Exercícios do Livro "Um curso de Cálculo, Volume 1. - Cálculo I, Guidorizzi, H. L. , LTC, Rio de Janeiro, RJ"

1. Calcule a derivada.

a) $y = 5x^2 + 6x - 1$

b) $s = \sqrt[5]{t} + \frac{3}{t}$

c) $x = \frac{t}{t+1}$

d) $y = t \cos t$

e) $y = \frac{u+1}{\ln u}$

f) $x = t^3 e^t$

g) $s = e^t \operatorname{tg} t$

h) $y = \frac{x^5 + 1}{\operatorname{sen} x}$

i) $y = \sqrt[3]{u} \sec u$

j) $x = \frac{3}{t} + \frac{2}{t^2}$

m) $x = e^t \cos t$

n) $u = 5v^2 + \frac{3}{v^4}$

o) $V = \frac{4}{3} \pi r^3$

p) $E = \frac{1}{2} y^2$

q) $E = \frac{1}{2} m v^2$, m constante

r) $U = \frac{a}{x^{12}} - \frac{b}{x^6}$, a e b constantes

2. Determine a derivada.

a) $y = \operatorname{sen} 4x$

b) $y = \cos 5x$

c) $f(x) = e^{3x}$

d) $f(x) = \cos 8x$

e) $y = \operatorname{sen} t^3$

f) $g(t) = \ln(2t + 1)$

g) $x = e^{\operatorname{sen} t}$

h) $f(x) = \cos e^x$

i) $y = (\operatorname{sen} x + \cos x)^3$

j) $y = \sqrt{3x + 1}$

l) $f(x) = \sqrt[3]{\frac{x - 1}{x + 1}}$

m) $y = e^{-5x}$

n) $x = \ln(t^2 + 3t + 9)$

o) $f(x) = e^{\operatorname{tg} x}$

p) $y = \operatorname{sen}(\cos x)$

q) $g(t) = (t^2 + 3)^4$

r) $f(x) = \cos(x^2 + 3)$

s) $y = \sqrt{x + e^x}$

t) $y = \operatorname{tg} 3x$

u) $y = \sec 3x$

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3. Derive.

a) $y = xe^{3x}$

b) $y = e^{x} \cos 2x$

c) $y = e^{-x} \operatorname{sen} x$

d) $y = e^{-2t} \operatorname{sen} 3t$

e) $f(x) = e^{-x^2} + \ln(2x+1)$

f) $g(t) = \dfrac{e^{t} - e^{-t}}{e^{t} + e^{-t}}$

g) $y = \dfrac{\cos 5x}{\operatorname{sen} 2x}$

h) $f(x) = (e^{-x} + e^{x^2})^3$

i) $y = t^3 e^{-3t}$

j) $g(x) = e^{x^2} \ln(1 + \sqrt{x})$

l) $y = (\operatorname{sen} 3x + \cos 2x)^3$

m) $y = \sqrt{e^{x} + e^{-x}}$

n) $y = \ln(x + \sqrt{x^2+1})$

o) $y = \sqrt{x^2 + e\sqrt{x}}$

p) $y = x \ln(2x+1)$

q) $y = [\ln(x^2+1)]^3$

r) $y = \ln(\sec x + \operatorname{tg} x)$

s) $y = \cos^3 x^3$

t) $f(x) = \dfrac{\cos x}{\operatorname{sen}^2 x}$

u) $f(t) = \dfrac{te^{2t}}{\ln(3t+1)}$

4. Calcule a derivada segunda.

a) $y = \operatorname{sen} 5t$

b) $y = \cos 4t$

c) $x = \operatorname{sen} \omega t$, ω constante

d) $y = e^{-3x}$

e) $y = e^{-x^2}$

f) $y = \dfrac{e^{x}}{x+1}$

g) $y = \ln(x^2+1)$

h) $y = \dfrac{x^2}{x-1}$

i) $y = e^{-x} - e^{-2x}$

j) $y = e^{-x} \cos 2x$

l) $y = \dfrac{x}{x^2+1}$

m) $y = \dfrac{3x+1}{x^2+x}$ NÃO TERMINEI "

GIGANTE!

