Homework #4

Problem 1: Dies by Calculation

Suppose a fair die is tossed three times.

a. Let X be the largest of the faces that appear. Write with justification the probability density function of X.

b. Let Y be the number of different faces that appear. Write with justification the probability density function and the cumulative distribution function F_Y of Y. Plot the graph of F_Y .

Solution.

a. Probability mass function of X:

x	0	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{216}$	$\frac{7}{216}$	$\frac{19}{216}$	$\frac{37}{216}$	$\frac{61}{216}$	$\frac{91}{216}$	$\frac{127}{216}$

To find the probabilities for each value of X, we use the formula $\frac{k^3}{6^3} - \frac{(k-1)^3}{6^3}$, since $P(x \leq k) - P(x \leq k-1) = P(x = k)$

b. Probability mass function of Y:

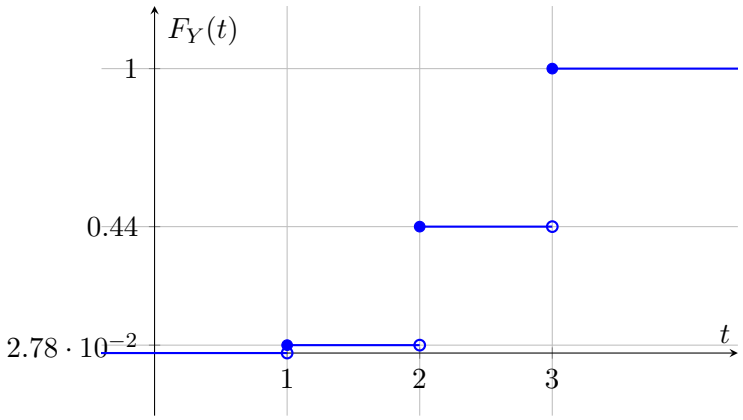
y	1	2	3
$P(Y=y)$	$\frac{6}{216}$	$\frac{90}{216}$	$\frac{120}{216}$

$P(x = 1) = 6 \cdot 1 \cdot 1 = 6$, since there must be only 1 distinct number. $P(x = 2) = 6 \cdot 5 \cdot 1 = 30$, since there

Cumulative distribution function of Y:

$$F_Y(t) = \begin{cases} 0 & t < 1 \\ \frac{1}{36} & 1 \leq t < 2 \\ \frac{4}{9} & 2 \leq t < 3 \\ 1 & 3 \leq t \end{cases}$$

Plot of F_Y:



Problem 2: Triple Flip Tally

A fair coin is flipped three times. Let Y be the number of heads minus the number of tails. Write with justification the probability density function and the cumulative distribution function F_Y of Y. Plot the graph of F_Y.

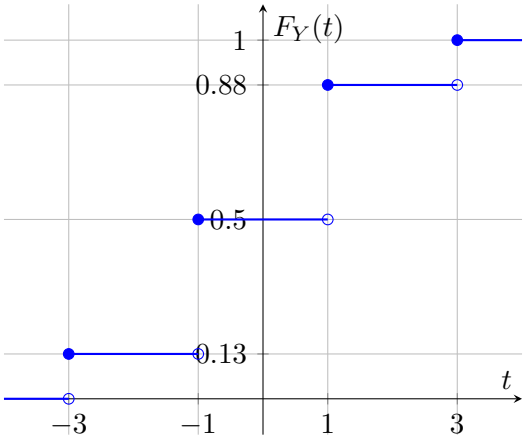
Solution. Probability mass function of Y:

y	-3	-1	1	3
$P(Y=y)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

To find the probability for each value of Y, we use the formula $\binom{3}{|x|} \cdot \frac{1}{2^3}$.
Cumulative distribution function of Y:

$$F_Y(t) = \begin{cases} 0 & t < -3 \\ \frac{1}{8} & -3 \le t < -1 \\ \frac{1}{2} & -1 \le t < 1 \\ \frac{7}{8} & 1 \le t < 3 \\ 1 & 3 \le t \end{cases}$$

Plot of F_Y:



Problem 3: Triple Flip Tally

Let X be a discrete random variable which can take only the value $x = 0, 1, 2, 3, 4, 5, 6$ such that the cumulative distribution function is defined by $F_X(x) = \dfrac{x^2 + x}{42}$ for the above values. Find the probability density function of X.

Problem 4

Suppose X is a random variable with binomial distribution $B\left(4, \frac{2}{3}\right)$. Find the probability density function of $2X + 1$.