Assessments of statistical measures of syncopation: Two approaches

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Abstract

We seek a measure of the perceived "strength" of a syncopation. Existing metrics to statistically assess the syncopation of a given rhythmic sequence consider various relationships between a rhythm and the metrical grid, thus capturing different combinations of complexity and syncopation. However, as most assessments of these measures with respect to perceptual data are accuracy comparisons, precise distinctions among these metrics have yet to be directly determined.

In the first part of this study, a sample from the space of rhythms consisting of 32 isochronous onsets was assessed for syncopation under a common-time meter at different sixteenth-note offsets, using a collection of commonly-applied measures. An exploratory factor analysis was run using this data, producing three primary factors, the first of which appears to be insensitive to offset. The weighted note-to-beat distance (WNBD), off-beatness, and Keith's measure all loaded primarily onto the first factor, while Longuet-Higgins and Lee's and Toussaint's measures loaded onto either the second or third, depending on the offset. These data suggest that syncopation measures not derived from a metric hierarchy are confounded by the overall density of the rhythm.

In the second part, rhythms were sampled from the space defined by the factors from the first study, and their syncopation was rated by participants on Amazon Mechanical Turk. The obtained ratings were most closely-associated with the Longuet-Higgins and Lee measure, were negatively correlated with the aggregated measures primarily associated with density, and positively correlated with the aggregated measures derived from a metric hierarchy. These results imply that syncopation involves multiple varieties of expectation, most notably schematic and dynamic, which in turn supports the theory that syncopation is a culturally-contingent phenomenon.

Introduction

Over the past few decades, the phenomenon of perceived organizational structure in music has come under increased scrutiny. This interest has been expressed in melodic (Krumhansl, 1990; Narmour, 1989), harmonic (Loui & Wessel, 2007; Schmuckler, 1989), and rhythmic contexts (Johnson-Laird, 1991), each of which features a strong reliance on theories of musical expectation. In the rhythmic context, expectation manifests as the development of perceived meter, a process often called beat or meter induction (Desain & Honing, 1999). Like any other theory of expectation, any deviations from an expected rhythmic pattern are of particular interest, and one of the primary forms of rhythmic deviation is syncopation.

Syncopation has been generally defined as a dissociation between rhythmic emphasis and metrical emphasis (Witek, Clarke, Wallentin, Kringelbach, & Vuust, 2015; Longuet-Higgins & Lee, 1984; Temperley, 1999), although this dissociation is somewhat difficult to quantize. More generally, syncopation can be characterised as a violation of a hierarchical rhythmic expectation, where that hierarchy may take the form of a Western metrical structure, but may entail one of a variety of alternate, culturally-specific structures. Much research into computational measures of syncopation focuses primarily on the location of note onsets (Toussaint, 2003; Dixon, 2001), with comparatively little attention paid to characteristics such as velocity or intensity (Leong, 2011). As a result, a range of computational measures for syncopation have been proposed over the past few decades, each of which entails a slightly difference interpretation of deviation from a metrical grid (Gómez, Melvin, Rappaport, & Toussaint, 2005).

This variety of approaches is further complicated by the intersection between studies of syncopation and studies of rhythmic complexity. Although these two concepts are related, they are not identical; more precisely, syncopation can be considered a subtype of complexity, so while measures of syncopation necessarily measure complexity, the converse (that measures of complexity necessarily measure syncopation) is not true. If a value is a measure of syncopation, as opposed to a more generic measure of rhythmic complexity, it should be sensitive to shifts in the metrical grid, most notably by shifting a fixed grid by some metrical subdivision. For instance, for a given rhythm in 4/4 time, any measure of syncopation should change if the metrically-emphasized downbeats are shifted by a 16^{th} note, as shown in Figure 1. However, to the best of our knowledge, no measure of syncopation has been directly assessed using this methodology, although they have been compared to perceptual or behavioral assessments of syncopation (Sioros, Holzapfel, & Guedes, 2012; Fitch & Rosenfeld, 2007; Smith & Honing, 2006; Gómez et al., 2005).

Since syncopation is a form of rhythmic surprise, it can be characterized probabilistically as well. Expectation is commonly represented with conditional probabilities of the form:

$$P\big(\text{stimulus}\big|\text{context}\big). \tag{1}$$

This representation is common in surprisal-based models of linguistic parsing (Jurafsky, 1996; Levy, 2008) as well as musical (Temperley, 2007; Margulis & Beatty, 2008). Notably, linguistic versions of such an analysis are relatively agnostic as to the nature of the context, while musical instantiations tend to be extremely particular. In this case, since syncopation is rhythmic surprise under some assumed hierarchy, we can represent it probabilistically using such a conditional structure with the context specifically defined to be the metrical hierarchy under consideration. For any particular onset r and rhythmic hierarchy (in the Western tradition, metrical hierarchy) M, we have:

$$S(r|M) = -\log P(r|M) \tag{2}$$

$$P(r|M) = \frac{P(M|r)P(r)}{P(M)} \tag{3}$$

The cognitive process underpinning these probabilities can likely be modelled using a Bayesian paradigm, since the nature of this hierarchical problem bears striking similarities to Bayesian theories of neural computation (Knill & Pouget, 2004; Bar, 2007; Friston, 2010; Friston et al., 2015) as well as categorization problems in the visual world (Lee & Mumford, 2003; Bar, 2004; O'Callaghan, Kveraga, Shine, Adams Jr., & Bar, 2017). However, this process will not be explicitly developed in this paper; rather, we are concerned with the mathematical correlates of this probabilistic structure. Since syncopation is surpise-based, we can say that added syncopation of an onset is proportional to the surprisal of that onset, which in turn implies:

$$\mathrm{Sync}(r|M) \propto S(r|M)$$
$$\mathrm{Sync}(r|M) \propto -\log P(r)$$
$$\mathrm{Sync}(r|M) \propto \log P(M)$$

Therefore, overall syncopation varies inversely with density and directly with hierarchical likelihood, although these are entangled concepts. Among the plethora of available computational measures of syncopation, some clustering behavior has already been noted (Gómez, Thul, & Toussaint, 2007). This may be due in part to differently-weighted contributions of density and hierarchy to these measures.

The purpose of this study was to investigate the relationships among disparate approaches to measuring syncopation, to assess the extent to which these approaches are sensitive to offsets in the metrical grid, and to determine how these measures of syncopation align with perceptual reports of syncopation. To determine these relationships, we performed two experiments, one with a computational framework and one with a behavioral paradigm, and both of which used initial rhythmic stimuli generated using the same probabilistic approach to allow for generalizations across all three experiments. In both cases, we used single-voice rhythms, since measuring the syncopation of a multi-voice rhythm is difficult and not well-described (Sioros et al., 2012).

Computational Experiment

We generated 10,000 rhythms using Monte Carlo simulation methods to create conditional probability tables and rhythms, seeking a uniform distribution of conditional entropy values where the probability of an onset at each 16$^{\text{th}}$ note location is dependent on the preceding four onsets.[1] Then, we calculated these nine measures of syncopation for each rhythm at metric offsets of 0, 1, 2, and 3 16$^{\text{th}}$ notes:

- Weighted note-to-beat distance (WNBD) at the whole note, half note, quarter note, and eighth note metric levels (Gómez et al., 2005);
- Keith's measure for syncopation (Keith, 1991);
- Off-beatness at the quarter note metric level (Toussaint, 2003);
- Toussaint's metrical complexity measure (Toussaint, 2002); and
- Longuet-Higgins and Lee's measure of syncopation,(Longuet-Higgins & Lee, 1984) implemented as in Smith and Honing (Smith & Honing, 2006) and as in Fitch and Rosenfeld (Fitch & Rosenfeld, 2007).

Results

This procedure resulted in 36 measures of syncopations on each of the 10,000 cases, along with a single conditional entropy measure for each of the rhythms, since entropy is not dependent on the metrical grid. Parallel analysis of the resulting data resulted in the scree plot in Figure 1, which clearly indicates a three-factor model should be an adequate starting point for further analysis. Exploratory factor analysis using a minimum residual method was performed on the resulting 36 measures, resulting in three major factors with the loadings in Table 1. Although no factor model was particularly well-fitting, no other model demonstrated sufficient improvements to justify increasing the number of factors.[2]

