¿NÃO FIZ!

uff Universidade Federal Fluminense
EGM - Instituto de Matemática
GMA - Departamento de Matemática Aplicada

LISTA 13 - 2008-1

Regra de L'Hôpital

Calcule os limites dos exercícios 1. a 10.

1. $\lim\limits_{x \to 0} \dfrac{\cos^2 x - 1}{e^{x^2} - 1}$

2. $\lim\limits_{x \to 1^+} (\ln x)^{x-1}$

3. $\lim\limits_{x \to +\infty} (x^2 - 1)\, e^{-x^2}$

4. $\lim\limits_{x \to +\infty} \dfrac{\ln(\ln x)}{\ln x}$

5. $\lim\limits_{x \to 0^+} \dfrac{\ln(\operatorname{arcsen} x)}{\cot x}$

6. $\lim\limits_{x \to 0} \dfrac{x}{\arctan x}$

7. $\lim\limits_{x \to +\infty} \left(\frac{2}{\pi} \arctan x\right)^x$

8. $\lim\limits_{x \to +\infty} \left(\cos \dfrac{2}{x}\right)^{x^2}$

9. $\lim\limits_{x \to 0^+} \left(\tan \dfrac{\pi}{x+2}\right)^x$

10. $\lim\limits_{x \to 0} \left(\frac{1}{e}(1+x)^{\frac{1}{x}}\right)^{\frac{1}{x}}$

Nos exercícios 11. e 12. encontre o valor de a que satisfaz a igualdade.

11. $\lim\limits_{x \to +\infty} \left(\dfrac{1 + e^{2x}}{2}\right)^{\frac{a}{x}} = \sqrt{e}$

12. $\lim\limits_{x \to +\infty} \left(\dfrac{x+a}{x-a}\right)^x = 4$

Nos exercícios 15. a 22. encontre, se existirem, as assíntotas horizontais e verticais do gráfico da função.

13. $f(x) = \dfrac{x}{\ln x}$

14. $f(x) = \dfrac{e^{-\frac{1}{x^2}}}{x}$

15. $f(x) = e^{\frac{1}{x}}$

16. $f(x) = x^2 \ln x$

17. $f(x) = e^{-x^2}$

18. $f(x) = xe^{-x}$

19. $f(x) = \pi^{x^3}$

RESPOSTAS

1. -1
2. 1
3. 0
4. 0
5. 0
6. 1
7. $e^{-\frac{2}{\pi}}$
8. e^{-2}
9. 1
10. $e^{-\frac{1}{2}}$
11. $\frac{1}{4}$
12. $\ln 2$

13. Assíntota vertical: $x = 1$

14. Assíntota horizontal: $y = 0$

15. Assíntota vertical: $x = 0$
Assíntota horizontal: $y = 0$

16. Não tem assíntotas

17. Assíntota horizontal: $y = 0$

18. Assíntota horizontal: $y = 0$

19. Assíntota horizontal: $y = 0$

GMA:

Lista 13 A:

① $\lim_{x\to 0} \dfrac{\cos^2 x - 1}{e^{x^2}-1} = \lim_{x\to 0} \dfrac{-\sin^2 x}{e^{x^2}-1} = \lim_{x\to 0} \dfrac{\dfrac{-\sin^2 x \cdot x^2}{x^2}}{(e^{x^2}-1)} \xrightarrow[L'H]{1^a R}$

$\boxed{\begin{matrix}\cos^2 x + \sin^2 x = 1 \\ \cos^2 x - 1 = -\sin^2 x\end{matrix}}$ $\lim_{x\to 0} \dfrac{-2x}{e^{x^2} \cdot 2x} = \dfrac{-1}{1} = -1$

② $\lim_{x\to 1^+} (\ln x)^{x-1} = \lim_{x\to 1^+} (x-1)\ln x \to e^{\lim_{x\to 1^+} (x-1)\ln x} = e^0 = 1$

$\lim_{x\to 1^+} (x-1)(\ln x) = 1 \cdot \ln x + (x-1) \cdot \dfrac{1}{x} = 0 + 1-1 \dfrac{1}{1} = 0$

③ $\lim_{x\to \infty} (x^2-1)e^{-x^2} = \lim_{x\to\infty} \ln[(x^2-1)e^{-x^2}] = \lim_{x\to\infty} \ln \dfrac{(x^2-1)}{e^{x^2}}$

$\xrightarrow[L'H]{2^a R} \lim_{x\to\infty} \dfrac{\dfrac{2x}{x^2-1}}{e^{x^2} 2x} = \lim_{x\to\infty} \dfrac{\cancel{2x}}{x^2-1} \cdot \dfrac{1}{(e^{x^2} \cdot \cancel{2x})} = \dfrac{1}{(x^2-1)(e^{x^2})}$

$\dfrac{1}{\infty} = 0$

④ $\lim_{x\to +\infty} \dfrac{\ln(\ln x)}{\ln x} = \lim \dfrac{\dfrac{\ln x'}{\ln x} \cdot \dfrac{1}{x}}{\dfrac{1}{x}} = \lim \dfrac{\dfrac{1/x}{\ln x} \cdot \dfrac{1}{x}}{1/x} = \lim \dfrac{\dfrac{\ln x}{\cancel{x}} \cdot \dfrac{1}{x}}{\cancel{1/x}}$

$= \lim_{x\to\infty} \dfrac{\ln x}{x} \xrightarrow[L'H]{2^a R} \lim_{x\to\infty} \dfrac{1/x}{1} = 0$

$\frac{1}{sen}$

$\cot g = \frac{\cos x}{sen\, 0} = \frac{1}{0} = \infty$

$\frac{\frac{1}{\sqrt{1-x^2}}}{arcsen(x)}$ $\quad sen^2 + cos^2 = 1$

$-cossec^2(x)$

2

$$\lim_{x \to 0} \frac{1}{\sqrt{1-x^2}\cdot arcsen(x)} = \lim_{x\to 0} \frac{1 \cdot sen^2(x)}{\sqrt{1-x^2}\cdot arcsen(x)} = \frac{0}{\sqrt{1-0}\; arcsen(0)} \quad \frac{0}{0}$$

$$-\frac{1}{sen^2(x)} \quad \lim_{x\to 0^+} \frac{-sen^2(x) \to 0}{arcsen(0)\cdot\sqrt{1-x^2}} = \frac{-0}{\frac{\pi}{2}\sqrt{1}} = 0 \cdot \frac{2}{\pi}$$

$$\lim_{x\to 0} \frac{1}{\frac{1}{1+x^2}} = 1$$

$$6) \lim_{x\to 0} \frac{x}{arctg(x)} \xrightarrow[L'H]{1°} \lim_{x\to 0} \frac{1}{\frac{1}{1+x^2}} = 1$$

$$7) \lim_{x\to +\infty} \left(\frac{2}{\pi}\, arctg\, x\right)^x = [1^\infty] \Rightarrow \lim_{x\to\infty} \ln\left(\frac{2}{\pi} arctg\, x\right)^x = x \ln\left(\frac{2}{\pi} arctg\, x\right)$$

$$\lim_{x\to\infty} x \ln\left(\frac{2}{\pi} arctg\, x\right)$$

2°

$$\underset{L'H}{\lim_{x\to\infty}} \frac{\ln\left(\frac{2}{\pi} arctg\, x\right)}{1/\ln\left(\frac{2}{\pi} arctg\, x\right)} = \lim_{x\to\infty} \frac{1}{\frac{\frac{1}{x^2+1}}{\frac{2}{\pi}arctg\, x}} = \lim_{x\to\infty} \frac{1}{\frac{1}{\frac{1}{(x^2+1)(arctg\, x)}}} = \frac{1}{(x^2+1)(arctg\, x)} =$$

$$8) \lim_{x\to+\infty} \left(\cos\left(\frac{2}{x}\right)\right)^{x^2} = \lim_{x\to\infty} 2x \cdot \ln\left[\cos\left(\frac{2}{x}\right)\right] = \lim_{x\to\infty} \frac{\ln\cos\left(\frac{2}{x}\right)}{\frac{1}{2x}} \underset{L'H}{\Rightarrow}$$

$$\lim_{x\to\infty} \frac{\frac{\left[-sen\left(\frac{2}{x}\right)\right]\cdot\frac{-2}{x^2}}{\cos\left(\frac{2}{x}\right)}}{-\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\frac{2\, sen\left(\frac{2}{x}\right)}{\cos\left(2/x\right)\cdot x^2}}{-1/x^2} = \lim_{x\to\infty} \frac{-2\, sen\left(\frac{2}{x}\right)\cdot x^2}{\cos\left(\frac{2}{x}\right)\cdot x^2}$$

$$\lim_{x\to\infty} -2\, tg\left(\frac{2}{x}\right) = -2\cdot 0 = 0$$

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$\cot g = \frac{\cos x}{\sin 0} = \frac{1}{0} = \infty$. 1

$\left(\frac{1}{2} + \frac{e^{2x}}{2}\right)^{a/x}$

11) $\lim_{x \to \infty} \left(\frac{1 + e^{2x}}{2}\right)^{\frac{a}{x}} = \sqrt{e}$

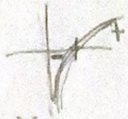
12) $\lim_{x \to \infty} \left(\frac{x+a}{x-a}\right)^{x} = 4$

$\cancel{x} - a \neq 0$

$x \neq a.$

$\left(\frac{x(1+a/x)}{x(1-a/x)}\right)$ $\quad x \ln\left(\frac{x+a}{x-a}\right) = \ln 4 \Rightarrow$

$x \cdot \left(\ln(\cancel{x}+a) - \ln(x-a)\right) = \ln 4$



13) $f(x) = \frac{x}{\ln x \neq 0}$

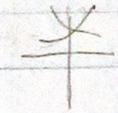
$e^0 \neq x$

$\boxed{1 \neq x}$

$\lim_{x \to 1} \frac{x}{\ln x}$. $\begin{cases} \lim_{x \to 1^+} \frac{1}{\ln x > 0} = +\infty \\ \lim_{x \to 1^-} \frac{1}{\ln x} = -\infty \end{cases}$ A.V em $\boxed{x = 1.}$

$\lim_{x \to -\infty} \frac{x}{\ln x} \nexists \to x > 0$ $\nexists$ AH

$\lim_{x \to \infty} \frac{x}{\ln x} = \infty$



14) $f(x) = \frac{e^{-\frac{1}{x^2}}}{x}$. $\lim_{x \to 0^+} \frac{e^{-1/x^2 < 0}}{x > 0} = -\infty$. A.V em $x = 0$

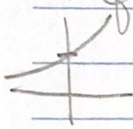
DOM: $x \neq 0$;

$\lim_{x \to 0^-} \frac{e^{-1/x^2 < 0}}{x < 0} = -\infty$

$\frac{1}{x \cdot e^{1/x^2}}$

$\frac{-1}{x^2} \Rightarrow$

15) $f(x) = e^{1/x^0} = e^{-x}$ DOM: $x \neq 0$.



~~HAs~~ $\nexists$ A $\lim_{x \to 0^-} e^{1/x} = e^{-\infty} = \infty$ A.V em $x = 0$.

$\lim_{x \to 0^+} e^{1/x} = e^{+\infty} = \infty$

$\lim_{x \to \infty} e^{\cancel{1/x}} = 1$

A.H em $y = 1$.

$\lim_{x \to -\infty} e^{\cancel{1/x}} = 1$

16) $f(x) = x^2 \ln x$ $\boxed{x > 0}$

$\lim_{x \to 0^+} x^2 \cdot \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x^2} \overset{L'H}{=} \lim_{x \to 0^+} \frac{1/x}{-2/x^3} = \frac{1}{x} \cdot \frac{x^{3}}{-2} = 0$

~~lim~~ $x = 0$ não está no Domínio $\Rightarrow$ Não tem A.V.

$\lim_{x \to \infty} x^2 \cdot \ln x = \infty \cdot \infty = \infty$ NÃO TEM A.H.

$\frac{1}{sen}$ $cotg = \frac{cos\,x}{sen\,0} = \frac{1}{-} = \infty$

(/ /)

$e^{x^2} \neq 0$

17) $f(x) = e^{-x^2} = \frac{1}{e^{x^2}}$ DOMÍNIO: $\mathbb{R}$

∄ A.V.

$\lim_{x\to-\infty} e^{-x^2} = \lim \frac{1}{e^{x^2\to\infty}} = 0$

A.H em $y=0$.

$\lim_{x\to\infty} e^{-x^2} = \lim_{x\to\infty} \frac{1}{e^{x^2\to\infty}} = 0$

18) $f(x) = xe^{-x} = x \cdot \frac{1}{e^x \neq 0}$ DOM $\mathbb{R}$

NÃO TEM A.V.

$\lim_{x\to\infty} x \frac{1}{e^x} \overset{2^a\, r\, L.H.}{=} \lim_{x\to\infty} \frac{1}{e^x} = 0$

A.H em $y=0$

$\lim_{x\to+\infty} \frac{x}{e^x} \overset{2^a\, r\, L.H}{\to} \lim_{x\to-\infty} \frac{1}{e^x} = 0$

19) $f(x) = \pi^{x^3}$ DOMÍNIO: $\mathbb{R}$.

∄ A. Verticais

$\pi^{x^3} \cdot [\cancel{3x}^3 . \ln x]' = \lim_{x\to\infty} \pi^{x^3} \cdot [3x^2 \ln x + \cancel{x^3}^{x^2} \frac{1}{\cancel{x}}] =$

$= \lim_{x\to\infty} \pi x^3 [x^2(3\ln x + 1)$

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