TO CORRECT: there are missing Y_3, Y_4 in the middle, plus the terms coming from D_0 that have $\partial Y_3, \partial Y_4$.

Planar case

A general term with $i <_1 j <_1 k$ has the form $\Gamma = Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2$, where Y_i's are clouds of $<_1$ and X_j's are clouds of $<_2$. Denoting by ∂Y_i the summands of $D_0 Y_i$ in which we split a cloud of $<_2$ belonging to Y_i, we have the following formulas:

$$
\begin{aligned}
D_1^i D_2^{jk}(Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2) = \; & Y_1|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2 \\
+ \; & Y_1|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|Y_2||Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|Y_2 \\
+ \; & Y_1|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_1 i X_2|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_3 j X_4|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|Y_2||Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_3 j X_4|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|Y_2||Y_1|X_3 j X_4|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_1 i X_2|X_5 k X_6|Y_2||Y_1|X_3 j X_4|X_5 k X_6|Y_2
\end{aligned}
$$

$$
\begin{aligned}
D_1^j D_2^{ik}(Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2) = \; & Y_1|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_1 i X_2|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_3 j X_4|Y_2||Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_5 k X_6|Y_2||Y_1|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_5 k X_6|Y_2||Y_1|X_3 j X_4|Y_2||Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_3 j X_4|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_3 j X_4|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_3 j X_4|Y_2||Y_1|X_1 i X_2|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_5 k X_6|Y_2
\end{aligned}
$$

$$
\begin{aligned}
D_1^k D_2^{ij}(Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2) = \; & Y_1|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2 \\
+ \; & Y_1|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_5 k X_6|Y_2||Y_1|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_1 i X_2|X_5 k X_6|Y_2||Y_1|X_3 j X_4|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_5 k X_6|Y_2
\end{aligned}
$$

$$
\begin{aligned}
D_2^{ij} D_1^k(Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2) = \; & Y_1|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2 \\
+ \; & Y_1|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_5 k X_6|Y_2 \\
+ \; & Y_1|X_1 i X_2|X_3 j X_4|Y_2||Y_1|X_1 i X_2|X_3 j X_4|X_5 k X_6|Y_2||Y_1|X_5 k X_6|Y_2
\end{aligned}
$$

$$D_2^{ik}D_1^j(Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2) = Y_1|X_3jX_4|Y_2||Y_1|X_1iX_2|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_3jX_4|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_3jX_4|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_5kX_6|Y_2||Y_1|X_3jX_4|Y_2$$

$$D_2^{jk}D_1^i(Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2) = Y_1|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|Y_2$$
$$+ Y_1|X_1iX_2|Y_2||Y_1|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|Y_2$$

$$D_0D_3^{ijk}(Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2) = \partial Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_3jX_4|Y_2||Y_1|X_1iX_2|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_5kX_6|Y_2||Y_1|X_3jX_4|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|Y_2||Y_1|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5X_6|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|\partial Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|X_5X_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||\partial Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_3jX_4|Y_2||Y_1|X_1iX_2|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_5kX_6|Y_2||Y_1|X_3jX_4|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|Y_2||Y_1|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5X_6|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|\partial Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|X_5X_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|Y_2||Y_1|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_3X_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3X_4|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_1X_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1X_2|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|Y_2||Y_1|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3jX_4|X_3X_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_3X_4|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1iX_2|X_1X_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$
$$+ Y_1|X_1X_2|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2$$

$$\begin{aligned}
D_3^{ijk}D_0(Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2) = {} & \partial Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2 \\
& + Y_1|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_5X_6|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_5kX_6|\partial Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_5kX_6|X_5X_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||\partial Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5X_6|X_5kX_6|Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|\partial Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|X_5X_6|Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_3X_4|X_5kX_6|Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3X_4|X_3jX_4|X_5kX_6|Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_1X_2|X_3jX_4|X_5kX_6|Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1X_2|X_1iX_2|X_3jX_4|X_5kX_6|Y_2 \\
& + Y_1|X_1iX_2|X_3jX_4|X_3X_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2 \\
& + Y_1|X_1iX_2|X_3X_4|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2 \\
& + Y_1|X_1iX_2|X_1X_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2 \\
& + Y_1|X_1X_2|X_1iX_2|X_3jX_4|X_5kX_6|Y_2||Y_1|X_1iX_2|X_3jX_4|X_5kX_6|Y_2
\end{aligned}$$