Although the improvements in fit from using this model are weak, it does point to some similarities among the measures being analyzed. In particular, these results imply a division between measures with high positive loadings onto factor 1 and measures with a relatively small-magnitude (< 0.3) loading on that first factor, with the latter category clearly segregating into factors 2 and 3 based on where their offset lands in the eighth-note metrical grid. In addition, measures that primarily load onto the first factor occasionally have small negative loadings onto factors 2 and 3, with offsets of 0 and 2 uniformly loading to factor 3 and offsets of 1 and 3 loading onto factor 2. This implies that some measures of syncopation prioritize rhythmic complexity without a specific metrical context, that the second pair of factors is associated with syncopation with respect to the eighth-note metrical grid, and that measures (most notably the instantiations of Longuet-Higgins and Lee's measure of syncopation) which are less associated with overall complexity have stronger associations to the metrical grid. In addition, the WNBD measures for metrical levels higher than a quarter note were not implicated in metrical shifts at all, and the WNBD at the quarter note level was only implicated in factor 2 with a negative loading.

[1] The first four onsets of each rhythm were generated from a uniform probability distribution, since the conditional probability tables could not be applied.

[2] This lack of fit may be due to significant correlations between the variables in the model, as the correlation matrix was not computationally positive definite and had to be smoothed during the analysis.

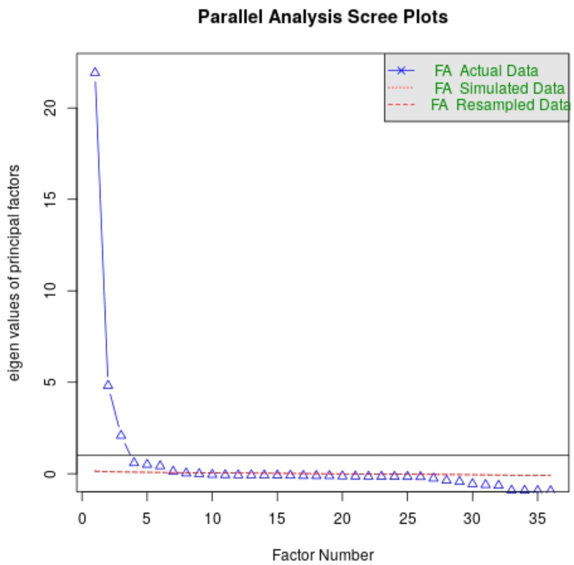

Figure 1: Scree plot for parallel analysis of possible factors for the syncopation data.

Discussion

These results imply that non-hierarchical measures like WNBD and off-beatness are only implicated in metrically-dependent syncopation when their beat resolution is sufficiently high. By contrast, hierarchical measures such as metrical complexity and the Longuet-Higgins and Lee metric, both of which are derived from Lerdahl and Jackendoff's beat hierarchy (Lerdahl & Jackendoff, 1983), are highly sensitive to metrical shifts, but comparatively naïve to overall rhythmic complexity. These two groupings can be interpreted as corresponding to density-based expectation and hierarchy-based expectation, both of which contribute to the probabilistic view of syncopation detailed earlier. The subsequent experiment was designed to investigate how these distinctions among various measures of syncopation were related to perceptual ratings of syncopation.

Behavioral Experiment

The same measures from the computational experiment were computed for 10,000 randomly generated rhythms consisting of 2 measures of 16th-note subdivisions and converted into z-scores. These z-scores were averaged for the measures primarily connected to density (WNBD at each metric level, Keith's measure, and off-beatness) and those connected to hierarchy (metrical complexity and both instantiations of the Longuet-Higgins and Lee measure), producing a two-dimensional space.[3] Twenty-five rhythms were then chosen to be evenly distributed in that space, and a minimally-syncopated and maximally-syncopated rhythm were also generated; the sample space and selected test rhythms are shown in Figure 2. In addition, five attention-check rhythms were manually generated to exhibit obviously minimal levels of syncopation.

