

TO CORRECT: there is a missing Y_3 in the middle that must be adjoined, together with the D_0 terms coming from ∂Y_3 .

Right mixed case

In case $i <_1 j <_2 k$, we can write $\Gamma = Y_1|X_1iX_2|X_3jX_4kX_5|Y_2$, where Y_i 's are clouds of $<_1$ and X_j 's are clouds of $<_2$. Denoting by ∂Y_i the summands of D_0Y_i in which we split a cloud of $<_2$ belonging to Y_i , we have the following formulas:

$$\begin{aligned} D_1^i D_2^{jk} (Y_1|X_1iX_2|X_3jX_4kX_5|Y_2) = & Y_1|X_1iX_2|X_3jX_4X_5|Y_2||Y_1|X_1iX_2|X_3X_4kX_5|Y_2||Y_1|X_3jX_4kX_5|Y_2 \\ & + Y_1|X_1iX_2|Y_2||Y_1|X_1iX_2|X_3X_4kX_5|X_3jX_4X_5|Y_2||Y_1|X_3jX_4kX_5|Y_2 \\ & + Y_1|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4X_5|Y_2||Y_1|X_1iX_2|X_3X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2|X_3X_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4X_5|Y_2||Y_1|X_3jX_4kX_5|Y_2 \\ & + Y_1|X_3X_4kX_5|X_3jX_4X_5|Y_2||Y_1|X_1iX_2|Y_2||Y_1|X_1iX_2|X_3jX_4kX_5|Y_2 \\ & + Y_1|X_3X_4kX_5|X_3jX_4X_5|Y_2||Y_1|X_1iX_2|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|Y_2 \\ & + Y_1|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4X_5|X_3X_4kX_5|Y_2||Y_1|X_1iX_2|Y_2 \\ & + Y_1|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|Y_2||Y_1|X_1iX_2|X_3jX_4X_5|X_3X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2|X_3X_4kX_5|X_3jX_4X_5|Y_2||Y_1|X_1iX_2|Y_2||Y_1|X_3jX_4kX_5|Y_2 \\ & + Y_1|X_1iX_2|Y_2||Y_1|X_1iX_2|X_3jX_4kX_5|Y_2||Y_1|X_3jX_4X_5|X_3X_4kX_5|Y_2 \\ & + Y_1|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|X_3X_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4X_5|Y_2 \\ & + Y_1|X_1iX_2|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|Y_2||Y_1|X_3jX_4X_5|X_3X_4kX_5|Y_2 \end{aligned}$$

$$\begin{aligned} D_1^j D_2^{ik} (Y_1|X_1iX_2|X_3jX_4kX_5|Y_2) = & Y_1|X_1iX_2|X_3X_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4kX_5|Y_2||Y_1|X_3jX_4X_5|Y_2 \\ & + Y_1|X_1iX_2|X_3X_4kX_5|Y_2||Y_1|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4X_5|Y_2 \\ & + Y_1|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4X_5|Y_2||Y_1|X_1iX_2|X_3X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2|X_3X_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4X_5|Y_2||Y_1|X_3jX_4kX_5|Y_2 \\ & + Y_1|X_1iX_2|X_3X_4kX_5|Y_2||Y_1|X_3jX_4X_5|Y_2||Y_1|X_1iX_2|X_3jX_4kX_5|Y_2 \\ & + Y_1|X_3jX_4X_5|Y_2||Y_1|X_1iX_2|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|X_3X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2|X_3jX_4X_5|Y_2||Y_1|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|X_3X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2|X_3jX_4kX_5|Y_2||Y_1|X_3jX_4X_5|Y_2||Y_1|X_1iX_2|X_3X_4kX_5|Y_2 \end{aligned}$$

$$\begin{aligned} D_1^k D_2^{ij} (Y_1|X_1iX_2|X_3jX_4kX_5|Y_2) = & Y_1|X_1iX_2|X_3jX_4kX_5|Y_2||Y_1|X_3X_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4X_5|Y_2 \\ & + Y_1|X_1iX_2|X_3jX_4X_5|Y_2||Y_1|X_1iX_2|X_3X_4kX_5|Y_2||Y_1|X_3jX_4kX_5|Y_2 \\ & + Y_1|X_1iX_2|X_3X_4kX_5|Y_2||Y_1|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4X_5|Y_2 \\ & + Y_1|X_1iX_2|X_3jX_4X_5|Y_2||Y_1|X_1iX_2|X_3jX_4kX_5|Y_2||Y_1|X_3X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2|X_3jX_4X_5|Y_2||Y_1|X_3X_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4kX_5|Y_2 \\ & + Y_1|X_1iX_2|X_3jX_4X_5|Y_2||Y_1|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|X_3X_4kX_5|Y_2 \\ & + Y_1|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|X_3X_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4X_5|Y_2 \\ & + Y_1|X_3X_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4kX_5|Y_2||Y_1|X_1iX_2|X_3jX_4X_5|Y_2 \end{aligned}$$

