

TO CORRECT: terms in  $D_0$  that splits a 2-cloud in one part without constants are missing.

## Aligned case

A general term with  $i <_2 j <_2 k$  has the form  $\Gamma = Y_1|X_1iX_2jX_3kX_4|Y_2$ , where  $Y_i$ 's are clouds of  $<_1$  and  $X_j$ 's are clouds of  $<_2$ . Denoting by  $\partial Y_i$  the summands of  $D_0Y_i$  in which we split a cloud of  $<_2$  belonging to  $Y_i$ , we have the following formulas:

[illegible]

$$\begin{aligned}
D_1^j D_2^{ik} (Y_1 | X_1 i X_{2j} X_3 k X_4 | Y_2) = & Y_1 | X_1 i X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 k X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 k X_4 | X_1 X_2 X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 i X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 X_2 X_3 k X_4 | X_1 X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 i X_{2j} X_3 k X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 X_4 | X_1 X_2 X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 i X_2 X_3 k X_4 | Y_2 | Y_1 | X_1 i X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 X_4 | X_1 X_2 X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 i X_2 X_3 k X_4 | Y_2 | Y_1 | X_1 i X_{2j} X_3 X_4 | X_1 X_2 X_3 k X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 X_4 | Y_2 \\
& + Y_1 | X_1 i X_2 X_3 k X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 X_4 | X_1 X_{2j} X_3 k X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 X_4 | Y_2 \\
& + Y_1 | X_1 i X_2 X_3 k X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 i X_{2j} X_3 X_4 | X_1 X_2 X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 i X_2 X_3 k X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 X_4 | X_1 X_{2j} X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 i X_2 X_3 k X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 X_4 | X_1 X_2 X_3 k X_4 | Y_2 | Y_1 | X_1 i X_{2j} X_3 X_4 | Y_2 \\
& + Y_1 | X_1 i X_2 X_3 k X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 k X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 X_4 | X_1 X_{2j} X_3 X_4 | Y_2 \\
& + Y_1 | X_1 X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 i X_{2j} X_3 k X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 X_4 | X_1 X_2 X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 k X_4 | X_1 i X_2 X_3 X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 X_2 X_3 k X_4 | X_1 i X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 X_{2j} X_3 k X_4 | Y_2 | Y_1 | X_1 i X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 X_4 | X_1 X_2 X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 X_{2j} X_3 k X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 X_4 | X_1 i X_2 X_3 X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 X_{2j} X_3 k X_4 | X_1 i X_2 X_3 X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 X_2 X_3 k X_4 | X_1 i X_2 X_3 X_4 | Y_2 | Y_1 | X_1 i X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 k X_4 | Y_2 \\
& + Y_1 | X_1 X_2 X_3 k X_4 | X_1 i X_2 X_3 X_4 | Y_2 | Y_1 | X_1 i X_{2j} X_3 k X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 X_4 | Y_2 \\
& + Y_1 | X_1 X_2 X_3 k X_4 | X_1 i X_2 X_3 X_4 | Y_2 | Y_1 | X_1 X_{2j} X_3 k X_4 | Y_2 | Y_1 | X_1 i X_{2j} X_3 X_4 | Y_2 \\
& + Y_1 | X_1 X_2 X_3 k X_4 | X_1 X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 i X_{2j} X_3 X_4 | Y_2 | Y_1 | X_1 i X_2 X_3 k X_4 | Y_2
\end{aligned}$$



$$\begin{aligned}
D_2^{ik} D_1^j(Y_1|X_1 i X_2 j X_3 k X_4|Y_2) = & Y_1|X_1 i X_2 j X_3 k X_4|Y_2||Y_1|X_1 i X_2 X_3 X_4|X_1 X_2 X_3 k X_4|Y_2||Y_1|X_1 X_2 j X_3 X_4|Y_2 \\
& + Y_1|X_1 i X_2 X_3 k X_4|Y_2||Y_1|X_1 i X_2 j X_3 X_4|X_1 X_2 X_3 k X_4|Y_2||Y_1|X_1 X_2 j X_3 X_4|Y_2 \\
& + Y_1|X_1 i X_2 X_3 k X_4|Y_2||Y_1|X_1 i X_2 X_3 X_4|X_1 X_2 j X_3 k X_4|Y_2||Y_1|X_1 X_2 j X_3 X_4|Y_2 \\
& + Y_1|X_1 X_2 j X_3 X_4|Y_2||Y_1|X_1 i X_2 j X_3 k X_4|Y_2||Y_1|X_1 i X_2 X_3 X_4|X_1 X_2 X_3 k X_4|Y_2 \\
& + Y_1|X_1 X_2 j X_3 X_4|Y_2||Y_1|X_1 i X_2 X_3 k X_4|Y_2||Y_1|X_1 i X_2 j X_3 X_4|X_1 X_2 X_3 k X_4|Y_2 \\
& + Y_1|X_1 X_2 j X_3 X_4|Y_2||Y_1|X_1 i X_2 X_3 k X_4|Y_2||Y_1|X_1 i X_2 X_3 X_4|X_1 X_2 j X_3 k X_4|Y_2 \\
& + Y_1|X_1 X_2 j X_3 X_4|Y_2||Y_1|X_1 X_2 j X_3 k X_4|X_1 i X_2 X_3 X_4|Y_2||Y_1|X_1 i X_2 X_3 k X_4|Y_2 \\
& + Y_1|X_1 X_2 j X_3 X_4|Y_2||Y_1|X_1 X_2 X_3 k X_4|X_1 i X_2 j X_3 X_4|Y_2||Y_1|X_1 i X_2 X_3 k X_4|Y_2 \\
& + Y_1|X_1 X_2 j X_3 X_4|Y_2||Y_1|X_1 X_2 X_3 k X_4|X_1 i X_2 X_3 X_4|Y_2||Y_1|X_1 i X_2 j X_3 k X_4|Y_2 \\
& + Y_1|X_1 X_2 j X_3 k X_4|X_1 i X_2 X_3 X_4|Y_2||Y_1|X_1 i X_2 X_3 k X_4|Y_2||Y_1|X_1 X_2 j X_3 X_4|Y_2 \\
& + Y_1|X_1 X_2 X_3 k X_4|X_1 i X_2 j X_3 X_4|Y_2||Y_1|X_1 i X_2 X_3 k X_4|Y_2||Y_1|X_1 X_2 j X_3 X_4|Y_2 \\
& + Y_1|X_1 X_2 X_3 k X_4|X_1 i X_2 X_3 X_4|Y_2||Y_1|X_1 i X_2 j X_3 k X_4|Y_2||Y_1|X_1 X_2 j X_3 X_4|Y_2
\end{aligned}$$

$$\begin{aligned}
D_2^{jk} D_1^i(Y_1|X_1 i X_2 j X_3 k X_4|Y_2) = & Y_1|X_1 i X_2 j X_3 k X_4|Y_2||Y_1|X_1 X_2 j X_3 X_4|X_1 X_2 X_3 k X_4|Y_2||Y_1|X_1 i X_2 X_3 X_4|Y_2 \\
& + Y_1|X_1 i X_2 X_3 X_4|Y_2||Y_1|X_1 i X_2 j X_3 k X_4|Y_2||Y_1|X_1 X_2 j X_3 X_4|X_1 X_2 X_3 k X_4|Y_2 \\
& + Y_1|X_1 i X_2 X_3 X_4|Y_2||Y_1|X_1 i X_2 X_3 k X_4|X_1 X_2 j X_3 X_4|Y_2||Y_1|X_1 X_2 j X_3 k X_4|Y_2 \\
& + Y_1|X_1 i X_2 X_3 X_4|Y_2||Y_1|X_1 X_2 j X_3 k X_4|Y_2||Y_1|X_1 i X_2 j X_3 X_4|X_1 X_2 X_3 k X_4|Y_2 \\
& + Y_1|X_1 i X_2 X_3 X_4|Y_2||Y_1|X_1 X_2 j X_3 k X_4|Y_2||Y_1|X_1 X_2 j X_3 X_4|X_1 i X_2 X_3 k X_4|Y_2 \\
& + Y_1|X_1 i X_2 X_3 X_4|Y_2||Y_1|X_1 X_2 X_3 k X_4|X_1 i X_2 j X_3 X_4|Y_2||Y_1|X_1 X_2 j X_3 k X_4|Y_2 \\
& + Y_1|X_1 i X_2 X_3 X_4|Y_2||Y_1|X_1 X_2 X_3 k X_4|X_1 X_2 j X_3 X_4|Y_2||Y_1|X_1 i X_2 j X_3 k X_4|Y_2 \\
& + Y_1|X_1 i X_2 X_3 k X_4|X_1 X_2 j X_3 X_4|Y_2||Y_1|X_1 X_2 j X_3 k X_4|Y_2||Y_1|X_1 i X_2 X_3 X_4|Y_2 \\
& + Y_1|X_1 X_2 j X_3 k X_4|Y_2||Y_1|X_1 i X_2 j X_3 X_4|X_1 X_2 X_3 k X_4|Y_2||Y_1|X_1 i X_2 X_3 X_4|Y_2 \\
& + Y_1|X_1 X_2 j X_3 k X_4|Y_2||Y_1|X_1 X_2 j X_3 X_4|X_1 i X_2 X_3 k X_4|Y_2||Y_1|X_1 i X_2 X_3 X_4|Y_2 \\
& + Y_1|X_1 X_2 X_3 k X_4|X_1 i X_2 j X_3 X_4|Y_2||Y_1|X_1 X_2 j X_3 k X_4|Y_2||Y_1|X_1 i X_2 X_3 X_4|Y_2 \\
& + Y_1|X_1 X_2 X_3 k X_4|X_1 X_2 j X_3 X_4|Y_2||Y_1|X_1 i X_2 j X_3 k X_4|Y_2||Y_1|X_1 i X_2 X_3 X_4|Y_2
\end{aligned}$$

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