		Factors		
Measure	Offset	1	2	3
Offbeatness	0	0.91		−0.36
Offbeatness	1	0.91	−0.37	
Offbeatness	2	0.91		−0.36
Offbeatness	3	0.91	−0.37	
WNBD-1	0	0.88		
WNBD-1	1	0.87		
WNBD-1	2	0.89		
WNBD-1	3	0.89		
WNBD-2	0	0.91		
WNBD-2	1	0.9		
WNBD-2	2	0.91		
WNBD-2	3	0.92		
WNBD-4	0	0.93		
WNBD-4	1	0.92	−0.31	
WNBD-4	2	0.93		
WNBD-4	3	0.93	−0.3	
WNBD-8	0	0.87		−0.42
WNBD-8	1	0.87	−0.42	
WNBD-8	2	0.87		−0.42
WNBD-8	3	0.86	−0.44	
Keith	0	0.93		
Keith	1	0.93		
Keith	2	0.94		
Keith	3	0.94		
LHL (Smith and Honing)	0		0.85	
LHL (Smith and Honing)	1			0.86
LHL (Smith and Honing)	2		0.88	
LHL (Smith and Honing)	3			0.88
LHL (Fitch and Rosenfeld)	0		0.84	
LHL (Fitch and Rosenfeld)	1			0.85
LHL (Fitch and Rosenfeld)	2		0.87	
LHL (Fitch and Rosenfeld)	3			0.87
Metrical complexity	0		0.5	
Metrical complexity	1			0.59
Metrical complexity	2		0.64	
Metrical complexity	3			0.6

Table 1: Factor analysis on syncopation measures at 16^{th}-note offsets, with no loadings with magnitude under 0.3 shown (TLI = 0.216, RMSEA = 0.494).

For each rhythm, participants were played a click track at 120 bpm for one measure of 4/4 time to prime the metric context. After these two measures, the click track continued with the rhythm added for one cycle of the rhythm, consisting of two measures. The click and the rhythm were played in different voices to differentiate between the two. Participants were asked to rate the syncopation of the rhythm based on a provided definition and the previously-generated minimally- and maximally-syncopated rhythms. This experiment was constructed and deployed on Mechanical Turk using HTML and Javascript.

[3] These approximations of density- and hierarchy-based expectation are linearly related with a small effect size $\left(\beta = 9.461 \times 10^{-2}, p < 0.01\right)$, which is unsurprising given the nature of the individual measures involved. This relationship does not alter the subsequent interpretation of these data.

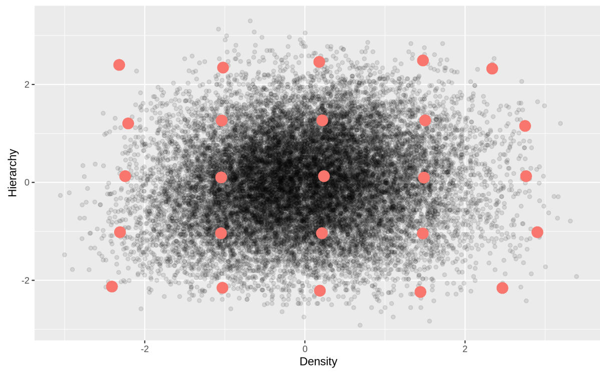

Figure 2: Sample space and selected rhythms. The red points are the 25 rhythms chosen as stimuli for the behavioral experiment.

Participants

Fifty participants (30 male, 19 female, 1 other) were recruited from Mechanical Turk, with the condition that they were using a United States IP address. They were paid for their time. Participants had a mean age of 34.28 years ($\sigma = 10.19$), and 11 self-reported as playing an instrument.

This sample was restricted based on performance on the attention-check rhythms: participants who rated more than one of the attention-check rhythms to be more syncopated than any of the test rhythms were excluded from further analysis. Due to these restrictions, the population tested in this study included 20 participants (14 male, 6 female) with a mean age of 32.8 ($\sigma = 8.06$), 7 of whom self-reported as playing an instrument and 14 of whom reported listening to music on a daily basis.

Results

We correlated the reported syncopation ratings with both the Longuet-Higgins and Lee measure (Figure 3) and metrical complexity (Figure 4); both correlations were positive, and both were significant ($p < 0.001$).

