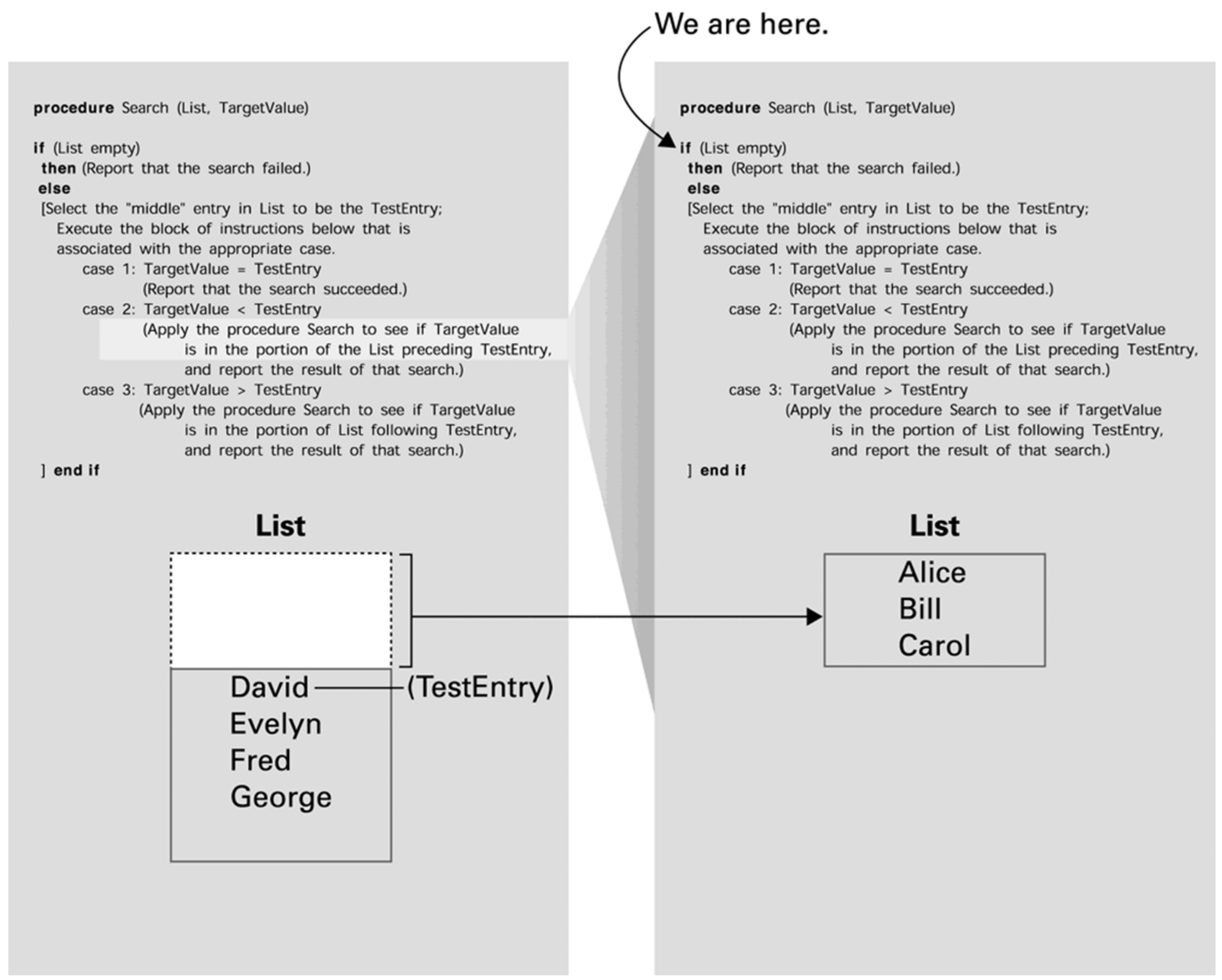
Figure 5.13 **A first draft of the binary search technique**

```
if (List empty)
 then
  (Report that the search failed.)
 else
   [Select the "middle" entry in the List to be the TestEntry;
    Execute the block of instructions below that is
      associated with the appropriate case.
        case 1: TargetValue = TestEntry
                    (Report that the search succeeded.)
        case 2: TargetValue < TestEntry
                    (Search the portion of List preceding TestEntry for
                          TargetValue, and report the result of that search.)
        case 3: TargetValue > TestEntry
                    (Search the portion of List following TestEntry for
                          TargetValue, and report the result of that search.)
   ] end if
```

Figure 5.14 The binary search algorithm in pseudocode

```
procedure Search (List, TargetValue)
if (List empty)
  then
     (Report that the search failed.)
   else
     [Select the "middle" entry in List to be the TestEntry;
      Execute the block of instructions below that is
         associated with the appropriate case.
            case 1: TargetValue = TestEntry
                    (Report that the search succeeded.)
            case 2: TargetValue < TestEntry
                    (Apply the procedure Search to see if TargetValue
                         is in the portion of the List preceding TestEntry,
                         and report the result of that search.)
            case 3: TargetValue > TestEntry
                   (Apply the procedure Search to see if TargetValue
                        is in the portion of List following TestEntry,
                        and report the result of that search.)
     ] end if
```

Figure 5.15



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procedure Search (List, TargetValue)

if (List empty)
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 else
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                    is in the portion of the List preceding TestEntry,
                    and report the result of that search.)
       case 3: TargetValue > TestEntry
               (Apply the procedure Search to see if TargetValue
                    is in the portion of List following TestEntry,
                    and report the result of that search.)
 ] end if
```

```
procedure Search (List, TargetValue)

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 then (Report that the search failed.)
 else
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   Execute the block of instructions below that is
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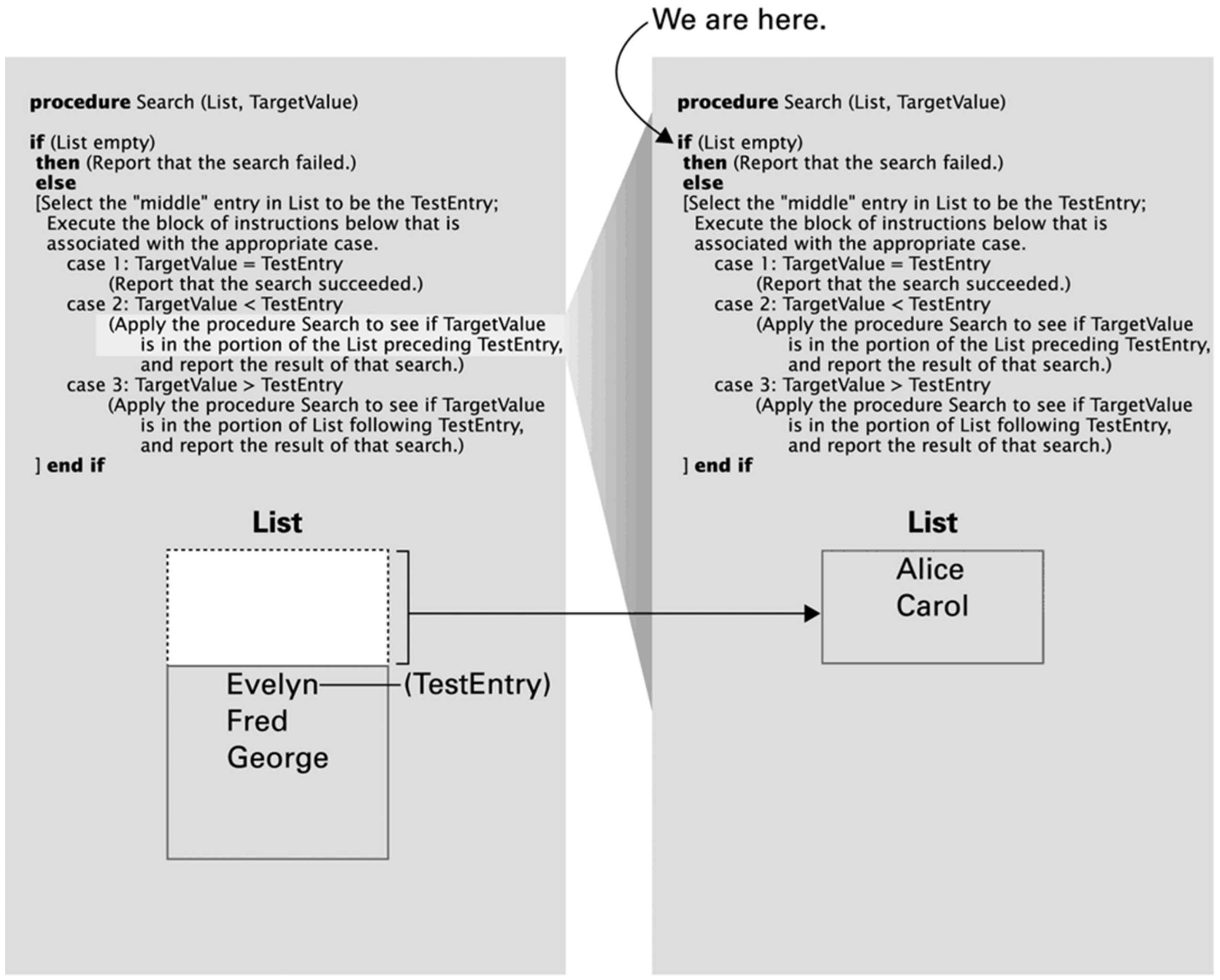
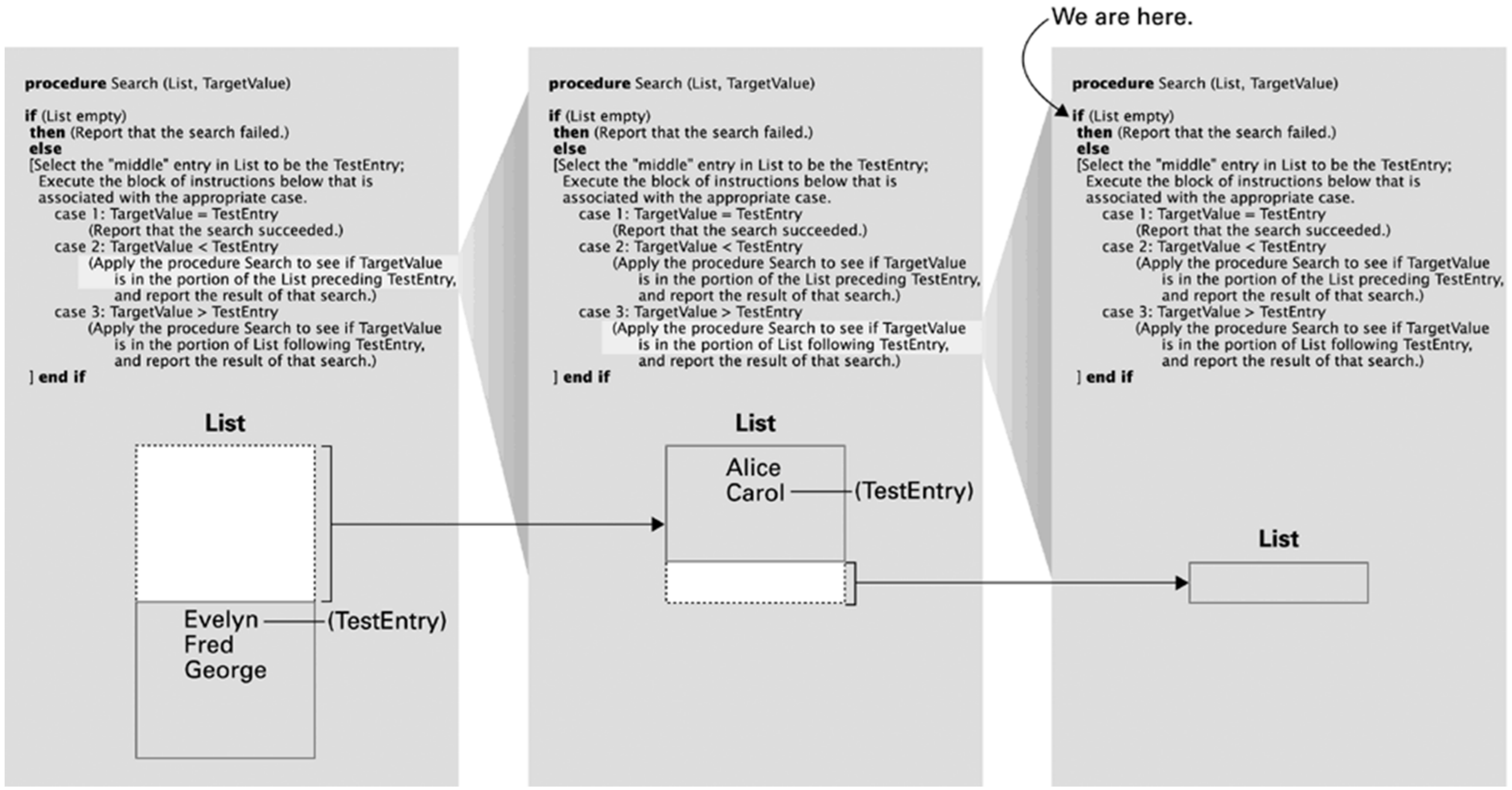
                    is in the portion of the List preceding TestEntry,
                    and report the result of that search.)
       case 3: TargetValue > TestEntry
               (Apply the procedure Search to see if TargetValue
                    is in the portion of List following TestEntry,
                    and report the result of that search.)
 ] end if
```

Figure 5.16



We are here.
procedure Search (List, TargetValue)
if (List empty)
then (Report that the search failed.)
else
[Select the "middle" entry in List to be the TestEntry;
Execute the block of instructions below that is associated with the appropriate case.
case 1: TargetValue = TestEntry
(Report that the search succeeded.)
case 2: TargetValue < TestEntry
(Apply the procedure Search to see if TargetValue is in the portion of the List preceding TestEntry, and report the result of that search.)
case 3: TargetValue > TestEntry
(Apply the procedure Search to see if TargetValue is in the portion of List following TestEntry, and report the result of that search.)
] end if
List
Evelyn
Fred
George
(TestEntry)
procedure Search (List, TargetValue)
if (List empty)
then (Report that the search failed.)
else
[Select the "middle" entry in List to be the TestEntry;
Execute the block of instructions below that is associated with the appropriate case.
case 1: TargetValue = TestEntry
(Report that the search succeeded.)
case 2: TargetValue < TestEntry
(Apply the procedure Search to see if TargetValue is in the portion of the List preceding TestEntry, and report the result of that search.)
case 3: TargetValue > TestEntry
(Apply the procedure Search to see if TargetValue is in the portion of List following TestEntry, and report the result of that search.)
] end if
List
Alice
Carol

Figure 5.17



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procedure Search (List, TargetValue)

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 else
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                 is in the portion of the List preceding TestEntry,
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            (Apply the procedure Search to see if TargetValue
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 ] end if
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            (Apply the procedure Search to see if TargetValue
                 is in the portion of List following TestEntry,
                 and report the result of that search.)
 ] end if
```

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if (List empty)
 then (Report that the search failed.)
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                 is in the portion of the List preceding TestEntry,
                 and report the result of that search.)
     case 3: TargetValue > TestEntry
            (Apply the procedure Search to see if TargetValue
                 is in the portion of List following TestEntry,
                 and report the result of that search.)
 ] end if
```

Algorithm Efficiency

- Measured as number of instructions executed
- Big theta notation: Used to represent efficiency classes
  - Example: Insertion sort is in $\Theta(n^2)$
- Best, worst, and average case analysis

Figure 5.18 **Applying the insertion sort in a worst-case situation**

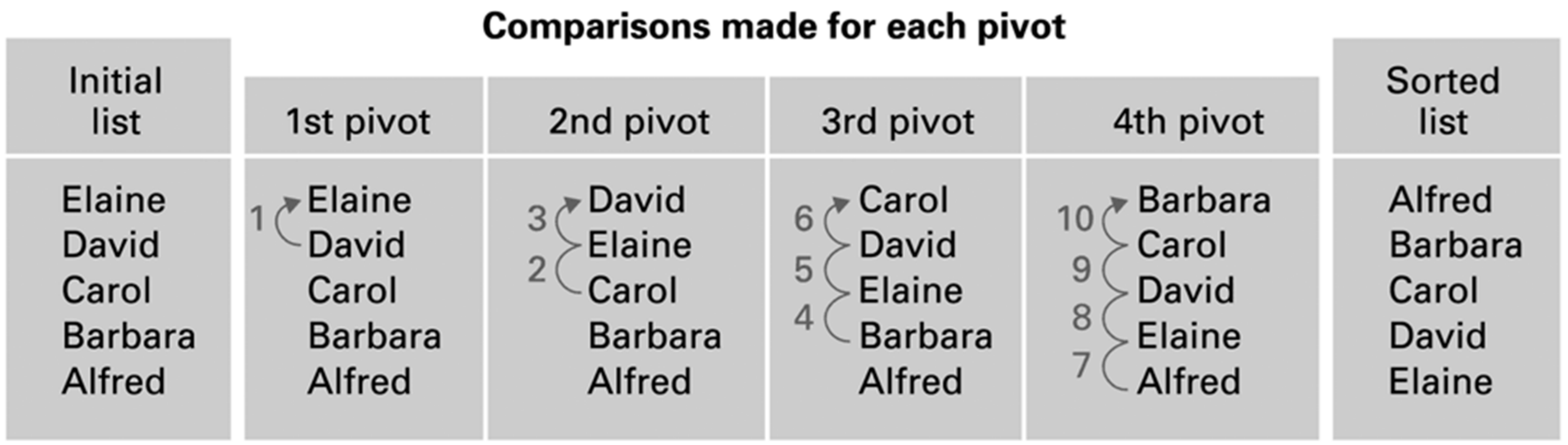


Figure 5.19 **Graph of the worst-case analysis of the insertion sort algorithm**

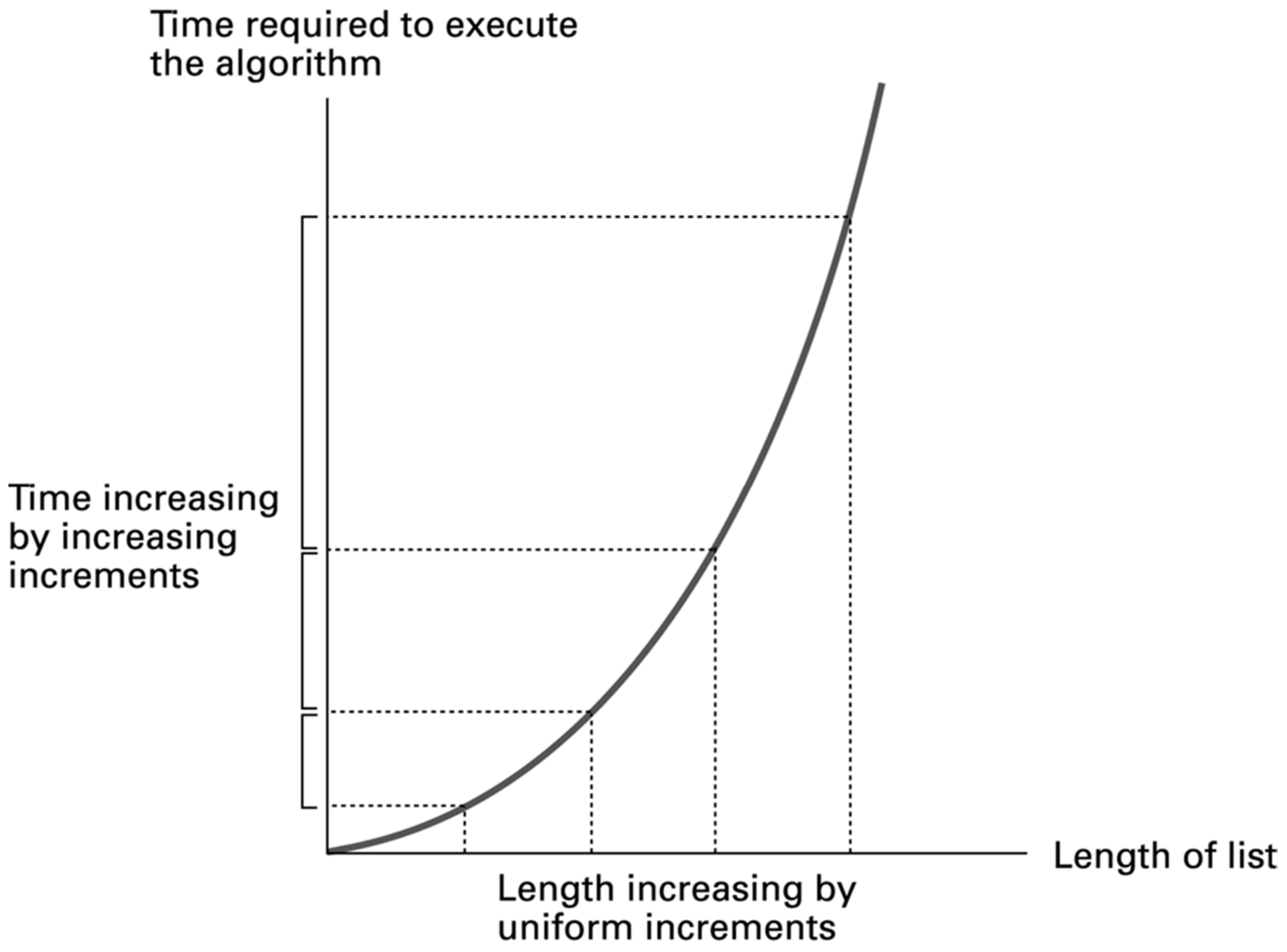
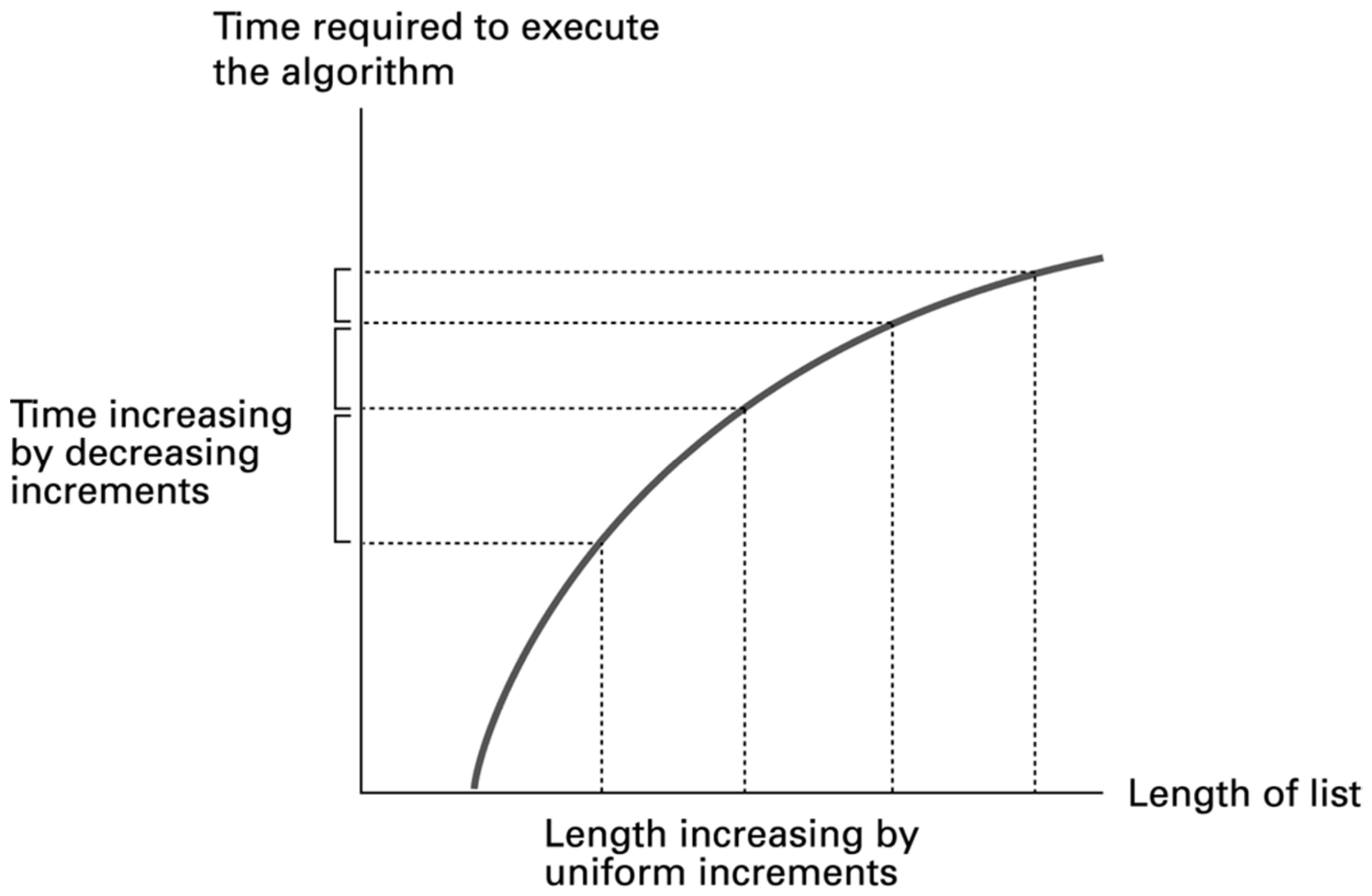


Figure 5.20 **Graph of the worst-case analysis of the binary search algorithm**



Software Verification

- Proof of correctness
  - Assertions
    - Preconditions
    - Loop invariants
- Testing

Chain Separating Problem

- A traveler has a gold chain of seven links.
- He must stay at an isolated hotel for seven nights.
- The rent each night consists of one link from the chain.
- What is the fewest number of links that must be cut so that the traveler can pay the hotel one link of the chain each morning without paying for lodging in advance?

Figure 5.21 **Separating the chain using only three cuts**

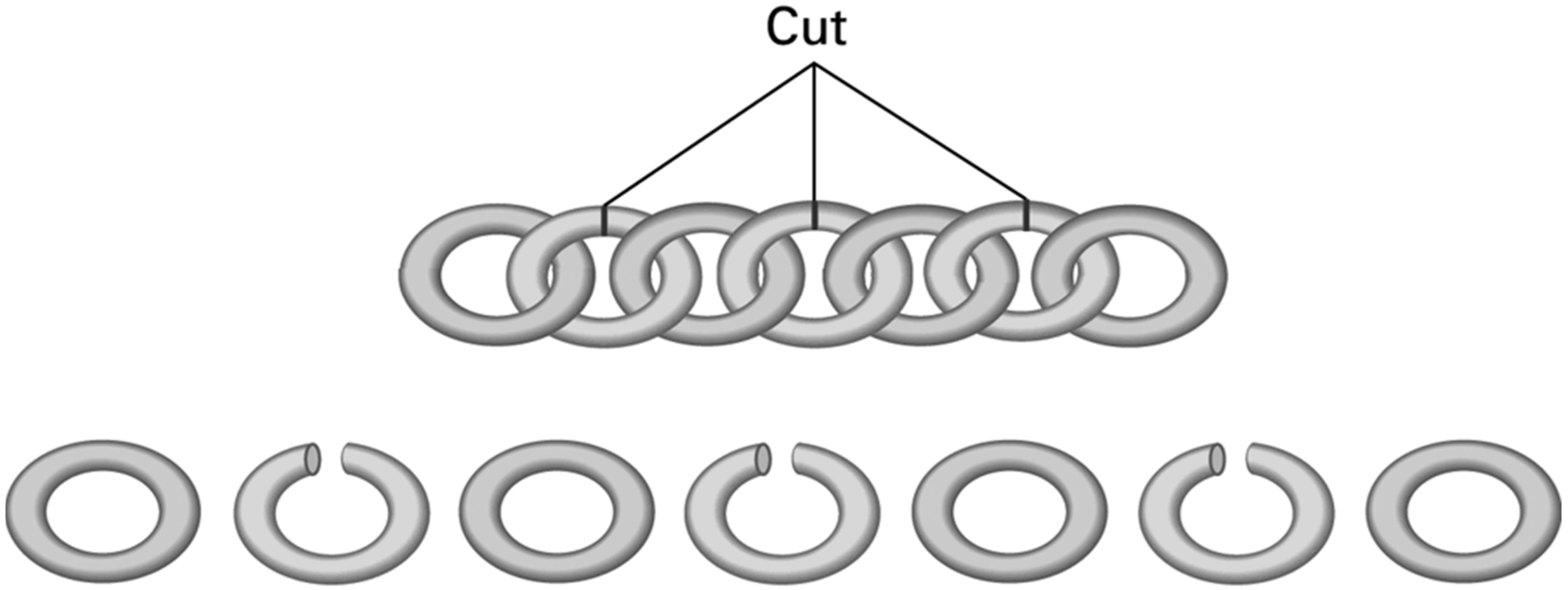


Figure 5.22 **Solving the problem with only one cut**

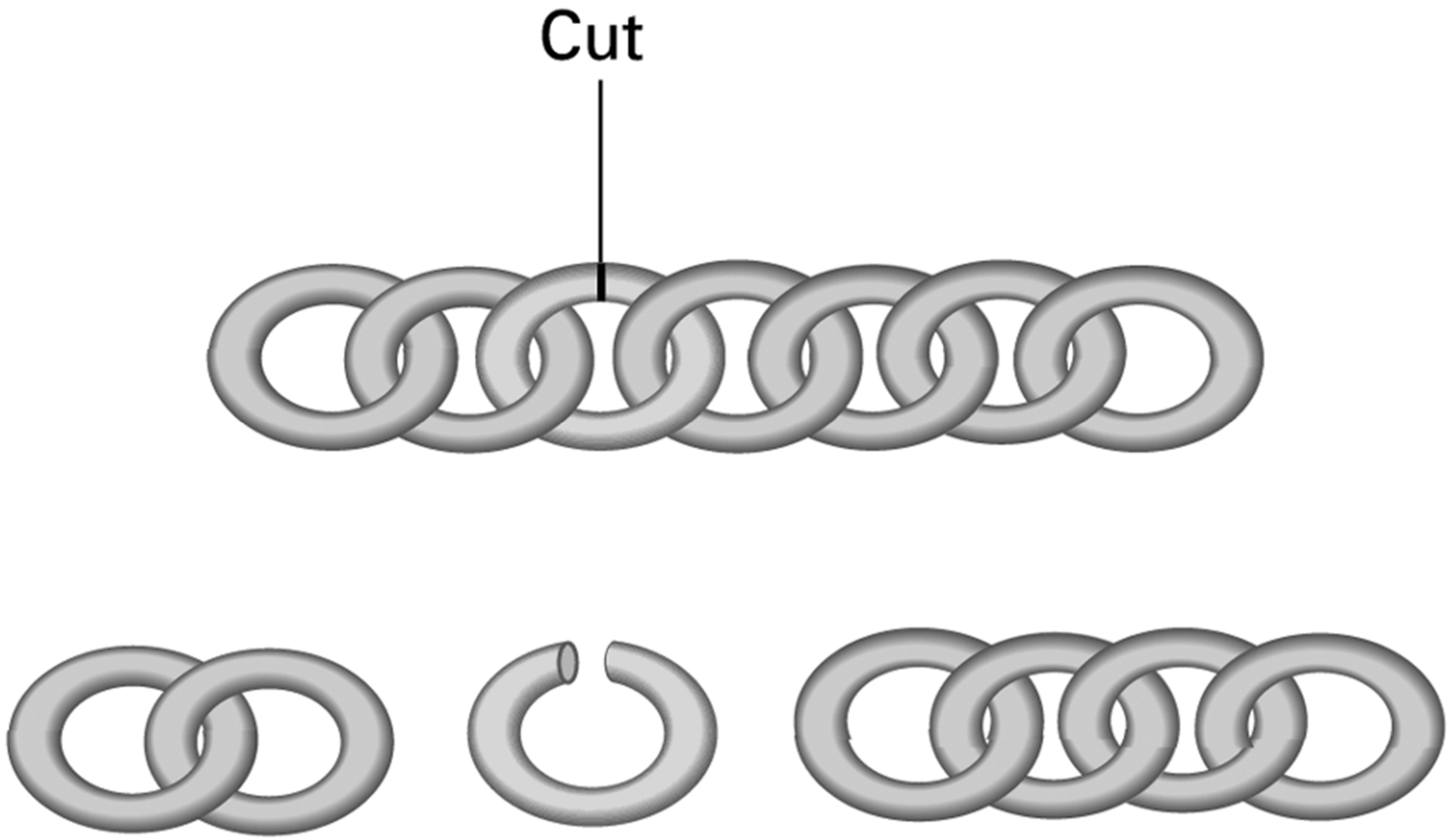
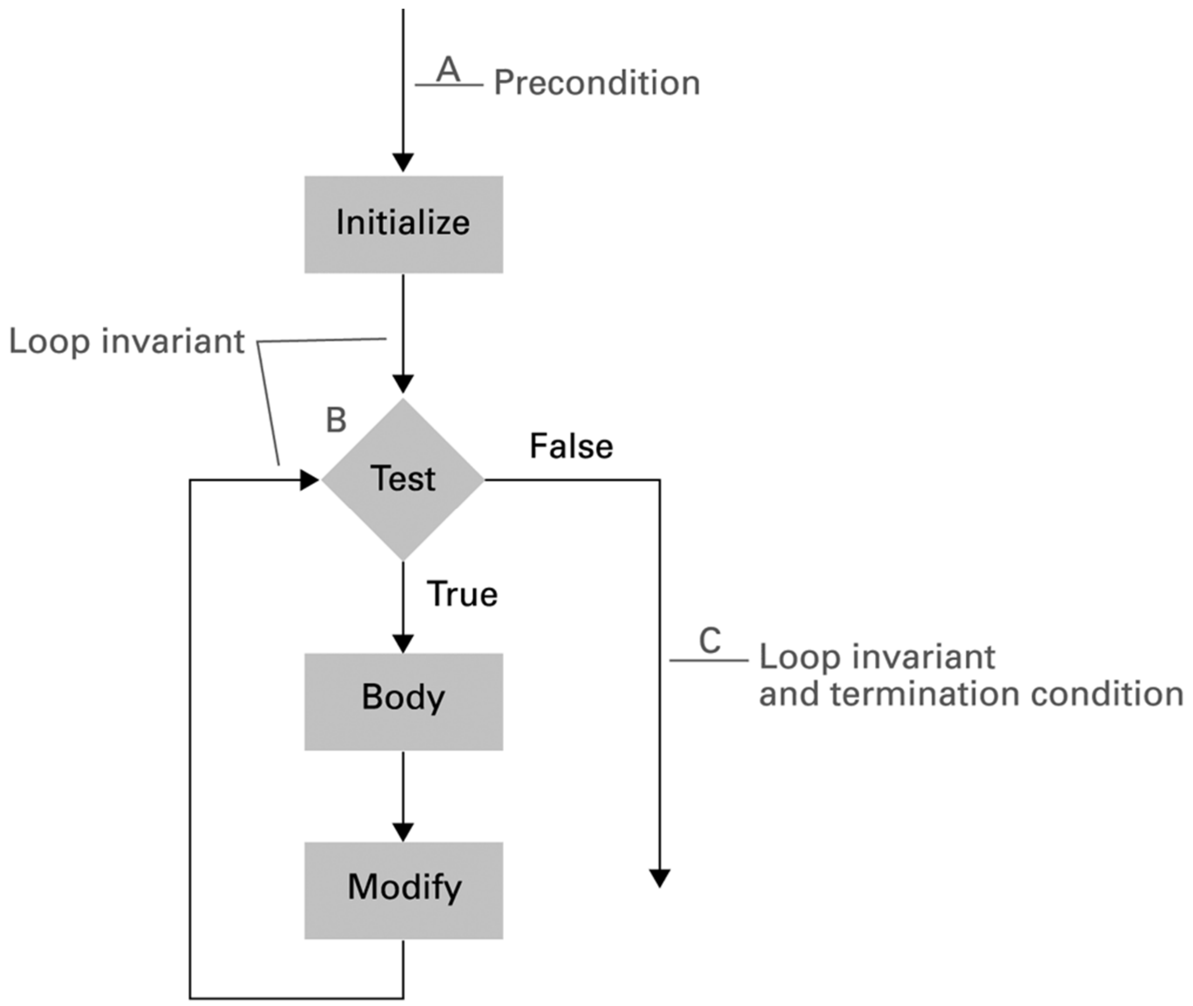


Figure 5.23 **The assertions associated with a typical while structure**


A
Precondition
Initialize
Loop invariant
B
Test
False
True
C
Loop invariant
and termination condition
Body
Modify