GIGANTE n) $y = \dfrac{\operatorname{sen} 3x}{e^{x}}$

o) $y = xe^{-2x}$

p) $y = \operatorname{sen}(\cos x)$

q) $f(x) = \dfrac{4x+5}{x^2-1}$ GIGANTE.

r) $y = xe^{\frac{1}{x}}$

s) $y = \dfrac{x^2}{x^2+x+1}$ GIGANTE

t) $g(t) = \sqrt{t^2+3}$

u) $y = x\sqrt[3]{x+2}$ GIGANTE!

5. Calcule a derivada

a) $y = \sqrt{1 + \sqrt{x}}$

b) $y = \ln\left(3x + \sqrt{1 + 9x^2}\right)$

c) $y = x\, 5^{x^2}$

d) $y = (2 + \operatorname{sen} x)^x$

e) $y = \ln \sqrt{\dfrac{1 + \operatorname{sen} x}{1 - \operatorname{sen} x}}$

f) $x = e^{t^2} \operatorname{sen} 3t$

g) $s = t \ln \dfrac{t^2 - 1}{t^2 + 1}$

h) $y = \dfrac{x^2 + 1}{\sqrt{x + 1}}$

i) $y = \dfrac{t^3}{(t^2 + 1)^2}$

j) $f(x) = \dfrac{x \operatorname{tg} 3x}{x^2 + 4}$

l) $y = \ln \dfrac{1 + \operatorname{tg} \frac{x}{2}}{1 - \operatorname{tg} \frac{x}{2}}$

m) $g(x) = \dfrac{e^{\sec \sqrt{x}}}{x}$

n) $y = e^{x^x}$

o) $y = \dfrac{1}{2} \operatorname{tg}^2 x + \ln \cos x$

p) $y = \ln\left[\dfrac{\sqrt{1 - x} + \sqrt{1 + x}}{\sqrt{1 - x} - \sqrt{1 + x}}\right] - \dfrac{\sqrt{1 - x^2}}{x}$

q) $y = \dfrac{2\,(4 + 3\sqrt[3]{x})\,(2 - \sqrt[3]{x})^{\frac{3}{2}}}{5}$

r) $s = \dfrac{2^{3t} - 2^{-3t}}{2^{3t} + 2^{-3t}}$

s) $f(x) = \ln \dfrac{\cos \sqrt{x}}{1 + \operatorname{sen} \sqrt{x}}$

t) $y = e^{-3x} (\cos 3x - \operatorname{sen} 3x)$

u) $y = \dfrac{1}{2} \operatorname{cotg}^2 5x + \ln \operatorname{sen} 5x$

Respostas:

1. a) $\dfrac{dy}{dx} = 15x^2 + 6$

b) $\dfrac{ds}{dt} = \dfrac{1}{5\sqrt[5]{t^4}} - \dfrac{3}{t^2}$

c) $\dfrac{dx}{dt} = \dfrac{1}{(t + 1)^2}$

d) $\dfrac{dy}{dt} = \cos t - t \operatorname{sen} t$

e) $\dfrac{dy}{du} = \dfrac{u \ln u - u - 1}{u (\ln u)^2}$

f) $\dfrac{dx}{dt} = t^2 e^t (3 + t)$

g) $\dfrac{ds}{dt} = e^t [\operatorname{tg} t + \sec^2 t]$

h) $\dfrac{dy}{dx} = \dfrac{3x^2 \operatorname{sen} x - (x^3 + 1) \cos x}{\operatorname{sen}^2 x}$

i) $\dfrac{dy}{du} = \dfrac{\sec u\, [1 + 3u \operatorname{tg} u]}{3\sqrt[3]{u^2}}$

