Homework #4

Problem 1: Dies by Calculation

Suppose a fair die is tossed three times.

   a. Let X be the largest of the faces that appear. Write with justification the probability density function of X.

   b. Let Y be the number of different faces that appear. Write with justification the probability density function and the cumulative distribution function F_Y of Y. Plot the graph of F_Y .

Solution.

   a. Probability mass function of X:

x	0	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{216}$	$\frac{7}{216}$	$\frac{19}{216}$	$\frac{37}{216}$	$\frac{61}{216}$	$\frac{91}{216}$	$\frac{127}{216}$

   To find the probabilities for each value of X, we use the formula $\dfrac{k^3}{6^3} - \dfrac{(k-1)^3}{6^3}$, since $P(x \le k) - P(x \le k-1) = P(x = k)$

   b. Probability mass function of Y:

y	1	2	3
$P(Y = y)$	$\frac{6}{216}$	$\frac{90}{216}$	$\frac{120}{216}$

   $P(x = 1) = 6 \cdot 1 \cdot 1 = 6$, since there must be only 1 distinct number. $P(x = 2)$ $= 6 \cdot 5 \cdot 1 = 30$, since there

   Cumulative distribution function of Y:

   $$F_Y(t) = \begin{cases} 0 & t < 1 \\ \frac{1}{36} & 1 \le t < 2 \\ \frac{4}{9} & 2 \le t < 3 \\ 1 & 3 \le t \end{cases}$$

   Plot of F_Y: