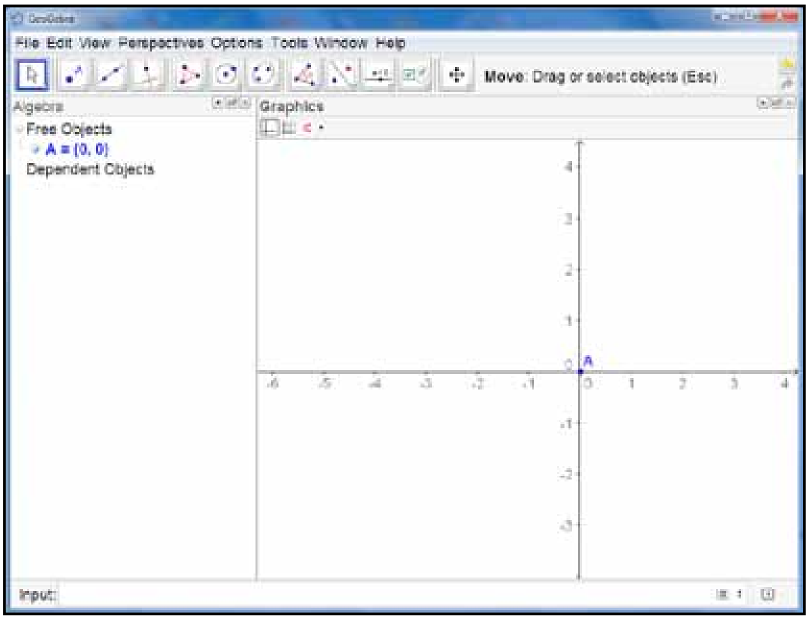
Notes:

In this chapter, we're going to take a bit of a break from programming and have a look at another extremely useful program that your Raspberry Pi can support.

GeoGebra is the scientific equivalent of Microsoft Office. But, instead of helping you write letters or create spreadsheets, it's designed for doing geometry, algebra and calculus. As with most things to do with computers, the best way to learn is to start by doing some experiments yourself. Below is what you will typically see when you start GeoGebra for the first time. Don't worry if what you see isn't exactly the same.

The GeoGebra interface.

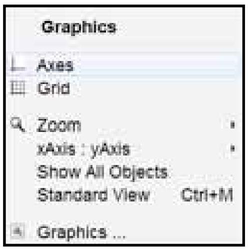


There are four different areas of the view, each of which has its own job to do. Along the top are the **Menus** and **Icons**, which allow you to make things happen. At the bottom is the **Input Bar**, where you can type stuff. In between, there is an **Algebra Window** on the left, which tells you about things you have created and, on the right, is a **Graphics Window**, which shows you what you have created.

Notes:

If we're going to get anywhere with geometry, the first challenge is to place a point on the screen. So, let's begin by learning how to create a point A.

Click on the Input Bar, type "A=(0,0)" and press Return. What happens? You should see that "A = (0, 0)" appears under the "**Free Objects**" section in the Algebra Window and the point (0, 0) is drawn in the Graphics Window and labelled "A". Now, see if you can create the point D at (-1, 0).



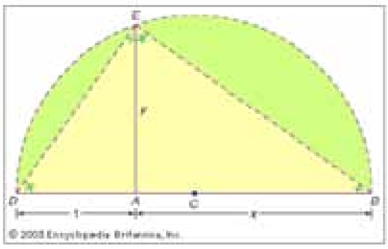
If your Graphics Window isn't showing any axes, you can right-click the mouse in it and click on the "**Axes**" icon in the pop-up box.

Our first experiment is going to be to make a computer model of a device invented by René Descartes to work out the square root of any number. At the time he found this, around 400 years ago, there were no calculators or computers, of course! The online Encyclopaedia Britannica gives us a drawing to work from.

http://www.britannica.com/EBchecked/media/67611/The-French-mathematician-Rene-Descartes-demonstrated-that-the-square-root

Descartes's calculation.

square root: Descartes's calculation



The French mathematician René Descartes (1596-1650) demonstrated that the square root of any line segment could be constructed by the simple, but ingenious, addition of a line segment with unit length.

Encyclopædia Britannica, Inc.

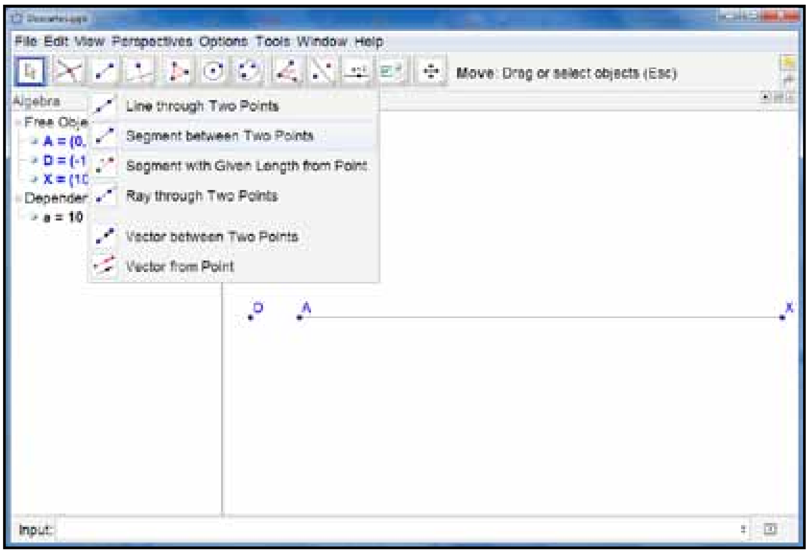
The basic principle here is that we can calculate the square root of any line segment by simply adding another line segment of unit length (length = 1). Sounds complicated? You'll soon see how it works!

To do this, we need to create a point B on the x-axis (x, 0), which we can slide about to compute any square root. Rather than let point B roam anywhere on the x-axis, we will create a segment AX as its home. So, enter "X=(10,0)". In order to see X, we need to adjust the Graphics Window. The last icon on the icon bar at the top shows four arrows. Click on this and then drag the Graphics Window so that D, A and X are roughly in the middle.

