## Planar degenerate case

Consider again two indices i < j. We suppose WLOG i < j with respect to the order of  $\Gamma$ , so that we can write  $\Gamma = Y_1 | X_1 i X_2 | Y_2 | X_3 j X_4 | Y_3$ , with the usual convention for clouds of points. Then we have:

$$\begin{split} &D_1^iD_2^{ij}(Y_1|X_1iX_2|Y_2|X_3jX_4|Y_3) = \\ &= Y_1|X_1iX_2|Y_2|X_3(j+2)X_4|Y_3||Y_1|X_1(i+1)X_2|Y_2|Y_3||Y_1|X_1(i+2)X_2|Y_2|X_3(j+3)X_4|Y_3 \\ &+ Y_1|X_1iX_2|Y_2|Y_3||Y_1|X_1(i+1)X_2|Y_2|X_3(j+2)X_4|Y_3||Y_1|X_1(i+2)X_2|Y_2|X_3(j+3)X_4|Y_3 \\ &D_2^{i,j+1}D_1^i(Y_1|X_1iX_2|Y_2|X_3jX_4|Y_3) = \\ &= Y_1|X_1iX_2|Y_2|X_3(j+2)X_4|Y_3||Y_1|X_1(i+1)X_2|Y_2|X_3(j+3)X_4|Y_3||Y_1|X_1(i+2)X_2|Y_2|Y_3 \\ &D_2^{i+1,j+1}D_1^i(Y_1|X_1iX_2|Y_2|X_3jX_4|Y_3) = \\ &= Y_1|X_1iX_2|Y_2|Y_3||Y_1|X_1(i+1)X_2|Y_2|X_3(j+2)X_4|Y_3||Y_1|X_1(i+2)X_2|Y_2|X_3(j+3)X_4|Y_3 \\ &D_1^{i+1}D_2^{ij}(Y_1|X_1iX_2jX_3|Y_3) = \\ &= Y_1|X_1iX_2|Y_2|X_3(j+2)X_4|Y_3||Y_1|X_1(i+1)X_2|Y_2|Y_3||Y_1|X_1(i+2)X_2|Y_2|X_3(j+3)X_4|Y_3 \\ &+ Y_1|X_1iX_2|Y_2|X_3(j+2)X_4|Y_3||Y_1|X_1(i+1)X_2|Y_2|X_3(j+3)X_4|Y_3||Y_1|X_1(i+2)X_2|Y_2|X_3(j+3)X_4|Y_3 \\ &+ Y_1|X_1iX_2|Y_2|X_3(j+2)X_4|Y_3||Y_1|X_1(i+1)X_2|Y_2|X_3(j+3)X_4|Y_3||Y_1|X_1(i+2)X_2|Y_2|Y_3 \end{split}$$