

# Homework #4

## Problem 1: Dies by Calculation

Suppose a fair die is tossed three times.

- Let  $X$  be the largest of the faces that appear. Write with justification the probability density function of  $X$ .
- Let  $Y$  be the number of different faces that appear. Write with justification the probability density function and the cumulative distribution function  $F_Y$  of  $Y$ . Plot the graph of  $F_Y$ .

**Solution.**

- Probability mass function of  $X$ :

$x$	0	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{216}$	$\frac{7}{216}$	$\frac{19}{216}$	$\frac{37}{216}$	$\frac{61}{216}$	$\frac{91}{216}$	$\frac{127}{216}$

To find the probabilities for each value of  $X$ , we use the formula  $\frac{k^3}{6^3} - \frac{(k-1)^3}{6^3}$ , since  $P(x \leq k) - P(x \leq k-1) = P(x = k)$

- Probability mass function of  $Y$ :

$y$	1	2	3
$P(Y = y)$	$\frac{6}{216}$	$\frac{90}{216}$	$\frac{120}{216}$

$P(x = 1) = 6 \cdot 1 \cdot 1 = 6$ , since there must be only 1 distinct number.  $P(x = 2) = 6 \cdot 5 \cdot 1 = 30$ , since there

Cumulative distribution function of  $Y$ :

$$F_Y(t) = \begin{cases} 0 & t < 1 \\ \frac{1}{36} & 1 \leq t < 2 \\ \frac{4}{9} & 2 \leq t < 3 \\ 1 & 3 \leq t \end{cases}$$