For the moment we will hide the axes and work just with lines and circles. So only the points D, A and X are showing. Now, click on the little arrow at the bottom right of the third icon to open up a dialog box for straight objects. Select the second option, "**Segment between Two Points**", click on A and then click on X. Now you should have the segment AX in the Graphics Window. When you have finished using one of the icons it is good practice to then click on the first icon (called "**Move**"), so you don't accidentally create any unwanted objects.

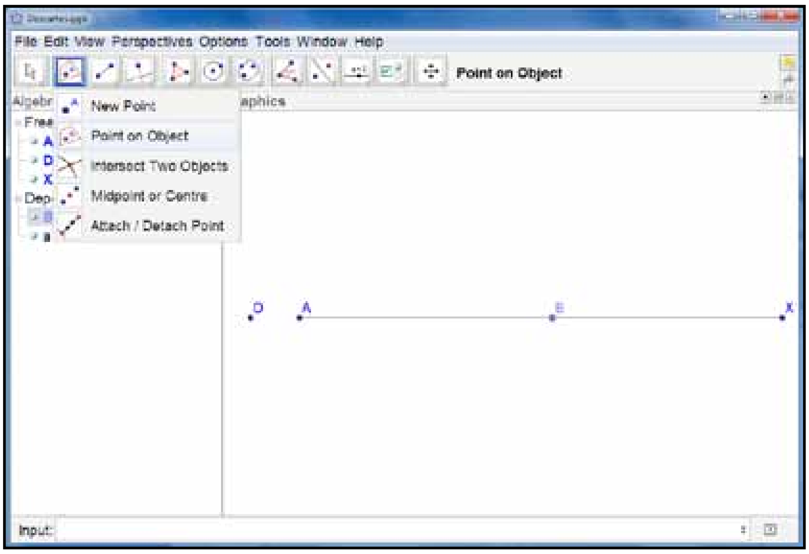
Notes:

Adding segment AX between the two points, A and X.



Next, we need to create a slideable point B on AX. Click on the little arrow on the second icon and select the option: "**Point on Object**". Click the mouse somewhere on AX. This should create a point B, which you can slide along AX.

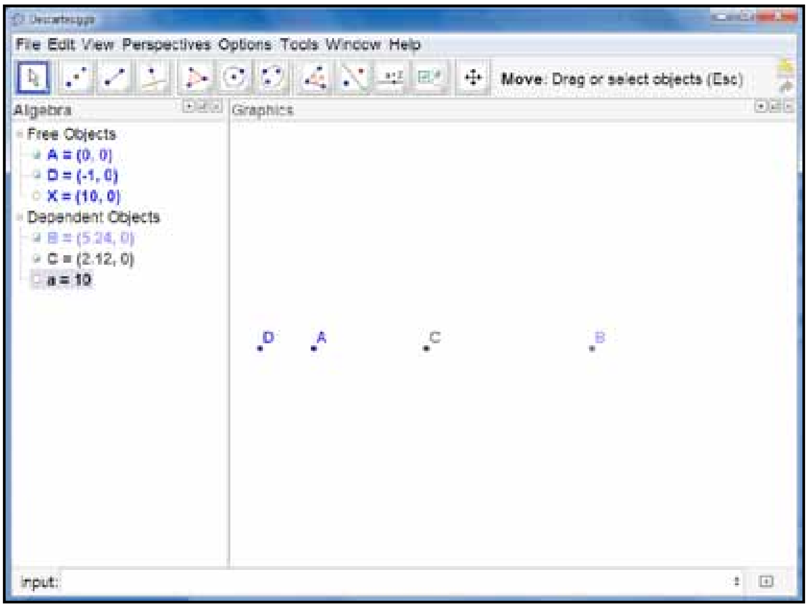
Adding a point B on the object AX.



We now need to create the circle on DB as diameter, so we need to find its centre, C, which is the mid-point of DB. Click on the arrow in the second icon and select the fourth option: "**Midpoint or Centre**".

Now we can tidy up the Graphics Window by hiding point X and segment a=AX. Just click on the little circle next to X in the Algebra Window, and do the same for the segment a.

We've found centre point C and then cleaned up X and the segment a

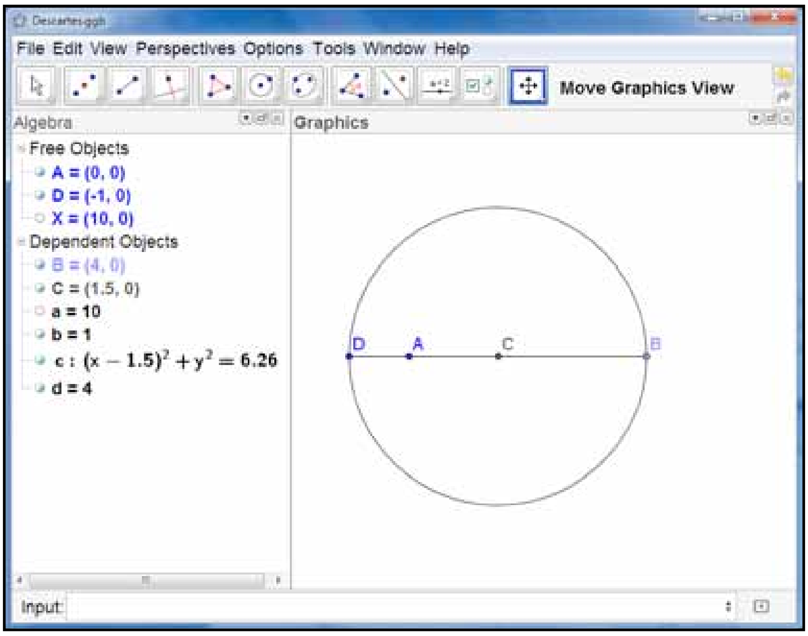


Notes:

Can you see how to draw the circle centre C through B? Try looking at the choices under the sixth icon. The first one is: "**Circle with Centre through Point**".

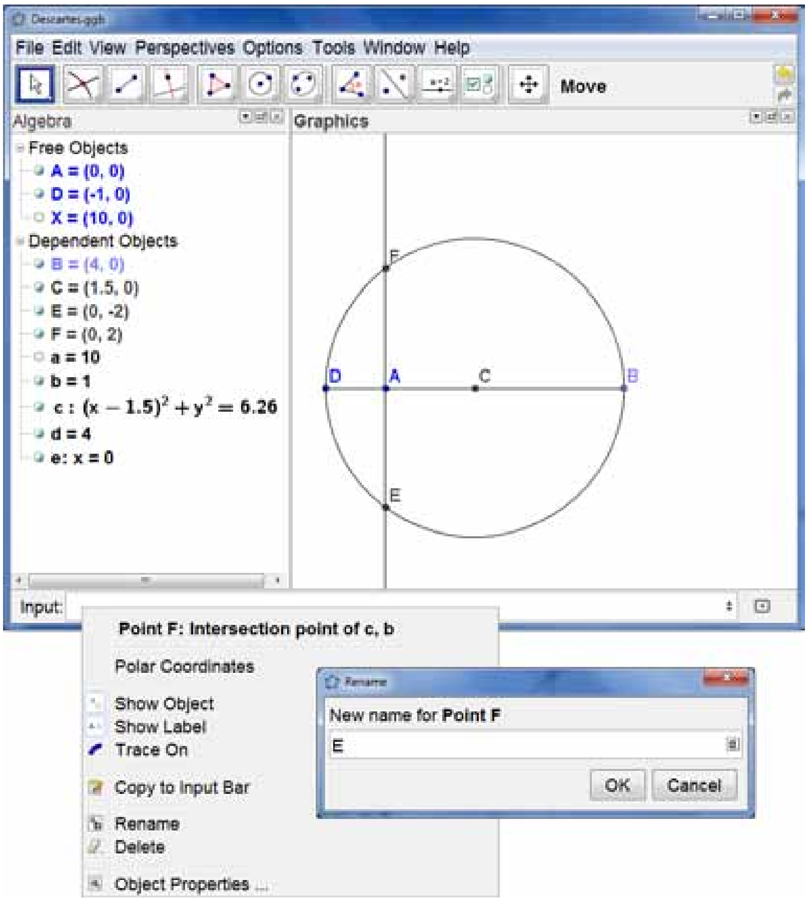
When you have created the circle you should see that its equation appears in the Graphics Window as the dependent object called c. Now, it's time for a little more tidying up. Create the segments AD and AB and slide B until it represents a number whose square root we already know, e.g. with x = 4. The length of segment AD is represented by the variable b and the length of AB by the variable d.

Circle c has been added, along with segments b=AD and d=AB.



Notes:

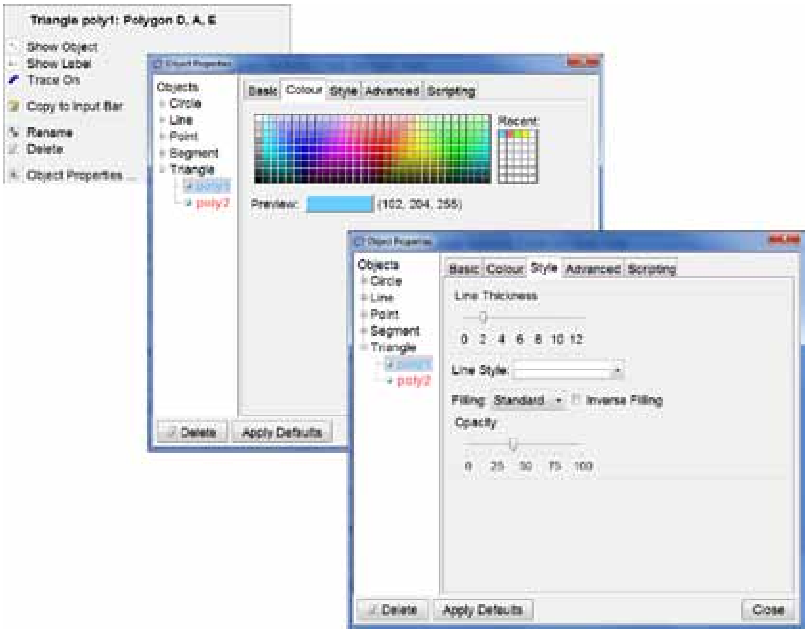
We now need to create the line perpendicular to BA through A. From the dialog box under the fourth icon select the first option, “**Perpendicular Line**”, then click on the point A and then anywhere on AB. Now use the “**Intersect Two Objects**” choice again from the second icon. Click on the perpendicular line through A and the circle centre C. This creates two points, labelled slightly differently from the original diagram. We want to rename the point shown as F, with the label E. Right click on the point F in the Graphics Window and select the option: “**Rename**”. Enter “E” in the dialog box. You should see that the other intersection point now has its name changed from E to E1.



Creating the perpendicular line through A.

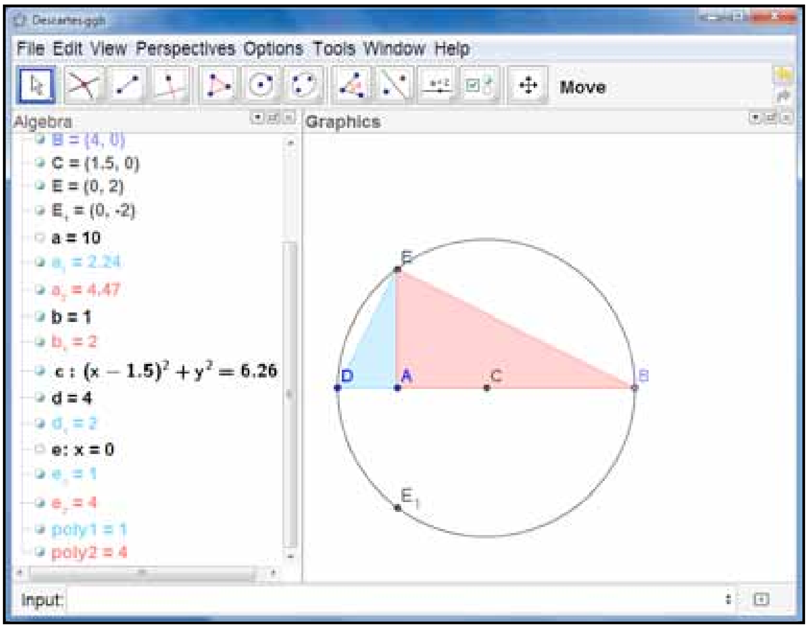
Now we have all the points we need. We will next divide the triangle DEB into the two triangles ADE and ABD. Click on the fifth icon and select the option: “**Polygon**”. Click in turn on A, D, E and then A again to create the triangle ADE. Right-click somewhere inside ADE and select the option “**Object Properties**” to change the colour and shading. Now create the triangle ABE.

