

# MATH 3423: ADV LINEAR ALG & OPTIMIZATION

## HOMEWORK 3

Let  $A$  be a  $(4 \times 3)$  randomly generated matrix

$$A = \text{np.random.randint}(0, 5, \text{size} = (4, 3))$$

with integer elements in the interval  $[0, 5)$ ; let  $b$  a  $(4 \times 1)$  randomly generated vector

$$b = \text{np.random.randint}(-1, 3, \text{size} = (4, 1))$$

with integer elements in the interval  $[-1, 3)$ ; and let  $c$  a  $(3 \times 1)$  randomly generated vector

$$c = \text{np.random.randint}(-1, 3, \text{size} = (3, 1))$$

with integer elements in the interval  $[-1, 3)$ .

1. Find the  $QR$  factorizations of  $A$  and  $A^T$  and use them solve  $Ax = b$  and  $A^Ty = c$ .
2. Find the  $SVD$  factorizations of  $A$  and  $A^T$  and use them solve  $Ax = b$  and  $A^Ty = c$ .
3. Solve  $A^TAx = A^Tb$  using the Cholesky factorization.
4. Solve  $A^TAx = A^Tb$  using the  $QR$  factorization.
5. Solve  $A^TAx = A^Tb$  using the  $SVD$  factorization.
6. Find the eigenvalues and the corresponding eigenvectors of  $A^TA$  and  $AA^T$ .
7. Show numerically that for any rectangular matrix:

$$AA^+A = A \quad \text{and} \quad A^+AA^+ = A^+$$

► E-MAIL YOUR WORK IN A SINGLE JUPYTER NOTEBOOK.