uff Universidade Federal Fluminense
EGM - Instituto de Matemática
GMA - Departamento de Matemática Aplicada

LISTA 12 - 2008-1
Função logarítmica
Função exponencial

1. Seja $f(x) = \dfrac{\ln(x^2-3)}{\sqrt{(x-1)(x+3)}}$. Determine o domínio de f, os valores de x onde a f se anula e os intervalos onde a f é positiva e onde a f é negativa.

Nos exercícios 2. a 5. esboce o gráfico da função.

2. $f(x) = \ln|x-4|$
3. $y = |\ln|x+1||$ Qual o passo?
4. $F(x) = e^{|x+2|}$
5. $g(t) = \frac{1}{2} - e^{-t}$

Derive as funções dos exercícios 6. a 16. (se for conveniente, use derivação logarítmica)

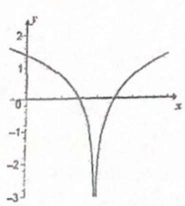
6. $f(x) = \dfrac{e^{\operatorname{sen} 2x}\sqrt{x}}{e^{\cos 3x}}$
7. $f(x) = e^{\sqrt{x}} \ln\sqrt{x}$
8. $f(x) = \ln\left(x\sqrt{x^2+1}\right)$
9. $f(x) = (e^x)^x$
10. $f(x) = e^{x^x}$
11. $f(x) = (x^x)^x$
12. $f(x) = \log_2 x^2$
13. $f(x) = (\operatorname{sen} x)^{\operatorname{arcsen} x}$
14. $f(x) = x^\pi + \pi^x$
15. $f(x) = (\ln x)^x\, x^{\ln x}$
16. $\ln \dfrac{\sqrt{x+1}}{(x-1)^3}$

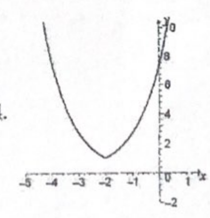
Calcule y' nos exercícios 17. a 19.

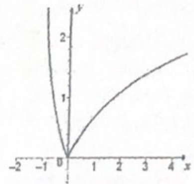
17. $\ln\left(\dfrac{x}{y} + \dfrac{y}{x}\right) = 5$
18. $\operatorname{sen} e^{xy} = x$
19. $\dfrac{y^2\cos x}{e^x} = 2^{\ln y}$, para $x = 0$ e $y = 1$

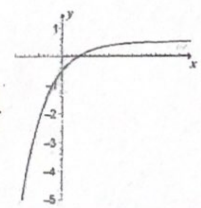
RESPOSTAS

1. Domínio $= (-\infty, -3) \cup (\sqrt{3}, \infty)$;
   $f = 0$ em $x = 2$
   $f > 0$ para $x < -3$ ou $x > 2$;
   $f < 0$ para $\sqrt{3} < x < 2$

2. 

4. 

3. 

5. 

Lista 12 **Cálculo I -A-** 2008-1

6. $f'(x) = \dfrac{(1 + 4x\cos 2x + 6x \operatorname{sen} 3x)e^{\operatorname{sen} 2x}}{2e^{\cos 3x}\sqrt{x}}$

7. $f'(x) == \dfrac{e^{\sqrt{x}}(1 + \sqrt{x}\ \ln\sqrt{x})}{2x}$

8. $f'(x) = \dfrac{2x^2 + 1}{x\,(x^2 + 1)}$

9. $f'(x) = 2xe^{x^2}$

10. $f'(x) = x^x e^{x^x}(1 + \ln x)$

11. $f'(x) = (x^x)^x\,(x + 2x\ln x)$

12. $f'(x) = \dfrac{2}{x\ln 2}$

13. $f'(x) = (\operatorname{sen} x)^{\operatorname{arcsen} x}\left(\cot x\ \operatorname{arcsen} x + \dfrac{\ln(\operatorname{sen} x)}{\sqrt{1-x^2}}\right)$

14. $f'(x) = \pi x^{\pi-1} + (\ln\pi)\pi^x$

15. $f'(x) = (\ln x)^x\,(x^{\ln x})\left(\dfrac{1}{\ln x} + \ln(\ln x) + \dfrac{2\ln x}{x}\right)$

16. $f'(x) = \dfrac{-(5x+7)}{2\,(x^2-1)}$

17. $y' = \dfrac{y}{x}$

18. $y' = \dfrac{1 - ye^{xy}\cos e^{xy}}{xe^{xy}\cos e^{xy}}$

19. $y' = \dfrac{1}{2 - \ln 2}$

GMA

Lista 12

$\ln_e(x^2-3)=0$

1) $f(x)=\dfrac{\ln(x^2-3)}{\sqrt{(x-1)(x+3)}}=y$

I) $(x-1)(x+3)>0$

$- \; - \;|\; + \; + \; +$ (1)

$- \; - \; - \;|\; + \; +$ (-3)

$+ \;|\; - \;|\; +$

$-3 \qquad 1$

II) $x^2-3>0$

$x^2>3$

$+ \quad - \quad +$

$-\sqrt{3} \qquad \sqrt{3}$

$1{,}7 \qquad 1{,}7$

I - -3, 1

II - $-\sqrt{3}$, $\sqrt{3}$

I∩II - -3, $-\sqrt{3}$, 1, $\sqrt{3}$

ANÁLISE DE SINAL

I: $+ \;|\; - \; - \; - \; - \; - \;|\; + \; +$ (-3, 1)

II: $+ \; + \; + \;|\; - \; - \; - \; - \;|\; + \; +$ ($-\sqrt{3}$, $\sqrt{3}$)

f: $+ \;|\; - \;|\; + \; + \;|\; - \;|\; + \; +$ (-3, $-\sqrt{3}$, 1, $\sqrt{3}$)

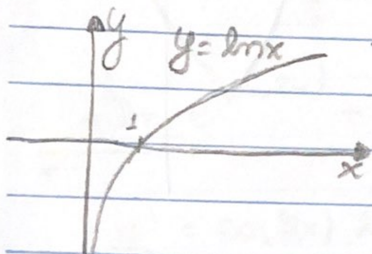
4) Dom $]-\infty,-3[\;\cup\;]\sqrt{3},+\infty[$

$f>0 \to x<-3$ ou $-\sqrt{3}<x<1$ ou $x>\sqrt{3}$

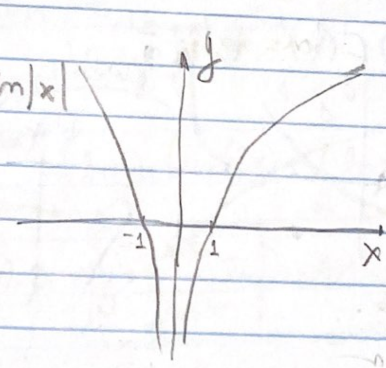
$f=0 \to x=-3$ ou $x=-\sqrt{3}$ ou $x=1$ ou $x=\sqrt{3}$

$f<0 \to -3<x<-\sqrt{3}$ ou $1<x<\sqrt{3}$

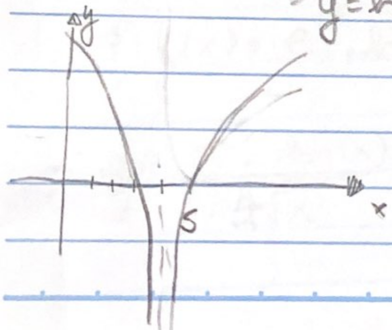
2) $f(x)=\ln|x-4|$



$\xrightarrow{f|x|}$ $y=\ln|x|$

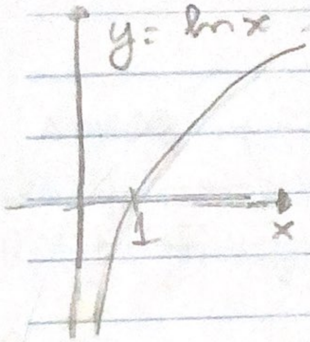


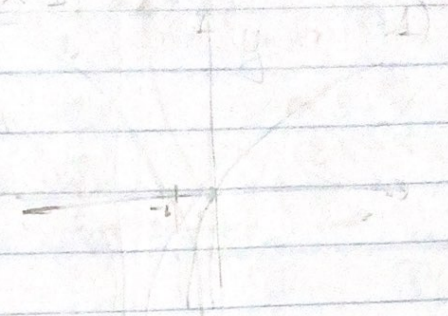
$\xrightarrow{f(x-4)}$ $y=\ln|x-4|$

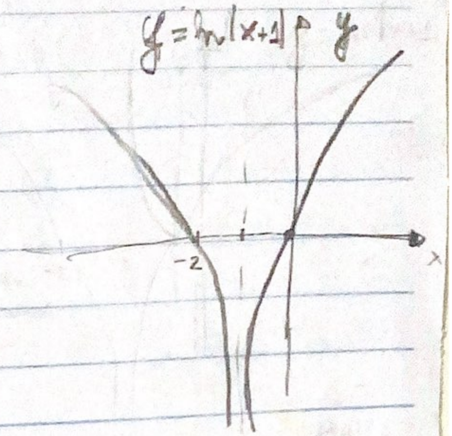


$x-(-1)$

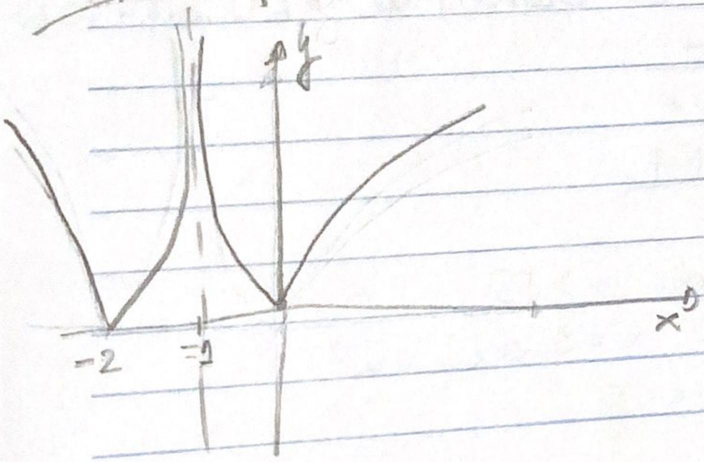
3) $y = |\ln|x+1||$



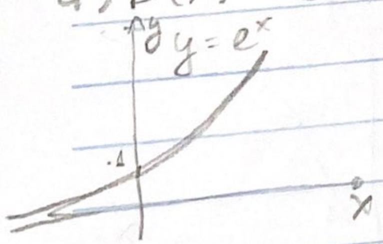




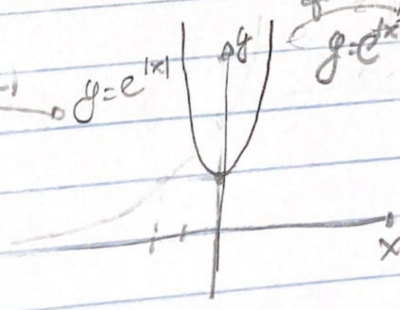
$|f(x)|$



4) $f(x) = e^{|x+2|}$

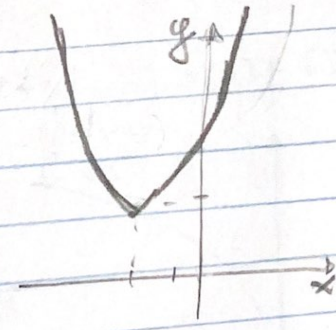


$f(|x|) - 1$ $y = e^{|x|}$

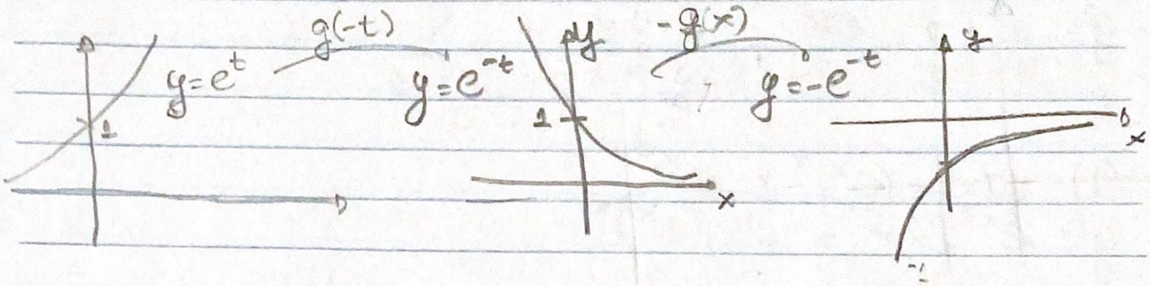


$f(x+2)$

$y = e^{|x+2|}$

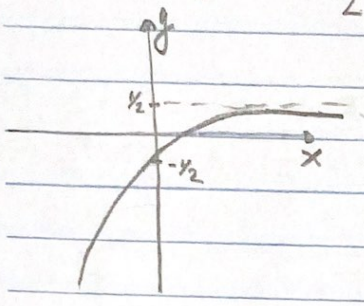


5) $g(t) = \frac{1}{2} - e^{-t} = y$



$g(x) + \frac{1}{2}$

$\rightarrow y = -e^{-t} + \frac{1}{2}$



6) $f(x) = \frac{e^{\sin 2x}\sqrt{x}}{e^{\cos 3x}} \Rightarrow \ln y = \ln\left(\frac{e^{\sin 2x} \cdot x^{1/2}}{e^{\cos(3x)}}\right) \Rightarrow$

$\ln y \Rightarrow \ln e^{(\sin 2x)} + \ln x^{1/2} - \ln e^{\cos(3x)} \Rightarrow (\ln y)' = \left(\sin(2x) + \frac{1}{2}\ln x - \cos(3x)\right)' =$

$\frac{y'}{y} = \cos(2x)\cdot 2 + \frac{1}{2}\,\frac{1}{x} + \sin(3x)\cdot 3 \Rightarrow \boxed{y' = \frac{e^{\sin 2x}\sqrt{x}}{e^{\cos 3x}} \cdot \left[2\cos(2x) + \frac{1}{2x} + 3\sin(3x)\right]}$

7) $f(x) = \frac{1}{2} e^{\sqrt{x}} \cdot \ln \overset{x^{1/2}}{\cancel{x}} \Rightarrow y' = \frac{1}{2}\left[\left(e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \cdot x\right) \ln x + \left(e^{\sqrt{x}} \cdot \frac{1}{x}\right)\right] =$

$y' = \frac{e^{\sqrt{x}} \cdot \ln(x)}{4\sqrt{x}} + \frac{e^{\sqrt{x}}}{2x} = e^{\sqrt{x}}\left(\frac{\ln(x)}{4\sqrt{x}} + \frac{1}{2x}\right)$

(/ /)

8) $f(x) = \ln(x\sqrt{x^2+1}) = \ln x + \frac{1}{2}\ln x^2+1 \Rightarrow$

$y' = \frac{1}{\cancel{x}} + \frac{1}{\cancel{2}} \cdot \frac{\cancel{2x}}{(x^2+1)} = \frac{1}{x} + \frac{x}{x^2+1}.$

9) $f(x) = (e^x)^x = e^{x^2}$

$y' = e^{x^2} \cdot 2x = 2x\,(e^x)^x$

10) $f(x) = (e^x)^x = e^{x^2}$

$\cancel{f(x) = (e^x)^x} = e^{x^2}[2x + \ln]x$

11 - $f(x) = (x^x)^x = x^{x^2}$

$\ln y = \ln x^{x^2} = x^2 \cdot \ln x$

$\frac{y}{y'} = 2x\ln x + x^{\cancel{2}}\frac{1}{\cancel{x}} \Rightarrow \boxed{y = x^{x^2}\left[2x\ln x + x\right]}$

12 - $f(x) = \log_2 x^2 = 2\log_2 x = \frac{2\ln x}{\ln 2}$

$y' = \frac{2}{\ln 2} \cdot \frac{1}{x} = \frac{2}{x\ln 2}$

13) $f(x) = (\operatorname{sen}(x))^{\operatorname{arcsen} x} \Rightarrow f'(x) = (\operatorname{sen}(x))^{\operatorname{arcsen}(x)} \cdot [(\operatorname{arcsen}(x)) \cdot \ln(\operatorname{sen}$

$y' = \operatorname{sen}(x)^{\operatorname{arcsen}(x)} \cdot \left(\frac{1}{\sqrt{1-x^2}} \cdot \ln(\operatorname{sen}(x)) + \operatorname{arcsen}(x) \frac{\overset{\cot g(x)}{(\cos(x))}}{\operatorname{sen}(x)} \right) =$

$y' = \operatorname{sen}(x)^{\operatorname{arcsen}(x)} \left(\frac{\ln(\operatorname{sen}(x))}{\sqrt{1-x^2}} + \operatorname{arcsen}(x) \cdot \cot g(x) \right)$

14) $f(x) = x^{\pi} + \pi^{x} \rightarrow \ln f(x) = (\ln(x^{\pi} + \pi^{x})) = v$

$\frac{y'}{y} = \pi x^{\pi - x} + (\pi^{x})[x \cdot \ln(\pi^{x})]' = \pi x^{\pi - x} + \pi^{x}(x \cdot \ln \pi)' = \pi$

14) $f(x) = x^{\pi} + \pi^{x}$

$\ln f(x) = \ln(x^{\pi} + \pi^{x})$

$\frac{y'}{y} = \frac{\pi x^{\pi - 1} + \pi^{x} \cdot 1}{x^{\pi} + \pi^{x}}$

15) $f(x) = \overbrace{(\ln x)^{x}}^{y} \cdot (x^{\ln x}) = $ [illegible] $; \ \cancel{f(x) = \frac{1}{x} x^{1+\ln x} + \ln x \cdot [x^{1+\ln x}]} \cdot \frac{1}{x} \cdot \ln(1+\ln x)$

$f(x) = \left\{ (\ln x)^{x} \cdot [1 + \ln(\ln x)](x^{\ln x}) \right\} + \left\{ (\ln x)^{x} \cdot [x^{\ln x} \left(\frac{1}{x} + \ln x \right) \right\}$

$(\ln x^{x})(x^{\ln x}) \left[1 + \ln(\ln x) + 1/x + \ln x \right]$

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