Setting up the triangle polygons.



In the Algebra Window you can see that we have created two new variables, called "poly1" and "poly2", shown in colours corresponding to their shading. Their values correspond to their areas. You could rename these as "ADE" and "ABC", if you like.

Colour-coded triangles.



Now, according to Descartes, the length of AE, shown as variable b_1 is the square root of the length of AB, shown as the variable d. Slide B to change d to 5 and see what happens to b_1. In the Input Bar, type "f=b_1^2" followed by Return, and see what is shown as the value of f in the Algebra Window.

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Notes:

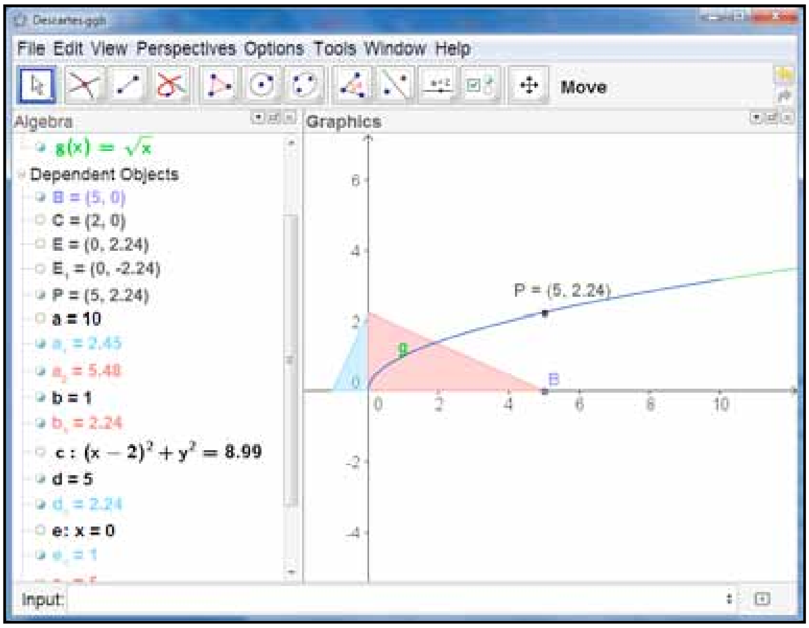
So we have verified that Descartes's technique holds for a couple of special cases. Can you prove why it works in all cases? Since BD is a diameter of a circle, what can you say about the size of the angle BED? You could check this by measuring it using the eighth icon and clicking in turn on B, E and D. We know that ADE and ABE are both right-angled triangles. So have you enough information to prove that the angles ADE and AEB must always be equal? The tangent of ADE is given by AE/AD = $b_1/1$, and the tangent of ABE is given by AB/AE = d/b_1. So, if we put these equal to each other, we can multiply both sides by b_1 to get $b_1^2 = d$.

We can finish off this first dip into GeoGebra by seeing how Descartes's geometric machine can be turned into the kind of graph we are more familiar with nowadays. First, we will define the point P, which has coordinates (b_1, d). In the Input Bar, type "P=(b_1,d)", followed by pressing Return. As you drag B along the x-axis, you should see that P moves on some sort of curve.

We could trace the positions of P as we move B by hand, but there is a smarter tool, called "Locus". From the fourth icon, select the last option: "**Locus**". Then click on P, and then on B. GeoGebra has calculated a large set of positions of P corresponding to different places for B and shown them connected by a smooth curve. If you right-click on this curve, you can change the properties of the locus called loc1, such as its colour.

Finally, in the Input Bar, type "y = sqrt(x)", then press Return. The Algebra Window shows the function g(x) and the Graphics Window shows the graph of the function. You can hide bits you no longer want, such as the circle and its centre, by clicking on the little blobs in the Algebra Window. You can zoom in and out of the Graphics Window using the mouse wheel (if it has one). You can also right-click on the graph to change its style and colour.

Using the locus tool.



Before we leave this example, you had better save your work, and maybe print it out using the "**File**" menu at the top left.

Lesson 5.2: A tale of two bridges (or "Some things to do with a camera")

Notes:

Below are photographs of a couple of bridges in Cambridge, over the "backs". The one on the left is called the "Mathematical Bridge" and is part of Queen's College. The other is the King's College bridge. We will use these as the background for some explorations with the graphics of GeoGebra.

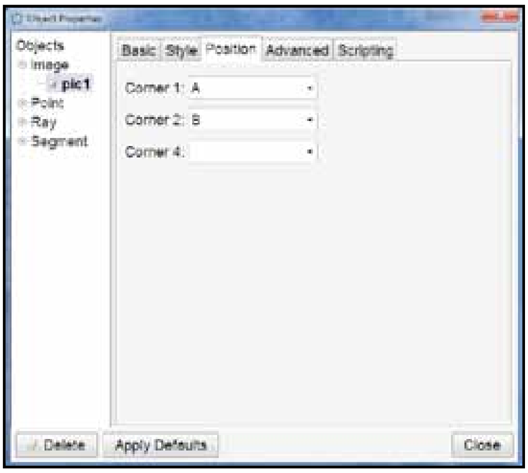
The Mathematical Bridge

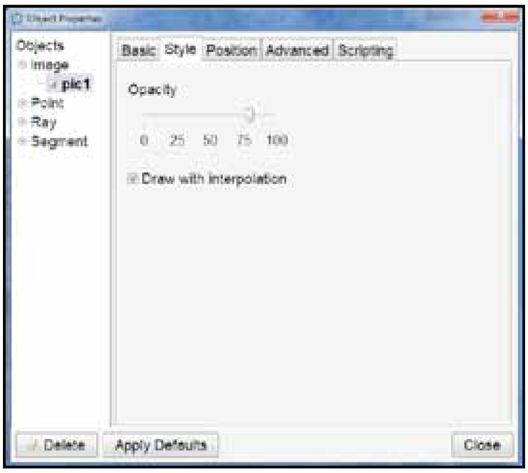
The King's College Bridge

The Mathematical Bridge

Modern digital cameras usually save photographs as images in a common picture file format, such as **jpg**. GeoGebra lets you import such files in to the Graphics Window. In order to manipulate them, it is convenient to make a segment in the window, which will be used for one edge of the image. To do this, hide the axes if they are showing, create two points (A and B), and the segment AB that joins them. Now, drop down the menu for the 10th icon and select the third option: "**Insert Image**". Click on the point A and a file menu appears of the images in the current directory. Navigate to find the file "*math bridge 3.jpg*" and load it.

