ME538: Finite Element Analysis
Final Project

1. Due date* : Thursday, June 29, 2017, 12 PM in Dr. Park's office (730 Commonwealth, Room 212)

2. Present your results in neat typed reports in the following order:
   a) Brief description of the problem. One paragraph will suffice.
   b) Description of what you did and your major results in a visually efficient way, i.e., using graphs or tables. Less IS good here.
   c) Most importantly, what did you learn? Why did certain elements perform better for certain problems? How do the displacements compare with the analytical solutions, and why? Compare the 4-node and 9-node solutions for the same meshes. Which one is better and why? Discuss convergence rates for the different elements.
   d) The modified parts of the MATLAB code (element stiffness routines for 4-node and 9-node elements, convergence codes for both 4-node and 9-node elements) need to be submitted with the report.

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* The due date is NOT flexible, except under special circumstances

ME 538 Finite Element Analysis, Fall 2014

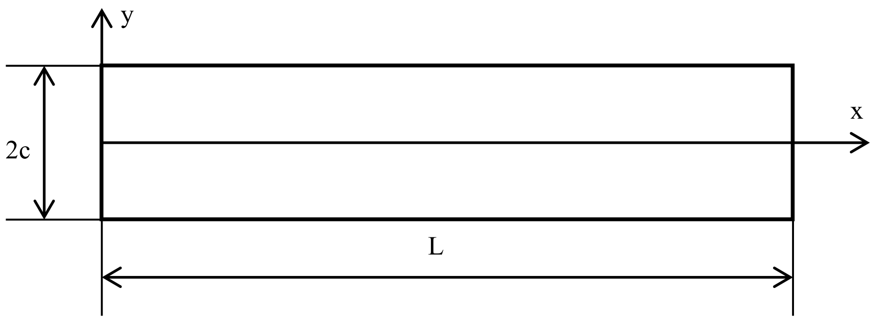

Figure 1a

Consider the equations of two-dimensional, linear isotropic elasticity theory on the domain illustrated in Figure 1. Assume there is no body force, and that the boundary conditions are given as follows:

Displacement: $u_1(0,0) = u_2(0,0) = 0$, $u_1(0,\pm c) = 0$

Traction: $h_1(x,\pm c) = h_2(x,\pm c) = 0$, $x \in (0, L)$

$h_1(L, y) = 0$, $y \in (-c, c)$

$h_2(L, y) = P(c^2 - y^2)/2I$, $y \in (-c, c)$

$h_1(0, y) = PLy/I$, $y \in (-c, 0) \cup (0, c)$

$h_2(0, y) = -P(c^2 - y^2)/2I$, $y \in (-c, 0) \cup (0, c)$

where P is a constant and $I=2c^3/3$.

The traction boundary conditions are those encountered in simple bending theory for a cantilever beam with root section at x = 0, i.e., parabolically varying end shear and linearly varying bending stress at the root. The displacement boundary conditions allow the root section to warp.

The exact solution of the problem is given as follows:

$$\sigma_{11} = -\frac{P\tilde{x}y}{I}, \sigma_{12} = \frac{P}{2I}\left(c^2 - y^2\right)$$

$$\frac{6E_0 I}{P}u_1(x,y) = -y\left[3\left(L^2 - \tilde{x}^2\right) + \left(2+\nu_0\right)\left(y^2 - c^2\right)\right]$$

$$\frac{6E_0 I}{P}u_2(x,y) = \left(\tilde{x}^3 - L^3\right) - \left[\left(4+5\nu_0\right)c^2 + 3L^2\right]\left(\tilde{x} - L\right) + 3\nu_0\tilde{x}y^2$$

where $\tilde{x} = L - x$

For plane strain and plane stress conditions, E_0 and ν_0 are as following:

	E_0	ν_0
Plane stress	E	ν
Plane strain	$\frac{E}{1-\nu^2}$	$\frac{\nu}{1-\nu}$

1. Use these data in your calculations: P=-1, L=16, c=2, $E=10^7$, $\nu=0.3$, i.e. run these parameters for both 4-node and 9-node elements
2. Use plane strain conditions only
3. Use several mesh(es) for each element type:

   For 4-node element, run four meshes: 4x2, 8x4, 16x8 and 32x16 elements.

   For 9-node element, run four meshes: 4x2, 8x4, 16x8 and 32x16 elements.
4. In each run, compare the computed vertical tip displacement $u_2(16,0)$ with the exact solution. Analyze how the displacements converge to the analytical solution for both 4-node and 9-node elements.
5. Calculate the error and convergence rate for both the 4-node and 9-node elements. What is the convergence rate for both? How and why are they different?
6. Repeat steps 1-5 if the Poisson's ratio v = 0.499. Elaborate on the differences between the 4-node and 9-node elements for this Poisson's ratio, and why any differences exist.

Implementation To Do:

1. Implement the 4-node bilinear element stiffness subroutine in the file elem1.m
2. Implement the 9-node quadratic element stiffness subroutine in the file elem4_9.m
3. Determine the appropriate gauss point positions (gauss) and weights (weight) for both the 4-node and 9-node element in the main program prob1.m
4. Calculate the L2 error for both the 4 and 9-node elements in the file dispnrm.m