$$\begin{aligned}
D_2^{ij} D_1^k (Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2) &= Y_1 | X_3 X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2 \\
&+ Y_1 | X_1 i X_2 | X_3 j X_4 X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2 || Y_1 | X_3 X_4 k X_5 | Y_2 \\
&+ Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 X_5 | Y_2 || Y_1 | X_3 X_4 k X_5 | Y_2 \\
&+ Y_1 | X_3 X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 X_5 | Y_2
\end{aligned}$$

$$\begin{aligned}
D_2^{ik} D_1^j (Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2) &= Y_1 | X_1 i X_2 | X_3 X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2 || Y_1 | X_3 j X_4 X_5 | Y_2 \\
&+ Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 X_4 k X_5 | Y_2 || Y_1 | X_3 j X_4 X_5 | Y_2 \\
&+ Y_1 | X_3 j X_4 X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 X_4 k X_5 | Y_2 \\
&+ Y_1 | X_3 j X_4 X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2
\end{aligned}$$

$$\begin{aligned}
D_2^{jk} D_1^i (Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2) &= Y_1 | X_1 i X_2 | Y_2 || Y_1 | X_1 i X_2 | X_3 X_4 k X_5 | X_3 j X_4 X_5 | Y_2 || Y_1 | X_3 j X_4 k X_5 | Y_2 \\
&+ Y_1 | X_1 i X_2 | X_3 X_4 k X_5 | X_3 j X_4 X_5 | Y_2 || Y_1 | X_3 j X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | Y_2 \\
&+ Y_1 | X_3 X_4 k X_5 | X_3 j X_4 X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | Y_2 \\
&+ Y_1 | X_1 i X_2 | Y_2 || Y_1 | X_3 j X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 X_5 | X_3 X_4 k X_5 | Y_2 \\
&+ Y_1 | X_3 j X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 X_5 | X_3 X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | Y_2 \\
&+ Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2 || Y_1 | X_3 j X_4 X_5 | X_3 X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 | Y_2 \\
&+ Y_1 | X_1 i X_2 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2 || Y_1 | X_3 j X_4 X_5 | X_3 X_4 k X_5 | Y_2 \\
&+ Y_1 | X_1 i X_2 | Y_2 || Y_1 | X_3 X_4 k X_5 | X_3 j X_4 X_5 | Y_2 || Y_1 | X_1 i X_2 | X_3 j X_4 k X_5 | Y_2
\end{aligned}$$

[illegible]

$$\begin{aligned}
D_3^{ijk} D_0(Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2) = & Y_1|X_1 i X_2|X_1 X_2|X_3 j X_4 k X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2 \\
& + \partial Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_1 X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 j X_4 k X_5|X_3 X_4 X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2||Y_1|Y_2 \\
& + Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||Y_1|X_1 i X_2|X_3 X_4 X_5|X_3 j X_4 k X_5|Y_2 \\
& + Y_1|X_1 X_2|X_1 i X_2|X_3 j X_4 k X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2||Y_1|Y_2 \\
& + Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||Y_1|X_1 i X_2|X_1 X_2|X_3 j X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|X_3 X_4 X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2||Y_1|X_1 X_2|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|\partial Y_2 \\
& + Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2||Y_1|X_1 i X_2|X_3 X_4 X_5|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||Y_1|X_1 X_2|X_1 i X_2|X_3 j X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 j X_4 k X_5|\partial Y_2||Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|X_3 X_4 X_5|Y_2 \\
& + \partial Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2 \\
& + Y_1|X_1 X_2|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||\partial Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2||Y_1|X_1 i X_2|X_1 X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 X_4 X_5|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 k X_5|\partial Y_2 \\
& + Y_1|Y_2||Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 X_4 X_5|X_3 j X_4 k X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2||\partial Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 k X_5|X_3 X_4 X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|\partial Y_2||Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2 \\
& + Y_1|Y_2||Y_1|X_1 i X_2|X_3 j X_4 k X_5|Y_2||Y_1|X_1 i X_2|X_3 j X_4 X_5|X_3 X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2||Y_1|X_1 i X_2|X_3 X_4 k X_5|X_3 j X_4 X_5|Y_2
\end{aligned}$$