It should have the point A at its bottom left corner but may be far too big. Position the mouse somewhere in the picture and right-click to bring up the dialog box for the object "Image pic1". Select "**Object Properties**" and click on the "**Position**" tab. You will see that A is already entered as "Corner 1". Enter B for "Corner 2". Click on the "**Style**" tab and adjust the slider to vary the opacity of the image – let's set it to 80%.

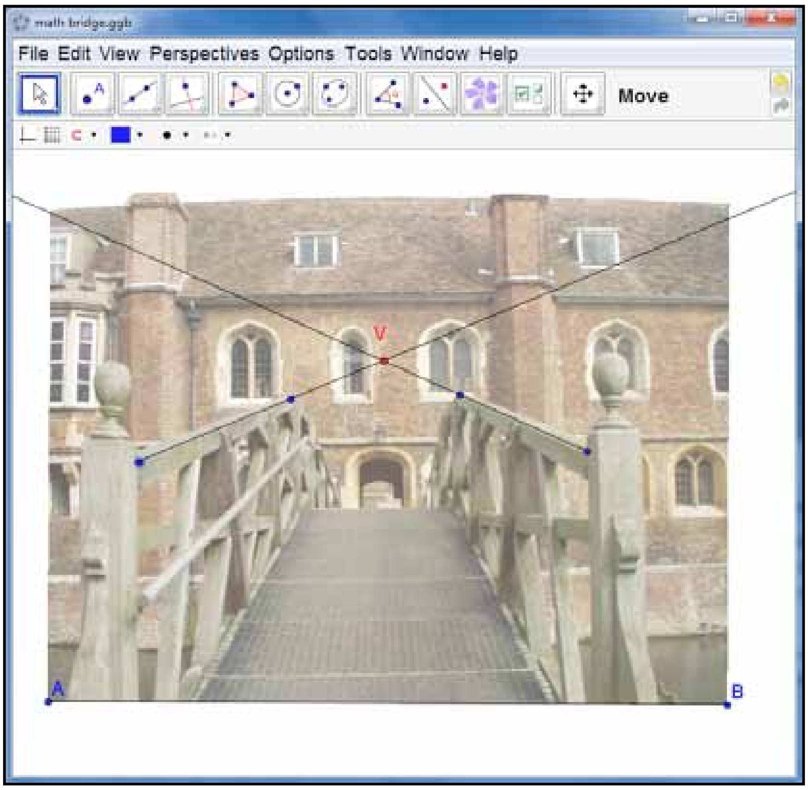




For the rest of this exercise we will only need the Graphics Window, so use the "**View**" menu to hide the Algebra Window and close the Input Bar. Use the 12th icon to move the image, the mouse-wheel to zoom in or out, and points A and B to position the bridge just where you want it.

Notes:

Establishing the vanishing point.



The reason for choosing this particular viewpoint for the photograph is to illustrate the idea of perspective – see ***http://mathworld.wolfram.com/Perspective.html***. This is where parallel objects going away from the eye, such as the top wooden rails, appear as segments of lines that meet at a point, called a "**vanishing point**".

To find this point, you need to do the following:

1. Create two points on the left-hand top rail and the ray from one through the other.
2. Repeat for two points on the right-hand top rail.
3. Find the point V, where the two rays meet.

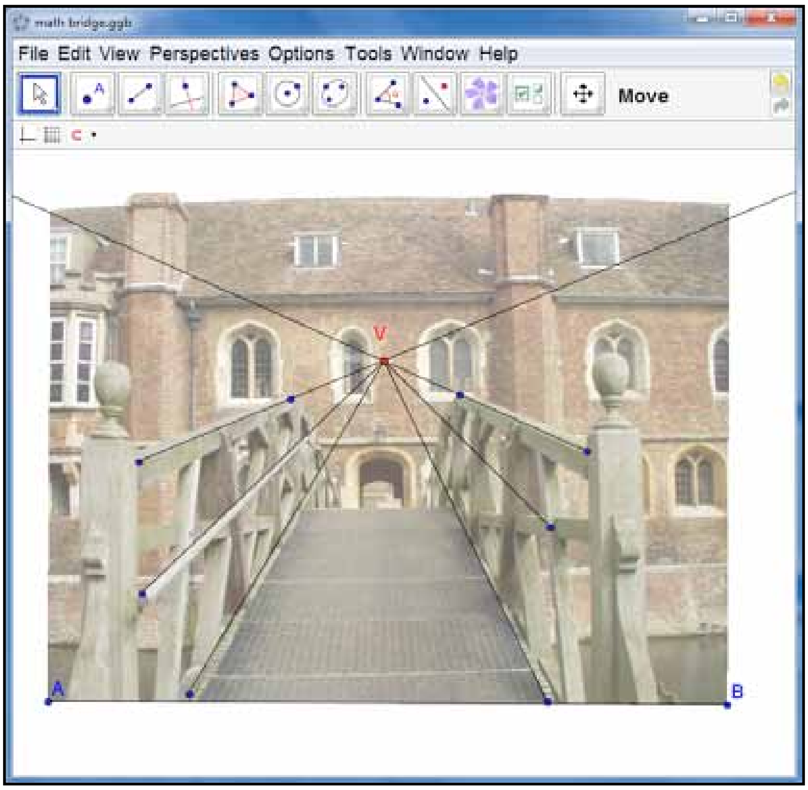
Add some more points on different parts of the bridge and connect them to V with segments. We can see that the hand-rail on the left, the mid-points of the trusses on the right and the edges of the footway are all parallel lines passing through point V.

The rules of perspective were discovered by the Florentine architect called F. Brunelleschi (1377-1446). Other sets of parallel lines going away from the lens will meet in different vanishing points, which lie on the horizon – a line through V perpendicular to the direction of view.

Now we can start creating imaginary objects on the bridge that conform to the rules of perspective – the basis of some of the special effects used in films and video games. The image below shows the outline of a box that has appeared from nowhere using four free points I, J, K and L. Segment EF represents a line across

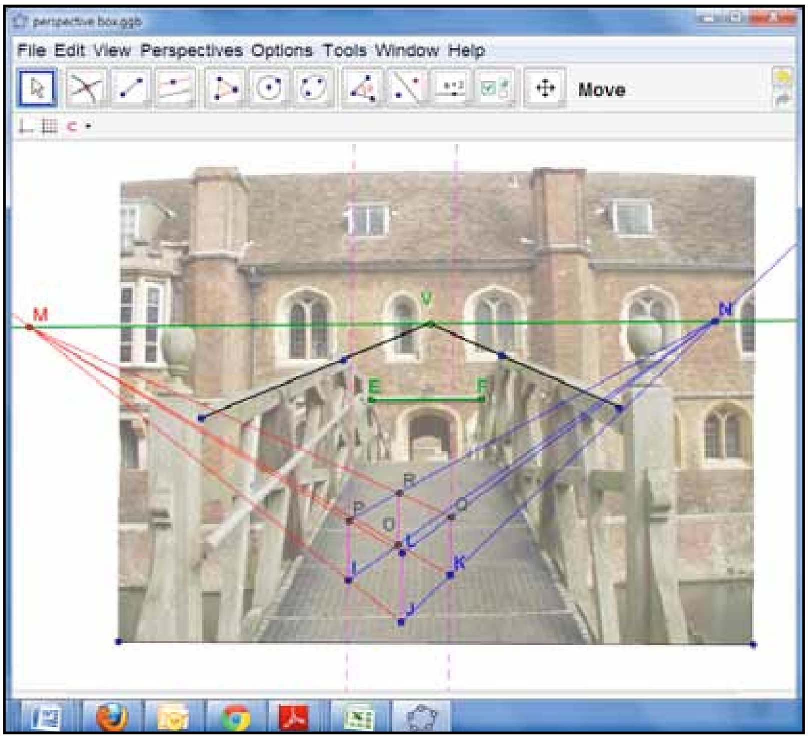
Notes:

Adding in some parallel lines.



the field of view, so the parallel to this through V represents the horizon. The ray JI meets the horizon at the vanishing point M, and the ray JK meets it at N. So, KM represents a line parallel to JM, and IN represents one parallel to JN. They meet at point O, the fourth vertex of the parallelogram IJKO. If JL represents a vertical edge of the box, then parallels to it through I, J, K and O represent the vertical edges. Hence, we can construct the remaining vertices P, R and Q by intersections with segments such as LM, QM, LN and PN.

Creating an imaginary shape using the vanishing point.



Now would be a good time to save the file.

Notes:

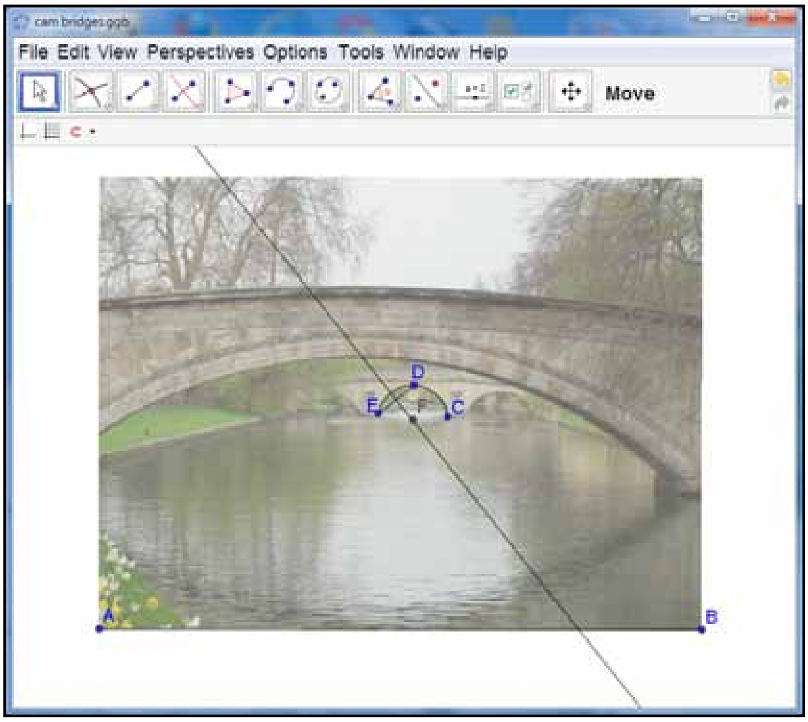
The King's College Bridge

Let's start a new file for use with the King's College bridge.

Again, create two points (A and B) and the segment AB. Then load in the picture file "*cam bridge 2.jpg*", using A as one corner. Adjust the "**Object Properties**" of the image so that B is the second corner and adjust the opacity. Hide the Algebra Window and the Input Bar. Manipulate the image to fill the Graphics Window. Actually, we now have two bridges for the price of one: Clare College Bridge appears beyond King's College Bridge. We will investigate the idea that the central span of Clare could be an arc of a circle – almost a semi-circle.

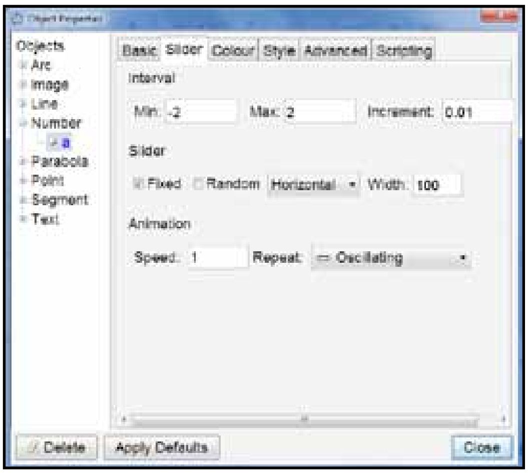
King's College Bridge, with Clare College Bridge in the background.