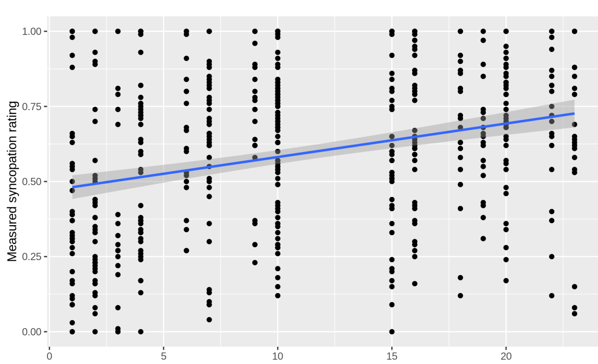

Figure 3: Correlation between Longuet-Higgins and Lee's syncopation measure and reported syncopation, including 95% confidence intervals.

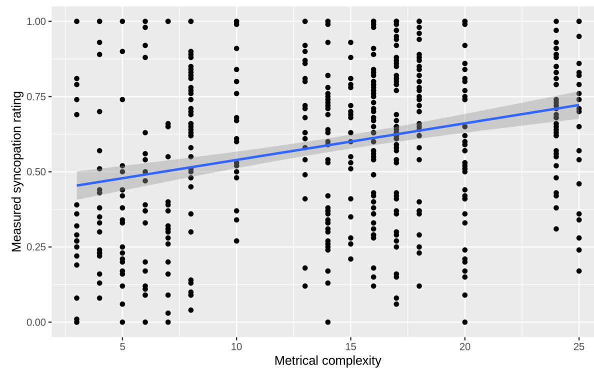

Figure 4: Correlation between metrical complexity and reported syncopation, including 95% confidence intervals.

Measure	β	AIC
LHL - Fitch & Rosenfeld	0.026	21.38
LHL - Smith & Honing	0.018	19.47
Metrical complexity	0.018	37.62
Keith	−0.012	48.23
WNBD (whole note)	−0.004	47.68
WNBD (half note)	−0.011	50.37
WNBD (quarter note)	−0.035	50.47
WNBD (eighth note)	−0.123	55.21

Table 2: Main effect coefficients and AIC values for individual measures. The reported coefficients are those relating the measure to the observed syncopation value directly.

We also ran a collection of linear mixed models predicting the obtained syncopation ratings with fixed effects for each of the eight primary measures (offbeatness was not included in this analysis) and all of the music engagement and experience variables, and compared the AIC goodness-of-fit measures for each of the resulting models. These results are summarized in Table 2.

It is apparent that the AIC values for the two LHL implementations are the lowest, and those for the density-based measures have a negative relationship with the syncopation levels reported by participants.

Lastly, we fit a linear mixed model to the obtained syncopation ratings with fixed effects for factors 1 and 2 and random effects for demographic variables pertaining to musical participation and engagement (whether the participant currently plays an instrument, how long they have played any instrument, how frequently they listen to music, and how frequently they practice). Participants were also asked how frequently they perform, but the subjects whose data was analyzed all provided the same response for this question, so it was not included in this model. The significant fixed effects of this analysis are summarized in Table 3. There were no significant interactions in these results.

This analysis was repeated with fixed effects for density and hierarchy separately, resulting in nearly identical effect

	β	SE	t	p
(Intercept)	0.39	0.06	7.05	$< 0.001***$
Density	-0.08	0.03	-3.24	$< 0.01**$
Hierarchy	0.10	0.03	3.627	$< 0.001***$
Does play	0.28	0.03	2.55	$< 0.05*$
Listens often	-0.24	0.10	-2.41	$< 0.05*$

Table 3: Significant fixed effects for linear mixed effects model. Density has a negative relationship with participant-reported syncopation ratings, while hierarchy has a positive relationship. In addition, playing an instrument leads to higher overall syncopation ratings, while listening often (as opposed to daily, sometimes, rarely, or never) leads to lower syncopation ratings.

sizes and probabilities.

Since whether or not participants currently played an instrument and how frequently participants listened to music both had significant main effects on the syncopation rating, we computed correlations between density and hierarchy and reported syncopation ratings, stratifying by different levels of those variables. These stratified correlations are shown in Figures 5 and 6, respectively.

