

TO CORRECT: there is a missing Y_3 in the middle, plus the terms coming from D_0 that have ∂Y_3 .

Left mixed case

In case $i <_2 j <_1 k$, we can write $\Gamma = Y_1|X_1iX_2jX_3|X_4kX_5|Y_2$, where Y_i 's are clouds of $<_1$ and X_j 's are clouds of $<_2$. Denoting by ∂Y_i the summands of D_0Y_i in which we split a cloud of $<_2$ belonging to Y_i , we have the following formulas:

$$\begin{aligned} D_1^i D_2^{jk} (Y_1|X_1iX_2jX_3|X_4kX_5|Y_2) = & Y_1|X_1iX_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2X_3|Y_2||Y_1|X_1X_2jX_3|X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2jX_3|Y_2||Y_1|X_1iX_2X_3|X_4kX_5|Y_2||Y_1|X_1X_2jX_3|X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2X_3|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|Y_2||Y_1|X_1X_2jX_3|X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2X_3|Y_2||Y_1|X_1iX_2jX_3|X_4kX_5|Y_2||Y_1|X_1X_2jX_3|X_4kX_5|Y_2 \\ & + Y_1|X_1X_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2X_3|Y_2 \\ & + Y_1|X_1X_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|Y_2||Y_1|X_1iX_2X_3|X_4kX_5|Y_2 \\ & + Y_1|X_1X_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2X_3|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|Y_2 \\ & + Y_1|X_1X_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2X_3|Y_2||Y_1|X_1iX_2jX_3|X_4kX_5|Y_2 \end{aligned}$$

$$\begin{aligned} D_1^j D_2^{ik} (Y_1|X_1iX_2jX_3|X_4kX_5|Y_2) = & Y_1|X_1iX_2jX_3|X_4kX_5|Y_2||Y_1|X_1X_2jX_3|Y_2||Y_1|X_1iX_2X_3|X_4kX_5|Y_2 \\ & + Y_1|X_1X_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|Y_2||Y_1|X_1iX_2X_3|X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2jX_3|Y_2||Y_1|X_1X_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2X_3|X_4kX_5|Y_2 \\ & + Y_1|X_1X_2jX_3|Y_2||Y_1|X_1iX_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2X_3|X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2X_3|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|X_4kX_5|Y_2||Y_1|X_1X_2jX_3|Y_2 \\ & + Y_1|X_1iX_2X_3|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|Y_2||Y_1|X_1X_2jX_3|X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2X_3|X_4kX_5|Y_2||Y_1|X_1X_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|Y_2 \\ & + Y_1|X_1iX_2X_3|X_4kX_5|Y_2||Y_1|X_1X_2jX_3|Y_2||Y_1|X_1iX_2jX_3|X_4kX_5|Y_2 \end{aligned}$$

$$\begin{aligned} D_1^k D_2^{ij} (Y_1|X_1iX_2jX_3|X_4kX_5|Y_2) = & Y_1|X_1iX_2jX_3|X_4kX_5|Y_2||Y_1|X_4kX_5|Y_2||Y_1|X_1iX_2X_3|X_1X_2jX_3|Y_2 \\ & + Y_1|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2X_3|X_1X_2jX_3|Y_2 \\ & + Y_1|X_1iX_2jX_3|Y_2||Y_1|X_1iX_2X_3|X_1X_2jX_3|X_4kX_5|Y_2||Y_1|X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2jX_3|Y_2||Y_1|X_1iX_2X_3|X_4kX_5|Y_2||Y_1|X_1X_2jX_3|X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2jX_3|Y_2||Y_1|X_1X_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2X_3|X_4kX_5|Y_2 \\ & + Y_1|X_1iX_2jX_3|Y_2||Y_1|X_4kX_5|Y_2||Y_1|X_1iX_2X_3|X_1X_2jX_3|X_4kX_5|Y_2 \\ & + Y_1|X_1X_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2X_3|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|Y_2 \\ & + Y_1|X_1X_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2X_3|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|Y_2 \\ & + Y_1|X_1iX_2X_3|X_4kX_5|Y_2||Y_1|X_1X_2jX_3|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|Y_2 \\ & + Y_1|X_4kX_5|Y_2||Y_1|X_1X_2jX_3|X_1iX_2X_3|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|Y_2 \\ & + Y_1|X_1X_2jX_3|X_1iX_2X_3|Y_2||Y_1|X_1iX_2jX_3|X_4kX_5|Y_2||Y_1|X_4kX_5|Y_2 \\ & + Y_1|X_1X_2jX_3|X_1iX_2X_3|Y_2||Y_1|X_4kX_5|Y_2||Y_1|X_1iX_2jX_3|X_4kX_5|Y_2 \end{aligned}$$