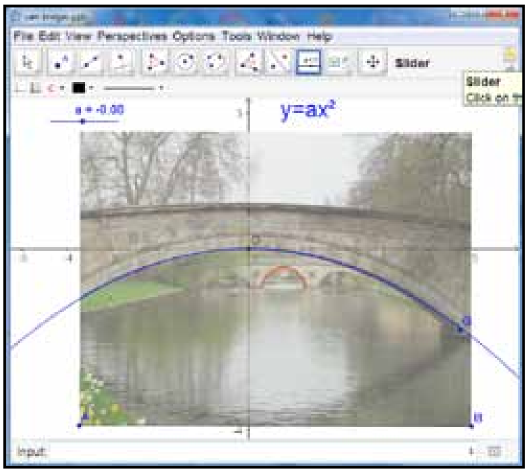
Does the central span of Clare form a semi-circle?



From the drop-down menu for the sixth icon, select the seventh option: "**Circumcircular Arc through Three Points**". Click on three points on the central arch to create the arc CDE. The segment DE is a chord of a circle, so its perpendicular bisector passes through the centre F, where the perpendicular bisectors of chords CD and DE meet. Looks good? Look up techniques for building stone or brick arches and see if circular arcs are likely. Gives some reasons why the arc CDE might not be a semi-circle.

The last experiment we'll try with the King's Bridge is to see if it could be modelled by a quadratic function – or the shape the ancient Greek geometers called the "**parabola**". Right click in the Graphics view and select "**Axes**". From the "**View**" menu, select "**Input**" and "**Show**". Type "O=(0,0)" and drag the axes to the centre of the screen. Then drag the segment AB to position the top of the arch under the point O. Create a new point, G, and drag it to the bottom-right of the arch. In the Input bar, type "y=a*x^2". You won't see any graph because the variable a doesn't yet have a value. This is where we can use a neat GeoGebra device called a "slider". From the 10th icon, select the first option: "**Slider**".





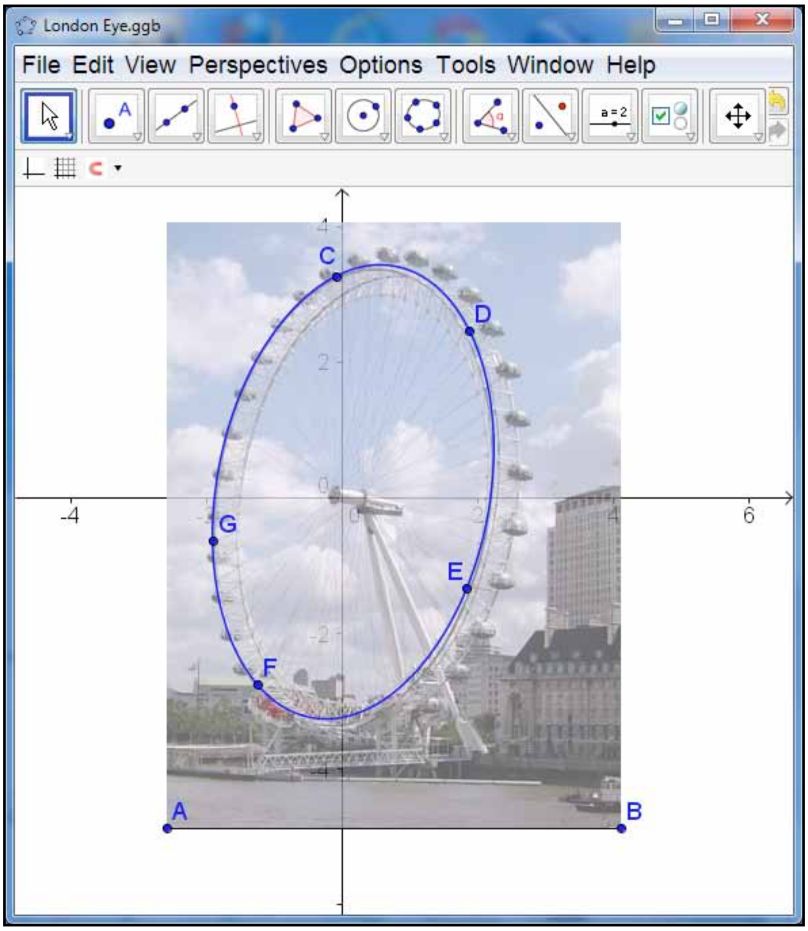
Click in the Graphics Window to create a slider for the variable a. Right-click on it and select "**Object Properties**". Set the value of "Min" to -2, "Max" to 2, and "Increment" to 0.01. Now, as you slide a along its segment, you should see the shape of the graph change. Can you find a value of a for which the graph passes as close as possible to G? Of course, the actual value of a depends on the units of measurement for the axes.

So, it would be helpful to know just how wide the river is as it passes under the King's bridge. From Google Earth, we can find that the span is roughly 20 metres, so the x-coordinate of G should be 10 instead of about 5, as shown. Can you adjust the size of the image so that G is in the right place – and find an estimate for the height of the bridge above the water?

Notes:

Notes:

The London Eye.