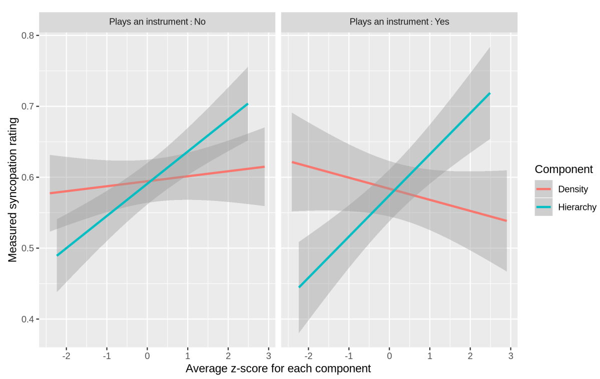

Figure 5: Correlation between both density and hierarchy and participant-reported syncopation, stratified by playing an instrument. The correlations are separated by whether the participant in question reported playing an instrument, and include 95% confidence intervals.

Discussion

The univariate correlations successfully replicated the results from both Smith and Honing (Smith & Honing, 2006) and Gómez et al. (Gómez et al., 2007), implying both that this implementation of their measure was accurate and that our syncopation ratings broadly align with the data used in their studies. Furthermore, the AIC comparison for the mxied models featuring each measure individually support the clustering of these measures into distinct density- and hierarchy-based groups, which are distinguished both by the sign of each measure's relationship with the participants' syncopation ratings and by the AIC values themselves. Generally speaking,

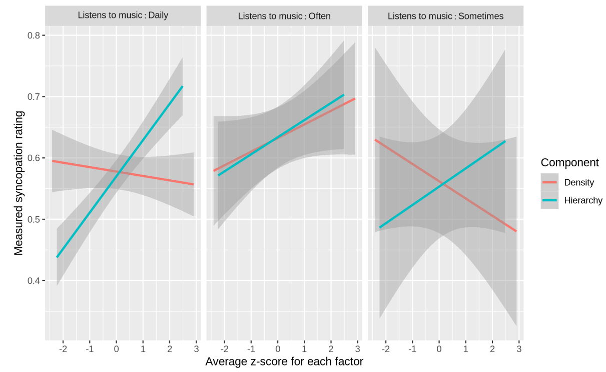

Figure 6: Correlation between both density and hierarchy and participant-reported syncopation, stratified by listening frequency. The correlations are separated by how often the participant in question reported listening to music, and include 95% confidence intervals.

hierarchy-based measures (both implementations of LHL and metrical complexity, in this study) are better-aligned with perceptual syncopation ratings, which is reflected by lower AIC values.[4] This result aligns with the clustering analysis performed by Gómez, which found a similar result.

The aggregate mixed effects model's results detailing the relationships among density, hierarchy, and perceived syncopation have a few primary implications. First, although playing an instrument and listening to music frequently both have significant impacts on syncopation ratings, there were no statistically significant interactions. This may well be due to problems with the sample size: only five participants reported listening to music "often" as opposed to "daily," and only one reported listening to music "sometimes." However, the strongly significant main effects of both density ($\beta = -0.08$) and hierarchy ($\beta = 0.10$) both align with the implications of the probabilstic model of syncopation.

It is likely that the syncopation ratings obtained in this study were affected by the four-onset click track which preceded each sequence. This click track likely primed participants to expect a four-beat metrical structure, which may have exaggerated the impact of hierarchical expectation on the results. However, the negative relationship between density-first measures and the participants' syncopation ratings would not have been affected by such a priming effect, which implies that the distinction between density- and hierarchy-based measures is robust to this possible interference.

In addition, the relatively high rejection rate in this study is cause for concern, and evidence that a larger initial sample would be necessary for future investigations. Given the ease of recruitment on platforms such as Mechanical Turk, this should not be a difficult hurdle to clear in subsequent work.

[4] These values are directly comparable in part because the number of parameters is identical across models, since the same demographic variables are included in each version.

General Discussion and Conclusions

Perhaps the most immediately apparent implication of these two experiments concerns the relative effectiveness of different measures of syncopation. Not only is it clear that they emphasize different components of syncopation, it is also apparent that some of them approximate Western perceptual ratings of syncopation more closely than others, and that the quality of a given measure's fit is tied directly to that measure's reliance on hierarchical expectation, and that the measures that include an explicitly-defined hierarchy have strong fits than those that rely on modular arithmetic to derive it. This indicates that incorporating information about the hierarchical nature of rhythmic perception is vital to accurately modelling the perception of syncopation.

