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Mathematical Model of Complete Electromagnetic Rotor Suspension

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Abstract

A mathematical description of complete (non-contact) electromagnetic suspension of the rotor representing an alternative approach to the bearing construction of high-speed electrical machine is discussed. This approach provides an accurate positioning of the rotor in the air gap, increasing system uptime compared with conventional bearing assemblies. Synthesized selective control law of rotor position in complete electromagnetic suspension is considered.

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Keywords: active magnetic bearing; complete electromagnetic suspension; selective control.

1. Introduction

In modern engineering are increasingly put the task of designing electromechanical systems with ultra-high speed of the rotor to create a high-tech installations (compressors, air-purge, turbo generators, refrigeration units, etc.). To solve the problem of achieving ultra-high speeds (150 000 - 200 000 rpm) it is necessary to use a special bearing supports capable of operating in a given speed range. Hydrodynamic, gas-dynamic or hydrostatic bearings are most commonly used at the present time. These bearings have very low friction coefficients - is much lower than the mechanical bearings. The main source of friction is the viscosity of the liquid or gas. However, the main disadvantage of the use of such supports is complex power system and limit rapidity due to friction. As an alternative approach of systems construction of high-speed electrical machine bearing supports, completely

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eliminates friction, is the application of active magnetic bearings (AMB). The use of these bearings allows not only to achieve the rotor speed, but also to implement a system that does not require lubrication.

Nomenclature

F	tractive force of the electromagnet
F_m	maximum tractive effort
I	coil current
I_2	current, taking into account the effect of the eddy currents
I_{m0}	maximum current in the coil at the central rotor position (z = 0)
$I\mu$	magnetizing current
I_σ	current, taking into account the effect of the eddy currents of leakage flux.
L	inductance corresponding to the magnetic flux in the air gap
$L0$	inductance in the air gap corresponding to the magnetic flux at the rotor central position
$L1$	inductance corresponding to leakage flux
Q	external component of the force acting on the rotor from the electromagnet
R	active resistance of winding
R_1	the active component of the eddy current loss of leakage flux.
U	voltage applied to the electromagnet coil
U_{m0}	voltage in the coil at the central rotor position (z = 0)
z	deviation from the central rotor position
δ	air gap between the rotor and the stator at the central rotor position
ψ	linkage flux
ψ_m	maximum linkage flux

2. Active magnetic bearing

Active magnetic bearing is a complex electronic device that allows to carry out contactless suspension of the rotor of electrical machine. The main feature of such a systems is that the design and operating principle of the AMB, realizing complete electromagnetic rotor suspension ensures operation electrical machine without friction over a wide speed range. The introduction the AMB technology in high-speed electrical machines leads to a reduction the costs of aggregates technology services and also improves lifetime of the device as a whole.

Further advantages of the AMB are relatively high load capacity, high mechanical strength, the ability of the non-contact rotor stable suspension, the possibility of changing the stiffness and damping in a wide range, with the possibility of using high rotational speeds, in vacuum, high and low temperatures, sterile technologies [1].

3. Description of the developed system

Significant complexity of the existing electromagnetic suspension (EMS) control system limits the potential scope of its application. The characteristic feature of EMS is that the designed system plant is a composite object "magnet rotor" having a complicated nonlinear mathematical description.

The principle of operation of EMF is that the rotor is held in a predetermined position, radial or axial, through the action of magnetic fields generated by electromagnets.

The electromagnets are located inside the stator of the electric machine and currents supplied to the windings are changed depending on the rotor position, measured by position sensors by a particular control law.

For the synthesis of the position control law a mathematical model of the magneto-mechanical system (complete electromagnetic suspension) consisting the following elements are developed:

- model of flexible horizontally oriented rotor
- model of radial active magnetic bearing (RAMB)
- model of axial (thrust) active magnetic bearing (AAMB)

Block diagram of the system is shown at Fig. 1.

