

**UFF** Universidade Federal Fluminense  
EGM - Instituto de Matemática  
GMA - Departamento de Matemática Aplicada

**LISTA 18 - 2008-1**  
Anti-derivada  
Integral indefinida  
Problema de valor inicial

Calcule as integrais dos exercícios 1. a 16.

1.  $\int ((\sqrt[3]{t})^2 - 2) dt$

6.  $\int \frac{x^9 - x^3}{x^4} dx$

12.  $\int \frac{x^2}{1+x^2} dx$

2.  $\int \frac{x - \sqrt{x}}{3} dx$

7.  $\int \frac{\sin 2\theta}{\cos \theta} d\theta$

13.  $\int \frac{\sqrt{1-x^2}}{1-x^2} dx$

3.  $\int \left( \frac{3}{x^2} - 1 \right) dx$

8.  $\int \frac{\cos x}{1 - \cos^2(x)} dx$

14.  $\int \frac{1}{1 + \sinh^2 y} dy$

4.  $\int \sqrt{\frac{2}{x}} dx$

9.  $\int \tan^2 u du$

15.  $\int (e^t - e^{-t}) dt$

5.  $\int (2-s)\sqrt{s} ds$

10.  $\int 1 + x^2 + \frac{1}{1+x^2} dx$

16.  $\int x(1 - \tanh^2 x) \cosh^2 x dx$

17. Encontre a expressão que define a função  $f$ , cujo gráfico contém o ponto  $(4, \frac{5}{3})$  e cuja derivada é  $f'(x) = \sqrt{x}(2\sqrt{x} - 1)$ .

Resolva os problemas de valor inicial dos exercícios 18. a 20.

18.  $\begin{cases} y' = \frac{1}{x^2} - \frac{1}{x^3} \\ y(1) = \frac{3}{2} \end{cases}$

19.  $\begin{cases} y' = \frac{1}{x} - \frac{1}{x^3} \\ y(1) = 2 \end{cases}$

20.  $\begin{cases} f'(x) = 2 \cos x - 3 \csc^2 x \\ f(\frac{\pi}{2}) = 8 \end{cases}$

21. Uma função tem derivada de segunda ordem  $f''(x) = 6x - 6$ . Encontre a expressão da  $f$ , sabendo que seu gráfico contém o ponto  $(2, 1)$  e que em tal ponto a reta tangente tem equação  $3x - y - 5 = 0$ .

# RESPOSTAS

1.  $\frac{3}{5} t^{\frac{5}{3}} - 2t + C$

7.  $-2 \cos \theta + C$

15.  $2 \cosh t + C = e^t + e^{-t} + C$

2.  $\frac{x^2}{6} - \frac{2\sqrt{x^3}}{9} + C$

8.  $-\csc x + C$

16.  $\frac{x^2}{2} + C$

3.  $-\frac{3}{x} - x + C$

9.  $-u + \tan u + C$

17.  $f(x) = x^2 - \frac{4}{3}\sqrt{x^3} - 9$

4.  $2x\sqrt{\frac{2}{x}} + C$

10.  $x + \frac{x^3}{3} + \arctan x + C$

18.  $y = 2 - \frac{1}{x} + \frac{1}{2x^2}$

5.  $\frac{4}{3} s^{\frac{3}{2}} - \frac{2}{5} s^{\frac{5}{2}} + C$

11.  $e^x - 3x + C$

19.  $\frac{3}{2} + \frac{1}{2x^2} + \ln|x|$

6.  $\frac{x^6}{6} - \ln|x| + C$

12.  $x - \arctan x + C$

20.  $6 + 2 \sin x + 3 \cot x$

13.  $\arcsen x + c$

14.  $\tanh y + C$

21.  $f(x) = x^3 - 3x^2 + 3x - 1$



GMA

$$\frac{2+1}{3}$$

Lista 18.  $\frac{-1+2}{2} = \frac{1}{2}$ 

$$1) \int (t^{1/3})^2 - 2 dt = \int t^{2/3} - 2 dt = \frac{3t^{5/3}}{5} - 2t + K.$$

$$2) \int \frac{x - \sqrt{x}}{3} dx = \frac{1}{3} \int x - (x)^{1/2} dx = \frac{x^2}{2} - \frac{2x^{3/2}}{3} + K.$$

$$3) \int \left( \frac{3}{x^2} - 4 \right) dx = \int 3x^{-2} - 4 dx = \int \frac{3x^{-1}}{-1} - 4x + K.$$

$$4) \int \sqrt{\frac{2}{x}} dx = \int (2x^{-1})^{1/2} dx = \int \sqrt{2} \cdot x^{-1/2} dx =$$

$$= \frac{\sqrt{2} \cdot x^{1/2}}{1/2} + K.$$

$$5) \int (2-s)\sqrt{s} ds = \int 2s^{1/2} - s^{3/2} ds =$$

$$= \frac{2 \cdot 2}{3} s^{3/2} - \frac{2}{5} s^{5/2} + K$$

$$6) \int \frac{x^9 - x^3}{x^4} dx = \int x^5 - x^{-1} dx = \frac{x^6}{6} - \ln|x| + K$$

$$7) \int \frac{\sin 2\theta}{\cos \theta} dx = \int \frac{2 \sin \theta \cos \theta}{\cos \theta} dx = 2 \cos \theta + K$$

$$8) \int \frac{\cos(x)}{1 - \cos^2(x)} dx = \int \frac{\cos(x)}{\sin^2(x)} dx =$$

$$= \int \cos x \cdot u^{-2} dx = \int u^{-2} du =$$

$$u = \sin(x)$$

$$du = \cos(x) dx$$

$$= \frac{u^{-1}}{-1} + K =$$

$$= \frac{1}{\sin(x)} + K$$



$$9) \int \tan(u)^2 du = \int \frac{\sin^2 u}{\cos^2 u} du =$$

$$\tan u = x$$

$$du = \frac{1}{1+x^2} dx$$

$$10) \int \frac{1+x^2 + 1}{1+x^2} dx = \int \frac{1+x^2}{1+x^2} + \frac{1}{1+x^2} dx = \int 1 + \frac{1}{1+x^2} dx = x + \arctan(x) + K$$

$$11) \int \frac{e^{2x} - 3e^x}{e^x} dx = \int e^x - 3 dx = e^x - 3x + K$$

$$12) \int \frac{x^2}{1+x^2} dx =$$

$$13) \int \frac{\sqrt{1-x^2}}{1-x^2} dx =$$



$$17) f'(x) = \sqrt{x} (2\sqrt{x} - 1)$$

$$\int 2 - \sqrt{x} dx = \int 2x^{-1/2} dx =$$

$$= \frac{2x^{1/2}}{1/2} - \frac{2}{3} x^{3/2} + K$$

$$(4, 5/3)$$

$$f(4) = 5/3 \Rightarrow 4^2 - \frac{2}{3} \sqrt{4^3} + K = \frac{5}{3} \Rightarrow 16 - \frac{16}{3} + K = \frac{5}{3}$$

$$\Rightarrow 16 - \frac{16}{3} + K = \frac{5}{3} \Rightarrow K = \frac{5}{3} - \frac{32}{3} = -\frac{27}{3} = -9$$

$$y = x^2 - \frac{2}{3} \sqrt{x^3} - 9$$

$$18) \begin{cases} y' = \frac{1}{x^2} - \frac{1}{x^3} \\ f(x) = \int x^{-2} - x^{-3} dx = \end{cases}$$

$$\begin{cases} y(1) = 3/2 \\ f(x) = \frac{x^{-1}}{-1} - \left( \frac{-x^{-2}}{+2} \right) + K = \end{cases}$$

$$f(x) = -\frac{1}{x} + \frac{1}{2x^2} + K = -\frac{1}{2} + \frac{1}{2} + K = \frac{3}{2} \Rightarrow$$

$$K = \frac{3}{2} + \frac{1}{2} = 2$$

$$y = -\frac{1}{x} + \frac{1}{2x^2} + 2$$

$$\cot \theta = 1$$

$$\tan$$



$$19) y' = \frac{1}{x} - \frac{1}{x^3} \quad f(x) = \int \frac{1}{x} - \frac{1}{x^3} dx = \int x^{-1} - x^{-3} dx =$$

$$f(x) = \ln|x| - \frac{x^{-2}}{-2} + K = \ln|x| + \frac{1}{2x^2} + K$$

$$f(1) = \ln(1) + \frac{1}{2 \cdot 1} + K = 2 \rightarrow K = 2 - \frac{1}{2} = \frac{3}{2}$$

$$f(x) = \ln|x| + \frac{1}{2x^2} + \frac{3}{2} //$$

$$20) \begin{cases} f(x) = 2\cos(x) - 3\sec^2 x \\ f(\pi/2) = 8 \end{cases}$$

$$f(x) = \int 2\cos(x) - 3\sec^2 x = \int 2\sin(x) - 3\cotg(x) + K$$

$$f(\pi/2) = 2\sin(\pi/2) - 3\cotg(\pi/2) + K = 8 \rightarrow$$

$$2 \cdot 1 + K = 8 \rightarrow K = 6$$

$$f(x) = 2\sin(x) - 3\cotg(x) + 6$$

$$21) f''(x) = 6x - 6 \quad (2, 1)$$

$$f'(x) = \int 6x - 6 dx = 3x^2 - 6x + K \Rightarrow f'(2) = 2 \Rightarrow 12 + K = 2$$

$$f'(x) = 3x^2 - 6x + 1$$

$$f(x) = \int 3x^2 - 6x + 1 dx = 3 \cdot \frac{x^3}{3} - 6 \cdot \frac{x^2}{2} + x + K \Rightarrow$$

$$f(2) = 2^3 - 3 \cdot 2^2 + 2 + K = 1 \Rightarrow 8 - 12 + 2 + K = 1 \Rightarrow$$

$$K = 1 + 2 = 3 \quad f(x) = x^3 - 3x^2 + x + 3 //$$