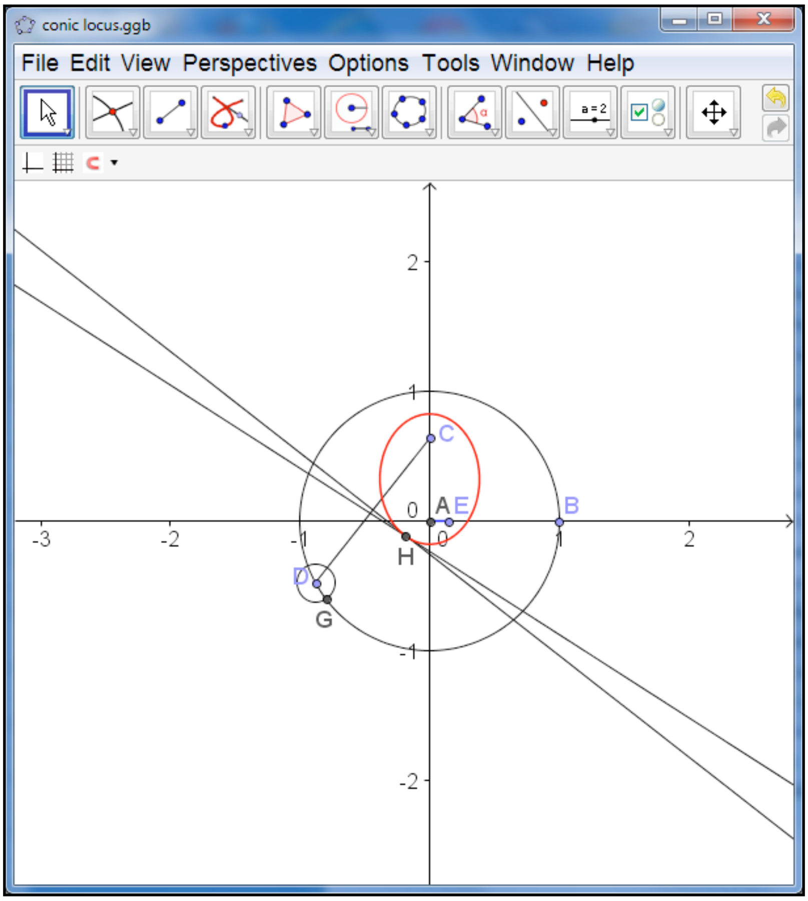


Here are a couple of ideas related to other classic curves. Find an image of the London Eye and import it to the Graphics Window (or you could take a photo of one of your bike's wheels). Adjust its transparency and position. From the seventh icon select the last option: "**Conic through Five Points**". Click on five places on the curve representing the outer edge of the wheel. The shape you have created is known as an "**ellipse**", which – along with the parabola and hyperbola – make up the class of curves called the "**conics**". We will see an easy construction that produces such curves, as well as introducing another natty GeoGebra idea – that of a "**locus**".

Save your work and start a new file. Create points A (0, 0) and B (1, 0). Construct the circle centre A through B. Create a point C on the y-axis. Create a point D on the circle. Construct the segment DC and its perpendicular bisector. Drag D round the circle and imagine that you make a fold along the bisector at regular intervals. What sort of shape would the folds enclose?

Create a point E on the x-axis, and the segment AE. From the fifth icon select the "**Compass**" tool. Use the segment AE and the centre D to create a new circle and find its intersections F and G with the unit circle. Hide F. Construct the perpendicular bisector of GC and find its intersection H with the bisector of DC. Drag D round the circle and follow the path of H – what sort of shape is it?

Finding classic curves.



From the fourth icon select the "**Locus**" option, click on H and then on D. Move E close to A, so that you have a little circle centre D. Right-click on the locus and adjust its colour and thickness. Now see what happens to the shape of the locus as you slide C up and down the y-axis.

When C is outside the unit circle, the shape of the locus is called a "**hyperbola**". Why not find out more about the ellipse, parabola and hyperbola – such as the path of planets around the sun or the shape of the mirror in a reflecting telescope? You often find parabolic shapes on the outside of houses – what function do they perform?

Notes:

Lesson 5.3: Pistons and cranks – linkages in action

Notes:

GeoGebra is being developed to support many aspects of science, technology, engineering and mathematics, including data-logging, 3D stereo vision and robotics. The final example in this short introduction is taken from mechanical engineering and is the basis of mechanisms such as pumps and the pistons that make car engines work. Here you will explore a simple-but-important mechanism that can turn circular motion into linear motion or vice versa – the piston and crank.

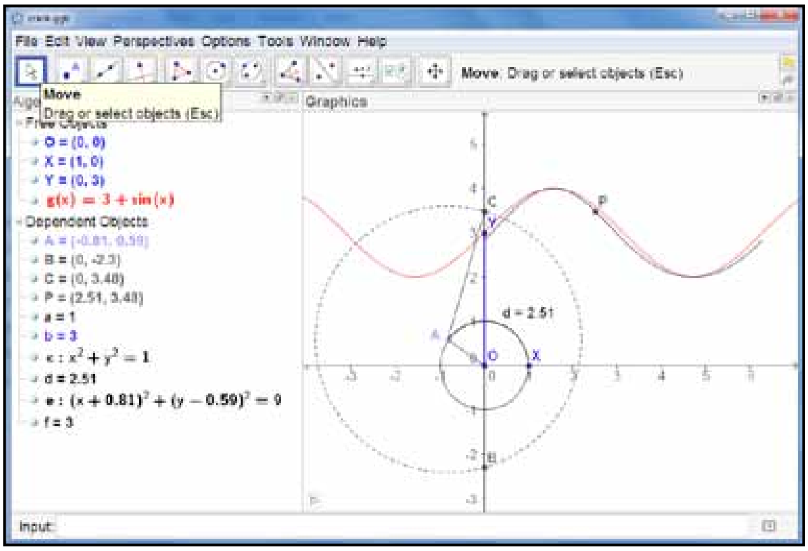
To start with, you just need to make a point rotate around a circle. So create points O (0, 0) and X (1, 0), and the circle centre O through X. Create a point A on this circle and the segment OA. Now, create the arc d, which joins X to A and measure its length. Select the "**Circular Arc with Centre between Two Points**" tool, and click in turn on X, A and O. In the Algebra Window, you will see that d has been created and measured.

Now, you need to know how to construct a point C on the y-axis above O, which is always the same fixed length from A. To do this, first create the point Y (0, 3) on the y-axis to define the length of the crank OY, which will be used to create C. Construct the segment OY. From the sixth icon, select the "**Compass**" tool. Click on the segment OY, and then click on the point A. This creates a circle centre A, whose radius is equal to OY. Construct the intersection points B and C of this circle with the y-axis.

Check that, as you drag the point A round its home circle, the point C moves up and down on the y-axis above O. Right-click on the point A and select "**Animation On**". All being well, you have created your first working mechanism! In order to know more about the motion of C on the y-axis, we can create a point P, whose x-coordinate is the arc-length of d and whose y-coordinate is that of C: P=(d, y(C)). Now, as you drag or animate A on its circle, you should see that P traces out a snake-like curved path. In order to reveal this, first select the "**Locus**" tool from the fourth icon, then click on P and then click on A.

You should see a section of a curve that has a smooth top part (around the maximum) and a smooth bottom part (around the minimum). The shape of the curve looks like that of a sine wave, but it isn't quite! In the Input Bar, enter "3+sin(x)" to see a sine-wave that has a mean value of 3 and an amplitude of 1. It should be quite a good fit, but not an exact one. Remember to save your work.

The result we have created is almost the same as a sine wave, but not quite.



You should now be in a good position to explore other mechanisms, as well as inventing ones of your own. A common mechanism is called the "**four-bar linkage**". See what you can find out about it and how it is used. Can you model it with GeoGebra? Happy inventing!

Notes: