

## Aligned degenerate case

In this case we only have two indices  $i < j$ . We suppose WLOG  $i < j$  with respect to the order of  $\Gamma$ , so that we can write  $\Gamma = Y_1|X_1iX_2jX_3|Y_2$ , with the usual convention for clouds of points. Then we have:

$$\begin{aligned} D_1^i D_2^{ij} (Y_1|X_1iX_2jX_3|Y_2) &= Y_1|X_1iX_2(j+2)X_3|Y_2||Y_1|X_1(i+1)X_2X_3|Y_2||Y_1|X_1(i+2)X_2X_3|X_1X_2(j+3)X_3|Y_2 \\ &\quad + Y_1|X_1iX_2X_3|Y_2||Y_1|X_1(i+1)X_2(j+2)X_3|Y_2||Y_1|X_1(i+2)X_2X_3|X_1X_2(j+3)X_3|Y_2 \\ &\quad + Y_1|X_1iX_2X_3|Y_2||Y_1|X_1X_2(j+2)X_3|X_1(i+1)X_2X_3|Y_2||Y_1|X_1(i+2)X_2(j+3)X_3|Y_2 \\ &\quad + Y_1|X_1X_2(j+2)X_3|X_1iX_2X_3|Y_2||Y_1|X_1(i+1)X_2X_3|Y_2||Y_1|X_1(i+2)X_2(j+3)X_3|Y_2 \end{aligned}$$

$$\begin{aligned} D_2^{i,j+1} D_1^i (Y_1|X_1iX_2jX_3|Y_2) &= Y_1|X_1iX_2(j+2)X_3|Y_2||Y_1|X_1(i+1)X_2X_3|X_1X_2(j+3)X_3|Y_2||Y_1|X_1(i+2)X_2X_3|Y_2 \\ &\quad + Y_1|X_1X_2(j+2)X_3|X_1iX_2X_3|Y_2||Y_1|X_1(i+1)X_2(j+3)X_3|Y_2||Y_1|X_1(i+2)X_2X_3|Y_2 \end{aligned}$$

$$\begin{aligned} D_2^{i+1,j+1} D_1^i (Y_1|X_1iX_2jX_3|Y_2) &= Y_1|X_1iX_2X_3|Y_2||Y_1|X_1(i+1)X_2(j+2)X_3|Y_2||Y_1|X_1(i+2)X_2X_3|X_1X_2(j+3)X_3|Y_2 \\ &\quad + Y_1|X_1iX_2X_3|Y_2||Y_1|X_1X_2(j+2)X_3|X_1(i+1)X_2X_3|Y_2||Y_1|X_1(i+2)X_2(j+3)X_3|Y_2 \end{aligned}$$

$$\begin{aligned} D_1^{i+1} D_2^{ij} (Y_1|X_1iX_2jX_3|Y_2) &= Y_1|X_1iX_2(j+2)X_3|Y_2||Y_1|X_1(i+1)X_2X_3|Y_2||Y_1|X_1(i+2)X_2X_3|X_1X_2(j+3)X_3|Y_2 \\ &\quad + Y_1|X_1iX_2(j+2)X_3|Y_2||Y_1|X_1(i+1)X_2X_3|X_1X_2(j+3)X_3|Y_2||Y_1|X_1(i+2)X_2X_3|Y_2 \\ &\quad + Y_1|X_1X_2(j+2)X_3|X_1iX_2X_3|Y_2||Y_1|X_1(i+1)X_2(j+3)X_3|Y_2||Y_1|X_1(i+2)X_2X_3|Y_2 \\ &\quad + Y_1|X_1X_2(j+2)X_3|X_1iX_2X_3|Y_2||Y_1|X_1(i+1)X_2X_3|Y_2||Y_1|X_1(i+2)X_2(j+3)X_3|Y_2 \end{aligned}$$