j) $\dfrac{dx}{dt} = -\dfrac{3}{t^2} - \dfrac{4}{t^3}$

m) $\dfrac{dx}{dt} = e^t [\cos t - \operatorname{sen} t]$

n) $\dfrac{du}{dv} = 10v - \dfrac{12}{v^5}$

o) $\dfrac{dV}{dr} = 4\pi r^2$

p) $\dfrac{dE}{dv} = v$

q) $\dfrac{dE}{dv} = mv$

r) $\dfrac{du}{dx} = -\dfrac{12a}{x^{13}} + \dfrac{6b}{x^7}$

2. a) $4 \cos 4x$

b) $-5 \operatorname{sen} 5x$

c) $3e^{3x}$

d) $-8 \operatorname{sen} 8x$

e) $3t^2 \cos t^3$

f) $\dfrac{2}{2t+1}$

g) $e^{\operatorname{sen} t} \cos t$

h) $-e^x \operatorname{sen} e^x$

i) $3(\operatorname{sen} x + \cos x)^2 (\cos x - \operatorname{sen} x)$

j) $\dfrac{3}{2\sqrt{3x+1}}$

l) $\dfrac{2}{3(x+1)^2} \sqrt[3]{\left(\dfrac{x+1}{x-1}\right)^2}$

m) $-5e^{-5x}$

n) $\dfrac{2t+3}{t^2+3t+9}$

o) $e^{\operatorname{tg} x} \sec^2 x$

p) $-\operatorname{sen} x \cos(\cos x)$

q) $8t(t^2+3)^3$

r) $-2x \operatorname{sen}(x^2+3)$

s) $\dfrac{1+e^x}{2\sqrt{x+e^x}}$

t) $3 \sec^2 3x$

u) $3 \sec 3x \operatorname{tg} 3x$

3. a) $e^{3x}(1+3x)$

b) $e^x(\cos 2x - 2 \operatorname{sen} 2x)$

c) $e^{-x}(\cos x - \operatorname{sen} x)$

d) $e^{-2t}(3 \cos 3t - 2 \operatorname{sen} 3t)$

e) $-2x e^{-x^2} + \dfrac{2}{2x+1}$

f) $\dfrac{4}{(e^t + e^{-t})^2}$

g) $-\dfrac{5 \operatorname{sen} 5x \operatorname{sen} 2x + 2 \cos 5x \cos 2x}{\operatorname{sen}^2 2x}$

h) $3(e^{-x} + e^{x^2})^2(-e^{-x} + 2x e^{x^2})$

i) $3t^2 e^{-3t}(1-t)$

j) $e^{x^2}\left[2x \ln(1+\sqrt{x}) + \dfrac{1}{2(\sqrt{x}+x)}\right]$

l) $3(\operatorname{sen} 3x + \cos 2x)^2 (3 \cos 3x - 2 \operatorname{sen} 2x)$

m) $\dfrac{e^x - e^{-x}}{2\sqrt{e^x + e^{-x}}}$

n) $\dfrac{1}{\sqrt{x^2+1}}$

o) $\dfrac{4x\sqrt{x} + e^{\sqrt{x}}}{4\sqrt{x^2 + x e^{\sqrt{x}}}}$

p) $\ln(2x+1) + \dfrac{2x}{2x+1}$

q) $\dfrac{6x\,[\ln(x^2+1)]^2}{x^2+1}$

r) $\sec x$

s) $-9x^2\cos^2 x^3 \operatorname{sen} x^3$

t) $-\dfrac{\operatorname{sen}^2 x + 2\cos^2 x}{\operatorname{sen}^3 x}$

u) $e^{2t}\;\dfrac{(1+2t)\ln(3t+1) - \dfrac{3t}{3t+1}}{[\ln(3t+1)]^2}$

4. a) $-25 \operatorname{sen} 5t$

b) $-16\cos 4t$

c) $-w^2 \operatorname{sen} wt$

d) $9e^{-3x}$

e) $2e^{-x^2}(2x^2-1)$

f) $\dfrac{e^x(x^2+1)}{(x+1)^3}$

g) $\dfrac{2(1-x^2)}{(x^2+1)^2}$

h) $\dfrac{2}{(x-1)^3}$

i) $e^{-x} - 4e^{-2x}$

j) $e^{-x}(4\operatorname{sen} 2x - 3\cos 2x)$

l) $\dfrac{2x(x^2-3)}{(x^2+1)^3}$

m) $\dfrac{2(3x^3+3x^2+3x+1)}{(x^2+x)^3}$

n) $\dfrac{-2\,[4\operatorname{sen} 3x + 3\cos 3x]}{e^x}$

o) $4e^{-2x}(x-1)$

p) $-\cos x\cos(\cos x) - \operatorname{sen}^2 x \operatorname{sen}(\cos x)$

q) $\dfrac{8x^3+30x^2+24x+10}{(x^2-1)^3}$

r) $\dfrac{e^{1/x}}{x^3}$

s) $\dfrac{2(-x^3-3x^2+1)}{(x^2+x+1)^2}$

t) $\dfrac{3}{(t^2+3)\sqrt{t^2+3}}$

u) $\dfrac{4x+12}{9\sqrt[3]{(x+2)^5}}$

Lista de Derivada (prof. Luciana)

1)

a) $f(t) = 5 \cdot t^{3/5}$:

$\frac{3}{5} - \frac{1}{1} = \frac{-2}{5}$

$f'(t) = \frac{3\cancel{5}}{\cancel{5}} t^{-2/5} = 3\, t^{-2/5}$ //

b) $y = \frac{t^6}{2} - 3t^4 - 5 \log(t)$

$f'(x) = \frac{6t^5}{2} - 3 \cdot 4t^3 - 5 \cdot \frac{1}{t} = 3t^5 - 12t^3 - \frac{5}{t}$ //

c) $y = \frac{\sqrt{10}}{t^4} = \sqrt{10} \cdot t^{-4}$

$f'(x) = -4\sqrt{10} \cdot t^{-5} = -\frac{4\sqrt{10}}{t^5}$ //

d) $y = \pi^2 + \cos(x) \cdot \operatorname{sen}(x)$.

$f'(x) = \cancel{(\pi^2)'}^{\,0} + (\cos(x) \cdot \operatorname{sen}(x))' = \cos'(x) \operatorname{sen}(x) + \operatorname{sen}'(x) \cdot \cos(x) =$

$= -\operatorname{sen}(x) \operatorname{sen}(x) + \cos(x) \cdot \cos(x) = -\operatorname{sen}^2 x + \cos^2 x = 2\cos^2 x - 1$ //

e) $y = x^2 e^x$

$f'(x) = 2xe^x + e^x x^2 = 2xe^x + x^2 e^x$ //

f) $g\left(\frac{e^t}{t^2+5t}\right)' \Rightarrow f(t) = \frac{(e^t)'(t^2+5t) - e^t(t^2+5t)'}{(t^2+5t)^2} =$

$= \frac{e^t(t^2+5t) - e^t(2t+5)}{(t^2+5t)^2}$ //

$g)\ (y)' = [(r^2-2r).e^r]' = (r^2-2r)'e^r + (r^2-2r)(e^r)' =$

$= (2r-2)e^r + (r^2-2r)e^r = e^r(r^2-2)$

$h)\ (y)' = \left(2x^3 - \frac{2}{x^2}\right)' = (2x^3)' - \left(\frac{2}{x^2}\right)' = 6x^2 - \left[\frac{\cancel{0.x^2} - 2x\,2}{(x^2)^2}\right] =$

$= 6x^2 + \frac{4x}{x^4}$

$i)\ (y)' = \left(3x - 2x^3 - \frac{2}{x^2+5}\right)' = (3x)' - (2x^3)' - \left(\frac{2}{x^2+5}\right)' =$

$= 3 - 6x^2 - \left(\frac{\cancel{0(x^2+5)} - 2(2x)}{(x^2+5)^2}\right) = 3 - 6x^2 + \frac{4x}{(x^2+5)^2}$

$j)\ (y)' = \left(\frac{2\ln(x) + 5x^2}{3x - \sqrt{2x}}\right)' = \frac{(2\ln(x)+5x^2)'(3x-\sqrt{2x}) - (2\ln(x)+5x^2)(3x-\sqrt{2x})'}{(3x-\sqrt{2x})^2}$

$= \frac{\left(\frac{2}{x} + 10x\right)(3x-\sqrt{2x}) - (2\ln(x)\ 5x^2)\left(3 - \frac{1}{2\sqrt{2x}}\right)}{(3x-\sqrt{2x})^2}$

$(*)\ 3x - (2x)^{1/2} = 3 - \frac{1(2x)^{-1/2}}{2} = 3 - \frac{2x^{-1/2}}{2} = 3 - \frac{1}{2.\sqrt{2x}}$

k) $(y)' = \left(\frac{x^2-1}{3x^4+sen(x)}\right)' = \frac{(x^2-1)'(3x^4+sen(x)) - (x^2-1)(3x^4 sen(x))'^{*}}{(3x^4+sen(x))^2} =$

$= \frac{2x\cdot(x^4+sen(x)) - (x^2-1)(12x^3 sen(x)+3x^4\cos(x))}{(3x^4+sen(x))^2}$ //

$*(3x^4)' sen(x)) = (3x^4)' sen(x) + (3x^4)(sen(x))' = 12x^3 sen(x) + 3x^4 \cos(x)$

l) $f'(t) = [(1-t)(1+t^2)^{-1}]' = \left(\frac{1-t}{1+t^2}\right)' = \frac{(1-t)'(1+t^2) - (1-t)(1+t^2)'}{(1+t^2)^2} =$

$= \frac{-1(1+t^2) - (1-t)t^2}{(1+t^2)^2} = \frac{-1 - t^2 - t^2(1-t)}{(1+t^2)^2}$ //

m) $f'(s) = \left(\frac{\sqrt{s}-1}{\sqrt{s}+1}\right)' = \left(\frac{s^{1/2}-1}{s^{1/2}+1}\right)' = \frac{(s^{1/2}-1)'(s^{1/2}+1) - (s^{1/2}-1)(s^{1/2}+1)'}{(\sqrt{s}+1)^2}$

$= \frac{\frac{1}{2} s^{-1/2}(\sqrt{s}+1) - (\sqrt{s}-1)\left(\frac{1}{2\sqrt{s}}\right)}{(\sqrt{s}+1)^2} = \frac{1}{2\sqrt{s}} \frac{(\sqrt{s}+1-\sqrt{s}+1)}{(\sqrt{s}+1)^2} = \frac{1}{\cancel{2}\sqrt{s}}\cdot\cancel{2} = \frac{1}{\sqrt{s}(\sqrt{x}+1)^2}$

n) $f(s) = 2\left(\frac{1}{\sqrt{s}} + \sqrt{s}\right)'\cdot\cos(s) = 2\left[\left(\frac{1}{\sqrt{s}}+\sqrt{s}\right)'\cdot\cos(x) + \left(\frac{1}{\sqrt{s}}+\sqrt{s}\right)\cos'(x)\right] =$

$= 2\left[\left(s^{-3/2}\right)\cdot\cos(x)\right] + \left(\frac{1}{\sqrt{s}}+\sqrt{s}\right)sen(x)$ //

$5 \cdot e^x = 0 + 5e^x$

o) $f(x) = \left(\frac{1}{(x^2+5e^x)(x^2\cos(x)+\ln(x))} \right)' =$

$= \frac{-[(x^2+5e^x)(x^2+\cos(x)+\ln(x)]'}{[(x^2+5e^x)(x^2\cos(x)+\ln(x)]^2} = -$

$= -\frac{[(x^2+\cancel{\cos}\,5e^x)'(x^2+\cos(x)+\ln(x))+(x^2+5e^x)(x^2+\cos(x))+\ln(}{[(x^2+5e^x)(x^2\cdot\cos(x)+\ln(x)]^2}$

$= -[(2x+5e^x)(\qquad)+(x^2+5e^x)$