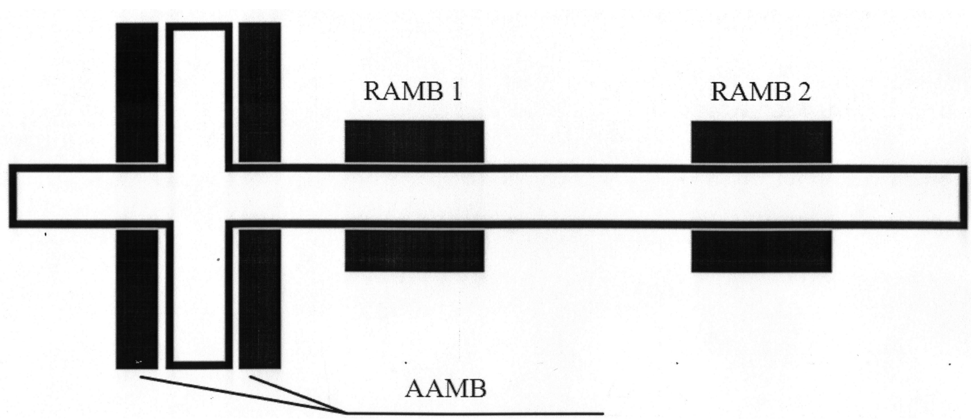

Fig. 1. Block diagram of a magnetic-mechanical systems of rotor in elastic supports

3.1. Mathematical model of axial active magnetic bearing

The approximated equivalent circuit of AAMB is shown at Figure 2. Circuit with a current I2 takes into account the physical nature of the changing processes of magnetic resistance and power losses in a massive ferromagnetic AAMB core. This could have a significant effect on the dynamic properties and AAMB control [2].

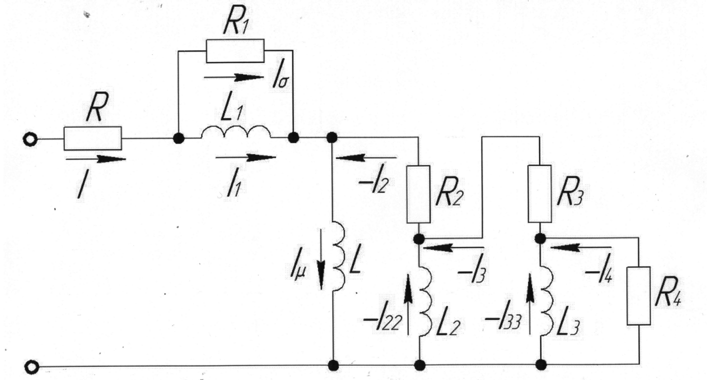

Fig. 2. Approximated equivalent circuit of AAMB

To move from the approximate equivalent circuit to the mathematical model is necessary to use the following equations of electromagnetic energy conversion:

$$m\frac{d^2z}{dt^2}-\frac{1}{2}\frac{dL}{dz}I_{\mu}^2=Q\,; \tag{1}$$

$$U=\frac{d\psi}{dt}+L1\frac{dI}{dt}+IR\,; \tag{2}$$

$$F=\frac{1}{2}\frac{dy}{dx}I_{\mu}^2\,; \tag{3}$$

$$\psi = LI_{\mu}^{2}, \tag{4}$$

The dependence of electromagnet inductance in the air gap, corresponding to the magnetic flux, is approximated by the expression:

$$L = \frac{L0}{1 - z/\delta} \tag{5}$$

The current I in the coil is given by:

$$I = I_{\mu} + I_{2} \ or \ I = I_{1} + I_{\sigma} \tag{6}$$

The equation for the electrical balance of circuit L1-R1:

$$L1\frac{dI_{1}}{dt} = R1I_{\sigma} \tag{7}$$

The equation for the electrical balance of circuit *L0 – R2 – R3 – R4:*

$$\begin{aligned}
&\frac{d\psi}{dt} = R2I_{2} + R3I_{3} + R4I_{4} \\
&I_{2} = I_{22} + I_{3} \\
&I_{3} = I_{33} + I_{4} \\
&L3\frac{dI_{33}}{dt} = R4I_{4} \\
&L2\frac{dI_{22}}{dt} = R3I_{3} + R4I_{4}
\end{aligned} \tag{8}$$

After the introduction of the differentiation operator *s* and algebraic manipulations, equation (8) takes the form:

$$s\psi = I_{4}\left[\frac{s^{2}L2L3(R2+R3+R4)+s(R2R3L3+R2R4L3+R2R4L2+R3R4L2)+R2R3R4}{s^{2}L2L3}\right] \tag{9}$$

Let us make the transition to the relative units:

$$\begin{aligned}
&z' = \frac{z}{\delta}; F' = \frac{F}{F_{m}}; Q' = \frac{Q}{F_{m}}; \\
&\psi' = \frac{\psi}{\psi_{m}}; U' = \frac{U}{U_{m0}}; I' = \frac{I}{I_{m0}}; \\
&I'_{\mu} = \frac{I_{\mu}}{I_{m0}}; I'_{2} = \frac{I_{2}}{I_{m0}}; I'_{22} = \frac{I_{22}}{I_{m0}}; I'_{3} = \frac{I_{3}}{I_{m0}}; \\
&I'_{33} = \frac{I_{33}}{I_{m0}}; I'_{4} = \frac{I_{4}}{I_{m0}}; I'_{1} = \frac{I_{1}}{I_{m0}}; I'_{\sigma} = \frac{I_{\sigma}}{I_{m0}};
\end{aligned} \tag{9}$$

The equations of electromagnetic energy conversion (1) - (4) and (9) assume a simpler form:

$$s^2 y' = \frac{F_m}{m\delta}(F' + Q')\,;\;\; s\psi' = \frac{U_{m0}}{\psi_m}U' - \frac{R}{L0}I'(T_1 s + 1)\,; \tag{10}$$

$$F' = \left(\psi'\right)^2\,;\;\; I'_\mu = (1 - z')\psi'\,;\;\; I' = I'_1\left(T_\sigma s + 1\right); \tag{11}$$

$$I'_4 = s\psi'\left[\frac{s^2 L0L2L3}{s^2 L2L3(R2 + R3 + R4) + s(R2R3L3 + R2R4L3 + R2R4L2 + R3R4L2) + R2R3R4}\right], \tag{12}$$

$$T_1 = \frac{L1}{R}\,;\;\; T_\sigma = \frac{L1}{R1} \tag{13}$$

A block diagram is made on mathematical models and presented in Figure 3. The inputs receive signals that simulate voltage of electromagnet coil *U* and the deviation from the rotor central position *z*. Then, these signals are converted into corresponding relative values *U'* and *z'*. The output variables of the model are the traction force F of the electromagnet, the coil current *I* and magnetizing current *Iμ*. The introduction of a time constant *τ = 0.1 T1* is caused by the necessity of exclude the excitation in the integrator feedback loop [3].

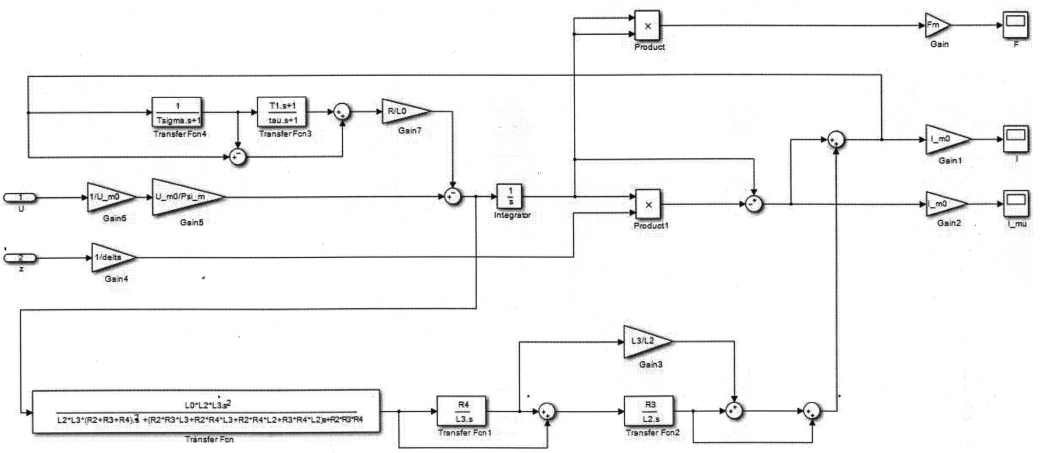

Fig. 3. Block diagram of AAMB in Matlab/Simulink

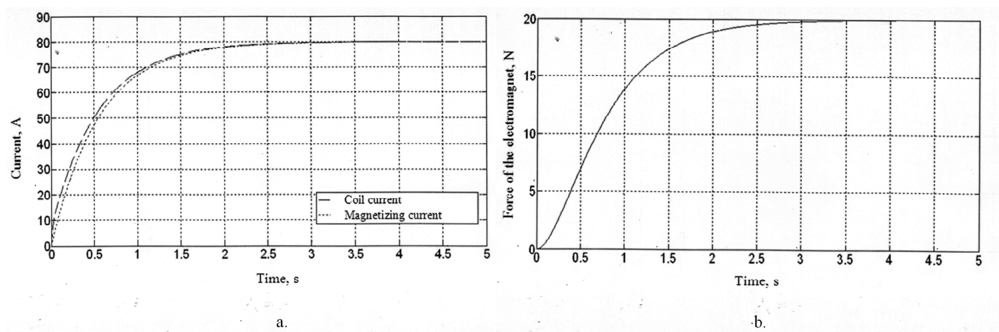

Fig. 4. Transient responses: a – the coil current and the magnetizing current of AAMB, b – the magnetic force of AAMB.

3.2. Mathematical model of radial active magnetic bearing

According to the mathematical model, as detailed in [4], block diagram in Matlab/Simulink is composed (see Figure 5). For the calculation of the current and the magnetic force, and also for the results visualization created m-file program is developed. When specifying certain parameters of controlled electromagnet simulation results are obtained and shown in Figure 6.

