APPUNTI 2

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DEFINIZIONE

UN INSIEME V SI DICE SPAZIO VETTORIALE SUL CAMPO F, SE ESISTONO DUE OPERAZIONI

$(+): V \times V \to V$

$(\cdot): F \times V \to V$

TALI CHE

1) $(V,+)$ È UN GRUPPO ABELIANO

2) $1_F \cdot v = v \quad \forall v \in V$ ELEMENTO NEUTRO

3) $\forall \alpha,\beta \in F, \forall v \in V, (\alpha\beta)v = \alpha(\beta v)$ ASSOCIATIVA SU F

4) $\forall \alpha,\beta \in F, \forall v \in V, (\alpha+\beta)v = \alpha v + \beta v$ DISTRIBUTIVA SU F

5) $\forall \alpha \in F, \forall v,w \in V; \alpha(v+w) = \alpha v + \alpha w$ DISTRIBUTIVA SU V

ESEMPIO

$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(a,b) \mid a,b \in \mathbb{R}\}$

$(+): (a,b) + (c,d) = (a+c, b+d)$

$(\cdot): \alpha \cdot (a,b) = (\alpha a, \alpha b)$

$\mathbb{R}^2$ È UNO SPAZIO VETTORIALE SU $\mathbb{R}$

1) $(\mathbb{R}^2, +)$ È UN GRUPPO ABELIANO

a) ASSOCIATIVA

$$((a,b)+(c,d)) + (e,f) \overset{?}{=} (a,b) + ((c,d)+(e,f))$$

$$(a+c) + (b+d) + (e,f) = (a,b) + (c+e, d+f)$$

ASSOCIATIVA IN $\mathbb{R}$

$$(a+c+e, b+d+f) = (a+c+e, b+d+f)$$

b) ELEMENTO NEUTRO

$$(0,0) + (a,b) = (0+a, 0+b) = (a,b)$$

$$(a,b) + (0,0) = (a+0, b+0) = (a,b)$$

$0_V = (0,0)$

c) OPPOSTI $(a,b)+(-a,-b) = (a+(-a), b+(-b)) = (0,0) = 0_V$

d) $(a,b)+(c,d) = (a+c, b+d) = (c+a, d+b) = (c,d)+(a+b)$

2) $\forall v \in V, \; 1_F \cdot v = v$

$$1\cdot(a,b) = (1a, 1b) = (a,b)$$

3) $\forall \alpha,\beta \in F, \; (\alpha\beta)v = \alpha(\beta\cdot v)$

$v = (a,b)$

$$(\alpha\cdot\beta)\cdot(a,b) = \alpha(\beta\cdot(a,b))$$
$$(\alpha\beta a, \alpha\beta b) = \alpha(\beta a, \beta b)$$
$$(\alpha\beta a, \alpha\beta b) = (\alpha\beta a, \alpha\beta b)$$

4) $\forall \alpha,\beta \in F, \; \forall v \in V, \; (\alpha+\beta)v = \alpha v + \beta v$

$v = (a,b)$

$$(\alpha+\beta)(a,b) = \alpha(a,b)+\beta(a,b)$$
$$\alpha(a,b)+\beta(a,b) = (\alpha a, \alpha b)+(\beta a, \beta b)$$
$$(\alpha a, \alpha b)+(\beta a, \beta b) = (\alpha a+\beta a, \alpha b+\beta b)$$
$$(\alpha a+\beta a, \alpha b+\beta b) = (\alpha a+\beta a, \alpha b+\beta b)$$

5) $\forall \alpha \in F, \; \forall v,w \in V, \; \alpha(v+w) = \alpha v+\alpha w$

$v = (a,b) \quad w = (c,d)$

$$\alpha((a,b)+(c,d)) = \alpha(a,b)+\alpha(c,d)$$
$$\alpha(a+c, b+d) = (\alpha a, \alpha b)+(\alpha c, \alpha d)$$
$$(\alpha a+\alpha c, \alpha b+\alpha d) = (\alpha a+\alpha c, \alpha b+\alpha d)$$