These disparities, and the nature of the relationships between density- and hierarchy-based measures and perceptual syncopation, provide evidence for the probabilistic view of syncopation. Both Keith's measure and weighted note-to-beat distance are directly related to the number of onsets in a rhythm, while metrical complexity controls for the number of onsets and the Longuet-Higgins and Lee measure's weighting scheme is explicitly tied to musical meter, whether derived from the generative theory of tonal music (Lerdahl & Jackendoff, 1983) or some other version of metrical structure (Palmer & Krumhansl, 1990; Smith & Honing, 2006). The closer fit of Longuet-Higgins and Lee's measure indicates that including some information about density does improve the quality of the measure.

The necessity of both density and hierarchy information to accurately represent syncopation also implies that syncopation relies on the synthesis of multiple varieties of expectation, most notably what Huron called schematic and dynamic expectations (Huron, 2006). Dynamic expectations are generated by the progress of the auditory signal itself, and correspond roughly to density, at least for novel rhythmic sequences. Schematic expectations, on the other hand, rely on generalized knowledge about how musical rhythms usually progress. This (often subconscious) awareness of typical rhythmic structure corresponds to hierarchical expectations, which is reflected in the Western tradition by the presence of musical meter, and was reinforced in this study by the use of a four-onset click track preceding the rhythm. The significant effect of listening frequency and instrument playing on syncopation ratings provides additional support for the idea that the perception of syncopation is connected to higher-level representations of rhythmic structure which are learned through exposure to and engagement with musical signals.

Because of this connection, the data gathered in these studies may support the hypothesis that much of musical expectation is culturally-dependent. Exposure to different varieties of music will result in different representations of hierarchical rhythmic structure. As a result, schematic expectations likely vary along with that prior musical exposure, and the consequent perception of syncopation will also vary. This is further reinforced by the comparative accuracy with which the Longuet-Higgins and Lee measure models syncopation ratings provided by participants using United States IP addresses, which may be assumed to have significant Western musical exposure.

This likely cultural dependency of syncopation also implies a tension between conceptualizations of syncopation as a perceptual phenomenon and as a music-theoretic phenomenon. Syncopation, defined in terms of metrical structure, can only exist in musical cultures that rely on meter for a rhythmic backbone. Rhythmic surprisal, on the other hand, is agnostic with respect to the specific structural parameters used to generate schematic expectations. Rather, it relies on established cognitive processes similar to those that have already been shown to be generalizable across cultural boundaries in language processing (Hagoort, 2005; Turken & Dronkers, 2011), and given the significant overlap between linguistic and musical neuroanatomy (Patel, 2003, 2006), this generalization to higher-level structures is likely paralleled in music perception. The data described in this study provide evidence supporting the cognitive-representation theory of syncopation, and therefore imply that music-theoretic analyses of syncopation are specific use-cases of the broader psychological phenomenon of temporal expectation.

Future Directions

There are a few broad areas of future investigation that seem especially relevant, given these results. It would be fruitful to investigate different cultural environments to probe the hierarchical aspect of syncopation. In order to minimize cross-cultural schematic interference, it may be necessary to move away from the web-based paradigm and towards a more traditional on-site ethnographic approach, such as that taken by Fritz et al. (Fritz et al., 2009) and Jacoby and McDermott (Jacoby & McDermott, 2017).

The interaction between onset timing and timbral change could also be worth examining, both in its own right and as a gateway to the study of rhythms with multiple voices. Most real-world musical signals include a variety of timbres and dynamic levels, so systematically assessing the effect of variations in these domains is necessary to improve the ecological validity of the results. Along the same lines, studies explicitly probing syncopation in metrically-ambiguous or polyrhythmic contexts are also of great interest. However, as the syncopation level of polyphonic music is much harder to characterize (Sioros et al., 2012), such research must initially proceed without the use of computational measures to provide a clearly-defined sample space.

Lastly, there is ample space for investigating the neurological correlates of rhythmic expectation, possibly drawing on Bayesian models of surprisal (Broderick, Anderson, Di Liberto, Cross, & Lalor, 2018; Fram, 2019) and established event-related potential (ERP) phenomena such as the mismatch negativity (Lieder, Klaas, Daunizeau, Garrida, & Friston, 2013). This could enable the study of surprisal on an onset-by-onset basis, and possibly lead to a much more thorough understanding of how rhythmic expectations are built throughout a musical signal.

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