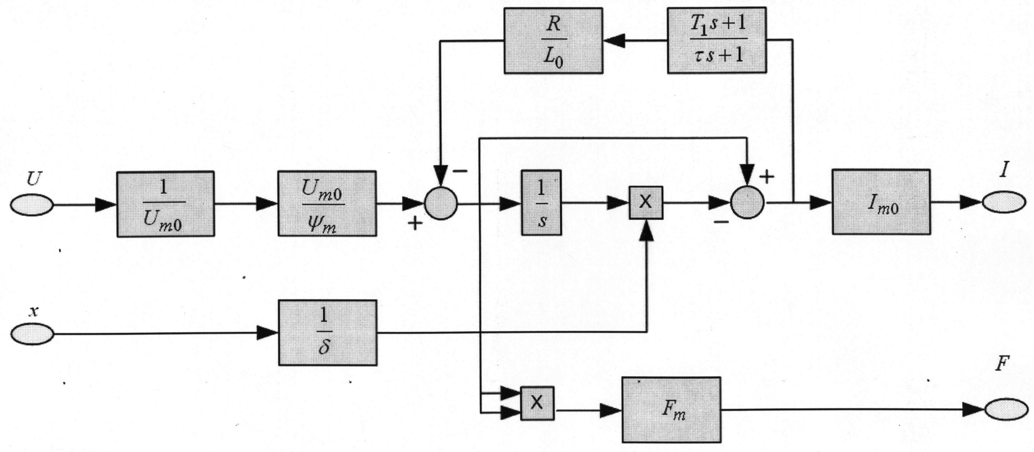

Fig. 5. Block diagram of RAMB in Matlab/Simulink

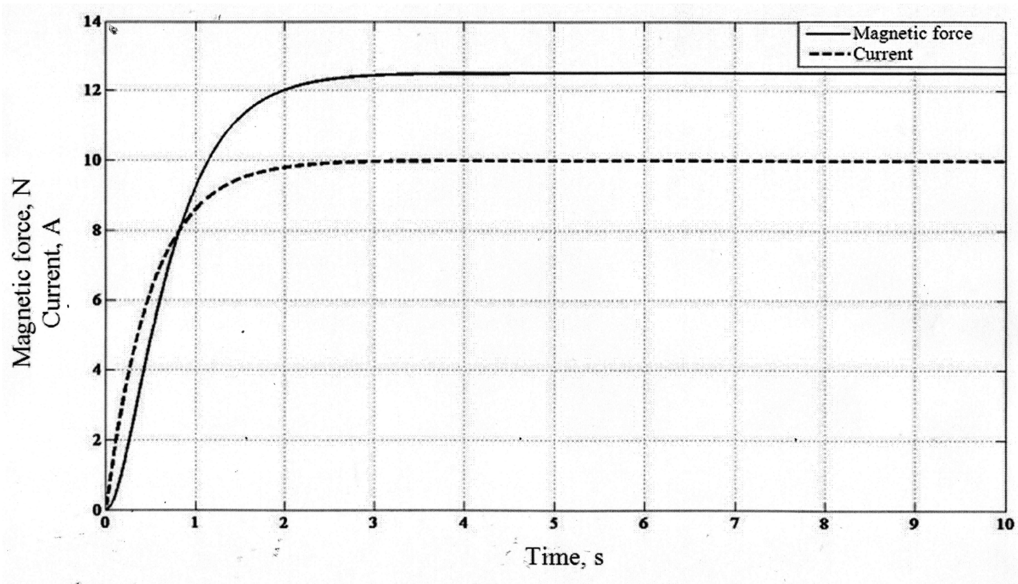

Fig. 6. The coil current and the magnetic force of RAMB

4. Control concept

The achievement of sustainable position of the rotor is carried out by magnetic attraction forces acting on the rotor [5]. AMB control system forms the electromagnet coils current by rotor displacement sensor signals or by the magnetic flux sensor signals.

During the synthesis of the rotor position control system in complete EMS, it is necessary to take into account the complex nonlinear dynamics of elastic supports and to evaluate their impact on the overall dynamics of the system. The main effect on the dynamics - is the appearance of resonance phenomena, precession and nutation that are must be previously investigated and the results have to be taken into account in the control law synthesis process.

In this paper selective algorithm rotor for position control is synthesized. The structure of the designed system is shown in Figure 7. It uses a mathematical model of rotor in the form of Lagrange equations of second type [6], mathematical model of RAMB and mathematical model of AAMB.

It should be noted that due to the complexity of object behavior under the influence of control signals selective control algorithm synthesis was carried out step by step for different modes of operation, listed in Table 1.

The basis for the transition between modes is the achievement of rotor speed the value of the critical frequency ($\omega 1$ .... $\omega 5$). By preliminary simulation it was found that, in the graph of displacement transient responses at the certain frequency f oscillations are appeared. Changing of operating mode, i.e. the connection of a particular controller (corresponding to a defined zone), takes place automatically based on the frequency of fixed oscillation $\{f1, f2, f3\}$ of transient responses process. Thus, the controller implements algorithm that is suitable for the current rotational speed (identified by oscillation frequency). An additional advantage of this approach is the possibility of usage only the position sensor signal and exception information channel of speed sensor.

Table 1. Operation modes.

#	Name	Description
1	Rotor lifting (hanging)	Formation of a control signal activating of AMB. Stabilization of rotor in the middle of the gap. The rotational rotor speed is equal to 0, the perturbing forces are not acting.
2	Zone 1	Formation of rotor position control signal in the speed range 0 ... $\omega 3$. Damping of oscillations is caused by generalized perturbing forces.
3	Zone 2	Formation of rotor position control signal in the range of speeds $\omega 3$ ... $\omega 4$. Damping of oscillations is caused by generalized perturbing forces.
4	Zone 3	Formation of rotor position control signal in the speed range $\omega 4$ ... $\omega 5$. Damping of oscillations is caused by generalized perturbing forces.

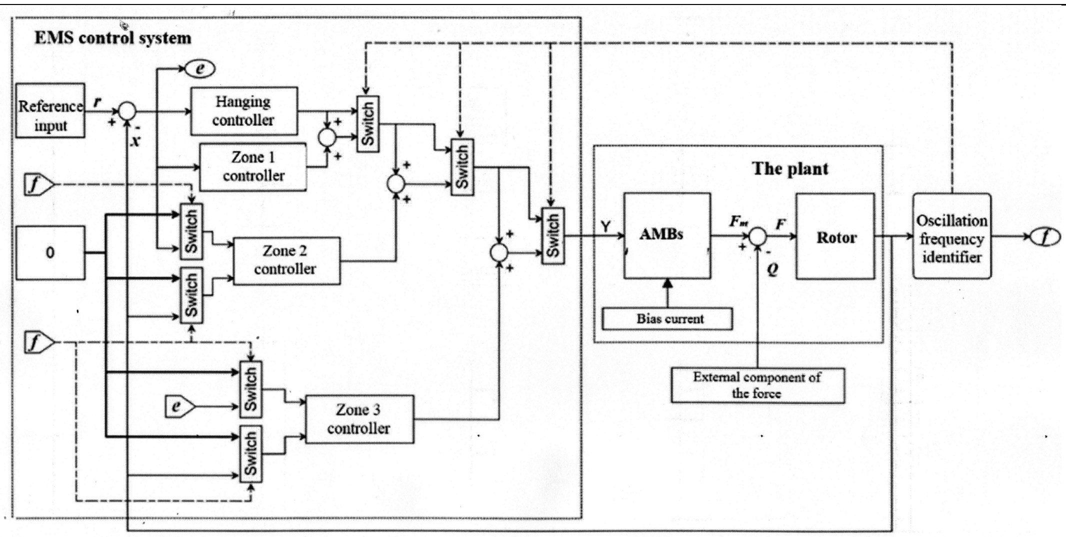

Fig. 7. Block diagram of designed control system

Selective control system consists of the following elements:

1. PID controllers:

- Option 1 – «hanging controller» with coefficients kP1, kI1, kD1;
- Option 2 – «zone 1 controller» with coefficients kP2, kI2, kD2

2. Cascade controllers

- robust law №1 - «zone 2 controller» with coefficients k0 ... k8;
- robust law №2 - «zone 3 controller» with coefficients k9 ... k17

Control law implemented by selective three-level control system (for one channel), is described by the expression:

$$Y = \begin{cases} \beta_1(r-x) + \beta_2\dfrac{1}{s}(r-x) + \beta_1 s(r-x), \quad f = 0 \\ (\beta_1+\gamma_1)(r-x) + (\beta_2+\gamma_2)\dfrac{1}{s}(r-x) + (\beta_3+\gamma_3)s(r-x), \quad f < f_1 \\ \delta_1 r + \delta_2 x + \delta_3 sr + \delta_4 sx + \delta_5\dfrac{r}{s} + \delta_6\dfrac{x}{s} + \delta_7 s^2 r + \delta_8 s^2 x + \delta_5\dfrac{r}{s^2} + \delta_6\dfrac{x}{s^2}, \quad f_1 < f < f_2 \\ \lambda_1 r + \lambda_2 x + \lambda_3 sr + \lambda_4 sx + \lambda_5\dfrac{r}{s} + \lambda_6\dfrac{x}{s} + \lambda_7 s^2 r + \lambda_8 s^2 x + \lambda_5\dfrac{r}{s^2} + \lambda_6\dfrac{x}{s^2} \quad f_2 < f < f_3 \end{cases} \tag{14}$$

5. Conclusion

In this work a mathematical model of axial AMB was developed on the basis of approximate equivalent circuit. As a result of the simulation electromagnetic characteristics of axial AMB and radial AMB were obtained. Mathematical model of axial AMB combined with a mathematical model of the radial AMB allowed us to form a five-channel model of complete electromagnetic rotor suspension and to synthesize selective control low for rotor position.

References

[1] H. Bleuler, Magnetic levitation: a challenge for control design in mechatronics, Toshiba Chair for Intelligent Mechatronics. 44 (2011) 578–583.

[2] Yu.N. Zhuravlev, Active magnetic bearings: Theory, calculation and application, Politehnica, SPb, 2003, pp. 22–30.

[3] V.P. Vereshchagin, V.A. Klabukov, A mathematical model of the axial magnetic bearing in view of eddy currents, Voprosi Electromechanici. 123 (2011) 3–8.

[4] N.D. Polyakhov, A.D. Stotckaia, Development and Investigation of Radial Active Magnetic Bearing's Mathematical Model, International Review of Electrical Engineering (I.R.E.E.). 3 (2014) 519–526.

[5] A.D. Stotckaia, Development and investigation of position control system for rotor in an electromagnetic suspension, Ph.D. diss., St. Petersburg, 2013.

[6] A.D. Stotckaia, T.O. Kuzmina, About the the robustness of the selective control of rotor position in the electromagnetic suspension, Izvestia LETI: Automation and control. 1 (2015) 51–58.