$$\begin{aligned}
D_2^{ij} D_1^k (Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2) = & Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 X_3 | X_1 X_2 j X_3 | Y_2 || Y_1 | X_4 k X_5 | Y_2 \\
& + Y_1 | X_1 i X_2 j X_3 | Y_2 || Y_1 | X_1 i X_2 X_3 | X_1 X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_4 k X_5 | Y_2 \\
& + Y_1 | X_1 X_2 j X_3 | X_1 i X_2 X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 j X_3 | Y_2 || Y_1 | X_4 k X_5 | Y_2 \\
& + Y_1 | X_1 X_2 j X_3 | X_1 i X_2 X_3 | Y_2 || Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_4 k X_5 | Y_2 \\
& + Y_1 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 X_3 | X_1 X_2 j X_3 | Y_2 \\
& + Y_1 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 j X_3 | Y_2 || Y_1 | X_1 i X_2 X_3 | X_1 X_2 j X_3 | X_4 k X_5 | Y_2 \\
& + Y_1 | X_4 k X_5 | Y_2 || Y_1 | X_1 X_2 j X_3 | X_1 i X_2 X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 j X_3 | Y_2 \\
& + Y_1 | X_4 k X_5 | Y_2 || Y_1 | X_1 X_2 j X_3 | X_1 i X_2 X_3 | Y_2 || Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2
\end{aligned}$$

$$\begin{aligned}
D_2^{ik} D_1^j (Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2) = & Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 X_2 j X_3 | Y_2 \\
& + Y_1 | X_1 i X_2 X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 X_2 j X_3 | Y_2 \\
& + Y_1 | X_1 X_2 j X_3 | Y_2 || Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 X_3 | X_4 k X_5 | Y_2 \\
& + Y_1 | X_1 X_2 j X_3 | Y_2 || Y_1 | X_1 i X_2 X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2
\end{aligned}$$

$$\begin{aligned}
D_2^{jk} D_1^i (Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2) = & Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 X_3 | Y_2 \\
& + Y_1 | X_1 X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 X_3 | Y_2 \\
& + Y_1 | X_1 i X_2 X_3 | Y_2 || Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 X_2 j X_3 | X_4 k X_5 | Y_2 \\
& + Y_1 | X_1 i X_2 X_3 | Y_2 || Y_1 | X_1 X_2 j X_3 | X_4 k X_5 | Y_2 || Y_1 | X_1 i X_2 j X_3 | X_4 k X_5 | Y_2
\end{aligned}$$

[illegible]

$$\begin{aligned}
D_3^{ijk} D_0(Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2) = & \partial Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2||\partial Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|Y_2 \\
& + \partial Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|Y_2||\partial Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2 j X_3|X_4 k X_5|\partial Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|\partial Y_2 \\
& + Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|\partial Y_2||Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 j X_3|X_4 k X_5|\partial Y_2 \\
& + Y_1|Y_2||Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|Y_2||Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|Y_2||Y_1|Y_2 \\
& + Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2||Y_1|Y_2 \\
& + Y_1|X_1 X_2 X_3|X_1 i X_2 j X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2||Y_1|X_1 X_2 X_3|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 X_2 X_3|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|Y_2||Y_1|X_1 X_2 X_3|X_1 i X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|Y_2||Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2 j X_3|X_1 X_2 X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_1 X_2 X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2 j X_3|X_4 X_5|X_4 k X_5|Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 X_5|X_4 k X_5|Y_2 \\
& + Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 X_5|X_4 k X_5|Y_2||Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 j X_3|X_4 X_5|X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2 j X_3|X_4 k X_5|X_4 X_5|Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 X_3|X_1 X_2 j X_3|X_4 k X_5|X_4 X_5|Y_2 \\
& + Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|X_4 X_5|Y_2||Y_1|X_1 i X_2 j X_3|X_4 k X_5|Y_2 \\
& + Y_1|X_1 X_2 j X_3|X_1 i X_2 X_3|X_4 k X_5|Y_2||Y_1|X_1 i X_2 j X_3|X_4 k X_5|X_4 X_5|Y_2
\end{aligned}$$